Optimal Timing of Technology Adoption

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Abstract

In a dynamic programming framework, this paper investigates the optimal timing of technology adoption by a competitive firm when technology choice is *irreversible* and the firm faces a stochastic innovation process with uncertainties about *both* the speed of the arrival and the degree of improvement of new technologies. A numerical example illustrates how the optimal timing decision is affected by changes in parameter values reflecting market conditions, the firm's initial technological attributes, and the characteristics of the stochastic innovation process. Some of these effects turn out to be in sharp contrast to common intuition. Contrasting the optimal decision rule derived here with the rule obtained under the net present value shows that the former implies a slower pace of adoption than implied by the latter. The optimal decision rule is generalized for the case of multiple technology switches and it is shown that for all the switching decisions *except the last one*, the optimal rule satisfies the net present value criterion.

Key words: Innovation adoption; Technological uncertainties; Optimal timing; Investment irreversibility; Waiting option value
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1 Introduction

A hallmark of the evolution of modern civilization has been an unceasing flow of technological innovations adoptable in industry, agriculture, services, or other branches of economic activity. Despite this abundance, however, adoption of new technologies appears to have been a slow and incremental process. Over any given period of time only a tiny fraction of available innovations have been actually adopted for mass production. Furthermore, adoption of new technologies with radical superiority to the prevailing state of the art seems to have taken place with very long delays. The history of technological advances in both industry and agriculture attests to these features of innovation adoption[1]. In the automobile industry, for instance, a notable example is the significant increase in the fuel efficiency of passengers vehicles brought about since 1970s by successive, and as yet only partial, adoptions of technical innovations in the areas of microelectronics, aerodynamics, and material substitution[2]. In agriculture, a classic example is the notable increase in productivity over time resulting from successive, and still continuing, adoptions of new mechanical and biochemical technologies that have each improved the grains yield incrementally. In both cases, commercial applications of new technologies have followed with significant delays. An example closer to present time is the delay in switching from the current fossil-fuel based energy technologies to the more efficient and less-polluting alternative technologies.

A natural question then is: What explains the apparently cautious approach of firms to technology adoption? Clearly, the answer lies chiefly in uncertainties of various kinds facing the firms. In deciding whether or not, and when, to adopt a new technology a firm is naturally concerned about uncertainties regarding future market conditions such as consumers' response to the new technology product, competition from rival producers, the cost of initial investment in the new technology, and costs of borrowing capital, hiring labor, and using other inputs. But importantly, it is also concerned about uncertainties surrounding the very process of technical innovation, which is often outside the firm's control.

The importance of the technological uncertainties becomes more evident once it is noted that the firm's decision about how soon to adopt innovations depends on how fast and by how much technology will advance over time. However, the process of technical innovation is inherently a stochastic one, so that in general there is not only uncertainty about the *speed* with which new technologies become available for adoption but also about the *extent* of *efficiency gains* of

[1]For historical accounts of the slow pace of adoption of technology innovations see, for example, Mansfield [20], Rosenberg [30], [31], Rosenberg and Birdzell [32], Mokyr [21], and Kindelberger [18].

[2]It is estimated that still a very large fuel efficiency, about 75 miles per gallon, can easily be gained by a slightly further use of technical innovations already on the shelf in vehicles of relatively conventional design. See Altshuler *et al* [1].

new technologies relative to the current state of the art. Furthermore, where technological change is rapid there is very little chance of fully recovering the cost of capital invested in any chosen new technology, so that the technology choice becomes largely an irreversible one. Choice of personal computer technology, whether in the area of software or hardware, is a prime example of innovation adoption under the conditions of technological uncertainty and irreversibility. Under such conditions, the technology adopter should weigh two types of costs against each other: on the one hand, the cost of making a mistake by adopting too soon (as the sunk cost cannot be recovered to be reinvested should a more efficient technology become available later on) and, on the other hand, the opportunity cost of waiting in anticipation of better future technologies (as potential payoffs will be foregone during the waiting period).

Thus, as will be shown formally here, even when there are no uncertainties about future market conditions, still the decision about technology adoption will be greatly influenced by combined considerations of irreversibility and *technological* uncertainties surrounding both the speed of arrival and the extent of efficiency improvements of new technologies. How these factors precisely affect the optimal timing of innovation adoption is the central question we investigate in this paper. The voluminous literature on technology adoption has devoted relatively little attention to the role of technological uncertainties and has mostly concentrated on the effects of uncertainties about market conditions[3]. Furthermore, the previous related works (see, e.g., Kamien and Schwartz [17], Dasgupta and Stiglitz [8], [9], and Jensen [15]) have typically relied on the standard net present value approach and have either considered only the case where a new technology with known efficiency characteristics arrives at an unexpected date, or assumed that the efficiency of technology improves deterministically, or ignored the irreversible nature of sunk costs. They also have neglected the value of option to postpone the adoption decision. This is not surprising since it is relatively recently that through the pioneering works of Baldwin [3], McDonald and Siegel [19], Bertola [4], Pindyck [22], and especially Pindyck [23], [24], [25], Dixit [10], [11], [12], and Dixit and Pindyck [13] an appropriate framework for the analysis of optimal investment decision under uncertainty and irreversibility has emerged.

Still, to our knowledge, there are very few studies which have focused on the specific question of the effect of technological uncertainty and irreversibility on innovation adoption. Choi [7] considers a two-period model to explore the implications of network externalities for consumers' sequential and irreversible technology choice when the technologies stochastically evolve over time. Stenbacka and Tombak [33] use a duopoly-game model of timing adoption to analyze

[3]In fact, some of the seminal works on the timing of adoption of new technologies, e.g. Reinganum [28], Quirmbach [27], and Fudenberg and Tirole [14], have abstracted from uncertainty. Some others, e.g. Jensen [16] and Bhattacharya *et al* [5], while allowing for uncertainty about market conditions, have analyzed the problem in a static framework. For a general survey of the models dealing with the firm's decision about timing of new technology adoption, see Bridges *et al* [6]. Also, see the survey in Reinganum [29].

the effect of uncertainty in the time lag between adoption of the new technology and its successful implementation. Purvis *et al* [26] develop an *ex ante* simulating model to quantify the deterring effects of irreversibility and uncertainties about investment cost, production, and environmental regulation on Texas dairy producer's incentive to switch from the conventional open lot to free-stall dairy technology. The closest work to ours is the insightful paper by Balcer and Lippman [2] which addresses a similar question, but differs from the present paper in several respects: (i) Rather than adopting a dynamic programming approach, it uses a model in which innovation potential, assumed to be integer-valued, changes according to a discrete time semi-Markov process. As such, it does not explicitly derive the option value of waiting; (ii) It assumes the profit function to be linear in firm's technology level and characterizes innovations by cost reductions; (iii) It does not therefore allow for uncertainty about the profitability of new technologies, and (iv) It is confined to the analysis of a single-switching case and does not consider the case of multiple-switchings. While the present work improves over the Balcer and Lippman's paper in these respects, theirs has the advantage that it also allows for the role of learning in development of technology by incorporating the time lapsed since the last innovation.

