Asymmetries of information in electronic systems[*]

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Abstract

We study the efficiency and the liquidity properties of a centralized, non-anonymous floor market where asymmetrically informed traders are active for two periods and can observe each other current and past orders. We show that the more precise the information the lower the incentive to reveal it in the first trading rounds. On the contrary, strategic competition pushes the less informed trader to reveal his information in the earliest stage. This implies that when differences in information quality are very important, the liquidity of the market decreases as we approach the date of public revelation. We are able to show that more transparent markets as the ones organized via electronic systems are not performing better than markets organized on anonymous floor trades in terms of revelation of information, due to the oligopolistic behavior of insiders.

Keywords: asymmetric information, liquidity, insider trading, strategic revelation.

JEL Classification: D 43, G 14.

1 Introduction

Electronic, non-anonymous quotation systems like the Toronto Stock Exchange CATS (Computer Assisted Trading System) have remarkable importance in terms of volume of trade. Their increasing practical relevance has lead microstructure theorists to study *pre-trade* transparency. In this literature, the broad conclusion seems to be that non-anonymous systems bring more informative prices and lower trading costs by allowing for widely spread information about investors' demands (with respect to anonymous floor systems). This belief is also shared by some regulators like the US Security and Exchange Commission or the UK Office of Fair Trading.

In this paper we argue that strong asymmetries of information may generate inefficient price dynamics which could explain the observed increase of trading cost before a public announcement (e.g., earnings or dividends). From this result we can then derive conditions under which the high degree of transparency of electronic systems is not beneficial in terms of reduced costs of trading or for informational efficiency. Madhavan, Porter and Weaver (2000) find related empirical evidence for the Toronto Stock Exchange. They analyze the effects of the introduction in April 1990 of a computerized system called Market by Price (MBP) which dramatically increased the level of pre-trade transparency. These authors observe an increased price volatility (perhaps allowing for an higher informational efficiency) and above all that the cost of trading does not reduce following the reform. Although they give a different interpretation to their empirical findings with respect to the increased competition in information, we stress here the similarity with our theoretical conclusions.

The key to our result is twofold. Firstly the transparency of an electronic quotation system enables brokers to observe the identities of other brokers submitting orders and therefore to acquire private information as soon as it is used by someone to perform a (profitable) trade: this provokes a direct competition among insiders. Secondly we model asymmetries of information between traders in a very natural way that accounts for differences in the *quality* of information which is treated as the main strategic variable.

The issue of information revelation in financial markets has long been studied in competitive markets but it has taken a new start with Kyle (1985). He shows that the optimal behavior of a monopolistic informed trader is to reveal information slowly so as to maintain a constant market depth (until the last few periods). But, as reported by Cornell and Sirri (1992) and Meulbroek (1992), private information can be disseminated among dozens of traders. Thus it makes sense to look at oligopolistic competition among equally informed traders as done by Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993) among others. These later models have a Bertrand flavor since they treat information as an *homogeneous* good. Although the ensuing "rat race" yields a quite efficient price dynamic, this approach remains unsatisfactory due to the discrepancy existing between the outcomes of the monopoly and duopoly settings.

Traders frequently disagree on the future value of an asset either because they have different

private information or because they interpret differently the same piece of news (due to different experience or education). Foster and Viswanathan (1996) (hereafter FV) pursue this venue and argue that private information is a peculiar good, intrinsically non homogeneous and spread asymmetrically among rational traders. Our present work starts from the same observation and yields similar results on some grounds. However there are fundamental differences in the two approaches.

Firstly we study a *non-anonymous electronic system* where traders directly observe their competitors orders while FV's model rather applies to *anonymous floor systems* where informed traders only observe the order flow (like market makers) and their own orders. From a strategic point of view, informed traders in FV do not compete one against the other but indirectly through the market maker. Secondly we consider *asymmetries in the quality* of information (variance of private signals) while FV use an identical variance of private signals. On the other hand these authors consider *asymmetries of opinions* by allowing for any kind of initial correlation structure (agreement or disagreement) while in our model there is always a positive correlation between signals (a form of agreement).

In defense of our choice, we believe that although traders process the same news differently they should nevertheless agree on the direction of the stock variation (up or down) and disagree only with respect on its magnitude.

In this article we analyze a two-stage game describing a centralized and order-driven market with three kinds of agents: informed traders (hereafter insiders), liquidity traders and risk neutral arbitrageurs. Insiders is a short hand for registered traders who can enjoy the information services provided by the market organizer. We study a market with high transparency and no designated market makers. Hence we assume that in each period of trade, the order book is electronically collected and insiders observe the orders of each others. This degree of transparency in the order flow is present in some stock exchanges like for example the Toronto CATS and the order system for the stocks quoted on the Euronext in Paris before 1998; moreover, most electronic systems display the 3 or 5 best bid and ask quotes together with the dealers identities. The insiders' information therefore consists of their observation of a signal correlated with the liquidation value of the asset, of the past history of prices and all the individual orders. Insiders also observe the amount of liquidity trade present in the market at each period.[1] The competition between risk neutral arbitrageurs makes the equilibrium price equal to the expected value of the asset given the observable order flows (semi strong price efficiency).

We characterize the set of equilibria in pure and linear strategies. A simple backward induction argument shows that one should always reveal its information in the last period. We then enquire about the incentives to reveal the private information in the first stage. On the one hand, using private information to trade with uninformed agents is beneficial but, by doing this, an insider gives

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[1]This is not allowed in Kyle (1985) and Holden & Subrahmanyam (1992) where insiders perfectly know the asset value because the price would immediatly incorporate all the information. One can motivate our assumption by invoking the possibility for the traders to estimate the liquidity order flow during the pre-opening period.

an advantage to its competitor for the last stage and further brings the price closer to its own estimate. Both effects reduce the profit opportunities for the second period.

We show (as in Admati and Pfleiderer (1988)) that the amount of liquidity trade present on the market can be considered as a "cake" to be divided between the insiders. If the second period cake is much larger than the first period one, then traders conceal their information in the first period: the size effect predominates over the duopoly competition. Choosing a comparable liquidity volume across periods we obtain other insightful results. If their private information is almost equally precise, the two traders reveal their signal in the first stage. However, if asymmetries are large, the better informed trader conceals and the opponent reveals in the first period. The better informed player is induced to trade less aggressively on his information because his opponent can detect perfectly his move. The less informed trader prefers to reveal his type immediately in order to exploit an informational advantage that will not be relevant in the following stage for the presence of a better informed opponent. The competition between asymmetric agents is not of the "Bertrand" kind because the information released by the traders is non-homogeneous.

In equilibrium, the price informativeness increases in the second period and the depth of the market reduces with time except when there is a strong information asymmetry leading the better informed insider to wait. We can then predict that in highly transparent systems, the information of the better quality comes to the market only at the latest stages, and the cost of trading increases before a public announcement which is value-relevant. Notice that this predictions are indirectly verified by Foucault, Moinas, Theissen (2002) in their observation of the pattern of the cost of trading in Euronext Paris before and after the reform of April 2001 that made such a system anonymous.

The paper is organized as follows: in section 2 we present the two stage game and describe the equilibrium concept. Section 3 tackles the main part of the analysis while in section 4 we present our results on equilibrium revelation of information and on the liquidity and the informativeness of prices in order to point out the differences of our model with the existing literature. Section 5 concludes.

2   A Model of Electronic Trading

In this section we first introduce our model. We then proceed to highlight the differences with the previous literature before defining the equilibrium concept used for the solution of the game. We conclude with an analysis of the case in which only one informed trader exists on the market (the monopoly of information), that will be used as a benchmark to give some intuitions of the results in the case of multiple informed traders.

2.1   Structure and notation

Consider a market for a risky asset where the exchanges occur during two rounds of trade between three kinds of agents: informed traders, noise traders and risk neutral arbitrageurs.

The risky asset has a random liquidation value v distributed according to the standard normal law $\mathcal{N}(0,1)$. At the beginning of the first trading round, insider i observes a private signal $s_i = v + \varepsilon_i$ where the error term ε_i has law $\mathcal{N}(0, \tau_i^{-1})$ and τ_i is interpreted as the ex-ante precision with which the trader can guess the true value of v. We study the case of two information insiders.[2] All random variables are assumed to be independently distributed.

In each of the two periods of trade, all agents submit market orders to an electronic order system, and this system creates automatically the order book. The system is fully non-anonymous, so that all registered market participants can observe the identification codes of every other trader. Therefore, all the participants are able to identify the orders of each others. We also assume the presence of a group of risk-neutral arbitrageurs who act as liquidity providers. They can observe each individual order but they limit themselves to the information given by the total order flow: indeed, they cannot identify the informed orders, since they do not know who are the privately informed traders.[3]

At each stage $t = 1, 2$, insiders choose the quantities they trade, $q_{t,i}$ and $q_{t,j}$ knowing their private signals s_i and s_j while liquidity traders submit an aggregated order u_t distributed according to the normal law $\mathcal{N}(\bar{u}, \tau_t^{-1})$. We normalize their average trade to $\bar{u} = 0$ and assume that liquidity trading is independent of all other random variables. Notice that the standard deviation $1/\sqrt{\tau_t}$ is a measure of the size of noise trading in stage t.[4]

Given the realized order flow $\omega_t = q_{t,i} + q_{t,j} + u_t$, the system electronically computes at the end of each stage the price at which all orders are filled. At the end of the second period of trading, the realized liquidation value of the asset is announced and holders of the asset are paid its realized value. The public information at stage $t = 1, 2$ is $\mathcal{H}_1 = \{\omega_1\}$ and $\mathcal{H}_2 = \{\omega_1, \omega_2\}$. We denote $\mathcal{H}_{t,i}$ the private information of insider i in stage t. The insider's ex-post stage profit is $q_{t,i}(v - p_t)$ while the ex-interim expectation, conditional on the private information $\mathcal{H}_{t,i}$, is $\Pi_{t,i} = E\left[q_{t,i}(v - p_t) \mid \mathcal{H}_{t,i}\right]$. We will consider also the ex-ante profit obtained by integrating $\Pi_{t,i}$ with respect to the joint measure of private signals and liquidity trade.

