Explaining Fashion Cycles: Imitators Chasing Innovators in Product Space*

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March 23, 2006

Abstract

This paper considers the problem of a fashion trend-setter confronting an imitator who can produce the same product at lower cost. A one-dimensional product space is considered, which is an abstraction of the key attribute of some consumer good.

Three broad strategies can be optimal for the fashion-leader: (1) Never innovate, which milks profits from the initially advantageous position but ultimately concede the market without a fight. (2) Innovate once but only once, which just temporarily defers conceding the market. (3) Cycle infinitely around product space, never letting the imitator catch up and capture the market. Sometimes the cycles start immediately; sometimes the innovator should wait for a time before beginning the cycles.

The optimal solution exhibits strong state-dependency, with so-called Skiba curves separating regions in state space where various of these strategies are optimal. There are even instances of intersecting Skiba curves. In most cases, analytical expressions can be stated that characterize these Skiba curves.

1 Introduction

This paper suggests a novel explanation for the existence of fashion cycles,namely movement around a 'product space' that is strategic on the part of a fashion trend-setter and imitative by low-cost competitor(s). The trend-setter defines

*The authors like to thank an anonymous referee and the editor, Cars Hommes, for their remarks.

what is fashionable, and off-label brands imitate them. The product design space is actually of very high dimension, leaving lots of room for complex trajectories that never settle down to a single steady state. We show, however, that such complexity is not essential to this story. Even in a one-dimensional abstraction of that space, the optimal solution may involve continual adaptation and imitation. One example of a one-dimensional product space is the width of neckties, which we all have observed to vary over time. Other examples include the extent to which accessories are flashy or understated, the width of labels on sport coats, or the length of skirts. An important benefit of sticking with a one-dimensional product space is that we can write explicit expressions describing the thresholds between initial conditions where different strategies are optimal.

The structure of the optimal solution depends on how much it costs to develop new designs and on the initial positions of both firms in the product space. For low design costs it is optimal for the firms to cycle around the product space indefinitely, with a Skiba curve separating the two possible directions for changing the design initially.

For intermediate design costs, two other Skiba curves circumscribe an area in the middle of the product space where it is optimal for the fashion setter not to innovate, at least initially. In that case either the imitator simply catches up and conquers the whole market or the fashion setter changes its design later.

If the cost of marketing new designs is still larger, two other Skiba curves arise, which separate a policy of "never changing the fashion" from one in which the fashion leader makes an initial major design change but no subsequent changes.

The paper is organized as follows. Section 2 reviews some relevant literature. The model is presented in Section 3. A short overview of the solution structure is provided in Section 4. Section 5 analyzes the solution structure when the costs of making new designs are so low that periodically changing designs forever is optimal. In Section 6 these costs are large enough that it is not optimal to change the design more than once . Section 7 considers parameter constellations where, depending on the initial situation, either periodic design changes or making no new designs at all can be optimal.

2 Literature Review

Firms engage in at least two kinds of product design innovation: technological innovation and stylistic or fashion innovation (Schweizer, 2003). The former improves the product. E.g., computers today are faster than they were ten years ago. Fashion innovation differentiates what's new from current models without improving functionality. For women's clothes, red may be "in" this year and blue may be "out", but that does not mean red is intrinsically better than blue. Certainly the color red is not a new invention per se. Furthermore, it is likely that in a few years blue will be in and red will be out, and a few years after that, red will be in again. The same happens for the width of men's ties, the length of skirts, and the popularity of one material relative to another.

In short, we observe that consumers are willing to pay more for one good (the "in" good) than another, functionally equivalent product (the one that is "out"). Economists have long been fascinated by this behavior, dubbed "Veblen effects" in honor of Thornstein Veblen's seminal inquiries into conspicuous consumption (Bagwell and Bernheim, 1996). Typical explanations involve "status" or "prestige" goods conferring utility on their consumers by allowing implicit association with other high-status consumers of that good. If a good is so expensive that only the rich can afford it, then onlookers can infer that anyone they see consuming it must be rich. Allowing others to make that inference may bring various benefits, and a variety of models have been developed under which it is optimal for consumers to behave in this way (e.g., Bikhchandani et al., 1992; Coelho and McClure, 1993; Bagwell & Bernheim, 1996; Frijters, 1998; Corneo and Jeanne, 1999; Bianchi, 2002).

Here we take this consumer behavior as given, rather than trying to "explain" it within a rational actor framework. We ask instead how firms might manage fashion innovation in order to exploit this behavior in order to maximize profits. There is a large management science/operations research literature providing practical guidance to fashion goods producers, but it does not treat the rate of fashion innovation as a decision variable of interest. Rather, these papers address manufacturing (Degraeve and Vandebroek, 1998; Jain and Paul, 2001), supply chain management (Donohue, 2000; Mantrala & Rao, 2001; Milner & Kouvelis, 2002), inventory policy (Fisher et al., 2001), pricing (Zhao and Zheng, 2000), and other management issues that arise in the context of a given fashion innovation's product life cycle.

Likewise there is a literature advising firms how quickly to make technological innovations (e.g., Paulson Gjerde et al., 2002), but little has been written about how to manage the rate of fashion innovation.

A partial exception is Swann's (2001) case study of the evolution of two prestige cars, the Rolls Royce and the Ferrari. However, that is an interesting descriptive analysis of a particular case more than an effort to derive prescriptive insights from a general model.

The closest analog to the current investigation is Pesendorfer's (1995) innovative paper. As in our model, Pesendorfer's fashion producer dynamically optimizes the timing of fashion innovations and, finds, as do we, that the optimal solution could involve introduction of new fashions at fixed, regular intervals whose period varies inversely with the cost of innovation. There are three significant differences, however. First, Pesendorfer explicitly models the behavior of two discrete types of individuals and the producer's decision about how to vary the price of a given fashion over time. In that sense, Pesendorfer's focus is on creating a rational-actor model of fashion that includes the incentives of producers not just consumers, whereas, again, we take consumer's taste for fashion (meaning products differentiated from low-cost alternatives) as a given.

Second, Pesendorfer thinks of innovations as discrete. At some fixed unit cost, the innovator can create a new design that is completely differentiated from the current design, and renders the current design instantly and completely obsolete. However, it seems more realistic to think of a product design space. Innovation implies moving one's product within that space, and one could move a little (minor innovation) or a long way (major innovation). One or the other might turn out to be optimal, depending on the particular circumstances, but the model should recognize that the producer has that choice, rather than assuming arbitrarily that all innovation must necessarily be draconian.

Likewise, we view allowing even a one-dimensional continuum to be an advance. Related to this, we view the cost of innovation as increasing in the "amount of innovation". Consider test marketing, for example. Understanding how consumers will react to modest variations might be relatively easy, but accurately predicting their response to a radical change might require more market research. Likewise, it might take more advertising to persuade people that what initially seems like a very extreme departure from current trends will in fact become de rigeur. Indeed, Barnett and Freeman (2001) found that firm mortality rates increase with the simultaneous introduction of multiple significant innovations.