The remainder of the paper is organized as follows. In Section 2, we present the basic model of a competitive firm which faces an exogenous stochastic innovation process and is allowed to switch to a new technology only once. Using Dixit and Pindyck's [13] framework, we derive the firm's optimal decision rule for timing of technology switching when there are both uncertainties about the arrival and the degree of improvements of new technologies. This is followed by a numerical example illustrating the comparative static effects of changes in values of various parameters reflecting market conditions, firm's initial technological attributes, and the characteristics of the stochastic innovation process. Some of these effects defy common intuition, and for which we provide economic explanation. In Section 3, we contrast the optimal decision rule derived in Section 2 with that arising from the net present value approach. Section 4 generalizes the basic analysis to the case where multiple technology switches are allowed. Concluding remarks are given in Section 5.

2 Single Switching Case

2.1 Basic Model

To abstract from uncertainties about market conditions and to focus on the effect of technological uncertainty on timing of adoption, we consider a perfectly competitive firm which produces a homogeneous good according to the simple production function[4]

$$h(v, \theta) = \theta v^a, \tag{1}$$

[4]Time subscripts are suppressed when no confusion arises.

where v is a variable input, a $(0 < a < 1)$ is the constant output elasticity, and θ is a technology-efficiency parameter whose value is determined stochastically (see below). Let p be the fixed price of output and w the fixed unit cost of a variable input.

We analyze a dynamic model with an infinite planning horizon. At $t = 0$, the firm produces with a technology designated by $\theta = \theta_0$. As time passes new technologies become available, and the firm has the opportunity to adopt a new technology. We assume that the process of technological evolution (innovation supply) is exogenous to the firm[5]. Technologies become more and more efficient over time, and the more efficient a technology the larger the associated parameter θ. However, the precise development of the efficiency level is a stochastic process in that whenever a new technology becomes available θ increases, but neither the precise arrival date of a new technology nor the associated increase in θ is known beforehand.

With this background, it is assumed that the parameter θ follows a jump process such that

$$d\theta = dq, \quad \theta(0) = \theta_0, \tag{2}$$

where

$$dq = \begin{cases} u \text{ with probability } \lambda dt, \\ 0 \text{ with probability } 1 - \lambda dt. \end{cases}$$

As already mentioned, the size of the jump is uncertain. We assume that u is uniformly distributed over the interval $(0, \overline{u})$ so that the expected value of u is $\frac{1}{2}\overline{u}$.[6]

We consider a risk-neutral firm which discounts the stream of future profits at a constant rate, r. At the moment that the firm adopts a new technology it incurs a sunk cost investment, I, which is assumed to remain constant. Along with the assumption of process innovation, we take it that the firm remains perfectly competitive after adopting a new technology, so that the price of its output p will not change after a technology switch.

The general problem facing the firm is to choose right moments to switch to new technologies. One extreme possibility is to switch to a new technology every time that one becomes available, but this would entail perhaps unaffordably large sunk cost investments. The other extreme possibility is never to switch, but then the opportunity cost of keeping on producing with an old inefficient technology

[5]Although innovation may generally involve both process and product, we are concerned here only with *process* innovation, as it seems to occur often in agriculture and also in some branches of industry such as electronics.

[6]These assumptions, although not too implausible, are made chiefly for their analytical convenience rather than realism. For example, we could assume the lower bound of u to be positive, in which case the assumption of uniform distribution of u over the range $[0 < \underline{u}, \overline{u}]$ may be interpreted to reflect situations in which the firm has a very good knowledge of the range of future technological improvements but is unable to decide which values within this range are more likely to occur than others.

(i.e. foregone potentially high payoffs from adopting new technologies) may be huge.

In this section, we consider the simple case where only a single switch is allowed, so that, once installed, the new technology will remain in use forever. We will study the more general case of multiple switching in Section 4.

Essentially, the problem facing the firm is an optimal stopping one where continuation is optimal for θ sufficiently low (i.e. the firm does not yet invest) and stopping is optimal for θ sufficiently large (i.e. the firm invests). Hence, intuition suggests that there must be a critical level θ^* such that it is optimal for the firm to invest in the new technology if $\theta > \theta^*$, and to refrain from investment if $\theta < \theta^*$.

2.2 Profit Flow and Termination Payoff

At every instant, the firm can either continue its current situation to get a *profit flow*, or stop and get a *termination payoff*. In order to derive an expression for the profit flow and the termination payoff, we first determine the value of the project for $\theta > \theta^*$, i.e. when the firm has already adopted the new technology. Let θ_1 be the value of θ associated with this new technology. Since the investment is a once-and-for-all decision here, the firm produces with this new technology during the remaining planning period. If we denote the value of the project by $V(\theta_1)$, then

$$V(\theta_1) = \int_{t=0}^{\infty} \max_v (p\theta_1 v^a - wv) e^{-rt} dt = \int_{t=0}^{\infty} f(\theta_1) e^{-rt} dt = \frac{f(\theta_1)}{r}, \tag{3}$$

where $f(\theta)$, the profit flow, is defined as:

$$f(\theta) = \max_v (p\theta v^a - wv). \tag{4}$$

The value of v that maximizes the term within brackets is given by

$$v^* = \left(\frac{ap\theta}{w}\right)^{\frac{1}{1-a}}, \tag{5}$$

which leads to the following expression for $f(\theta)$:

$$f(\theta) = (1-a)\left(\frac{a}{w}\right)^{\frac{a}{1-a}} p^{\frac{1}{1-a}} \theta^{\frac{1}{1-a}} = \varphi\theta^b, \tag{6}$$

where φ and b are defined by:

$$\varphi = (1-a)\left(\frac{a}{w}\right)^{\frac{a}{1-a}} p^{\frac{1}{1-a}}, \tag{7}$$

$$b = \frac{1}{1-a} > 1. \tag{8}$$

From (3), and (6) we obtain

$$V(\theta) = \frac{\varphi \theta^b}{r}. \tag{9}$$

The termination payoff for the firm is equal to $V(\theta) - I$.