2.2 Relation to the existing literature

The differences between our findings and those of Foster and Viswanathan (1996) (hereafter FV) originate in the statistical properties of the private informative signals received by the insiders. In our setting the signal received by an insider is the realized liquidation value v plus a white noise ε . Hence, the correlation between signals is always positive as opposed to FV. In other words, insiders do not have different "opinions" about the true value of the asset, they just receive information that is more or less

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[2]Notation: i stands for trader 1 or 2 and j for the other trader. Whenever a formula is given for i only, the j formula is obtained by interverting symbols i and j.

[3]Our view amounts to assume that arbitrageurs are bounded in their rationality: they do not try to infer from the observation of the individual orders which is the informed one. In a market in which a large number of individual orders is observed in each trading round, we believe such an assumption is not too restrictive.

[4]If liquidity trade satisfy $u_2 = xu_1$ and $\tau_1 = 1$ then $\tau_2^{-1} = x^2\tau_1^{-1} \Rightarrow 1/\sqrt{\tau_2} = x$.

noisy. Yet as in FV, the covariance of the signals with v is the same: $Cov(s_i, v) = Cov(s_j, v) = Var(v)$.

The covariance matrix of the private signals is

$$\Psi_0 = \begin{pmatrix} 1 + \tau_i^{-1} & 1 \\ 1 & 1 + \tau_j^{-1} \end{pmatrix}$$

where the diagonal terms are different as precisions differ. The expected value of the asset given the observation of the two private signals is

$$E\left[v \mid s_i, s_j\right] = \begin{pmatrix} 1 \\ 1 \end{pmatrix}^T \Psi_0^{-1} \begin{pmatrix} s_i \\ s_j \end{pmatrix} = \frac{\tau_i s_i + \tau_j s_j}{\tau_i + \tau_j + 1}$$

and a simple average of the signals $\frac{1}{2}(s_i + s_j)$ is not a sufficient statistic for the information known to all insiders unlike in the approach of FV.

This observation implies that the learning process of the arbitrageurs is different from that of an insider: they can, at best, infer an average of the signals from the order flow. Yet this statistic has less predictive power than the information known to each single insider. This has two important consequences: first, the strategic interaction between insiders is much more complex than in FV. Each insider has to optimally reply to the move of its informed opponent taking into account that the latter and the market will derive different statistics from his trade. Both learning processes have to be considered in formulating the optimal informative trade. Second, as arbitrageurs will not be able to infer precisely the individual signals (since they don't know who are the insiders), the variance of v given the whole history of prices and orders will still be positive, even after the second round of trade.

In the following subsections we proceed to illustrate first the pricing reaction of the market to the order flow, then we describe the strategies used by insiders and finally, we analyze the optimal revelation of information by an insider who has a monopolistic informational advantage, as in Kyle (1985).

2.3 The Market Pricing Rule

Our task is to assess how the revelation of private information in the order flow affects the pricing function of the market. It is relatively easy for the first period because there is no information contained in the previous day closing price, p_0. The second period is more complex as arbitrageurs can make inferences from the observation of both ω_2 and ω_1 which potentially contain private information.

The presence of risk-neutral arbitrageurs guarantees semi-strong price efficiency, so that $p_t = E\left[v \mid \mathcal{H}_t\right]$. Now, since all random variables in our model are normal, the pricing rule is linear in each period;[5] hence we can write

$$p_t(\omega_t, \omega_{t-1}) = \mu_t(\omega_{t-1}) + \lambda_t \omega_t \text{ for } t = 1, 2 \tag{1}$$

where λ_t is the market aggressiveness (inverse of market depth) and μ_t the memory effect (linear in the previous period order book).

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[5]This is usually stated as an assumption in the literature.

We shall later show that the order flows take the following functional forms:

$$\omega_1 - E[\omega_1] = \frac{k_i s_i + k_j s_j}{\lambda_1} + \eta_1 u_1 \tag{2}$$

$$\omega_2 - E[\omega_2] = \frac{m_i s_i + m_j s_j}{\lambda_2} + \eta_2 u_2 \tag{3}$$

According to the standard projection of normal variables, if $\tilde{x}$ and $\tilde{y}$ are independent with zero mean, then $E[\tilde{x} / \tilde{x} + \tilde{y}] = \frac{Var(\tilde{x})(\tilde{x}+\tilde{y})}{Var(\tilde{x})+Var(\tilde{y})}$. We apply this formula to $\tilde{x} = \lambda_1 (k_i + k_j)v$ and $\tilde{y} = \omega_1 - E[\omega_1] - \tilde{x}$ in (2) to obtain:

$$E[v \mid \omega_1 - E[\omega_1]] = \frac{k_i + k_j}{\lambda_1 Var(\omega_1)} (\omega_1 - E[\omega_1]). \tag{4}$$

We can now use (1) and (4) in the efficient price equation $p_1 = E[v \mid \mathcal{H}_1]$ to identify μ_1 and λ_1. We observe that μ_1 is proportional to[6] $E[\omega_1]$ and derive the following equation in λ_1:

$$\frac{k_i + k_j}{\lambda_1^2} = Var(\omega_1) = \left(\frac{k_i + k_j}{\lambda_1}\right)^2 + \frac{k_i^2}{\lambda_1^2 \tau_i} + \frac{k_j^2}{\lambda_1^2 \tau_j} + \frac{\eta_1^2}{\tau_1}$$

that can be solved to yield the degree of market aggressiveness in the first period as

$$\lambda_1^2 = \frac{\tau_1}{\eta_1^2} \left(k_i + k_j - (k_i + k_j)^2 - k_i^2 \tau_i^{-1} - k_j^2 \tau_j^{-1}\right) \tag{5}$$

where it can be noted that the ex-ante precision of exogenous trade (τ_1) negatively influences the depth of the market, λ_1^{-1}.

In the following, we will measure the informational efficiency of the market by the (remaining) variance of the asset value conditional on the information revealed. For the first stage, we have $\Sigma_1 \equiv E\left[(v - E[v \mid \omega_1])^2\right] = 1 - Var(E[v \mid \omega_1])$ as the conditional expectation is an orthogonal projector. Using (4) and $Var(\omega_1) = \frac{k_i + k_j}{\lambda_1^2}$ we derive

$$\Sigma_1 = 1 - k_i - k_j \tag{6}$$

To find λ_2, μ_2 and Σ_2 we use the law of iterated expectations into (1) and the semi-strong price efficiency (cf. (36) and (37) in appendix 5.1).

2.4 Insiders strategies and equilibrium concept

We now formally define the equilibrium concept and the strategy space of the insiders.

Due to the high degree of transparency of the electronic quotation system, insider i knows the history of all trades whether their motivation was liquidity or private information. The order he submits $q_{t,i}$ at period t thus depends on his forecast s_i of the realized value of v, on the trading decisions of the other insider j, and on the induced equilibrium pricing rule. Indeed the insider is aware that the market adjusts the sensitivity of its pricing rule according to the amount of information forecasted to be released in equilibrium.

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[6]The factor doesn't matter since it will be shown that there is no memory effect in the first period i.e., $E[\omega_1] = 0$.

Some important restrictive assumptions are necessary in order to be able to solve the model analytically. Firstly, we make the standard assumption that *insiders use strategies linear in their private information* i.e., $q_{t,i}(\mathcal{H}_{t,i}) = \alpha_{t,i} + \beta_{t,i}E[v \mid \mathcal{H}_{t,i}]$. Notice that this also implies that we do not analyze equilibria in mixed strategies.

A second problem arises when considering the Bayesian inference process of insiders and arbitrageurs. It is difficult to construct a Bayesian-Nash equilibrium of the game where the strategy of insider i at the second stage is required to be optimal not only when insider j plays his optimal strategy in the first period, but also for any other strategy. To construct such an equilibrium, we would have to be able to rule out all possible deviations. Consider for example the following strategy for insider j: in the first stage, he uses his optimal $\beta_{1,j}$, but pretends he received signal $\hat{s}_j \neq s_j$ (the true one). Insider i can detect this individual trade and will wrongly infer $\hat{s}_j$, while the market will construct the wrong statistic $E[v \mid \omega_1(q_{1,j}(\hat{s}_j))]$. This deviation could in principle give higher profits to j in the second period, where both his opponents have been mislead. The main problem is that in such an incomplete information game, some signalling activity between the two informed players could arise, and we are not sure that truthful revelation is indeed the only equilibrium, at least if we don't construct beliefs out of the equilibrium path supporting it.

Hence, we assume that any misleading activity is ruled out from the strategy space: in this sense, the equilibrium we characterize is restricted to what we call *truthful revelation strategies*[7].

