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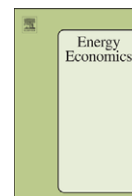
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Electricity portfolio management: Optimal peak/off-peak allocations

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ABSTRACT

Electricity purchasers manage a portfolio of contracts in order to purchase the expected future electricity consumption profile of a company or a pool of clients. This paper proposes a mean-variance framework to address the concept of structuring the portfolio and focuses on how to optimally allocate positions in peak and off-peak forward contracts. It is shown that the optimal allocations are based on the difference in risk premiums per unit of day-ahead risk as a measure of relative costs of hedging risk in the day-ahead markets. The outcomes of the model are then applied to show (i) that it is typically not optimal to hedge a baseload consumption profile with a baseload forward contract and (ii) that, under reasonable assumptions, risk taking by the purchaser is rewarded by lower expected costs.

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1. Introduction

In many countries electricity markets are liberalized. As a result large electricity purchasers, e.g. large industrial consumers and electricity retail distribution companies, need to contract the future expected electricity consumption (load) for their own company or for a pool of clients. In liberalized electricity markets, they can do so by managing a hedging portfolio of contracts that involve delivery of electricity in future time periods and/or financially settle the difference between a fixed and a variable price. Examples of such contracts are day-ahead contracts, derivatives such as forwards, futures, swaps, variable volume or swing options and direct or indirect investments in energy production facilities.¹ Proper management of the load hedging portfolio involves a continuous assessment of (a) the types of instruments (contracts) to buy or sell and (b) at what moment the portfolio needs to be rebalanced according to the risks the electricity purchaser prefers to take. An obvious objective of the purchaser is to incur the lowest expected costs for the expected electricity load, given a specific risk target.

Since the beginning of the liberalization of energy markets, researchers have primarily focused on the price characteristics of different energy commodities and the valuation of derivative contracts. Traditionally the academic literature has dealt with proposing optimal hedging strategies using commodity futures.² The issue of constructing efficient portfolios for electricity purchasers has received much less attention in the academic literature. Given the sometimes extreme price fluctuations in energy commodities, we feel that this issue is grossly undervalued. Poorly constructed portfolios exhibit either too high expected costs at a given risk level or, alternatively, too much risk for the current level of expected costs. This paper focuses on optimal instrument selection for a rational electricity purchaser that cares about the mean and variance of the future sourcing costs. It specifically tackles the question how electricity purchasers should choose between peak and off-peak forward contracts in order to structure their portfolios optimally. To do so, we construct a simple one-period framework and cast the allocation problem in a portfolio framework to find the optimal allocations to the forward contracts and the day-ahead market.

The paper is organized as follows. Section 2 discusses the literature on energy portfolio management. In Section 3 we present our model. Section 4 highlights some managerial implications of the model and provides answers to the questions how a company should purchase a baseload consumption profile and whether taking risk is rewarded by lower expected costs. Section 5 concludes.

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¹ Indirect investments in power plants often take the form of virtual power plants or tolling agreements, both being power purchasing agreements in which the owner purchases electricity from a power plant and pays according to a formula that relates the price to, among others, fuel prices.

² See for example Moschini and Myers (2002) and Alizadeh et al. (2008), and the references therein.

2. Portfolio structuring in electricity markets

In order to facilitate trading of power contracts many countries have established over-the-counter (OTC) and centralized markets. The two most prevalent markets are the day-ahead and forward/futures markets. On the day-ahead market, traders can submit bids and offers for amounts of electricity to be delivered in the individual hours of the next day. This market is the closest equivalent to a spot market.³ Electricity purchasers use day-ahead markets for buying (a part of) their electricity consumption, but the amount of price variation in these markets is substantial. As electricity cannot be stored in an efficient way, prices are very volatile, and seasonal and price spikes frequently occur.⁴

Instead of taking the risk of price variations in the day-ahead market, purchasers seek protection, depending on their risk appetite, and manage a portfolio of derivative contracts that involve delivery at future dates against fixed prices. Popular contracts are the so-called baseload and peakload forward and future contracts that can be traded on all OTC markets (forwards) and exchanges (futures).⁵ On many electricity markets around the world electricity can be traded by using contracts that apply specifically to peak and off-peak hours.⁶ Baseload contracts involve the delivery of 1 MW in all hours of the delivery period against a price fixated at the moment at which the transaction occurs. Peakload contracts are defined similarly, but involve delivery only in the peak hours of the delivery period. Delivery periods range from weeks, months, quarters to calendar years, sometimes up to six years ahead. By holding a portfolio of these contracts, the purchaser can already lock in the acquisition of (a part of) the expected future consumption long before the actual delivery and consumption period against fixed prices and can thereby manage the risks faced from price variations in the day-ahead market.

The prices of baseload and peakload forward contracts exhibit different characteristics than day-ahead prices. According to the expectation theory,⁷ forward prices for non-storable commodities reflect the expectation of market participants on the (average) spot price in the delivery period and a risk premium that compensates producers for bearing the uncertainty of committing to sell against fixed prices. In electricity markets, risk premiums can be positive and negative. For instance, Bessembinder and Lemmon (2002) and Karakatsani and Bunn (2005) find negative risk premiums in low-demand off-peak hours due to power producers who are willing to pay a premium for not having to cut down production from plants with long ramp-up and ramp-down times in order to be able to produce more in the high-demand and more expensive peak hours.