The third and most important difference between our model and Pesendorfer's pertains to the existence and behavior of other producers. Pesendorfer focuses on the monopolistic case, and, even in his competition case, Pesendorfer (1995, 773) did not "*allow imitation of successful designs. Imitation would give designers an additional incentive to create new fashions periodically. Clearly imitation is an important force behind the creation of new designs. However, through the creation of brand names, designers can at least partially insulate themselves from competition with potential imitators. In this paper I consider the case in which the designer has well-defined property rights over his innova-*

tions."

Pesendorfer's no-imitation case is of interest, but so is allowing imitation because "knocking off" expensive designers is pervasive. Protecting intellectual property rights concerning fashion goods can be difficult, at least in the US. (Some European countries may have stronger protections.) Fashion innovations are ineligible for patent protection because they do not advance prior art in a non-obvious way. Something like reintroducing the color mauve in 2003, when it was popular in the 1980s but fell out of favor in the 1990s clearly does not meet that test. Likewise copyright protection cannot be afforded in the US to "useful articles", so it can only protect design elements that can be identified separately from and can exist independently of the utilitarian aspects of the article. Practically, that means that fashion designs in apparel (as opposed to accessories) are hard to copyright. Finally, although defending a fashion trademark against counterfeiting is relatively straightforward, defending trade dress against imitation is more difficult. (The Lanham Act differentiates between "trademarks", which are words, emblems, logos, or symbols used to identify goods and distinguish them from those sold by others and "trade dress", which refers to the product's overall image or appearance including shape, size, color, packaging, and marketing.)

To give a concrete example, Abercrombie & Fitch sued American Eagle Outfitters in 1998 for "intentional and systematic copying of its brand, images and business practices, including its merchandise, marketing and catalog" (Seiling, 1998). However, both the lower court and the Pennsylvania Sixth Circuit Court of Appeals sided with American Eagle Outfitters because the clothing designs for which Abercrombie & Fitch sought protection were functional as a matter of law and therefore not protectable under trade dress (Catalog Age, 2002). Abercrombie & Fitch then filed suit seeking just to prevent American Eagle Outfitters from using the number '22' on its clothing, arguing that it had a common law trademark on that number (Associated Press, 2003). That suit has yet to be resolved, but even if Abercrombie & Fitch wins, it would only affect a minor aspect of American Eagle Outfitters' alleged imitation.

Our model assumes there is a single innovative "fashion czar" that defines what is fashionable within the product space. This is an outcome Pesendorfer found to be among the plausible competitive equilibria. Essentially if all consumers believe that only the fashion czar is capable of creating fashion, then this will be the equilibrium outcome. The fashion czar's product is imitated by low-cost producers who are not strategic about their fashion innovation. The fashion czar might invest in costly activities that support innovation such as "cool hunting" (gathering intelligence about trend-setting consumers' preferences, see, e.g., Gladwell, 1997), "depth test" marketing preliminary designs with bellwether groups (e.g., Fisher and Rajaram, 2000), or advertising heavily to mold expectations about what will be "in" (Pastine & Pastine, 2002). The imitators follow the simpler, low-cost strategy of adapting their designs to conform to those of the fashion czar, whatever those designs are.

Note two differences with some articles in the literature. First, some models of fashion cycles are based on innovation and imitation by consumers who are conformists or non-conformists with regard to purchasing decisions (e.g., Matsuyama, 1992). Here it is producers who innovate or imitate. Second, portions of the literature assume that unit production costs are identical for "in" and "out" products since they are functionally identical; in our model it is allowable and perhaps more reasonable to think of the fashion czar as having higher production costs.

Thus we imagine a market populated by heterogeneous firms. One firm optimizes, at some nontrivial information processing cost; others follow heuristic strategies that are cheaper to implement. This structure is akin to that of Conlisk (1980), Sethi & Franke (1995), and Brock and Hommes (1997). The key point of these models is that information processing costs may trigger cycles (or even chaos) due to optimal (or boundedly rational) switching behavior. See also the discussion in Hommes (2006, section 7). Our model differs in that it is continuous time and the producers' decisions pertain to product design, specifically where they position their products in some product space.

3 The Model

State variable X represents the fashion leader/decision maker's position in some consumer product space, and Y represents the position of a competitor in that same space.

The competitor's corporate strategy is simply to imitate the market leader (X), so Y always chases X. Pesendorfer (1995) suggests that one can think of Y

not just as a single follower, but rather as a group of followers, which motivates the absence of strategic behavior on their part. Not only do they lack market research capability and other prerequisites to strategic behavior, but they may also each be small, making it hard to amortize the fixed costs of such strategic infrastructure. Hence, we model an optimal control problem, not a dynamic game.

The system dynamics for this model would be simply:

$$\dot{X} = u, \tag{1}$$

$$\dot{Y} = X - Y, \tag{2}$$

where u is the control variable. The product space is constrained by zero and one (neckties of infinite width make no sense), so that

$$0 \leq X \leq 1. \tag{3}$$

Since Y just follows X, expression (3) implies that Y is also effectively constrained to be between zero and one without needing to make this explicit.

The fashion leader's objective function balances the cost of fashion innovation against the benefits of being well-differentiated in product space from the low-cost imitator's product. For simplicity we assume that profits grow as the square of the distance between the innovator's and imitator's products. Note profits in this model do not depend on the absolute location of either firm in product space since fashion is not useful per se. Extensions in which consumers care not only about differentiation but also about the absolute position in product space could be an interesting topic for further research. The greater the rate of innovation, the more costly that innovation is. In particular, it is assumed here that the cost of innovation is linear in the rate of innovation. Hence, the fashion-setter's objective is:

$$\max \int_0^\infty e^{-rt} \left[\frac{1}{2} (Y - X)^2 - c\,|u| \right] dt. \tag{4}$$

and the decision maker seeks to optimize (4), subject to the system dynamics (1)-(2) and the state constraint (3).

In (2) the difference between X and Y could have been premultiplied by some constant $k > 0$ measuring the speed of convergence. However, by an appropriate time transformation it can be shown that increasing k has the same effect as jointly increasing the switching cost c and decreasing the discount rate r. Hence, nothing is lost by normalizing k equal to 1, which has the advantage of reducing the number of parameters.

4 Properties of Optimal Solutions

Before beginning detailed analysis, it is useful to make some observations about the nature of optimal solutions to this problem. Most are self-evident or require only a brief explanation, but collectively they delineate the space within which one must search for optimal solutions.

Proposition 1 *If it is optimal to exercise control, it is optimal to jump all the way to a boundary ($X = 0$ or $X = 1$).*

This follows from the cost of moving being linear in the distance moved, while the benefit is convex. An immediate implication is that once the fashion leader has reached a boundary, the leader will subsequently always be at one boundary or the other.