Allowing only for full revelation or concealing of the private signal, the second stage can start with four possible information structures. Strategies in the first stage denoted by $\sigma_{1,i}$ and $\sigma_{1,j}$ are either to reveal R or conceal C. The information sets are therefore[8]:

▶ Only i reveals $(\sigma_{1,i} = R) \Rightarrow \mathcal{H}_{2,i} = \{\omega_1, \omega_2, s_i\}, \mathcal{H}_{2,j} = \{\omega_1, \omega_2, s_i, s_j\}$.
▶ Only j reveals $(\sigma_{1,j} = R) \Rightarrow \mathcal{H}_{2,i} = \{\omega_1, \omega_2, s_i, s_j\}, \mathcal{H}_{2,j} = \{\omega_1, \omega_2, s_j\}$.
▶ No revelation $(\sigma_{1,i} = \sigma_{1,j} = C) \Rightarrow \mathcal{H}_{2,i} = \{\omega_1, \omega_2, s_i\}, \mathcal{H}_{2,j} = \{\omega_1, \omega_2, s_j\}$.
▶ Both reveal $(\sigma_{1,i} = \sigma_{1,j} = R) \Rightarrow \mathcal{H}_{2,i} = \{\omega_1, \omega_2, s_i, s_j\}, \mathcal{H}_{2,j} = \{\omega_1, \omega_2, s_i, s_j\}$.

In the symmetric revelation case denoted RR, the second period is a Cournot duopoly with complete information i.e., each pair $\{s_i, s_j\}$ defines a proper subgame. The game is solved for any couple of given private signals in the space of linear strategies. When only trader i reveals his information in stage one, denoted RC, trader j behaves as in the previous case, since he knows $\{s_i, s_j\}$. In equilibrium, trader i, despite the fact that he does not know the signal s_j, anticipates the rule $q_{1,j}(s_j, s_i)$ used by his opponent. Hence, we deal with a form of Stackelberg game. When no revelation has occurred, (CC), traders play a game with incomplete information on both sides. Each trader has to optimize against a rule and not against a single order.

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[7]Studying only truthful revealing strategies implies that we will find an upper bound in terms of informational efficiency of the market. All our results should then be read keeping this *caveat* in mind.

[8]Observe that $\mathcal{H}_{2,i}$ is always finer than the observation of the total order flow ω_1 which can therefore be safely ignored in the second stage calculations.

An obvious consequence of the finite number of stages is that insiders have an incentive to use their private information in stage 2, thus ω_2 certainly conveys information about the underlying liquidation value of the asset. Yet, if one or both insiders have revealed information during the first stage, then the order flow ω_1 is also an informative statistic for the market.

According to our restrictions on the strategy spaces, we analyze the 2×2 matrix game and for any pair of strategies $h \in \{CC, RC, CR, RR\}$, the ex-ante global profit is the sum of profits obtained in each active trading stage $\Pi_i^h = \Pi_{1,i}^h + \Pi_{2,i}^h$.

Definition 1 *A Bayesian-Nash equilibrium in linear, truthfully revealing strategies of the trading game is a vector of strategies (σ_i, σ_j, p) such that:*

1. For any trader $i = 1, 2$, period $t = 1, 2$ and linear strategy $\hat{\sigma}_i = (\hat{\sigma}_{1,i}, \hat{\sigma}_{2,i})$,

$$E\left[\Pi_{t,i}^h(\sigma_i, \sigma_j, p) \mid \mathcal{H}_{t,i}, \omega_{t-1}, \omega_t\right] \geq E\left[\Pi_{t,i}^h(\hat{\sigma}_i, \sigma_j, p) \mid \mathcal{H}_{t,i}, \omega_{t-1}, \omega_t\right].$$

2. For any period $t = 1, 2$, $p_t = E\left[\tilde{v} \mid \mathcal{H}_t\right].$

2.5 The Monopoly Benchmark

We develop in full length the analysis of the monopoly of information i.e., the case of a single insider. This will enable us to gain concision in presenting the various duopoly cases by concentrating on the differences brought by strategic interaction into the key equations of this monopoly benchmark. We drop indices i and j when referring to the monopoly. As explained before an insider can either conceal his information in the first stage or reveal it. We consider each case in turn before comparing the total resulting payoffs.

2.5.1 Concealing

If the insider conceals his information during the first period he places a market order q_1 constant for any private signal s he received. The first stage order flow $\omega_1 = q_1 + u_1$ is therefore non informative. Hence the expected value of the asset conditional on the order flow is $E[v \mid \omega_1] = E[v \mid p_0] = p_0$, the closing price of the previous day which is normalized to zero (as it is assumed to be efficient).

Arbitrage forces drive the equilibrium price p_1 towards zero so that any q_1 is indeed optimal for the insider whose expected profit is then $\bar{\Pi}_1^C = 0$. The intensity parameter in (2) is $k = 0$ and the noise trading parameter is $\eta_1 = 1$. We now use (5) with $k_i = k_j = 0$ to obtain the memory effect $\mu_1^C = 0$, the market aggressiveness $\lambda_1^C = 0$ as well as the remaining variance of the asset value $\Sigma_1^C = 1$. No information is enclosed in the order flow so that the market has an infinite depth and the variance of the asset value remains at the previous day level.

It is obviously a dominant strategy for the insider to use (and reveal) its information in the last period before the public announcement. Letting $\mathcal{H}_2$ be the information set of the insider, its second

period profit writes

$$\Pi_2 = q_2 E\left[v - p_2 \mid \mathcal{H}_2\right] \tag{7}$$

where $p_2 = \mu_2 + \lambda_2(q_2 + u_2)$ by (1). The FOC of profit maximization is $2q_2 = E\left[v \mid \mathcal{H}_2\right]/\lambda_2 - \mu_2/\lambda_2 - u_2$. The projection theorem for normal random variables yields $E\left[v \mid \mathcal{H}_2\right] = E\left[v \mid s\right] = \frac{\tau s}{\tau+1}$, hence the optimal order is $q_2(s) = \frac{1}{2\lambda_2}\frac{s\tau}{\tau+1} - \frac{\mu_2}{2\lambda_2} - \frac{u_2}{2}$.

The derived order flow is $\omega_2 = \frac{1}{2\lambda_2}\frac{s\tau}{\tau+1} - \frac{\mu_2}{2\lambda_2} + \frac{u_2}{2} = E\left[\omega_2\right] + ms/\lambda_2 + \eta_2 u_2$ so that the identification of the coefficients yields $m = \frac{\tau}{2(\tau+1)}$ and $\eta_2 = \frac{1}{2}$. The market depth obtained from (36) or (5) (if there is no revelation in the first period these two formulae are identical) simplifies to $\lambda_2^C = \frac{\sqrt{\tau_2}\sqrt{m - m^2 - m^2\tau^{-1}}}{\eta_2}$ $= \sqrt{\frac{\tau_2\tau}{\tau+1}}$ while the memory effect is $\mu_2^C = 0$ (as shown in (37), μ_2 is proportional to $\lambda_1^C \omega_1$).

Rewriting the optimal order as $q_2^C(s) = \frac{1}{2}\left(\frac{s\tau}{\tau+1}/\lambda_2^C - u_2\right)$ we observe that $E\left[v - p_2 \mid s\right] = \lambda_2 q_2^C(s)$; hence we can substitute into (7) to obtain the expected profit in the second stage, conditional on the private signal,

$$\bar{\Pi}_2^C(s) = \lambda_2^C q_2^C(s)^2 = \frac{1}{4\lambda_2^C}\left(\frac{s\tau}{\tau+1} - u_2\lambda_2^C\right)^2.$$

We then use $E\left[s^2\right] = \frac{1+\tau}{\tau}$, $E\left[u_2^2\right] = \tau_2^{-1}$ and the independence of s and u_2 to compute the ex-ante payoff (prior to the realization of s and u_2):

$$\bar{\Pi}_2^C \equiv E\left[\Pi_2^C(s)\right] = \frac{1}{2\sqrt{\tau_2}}\sqrt{\frac{\tau}{\tau+1}}. \tag{8}$$

The complete payoff is $\bar{\Pi}^C = \bar{\Pi}_1^C + \bar{\Pi}_2^C$ and since $\bar{\Pi}_2^C > 0$ it was indeed optimal to reveal information in the last stage.[9]

2.5.2 Revealing

If the monopolist insider decides to use and reveal immediately his private information then the first period is exactly the second period of the previous case. We then have $\lambda_1^R = \sqrt{\frac{\tau_1\tau}{\tau+1}}$, $\bar{\Pi}_1^R(\tau, \tau_1) = \frac{1}{2}\sqrt{\frac{\tau}{\tau_1(\tau+1)}}$ and $\omega_1 = \frac{1}{2}\left(\frac{s\tau}{\tau+1}/\lambda_1^R + u_1\right)$.

The optimal second period order q_2^R remains $\frac{1}{2\lambda_2}\left(\frac{\tau s}{\tau+1} - \mu_2 - u_2\lambda_2\right)$ but we now have $\lambda_2^R = \sqrt{\frac{\tau_2\tau}{2(\tau+1)}}$ and a positive memory effect $\mu_2^R = \frac{1}{4}\left(\frac{\tau s}{\tau+1} + u_1\lambda_1^R\right)$.