For the portfolio manager forward contracts make it possible to fix delivery prices, thereby reducing the exposure to the price fluctuations in the day-ahead market. Purchasing power with forward contracts boils down to hedging the risk faced from the day-ahead market with the expected hedging costs being equal to the risk premium embedded in the forward price.

Given a set of forward and future contracts that can be traded every day, the task of the portfolio manager is to determine the optimal

³ Many countries also run imbalance markets in which power changes hand in real-time. The liquidity of these markets is rather limited and prices are in some cases set by the imbalance operator at their discretion. Karakatsani and Bunn (2005) show that there may be an effect of the imbalance market on the day-ahead market, although they do not provide a clear economic rationale. In this paper we refrain from this relation and leave the balancing markets out of the analysis.

⁴ We refer to Bunn and Karakatsani (2003), Huisman et al. (2007), among others, for an overview on (hourly specific) day-ahead price characteristics.

⁵ In this paper, we do not differentiate between forwards and futures and only mention forward contracts, although in reality small price differences might occur due to differences in settlement procedures and margining schemes.

⁶ See for example the overview in Eydeland and Wolyniec (2003) regarding North-American electricity markets. For Europe, see Brand et al. (2002).

⁷ We refer to Fama and French (1987) for an overview of forward premiums in commodity markets.

selection of forward contracts to hold for various delivery periods. The optimal selection depends on a risk assessment of the day-ahead market, an expectation regarding the expected price in the day-ahead market in the delivery period, the amount of risk premium she needs to pay and her personal (or the company's) appetite for taking risk. The goal of the portfolio manager is to maintain such a portfolio that yields lowest expected costs for electricity consumption while respecting her risk appetite.

This paper builds on the original ideas from Markowitz (1952), who proposes a methodology to construct efficient investment portfolios based on investors' goal to maximize expected future returns on their investments given a certain level of risk. The idea to use portfolio theory to construct energy hedging portfolios is not new. Several researchers have followed the Markowitz methodology to address the hedging decision process, particularly Näsäkkälä and Keppo (2005) and Woo et al. (2004). Both studies focus on the interaction between stochastic consumption volumes and electricity prices (day-ahead and forward contracts) and propose a Markowitz-style mean-variance framework to determine optimal hedging strategies. Näsäkkälä and Keppo (2005) apply static forward hedging strategies in a representative agent setting. The main result from this paper is that agents who are confronted with high load uncertainties will postpone their hedging strategies in the expectation of load uncertainties resolving over time. Their results crucially depend on the assumption on the correlation between forward prices and load estimates. In our view these correlation assumptions are difficult to maintain, especially as we feel no direct causal relation between these variables exists. In general, the relation between volumes, like load or demand, and prices is weak in electricity markets. See for example the evidence in Mount et al. (2006) and Kanamura and Ohashi (2007), who show that only in very extreme cases, where demand is extraordinarily high, that prices react substantially. In normal circumstances the impact of load on day-ahead electricity prices is statistically significant, but the economic significance of this result is limited.

Vehviläinen and Keppo (2004) take the viewpoint of a generating company and solve for hedging strategies using Value at Risk (VaR) as a risk measure instead of standard deviation. Their analysis based on both stochastic consumption and prices is meaningful, yet complex. Even after a number of simplifying assumptions (Vehviläinen and Keppo, 2004) need simulation techniques to solve for the optimal hedging strategies. Again, it is questionable if electricity prices are elastic with respect to consumption patterns. Furthermore, even if prices would be elastic consumption patterns are highly persistent, and therefore less informative for price explaining price behavior.

The paper that is closest to ours is Woo et al. (2004). These authors provide a general strategy of hedging shorter-dated price exposures with forward contracts. They also focus on finding the efficient frontier for trading off expected costs and risks. Our approach is different in the sense that we extend the set of hedging instruments by differentiating between peak and off-peak hours during the day. We find that this feature of the electricity market cannot be ignored. As a result we expect that efficiency of the expected cost-risk trade-off can be enhanced.

With respect to the studies mentioned above this paper focuses on providing analytical insight in the optimal hedging amounts for an electricity purchaser in a setting that allows for different types of forward contracts, day-ahead prices and risk attitudes. Special attention is given to the relative impact on the optimal hedging decisions of forward risk premia.

3. The purchase decision in a one-period framework

At time t , consider an electricity purchaser who has to decide on how to purchase the consumption for future delivery of electricity at day T . We have that $t < T - 1$, i.e. the hedging decision needs to be made

at least 2 days before the delivery period.⁸ Delivery takes place in all hours of day T . The actual consumption volume during hour h on day T equals $v(h, T)$ MW h ($h=1, \dots, 24$).