Proposition 2 *Once the fashion leader has reached a boundary, and hence will always be at a boundary, by symmetry the problem becomes a one-dimensional dynamic program whose state is D the distance the imitator is away from that boundary, with value function V^*which satisfies*

$$V^*(D) = \max\{D^2 dt + e^{-rdt}V^*(De^{-dt}), -c + e^{-rdt}V^*(1 - D)\}.$$

Proposition 3 *Assume the fashion leader is at a boundary ($X = 0$ or 1). Let D_0 be some distance such that when the imitator is that distance away, the fashion setter prefers jumping to the opposite boundary over staying in place. Then for all $D < D_0$, the fashion setter would also rather jump to the opposite boundary than stay in place.*

Proof. This is because the cost of moving is the same while the revenue is higher when $D < D_0$, i.e., $V(D, jump) > V(D_0, jump)$. On the other hand, clearly, $V(D, stay) < V(D_0, stay)$, while by definition $V(D_0, stay) < V(D_0, jump)$. ■

Corollary 4 *If it is ever optimal to jump away from a boundary, it is optimal to continue to jump forever.*

This follows because all jumps leave the fashion leader at the boundary, and eventually the shadow approaches that boundary arbitrarily close.

Corollary 5 *If it is optimal to jump more than once, it is optimal to jump forever.*

This follows from Proposition 1 (all jumps are to a boundary) and Corollary 4.

Corollary 6 *When the fashion leader is at a boundary, the optimal strategy is fully characterized by a single distance parameter D_0. If the imitator's distance from the boundary $D \leq D_0$, then the fashion leader should jump immediately. Otherwise, the fashion leader should wait until D decreases to D_0 and then jump.*

Corollary 7 *Only seven strategies are candidates for optimality:*

1) Never moving

2) Jumping once to $X = 0$

3) Jumping once to $X = 1$

4) Jumping forever, with the first jump to $X = 0$

5) Jumping forever, with the first jump to $X = 1$

6) Waiting for some time and then jumping forever, with the first jump to $X = 0$

7) Waiting for some time and then jumping forever, with the first jump to $X = 1$.

In the next sections we show that all seven strategies actually occur for some parameter values.

Proposition 8 *Strategy pair #2/#3 and quadruple #4/#5/#6/#7 are incompatible in the following sense. For any given set of parameters, if there exist* *initial conditions such that any of strategies #4, #5, #6 or #7 is optimal, then there do not exist initial conditions such that strategies #2 or #3 are optimal.*

To see this, note that if either Strategy #4, #5, #6 or #7 are optimal for some set of initial conditions, then there must exist a D_0 such that $V(D_0, jump) > V(D_0, stay)$. Since if the fashion leader were to jump only once, eventually D would become less than D_0, making it no longer optimal to stay.

As will be illustrated below, all other combinations of strategies can co-exist in the sense that there exist parameter values such that any one of those strategies can be optimal depending on the initial conditions. Analytic expression can be written fully characterizing most of the boundaries separating the regions where each of the alternative strategies is optimal. These boundaries are found by equating the value functions computed under each candidate optimal strategy. The discussion turns next to the computation of those value functions.

5 Solution structure for low unit cost

To begin suppose the market leader's product is initially at one of the boundaries of product space and, without losing generality, suppose it is at the lower end, $X = 0$, and $Y < 1/2$. (If initially $X = 0$ and $Y > 1/2$ then any innovation would be both costly and revenue-reducing, so clearly the fashion-setter should do nothing at least until $Y < 1/2$.) If the cost of making new designs is low, then when the imitator gets close enough to 0, the decision maker will move X from zero to one. Later, when the imitator comes sufficiently close to one, the trend-setter will jump from one back to zero. Since the control variable appears linearly in the optimization problem and is unbounded, these movements take the form of discrete jumps, yielding the solution structure depicted in Figure 1.

The next subsections investigate the properties of the cycle in Figure 1 and obtains an upper bound on the unit cost for which this solution structure is in fact optimal. The second subsection finds all points (X, Y) at which the decision maker is indifferent between jumping to zero and jumping to one. It turns out that all these points are situated on a curve, which in the optimal control literature is called a Skiba-curve. Skiba (1978), in a one state optimal control model, detected a threshold (the *Skiba point*) at which the decision maker is indifferent between converging to a positive steady state or converging to zero. Haunschmied et al. (2003) extended this analysis to a two state optimal control model so, as in the present paper, due to the extra dimension, the Skiba point becomes a Skiba curve. For other recent research concerning Skiba surfaces the reader is directed to Wagener (2005a, b) and Deissenberg et al. (2004).

5.1 Properties of the Cycle

On the cycle $X = 1$ or $X = 0$ (see Figure 1). Let Y_0 be the position of the imitator in the product space at which the decision maker is indifferent between staying at zero or jumping from zero to one, and, analogously, Y_1 is the imitator's position for which the market leader is indifferent between staying at one or jumping from one to zero. Defining T_1 to be the length of the time interval at which $X = 1$, and T_0 is the time interval length at which $X = 0$, it can be obtained from expression (2) that

$$X = 1,\ Y = 1 - (1 - Y_0)e^{-t} \quad \text{for} \quad 0 < t < T_1,$$

$$X = 0,\ Y = Y_1 e^{T_1 - t} \quad \text{for} \quad T_1 < t < T_1 + T_0.$$

Only the relative positions in the product space matter, which implies that $T_1 = T_0 = T$ and $Y_1 = 1 - Y_0$. Since Y has the same value at the end of the interval where $X = 1$, and at the beginning of the interval where $X = 0$, it holds that:

$$e^{-T} = Y_0 \left(1 + e^{-T}\right). \tag{5}$$

Next, we determine Y_0 by choosing that value of Y_0 that maximizes the objective.

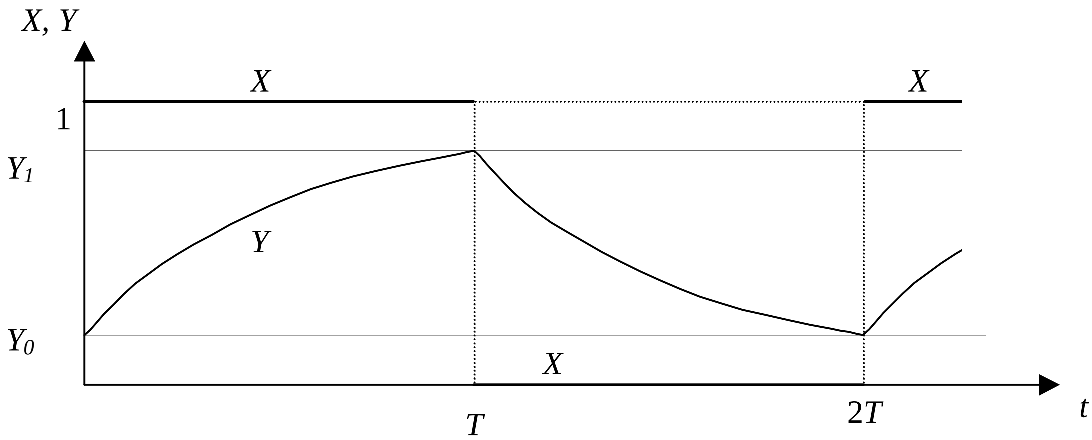

Figure 1: The optimal periodic solution structure.