Unlike the previous case, $q_2^R(s)$ develops into $\frac{1}{2\lambda_2^R}\left(\frac{3\tau s}{4(\tau+1)} - \frac{u_1}{4}\lambda_1^R - u_2\lambda_2^R\right)$ so that the conditional profit is $\bar{\Pi}_2^R(s) = \lambda_2^R q_2^R(s)^2 = \frac{1}{4\lambda_2^R}\left(\frac{3\tau s}{4(\tau+1)} - \frac{u_1}{4}\lambda_1^R - u_2\lambda_2^R\right)^2$. As before we use $E\left[s^2\right] = \frac{1+\tau}{\tau}$, $E\left[u_1^2\right] = \tau_1^{-1}$, $E\left[u_2^2\right] = \tau_2^{-1}$ and the independence of these random variables to obtain:

$$\bar{\Pi}_2^R = \frac{9}{16\sqrt{2\tau_2}}\sqrt{\frac{\tau}{\tau+1}} \tag{9}$$

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[9]Notice also that if the insider were to observe two signals of precisions τ_i and τ_j, the payoff would be $\frac{1}{2\sqrt{\tau_2}}\sqrt{\frac{\tau_i+\tau_j}{\tau_i+\tau_j+1}} > \frac{1}{2\sqrt{\tau_2}}\sqrt{\frac{\tau_i}{\tau_i+1}}$ the payoff corresponding to the observation of a single signal. More information is always beneficial in a monopoly.

which is roughly 80% of $\bar{\Pi}_1^R = \bar{\Pi}_2^C$ (cf. equation (8)). The final payoff is $\bar{\Pi}^R = \bar{\Pi}_1^R + \bar{\Pi}_2^R = \frac{1}{2}\sqrt{\frac{\tau}{\tau+1}}\left(\frac{1}{\sqrt{\tau_1}} + \frac{9}{8\sqrt{2\tau_2}}\right)$.

2.5.3 Optimal revelation timing

Notice that in our model $\lambda_2^R < \lambda_1^R$ i.e., the market becomes less reactive to the release of private information in the second period; this is because the insider's private information was partly impounded into the first period price. Although $\lambda_2^R < \lambda_1^R$ would suggest that the insider profit may increase in the second period, the memory effect plays a strong role in reversing this intuition since the actual ranking is $\bar{\Pi}_2^R < \bar{\Pi}_1^R$.

Intuition would suggest that if the volume of liquidity trade is identical in the two periods ($\tau_1 = \tau_2$) then the best strategy for a monopolist insider is to reveal immediately his signal in order to benefit from informed trading in the two periods. Indeed, a direct calculation shows that $\bar{\Pi}^R > \bar{\Pi}^C \Leftrightarrow \sqrt{\frac{\tau_2}{\tau_1}} > 0.20$, hence:

Lemma 1 *In a market where noise trading in the first period is at least 20% of that in the second period, the monopolist insider optimally chooses to reveal immediately its information.*

3 The equilibria of the duopoly of information

3.1 Construction of the payoffs of the trading game

As we have illustrated in the monopoly benchmark, the revelation of information by the insider triggers a reaction of the market, increasing the aggressiveness λ of the pricing rule. In addition to this effect, two competing insiders with positively correlated signals also have to take into account that revealing at the first stage gives their opponent an informational advantage in the second period: the trade-off is then between using its own information immediately, getting a higher profit in period 1 but suffering tough competition in period 2, or concealing it, so that the profit in stage 1 is lower but the competition in 2 is softer.

To construct the payoffs of the game, and then the equilibrium strategies,[10] we consider the four possible combination of insiders behavior in the first period:[11] both conceal their signals (CC), i reveals and j conceals (RC), the symmetric (CR) and finally both reveal (RR). In solving the game by backward induction we shall develop the minimal amount of calculation in the body of the text and defer complex computations to the appendices.

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[10]We use backward induction as a solution concept to point out the strategic effect of the information revelation, as in Stackelberg competition. Notice however that, since the strategies we allow are markovian, and the state of the game is the only relevant statistic at the beginning of the second period, this solution concept is equivalent to the dynamic programming method more used in the microstructure literature (see for ex. Foster and Viswanathan (1996)).

[11]Recall that revealing its own information in the second period is a dominant strategy.

3.1.1 Symmetric Concealing (CC)

In this subsection we derive the optimal trading strategies conditional on the decision to conceal information in the first stage.

First period market outcome In this CC case, insiders place market orders independent of their private signals. The first stage order flow contains no information on the underlying value of the asset. The equilibrium price is p_0 (normalized to zero) so that any fixed quantity is an optimal order for an insider. The case is exactly analogous to the monopolist setting with no revelation in the first stage. The commitment by informed traders to conceal their information drives their first period profits to zero on average.

Second period market outcome The second stage starts with an empty public information set $\mathcal{H}_2 = \{\emptyset\}$ and private information set $\mathcal{H}_{2,i} = \{s_i\}$. It is like a first stage where insiders immediately exploit their information.

The expected profit for insider i conditional on $\mathcal{H}_{2,i}$ is $\Pi_{2,i}(s_i) = q_{2,i}(s_i)E\left[v - p_2 \mid \mathcal{H}_{2,i}\right]$. The FOC of maximization is altered, with respect to the monopoly case, by the presence of the competitor's order $q_{2,j}$; and the optimal order for i is then

$$q_{2,i} = \frac{1}{2}\left(\lambda_2 E\left[v \mid \mathcal{H}_{2,i}\right] - E\left[q_{2,j} \mid \mathcal{H}_{2,i}\right] - \lambda_2\mu_2 - u_2\right). \tag{10}$$

We use the projection theorem for normal random variables stating $E\left[E\left[v \mid \mathcal{H}_{2,j}\right] \mid \mathcal{H}_{2,i}\right] = \frac{\tau_j}{\tau_j+1}E\left[v \mid \mathcal{H}_{2,i}\right]$ and the fact that trader j's strategy is linear in the form $q_{2,j}(\mathcal{H}_{2,j}) = \alpha_{2,j} + \beta_{2,j}E\left[v \mid \mathcal{H}_{2,j}\right]$ to identify the intercept and the slope of the linear strategy in (10):

$$\alpha_{2,i} = -\frac{1}{2}\left(\lambda_2\mu_2 + \alpha_{2,j} + u_2\right) \quad \text{and} \quad \beta_{2,i} = \frac{1}{2}\left(\lambda_2 - \frac{\tau_j\beta_{2,j}}{\tau_j + 1}\right). \tag{11}$$

Together with the symmetric equations for insider j, we have a system whose solution is

$$\alpha_{2,i} = -\frac{u_2 + \mu_2\lambda_2}{3} \quad \text{and} \quad \beta_{2,i} = \frac{\lambda_2(\tau_j + 2)\left(\tau_i + 1\right)}{(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)}. \tag{12}$$

The order flow is therefore

$$\omega_2 = \frac{\beta_i\tau_i s_i}{\tau_i + 1} + \frac{\beta_j\tau_j s_j}{\tau_j + 1} + \frac{u_2}{3} - \frac{2\mu_2\lambda_2}{3}. \tag{13}$$

Given the form of the order flow assumed in (3), we can identify the coefficients of the various independent random variables to obtain $m_i = \frac{(\tau_j+2)\tau_i}{3\tau_i\tau_j+4\tau_j+4\tau_i+4}$ and $\eta_2 = \frac{1}{3}$.

Dynamic interaction In the first period there is no revelation so that $\mu_2^{CC} = 0$ and $\lambda_2^{CC} = \frac{3\sqrt{\tau_2 Z(\tau_i,\tau_j)}}{3\tau_i\tau_j+4\tau_j+4\tau_i+4}$.[12] By considering symmetrically informed insiders ($\tau_i = \tau_j = \tau$) we observe that

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[12] $Z(\tau_i,\tau_j) = 4\left(\tau_i + 1 + \tau_j\right)\left(\tau_i + \tau_j\right) + \tau_i\tau_j\left(2\tau_i\tau_j + 5\tau_j + 5\tau_i\right)$ is found by plugging $m_i = \frac{(\tau_j+2)\tau_i}{3\tau_i\tau_j+4\tau_j+4\tau_i+4}$ and $\eta_2 = \frac{1}{3}$ into equation (36).

$\lambda_2^{CC} = 3\sqrt{2\tau_2}\sqrt{\frac{\tau(\tau+1)}{(3\tau+2)^2}} > \lambda_2^C = \sqrt{\frac{\tau_2\tau}{\tau+1}}$ meaning that competition among insiders increases the market reactiveness to strategic orders (with respect to the monopoly case).[13]

We are now able to compute the expected profits of the players. From (13), we derive the equilibrium orders

$$q_{2,i}(s_i) = \alpha_{2,i} + \beta_{2,i}\frac{\tau_i s_i}{\tau_i + 1} = \frac{1}{3}\left(\frac{(\tau_j+2)\tau_i}{\sqrt{\tau_2 Z(\tau_i,\tau_j)}}\,s_i - u_2\right) \tag{14}$$

and like in the monopoly case, $q_{2,i}(s_i) = \lambda_2 E\left[v - p_2 \mid s_i\right]$, so that the profit conditional on private information $\Pi_{2,i}^{CC}(s_i) = q_{2,i}(s_i)^2/\lambda_2^{CC}$. The ex-ante profit is then

$$\Pi_i^{CC} = \Pi_{2,i}^{CC}(\tau_2,\tau_i,\tau_j) = E\left[\Pi_{2,i}^{CC}(s_i)\right] = \frac{(\tau_j+2)^2(1+\tau_i)\tau_i + Z(\tau_i,\tau_j)}{3(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)\sqrt{\tau_2 Z(\tau_i,\tau_j)}}. \tag{15}$$

We can verify that the payoff increases with noise trading volume ($\frac{\partial\Pi_i^{CC}}{\partial\tau_2} < 0$), with the quality of one's information ($\frac{\partial\Pi_i^{CC}}{\partial\tau_i} > 0$) and with the quality of the opponent's information ($\frac{\partial\Pi_i^{CC}}{\partial\tau_j} > 0$) except when being seriously disavantaged ($\tau_i < 0.5$), this last statement using a graphical representation. Notice also that a greater precision on information in the sense of $\tau_i > \tau_j$ pays more i.e., $\Pi_i^{CC} > \Pi_j^{CC}$.

