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Environmental effects of tourism industry investments: an inter-temporal trade-off

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SUMMARY

Efficient investment programmes in touristic infrastructure have to take into consideration that any kind of tourism reduces the environmental quality. Since pollution shows negative repercussions as concerns the attractiveness of a touristic region, tourism planners have to determine a trade-off between adequate services for tourists and a clean environment. To deal with this problem in a dynamic context, a three-state optimal control model is formulated. It turns out that, even if pollution reduction is not a goal in itself, the profit-maximizing tourism industry should care for ecological conservation. The paper further shows that persistent periodic investment policies are optimal for realistic parameter sets, and provides an economic intuition for such behaviour. From an economic point of view, this result implies that expansionary periods with high investment are followed by periods of stagnation with low investment. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: tourism industry; environment; optimal control; Pontryagin's maximum principle; limit cycles

1. INTRODUCTION

Since tourism is likely to become the largest single sector of world trade early in the next century [1], it is important to establish a theoretical framework for investment in the touristical infrastructure. Nevertheless, in the literature no contributions can be found that address this subject within a decision oriented optimization model. This paper is a first attempt to fill this gap.

In reality, it is impossible to imagine that any kind of tourism activity is developed and then operates without, in some way, reducing the quantity of natural resources somewhere. As a

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good example, the adverse environmental impact of tourism in mountains can be mentioned (see, e.g. Reference [2]). Human activities, particularly tourism, have caused damages in the fragile mountain ecosystem, especially in wetlands. In Reference [3] it is strongly argued that there must be substantial changes in the way mountain regions are used by tourism if the landscape, which forms the main resource for tourism, is not to be adversely and permanently affected.

Another example of bad environmental impact by tourism is wildlife being harmed by photographic tourism. As masses of tourism swarm around fewer and fewer animals (see Reference [4]) can be observed. Although the Royal Bardia National Park in Nepal is relatively inaccessible to tourists, their ecological impact, in terms of disturbance of habitat and of wildlife, is significant in the accessible area around the phantas (see Reference [5]).

All this inevitably leads to the conclusion that any meaningful analysis concerning investment in tourism must take account of its environmental implications.

As argued in Reference [1], the question of who decides on the most appropriate pathway of tourism development is crucial for the future development of this branch. Ecological conservation objectives may not be compatible with the desire of local communities. It may well be that different levels of community involvement in tourism development decision-making are appropriate for different pathways of sustainable tourism.

In our framework we assume that a tourism planner decides about the investment strategy. This planner represents different groups interested in attracting tourists, like private investors and the (local) government, who generate conditions for touristic investment projects. The objective of the planner then is to maximize revenues from the tourism industry, which implies that reducing pollution is not a goal in itself. However, in the tourists' decision-making process, a clean environment plays a central role in the choice of destination.

For example, an exploratory study in Switzerland, Austria and Bavaria has shown that traffic-free mountain holiday resorts have an above-average occupancy rate, and resorts where the environmental burden due to traffic is relatively low have a higher occupancy rate than comparable resorts where the burden is much higher. In hotels there are also some clearly noise-related losses (see Reference [6]). Tourists visiting Sochi (Black Sea Coast, Russia) complain about the high level of noise produced by the motorway, about aggressive odors, discomfort, noise and vibration produced by railway and air routes, air pollution, etc. (see Reference [7]).

In our model, we take into account that pollution negatively affects the number of tourists, and thus has an adverse impact on revenues from the tourist industry. Hence, in this respect it is still in the interest of the planner to preserve a clean environment. We are able to take this aspect into account by developing a dynamic model, since in such a model today's actions influence the state of the future. Thus, investments today attract more tourism. This, however, increases environmental pollution which, in turn, has a negative impact on the number of tourists in the future.

The paper is organized as follows. In Section 2 the model is formulated. Section 3 analyses the first-order necessary optimality conditions. In order to gain further insights into the optimal solution paths examples are studied in Section 4. The local stability analysis of the steady state yields saddle points (Section 4.1) and limit cycles (Section 4.2), whose economic meaning is discussed in Section 4.3. Bifurcation analysis in Section 4.4 gives some more general qualitative insights as to how generic cyclical investment policies are. Finally, Section 5 contains some concluding remarks as well as proposals for several extensions.

2. THE MODEL

We consider the optimal investment path for a tourism planner who invests in touristic infrastructure in order to attract tourists. Touristic infrastructure $S(t)$ evolves according to

$$\dot{S}(t) = I(t) - \delta S(t), \;\; S(0) = S_0 > 0 \tag{1}$$

in which $I(t)$ equals investment in infrastructure and $\delta \geqslant 0$ is the depreciation rate, which is assumed to be constant.

Not only a missing touristic infrastructure but also a polluted environment discourages tourists from visiting a tourist area—tourists are not only interested in touristic infrastructure but are also attracted by a clean environment. Therefore, we suppose that both touristic infrastructure as well as the stock of pollution $P(t)$ affect the evolution of the number of tourists $T(t)$, which we model mathematically by the following differential equation

$$\dot{T}(t) = a(S(t), k_1, P(t)) - b\,T(t), \quad T(0) = T_0 > 0 \tag{2}$$

The attractiveness function $a(S(t), k_1, P(t))$ satisfies $a_S(\cdot) > 0$, $a_{SS}(\cdot) < 0$, i.e. touristic infrastructure certainly attracts tourists, however, with diminishing intensity. The parameter k_1 stands for service expenditures for personnel per unit of touristic infrastructure and we posit $a(S, 0, P) = 0$. It is obvious that a stock of touristic infrastructure such as hotels or ski lifts by itself does not attract tourists unless it is staffed. Therefore, service expenditures and infrastructure are complementary and the service expenditures are proportional to the stock of infrastructure. That means, we suppose that the stock of infrastructure S needs k_1 service expenditures per unit in order to be operated efficiently.

Moreover, we assume that the stock of pollution negatively affects the number of tourists, $a_P(\cdot) < 0$ and that this function is S-shaped, i.e. $a_{PP} < 0$ for P small and $a_{PP} > 0$ for P large. That specification implies that an additional unit of pollution shows an increasing negative repercussion to the attractiveness of the region as long as that region is relatively clean, but small negative effects on the attractiveness, if pollution is relatively high. As to the cross-derivative a_{SP} we assume $a_{SP} < 0$, so that a_S, which we assume to be positive, increases with a decrease of P. From an economic point of view this means that additional touristic infrastructure attracts more additional tourists the cleaner the environment is. Alternatively, $a_{SP} < 0$ implies that for higher values of S one additional unit of pollution has a higher negative impact on the tourism increase. This is reasonable since, given P, more infrastructure, thus a higher value of S, gives a higher value of $a(S, k_1, P)$. Now, if a marginal unit of P gives an x per cent reduction in attractiveness, the absolute reduction in attractiveness is higher for larger values of S. The parameter $b \geqslant 0$ reflects the decline in the number of tourists due to crowding effects. If a lot of tourists invade a region, congestion makes that region less attractive and will lead to a decrease in tourists over time.