At time t , the purchaser has the opportunity to enter in forward or future contracts that facilitate delivery on day T at prices fixated at time t . We assume that two forwards contracts exist that deliver in day T : a peak contract that delivers 1 MW of electricity in all the peak hours of day T and an off-peak contract that delivers 1 MW in all the off-peak hours of day T .⁹ Let H_p be the set of peak hours in day T and let H_o be the set of off-peak hours. The number of peak and off-peak hours at day T equal N_p and N_o , respectively. The purchaser has to decide upon the number of peak contracts, θ_p , and off-peak contracts, θ_o . We assume that the forward contracts can be traded in any size with perfect liquidity with no short-sale constraints and zero transaction costs. At time t , the observed market prices for the forward contracts equal $f_o(t, T)$ for the off-peak contract and $f_p(t, T)$ for the peak contract.

In addition to entering in forward contracts, the purchaser can purchase electricity for delivery during day T in the day-ahead market at time $T-1$. We assume that the day-ahead market offers the last opportunity before delivery to purchase electricity. Therefore, the purchaser will use this market to settle the difference between expected consumption volume and the volume already contracted with forward contracts.¹⁰ Let $s(h, T-1)$ be the price in the day-ahead market at $T-1$ for delivery of 1 MW in hour h of day T . Given the above assumptions, the total costs for electricity consumption at day T , $C(T)$, equals:

$$C(T) = N_o \theta_o f_o(t, T) + N_p \theta_p f_p(t, T) + \sum_{h \in H_o} (v(h, T) - \theta_o) s(h, T-1) + \sum_{h \in H_p} (v(h, T) - \theta_p) s(h, T-1). \quad (1)$$

The total cost function is equal to the sum of costs from the volumes purchased with peak and off-peak forward contracts at time t and the costs of the remaining purchases in the day-ahead market. Note that we do not consider quantity risk, i.e. the possibility that trading larger amounts of electricity would affect prices. We refer again to Mount et al. (2006) and Kanamura and Ohashi (2007) who show that only in extreme cases the underlying quantities will affect day-ahead electricity prices.

At time t , the day-ahead prices and the consumption volumes are uncertain. Therefore, the total costs $C(T)$ are uncertain too. The objective of the purchaser is then to construct an optimal allocation over the peak and off-peak forward contracts that yield the lowest expected costs. However, in dealing with the uncertainty, we assume that the purchaser is not willing to take more risk than her risk appetite specifies. This objective is in line with the framework initially proposed by Markowitz (1952) for investment portfolios. In that world an investor first decides on the amount of risk he is willing to take and then to find the portfolio that yields the highest expected return at that risk level. In this paper, we assume that the goal of the purchaser is to achieve lowest expected costs at a risk level that meets her risk

appetite. The risk appetite is defined in terms of a maximum variance level σ_{\max}^2 . Then, the optimization problem of the purchaser becomes:

$$\min_{\theta_o, \theta_p} E_t \{C(T)\} \quad (2)$$

$$\text{s.t. } \text{var}_t \{C(T)\} \leq \sigma_{\max}^2,$$

where $\text{var}_t \{C(T)\}$ is the variance of the total costs anticipated at time t . To further specify the optimization problem and the uncertainty from variations in prices and volumes, we assume that the statistical properties of day-ahead prices and volumes can be described by the first two moments.¹¹ For convenience, we switch to matrix notation.

Let $\mathbf{s}(T-1)$ be a 24×1 vector with the stacked day-ahead prices. $\mathbf{s}(T-1)$ is distributed with a 24×1 mean vector μ_s and a 24×24 hourly covariance matrix Ω_s . Let \mathbf{f}_t be a 2×1 vector with the stacked forward prices $(f_o(t, T) \ f_p(t, T))'$. Likewise, let θ_t be the 2×1 vector $(\theta_o \ \theta_p)'$ and \mathbf{N} the 2×1 vector with $(N_o \ N_p)'$.

The hourly volumes $v(h, T)$ are stacked in the 24×1 vector $\mathbf{v}(T)$. The stochastic properties of the hourly volumes are represented by the 24×1 vector with hourly means μ_v , and the hourly 24×24 covariance matrix Ω_v . Lastly, we introduce a 24×2 selection matrix \mathbf{B} with row h equaling (1 0) when h is an off-peak hour and (0 1) when h is a peak hour ($h=1 \dots 24$).

In matrix notation the cost function $C(T)$ from (1) can then be written as

$$C(T) = \theta_t' (\mathbf{N} \cdot \mathbf{f}_t - \mathbf{B}' \mathbf{s}(T-1)) + \mathbf{v}(T)' \mathbf{s}(T-1), \quad (3)$$

with ‘ \cdot ’ the element-wise matrix multiplication operator. In order to proceed to an analytic solution we assume that the day-ahead prices and the actual consumption volumes are expectation-independent and variance-independent (Bohrnstedt and Goldberger, 1969). This implies among others that all co-variances between day-ahead prices and volumes are zero, i.e. a deviation in the actual consumption volume from its expected value does not necessary lead to a change in day-ahead prices. Of course, this assumption does not hold when a large increase in volumes would lead to a different marginal fuel in the merit order. However, for relatively small changes and under normal market conditions, the assumption is valid in our opinion. Using the results from Bohrnstedt and Goldberger (1969), we have that $E_t \{\mathbf{v}(T)' \mathbf{s}(T-1)\} = \mu_v' \mu_s$ and $\text{var}_t \{\mathbf{v}(T)' \mathbf{s}(T-1)\} = \text{tr}(\Omega_s \Omega_v) + \mu_s' \Omega_v \mu_s + \mu_v' \Omega_s \mu_v$, with $\text{tr}(\cdot)$ the matrix trace operator. From Eq. (3) and the above statistical properties, we can write the expectation and variance of the total cost, conditional on information available at t , as:

$$E_t \{C(T)\} = \theta_t' (\mathbf{N} \cdot \mathbf{f}_t - \mathbf{B}' \mu_s) + \mu_v' \mu_s, \quad (4)$$

and

$$\text{var}_t \{C(T)\} = \theta_t' \mathbf{B}' \Omega_s \mathbf{B} \theta_t + \text{tr}(\Omega_s \Omega_v) + \mu_s' \Omega_v \mu_s + \mu_v' \Omega_s \mu_v + 2 \theta_t' (\mathbf{B}' \mu_s) (\mu_v' \mu_s). \quad (5)$$

⁸ If the decision would be made at $t=T-1$ only the day-ahead market is available to the purchaser. As a result the optimization problem becomes trivial.

⁹ In most electricity markets, off-peak contracts cannot be traded directly but can be constructed synthetically by simultaneously buying one baseload contract (that delivers a fixed volume in each hour of the delivery period) and selling one peak contract. The assumption that an off-peak contract can be traded in the market is made for simplicity and does not lead to a loss in generalization.

¹⁰ In real life, imbalance and other intraday markets exist where electricity can be purchased that will be delivered in the same day. However, the liquidity of these markets is thin relative to the day-ahead markets and practitioners use these intraday markets to settle slight changes in volume that occur in the delivery day. Therefore, our assumption that the purchaser uses the day-ahead to settle the differences between consumption volume and previously contracted volume does not deviate too far from reality.

¹¹ Electricity prices exhibit strong higher moment characteristics, most noteworthy skewness and strong leptokurtosis. See for example, Huisman and Mahieu (2003) and Bunn and Karakatsani (2003). However, in this paper we concern with finding an optimal allocation in a mean-variance (two moments) framework. We refer to Levy and Markowitz (1979) and Kroll et al. (1984), who show that in many typical portfolio optimization cases the first two moments suffice in determining the optimal allocation. Building on these insights we assume that higher moment characteristics of electricity prices do not play a role in our analysis. We leave the implication of higher moments in the optimal allocation for further research.

In order to solve Eq. (2) for the optimal allocations θ_t we specify the Lagrangian:

$$\mathcal{L} = \theta'_t (\mathbf{N} \cdot \mathbf{f}_t - \mathbf{B}'\mu_s) + \mu'_v \mu_s + \lambda \{ \theta'_t \mathbf{B}' \Omega_s \mathbf{B} \theta_t + \text{tr}(\Omega_s \Omega_v) + \mu'_s \Omega_v \mu_s + \mu'_v \Omega_s \mu_v + 2\theta'_t (\mathbf{B}'\mu_s) (\mu'_v \mu_s) - \sigma_{\max}^2 \}, \quad (6)$$

with λ a Lagrange multiplier. The first-order Karush–Kuhn–Tucker conditions are

$$\frac{\partial \mathcal{L}}{\partial \theta_t} \Big|_{\hat{\theta}_t, \hat{\lambda}} = \mathbf{N} \cdot \mathbf{f}_t - \mathbf{B}'\mu_s + 2\lambda \theta'_t \mathbf{B}' (\mathbf{W}\mathbf{B} + \mu_s (\mu'_v \mu_s)) = 0 \quad (7a)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} \Big|_{\hat{\theta}_t, \hat{\lambda}} = \theta'_t \mathbf{B}' \Omega_s \mathbf{B} \theta_t + \text{tr}(\Omega_s \Omega_v) + \mu'_s \Omega_v \mu_s + \mu'_v \Omega_s \mu_v + 2\theta'_t (\mathbf{B}'\mu_s) (\mu'_v \mu_s) - \sigma_{\max}^2 \leq 0 \quad (7b)$$

$$\{ \theta'_t \mathbf{B}' \Omega_s \mathbf{B} \theta_t + \text{tr}(\Omega_s \Omega_v) + \mu'_s \Omega_v \mu_s + \mu'_v \Omega_s \mu_v + 2\theta'_t (\mathbf{B}'\mu_s) (\mu'_v \mu_s) - \sigma_{\max}^2 \} \hat{\lambda} = 0 \quad (7c)$$

$$\hat{\lambda} \geq 0 \quad (7d)$$

If the Lagrange multiplier $\hat{\lambda} = 0$ then the variance restriction is not binding. This implies that the risk limit σ_{\max}^2 cannot be met. In that case the optimal allocation to forward contracts should be found from Eq. (7a). Inspection of this condition shows that depending on the sign of the composite forward premia $\mathbf{N}_o \mathbf{f}_o(t, T) - \sum_{h=1}^{24} \mathbf{B}_{h1} s(h, T-1)$ for off-peak hours and $\mathbf{N}_p \mathbf{f}_p(t, T) - \sum_{h=1}^{24} \mathbf{B}_{h2} s(h, T-1)$ for peak hours the positions would become unlimited. Clearly this is not possible. As a result the variance restriction is binding: $\hat{\lambda} > 0$. In that case we can solve for the optimal allocation $\hat{\theta}_t$ from Eq. (7a) and the equality

$$\theta'_t \mathbf{B}' \Omega_s \mathbf{B} \theta_t + \text{tr}(\Omega_s \Omega_v) + \mu'_s \Omega_v \mu_s + \mu'_v \Omega_s \mu_v + 2\theta'_t (\mathbf{B}'\mu_s) (\mu'_v \mu_s) = \sigma_{\max}^2.$$