The residual variance of the asset is $\Sigma_2^{CC} = E\left[(v - E\left[v \mid \mathcal{H}_2\right])^2 \mid \mathcal{H}_2\right] = 1 - m_j - m_i = \frac{\tau_i\tau_j + 2\tau_j + 2\tau_i + 4}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4}$ which is a convex decreasing function of both precisions.

3.1.2   Asymmetric Behavior (RC or CR)

One player conceals his private information in stage 1 to better use it in stage 2. In what we call the RC case, trader i reveals optimally while trader j conceals optimally. Since $\Pi_i^{CR}(\tau_j,\tau_i) = \Pi_j^{RC}(\tau_i,\tau_j)$ we need only compute both traders payoffs in the RC case.

First period market outcome   The two maximization problems of insiders yield

$$q_{1,i} = -\frac{\mu_1}{3\lambda_1} - \frac{u_1}{3} + \frac{s_i\tau_i}{2\lambda_1(\tau_i + 1)} \tag{16}$$

$$q_{1,j} = -\frac{\mu_1}{3\lambda_1} - \frac{u_1}{3} \tag{17}$$

where $q_{1,j}$ is independent of s_j because insider j voluntary ignores his private information at stage 1. Nevertheless he optimally uses the noise trade u_1 by trading against it. The order flow equation (3) is now $\omega_1 - E\left[\omega_1\right] = \frac{1}{\lambda_1}k_i(v + \varepsilon_i) + \frac{u_1}{3}$, thus we derive $k_i = \frac{\tau_i}{2(1+\tau_i)}$, $k_j = 0$ and $\eta_1 = \frac{1}{3}$. Applying formula (5) we obtain

$$\lambda_1^{RC} = \frac{3}{2}\sqrt{\frac{\tau_1\tau_i}{\tau_i + 1}} \tag{18}$$

which is greater than $\lambda_1^R = \sqrt{\frac{\tau_1\tau}{\tau+1}}$ (monopolist insider). Although the revealing insider acts exactly like a monopolist, the presence of an additional insider trading against the noise trade ($\eta_1^{RC} = \frac{1}{3} < \eta_1^R = \frac{1}{2}$) augments the agressiveness of the market rule. As expected, λ_1^{RC} is increasing in the revealed

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[13]The inequality is equivalent to $18 > \frac{(3\tau+2)^2}{(\tau+1)^2}$ which is true since $(3\tau + 2)^2 < 9(\tau + 1)^2$.

precision τ_i because the more precise the signal, the more aggressive the revealing trader and the more aggressive the market response which ultimately reduces the depth of the market.

We now compute the expected profits for the players. Integrating (18) into optimal demands (23) and (24), we get the ex-interim profits

$$\Pi_{1,i}^{RC}(s_i) = \lambda_1^{RC}\left(\frac{s_i\tau_i}{2\lambda_1^{RC}(1+\tau_i)} - \frac{u_1}{3}\right)^2 \quad \text{and} \quad \Pi_{1,j}^{RC} = \lambda_1^{RC}\frac{u_1^2}{9}. \tag{19}$$

Again we use $E\left[s_i^2\right] = \frac{1+\tau_i}{\tau_i}$, $E\left[u_1^2\right] = \tau_1^{-1}$ and the fact that s_i and u_2 are independent to compute the ex-ante expectations of the profits:

$$\Pi_{1,i}^{RC} = \frac{1}{3\sqrt{\tau_1}}\sqrt{\frac{\tau_i}{\tau_i+1}} \quad \text{and} \quad \Pi_{1,j}^{RC} = \frac{1}{2}\Pi_{1,i}^{RC}. \tag{20}$$

As intuition would suggest, there is a first stage advantage to use its information when compared to concealment $\left(\Pi_{1,i}^{CC} = 0\right)$. Moreover, the revealing trader is earning more than the concealing one who nevertheless takes advantage of the quality of the information revealed ($\Pi_{1,j}^{RC}$ increases with τ_i).

Second period market outcome At the beginning of stage 2, information sets are $\mathcal{H}_{2,i} = \{s_i\}$ and $\mathcal{H}_{2,j} = \{s_j, s_i\}$ so that $E\left[E\left[v \mid \mathcal{H}_{2,j}\right] \mid \mathcal{H}_{2,i}\right] = E\left[v \mid \mathcal{H}_{2,i}\right] = \frac{s_i\tau_i}{\tau_i+1}$ and $E\left[E\left[v \mid \mathcal{H}_{2,i}\right] \mid \mathcal{H}_{2,j}\right] = E\left[v \mid \mathcal{H}_{2,j}\right] = \frac{s_i\tau_i+s_j\tau_j}{\tau_j+\tau_i+1}$.[14]

The FOC for trader i is still

$$2\lambda_2 q_{2,i}(\mathcal{H}_i) = E\left[v \mid \mathcal{H}_{2,i}\right] - \mu_2 - \lambda_2 u_2 - \lambda_2 E\left[q_j \mid \mathcal{H}_{2,i}\right] \tag{21}$$

but as trader j knows $\mathcal{H}_{2,i}$, its FOC reads

$$2\lambda_2 q_{2,j}(\mathcal{H}_{2,j}) = E\left[v \mid \mathcal{H}_{2,j}\right] - \mu_2 - \lambda_2 u_2 - \lambda_2 q_{2,i}(\mathcal{H}_{2,i}) \tag{22}$$

We proceed very much like in section 3.1.1 to derive the insiders orders. Trader i takes into account the fact that he is revealing his information to trader j; he therefore integrates (22) into (21) to find his optimal response:[15]

$$q_{2,i}(s_i) = \frac{s_i\tau_i}{3\lambda_2(\tau_i+1)} - \frac{u_2}{3} - \frac{\mu_2}{3\lambda_2} \tag{23}$$

The presence of another insider (j) forces the revealing insider (i) to trade less on its information in the second period (factor $\frac{1}{3}$ against $\frac{1}{2}$), for an equal market depth in both periods. This is an effect we already saw when comparing the CC case with the monopoly.

We can now solve for the more informed trader j substituting (23) in (22)

$$q_{2,j}(s_i, s_j) = \frac{(s_i\tau_i + s_j\tau_j)}{2\lambda_2(\tau_j+\tau_i+1)} - \frac{s_i\tau_i}{6\lambda_2(\tau_i+1)} - \frac{u_2}{3} - \frac{\mu_2}{3\lambda_2} \tag{24}$$

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[14]We use the normal variables linear decomposition property presented p 21 by equations (31) and (32).

[15]We could also posit $q_i = a_1 + b_1 s_i$ and $q_j = a_2 + b_2 s_i + b_3 s_j$ and derive the coefficients from the FOCs.

Dynamic interaction Having solved the price equilibrium of both stages, we can find out the updating performed by the market in the second period. Using the equilibrium values of $k_i, k_j, \eta_1, m_i, m_j$ and η_2 we obtain $\lambda_2^{RC} = \frac{1}{\sqrt{\tau_2}} \sqrt{\frac{4(1+\tau_i)(\tau_i+\tau_j)+5\tau_j}{4(\tau_i+\tau_j+1)(1+\tau_i)}}$ (cf. (36) in appendix). The positiveness indicates that the market spreads its reactiveness towards adverse selection over the two periods of trade.

The novelty of the RC case with respect to the CC case is the *memory effect* within the second period pricing rule; it is given by $\mu_2^{RC} = \frac{\tau_i s_i}{6(1+\tau_i)} + \frac{\lambda_1^{RC} u_1}{9}$ which affects the second period profits. The ex-interim profits in stage two are

$$\Pi_{2,i}^{RC}(s_i) = \lambda_2^{RC} q_{2,i}(s_i)^2 = \frac{1}{9\lambda_2^{RC}} \left(\frac{\tau_i s_i}{\tau_i+1} - \frac{\tau_i s_i}{6(1+\tau_i)} - \lambda_1^{RC} \frac{u_1}{9} - \lambda_2^{RC} u_2 \right)^2 \tag{25}$$

$$\Pi_{2,j}^{RC}(s_j, s_i) = \lambda_2^{RC} q_{2,j}(s_i, s_j)^2 = \frac{1}{9\lambda_2^{RC}} \left(\frac{3(s_i\tau_i + s_j\tau_j)}{2(\tau_i+\tau_j+1)} - \frac{\tau_i s_i}{2(\tau_i+1)} - \frac{\tau_i s_i}{6(1+\tau_i)} - \lambda_1^{RC} \frac{u_1}{9} - \lambda_2^{RC} u_2 \right)^2 \tag{26}$$

and their ex-ante expectations are

$$\Pi_{2,i}^{RC} = \frac{62\tau_i(\tau_i+\tau_j+1)+81\tau_j}{162\sqrt{\tau_2}\sqrt{(4\tau_i(\tau_i+\tau_j+1)+9\tau_j)(1+\tau_i)(\tau_i+\tau_j+1)}} \tag{27}$$

$$\Pi_{2,j}^{RC} = \frac{62\tau_i(\tau_i+\tau_j+1)+162\tau_j}{162\sqrt{\tau_2}\sqrt{(4\tau_i(\tau_i+\tau_j+1)+9\tau_j)(1+\tau_i)(\tau_i+\tau_j+1)}} \tag{28}$$

Finally, the variance of v not explained by the private signals is $\Sigma_2^{RC} = \frac{\tau_i^2+\tau_i\tau_j+7\tau_i+6+3\tau_j}{6(\tau_i+\tau_j+1)(1+\tau_i)}$.

