Optimal firm investment in security*

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In this paper, we analyze the problem of an individual firm that has to deal with losses from criminal activities. It is assumed that the firm can protect itself by investing in security equipment. Two different models are considered. In the first model, the firm has the possibility to spend money on production and on security investment. More production increases revenue but also criminal losses, while the latter can be decreased by investing in security. It turns out that the optimal production level increases with security equipment and is determined such that marginal revenue, net from criminal losses, equals marginal cost. For the optimal level of security investment it holds that, in the case of the existence of a long-run steady-state equilibrium, the properly discounted future reductions in criminal losses, which are due to an additional unit of security investment, exactly balances the initial outlay necessary to acquire an extra unit of security investment. In the second model, we extend this analysis by considering the effect that the firm's reputation has in the criminal world. If the firm has produced a lot in the past without having invested in security equipment, this firm is known to be a fruitful target for criminals. Therefore, more criminals will try to rob this firm, and this will increase future criminal losses.

Keywords: economics of crime, security investment, optimal control

1. Introduction

According to an article in a Dutch journal [4], in the Netherlands the number of raids on firms doubled during the period 1989–1993. Furthermore, the total revenue of security companies in the Netherlands rose from 390 million dutch guilders in 1989 to 700 million guilders in 1992. These data seem to us to document the importance of studying the so-called security investment problem.

According to the same journal article [4], growth in the security market first took place in a quantitative way (number of people), but recently, a more qualitatively

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oriented growth occurs (equipment). Nowadays, firms concentrate on finding the ideal combination between people and equipment. In several cases, the required number of people is very low. Think of, for instance, the remote control of security equipment such as camera systems, to close and open doors, and to switch lighting on and off. These mechanisms have in common that they can be controlled from one central point. Other examples of security equipment include "advanced hydraulic pressure systems", which are used to detect whether somebody is present somewhere on the firm's property, "infra-red equipment" that reports certain movements, and "seismic movement detectors".

The aim of this paper is to study the optimal security investment decision of an individual firm. For reasons of mathematical tractability, we do not distinguish various types of security equipment. Instead, a homogeneous stock of security capital is considered, and the investment problem is to determine the optimal development of this security capital stock. To the best of our knowledge, this paper is the first attempt to tackle this problem within a dynamic model of the firm. In this way, the ideas put forth by Becker [1] are applied within a micro-economic framework.

In the framework that we consider in this paper, security equipment has both a direct and an indirect effect on (future) criminal losses of the firm. The direct effect is associated with the fact that if a firm has installed more security equipment, it becomes more difficult for thieves to steal from this firm, so that criminal losses are reduced. The indirect effect has to do with the reputation of the firm in the criminal world. A firm is known to be a fruitful target for criminals, which implies that the risk of being robbed is large, when it has a high production without having invested sufficiently in security equipment. Hence, by investing in security equipment, the firm diminishes its reputation of being a fruitful criminal target now and in the future.[1)] This implies that the number of criminals wanting to rob this firm reduces, so that criminal losses will decrease.

The paper is organized as follows. In section 2, only direct effects of security equipment are considered. Indirect effects are added to this framework in section 3. Section 4 concludes the paper.

2. The direct effects of security equipment

2.1. Model formulation

We consider a firm that uses a variable input $V(t)$ (e.g. labor) to produce an amount of homogeneous goods $Q(V(t))$, such that

$$Q(0) = 0, \ Q'(V(t)) > 0, \ Q''(V(t)) < 0. \tag{1}$$

[1)] In the Netherlands, the number of bank robberies decreased during recent years. This is because criminals know that banks are highly protected.