An analytical solution could be found for θ_t , but these are not straightforward and do not provide any intuition.¹² In order to provide analytical results that provide insight, we continue in a more stylized setting.

3.1. One peak and one off-peak hour

Huisman et al. (2007) show that the covariance matrix of hourly day-ahead prices exhibits a clear block structure of high correlations among either the peak hours and the off-peak hours and near-zero correlations between peak and off-peak hours. To some extent, hourly day-ahead prices can be seen to behave in independent peak and off-peak blocks. Motivated by this result, we simplify the above model by bringing the number of hours back to just two: one representative off-peak and one representative peak hour. Assuming 2 h in the delivery day and zero correlation between the peak and off-peak hour, we rewrite the model as follows. The expected cost function (4) becomes:

$$E_t \{ C(T) \} = \theta_o f_o(t, T) + \theta_p f_p(t, T) + [\mu_{vo} - \theta_o] \mu_{so} + [\mu_{vp} - \theta_p] \mu_{sp}, \quad (8)$$

with $\mu_{vo} (\mu_{vp})$ the mean of the volumes in the representative off-peak (peak) hour and $\mu_o (\mu_p)$ the mean of the day-ahead prices that are representative for the off-peak (peak) hour. The variance Eq. (5) becomes:

$$\text{var}_t \{ C(T) \} = \mu_{so}^2 \sigma_{vo}^2 + (\mu_{vo} - \theta_o)^2 \sigma_{so}^2 + \sigma_{vo}^2 \sigma_{so}^2 + \mu_{sp}^2 \sigma_{vp}^2 + (\mu_{vp} - \theta_p)^2 \sigma_{sp}^2 + \sigma_{vp}^2 \sigma_{sp}^2. \quad (9)$$

In the above equation, σ_{so} and σ_{sp} represent the standard deviation of the off-peak and peak prices and σ_{vo} and σ_{vp} reflect the standard deviations of the off-peak and peak volumes. The first-order conditions that result from the minimization of the expected cost with respect to the variance restriction equal:

$$\frac{\partial \mathcal{L}}{\partial \theta_o} \Big|_{\hat{\theta}_o, \hat{\lambda}} = f_o - \mu_{so} - 2\hat{\lambda} (\mu_{vo} - \hat{\theta}_o) \sigma_{so}^2 = 0, \quad (10a)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_p} \Big|_{\hat{\theta}_p, \hat{\lambda}} = f_p - \mu_{sp} - 2\hat{\lambda} (\mu_{vp} - \hat{\theta}_p) \sigma_{sp}^2 = 0, \quad (10b)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} \Big|_{\hat{\theta}_o, \hat{\theta}_p, \hat{\lambda}} = \mu_{so}^2 \sigma_{vo}^2 + (\mu_{vo} - \hat{\theta}_o)^2 \sigma_{so}^2 + \sigma_{vo}^2 \sigma_{so}^2 + \mu_{sp}^2 \sigma_{vp}^2 + (\mu_{vp} - \hat{\theta}_p)^2 \sigma_{sp}^2 + \sigma_{vp}^2 \sigma_{sp}^2 - \sigma_{\max}^2 = 0. \quad (10c)$$

Rearranging Eqs. (10a) and (10b) yields

$$\frac{\mu_{vp} - \hat{\theta}_p}{\mu_{vo} - \hat{\theta}_o} = \frac{(f_p - \mu_{sp}) / \sigma_{sp}^2}{(f_o - \mu_{so}) / \sigma_{so}^2}. \quad (11)$$

The right hand side of Eq. (11) is the ratio of the hedging costs per unit of (variance) risk in the peak hour over the off-peak hour. The hedging costs per unit of risk equals the forward premium $(f - \mu_s)$ divided by the amount of (variance) risk σ_s^2 . The higher this number is, the more a purchaser pays for hedging away day-ahead price risk. Eq. (11) yields the intuitive result that the ratio of open positions equals the ratio of relative hedging costs: the more expensive it is to hedge in 1 h, the bigger the open position in that hour relative to the other hour.