Furthermore, we suppose that both the touristic infrastructure as well as the number of tourists lead to an increase in the stock of pollution. However, we also posit that the environment is endowed with the ability to absorb a certain amount of polluting activities without being harmed. Formally, that effect is modelled by introducing an absorption capacity. The stock of pollution then evolves according to

$$\dot{P}(t) = \sigma S(t) + \tau T(t) - \alpha(P(t)), \quad P(0) = P_0 > 0 \tag{3}$$

$\sigma > 0$ and $\tau > 0$ represent the contribution of one additional unit of infrastructure and one additional tourist to the stock of pollution, respectively. $\alpha(P(t))$ reflects the absorption capacity. One feasible specification as to the function $\alpha(\cdot)$ is a linear relationship, $\alpha(P(t)) = \alpha_1 * P(t)$, implying that nature absorbs a constant proportion of the stock of pollution (for a survey of how to model pollution in a formal model see e.g. Reference [8]). A different plausible specification for pollution absorption capacity is $\alpha(P(t)) = mP(t)e^{-P(t)/\bar{P}}$, with $m > 0$ and $\bar{P} > 0$, i.e. a specification shaped akin to the shape of an environmental Kuznets curve.

That implies that for values of $P(t)$ lower than $\bar{P}$ the absorption capacity is low and rises with $P(t)$ or, formulated in a different way, if not much pollution is generated, then the amount of cleaning can also be not that large. The absorption capacity reaches a peak for $P(t) = \bar{P}$ and declines again, meaning that nature cannot regenerate if the stock of pollution is high.

The objective of the tourism planner then is to maximize the discounted stream of cash flows generated by the tourist industry. We suppose that the planner is composed of different groups who are interested in attracting tourists, as mentioned in the introduction. So, there are private investors, who decide about the total amount to be invested within a region, and there are elected representatives of the inhabitants in that region, who try to attract private investors and determine whether a certain project is carried out. Formally, the planner solves (from now on we suppress the time argument t).

$$\max_I \int_0^\infty e^{-rt}(pT - c(I) - (k_1 + k_2)\, S)\, dt \tag{4}$$

subject to (1)–(3). p represents the income generated by one tourist which is assumed to be given exogenously. From the economic point of view, that assumption can be justified by strong competition among different touristic regions. The service expenditures are equal to $k_1 S$, and we suppose that k_1 is not a control variable for our planner. We do that because total services expenditures are completely determined by the stock of touristic infrastructure, since they are proportional to it, as mentioned above.

We also posit that touristic infrastructure causes maintenance costs and brings about pollution, which requires abatement activities, like refuse collection, water carriage system etc. Both maintenance costs and abatement activities cause expenditures for the touristic region, which we assume being linearly dependent on the stock of infrastructure and given by $k_2 S$. An increase of k_2 would lead to a reduction of abatement activities and thus to an increase of the parameter σ in (3). Operating sewage works, for instance, would decrease the parameter σ and τ in (3) and increase the parameter k_2.

We would like to point out that the discount rate, denoted with r, can be conceived of as composed of two factors: an interest factor and risk factor with the latter expressing the risk or uncertainty of future periods' profit flows.‡ In principle an infinite discount rate could be imagined, too. Then, the planner's objective would be equivalent to a static optimization problem (see Reference [10]). Finally, we model the investment cost function $c(I)$ to be increasing and convex, i.e. $c'(\cdot) > 0$; $c''(\cdot) > 0$.

‡For the relation of risk and the discount factor cf. Reference [9].

3. THE DYNAMIC BEHAVIOUR

In what follows we apply Pontryagin's maximum principle to derive insight into the structure of an optimal trajectory.[§] To do so, we start out by introducing the current-value Hamiltonian

$$H(\cdot) = pT - c(I) - (k_1 + k_2)S + \lambda_1(I - \delta S) + \lambda_2(a(S, k_1, P) - b\,T) + \lambda_3(\sigma S + \tau T - \alpha(P))$$

where λ_i, $i = 1, 2, 3$, represent the co-state variables belonging to S, T and P, respectively. The necessary optimality conditions are then given by

$$c'(I) = \lambda_1 \tag{5}$$

$$\dot{\lambda}_1 = (r + \delta)\lambda_1 + k_1 + k_2 - \lambda_2 a_S(\cdot) - \lambda_3 \sigma \tag{6}$$

$$\dot{\lambda}_2 = (r + b)\lambda_2 - p - \lambda_3 \tau \tag{7}$$

$$\dot{\lambda}_3 = (r + \alpha'(\cdot))\lambda_3 - \lambda_2 a_P(\cdot) \tag{8}$$

From (5) it is obtained that infrastructure investment I is an implicit function of the shadow price of touristic infrastructure λ_1, with $\mathrm{d}I/\mathrm{d}\lambda_1 = 1/c''(\cdot) > 0$. The higher the value of an additional unit of touristic infrastructure, the higher the level of investment will be, which is intuitively plausible.

Assuming that a stationary point exists, for our model the local stability properties can be analysed by linearization around the rest point. A stationary point for our model has to satisfy the following set of equations:

$$I(\lambda_1^*) = \delta S^* \tag{9}$$

$$a(S^*, k_1, P^*) = bT^* \tag{10}$$

$$\alpha(P^*) = \sigma S^* + \tau T^*, \tag{11}$$

$$(r + \delta)\lambda_1^* = \lambda_2^* a_S(\cdot) + \lambda_3^* \sigma - k_1 - k_2 \tag{12}$$

$$(r + b)\lambda_2^* = p + \lambda_3^* \tau \tag{13}$$

$$(r + \alpha'(\cdot))\lambda_3^* = \lambda_2^* a_P(\cdot) \tag{14}$$

with $*$ denoting stationary values.