2.3 Optimal Switching Level

To derive the optimal switching level θ^*, we turn to the case where $\theta^* - \overline{u} < \theta \leq \theta^*$, so that there is a positive probability that investing will be optimal after the next jump. First we derive the Bellman equation $F(\theta)$. Combining the Bellman equation at $\theta = \theta^*$ and the value-matching condition (see, e.g. Dixit and Pindyck [13]) at $\theta = \theta^*$, we arrive at the equation from which the optimal switching level can be calculated.

The possible switch to a new technology will always occur just after an upward jump of θ. If not, due to discounting, the firm could always do better by making the investment sooner for the same θ. Hence, we can distinguish between two situations: one situation where the value of θ after the jump is still below or equals θ^* (i.e. this holds for the size of the jump: $0 \leq u \leq \theta^* - \theta$) so that no investment will take place, and one situation where θ exceeds θ^* after the jump (i.e. $\theta^* - \theta < u \leq \overline{u}$) so that investment just after the jump will be optimal.

The Bellman equation is

$$F(\theta) = f(\theta_0)dt + \frac{1}{(1 + rdt)} E[F(\theta + d\theta)], \tag{10}$$

where

$$E[F(\theta + d\theta)] = F(\theta) + \lambda dt \left\{ \int_{u=0}^{\theta^* - \theta} F(\theta + u)\frac{1}{\overline{u}} du + \int_{u=\theta^* - \theta}^{\overline{u}} (V(\theta + u) - I)\frac{1}{\overline{u}} du - F(\theta) \right\}. \tag{11}$$

Equations (10) and (11) lead to (ignoring terms of dt raised to powers higher than one)

$$(1 + rdt)F(\theta) = f(\theta_0)dt + F(\theta) + \lambda dt \left\{ \int_{u=0}^{\theta^* - \theta} F(\theta + u)\frac{1}{\overline{u}} du + \int_{u=\theta^* - \theta}^{\overline{u}} (V(\theta + u) - I)\frac{1}{\overline{u}} du - F(\theta) \right\}. \tag{12}$$

If we divide (12) by $(r + \lambda)\, dt$ we get

$$F(\theta) = \frac{f(\theta_0)}{(r + \lambda)} + \frac{\lambda}{(r + \lambda)} \left\{ \int_{u=0}^{\theta^* - \theta} F(\theta + u)\frac{1}{\overline{u}} du + \int_{u=\theta^* - \theta}^{\overline{u}} (V(\theta + u) - I)\frac{1}{\overline{u}} du \right\}. \tag{13}$$

If $\theta = \theta^*$ we are sure that investing will be optimal after the next jump. From (13) we obtain the following Bellman equation for $\theta = \theta^*$:

$$F(\theta^*) = \frac{f(\theta_0)}{(r+\lambda)} + \frac{\lambda}{(r+\lambda)} \int_{u=0}^{\overline{u}} (V(\theta^* + u) - I) \frac{1}{\overline{u}} du. \tag{14}$$

Using (6), and (9) we can write (14) as

$$F(\theta^*) = \frac{\varphi \theta_0^b}{(r+\lambda)} + \frac{\lambda \varphi (\theta^* + \overline{u})^{b+1}}{\overline{u} r (r+\lambda)(b+1)} - \frac{\lambda \varphi (\theta^*)^{b+1}}{\overline{u} r (r+\lambda)(b+1)} - \frac{\lambda}{(r+\lambda)} I. \tag{15}$$

The value-matching condition indicates that for $\theta = \theta^*$ the firm is indifferent between investing now and waiting for a more efficient technology to occur. This leads to the following equation:

$$F(\theta^*) = V(\theta^*) - I = \frac{\varphi (\theta^*)^b}{r} - I. \tag{16}$$

Substitution for $F(\theta^*)$ from (15) in (16) gives

$$\frac{\lambda \varphi}{\overline{u} r (b+1)} \left((\theta^* + \overline{u})^{b+1} - (\theta^*)^{b+1} \right) - \frac{(r+\lambda)\varphi}{r} (\theta^*)^b + \varphi \theta_0^b + rI = 0. \tag{17}$$

This is the basic equation that implicitly determines θ^*, the efficiency level of a new technology that triggers adoption.

2.4 Expected value of θ at time t

If we denote the number of new technologies that arrive over the interval $[0, t)$ by $N(t)$, the following holds for the technology-efficiency parameter θ at time t:

$$\theta(t) = \theta_0 + \Pr(N(t) > 0) \sum_{n=1}^{N(t)} u_n, \tag{18}$$

where u_n is the n-th upward jump of θ (the stochastic variables $(u_n)_{n=1}^{\infty}$ are independent and identically uniformly distributed over the interval $(0, \overline{u})$). Since the stochastic variable $N(t)$ is distributed according to a Poisson distribution with parameter λt, the probability that no new technology arrives during the interval $[0, t)$, $\Pr(N(t) = 0)$, is equal to $e^{-\lambda t}$. Using the fact that the stochastic variable $N(t)$ is independent from the stochastic variables $(u_n)_{n=1}^{\infty}$ we derive the following expression for the expected value of θ at time t:

$$E[\theta(t)] = \theta_0 + \frac{1}{2} \left(1 - e^{-\lambda t} \right) \lambda \overline{u} t. \tag{19}$$

2.5 An example: Comparative Statics

We assume that the output elasticity $a = 0.5$ (implying the production function $h(v, \theta) = \theta v^{\frac{1}{2}}$), the output price is $p = 200$, and the unit cost of the variable input is $w = 50$. The firm currently produces with a technology whose efficiency level is indexed at $\theta_0 = 1$. The parameters of the jump process governing the technology evolution are set at $\lambda = 1$, which means that, on average, every year a new technology comes on the market, and $\overline{u} = 0.2$. The firm's discount rate is $r = 0.10$, and the sunk cost investment in a new technology is $I = 1600$. Solving equation (17) with these parameter values gives the optimal switching efficiency level $\theta^* = 2.7127$. Using simulation we can calculate that the expected value t^* (the length of the waiting period before switching to the new technology) is equal to 17.79 years, while the standard deviation equals 4.87. From (19) we calculate the expected value of θ at time $t = 17.79$ to be 2.7790. This is higher than the optimal switching level θ^* due to the jump process that θ follows.