For convenience, we let η be the ratio of the hedging costs per unit of (variance) risk in the peak hour over the off-peak hour, that is:

$$\eta = \frac{(f_p - \mu_{sp}) / \sigma_{sp}^2}{(f_o - \mu_{so}) / \sigma_{so}^2}. \quad (12)$$

The ratio of expected open positions, the left-hand-side of Eq. (11), shows how the purchaser will divide her resources over the off-peak and peak hours in the day-ahead market. If this ratio equals one ($\eta = 1$) then

$$\mu_{vp} - \hat{\theta}_p = \mu_{vo} - \hat{\theta}_o \iff \mu_{vp} - \mu_{vo} = \hat{\theta}_p - \hat{\theta}_o.$$

The purchaser buys electricity in the forward market such that the difference between the off-peak and peak hour volumes matches the difference between the expected volumes in both hours. If the ratio is higher (lower) than one, then hedging cost in the peak hour is relatively higher (lower) than in the off-peak hour, which leads the purchaser to a relatively smaller (larger) forward position in the peak (off-peak) hour.

Eq. (11) reveals another important result. The risk appetite of the purchaser, σ_{\max}^2 does not affect the ratio of the open positions. That is, every purchaser will structure her optimal portfolio such that Eq. (11) holds, given that they do not disagree on the first and second moments of the electricity spot prices.¹³ Therefore, to determine the optimal portfolio, the purchaser follows a two-step approach: firstly, she determines the optimal relative allocations to the peak and off-peak hour and, secondly, she will set the exact allocation levels based on her risk appetite. This result is in line with the *separation principle* that

¹³ If purchasers hold heterogeneous beliefs then each of them will have a different η . Note that in this paper we do not strive for finding allocations that hold in equilibrium; our analysis is partial and applies in general to individual purchasers, unless we assume that all purchasers have similar beliefs on the statistical distributions of electricity spot prices and volumes.

¹² Note that one can easily find a numerical solution to the equality.

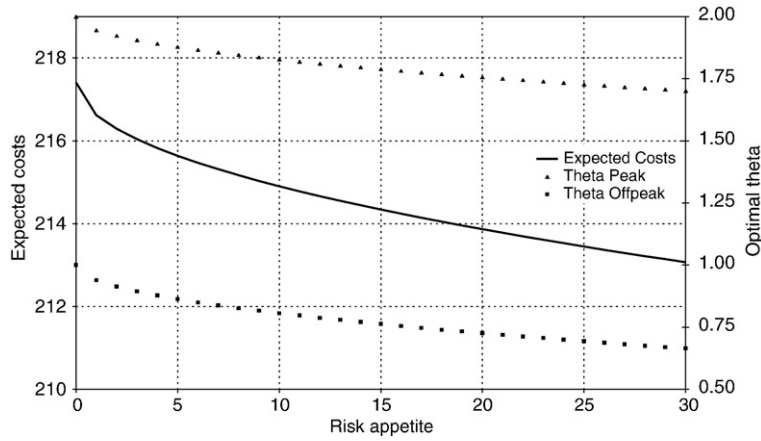


Fig. 1. The relation between risk appetite and expected costs.

results from the modern portfolio theory of Markowitz (1952). The optimal investment portfolio is constructed in two subsequent steps. First, the most efficient portfolio is determined (referred to as the market portfolio by Markowitz). Secondly, the investor chooses between investing in the market portfolio and the risk-free interest rate in such a way that her resulting portfolio reflects her risk appetite.¹⁴

In order to determine the optimal allocations θ_o and θ_p , we first define the excess risk appetite σ_e^2 being the difference between the risk appetite of the electricity purchaser σ_{\max}^2 and the risks associated with the variation in the max consumption volume that cannot be hedged by taking positions in forward contracts (but could be managed by improving consumption volume forecasting methods).

$$\sigma_e^2 = \sigma_{\max}^2 - (\mu_{so}^2 + \sigma_{so}^2) \sigma_{vo}^2 - (\mu_{sp}^2 + \sigma_{sp}^2) \sigma_{vp}^2.$$

We assume that the purchaser is aware of the volume risk and that she accounts for that in her risk appetite. Therefore, we assume that $\sigma_e^2 \geq 0$.

Substituting Eq. (11) in Eq. (10c) and solving for θ_p , we find that the optimal allocation for the peak hour:

$$\hat{\theta}_p = \mu_{vp} - \sqrt{\frac{\sigma_e^2}{\sigma_{so}^2/\eta^2 + \sigma_{sp}^2}}, \quad (13)$$

and the optimal solution for the off-peak hour $\hat{\theta}_o$:

$$\hat{\theta}_o = \mu_{vo} - \sqrt{\frac{\sigma_e^2}{\sigma_{so}^2 + \sigma_{sp}^2 \eta^2}}. \quad (14)$$

The optimal hedge ratios θ_o and θ_p depend on the hourly expected consumption volumes, the excess risk appetite, the variances of the hourly prices and the ratio of relative hedging costs η . Solving for the optimal total expected costs, we insert the optimal hedging positions $\hat{\theta}_o$ and $\hat{\theta}_p$ in the expected total cost function (8). We obtain:

$$E_t\{C(T)\} = \mu_{vo} f_o + \mu_{vp} f_p - \frac{(f_o - \mu_{so}) \sigma_e}{\sqrt{\sigma_{so}^2 + \sigma_{sp}^2 \eta^2}} - \frac{(f_p - \mu_{sp}) \sigma_e}{\sqrt{\sigma_{so}^2/\eta^2 + \sigma_{sp}^2}} \quad (15)$$

From this expression we immediately see that positive risk premia ($f - \mu_s$) lead to lower expected costs.