Due to the one-to-one correspondence implied by (5), this also gives the cycle length $2T$. Evaluating the objective on the initial interval $[0, T]$ (starting with

$Y(0) = Y_0$ and $X(0) = 1$) gives:

$$\int_0^T e^{-rt} \left(\frac{1}{2}(1-Y)^2\right) dt - e^{-rT} c = (1-Y_0)^2 \frac{1-e^{-T(r+2)}}{2(r+2)} - e^{-rT} c.$$

For reasons of symmetry, excluding discounting, the objective has the same value on the second interval $[T, 2T]$ (starting with $Y(T) = 1 - Y_0$ and $X(T) = 0$) as on the first interval. Equality modulo discounting also holds for all consecutive intervals of time length T. Therefore, the objective value becomes:

$$\begin{aligned} V &= \sum_{n=0}^{\infty} e^{-nrT} \left((1-Y_0)^2 \frac{1-e^{-T(r+2)}}{2(r+2)} - e^{-rT} c \right) \\ &= \frac{1}{2(r+2)} \frac{(1-Y_0)^{r+2} - Y_0^{r+2}}{(1-Y_0)^r - Y_0^r} - \frac{Y_0^r}{(1-Y_0)^r - Y_0^r} c. \end{aligned} \tag{6}$$

The first order condition eventually leads to

$$c = \frac{1}{r(r+2)} \left(\frac{Y_0^{r+2}}{(1-Y_0)^{r-1}} - \frac{(1-Y_0)^{r+2}}{Y_0^{r-1}} + \frac{r - 2Y_0(r-1) - 6Y_0^2 + 4Y_0^3}{2} \right). \tag{7}$$

This equation implicitly determines Y_0 as a function of parameters r and c, as depicted in Figure 2. This figure shows that the market leader will not change the design very often (Y_0 is low) if the cost of changing the design, c, is large. The same holds for the relation between Y_0 and the discount rate, because a large discount rate implies that the decision maker is more influenced by the immediate costs of design change. Furthermore, the figure shows that for large discount rates, Y_0 depends heavily on the rate, while for smaller discount rates

Y_0 is insensitive to changes in the discount rate. In fact, in this figure the curve for $r = 0.001$ could not be distinguished from that for $r = 0.01$.

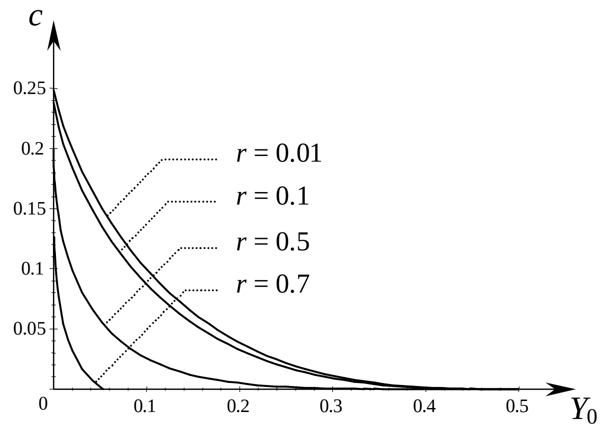

Figure 2: Optimal Switching Threshold Y_0 as a function of c and r.

Clearly it holds that $Y_0 \leq 1/2$ for all $c > 0$. For very small values of c the following result is established.

Proposition 9 *i) When changing the design is costless, i.e.* $c = 0$, *the threshold* Y_0 *equals* $1/2$. *ii) However, for very low values of* c, Y_0 *approaches* $1/2$ *only for* $r \leq 0.5$, *i.e.,*

$$\begin{aligned} \lim_{c \to 0} Y_0(c) &= 0.5 \quad \textit{for} \quad r \leq 0.5, \\ \lim_{c \to 0} Y_0(c) &< 0.5 \quad \textit{for} \quad r > 0.5. \end{aligned}$$

Proof. From (7) it can be obtained that $Y_0 = 1/2$ for $c = 0$. Due to this same expression it can also be shown that for $Y_0 = 1/2$ it holds that $\frac{dc}{dY_0} = 0$, $\frac{d^2c}{dY_0^2} = 0$, and $\frac{d^3c}{dY_0^3} = 16\left(r - \frac{1}{2}\right)$. Thus, if $r > 0.5$ the c-curve in Figure 2 lies below the

Y_0-axis for values of Y_0 less than but close to 1/2. Since $c = 1/2\,(r+2) > 0$ for $Y_0 = 0$, this proves that $\lim_{c\to 0} Y_0\,(c) < 0.5$ for $r > 0.5$. ■

Result i) is intuitively plausible since – if changing the design is costless – the market leader's strategy is simply always to maximize the distance between X and Y, which leads to chattering around 1/2. It is clear that $Y_0 = 0.5$ implies that the cycle length equals zero. Hence, the proposition implies that

$$\begin{aligned} \lim_{c\to 0} T\,(c) &= 0 \quad \text{for} \quad r \le 0.5, \\ \lim_{c\to 0} T\,(c) &> 0 \quad \text{for} \quad r > 0.5. \end{aligned}$$

Figure 2 illustrates result ii) for $r = 0.7$, where $\lim_{c\to 0} Y_0\,(c) = 0.053$. Apparently, for very large discount rates, immediate switching is not optimal even when the cost of design change is almost zero.

The following result gives an upper bound on the design change cost above which it is not optimal to have a solution structure as depicted in Figure 1.

Proposition 10 *Exactly for*

$$c > \frac{1}{2\,(r+2)} \tag{8}$$

Y_0 *does not exist and for* $c = \frac{1}{2(r+2)}$ *it holds that* $Y_0 = 0$.

Proof. By letting $Y_0 \to 0$ in (7), we obtain that $c = \frac{1}{2(r+2)}$ and that a non-negative value for Y_0 does not exist for $c > \frac{1}{2(r+2)}$.

■

For $c \geq \frac{1}{2(r+2)}$ changing the design is so expensive that the same design is kept forever when the decision maker finds itself at one of the boundaries of the product space ($X = 0$ or $X = 1$). One simply stays there while the imitator's product becomes more and more similar.

In the example with $r = 0.7$, $r = 0.5$, $r = 0.1$, and $r = 0.01$, the maximum value of c for which periodic design change is optimal is $c = 0.1852$, $c = 0.2$, $c = 0.2381$, and $c = 0.2488$, respectively. If the discount rate is large the decision maker is reluctant to incur immediate costs. Therefore, the upper bound on c goes down as r increases.