Next, we illustrate the direction and extent to which the optimal switching efficiency level, θ^*, and hence the firm's incentive to adopt, is affected by changes in parameter values. Three different groups of parameters are distinguished: (p, w, I, r) reflecting market conditions, $(\lambda, \overline{u})$ describing technology evolution, and (θ_0, a) representing the firm's initial technological attributes. These comparative static effects are presented respectively in Figures 1.1-1.4, 2.1-2.2, and 3.1-3.2.

As Figures 1.1-1.4 illustrate, the switching efficiency level, θ^*, will be lower, implying that the firm will adopt innovation sooner, the higher the price of output, the lower the unit cost of variable input, the smaller the initial investment cost, and the higher the discount rate (assumed equal to the market interest rate). The intuition for these effects is straightforward. The opportunity cost of waiting in anticipation of a still more efficient new technology (i.e. one with a higher expected θ) is the forgone profits during the waiting period, which clearly will be greater the higher is p, the lower is w, or the smaller is I. As regards the discount rate, a higher rate lowers the value of payoffs from more efficient, but also more distant, future technologies, and therefore reduces the value of the option to delay. It is worth noting that because of this, here the effect of the discount rate is the opposite of the conventional one under the net present value approach, namely the higher the discount rate the higher the trigger level of θ. In fact, as can be readily verified from Figure 1.4 and the analysis of Section 3, in the limiting case where the discount rate is raised to infinity the option value of waiting declines to zero and hence the optimal trigger levels of θ coincide under the alternative approaches.

Figures 2.1 and 2.2 depict θ^* as functions of λ and $\overline{u}$ respectively. Contrary to what intuition might suggest, they show that the optimal triggering efficiency level will be lower, and hence innovation adoption will occur sooner, the *smaller* the probability that a more efficient technology becomes available within a given time period, or the *smaller* the expected maximum efficiency improvements in future technologies. In both cases, however, the explanation is simple. For a given expected efficiency improvement $\left(\frac{1}{2}\overline{u}\right)$, a smaller chance of a new technology arriving within a certain time interval raises the opportunity cost of waiting by prolonging the average waiting period needed for an innovation to occur. This lowers the value of option to wait and hence the optimal trigger level. A rather striking implication of this result is that, contrary to common wisdom, a lower probability of arrival of new improved technologies (as indicated by a lower mean rate of arrival) may well *speed up*, rather than delay, adoption. Inversely, a faster rate of innovation arrival may well induce the firm to postpone adoption as the firm would hesitate to lock itself into a relatively less efficient technology by an early adoption while better technologies are highly likely to appear later. Similarly, for a given λ, a lower $\overline{u}$ implies a smaller expected efficiency gain when a new technology arrives, thus reducing the value of option to wait and hence quickening the adoption.

Alternative interpretations of what these comparative statics results imply are that: *all else equal*, (i) innovations with smaller expected efficiency improvements should be expected to be adopted sooner than those with radically superior expected efficiency gains; (ii) innovations with greater chances of arrival should be expected to be adopted more slowly than those with lower arrival probability. Given the stylized fact of *incremental* technological improvements, the former implication appears to accord well with actual experience. Whether the same thing can be said of the latter is a rather moot question, particularly in view of the simplifying assumption made here that the probability of arrival of innovations does not dependent on the expected extent of efficiency improvement (specifically, λ is assumed to be independent of $\overline{u}$). More realistically, the probability of a new improved technology to arrive is likely to be a decreasing function of the associated expected efficiency improvement. In that case, it may well be that an innovation with a higher probability of arrival but lower expected efficiency gain will be adopted sooner than one with a radically superior efficiency but much lower chance of arrival[7].

As seen from Figure 3.1, the lower the efficiency level of the prevailing technology, θ_0, the lower will be the optimal switching efficiency level, and therefore the sooner the optimal timing of switching to a new technology. This is not surprising because with a relatively highly efficient technology currently in use the opportunity cost of switching will be relatively large, so that for switching to be optimal the trigger efficiency level of a new technology should be higher than would be the case if the efficiency of the prevailing technology was low. An implication of this result is that, *all else equal*, technology adoption is likely to be *slower* for firms which are already at the cutting edge of technological efficiency than for those whose current technologies lag behind.

[7] A more general model would incorporate "learning effects" by allowing the probability distribution of innovations arrival or of efficiency improvements to be revised as time passes. For a formulation of the former possibility, see Balcer and Lippman [2].

Finally, Figure 3.2 shows θ^* as a function of a, output elasticity, which is another indicator of the firm's production efficiency and, for our specification of production function, is independent of the technology evolution. It is worth noting that in the extreme case where $a \longrightarrow 1$, the optimal switch level θ^* goes to infinity, implying that it will never be optimal to switch. The reason is that for $a = 1$ the firm's production function becomes $h(v, \theta) = \theta v$ so that the marginal product of the variable input is $\frac{\partial}{\partial v} h(v, \theta) = \theta$. Then, the optimal level of the variable input v^* will be 0 if $p\theta < w$ and infinity if $p\theta > w$. In the former case, the optimal trigger level is $\theta^* \geq \frac{p}{w}$, so that it will be optimal for the firm to *refrain from* production for an initial period and wait until such a time when technology has sufficiently improved to reach the trigger efficiency level $\theta^* \geq \frac{p}{w}$. In the latter case, the profit, and hence θ^* will be indeterminately large ($\theta^* \longrightarrow \infty$), so that it will never pay to switch. For the chosen parameter values, this case holds in our example.

Focusing on the more plausible cases where $0 < a < 1$, it is seen from Figure 3.2 that in contrast to the previous comparative static effects, here θ^* is *not* a monotonic function of a. For relatively large values of a, the optimal switching level rises with a, implying, analogous to the case of θ_0, that the higher a firm's *input* efficiency (or output elasticity) the slower will be its optimal timing of innovation adoption. Strikingly, however, for relatively small values of a, the optimal switching level rises as *a declines*, so that when the firm's input efficiency is below a certain level, then the lower the input efficiency the higher will be the optimal switching level. A rather interesting implication of this result is that, as for the firms with high input efficiency, firms with a very low input efficiency also tend to be slow in adopting innovations, thus suggesting a kind of "low-efficiency technology trap".