The shadow price of touristic infrastructure and tourists at the stationary state, denoted by the co-states λ_1^* and λ_2^*, are expected to be positive because a rise in touristic infrastructure and in the number of tourists increases the cash flow. But, of course, a better touristic infrastructure by itself does not raise the cash flows directly but only indirectly through attracting more

[§] An introduction to the optimality conditions can be found in References [11,12].

tourists. The shadow price of pollution at the stationary state, λ_3^*, is expected to be negative since a marginal rise in pollution leads to a decline in the number of tourists, ceteris paribus, and, thus, to a lower cash flow. A sufficient but not necessary condition for a negative λ_3^* is $\alpha'(\cdot) \geqslant 0$.

4. NUMERICAL EXAMPLES

The model is investigated for different functional specifications. However, it should be noted that in the first two examples the attractiveness function $a(S, k_1, P)$ does not satisfy the requirements as formulated in Section 2 (in the first example $a_{SS} = 0$ and $a_{SP} = 0$, while the latter also holds in the second example). Still we studied these examples in order to gain insights concerning the dynamic behaviour of the optimal trajectories. In particular it will be shown that $a_{SP} < 0$ is a necessary condition for the stable limit cycles to occur. Throughout all examples we assume that $\alpha(P)$ is linear in P, i.e. $\alpha(P) = \alpha_1 P, \alpha_1 > 0$.

4.1. Steady states

Firstly, we investigate a simple framework. We specify the attractiveness function $a(S, k_1, P) = k_1(\bar{a}_1 S - \bar{a}_2 P + \bar{a}_3) = a_1 S - a_2 P + a_3$ linearly in S and P, while the investment cost function is given by a quadratic function $c(I) = \frac{1}{2} I^2$. Now the canonical system is a system of six linear differential equations:

$$\dot{S} = \lambda_1 - \delta S$$

$$\dot{T} = a_1 S - a_2 P + a_3 - b\,T$$

$$\dot{P} = \sigma S + \tau T - \alpha_1 P$$

$$\dot{\lambda}_1 = (r + \delta)\lambda_1 + k_1 + k_2 - a_1 \lambda_2 - \sigma \lambda_3$$

$$\dot{\lambda}_2 = (r + b)\lambda_2 - p - \tau \lambda_3$$

$$\dot{\lambda}_3 = (r + \alpha_1)\lambda_3 + a_2 \lambda_2$$

It is not difficult to show that for any reasonable set of parameters there is a unique steady state, which is characterized by a three-dimensional stable invariant manifold (see Appendix A for the eigenvalues, which we have symbolically computed with Mathematica, [13]). Concerning the optimal trajectories it holds that with the aid of this stable invariant manifold the optimal trajectories are defined, because sufficiency conditions are fulfilled. Let us formulate this result in a less technical way: starting from an initial stock of service facilities, tourists and pollution, the optimally controlled system oscillates to a unique steady state.

In our second model the attractiveness function $a(S, k_1, P) = a_1 S^\gamma - a_2 P + a_3, 0 < \gamma < 1$ does no longer depend linearly on S. However, the qualitative behavior of the canonical system does not change (see Appendix A for the eigenvalues). Again, there is a unique steady state with a three-dimensional stable invariant manifold. The only difference with the linear case is that formerly real eigenvalues can now become complex; in other words the optimal controlled system may converge to the steady state in an oscillatory way.

Finally, we now consider specifications that fulfil all characteristics described in Section 2. We define a more general cost function of cost investment, i.e. $c(I) = c_1 I + (c_2/h) I^h$, with $c_1, c_2 > 0$, $h > 1$. More crucial, the attractiveness function $a(S, k_1, P)$ is assumed to be

$$a(S, k_1, P) = f(k_1)\, S^{\gamma} e^{-\nu P^2}$$

with $f(\cdot) \geqslant 0$, $f'(\cdot) > 0$, $f(0) = 0$, $\nu > 0, 0 < \gamma < 1$. The maximum principle then gives

$$I = ((\lambda_1/c_2) - (c_1/c_2))^{1/(h-1)} \tag{15}$$

The resulting dynamic system is equal to

$$\dot{S} = ((\lambda_1/c_2) - (c_1/c_2))^{1/(h-1)} - \delta S \tag{16}$$

$$\dot{T} = f(k_1) S^{\gamma} e^{-\nu P^2} - b T \tag{17}$$

$$\dot{P} = \sigma S + \tau T - \alpha_1 P \tag{18}$$

$$\dot{\lambda}_1 = (r + \delta)\lambda_1 + k_1 + k_2 - \lambda_2 f(k_1) \gamma S^{\gamma - 1} e^{-\nu P^2} - \sigma \lambda_3 \tag{19}$$

$$\dot{\lambda}_2 = (r + b)\lambda_2 - p - \tau \lambda_3 \tag{20}$$

$$\dot{\lambda}_3 = (r + \alpha_1)\lambda_3 + 2 f(k_1)\, \nu P e^{-\nu P^2} S^{\gamma}\, \lambda_2 \tag{21}$$

Proposition 1.

The tourism optimal control model (1)–(4) has a unique steady state with a positive stock of tourists.

Proof. See Appendix B.

Determining the eigenvalues of the Jacobian matrix (see Appendix A) is now a lot more complicated than in the previous two examples. What we did is numerically computing the steady-state values and the corresponding eigenvalues of the Jacobian matrix (which we have done with the software package Mathematica [13]). We set the discount rate to 10 per cent, i.e. $r = 0.1$. The income generated by one tourist p equals 2.5, the convexity parameter of the cost function h is 1.06, and c_1 and c_2 are set to 0.61 and 0.58, respectively. We assume $\delta = 0.085$, $\gamma = 0.95$, $\nu = 0.086501$, $b = 0.076$, $\sigma = 0.6$, $\tau = 0.88$ and $\alpha_1 = 0.064$. The aggregated per service unit cost $k_1 + k_2$ equals 0.3257323, and $f(k_1) = 0.9$. For these parameter values the stationary point is given by $S^* = 0.158268$, $T^* = 0.250745$, $P^* = 4.93151$, $\lambda_1^* = 1.05787$, $\lambda_2^* = 9.49703$, $\lambda_3^* = -0.941502$. The steady state value of Investment in touristic infrastructure is $I^* = 0.0134527$.

The eigenvalues of the Jacobian matrix evaluated at the steady-state show that the stationary point is a saddle point with a three-dimensional stable-invariant manifold. From an economic point of view, saddle point stability means that in the long run the region converges to a stationary state where infrastructure investment just equals the depreciation of that stock. Further, the number of tourists remains constant over time and the addition of new pollution to the stock is neutralized by the absorptive capacity so that, in a way, we may speak of sustainable development in our region. As the attractiveness function a is no longer concave in S and P, the Hamiltonian is not concave in the state variables P and S; hence, we get here only candidates for an optimal solution.