Proposition 10 does not imply that designs will never be changed if $c > \frac{1}{2(r+2)}$. If initially the market leader's product is in the interior of the product space ($0 < X < 1$), an initial design change to one of the boundaries may still be optimal. This possibility will be explored in Section 6.

5.2 Skiba Curve

This subsection finds a curve in the (X, Y) −plane on which the fashion trendsetter is indifferent between choosing the design $X = 0$ or $X = 1$. By definition the outside points of this curve are $(0, Y_0)$ and $(1, 1 - Y_0)$. It is also clear that $(1/2, 1/2)$ must be part of the Skiba curve.

To find an analytical expression for the whole curve we consider an arbitrary point $\left(\bar{X}, \bar{Y}\right)$, with $Y_0 < \bar{Y} < 1 - Y_0$. Then we evaluate the objective values for an immediate upward jump to 1 and an immediate downward jump to 0. Those points $\left(\bar{X}, \bar{Y}\right)$, for which both values are equal, belong to the Skiba curve.

First we consider an immediate upward jump from $\bar{X}$ to 1. On the initial time interval, say $0 < t < \bar{t}$, Y increases from $\bar{Y}$ to $1 - Y_0$. Then, from that moment on, the solution structure depicted in Figure 1 applies, the objective value of which is given by V (see (6)). This implies that the value of the objective, $V^{up}\left(\bar{X}, \bar{Y}\right)$, corresponding to "jumping upward" is

$$V^{up}\left(\bar{X}, \bar{Y}\right) = -c\left(1 - \bar{X}\right) + \int_0^{\bar{t}} e^{-rt}\left(\frac{1}{2}(1 - Y)^2\right) dt - e^{-r\bar{t}}c + e^{-r\bar{t}}V$$

which can be rewritten as

$$V^{up}\left(\bar{X}, \bar{Y}\right) = -c\left(1 - \bar{X}\right) + \left(\frac{Y_0}{1 - \bar{Y}}\right)^r (V - c) + \frac{1}{2}(1 - \bar{Y})^2 \frac{1 - \left(\frac{Y_0}{1 - \bar{Y}}\right)^{r+2}}{r + 2}. \quad (9)$$

Analogously,

$$V^{down}\left(\bar{X}, \bar{Y}\right) = -c\bar{X} + \left(\frac{Y_0}{\bar{Y}}\right)^r (V - c) + \frac{1}{2}\bar{Y}^2 \frac{1 - \left(\frac{Y_0}{\bar{Y}}\right)^{r+2}}{r + 2}. \quad (10)$$

To obtain the Skiba curve, we equate the objective values of jumping upwards and downward. The appendix shows this gives:

$$\bar{X} = -\frac{1}{4(r+2)c}\left[(1 - \bar{Y})^2\left(1 - \left(\frac{Y_0}{1 - \bar{Y}}\right)^{r+2}\right) - \bar{Y}^2\left(1 - \left(\frac{Y_0}{\bar{Y}}\right)^{r+2}\right)\right] \quad (11)$$
$$+\frac{1}{2}\left(\left(\frac{1}{\bar{Y}}\right)^r - \left(\frac{1}{1 - \bar{Y}}\right)^r\right)\frac{Y_0^r}{(1 - Y_0)^r - Y_0^r}\left(\frac{(1 - Y_0)^{r+2} - Y_0^{r+2}}{2(r+2)c} - (1 - Y_0)^r\right) + \frac{1}{2}.$$

This is the Skiba curve. Although there is no explicit expression for Y_0, it is implicitly given by (7). For the parameter values $r = 0.1$ and $c = 0.1$, the

Skiba curve is depicted in Figure 3.

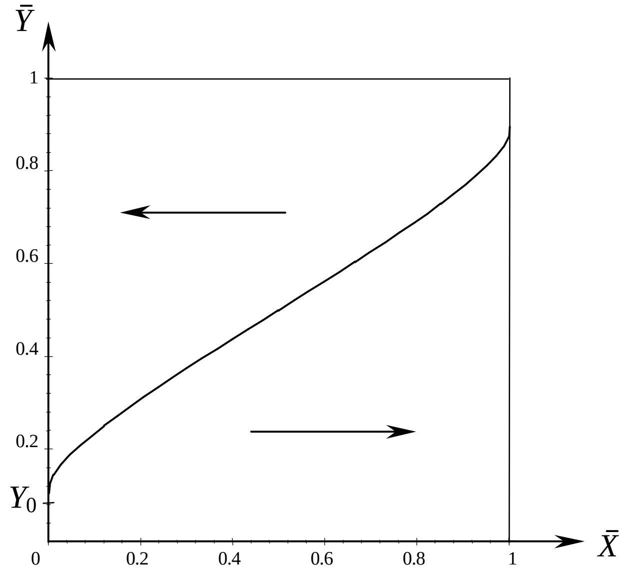

Figure 3: Skiba curve for $r = 0.1$ and $c = 0.1$.

6 Solution structure for large unit cost

From Proposition 10 the cyclical solution structure of Figure 1 cannot be optimal when the cost of changing the design is large, i.e. when $c > \frac{1}{2(r+2)}$. This implies that then it can never be optimal to change the design in such a way that X jumps from a level lower than Y to a level that is higher than Y, or vice versa. After excluding such design changes, three candidate policies are left, namely:

- starting from a situation where $\bar{X} > \bar{Y}$, jump up to $\bar{X} = 1$ but make no subsequent design changes in the future. The value of the objective that results from this policy is

$$V^{1up}\left(\bar{X},\bar{Y}\right)=-c\left(1-\bar{X}\right)+\frac{1}{2}(1-\bar{Y})^2\frac{1}{r+2}.$$

- starting with $\bar{X}<\bar{Y}$, jump down to $\bar{X}=0$ but make no subsequent design changes in the future. Then the value of the objective is

$$V^{1down}\left(\bar{X},\bar{Y}\right)=-c\bar{X}+\frac{1}{2}\bar{Y}^2\frac{1}{r+2}.$$

- stay at $\bar{X}$, which gives

$$V^{stay}\left(\bar{X},\bar{Y}\right)=\frac{1}{2}\left(\bar{Y}-\bar{X}\right)^2\frac{1}{r+2}.$$

The fashion trend-setter is indifferent between making only an initial design change leading to an upward jump of X and making no change at all, when $V^{1up}\left(\bar{X},\bar{Y}\right)$ equals $V^{stay}\left(\bar{X},\bar{Y}\right)$. This leads to the following Skiba-curve:

$$\bar{Y}=\frac{1}{2}\bar{X}-\left(c\left(r+2\right)-\frac{1}{2}\right).$$

This is an upward sloping line, which lies below the 45^0 line, because the policy with the initial upward jump can only occur if $\bar{X}>\bar{Y}$. The Skiba curve only occurs in the relevant region if $\bar{X}<1$ for $\bar{Y}=0$, which leads to the conclusion that this Skiba curve only exists in case $c<\frac{1}{r+2}$.