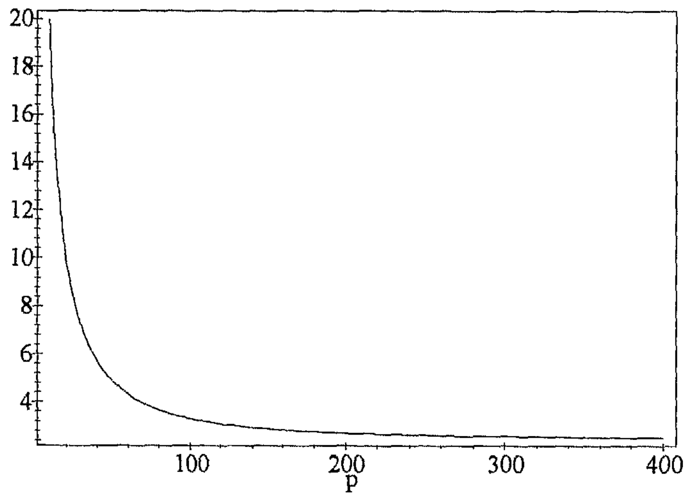

Fig. 1.1. Optimal switching level θ^* as function of p.

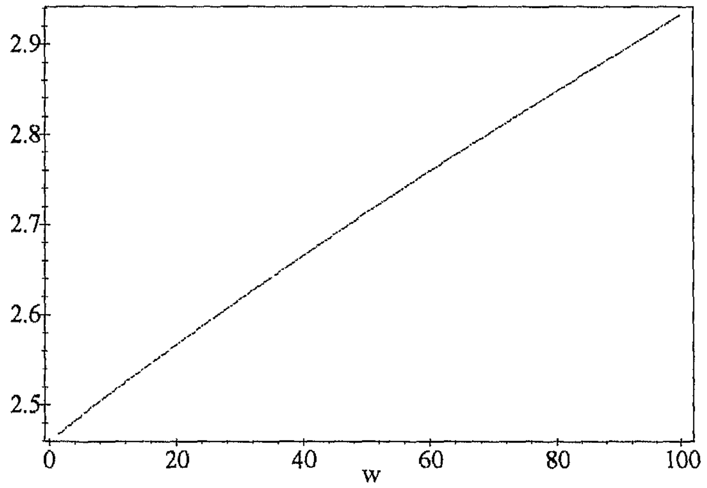

Fig. 1.2. Optimal switching level θ^* as function of w.

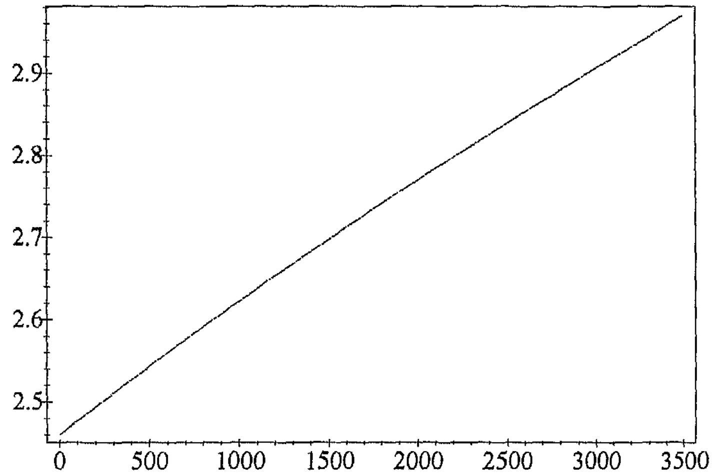

Fig. 1.3. Optimal switching level θ^* as function of I.

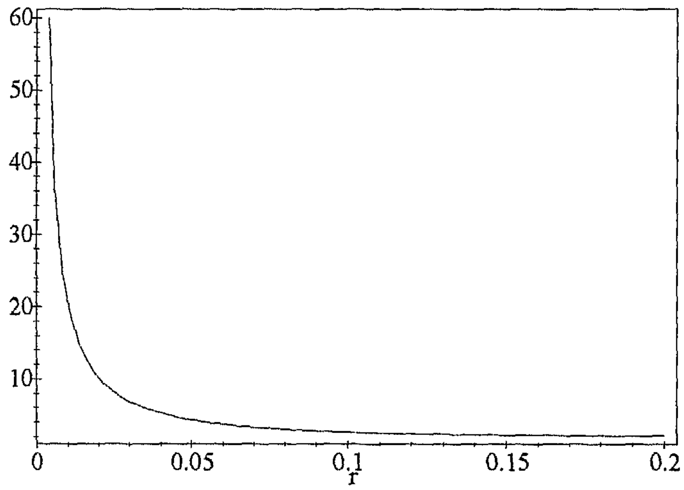

Fig. 1.4. Optimal switching level θ^* as function of r.

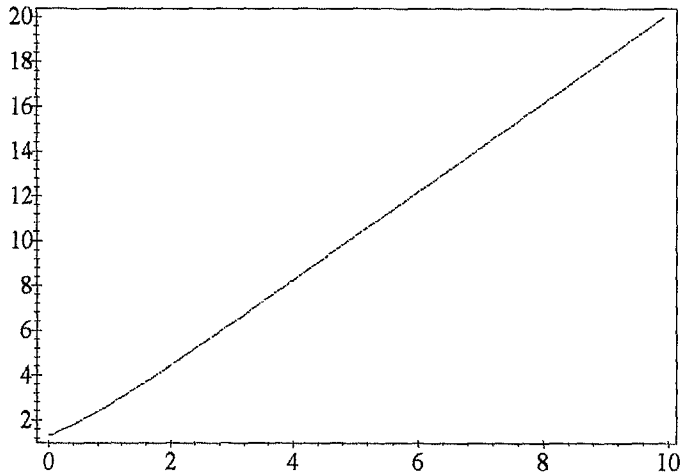

Fig. 2.1. Optimal switching level θ^* as function of λ.