4.2. *Limit cycles*

Taking the discount rate as bifurcation parameter we observe that for $r_{\text{crit}} = 0.0827797651$ two eigenvalues become purely imaginary and give rise to a Hopf bifurcation which leads to stable limit cycles¶ for $r < r_{\text{crit}}$.

Making use of COLSYS [14] such a stable cycle can be calculated for $r = 0.0824499$. The steady state is given by $S^* = 0.1540850$, $T^* = 0.2512494$, $P^* = 4.8992263$, $\lambda_1^* = 1.0571557$, λ_2 $^* = 9.7770773$, $\lambda_3^* = -1.0804808$, $I^* = 0.0130972$. Figure 1 shows the limit cycle in the P–T phase diagram. The next section provides a detailed economic explanation of this solution.

When limit cycles occur in our model the touristic region does not converge to a situation with a constant number of tourists in the long run. Instead, cyclical fluctuations in that variable can be observed which may be explained as follows. Within a region with many tourists much pollution is generated, which has a negative impact on the attractiveness of that region. Further, a high stock of pollution also tends to lower investment in infrastructure because one additional unit of touristic infrastructure attracts more additional tourists the cleaner the environment is (follows from $a_{SP} < 0$). As a consequence, the number of tourists will diminish. A decline in the number of tourists, however, reduces pollution which tends to attract more tourists. Moreover, a decline in the stock of pollution will lead to a lower shadow price of pollution which for its part raises the shadow price of infrastructure and, thus, investment activities. That causes a higher stock of infrastructure capital which acts positively on the attractiveness of the region and the number of tourists will rise again. In the next section, we give a more thorough discussion of the different regimes which can be identified.

¶Using BIFDD [15] we were able to proof numerically the existence of stable cycles for $r < r_{\text{crit}}$; the coefficients of the canonical form are

$$A = -3.2903698 \pm 0.1330538$$

$$B = -10.1109477 \pm 0.1376684$$

$$C = -0.0601466 \pm 3.8391187 \times 10^{-9}$$

$$D = -3.9904840 \pm 1.7839538 \times 10^{-6}$$

$$\omega = 0.2984661$$

Hence at r_{crit} we observe a supercritical Hopf bifurcation.

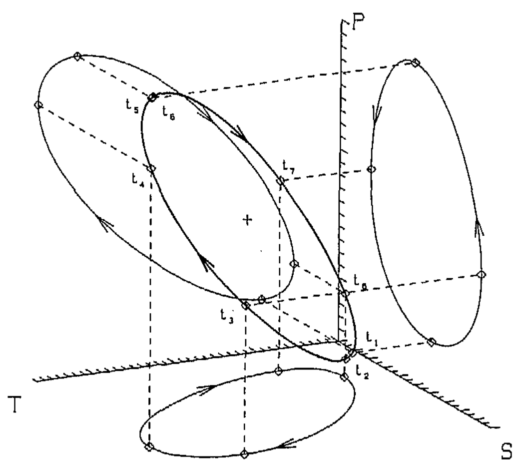

Figure 1. Phase portrait of the (S, T, P) state space.

4.3. *Discussion of the limit cycles*

In Figure 2 we depict the trajectories of S, T, and P corresponding to the cycle in Figure 1. Furthermore, we add the corresponding (optimal) investment policy I.

Given the initial values of S, T, and P at time t_1 as in Figure 1 and following the optimal investment policy I the current cash flow follows the path of the curve, which is denoted by 'cycle' in Figure 3. The straight line in Figure 3 depicts the current cash flow, if the initial values of touristic infrastructure S, pollution P and the number of tourists T are fixed to the values of the steady-state equilibrium and the decision-maker follows the (constant) optimal investment rule I.

4.3.1. Prosperity regime. The cycle starts at time t_1. Here, the environment is clean (i.e. low P), the touristic infrastructure S is medium-sized, but the number of tourists T is still relatively low, and, hence, the current cash flow is low, too, as Figure 3 shows. Relatively low environmental pollution and satisfactory infrastructure attract many additional tourists, so the touristic planner is eager to invest at a very high rate I casting a beady eye on huge cash flows in the near future. Because infrastructure reaches a high standard, the tourism planner tends to cut down investment. And in addition, he/she recognizes that the increasing touristic sector leads to serious environmental pollution. He/she reduces infrastructural investment I, whether to increase the current cash flow (cf. Figure 3) or to avoid an environmental (and touristic) disaster. The latter argument is supported by the observation, that at time t_1, t_2 two turning-points(increase to decrease for investment and decrease to increase for pollution) almost coincide. There is now a period, in which investment decreases, but it is high enough to ensure an increase in S, at least until t_3. Afterwards a low level investment is optimal, however, tourism is booming due to a good infrastructure and—still—due to a relatively clean environment. Low investment and a lot of tourists guarantee a high current cash flow, as we can see in Figure 3.

We denote this time period, that lasts until time t_4, as prosperity regime, since the tourism industry is expanding and prospering, which is seen from the increasing number of tourists.

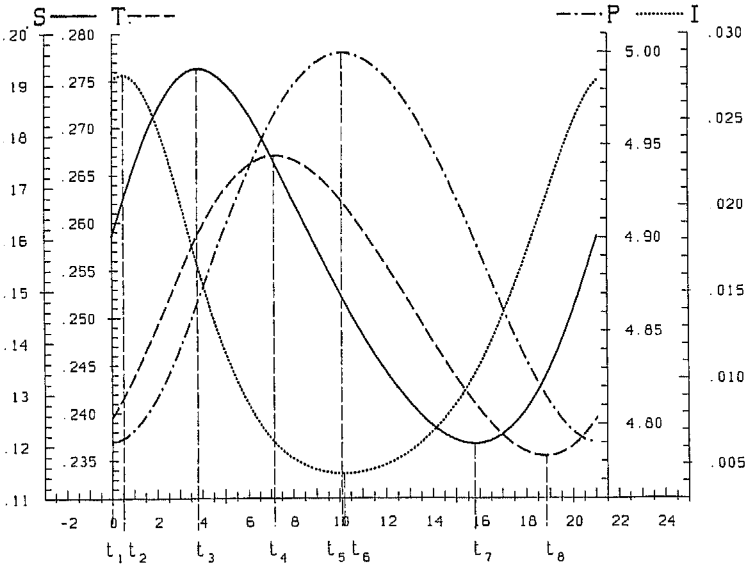

Figure 2. Time paths of the control variable I, and the state variables S, T and P.