The unit cost of this input is w ($w > 0$ and constant). Furthermore, we assume that the firm is "small", i.e., the product can be sold on the market against a fixed market price p. For reasons of convenience, we only consider scenarios where marginal revenue in case of zero production exceeds the unit cost of the variable input,

$$pQ'(0) > w. \tag{2}$$

Of course, $V(t)$ is always non-negative,

$$V(t) \geq 0. \tag{3}$$

Nowadays, a producing firm has to fear that it becomes a victim of criminal activities, but the firm has the possibility to protect itself by investing in security equipment. Let $S(t)$ be the stock of security capital, which increases with investment $I(t)$ and decreases with depreciation, where δ is the constant depreciation rate ($\delta > 0$ and constant). Then the dynamic equation for S becomes

$$\dot{S}(t) = I(t) - \delta S(t),\ S(0) = S_0 > 0. \tag{4}$$

The firm has to deal with losses from criminal activities. It is reasonable that criminal losses increase with production, because the more a firm produces, the more can be stolen. For simplicity reasons, we assume that these criminal losses depend only on the current production rate. Moreover, they are linearly dependent on the firm's revenue, which is pQ. On the other hand, the firm can reduce criminal losses by building up a stock of security capital. Hence, criminal losses decrease with S. It seems reasonable that there are decreasing returns to scale with respect to security capital, because for a highly protected firm, an additional investment in security equipment will have less effect in terms of reducing criminal losses than for a firm with a relatively low security capital stock.

We conclude that in our model the firm's criminal losses at a certain time t are given by the expression $L(S(t))pQ(t)$, where $L(S(t))$ satisfies

$$L(S(t)) > 0,\ L'(S(t)) < 0,\ L''(S(t)) > 0. \tag{5}$$

It is clear that the value of the stolen products will never exceed the value that the firm can obtain by selling the produced goods on the market, which implies that

$$L(0) \leq 1. \tag{6}$$

(Notice that $L(0) = 1$ means that every produced good will be stolen in the case the firm has not installed any security equipment.)

The objective of the firm is to maximize the net cash flow stream,

$$\text{maximize} \int_0^\infty e^{-rt}[pQ(V(t)) - L(S(t))pQ(V(t)) - wV(t) - C(I(t))]dt, \tag{7}$$

where r is the constant discount rate, and $C(I)$ is a convex cost function of security investment ($C' > 0$, $C'' > 0$). The decision problem of the firm is to determine a path for the variable input and investment in security $\{V(t), I(t)\}$ over an infinite planning period $[0, \infty)$ such that the objective functional in (7) is maximal subject to constraints (3) and (4).

2.2. *Mathematical analysis and economic interpretations*

Since the control V does not enter the system dynamics (4), this model can be treated by applying a two-step approach (cf. [3]):

2.2.1. *The Step 1 problem*

To find the optimal level of the variable input for a given S, we solve the following static (Step 1) problem:

$$\begin{aligned} &\underset{V}{\text{maximize}} \quad [pQ(V)\{1 - L(S)\} - wV] \\ &\text{subject to} \quad V \geq 0. \end{aligned} \tag{8}$$

The solution is easily obtained by using the Kuhn–Tucker conditions,

$$\begin{aligned} \mathcal{L} &= pQ(V)\{1 - L(S)\} - wV + \mu V, \\ \mathcal{L}_V &= pQ'(V)\{1 - L(S)\} - w + \mu = 0, \end{aligned} \tag{9}$$

as well as $\mu \geq 0$ and $\mu V = 0$. This yields

$$V(S) = \begin{cases} 0 & \text{when } Q'(0)\{1 - L(S)\} \leq w/p, \text{ i.e., } S \leq \tilde{S}, \\ V_c(S) & \text{when } Q'(0)\{1 - L(S)\} > w/p, \text{ i.e., } S > \tilde{S}, \end{cases} \tag{10}$$

where $V_c(S)$ is an implicit function such that

$$pQ'(V)\{1 - L(S)\} = w \tag{11}$$

and $\tilde{S}$ is the largest security capital stock for which it is optimal to have zero production,

$$pQ'(0)\{1 - L(\tilde{S})\} = w, \quad \text{i.e., } \tilde{S} = L^{-1}\left(1 - \frac{w}{pQ'(0)}\right). \tag{12}$$

From (2), we obtain that $\tilde{S}$ always exists.