Being indifferent between an initial downward jump or refraining from any design change, thus equating $V^{1down}\left(\bar{X},\bar{Y}\right)$ and $V^{stay}\left(\bar{X},\bar{Y}\right)$, gives the following Skiba-curve:

$$\bar{Y} = \frac{1}{2}\bar{X} + c\,(r+2)\,.$$

This is an upward sloping line above the 45^0 line. This curve is only relevant if $Y < 1$ for $X = 0$, which again gives $c < \frac{1}{r+2}$.

The results are summarized in the following proposition.

Proposition 11 *(a) Consider the c-region* $\frac{1}{2(r+2)} < c < \frac{1}{r+2}$*. Then the optimal policy is*

- $\bar{X} < 2\bar{Y} - 2c\,(r+2)$: *make an initial design change to* $X = 0$*. Then refrain from doing any changes afterwards.*
- $2\bar{Y} - 2c\,(r+2) < \bar{X} < 2\bar{Y} + 2c\,(r+2) - 1$: *make no design changes.*
- $\bar{X} > 2\bar{Y} + 2c\,(r+2) - 1$: *make an initial design change to* $X = 1$*. Afterwards make no design changes.*

(b) Consider the c-region $c > \frac{1}{r+2}$*. Then the optimal policy is to make no design changes at all.*

The next figure illustrates case (a). After choosing $r = 0.1$ we must have $0.2381 < c < 0.4762$ to be in the relevant parameter region. The figure is drawn for $c = 0.3$.

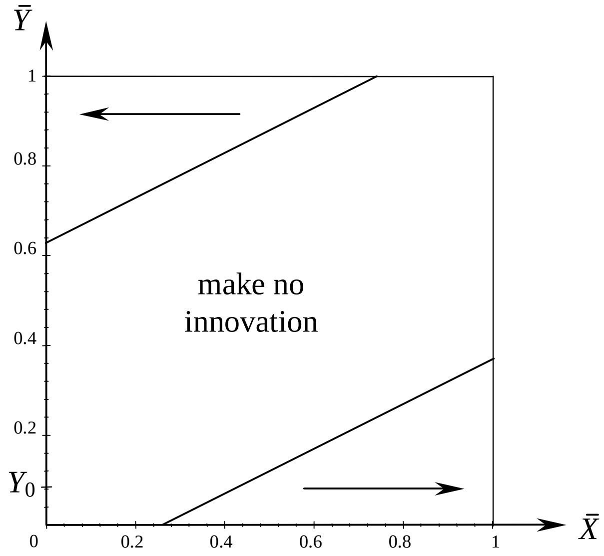

Figure 4: Skiba curves for $r = 0.1$ and $c = 0.3$.

7 Intermediate c

In the hairline case where $c = \frac{1}{2(r+2)}$, it holds that $Y_0 = 0$, and the Skiba curve (11), that separates the two policies of initially jumping up or down followed by periodic design changes as depicted in Figure 1, becomes $\bar{X} = \bar{Y}$. This hairline case is presented in Figure 5.

Note that the Skiba curve $\bar{X} = \bar{Y}$, denoted by (11), is not really relevant since here the unit cost is too large for periodic design changes to be optimal. Instead, the fashion leader refrains from doing any design changes when $X = Y$.

This raises the question whether for values of the unit cost c a little bit below $\frac{1}{2(r+2)}$ (implying that Y_0 is close to zero) making no changes at all could be better than jumping to the cycle for some initial values of $\bar{X}$ and $\bar{Y}$. After

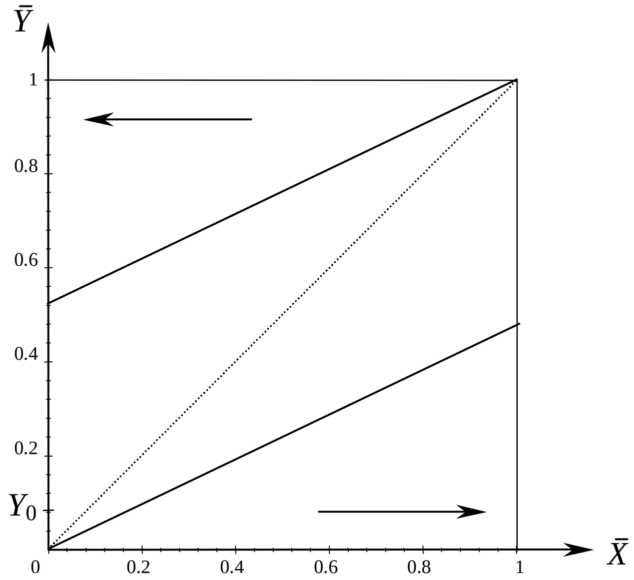

Figure 5: Hairline case $c = \frac{1}{2(r+2)}$ for $r = 0.1$.

all, in Section 3 we only compared the objective values resulting from periodic design changes after initial upward and downward jumps, without checking their absolute values. In case these objective values are negative, then a policy of making no design changes at all would be preferable.

The point where it is least attractive to jump to the cycle is $(1/2, 1/2)$. This implies that if a policy of making no new designs would be optimal anywhere, it would certainly be optimal for $\bar{X} = 0.5$ and $\bar{Y} = 0.5$. In Figure 6 we plot the difference in the objective values for jumping to the cycle and staying: $V^{up} - V^{stay}$, where $Y_0 = 0.01$. This figure shows that indeed for this Y_0 staying at $\bar{X} = \bar{Y} = 0.5$ is optimal for an interval that includes r-values between 0.2 and 0.4, while this does not occur for $r = 0.1$. The thin line proves that the

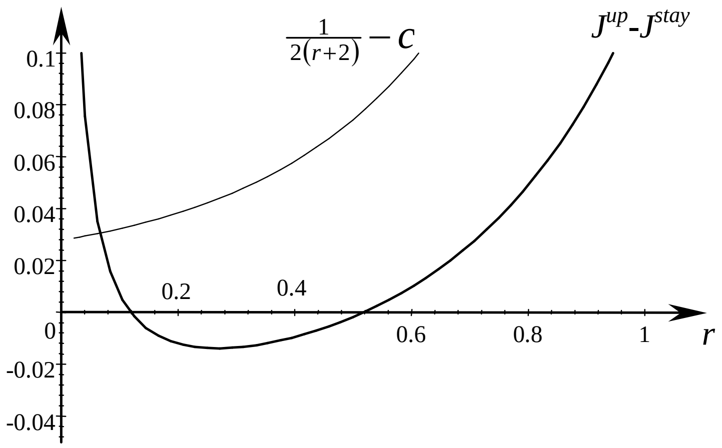

Figure 6: Difference in objective values for jumping to the cycle and staying, when $X = Y = 0.5$ and $Y_0 = 0.1$.

parameters are still in the relevant region, i.e., $\frac{1}{2(r+2)} - c > 0$.