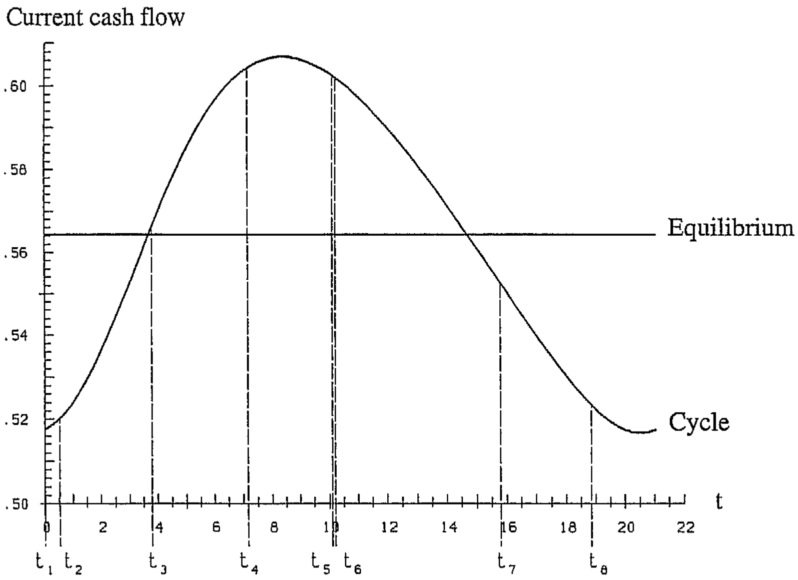

Figure 3. Current cash flow evaluated in the steady state equilibrium and along the cycle.

4.3.2. Saturation regime. It is pay-day—Figure 3 shows that during the time period beginning at t_4 and lasting till t_6 current cash flow peaks. The number of tourists saturates, because the accumulation of pollution limits its growth. More and more a reduction of the number of tourists and touristic services is optimal in order to decrease the speed of the increase of environmental pollution or even to reduce environmental pollution.

4.3.3. Declining regime. The recovery of the environment is the main topic of the time period from t_6 to t_8. With a very small lag behind the peak of pollution P, it is optimal to increase investment in touristic infrastructure. The numerical example reveals the fact that cyclical pattern of P and I is almost 'opposite' (with a very small lag of I behind P): if P is very high (low), I is very low (high). Note that such behaviour makes economic sense and provides an intuitively appealing managerial rule. Of course, increasing investments cut down current cash flow—as Figure 3 depicts—especially in a time period, where the earnings from the tourism decline. In the later part of this regime —t_7 to t_8—increasing investment and increasing infrastructure announce already the following phase of recovering.

4.3.4. Recovering regime. The low of the current cash flow is reached just after t_8, however, increasing touristic infrastructure standard S—due to a high investment level I—and a clean environment promise an increase in the near future. Finally, at t_1 the cycle start again.

4.4. Bifurcation diagram

In order to investigated when cyclical investment policies actually occur we provide a bifucation diagram with γ and ν on the axes, while keeping the other parameters at the same level as before. The parameters γ and ν govern the attractiveness function $a(S, k_1, P)$; γ influences the increase of tourists due to additional service facilities, whereas ν influences the negative impact of pollution on the attractiveness of existing service facilities. The bifurcation diagram in Figure 4 shows that for small γ and ν there exists a (unique) attracting steady state. Increasing both γ and

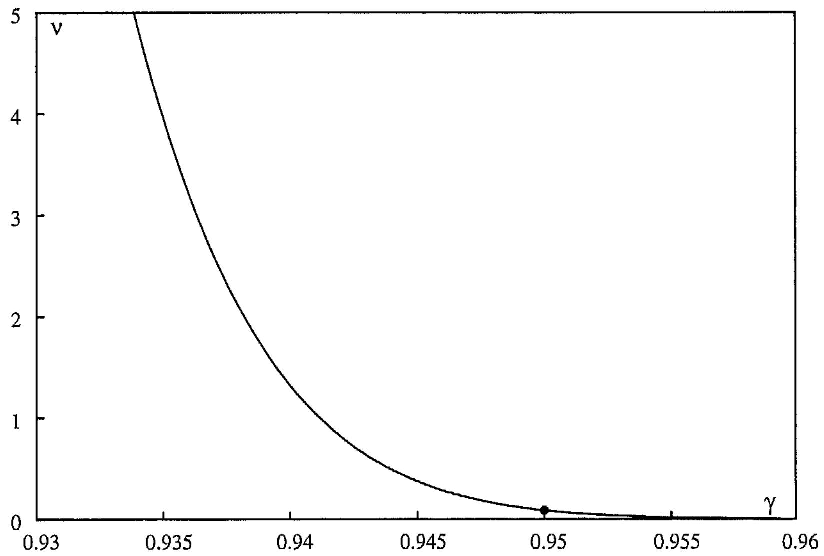

Figure 4. Bifurcation diagram.

ν, i.e. raising the increase of tourists due to extra infrastructure and the impact of pollution, eventually leads to cyclical optimal investment policies. Or, in other words, limit cycles especially occur when the product of γ and ν is large, thus when Abs (a_{SP}) is large; as already became clear from Section 4.1, a_{SP} being negative is a necessary condition for stable limit cycles to occur.

5. CONCLUSIONS AND EXTENSIONS

Nowadays, the tourism industry has to deal with the following dilemma: on the one hand tourists are attracted both by a clean environment and a good touristic infrastructure, but, on the other hand, the tourism industry is one of the main polluters in the relevant regions. The aim of this paper is to find optimal investment policies that guarantee a flourishing tourism industry. Since polluted regions distract tourists, the tourism planner has to take care of the environment at the same time. Our analysis is based on rather simple and general assumptions as concerns the interactions between the three main components of the system: infrastructure, tourists, and environment. While Reference [16] used a similar descriptive model to predict the economic and environmental impact of given policies, the present analysis wants to determine optimal investment policies for maximizing tourism industry profits, while taking into account the negative effect of pollution on the number of tourists.