As intuition suggests, the insider who benefited from the revelation of its competitor in the first period earns more than him in the second stage, since he is better informed than its competitor, possessing two informative signals ($\Pi_{2,j}^{RC} - \Pi_{2,i}^{RC} \propto 81\tau_j$).

Comparing the total expected payoffs for i when insider j conceals we can state a first result:

Lemma 2 *In a market where noise trading in the first period is at least 40% of that in the second period, an insider facing a concealing insider decides to reveal immediately its information.*

Proof: We have $\Pi_i^{CC} = \Pi_{2,i}^{CC} = \frac{(\tau_j+2)^2(1+\tau_i)\tau_i+Z(\tau_i,\tau_j)}{3\sqrt{\tau_2 Z(\tau_i,\tau_j)}(3\tau_i\tau_j+4\tau_j+4\tau_i+4)}$ and $\Pi_i^{RC} = \frac{1}{3\sqrt{\tau_1}}\sqrt{\frac{\tau_i}{\tau_i+1}} + \Pi_{2,i}^{RC}$. It can be algebraically checked that $\sqrt{\tau_2}\left(\Pi_{2,i}^{RC} - \Pi_{2,i}^{CC}\right)$ is a polynomial fraction F of τ_i and τ_j with positive coefficients only, hence

$$\Pi_i^{RC} > \Pi_i^{CC} \Leftrightarrow \frac{1}{3\sqrt{\tau_1}}\sqrt{\frac{\tau_i}{\tau_i+1}} > \Pi_{2,i}^{RC} - \Pi_{2,i}^{CC} \Leftrightarrow \sqrt{\frac{\tau_2}{\tau_1}} > S(\tau_i,\tau_j) \equiv 3F(\tau_i,\tau_j)\sqrt{\frac{\tau_i+1}{\tau_i}}.$$

As the latter expression is bounded above by 0.4 we obtain the lemma. ∎

3.1.3 Symmetric Revealing (RR)

In this final case, traders play in the first period as if they were in the second period without any previously revealed information. Accordingly, the analysis proceeds in a relative straightforward way. The second stage analysis of the CC case yields immediately the first stage of the RR case by changing τ_2 into τ_1 i.e., $\Pi_{1,i}^{RR}(\tau_1, \tau_i, \tau_j) = \Pi_{2,i}^{CC}(\tau_1, \tau_i, \tau_j)$. We obtain $k_i > 0, k_j > 0$ and $\eta_1 > 0$ i.e., traders

use their information against the market and also trade against the liquidity traders. The market aggressiveness is positive ($\lambda_1^{RR} > 0$) and there is no history effect ($\mu_1 = 0$).[16]

In the second stage, the private information sets are $\mathcal{H}_{2,i} = \mathcal{H}_{2,j} = \{s_j, s_i\}$. Using the above resolution method, we find the parameters of the optimal revealing strategies $\alpha_{2,i} = -\frac{u_2}{3} - \frac{\mu_2}{3\lambda_2}$ and $\beta_{2,i} = \frac{1}{3\lambda_2}$. Writing the order book in two different ways

$$\frac{u_2}{3} - \frac{2\mu_2}{3\lambda_2} + (\beta_i + \beta_j)\frac{s_i\tau_i + s_j\tau_j}{\tau_j + \tau_i + 1} = \omega_2 - E[\omega_2] = \frac{(m_is_i + m_js_j)}{\lambda_2} + \eta_2 u_2 \tag{29}$$

we are able to identify $m_i = \frac{\tau_i}{3(\tau_j + \tau_i + 1)}$ and $\eta_2 = \frac{1}{3}$. This leads to a complex formulae for λ_2^{RR}, μ_2^{RR} and the ex-ante profit $\Pi_{2,i}^{RR}$. The overall expected payoff $\Pi_i^{RR} = \Pi_{1,i}^{RR} + \Pi_{2,i}^{RR}$ is an increasing and concave function of the precision of its own signal.

3.2 The revelation of information in equilibrium

The previous analysis allows us to characterize the equilibrium of our trading game. The existing literature (Kyle (1989), Holden and Subrahmanyam (1992), Foster and Viswanathan (1993) and (1996)) has shown that increasing competition between informed traders leads to more efficient prices. The following proposition is the central result of the paper: it shows for which parameters of private signal precision (τ_i, τ_j) and volume of liquidity trade (τ_1, τ_2) one of the insiders prefers not to reveal his own information in the first period at equilibrium.

Let us define $z = \sqrt{\frac{\tau_2}{\tau_1}}$ as the ratio between the first period expected volume of noise trade and the second period one.

Theorem 3 *The equilibrium outcome of the duopoly game depends on the precisions of insiders' signals and the relative size z of noise trading in the first period as compared to the second period:*
(i) If $z \geq 60\%$ then both insiders reveal immediately ("rat race");
(ii) If $z \leq 20\%$ then everybody waits ("waiting game");
(iii) If $20\% \leq z \leq 60\%$ the equilibrium is asymmetric:

 (a) if precisions are dissimilar the better informed insider conceals his information while the other one reveals.

 (b) if precisions are similar one insider conceals his information while the other reveals.

Proof: Algebraic manipulations enable us to show that $\Pi_i^{RR} > \Pi_i^{CR} \Leftrightarrow z > T(\tau_i, \tau_j)$ where T is a polynomial fraction of precisions. The figure below represents the surfaces $T(\tau_i, \tau_j)$ (on top) and $S(\tau_i, \tau_j)$ (below) where the surface $S(\tau_i, \tau_j)$ represents the set of z such that $\Pi_i^{RC} > \Pi_i^{CC} \Leftrightarrow z > S(\tau_i, \tau_j)$.

---

[16]As intuition suggests, there is a first period advantage to reveal its information: $\Pi_{1,i}^{RR} > \Pi_{1,i}^{CR}$ (this difference is a polynomial in τ_i and τ_j with positive coefficients). Furthermore, since trader j is always revealing, the better informed j is, the better it is for trader i to use that information for himself; this is why $\Pi_{1,i}^{RR} - \Pi_{1,i}^{CR}$ increases with τ_j. It can also be noted that $\Pi_{1,i}^{RR}(0, \tau_j) = \Pi_{1,i}^{CR}(\tau_i, \tau_j)$ i.e., revealing an empty information is like not revealing.

It is immediate to see that the equilibrium is RR if $z > \max\left(T(\tau_i, \tau_j), T(\tau_j, \tau_i)\right)$ i.e., when both points (τ_i, τ_j, τ_2) and (τ_j, τ_i, τ_2) are above the T map. Indeed, as $T(\tau_i, \tau_j) > S(\tau_i, \tau_j)$, we have $\Pi_i^{RC} > \Pi_i^{CC}$ and $\Pi_i^{RR} > \Pi_i^{CR}$ for $i = 1, 2$ thus revelation is a dominant strategy for both insiders. A similar but reversed reasoning applies if $z < \min\left(S(\tau_i, \tau_j), S(\tau_j, \tau_i)\right)$ i.e., when (τ_i, τ_j, τ_2) and (τ_j, τ_i, τ_2) both lie below the S map. The concealment of private information is a dominante strategy: the unique equilibrium is C, C.

Finally we have to consider intermediate situations where points are in between the two maps:

1. If $\max\left(S(\tau_i, \tau_j), S(\tau_j, \tau_i)\right) < z < \min\left(T(\tau_i, \tau_j), T(\tau_j, \tau_i)\right)$ i.e., when both points are in between the maps S and T then both RC and CR are equilibria i.e., no one imitates C but given that someone plays R the other one does not wish to imitate.

2. If (τ_i, τ_j, τ_2) lies below the S map (right side of the figure) but (τ_i, τ_j, τ_2) is above (left side of the figure) then insider i will never reveal and insider j reveals against a concealing opponent. Now this is enough to fully characterize the equilibrium which is CR.