To determine the size of the region where making no design changes is optimal we determine two other Skiba curves. The first includes those points where jumping up followed by periodic design changes gives the same objective value as making no changes. The second is its mirror image. Figure 7 gives a numerical example in which these curves occur.

In Figure 7 two other regions occur, where it is in fact optimal to have an initial period of making no design change followed by a jump to the cycle. Here the idea is that when the first jump is upward (downward) it is only optimal to make this design change after the imitator has moved in the downward (upward) direction. Only then does jumping create a large enough difference between the two designs for the trend-setter to make enough profits to offset the cost of

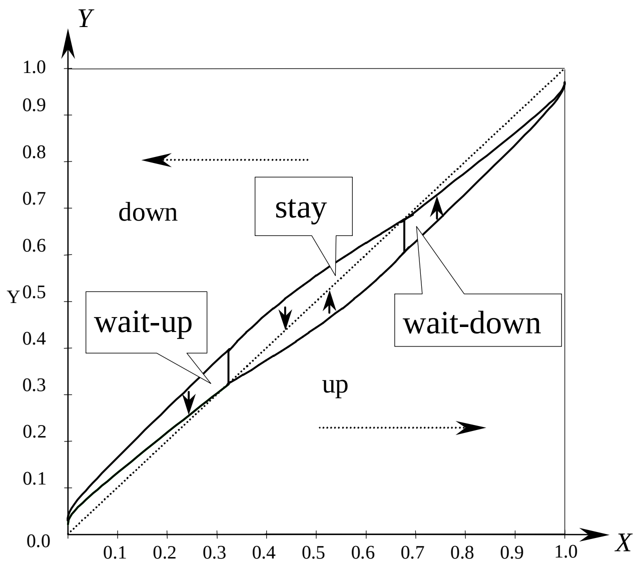

Figure 7: Eight Skiba curves for the case that $Y_0 = 0.01$, $r = 0.2$, and $c = 0.189$.

jumping. Within the region "wait-up" ("wait-down") the upward (downward) movement takes place at the moment that Y reaches the lower (upper) boundary of this region. Appendix B provides details on the computation of these Skiba curves.

8 Conclusions

Fashions change and even cycle. A variety of models have been advanced to explain this. Most have focused on consumers' tastes and behavior. Pesendorfer (1995) introduced perhaps the best-known model that explicitly considers optimal dynamic strategies for suppliers of fashion goods. It focuses on the monopoly case. However, it is not always easy to protect intellectual property claims concerning fashion (as opposed to technical) innovations, so competition is the norm. Indeed, two basic elements of the fashion industry are constant innovation by high-end designer labels and low-cost "knock-off" brands striving to offer products that look like those of the trend-setters.

Here we introduce a model whose solution describes how a high-end trend-setter ought to respond to competition from one or more low-cost imitators when consumers value one design over another only to the extent that it is distinguishable from the low-cost products. That is, consumers have no intrinsic preference for one design over another.

The optimal strategy depends on the parameter values and, in many cases, there is state-dependency. When the costs of innovation are low enough, the trend-setter should innovate indefinitely even though the product space is bounded. I.e., it is optimal to create fashion cycles. Because neither side of product space is intrinsically better or worse than the other, the optimal initial direction of innovation depends on the innovator and imitator's initial positions in product space. In particular, a two-dimensional Skiba curve separates regions in state space within which it is optimal for the innovator to begin by moving "left" or "right" in product space.

Not surprisingly, when the costs of innovation are high enough, the optimal strategy involves no innovation. The trend-setter simply milks the profits available because of its initial product differentiation, but is eventually overtaken by imitators who by virtue of their lower cost structure take over the entire market. Sometimes it is optimal for the trend-setter to extend this transient leadership with a single innovation.

For intermediate costs there are initial positions such that the innovator is indifferent between embarking on a long-term strategy of innovating indefinitely and one of these alternate strategies. Again, the collection of these indifference points constitute Skiba curves. Indeed, there are places in state space where several different Skiba curves meet. Furthermore, in most instances it is possible to write explicit analytical expressions characterizing these two-dimensional Skiba curves and to explore how they depend on various model parameters.

Given our one-dimensional abstraction of product space, some aspects of the optimal solution are artificial, such as the idea that fashion bounces forever between the same two points. In reality, there may be simple cycling in one dimensional projections of a higher dimensional product space (e.g., a color can be in, then out of fashion, then back in again), the true product space is of much higher-dimension. Translating the insights of our stylized model into a richer image of the variety possible in fashion goods, we would obtain the following prediction. The fashion leader should make bold moves (equivalent to jumping from one boundary to another) in directions that maximally differentiate it from the followers, while still remaining within the realm of what is "feasible" in customers' minds. That is in fact not a bad characterization of what is done at fashion shows.

9 Appendix A: Derivation of the Skiba Curve of Section 3

From (9) and (10) it is obtained that equating the objective values of jumping upwards and downward gives:

$$\begin{aligned}
& -c\bar{X} + \left(\frac{Y_0}{\bar{Y}}\right)^r (V-c) + \frac{1}{2}\bar{Y}^2 \frac{1-\left(\frac{Y_0}{\bar{Y}}\right)^{r+2}}{r+2} \\
= \quad & -c\left(1-\bar{X}\right) + \left(\frac{Y_0}{1-\bar{Y}}\right)^r (V-c) + \frac{1}{2}(1-\bar{Y})^2 \frac{1-\left(\frac{Y_0}{1-\bar{Y}}\right)^{r+2}}{r+2} \\
\Leftrightarrow \quad & c\left(1-\bar{X}\right) - c\bar{X} + \left(\left(\frac{Y_0}{\bar{Y}}\right)^r - \left(\frac{Y_0}{1-\bar{Y}}\right)^r\right)(V-c) \\
= \quad & \frac{1}{2\,(r+2)}\left[(1-\bar{Y})^2\left(1-\left(\frac{Y_0}{1-\bar{Y}}\right)^{r+2}\right) - \bar{Y}^2\left(1-\left(\frac{Y_0}{\bar{Y}}\right)^{r+2}\right)\right] \\
\Leftrightarrow \quad & c\left(1-2\bar{X}\right) + \left(\left(\frac{1}{\bar{Y}}\right)^r - \left(\frac{1}{1-\bar{Y}}\right)^r\right)(V-c)\,Y_0^r \\
= \quad & \frac{1}{2\,(r+2)}\left[(1-\bar{Y})^2\left(1-\left(\frac{Y_0}{1-\bar{Y}}\right)^{r+2}\right) - \bar{Y}^2\left(1-\left(\frac{Y_0}{\bar{Y}}\right)^{r+2}\right)\right]
\end{aligned}$$