Essentially, two types of long-run behaviour turn out to be optimal. First, the long-run equilibrium may be a saddle point. In this case, the touristic region converges to a situation with constant investments, a constant number of tourists and a constant level of pollution, which means that the instantaneous pollution flow is neutralized by the absorptive capacity. Thus, the level of investment as well as all other variables are constant in the long-run.

Second, and more interesting, we identify a scenario under which persistent oscillations are optimal. This means that the optimal investment rate fluctuates persistently over time implying that not only the touristic infrastructure and the number of tourists oscillate but also the quality of the environment. In this case, periods with over-investment in touristic infrastructure occur, alternated by periods in which investments are lower than the level that maintains the steady state.

It is a well-known fact that, due to environmental pollution, after a flourishing period tourists avoid some regions to visit more attractive sites. In order to compensate this instability, tourism managers could increase investment and develop special services to attract tourists. Sometimes they are successful with such programs, but at the expense of the environment, which is heavily deteriorated (see Reference [16]). Another policy, however, would be to reduce the touristic activities to give the environment a chance to recover, which in fact happens in our cyclical solution. Of course, environmentalists would advocate such behavior, but here we showed that this could also be optimal from a profit maximizing point of view.

It would be an interesting task to derive general conditions under which persistent cycles turn out to be optimal. However, we are rather skeptical that such conditions can be obtained in a general setting. Thus, the only thing we can assert is that optimal cycles exist for certain parameter constellations.

The problem that an infinitesimal change of a parameter (here: the discount rate) may lead to a sudden change of the qualitative behaviour of the solution paths (from a saddle-point to a limit cycle) is ubiquitous. Benhabib [17] wrote that there does not exist a real economic interpretation for a sudden qualitative change: 'The Hopf Bifurcation Theorem brings out a qualitative change in the topological structure of a dynamic system when a parameter is varied continuously. However, there does not seem to be a corresponding qualitative change in the economic structure as the orbits appear. We cannot associate the appearance of orbits with, say, the emergence of the inferiority of present or past consumption or satiation in the utility function'.

It is certainly admitted that the proposed model is highly stylized, and, probably much too simple to model a concrete planning situation. It shows, however, that under some circumstances alternating periods of high and low investment reassures environmental quality without reducing touristic activities too much. Our analysis might be seen as a first step to prove the superiority of cyclical investment strategies compared with a long-run steady control. Since we do not deal with a concrete planning case, no effort has been made at this stage to validate our model with empirical data. However, the latter task should clearly be a main issue of future research. Another important extension would be to include another investment opportunity, which is more costly but less deteriorating.

Our planner is a profit maximizing tourism manager. He/she cares on environmental quality only insofar as pollution deters potential tourists. Most of the decision-making oriented literature (cf. for example References [18–20], or for a survey [8]) deals with abatement policies or other activities to reduce environmental pollution, both on the micro- and macroeconomic level. Thus, it would be quite natural to include those activities as a second control variable, $A(t)$, influencing the dynamics of the stock of environmental pollution, $P(t)$. Since abatement is costly, an additional cost term has to be included in the objective functional. As a 'subcase' of such an extension the optimal allocation of a budget between investment into services and pollution control could be considered. This budget may be either constant or depend on the income generated by the tourists.

Another variant of the endless story of tourism and environment would be the explicit inclusion of the pollution disutility in the objective functional. We might consider a central planner who pursues environmentalistic goals not only in order to maximize touristic revenues, but who draws explicit utility from a clean environment since forests, lakes and mountains are of value not only for tourists but also for local people. Then, an interesting question would be how the optimal investment policy into touristic industry is affected by such an explicit negative assessment of pollution.

APPENDIX A

The maximum principle results in

$$\lambda_1 = c'(I) \Rightarrow I = I(\lambda_1)$$

and the canonical system is equal to $(\alpha(P) := \alpha_1 P)$:

$$\dot{S} = I(\lambda_1) - \delta S$$

$$\dot{T} = a(S, k_1, P) - bT$$

$$\dot{P} = \sigma S + \tau T - \alpha_1 P$$

$$\dot{\lambda}_1 = (r+\delta)\lambda_1 + k_1 + k_2 - a_S(S,k_1,P)\lambda_2 - \sigma\lambda_3$$

$$\dot{\lambda}_2 = (r+b)\lambda_2 - \tau\lambda_3 - p$$

$$\dot{\lambda}_3 = (r+\alpha_1)\lambda_3 - a_P(S,k_1,P)\,\lambda_2$$

Firstly, a linear: The six eigenvalues of the Jacobian matrix of the canonical system are:

$$-\delta$$

$$r+\delta$$

$$\tfrac{1}{2}\left(-b-\alpha_1 \pm \sqrt{(b-\alpha_1)^2 - 4\tau a_2}\right)$$

$$\tfrac{1}{2}\left(b+2r+\alpha_1 \pm \sqrt{(b-\alpha_1)^2 - 4\tau a_2}\right)$$

The canonical systems for different, reasonable set of parameters are topologically equivalent.

Secondly, a concave in S and linear in P: The six eigenvalues of the Jacobian matrix of the canonical system evaluated at the steady state $(S^*, T^*, P^*, \lambda_1^*, \lambda_2^*, \lambda_3^*)$ are

$$\tfrac{1}{2}\left(r \pm \sqrt{(r+2\delta)^2 + 4\gamma(\gamma-1)S^{*\gamma-2}\lambda_2^*}\right)$$

$$\tfrac{1}{2}\left(-b-\alpha_1 \pm \sqrt{(b-\alpha_1)^2 - 4\tau a_2}\right)$$

$$\tfrac{1}{2}\left(b+2r+\alpha_1 \pm \sqrt{(b-\alpha_1)^2 - 4\tau a_2}\right)$$

From the viewpoint of topological equivalence there is no change to the linear case.