We see that there is no production (i.e., $V = 0$) when the security capital stock is low. This makes sense, because then the firm's criminal losses will be large, so that marginal revenue from selling the few goods that are not stolen on the market falls below the unit cost of the variable input. From equation (11), we derive that whenever V is positive, the amount of variable input will be fixed such that marginal revenue net from criminal losses equals the unit cost of the variable input. Furthermore, due to the same equation, it is easily seen that the use of this input decreases with its unit cost and is increasing with the market price of the final product, as can be expected.

From the implicit function theorem, we can compute the derivatives of $V_c(S)$ for $S > \tilde{S}$,

$$V_c'(S) = \frac{L'(S)Q'(V)}{\{1 - L(S)\}Q''(V)} > 0, \tag{13a}$$

$$V_c''(S) = \frac{\{1 - L\}Q''\{L''Q' + L'Q''V_c'\} - L'Q'\{-L'Q'' + \{1 - L\}Q'''V_c'\}}{(1 - L)^2(Q'')^2}. \tag{13b}$$

By substituting (13a) for V_c', we obtain

$$V_c''(S) = \frac{Q'}{(1 - L)^2(Q'')^3}[(1 - L)(Q'')^2 L'' + (L')^2\{2(Q'')^2 - Q'Q'''\}]. \tag{14}$$

Unfortunately, the sign of $V_c''(S)$ is ambiguous because the sign of Q''' is, in general, unknown.

Summing up the results of Step 1, we can formulate

Proposition 1. For given values of the final product's market price, p, and the unit cost of the variable input, w, the optimal level of the variable input only depends on the stock of security capital. If $S < \tilde{S}$, with $\tilde{S}$ from (12), then the firm will not use any variable inputs, so that production equals zero. For $S > \tilde{S}$, the variable input is positive and increases with S.

2.2.2. The Step 2 problem

With $V(S)$ computed in Step 1, solve the following (Step 2) control problem:

$$\underset{I(T)}{\text{maximize}} \int_0^\infty e^{-rt}[pQ(V(S(t)))\{1 - L(S(t))\} - wV(S(t)) - C(I(t))]dt \tag{15}$$

$$\text{subject to} \quad \dot{S} = I - \delta S, \; S(0) = S_0. \tag{16}$$

This leads to the following current-value Hamiltonian:

$$H = pQ(V(S))\{1 - L(S)\} - wV(S) - C(I) + \lambda(I - \delta S), \tag{17}$$

in which $\lambda = \lambda(t)$ is the costate variable. The necessary optimality conditions are

$$I = \underset{I}{\arg\max}\, H, \quad \text{i.e., } \lambda = C'(I), \tag{18}$$

$$\dot{\lambda} = r\lambda - H_S = (r + \delta)\lambda + pQ(V(S))L'(S) + V'(S)\{-pQ'(V(S))\{1 - L(S)\} + w\},$$

where λ is the current-value costate variable of capital stock. From (10) and (11), it follows that

$$\dot{\lambda} = (r + \delta)\lambda + pQ(V(S))L'(S). \tag{19}$$

2.2.3. Qualitative analysis in the phase plane

In order to perform a phase plane analysis in the (S, I) plane, we first observe that the $\dot{S} = 0$ isocline is the straight line $I = \delta S$. Next, we have to derive a differential equation for I from the adjoint equation (19). This is done by differentiating (18) with respect to time t and subsequent elimination of the costate λ. This yields

$$\dot{I} = \frac{(r + \delta)C'(I)}{C''(I)} > 0 \quad \text{for } S \leq \tilde{S}, \tag{20}$$

$$\dot{I} = \frac{1}{C''(I)}[(r + \delta)C'(I) + pQ(V_c(S))L'(S)] \quad \text{for } S > \tilde{S}. \tag{21}$$