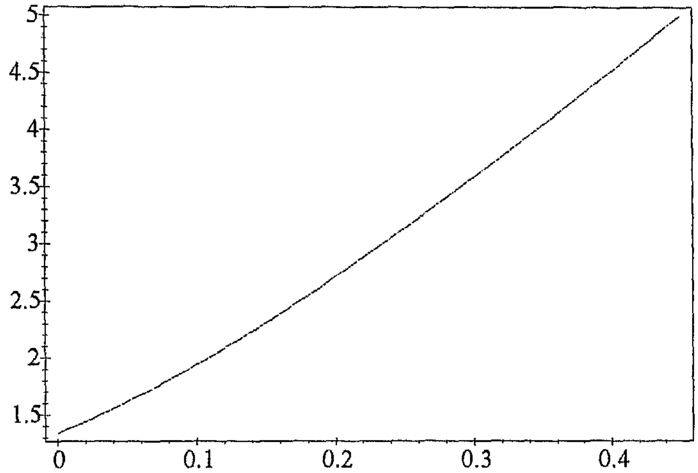

Fig. 2.2. Optimal switching level θ^* as function of $\overline{u}$.

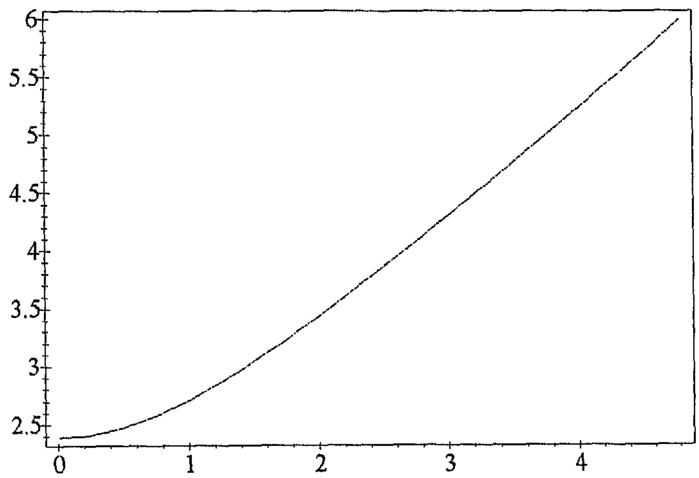

Fig. 3.1. Optimal switching level θ^* as function of θ_0.

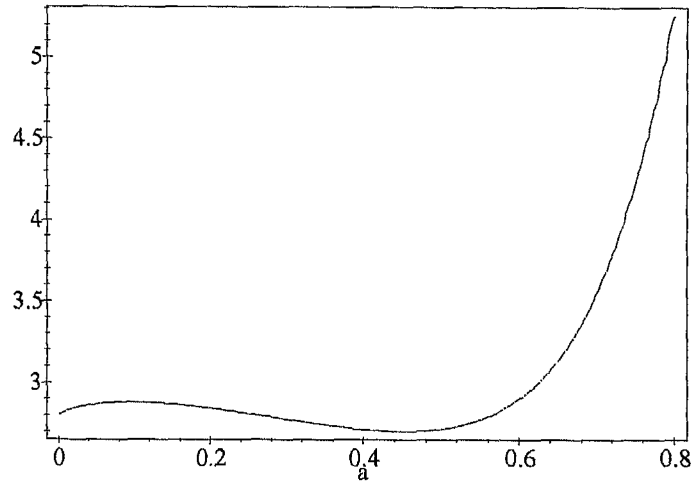

Fig. 3.2. Optimal switching level θ^* as function of a.

3 Net Present Value Method

In this section we derive the switching efficiency level according to the net present value method and contrast it with the optimal level derived in the previous section.

3.1 Optimal Switching Level

According to the net present value criterion, an investment should be undertaken if the present value of the cash flow stream it generates exceeds the investement cost. As Dixit and Pindyck [13] point out, most investment problems do not satisfy the implicit assumptions of the standard net present value rule; namely that either the investment is reversible, or if irreversible, it is a now or never proposition. In reality, however, irreversibility and the possibility to delay are inherent characteristics of most investments.

A firm with an opportunity to invest is holding an *option* analogous to a financial call option. The firm has the right but not the obligation to buy an asset at some future time of its choosing. When a firm makes an irreversible investment expenditure, it exercises its option to invest. This lost option value is an opportunity cost that must be included as part of the cost of the investment. Thus, the net present value rule, which ignores the waiting option value, is incorrect.

If at time, say t_0, investment is delayed in our model, there is a positive probability that the firm can invest later in a technology with a higher efficiency level than if it had invested at time t_0. Therefore the value of the option to postpone the adoption of a new technology will be positive. Consequently, the optimal switching level of θ determined by the net present value method will be smaller than that determined by equation (17).

From (3) we know that the value of the firm using technology θ is equal to $V(\theta)$. According to the net present value method the following holds for the optimal switch level θ^*_{NPV}:

$$V(\theta^*_{NPV}) - I = V(\theta_0). \tag{20}$$

Using (9) we get

$$\frac{\varphi\left(\theta^*_{NPV}\right)^b}{r} - I = \frac{\varphi\left(\theta_0\right)^b}{r}. \tag{21}$$

Rewriting this gives

$$\theta^*_{NPV} = \left(\frac{\varphi\theta_0^b + rI}{\varphi}\right)^{\frac{1}{b}}. \tag{22}$$

3.2 An example

For the same parameter values used in the example of subsection 2.4, we obtain $\theta^*_{NPV} = 1.3416 < \theta^* = 2.7127$, implying that under the net present value rule the firm will suboptimally adopt a new technology too soon. We can also calculate the firm's value of the option to delay to be $V(\theta^*) - I - V(\theta_0) = 11117$, or 87.4 percent of the value of the new investment! As in subsection 2.4 we also calculate the expected value and the standard deviation of t^*_{NPV} (the length of the waiting period before switching to the new technology, if the firm switches according to the net present value rule). These are equal to 4.08 and 2.33 respectively. Thus, here the firm is expected to switch to a new technology as early as a little after four years, optimal decision is to adopt a new technology only after more than seventeen years.

Figure 4.1 illustrates how the switching efficiency level, θ^*_{NPV}, is affected by a change in the firm's output elasticity a. It is seen that in stark contrast to the corresponding effect analysed in the previous section (see Fig. 3.2), under the *net present value method* the switching level declines with a for sufficiently large values of a. The reason for this difference is that under the net present value method investment in a new technology involves no lost option value of waiting. So, a larger value of a simply means a higher profit flow (see (6) and (3)) and therefore a lower level of θ^*_{NPV} which is needed to trigger the technology switch. In the previous section, however, a larger value of a raises the option value of waiting and hence the trigger efficiency level θ^*.