Plugging in V from (6) results in:

$$
\begin{aligned}
& c\left(1-2\bar{X}\right)+\left(\left(\frac{1}{\bar{Y}}\right)^{r}-\left(\frac{1}{1-\bar{Y}}\right)^{r}\right)(V-c)\,Y_0^r \\
= \quad & \frac{1}{2\,(r+2)}\left[(1-\bar{Y})^2\left(1-\left(\frac{Y_0}{1-\bar{Y}}\right)^{r+2}\right)-\bar{Y}^2\left(1-\left(\frac{Y_0}{\bar{Y}}\right)^{r+2}\right)\right] \\
\Longleftrightarrow \quad & c\left(1-2\bar{X}\right) \\
& +\left(\left(\frac{1}{\bar{Y}}\right)^{r}-\left(\frac{1}{1-\bar{Y}}\right)^{r}\right)\left(\frac{(1-Y_0)^{r+2}-Y_0^{r+2}}{(1-Y_0)^r-Y_0^r}\frac{1}{2\,(r+2)}-\frac{Y_0^r}{(1-Y_0)^r-Y_0^r}c-c\right)Y_0^r \\
= \quad & \frac{1}{2\,(r+2)}\left[(1-\bar{Y})^2\left(1-\left(\frac{Y_0}{1-\bar{Y}}\right)^{r+2}\right)-\bar{Y}^2\left(1-\left(\frac{Y_0}{\bar{Y}}\right)^{r+2}\right)\right] \\
\Longleftrightarrow \quad & c\left(1-2\bar{X}\right) \\
& +\left(\left(\frac{1}{\bar{Y}}\right)^{r}-\left(\frac{1}{1-\bar{Y}}\right)^{r}\right)\frac{1}{(1-Y_0)^r-Y_0^r}\left(\frac{(1-Y_0)^{r+2}-Y_0^{r+2}}{2\,(r+2)}-(1-Y_0)^r\,c\right)Y_0^r \\
= \quad & \frac{1}{2\,(r+2)}\left[(1-\bar{Y})^2\left(1-\left(\frac{Y_0}{1-\bar{Y}}\right)^{r+2}\right)-\bar{Y}^2\left(1-\left(\frac{Y_0}{\bar{Y}}\right)^{r+2}\right)\right] \\
\Longleftrightarrow \quad & \bar{X}=-\frac{1}{4\,(r+2)\,c}\left[(1-\bar{Y})^2\left(1-\left(\frac{Y_0}{1-\bar{Y}}\right)^{r+2}\right)-\bar{Y}^2\left(1-\left(\frac{Y_0}{\bar{Y}}\right)^{r+2}\right)\right] \\
& +\frac{1}{2}\left(\left(\frac{1}{\bar{Y}}\right)^{r}-\left(\frac{1}{1-\bar{Y}}\right)^{r}\right)\frac{Y_0^r}{(1-Y_0)^r-Y_0^r}\left(\frac{(1-Y_0)^{r+2}-Y_0^{r+2}}{2\,(r+2)\,c}-(1-Y_0)^r\right)+\frac{1}{2},
\end{aligned}
$$

which is the Skiba curve given by expression (11). QED

10 Appendix B: Technical Details of Section 5

We now discuss some technical details that relate to Figure 7. Ignoring for the moment the regions "wait-up" and "wait-down", we arrive at Figure 8.

Note that at point A the curve $V^{stay} = V^{up}$ crosses the 45° line $X = Y$. This means that to the left of point A, the curve $V^{stay} = V^{up}$ is not relevant anymore. Consider e.g. point B. Here $V^{stay} = V^{up}$ but both policies "stay

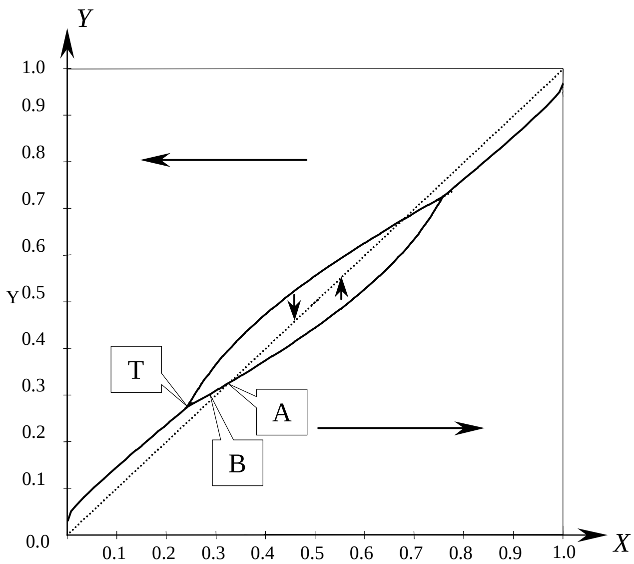

Figure 8: Figure 7 without regions "wait-up" and "wait-down".

forever" and "jump up and follow cycle" are not optimal anymore. The reason is that from B onwards, staying at the current value of X one immediately enters the region where "jumping up" is better than "staying". This implies that a jump taking place at any point in time after leaving B is better than staying forever or jumping immediately. This means that to the left of A and around the "naive triple point" T another policy has to be considered, namely "wait until Y has reached $\tilde{Y}$ and then jump up". The value of this policy is

$$V^{wait-up} = \int_0^{\tilde{t}} e^{-rt} \frac{1}{2} (X - Y)^2 \, dt + e^{-r\tilde{t}} V^{up} \left(X, \tilde{Y} \right), \tag{12}$$

where $\tilde{t}$ is given by the time that Y reaches $\tilde{Y}$, i.e.,

$$\tilde{t} = \ln\left(\frac{Y^{start} - X}{\tilde{Y} - X}\right) \tag{13}$$

Since $Y = X - \left(X - Y^{start}\right) e^{-t}$, the integral in (12) equals

$$\int_0^{\tilde{t}} e^{-rt}\frac{1}{2}\left(X - Y\right)^2 dt = \frac{\left(X - Y^{start}\right)^2}{2\left(r+2\right)}\left[1 - \left(\frac{\tilde{Y} - X}{Y^{start} - X}\right)^{r+2}\right]$$

and (12) becomes

$$V^{wait-up} = \frac{\left(X - Y^{start}\right)^2}{2\left(r+2\right)} + \frac{1}{\left(Y^{start} - X\right)^r}\left[-\frac{\left(\tilde{Y} - X\right)^{r+2}}{2\left(r+2\right)} + \left(\tilde{Y} - X\right)^r V^{up}\left(X, \tilde{Y}\right)\right]. \tag{14}$$

Since the term in brackets (depending on $\tilde{Y}$) does not depend on Y^{start}, the maximization of $V^{wait-up}$ w.r.t. $\tilde{Y}$ does not depend on Y^{start}. Clearly this value $\tilde{Y}$ will be below the A-B-T line in Figure 8. At point T, i.e., for $Y = 0.27431$ and $X = 0.242\,17$ in Figure 8 we numerically obtain $\tilde{Y} = 0.2475$ which means that $\tilde{Y}$ is closer to the 45° line than to T.

It should be noted that the difference $V^{wait-up} - V^{up} > 0$ can be interpreted as the option value of waiting.

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