From (20), we conclude that the $\dot{I} = 0$ isocline does not exist for $S \leq \tilde{S}$. With (21), we can compute the slope of the $\dot{I} = 0$ isocline for $S > \tilde{S}$. In this case, we obtain from $(r + \delta)C'(I) + pQ(V_c(S))L'(S) = 0$ that

$$\left.\frac{dI}{dS}\right|_{\dot{I}=0} = \frac{-p(QL'' + Q'V_c'L')}{(r + \delta)C''}$$

$$= \frac{-p\{(1 - L)Q''QL'' + (L')^2(Q')^2\}}{(r + \delta)C''(1 - L)Q''}. \tag{22}$$

The sign of this expression is ambiguous.

Let us now consider the steady state (S^*, I^*), which is defined by

$$I^* = \delta S^*,\ (r + \delta)C'(I^*) = -pQ(V_c(S^*))L'(S^*). \tag{23}$$

The second equation in (23) says that in the steady state, marginal costs of security equipment equals marginal revenue, where the latter equals the reduction of criminal losses due to one extra unit of security equipment.

The determinant of the Jacobian of the dynamical system (16) and (21) evaluated at the steady state is equal to

$$\det J = (r + \delta)\left(-\delta + \left.\frac{dI}{dS}\right|_{\dot{I}=0}\right). \tag{24}$$

Since δ is the slope of the $\dot{S} = 0$ isocline and $\det J$ is negative in the case of a saddle point, the following proposition holds.

Proposition 2. The steady state (S^*, I^*) is

- a saddle point, if the $\dot{I} = 0$ isocline hits the $\dot{S} = 0$ isocline from above;
- unstable, if the $\dot{I} = 0$ isocline hits the $\dot{S} = 0$ isocline from below.

As it is now, the sign of the slope of the $\dot{I} = 0$ isocline is undetermined. Therefore, we need to specify some functions in order to proceed with the analysis. Let us consider the following scenario:

$$L(S) = L(0)e^{-bS}, \quad b > 0, \tag{25}$$

$$Q(V) = V^a, \qquad 0 < a < 1. \tag{26}$$

From (26), we obtain that $Q'(0) = \infty$, which implies via (10) that

$$V(S) = V_c(S) \quad \forall S \geq 0. \tag{27}$$

Economically, this means that for every value of S, it is optimal for the firm to produce goods.

Substitution of (25) and (26) into (22) leads to the following expression for the slope of the $\dot{I} = 0$ isocline:

$$\left.\frac{dI}{dS}\right|_{\dot{I}=0} = \frac{pb^2 V^a L(0)}{(r+\delta)C''(1-a)\{e^{bS} - L(0)\}}\{a - 1 + L(0)e^{-bS}\}. \tag{28}$$

From (28), we conclude that the $\dot{I} = 0$ isocline is decreasing if S is sufficiently large. Furthermore, due to (21), we get that the $\dot{I} = 0$ isocline intersects the I-axis for positive I in the case it holds that

$$(r+\delta)C'(0) < -pQ(V_c(0))L'(0), \tag{29}$$

or marginal revenue, which is the reduction in criminal losses due to one additional unit of S, exceeds marginal costs when $S = 0$. This condition implies that it is always optimal for the firm to own some security capital stock. The phase diagram that holds under this condition is depicted in figure 1.[2)]

Since the $\dot{I} = 0$ isocline is decreasing in the steady state while the $\dot{S} = 0$ isocline increases everywhere, we can conclude from proposition 2 that this steady state is a saddle point.