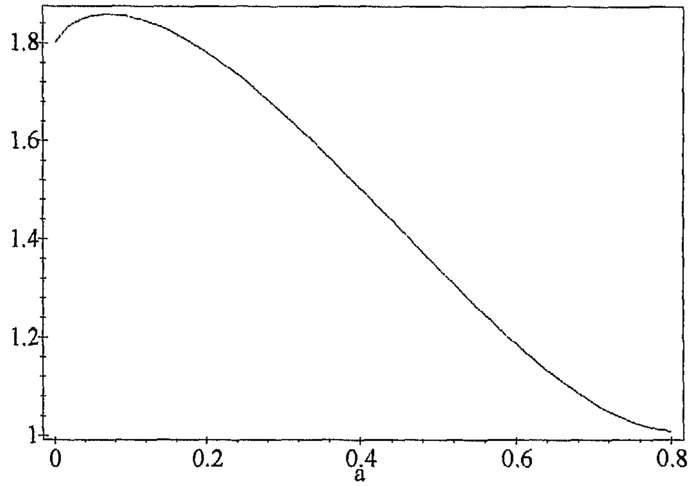

Fig. 4.1. Switching level θ^*_{NPV} as function of a.

4 Generalization to Multiple Switching Case

The problem of optimal timing of technology switching when a firm can switch n times is one of n-phase optimal stopping problem. For the n-th phase, the problem will be the same as that analyzed in Section 2. In each phase continuation is optimal (i.e. the firm does not invest yet and gets a profit flow) if θ is too low and stopping is optimal (i.e. the firm invests and gets a termination payoff) if θ is sufficiently large . Hence, intuition suggests that there are n trigger levels: $\theta_1^*, \ldots, \theta_n^*$, such that for each θ_i^*, it is optimal for the firm to switch to a new technology for the i-th time if $\theta > \theta_i^*$, and to delay switching if $\theta < \theta_i^*$.

As in Section 2 we use the expressions for the profit flow and the termination payoff to derive the optimal switching levels. We assume that the firm produces with technology θ_i after it makes the i-th switch. From Section 2 we also know that if the firm produces with technology θ_i its profit flow is equal to $f(\theta_i)$. Denoting by $F_i(\theta)$ the value of the firm after the i-th but before the $(i+1)$-th switch, the termination payoff at the i-th switch is equal to $F_i(\theta_i) - I$. Further we know from Section 2 that after the n-th switch, i.e. $\theta > \theta_n^*$, the value of the firm will be $V(\theta) = \frac{\varphi\theta^b}{r}$, so that the termination payoff at the last switch is equal to $V(\theta_n) - I$.

To derive the optimal switching levels, we first derive the Bellman equations for $F_i(\theta)$. Substitution of the Bellman equations $F_{i-1}(\theta_i^*)$, and $F_i(\theta_i^*)$ into the value-matching condition at $\theta = \theta_i^*$ gives the expression for the optimal switching level θ_i^*.

The Bellman equation for $\theta_i^* \leq \theta \leq \theta_{i+1}^* - \overline{u}$, so that the probability of switching for the $(i+1)$-th time after the next jump is zero, is

$$F_i(\theta) = \frac{f(\theta_i)}{(r+\lambda)} + \frac{\lambda}{(r+\lambda)} \int_{u=0}^{\overline{u}} F_i(\theta+u)\frac{1}{\overline{u}}du, \tag{23}$$

where $i = 0, 1, \ldots, n-1$, and $\theta_0^* = \theta_0$. The Bellman equation for $\theta_{i+1}^* - \overline{u} < \theta \leq \theta_{i+1}^*$ is

$$F_i(\theta) = \frac{f(\theta_i)}{(r+\lambda)} + \frac{\lambda}{(r+\lambda)} \left\{ \int_{u=0}^{\theta_{i+1}^*-\theta} F_i(\theta+u)\frac{1}{\overline{u}}du + \int_{u=\theta_{i+1}^*-\theta}^{\overline{u}} \left(F_{i+1}(\theta+u) - I\right)\frac{1}{\overline{u}}du \right\}. \tag{24}$$

If $\theta = \theta_i^*$ we are sure that investing for the i-th time is optimal after the next jump. From (24) we obtain the Bellman equation for $\theta = \theta_i^*$, with $i = 1, 2, \ldots, n-1$:

$$F_{i-1}(\theta_i^*) = \frac{f(\theta_{i-1})}{(r+\lambda)} + \frac{\lambda}{(r+\lambda)} \int_{u=0}^{\overline{u}} \left(F_i(\theta_i^*+u) - I\right)\frac{1}{\overline{u}}du. \tag{25}$$

The value-matching condition is

$$F_{i-1}(\theta_i^*) = F_i(\theta_i^*) - I. \tag{26}$$

Substitution of (26) in (25) gives

$$F_i(\theta_i^*) - I = \frac{f(\theta_{i-1})}{(r+\lambda)} + \frac{\lambda}{(r+\lambda)} \int_{u=0}^{\overline{u}} \left(F_i(\theta_i^*+u) - I\right)\frac{1}{\overline{u}}du. \tag{27}$$

Combining (23), in which $\theta = \theta_i^*$, and (27) leads to[8]

$$f(\theta_i^*) = f(\theta_{i-1}) + rI. \tag{28}$$

Substitution of (6) in (28) gives us the optimal trigger levels θ_i^*, for $i = 1, 2, \ldots, n-1$:

$$\theta_i^* = \left(\theta_{i-1}^b + \frac{rI}{\varphi}\right)^{\frac{1}{b}}. \tag{29}$$

The next task is to determine the optimal trigger level for the last (i.e. the n-th) technology switch. To do so we consider the case where the firm has switched $(n-1)$ times, and where it holds that $\theta_n^* - \overline{u} < \theta \leq \theta_n^*$. Then the Bellman equation is

[8] We assume that $\theta_i^* + \overline{u} < \theta_{i+1}^*$, so that immediate consecutive switchings are ruled out. This will always be true for sufficiently large values of I.

$$F_{n-1}(\theta) = \frac{f(\theta_{n-1})}{(r+\lambda)} + \frac{\lambda}{(r+\lambda)} \left\{ \int_{u=0}^{\theta_n^*-\theta} F_{n-1}(\theta+u)\frac{1}{\overline{u}}du + \int_{u=\theta_n^*-\theta}^{\overline{u}} \left(V(\theta+u)-I\right)\frac{1}{\overline{u}}du \right\}. \tag{30}$$