4. Managerial implications

In this section, we apply the outcomes of the model from the previous section to address two fundamental questions that purchasers deal with in practice.

4.1. How to use baseload contracts to purchase a baseload profile?

Some electricity consumers have a baseload consumption profile; they consume the same expected amount of electricity in each hour of the day (i.e. $\mu_{vo} = \mu_{vp} \equiv \mu_v$). Examples of such companies are chemical plants and industrial companies that work 24 h per day or supermarkets that are open all day. Baseload contracts (OTC forwards or futures) deliver in each hour of the delivery period and therefore perfectly match the baseload consumption profile. At first glance it seems straightforward to purchase the baseload profile directly with a baseload contract. However, from Eq. (11) it appears that hedging with baseload contracts exclusively ($\hat{\theta}_p = \hat{\theta}_o$) may not be the most optimal strategy. For a baseload profile, Eq. (11) can be rewritten as:

$$\hat{\theta}_p = \mu_v - (\mu_v - \hat{\theta}_o) \eta, \quad (16)$$

with η as defined in Eq. (12). It is obvious that $\hat{\theta}_p$ only equals $\hat{\theta}_o$ when η equals 1. The latter occurs when the relative risk-adjusted costs of hedging is the same for both hours. In all other cases, a baseload contract is not the optimal strategy and the purchaser would choose a combination of peak and off-peak contracts and a position on the day-ahead market instead. A purchaser who only uses baseload contracts in a world where the relative costs of hedging are not equal, could obtain lower expected costs level by applying a more differentiated hedging strategy by using an appropriate combination of peak and off-peak contracts.

4.2. Will taking more risk be rewarded with lower expected costs?

In order to provide insight in this issue, we need the first-order partial derivative of the optimal expected costs function (15) with respect to the risk appetite σ_{\max}^2 of the purchaser. Recall that in the above formulas, we used the risk appetite in excess of the volumetric risk, σ_e^2 , instead of the risk appetite itself. As the excess risk appetite depends linearly on the risk appetite, we will proceed with the excess risk appetite. From Eq. (15), it can be seen that the first-order derivative of the optimal total costs with respect to σ_e is negative assuming that the expected risk premiums are positive. This implies that taking risk is rewarded by lower expected costs when expected risk premiums are positive, and vice versa.

¹⁴ Remember that these portfolio allocation separation results hold under the assumption of a representative agent. For more information on these theorems in a setting with heterogeneous beliefs, see Chabi-Yo et al. (2008).

The following real-life case provides more insight in the risk versus expected costs relation and the hedging decision of the purchaser. Consider a power purchaser who needs to purchase the expected consumption of 1 MW in the off-peak hour and 2 MW in the peak hour of a specific delivery day in 2009. We assume that she faces no volumetric risk: $\sigma_{v_o}^2 = \sigma_{v_p}^2 = 0$. The delivery takes place in the American South Californian SP-15 market and hence she considers the optimal allocations in an off-peakload and peakload calendar year 2009 future contract (CAL09), which we assume is representative for the delivery day, or she can wait and purchase the electricity needed in the day-ahead market. On the SP-15 market, among some other markets, one can trade directly off-peak and peak contracts on both day-ahead and longer term delivery bases and therefore exactly matches our framework. We have, without any specific reason for selecting the date, used prices observed on January 10th, 2008. On that day, the off-peak futures CAL09 closed at \$56.73 and the peak contract closed at \$80.34. If the purchaser would buy the total volume using one off-peak contract and two peak contracts, her total cost equals \$217.41 euros. This cost represents a zero-risk portfolio, assuming no volumetric risk. In order to examine whether the purchaser can improve upon these expected expenditures, we apply the allocation model from the previous section. To that end we have to determine values for the expected day-ahead prices and the associated standard deviations. For simplicity reasons, we assume that the average day-ahead prices on January 10th (\$54.28 per MWh for off-peak delivery and \$68.54 per MWh for peak load hours) equal the expected day-ahead prices in the delivery period. For the standard deviations of the prices, we calculated the standard deviations of the daily average off-peak and peak prices over the previous calendar year 2007. The standard deviations are $\sigma_{sp} = 16.53$ \$ per MW h for the peak hours and $\sigma_{so} = 7.10$ \$ per MW h for the off-peak hours. Fig. 1 presents the outcomes of the model for different values for the risk appetite. The solid line in Fig. 1 represents the expected cost. In case the risk appetite is zero, $\sigma_{max}^2 = 0$, the purchaser is not willing to take risk and the total expected cost equal 217.41 \$ per MW h. In case the purchaser wants to take more risk, the expected costs decline: taking more risk is rewarded by lower expected cost. The lower expected costs are obtained by lowering the number of forward contracts in the portfolio, to profit from the lower expected prices in the day-ahead market. While increasing the risk appetite, the optimal θ_o and θ_p decline, but the extent to which they decline is different. This depends on the ratio of relative risk premia η in Eq. (12). In our example, the value of this ratio equals 0.89. This implies that the risk premium in the peak contract is lower per unit of risk than the risk premium for the off-peak hours. Therefore, the purchaser can take more risk by lowering the number of off-peak forward contracts faster than the number of peak contracts, as the risk premium in the off-peak contract is higher per unit of risk.¹⁵