3. If (τ_i, τ_j, τ_2) lies between the T and S maps while (τ_i, τ_j, τ_2) is above the T map, then insider j will always reveal but insider i will conceal against a revealing opponent. Hence the equilibrium is once more CR. Q.E.D.
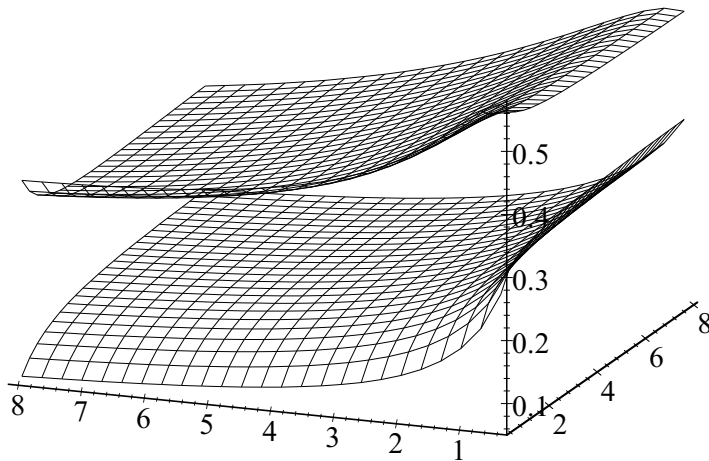

Figure 1: The S and T surfaces

A few comments on Theorem 3 are in order. The volume of liquidity trade on the equilibrium revelation of information has already been pointed out by Admati and Pfleiderer (1988) in order book markets (the "size" effect of liquidity trade). As intuition suggests, the bigger the volume of trade present in the market for hedging or other liquidity reasons, the higher the profit insiders can earn trading on their information signal. Hence, if stage 2 is significantly more liquid than stage 1, traders have an incentive to hide their information in order to exploit it successfully in the last stage. Let

us stress that such an inefficient release of information can arise also in highly transparent markets if the trading day involves moments of low and high activity. This is the case for instance in European stock exchanges at the opening of American markets.

For high levels of z or comparable liquidity trade among periods, our model displays what Holden and Subrahmanyam (1992) call the "rat race": if a well informed insider decides to reveal his signal, then the other insider also reveals. This happens because revealing a good piece of information generates a strong market reaction which will dramatically reduce future profits. Hence there is no more gain to wait and revealing becomes the best reply to adopt.

The volume of liquidity trade across periods is not the only equilibrium driver. For intermediate levels of z, the difference in the signal quality matters. When an insider with a low quality signal decides to reveal it, a better informed opponent conceals his own in equilibrium. For the badly informed insider, it is intuitive that using his information in a period in which he is the sole insider is optimal (the market rule is not too aggressive since his signal is imprecise); this first period advantage of revealing his information more than compensate the second period loss. For the better informed insider the trade-off is more complex. Revealing his information in the first period would certainly profit in the same period, but it would turn his opponent into a copy of himself for the second period: the opponent would then play aggressively on this acquired information in the second period, reducing dramatically the payoff for the first insider. If the size of the second period market is large enough, revealing at the first stage is not a best reply for the better informed insider.

The main practical conclusion we can draw from Theorem 3 is that in electronic and transparent order books, efficient revelation of information takes place only during trading phases with a high level of noise arriving to the market. In less liquid phases, only very imprecise information is revealed to the market. This effect is a consequence of the transparent microstructure of the exchanges, and it is not present in previous model as in Foster and Viswanathan (1996), where insiders wait to reveal their signals only when these are negatively correlated among themselves.

3.3 Informational efficiency and liquidity of the market

The main motivation to study a different microstructure of exchanges and a different information structure from the existing literature builds on the consequences that these two elements induce on the equilibrium liquidity dynamics and the degree of price efficiency. We describe these dynamics pointing out the difference with the predictions of existing works, especially Holden and Subrahmanyam (1992) and Foster and Viswanathan (1996). With this comparison, we can better understand the effect of the particular asymmetry of information assumed here, as well as the effect of the strategic interaction between insiders in a quotation system with extreme transparency.

The degree of price efficiency is given by Σ_t, the residual variance after the information publicly revealed has been impounded into the price. The following corollary is proved in the appendix:

Corollary 4 *In any equilibrium of the game, price informativeness increases in the second period, i.e. there is information revelation at the second stage: $\Sigma_1^{RC} \geq \Sigma_2^{RC}$ and $\Sigma_1^{RR} \geq \Sigma_2^{RR}$. The informativeness in the RR equilibrium is greater than in the RC one since $\Sigma_1^{RC} \geq \Sigma_1^{RR}$ and $\Sigma_2^{RC} \geq \Sigma_2^{RR}$.*

It is readily observed that the second period of the game is informative, under any condition, since the insiders will then have the last possibility to trade on a better position than the public. Yet, the difference in the signals quality can induce an equilibrium RC in which case some relevant information is not impounded in the first period price p_1; this negatively impacts the informativeness of the market.

The liquidity of the market, measured here by the price sensitivity λ (inverse of market depth) gives us an idea of the cost of trading: a very thin market (high λ) generates a high cost of trading via the effect that individual trading has on price. It is immediate to see that whatever the equilibrium, the market depth is never constant across periods as opposed to the monopolist case of Kyle (1985) but similarly to the rest of the literature of competition among insiders. Since the degree of correlation between signals and the length of the game are both fixed, we can make some comparative statics analysis changing the precisions τ_i and τ_j.

The meaningful measures of market sensitivity over the trading day that avoid volume effects are $\sqrt{\frac{1}{\tau_1}}\lambda_1$ and $\sqrt{\frac{1}{\tau_2}}\lambda_2$. The sensitivity of the market pricing rule is likely to decrease with time since the reaction to information revelation is stronger when private information is first used. We obtain indeed $\sqrt{\frac{1}{\tau_2}}\lambda_2^{RR} \leq \sqrt{\frac{1}{\tau_1}}\lambda_1^{RR}$ for all possible combinations of precision parameters (cf. Appendix 5.2). However when the RC equilibrium takes place the reverse inequality $\sqrt{\frac{1}{\tau_2}}\lambda_2^{RC} \geq \sqrt{\frac{1}{\tau_1}}\lambda_1^{RC}$ may hold if $\tau_j \geq \frac{5\tau_i(1+\tau_i)}{9-5\tau_i}$ and $\tau_i \leq \frac{9}{5}$.

Corollary 5 *A strong asymmetry of information which leads to an asymmetric equilibrium pattern of revelation can generate higher trading costs in the second period in which most of the information revelation occurs.*

As for comparison of the market sensitivity across equilibria we may observe that $\lambda_2^{RC} > \lambda_2^{RR}$; this is simply because the second period in the RC equilibrium is the moment where most information is revealed to the market thereby generating a strong reaction. In the first period the reverse inequality $\lambda_1^{RC} < \lambda_1^{RR}$ holds[17] since it is the moment where most information is revealed to the market in the play of the RR equilibrium triggering an intense response from the market.

We may say that the trading cost is higher the greater the revelation of private information. This effect is well known in the literature: in equilibria with high adverse selection, the cost of trading is higher. Yet as shown in Corollary 5, an ineffiecnt pattern of information revelation may occur making trading costlier with time.

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[17]The relation $\lambda_1^{RC} > \lambda_1^{RR}$ can hold true but only if it is the best informed insider who reveals and it is been shown that this does not happen in equilibrium.

An increasing sensitivity factor λ_t is also obtained by Foster and Viswanathan (1996), but only when the information of the insiders is negatively correlated which is not the case here. The reason for this result in Foster and Viswanathan (1996) is that informed trader i makes a small trade in the first period as he thinks that informed trader j is pulling the stock price in the wrong direction and perceive the possibility of making large profits in the future by postponing its trade (that otherwise would pull the price in the "right" direction immediately).

In our model the rationale is different; the player who receives the most valuable information (that is the most precise signal), prefers to wait to post a trade dependent on that signal, since he knows that in the second period he will be able to update his own information much better than the average (i.e. the market makers). Indeed he will possess a high informational advantage then, not having revealed his signal to the opponent. This waiting behavior enhances a more aggressive pricing rule by the market makers in the second period. Notice that our result arises from a strategic interaction, and it is not the consequence of a statistical characteristic of the model. We may interpret the competition in information as a competition between oligopolist with different cost structures where the most efficient acts as a leader, taking a first mover advantage.

Using the terminology of Foster and Viswanathan (1996), we can conclude that even if the insiders have the same opinion about the value of an asset, two effects arise: a competition effect ("rat race") and a strategic leadership effect ("waiting game"): when the quality of the information signals is similar across insiders, the first effect prevails, otherwise we can observe the second phenomenon.

4   Conclusion

In the studies that address the problem of aggregation of information by equilibrium prices, the role of asymmetries between informed traders has been rarely analyzed. The results of Kyle (1985), Holden and Subrahmanyam (1992) consider the equilibrium in a centralized, order-driven continuous financial market with a monopolist of information or more competitors with the same information. Foster and Viswanathan (1996) study the competition between asymmetrically informed traders, but consider a signal space where insiders receive signals with the same precision but with some positive or negative correlation.

In this article we have shown that the role of asymmetries is crucial in order to assess the efficiency properties of prices. We have studied the strategic interaction between insiders who are able to observe the orders of each other, as in transparent electronic order-book markets such as the Toronto Stock Exchange. In our model this feature can induce a slow process of information revelation if the private signals, even being positively correlated, have highly different precisions. We characterize the linear, pure strategies equilibrium set as a function of the precision of private signals and the volume of liquidity trade present on the market. Keeping aside the influence of the latter we show that the more precise the signal, the lower the incentive to reveal it at the first stage. Yet the optimal response

of a better informed trader can be to hide its own information during the first stage. Asymmetric equilibria hence arise if the insides have considerably different precision. In the symmetric case, we find the result of Holden and Subrahmanyam (1992) as a special case.

The higher transparency structure of electronic exchanges causes then a sophisticated interaction between informed traders, and the overall effect on the amount of information incorporated into prices can be negative.

Our result is also supported by the empirical evidence of Madhavan, Porter and Weaver (2000) who observed an increase in trading costs in the Toronto CATS after the reform of 1990 that increased the transparency in that market structure. The same result, but concerning a market which became anonymous, is described by Foucault, Moinas and Theissen (2002) about the reform of April 2001 in the Euronext Paris system.