From (30) we obtain the Bellman equation for $\theta = \theta_n^*$:

$$F_{n-1}(\theta_n^*) = \frac{f(\theta_{n-1})}{(r+\lambda)} + \frac{\lambda}{(r+\lambda)} \int_{u=0}^{\overline{u}} \left(V(\theta_n^*+u)-I\right)\frac{1}{\overline{u}}du. \tag{31}$$

The value-matching condition is

$$F_{n-1}(\theta_n^*) = V(\theta_n^*) - I. \tag{32}$$

Substitution of (32) in (31) gives

$$V(\theta_n^*) - I = \frac{f(\theta_{n-1})}{(r+\lambda)} + \frac{\lambda}{(r+\lambda)} \int_{u=0}^{\overline{u}} \left(V(\theta_n^*+u)-I\right)\frac{1}{\overline{u}}du. \tag{33}$$

Using (6), and (9) we can rewrite (33) as

$$\frac{\lambda\varphi}{\overline{u}r(b+1)}\left((\theta_n^*+\overline{u})^{b+1} - (\theta_n^*)^{b+1}\right) - \frac{\varphi(r+\lambda)}{r}(\theta_n^*)^b + \varphi\theta_{n-1}^b + rI = 0. \tag{34}$$

Analogous to equation (17) derived for the single switching case, equation (34) implicitly determines, θ_n^* in the present multiple switching case. We conclude that, with θ_0 given, equations (29) and (34) together yield the optimal switching levels $\theta_1^*, \ldots, \theta_n^*$.

Contrasting (29) with (22), immediately reveals that the first $(n-1)$ optimal trigger levels are in fact the ones which would be obtained by applying the net present value method:

$$\theta_i^* = \theta_{i,NPV}^* = \left(\theta_{i-1}^b + \frac{rI}{\varphi}\right)^{\frac{1}{b}}, \qquad i = 1, 2, \ldots, n-1. \tag{35}$$

The explanation for this rather striking result is simple. By ignoring the fact that once the firm invests it gives up its option to delay, the net present value rule fails to account for the cost of this lost option value and therefore results in a lower trigger level than would be optimal. However, in the present case where the firm has the opportunity to make multiple switches, the firm has *not* given up anything once it has invested for the m-th time, if $m < n$, because it can invest again once θ increases sufficiently. Of course, the matter is very different for the last switch: once the firm has invested for the n-th time, no longer will it be possible to invest again when θ increases in the future. So, here there is a option value of waiting, and as a result the net present value method fails: $\theta_n^* > \theta_{n,NPV}^*$.

5 Conclusions.

Taking the dynamic programming approach, à la Dixit and Pindyck [13], in this paper we have analyzed the optimal timing of technology adoption by a competitive firm when investment in a new improved technology is an irreversible decision and technology evolves stochastically over time. Contrasting the optimal decision rule derived under this approach with that obtained under the net present value method, it is shown that, much in accord with the real world experience, the former implies a more cautious and slower pace of adoption than implied by the latter. And, as is illustrated by the numerical example, this difference in the timing of adoption under the alternative approaches can well be very significant. The reason for the the difference is simple: the conventional net present value method only takes into account cash flows and ignores the option value of waiting for more efficient future technologies, thus failing to account for this opportunity cost component when an investment decision is made.

A central focus of our analysis has been the important question of how the optimal timing of adoption is affected by uncertainties inherent in the process of technological innovation; that is, uncertainties about *both* the speed of arrival and the extent of efficiency improvements of new technologies. Not surprisingly, we have shown that even in the absence of other kinds of uncertainties, *e.g.* uncertainties about market conditions, a firm's optimal timing of adoption is greatly influenced by *technological* uncertainties. Interestingly, the comparative static results illustrated by the numerical example indicate that some effects are in stark contrast to what common intuition might at first suggest. Specifically, we found that (i) contrary to what is the case with the conventional net present value method, here the higher the discount rate the lower the trigger efficiency level of technology and thus the *quicker* the timing of adoption; (ii) the slower the expected pace at which more efficient technologies arrive, or the smaller the expected maximum improvements in future technologies, the *lower* the trigger efficiency level of technology; (iii) innovation adoption will be *slower* for firms which are already at the forefront of technological efficiency (high θ_0) than for those currently using relatively inefficient technologies (low θ_0); and (iv) when the *input* efficiency (here equivalent to the elasticity of output, a) is below a certain level, then the lower the input efficiency of a firm the *slower* will be the innovation adoption, thus suggesting something akin to a "low-efficiency technological trap". Perhaps strikingly, and contrary to the case under the net present value method, at relatively high input efficiency levels, the higher a firm's input efficiency the slower the innovation adoption. According to (iv), *all else equal*, innovation adoption is likely to be relatively slow *both* for firms with very low and very high input efficiency levels. Whether these theoretical implications are anywhere near the truth is obviously a purely empirical question, and a subject for future research; although, taking them at face value, they seem to be supported by many real-world examples.

We have also generalized the optimal decision rule when only a single technology switch is allowed, which may, for example, apply for small firms with very limited financial resources, to the case where a firm is able to make multiple switches. In doing so, we have shown that for all the switching decisions *except the last one*, the optimal rule sastisfies the net present value criterion. This is not surprising for as long as the firm still has an opportunity to make a future investment, there will be no lost option value associate with making an investment. So, the optimal investment decisions coincide under the alternative approaches.

A model as simple as that analyzed in this paper is bound to have many limitations, thus calling for further research in several respects. Depending on the specific technological innovation process under study, both of our simplifying assumptions of Poisson arrival process and uniform probability distribution of the extent of technological improvement can be appropriately replaced by more realistic ones. Also, our assumption of constant sunk investment cost can be relaxed to allow for the more realistic situations where the investment cost declines over time or rises with the expected efficiency improvement of new technology. The former would accentuate the option value of delaying adoption while the latter mitigates it. More importantly, the assumption of fixed probability of the arrival of innovations made here may be relaxed by allowing for learning, for example, as an increasing function of time lapsed since the last adoption (as in Balcer and Lippman [2]), or of the cumulative investment in research and development. Further, our assumption of competition in the face of technological innovation is admittedly restrictive and may hold only when the innovation adopting firm is too small and the innovations are public (as in the case of research by universities or public agencies, for example). More realistically, the present model can be enriched by drawing on the existing literature on optimal supply of innovation to let the innovation process be internal to the firm's decision and a source of its market power.

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