5. Concluding remarks

In this paper we have introduced a one-period framework to examine the optimal allocations to peak and off-peak forward contracts of a rational electricity purchaser who wants to hedge both price and volumetric risks. The results show that building an optimal portfolio with electricity forward contracts is a two-step procedure. First, purchasers find the optimal allocation to peak contracts relative to off-peak

contracts in order to profit from differences in the relative hedging cost efficiency involved in both contracts. These relative positions are the same for every purchaser as they are not influenced by individual risk appetites. Secondly, the purchaser chooses the exact allocations, including positions in the day-ahead market, to meet her risk appetite.

We apply the model to focus on two important empirical cases in purchasing electricity. The first case shows that it is only optimal to source a baseload consumption profile with a baseload forward contract when the hedging costs per unit of day-ahead risk are the same for both peak and off-peak contracts. In practice, these marginal hedging costs are not likely to be the same at all times, indicating that a purchaser would be better off in holding a different portfolio with peak and off-peak contracts instead. The second case reveals that purchasers with a higher risk appetite on the day-ahead market are rewarded with lower expected purchasing costs, provided that expected risk premiums are positive.

References

- Alizadeh, A.H., Nomikos, N.K., Poulisis, P.K., 2008. A Markov regime switching approach for hedging energy commodities. *Journal of Banking and Finance* 32 (9), 1970–1983.
- Bessembinder, H., Lemmon, M., 2002. Equilibrium pricing and optimal hedging in electricity forward markets. *The Journal of Finance* 57 (3), 1347–1382.
- Bohrstedt, G., Goldberger, 1969. On the exact covariance of products of random variables. *Journal of The American Statistical Association* 64 (328), 1439–1442.
- Brand, H., Thorin, E., Weber, C., Madlener, R., Kaufmann, M., Kossmeier, S., 2002. Market analysis and tool for electricity trading. Tech. rep., OSCOGEN.
- Bunn, D., Karakatsani, N., 2003. Forecasting electricity prices. Tech. rep., London Business School Working Paper.
- Chabi-Yo, F., Ghysels, E., Renault, E., 2008. On portfolio separation theorems with heterogeneous beliefs and attitudes towards risk, Bank of Canada Working Paper 2008-16.
- Eydeland, A., Wolyniec, K., 2003. Energy and Power Risk Management: New Developments in Modeling, Pricing, and Hedging. John Wiley & Sons, New Jersey, U.S.A.
- Fama, E., French, K., 1987. Commodity futures prices: some evidence on forecast power, premiums, and the theory of storage. *Journal of Business* 60 (1), 55–73.
- Huisman, R., Mahieu, R., 2003. Regime jumps in electricity prices. *Energy Economics* 25 (5), 425–434.
- Huisman, R., Huurman, C., Mahieu, R., 2007. Hourly electricity prices in day-ahead markets. *Energy Economics* 29 (2), 240–248.
- Kanamura, T., Ohashi, K., 2007. A structural model for electricity prices with spikes: measurement of spike risk and optimal policies for hydropower plant operation. *Energy Economics* 29, 1010–1032.
- Karakatsani, N., Bunn, D., 2005. Diurnal reversals of electricity forward premia. Tech. rep., London Business School Working Paper.
- Kroll, Y., Levy, H., Markowitz, H.M., 1984. Mean-variance versus direct utility maximization. *Journal of Finance* 34 (1), 47–61.
- Junelevy, H., Markowitz, H., 1979. Approximating expected utility by a function of mean and variance. *American Economic Review* 69 (3), 308–317.
- Markowitz, H., 1952. Portfolio selection. *The Journal of Finance* 7 (1), 77–91.
- Moschini, G., Myers, R.J., 2002. Testing for constant hedge ratios in commodity markets: a multivariate GARCH approach. *Journal of Empirical Finance* 9, 589–603.
- Mount, T.D., Ning, Y., Cai, X., 2006. Predicting price spikes in electricity markets using a regime-switching model with time-varying parameters. *Energy Economics* 28, 62–80.
- Näsäkkälä, E., Keppo, J., 2005. Electricity load patterns hedging with static forward strategies. *Managerial Finance* 31 (6), 116–137.
- Vehviläinen, I., Keppo, J., 2004. Managing electricity market price risk. *European Journal of Operational Research* 145 (1), 136–147.
- Woo, C., Horowitz, I., Horii, B., Karimov, R., 2004. The efficient frontier for spot and forward purchases: an application to electricity. *The Journal of the Operational Research Society* 55 (11), 1130–1136.

¹⁵ We carried out a similar exercise for the German EEX market with data from 2007 yielding the same qualitative results. The difference between the SP-15 market and the EEX is that in the latter market only hourly specific day-ahead contracts and baseload and peakload forward contracts can be traded. Therefore, the EEX case does not directly match our framework. However, it did not lead to different results.