Thirdly, a concave in S and concave-convex in P; $a_{SP}<0$: The Jacobian matrix of the canonical system evaluated in $(S, T, P, \lambda_1, \lambda_2, \lambda_3)$ is

$$J = \begin{pmatrix} -\delta & 0 & 0 & dI/d\lambda_1 & 0 & 0 \\ f(k_1)\gamma S^{\gamma-1}e^{-\nu P^2} & -b & -2\nu P e^{-\nu P^2} f(k_1) S^\gamma & 0 & 0 & 0 \\ \sigma & \tau & -\alpha_1 & 0 & 0 & 0 \\ \begin{array}{c}-\lambda_2 f(k_1)\gamma(\gamma-1)\cdot \\ S^{\gamma-2}e^{-\nu P^2}\end{array} & 0 & \begin{array}{c}2\nu P e^{-\nu P^2}\lambda_2\cdot \\ f(k_1)\gamma S^{\gamma-1}\end{array} & r+\delta & \begin{array}{c}-f(k_1)\gamma\cdot \\ S^{\gamma-1}e^{-\nu P^2}\end{array} & -\sigma \\ 0 & 0 & 0 & 0 & r+b & -\tau \\ \begin{array}{c}2\nu P e^{-\nu P^2}\lambda_2\cdot \\ f(k_1)\gamma S^{\gamma-1}\end{array} & 0 & \begin{array}{c}2\lambda_2\nu f(k_1)S^\gamma e^{-\nu P^2}\cdot \\ (1-2\nu P^2)\end{array} & 0 & \begin{array}{c}2\nu P e^{-\nu P^2}\cdot \\ f(k_1)S^\gamma\end{array} & r+\alpha_1 \end{pmatrix}$$

Using the values of parameter specified in the text the eigenvalues of the Jacobian matrix evaluated at the steady state are $\mu_1 = 0.642595$, $\mu_2 = -0.542595$, $\mu_{3,4} = 0.140112 \pm 0.297614\,i$, and $\mu_{5,6} = -0.0401124 \pm 0.297614i$.

APPENDIX B

Lemma B1.

At an interior steady state $(S^*, T^*, P^*, \lambda_1^*, \lambda_2^*, \lambda_3^*)$ of the system of differential equations (15)–(21) the steady state values S^*, P^* are solutions of the following two equations:

$$F(S,P) = \sigma S + \frac{\tau}{b} f(k_1) S^{\gamma} e^{-\nu P^2} - \alpha_1 P = 0 \tag{B1}$$

$$P^2 - 2\left(\frac{\sigma S}{2\alpha_1} + \frac{V}{2\sigma S U(S)}\right) P + \frac{V}{\alpha_1 U(S)} + \frac{W(S)}{\alpha_1 U(S)} = 0 \tag{B2}$$

with

$$\tilde{K}(S) := (r+\delta)\, c'(\delta S) + k_1 + k_2$$

$$U(S) := 2\nu b \left(K(S) + \frac{p\sigma}{\tau}\right)$$

$$V := b\gamma\,(r+\alpha_1)\frac{p\sigma}{\tau}$$

$$W(S) := K(S)\,(r+\alpha_1)\,(r+b)$$

From S^*, P^* the remaining steady-state values are given as

$$T^* = \frac{1}{b} f(k_1) S^{*\gamma} e^{-\nu P^{*2}}$$

$$\lambda_1^* = c'(\delta S^*)$$

$$\lambda_2^* = \frac{p\,(r+\alpha_1)}{\Delta(S^*, P^*)}$$

$$\lambda_3^* = \frac{-2\nu p P^* f(k_1) S^{*\gamma} e^{-\nu P^{*2}}}{\Delta(S^*, P^*)}$$

with

$$\Delta(S^*, P^*) := (r+\alpha_1)\,(r+b) + 2\,\nu\,\tau P^* f(k_1) S^{*\gamma} e^{-\nu P^{*2}}$$

Proof. The equations $\dot{\lambda}_2 = 0$ and $\dot{\lambda}_3 = 0$ are linear in λ_2 and λ_3, which implies

$$\lambda_2 = \frac{p\,(r+\alpha_1)}{\Delta(S,P)}$$

$$\lambda_3 = \frac{-2\nu p P f(k_1) S^{\gamma} e^{-\nu P^2}}{\Delta(S,P)}$$

and from $\dot{S} = 0$ and $\dot{T} = 0$ we get

$$\lambda_1 = c'(\delta S) \qquad T = \frac{1}{b} f(k_1) S^{\gamma} e^{-\nu P^2}$$

Substituting into $\dot{P} = 0$ and $\dot{\lambda}_1 = 0$ yields

$$\sigma S + \frac{\tau}{b} f(k_1) S^{\gamma} e^{-\nu P^2} - \alpha_1 P = 0 \tag{B3}$$

$$\tilde{K}(S) - \frac{p f(k_1) S^{\gamma-1} e^{-\nu P^2} ((r + \alpha_1)\gamma - 2\sigma\nu PS)}{\Delta(S,P)} = 0 \tag{B4}$$

Taking only the numerator of (B4) gives

$$\tilde{K}(S)\left((r+\alpha_1)(r+b) + 2\nu\tau P f(k_1) S^{\gamma} e^{-\nu P^2}\right) - p f(k_1) S^{\gamma-1} e^{-\nu P^2} ((r+\alpha_1)\gamma - 2\sigma\nu PS) = 0 \tag{B5}$$

or

$$\overbrace{\tilde{K}(S)(r+\alpha_1)(r+b)}^{=:W(S)} - f(k_1) S^{\gamma-1} e^{-\nu P^2} (p(r+\alpha_1)\gamma - 2\sigma\nu pPS - 2\nu\tau PSK(S)) = 0 \tag{B6}$$

From (B3) it follows that

$$f(k_1) S^{\gamma-1} e^{-\nu P^2} = \frac{b}{\tau}\left(\alpha_1 \frac{P}{S} - \sigma\right) \tag{B7}$$

Substituting (B7) into (B6) gives

$$W(S) - \frac{b}{\tau}\left(\alpha_1 \frac{P}{S} - \sigma\right)(p(r+\alpha_1)\gamma - 2\nu(\tau\tilde{K}(S) + p\sigma)SP) = 0 \tag{B8}$$

which is for fixed S a quadratic equation in P. Expanding and collecting terms yields Equation (B2). □

Lemma B2.

For fixed S the equation $F(S,P) = 0$ implicitly defines a function $P_3(S)$, which has the following properties:

- $P_3(0) = 0$; $\lim_{S\to\infty} P_3(S) = \infty$.
- $P_3(S)$ is monotonically increasing.
- $P_3(S)$ is defined $\forall S \geqslant 0$.
- $P_3(S)$ has a vertical asymptote at the origin for $\gamma < 1$.