5   Appendix

5.1   Second Period Market pricing

We derive here the market marker pricing rule in the second period depending on the order flow ω_1 and ω_2. For normal variables the conditional expectation is a linear function of the observations, thus

$$p_2 = E\left[v \mid \omega_1, \omega_2\right] = a_1\omega_1 + a_2\omega_2 \tag{30}$$

and by the law of iterated expectations, we have

$$E\left[v \mid \omega_1\right] = E\left[E\left[v \mid \omega_1, \omega_2\right] \mid \omega_1\right] = a_1\omega_1 + a_2E\left[\omega_2 \mid \omega_1\right] \tag{31}$$

$$E\left[v \mid \omega_2\right] = E\left[E\left[v \mid \omega_1, \omega_2\right] \mid \omega_2\right] = a_2\omega_2 + a_1E\left[\omega_1 \mid \omega_2\right] \tag{32}$$

To solve (31), we use (2) and (3) to write

$$
\begin{aligned}
E\left[\omega_2 \mid \omega_1\right] &= E\left[\lambda_2(m_i + m_j)v \mid \omega_1\right] + E\left[\lambda_2 m_i\varepsilon_i \mid \omega_1\right] + E\left[\lambda_2 m_j\varepsilon_j \mid \omega_1\right] + E\left[\eta_2 u_2 \mid \omega_1\right]\\
&= \frac{\lambda_2(m_i+m_j)}{\lambda_1(k_i+k_j)}E\left[\lambda_1(k_i + k_j)v \mid \omega_1\right] + \frac{\lambda_2 m_i}{\lambda_1 k_i}E\left[\lambda_1 k_i\varepsilon_i \mid \omega_1\right] + \frac{\lambda_2 m_j}{\lambda_1 k_j}E\left[\lambda_1 k_j\varepsilon_j \mid \omega_1\right]\\
&= \left(\frac{\lambda_2(m_i+m_j)}{\lambda_1(k_i+k_j)}\left(\lambda_1(k_i + k_j)\right)^2 + \frac{\lambda_2 m_i}{\lambda_1 k_i}\left(\lambda_1 k_i\right)^2 + \frac{\lambda_2 m_j}{\lambda_1 k_j}\left(\lambda_1 k_j\right)^2\right)\frac{\omega_1}{Var(\omega_1)}\\
&= \frac{\lambda_1\lambda_2 Q\omega_1}{Var(\omega_1)} \text{ where } Q \equiv (m_i + m_j)(k_i + k_j) + k_i m_i\tau_i^{-1} + k_j m_j\tau_j^{-1} \tag{33}
\end{aligned}
$$

Combining (4) and (33), (31) becomes equivalent to $a_1\omega_1 + a_2\omega_1\frac{Q}{\lambda_1\lambda_2 Var(\omega_1)} = \frac{k_i+k_j}{\lambda_1}\frac{\omega_1}{Var(\omega_1)}$, hence $a_2 = \frac{\lambda_2(k_i+k_j-a_1\lambda_1 Var(\omega_1))}{Q}$. Symmetrically, (32) leads to $a_1 = \frac{\lambda_1(m_i+m_j-a_2\lambda_2 Var(\omega_2))}{Q}$. Solving this system yields

$$a_1 = \lambda_1\frac{Q(m_i + m_j) - (k_i + k_j)\lambda_2^2 Var(\omega_2)}{Q^2 - (k_i + k_j)\lambda_2^2 Var(\omega_2)} \tag{34}$$

$$a_2 = \lambda_2\frac{(k_i + k_j)\left(Q - (m_i + m_j)\right)}{Q^2 - (k_i + k_j)\lambda_2^2 Var(\omega_2)} \tag{35}$$

Combining the linear market pricing rule $p_2(\omega_1, \omega_2) = \mu_2 + \omega_2\lambda_2$ with (30) we obtain $\mu_2 + \omega_2\lambda_2 = a_1\omega_1 + a_2\omega_2$. Identifying the coefficients, we derive $\mu_2 = a_1\omega_1$ and $\lambda_2 = a_2$. Hence from (35) we get $\lambda_2^2 Var(\omega_2)(k_i + k_j) = Q^2 - (k_i + k_j)(Q - (m_i + m_j))$ and since, by construction, $Var(\omega_2) = \left(\frac{m_i + m_j}{\lambda_2}\right)^2 + \frac{m_i^2}{\lambda_2^2 \tau_i} + \frac{m_j^2}{\lambda_2^2 \tau_j} + \frac{\eta_2^2}{\tau_2}$ we finally obtain

$$\lambda_2^2 = \lambda_2 = \frac{\sqrt{\tau_2}}{\eta_2}\sqrt{\frac{Q^2}{(k_i + k_j)} - Q + m_i + m_j - (m_i + m_j)^2 - m_i^2\tau_i^{-1} - m_j^2\tau_j^{-1}} \tag{36}$$

Notice that when k_i and k_j tend to zero, Q and $Q^2/(k_i + k_j)$ tend also to zero, thus (36) is the exact counterpart to (5) the market depth in the first stage. We can also simplify (34) to obtain

$$\mu_2 = a_1\omega_1 = \frac{Q(m_i + m_j + k_i + k_j - Q) - (m_i + m_j)(k_i + k_j)}{Q(k_i + k_j) - (m_i + m_j)(k_i + k_j)}(k_is_i + k_js_j + \lambda_1\eta_1u_1) \tag{37}$$

which is nil whenever no information is revealed in stage one ($k_i = k_j = 0$ and $\lambda_1 = 0$).

Using $E[v \mid \omega_1, \omega_2] = a_1\omega_1 + a_2\omega_2$ we are able to compute

$$\Sigma_2^{RR} = 1 - Var(E[v \mid \omega_1, \omega_2]) = (k_i + k_j - 1)(m_i + m_j - 1) + \frac{k_jm_j\tau_i + k_im_i\tau_j}{\tau_i\tau_j}.$$

5.2 Proof of Corollaries 4 and 5

- $\Sigma_1^{RC} \geq \Sigma_2^{RC} \Leftrightarrow \frac{\tau_i + 2}{2(1 + \tau_i)} \geq \frac{\tau_i^2 + \tau_i\tau_j + 7\tau_i + 6 + 3\tau_j}{6(\tau_i + \tau_j + 1)(1 + \tau_i)} \Leftrightarrow$

$$\begin{aligned}
0 &\leq 6(\tau_i + \tau_j + 1)(1 + \tau_i)(\tau_i + 2) - 2(1 + \tau_i)(\tau_i^2 + \tau_i\tau_j + 7\tau_i + 6 + 3\tau_j) \\
&= 8\tau_i^2 + 4\tau_i + 4\tau_i^3 + 10\tau_i\tau_j + 6\tau_j + 4\tau_i^2\tau_j
\end{aligned}$$

- $\Sigma_1^{RR} \geq \Sigma_2^{RR} \Leftrightarrow \tau_i\tau_j + 2\tau_j + 2\tau_i + 4 \geq \frac{(\tau_j + \tau_i + 7)(\tau_i\tau_j + 2\tau_j + 2\tau_i) + 12}{3(\tau_i + \tau_j + 1)}$

$$\begin{aligned}
\Leftrightarrow 0 &\leq 3(\tau_i + \tau_j + 1)(\tau_i\tau_j + 2\tau_j + 2\tau_i + 4) - ((\tau_j + \tau_i + 7)(\tau_i\tau_j + 2\tau_j + 2\tau_i) + 12) \\
&= 2\tau_j\tau_i(\tau_j + \tau_i) + 4\left(\tau_i\tau_j + \tau_i^2 + \tau_j^2 + \tau_i + \tau_j\right)
\end{aligned}$$

- $\Sigma_1^{RC} \geq \Sigma_1^{RR} \Leftrightarrow 0 \leq (3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)(\tau_i + 2) - 2(1 + \tau_i)(\tau_i\tau_j + 2\tau_j + 2\tau_i + 4) = \tau_i^2\tau_j + 4\tau_i\tau_j + 4\tau_j$.

- $\Sigma_2^{RC} \geq \Sigma_2^{RR} \Leftrightarrow$

$$\begin{aligned}
0 &\leq (3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)(\tau_i^2 + \tau_i\tau_j + 7\tau_i + 6 + 3\tau_j) - 2(1 + \tau_i)((\tau_j + \tau_i + 7)(\tau_i\tau_j + 2\tau_j + 2\tau_i) + 12) \\
&= \tau_j\left(\tau_i^3 + \tau_i^2\tau_j + 5\tau_i^2 + 12\tau_i + 7\tau_i\tau_j + 8 + 8\tau_j\right)
\end{aligned}$$

- $\sqrt{\frac{1}{\tau_2}}\lambda_2^{RR} \leq \sqrt{\frac{1}{\tau_1}}\lambda_1^{RR} \Leftrightarrow$

$$\begin{aligned}
0 &\leq 9Z(\tau_i, \tau_j)X(\tau_i, \tau_j) - 2Y(\tau_i, \tau_j)(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)^2 \\
&\propto \tau_i\tau_j\left((12\tau_i\tau_j + 101)(\tau_i + \tau_j) + 25(\tau_j^2 + \tau_i^2) + 24\right) + 20\tau_j\left(\tau_j^2 + 2\tau_i^2\tau_j + 2\tau_j + 1\right) \\
&\quad + 20\tau_i\left(\tau_i^2 + 2\tau_i + 2\tau_i\tau_j^2 + 1\right)
\end{aligned}$$

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