Proof. The partial derivatives $\partial_S F(S,P) = \sigma + \frac{\tau}{b}\gamma f(k_1) S^{\gamma-1} e^{-\nu P^2} > 0$ and $\partial_P F(S,P) = -\frac{\tau}{b}\left(2\nu P f(k_1) S^{\gamma} e^{-\nu P^2}\right) - \alpha_1 < 0$ gives

$$P_3'(S) = \frac{\sigma b + \tau\gamma f(k_1) S^{\gamma-1} e^{-\nu P^2}}{\alpha_1 b + 2\nu\tau P f(k_1) S^{\gamma} e^{-\nu P^2}} > 0$$

Obviously $P_3'(S) \to \infty$ for $(S,P) \to (0,0)$ in case that $\gamma < 1$. □

For fixed S the solutions of the quadratic equation (B2) are given by

$$P_1(S) = \frac{\sigma S}{2\alpha_1} + \frac{V}{2\sigma S U(S)} + \sqrt{\left(\frac{\sigma S}{2\alpha_1} - \frac{V}{2\sigma S U(S)}\right)^2 - \frac{W(S)}{\alpha_1 U(S)}}$$

$$P_2(S) = \frac{\sigma S}{2\alpha_1} + \frac{V}{2\sigma S U(S)} - \sqrt{\left(\frac{\sigma S}{2\alpha_1} - \frac{V}{2\sigma S U(S)}\right)^2 - \frac{W(S)}{\alpha_1 U(S)}}$$

For the functions $P_1(S)$ and $P_2(S)$ the following lemma holds:

Lemma B3.

There are two constants S_1 and S_2 such that

(1) The term under the squareroot is positive and monotonically decreasing $\forall S \in (0, S_1]$, and negative $\forall S \in (S_1, S_2)$.

(2)
$$\frac{\sigma S}{2\alpha_1} - \frac{V}{2\sigma S U(S)} < 0 \quad \forall S \in (0, S_1] \quad \text{and} \quad \frac{\sigma S}{2\alpha_1} - \frac{V}{2\sigma S U(S)} > 0 \quad \forall S \in [S_2, \infty)$$

(3)
$$\lim_{S\to 0^+} P_1(S) = +\infty, \quad \lim_{S\to 0^+} P_2(S) = 0, \quad \lim_{S\to 0^+} P_2'(S) < +\infty$$

Proof. Property (3) is obvious and to prove properties (1) and (2) it is enough to note that

$$\lim_{S\to 0^+} \frac{\sigma S}{2\alpha_1} - \frac{V}{2\sigma S U(S)} = -\infty$$

and that

$$\frac{\sigma S}{2\alpha_1} - \frac{V}{2\sigma S U(S)} \quad \text{as well as} \quad \frac{W(S)}{\alpha_1 U(S)} = \frac{(r+\alpha_1)(r+b)}{2vb\alpha_1} \frac{K(S)}{K(S) + (p\,\sigma)/\tau}$$

are monotonically increasing in S. □

Lemma B4.

The optimal control model (4) together with the dynamics (1)–(3) cannot have a steady state with $S^* \geqslant S_2$.

Proof. Suppose that at the steady state $S^* \geqslant S_2$ holds. This implies

$$P_2(S^*) \leqslant P_1(S^*) \leqslant \frac{\sigma S^*}{2\alpha_1} + \frac{V}{2\sigma S^* U(S^*)} + \sqrt{\left(\frac{\sigma S^*}{2\alpha_1} - \frac{V}{2\sigma S^* U(S^*)}\right)^2} = \frac{\sigma S^*}{\alpha_1}$$

and thus $P^* \leqslant \sigma S^*/\alpha_1$. Hence,

$$F(S^*, P^*) = \sigma S^* + \frac{\tau}{b} f(k_1) S^{*\gamma} e^{-\nu P^{*2}} - \alpha_1 P^* \geqslant \frac{\tau}{b} f(k_1) S^{*\gamma} e^{-\nu P^{*2}} > 0,$$

which is a contradiction to Equation (B1). □

Lemma B5.

The graphs of the functions $P_2(S)$ and $P_3(S)$ do not intersect for $S > 0$.

Proof. The function $P_2(S)$ is defined on the interval $[0, S_1]$; for any $S^* \in [0, S_1]$ it holds:

$$P_2(S^*) \leqslant \frac{\sigma S^*}{2\alpha_1} + \frac{V}{2\sigma S^* U(S^*)} - \sqrt{\left(\frac{V}{2\sigma S^* U(S^*)} - \frac{\sigma S^*}{2\alpha_1}\right)^2} = \frac{\sigma S^*}{\alpha_1}$$

Hence,

$$F(S^*, P_2(S^*)) = \sigma S^* + \frac{\tau}{b} f(k_1) S^{*\gamma} e^{-\nu P_2^2(S^*)} - \alpha_1 P_2(S^*) \geqslant \frac{\tau}{b} f(k_1) S^{*\gamma} e^{-\nu P_2^2(S^*)},$$

which is positive, when $S > 0$. □

Proposition 1 (equivalent formulation)

The dynamical system (15)–(21) has a unique equilibrium with $S > 0$.

Proof. Lemma B4 says that steady-state values (S^*, P^*) have to lie in the region $S \leqslant S_1$, and Lemma B5 says, respectively, that they are given as intersections of $P_2(S)$ and $P_3(S)$. As $P_1(S)$ decreases monotonically from ∞ to $P_1(S_1)$ and as $P_3(S)$ increases monotonically from 0 to ∞, there is a unique steady state for $S > 0$ (Figure B1). □

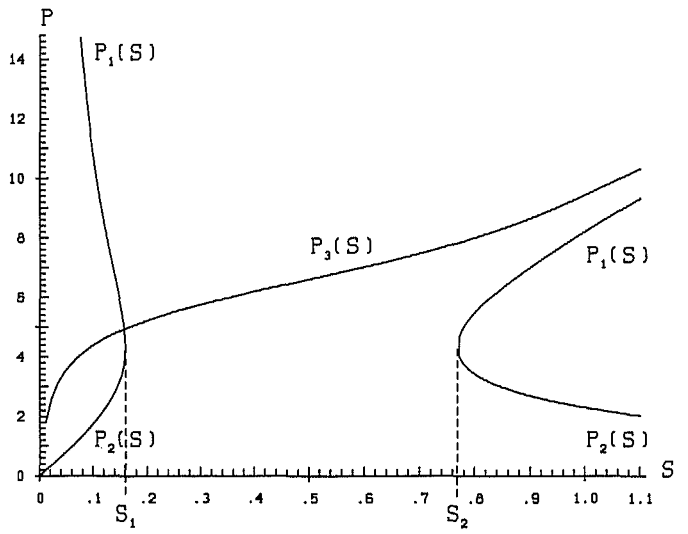

Figure B1. Graphs of the functions $P_1(S), P_2(S)$ and $P_3(S)$.

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