So far, we have only discussed the steady state and have not considered what happens on the path towards the steady state. It would be interesting to know how the investment rate is determined on this path. After solving the differential equation (19), substituting (18) into this relation, and using (23) as a fixed point, we obtain the following condition for the security investment rate which holds for every t:

$$\int_t^\infty e^{-(r+\delta)(s-t)}\{-pQ(V(S(s))L'(S(s)))\}ds - C'(I(t)) = 0, \tag{30}$$

[2)] From (28), we obtain that for the $\dot{I} = 0$ isocline to increase for $S = 0$, it must hold that $L(0) > 1 - a$. In the case the reverse is true, the $\dot{I} = 0$ isocline is globally decreasing. However, this does not change the saddle point property of the solution.

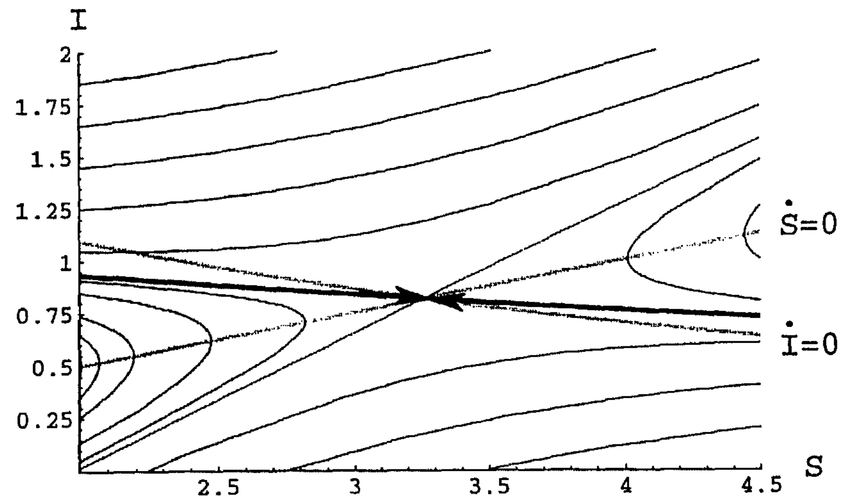

Figure 1. Bounded and unbounded trajectories of the canonical system for the one-state problem and parameters $L(0) = \frac{3}{4}$, $b = \frac{1}{10}$, $a = \frac{1}{2}$, $p = 2$, $r = \frac{4}{100}$, $\omega = 1$, $\delta = \frac{1}{4}$, $C(I) = \frac{1}{2}I^2$.

where the left-hand side is the net present value of marginal investment. This equation implies that the level of security investments is such that marginal security investment expenses (C') balance the discounted decrease in criminal losses over the whole planning period, caused by an extra unit of security investment at time t. Namely, an extra unit of security investment increases the stock of security capital S, which in turn decreases the criminal losses now and in the future with $-pQL'$. This stream of reductions of criminal losses is corrected for depreciation by multiplication by $e^{-\delta(s-t)}$. Therefore, condition (30) states that the net present value of marginal security investment equals zero, so that the fundamental economic principle of balancing marginal revenue with marginal expenses applies.

3. Optimal security investment including indirect effects

3.1. Model formulation

In this section, the framework of section 2 is extended by including the indirect effect on the firm's criminal losses of security investment. As already stated in the introduction, the indirect effect has to do with the reputation of the firm in the criminal world. If the firm has the reputation of being a fruitful target for criminals, then the risk of being robbed is large. We suppose that the state variable $R(t)$ provides information about how serious this risk is. The risk increases with the firm's revenue from production, because the more valuable production is, the more criminals are interested in robbing the firm. On the other hand, criminals are less interested in this firm in the case the firm is known to be highly protected. Hence, the firm can reduce the risk of being robbed by building up a stock of security capital. Let $\gamma(S)$ denote the amount that the risk will be decreased, where we assume that γ increases with $S(\gamma' > 0)$, but that there are also decreasing returns to scale with respect to security capital ($\gamma'' < 0$). Now the dynamic equation for R is equal to

$$\dot{R}(t) = c_1\{pQ(V(t)) - \gamma(S(t))\}, \quad R(0) = R_0 \geq 0, \tag{31}$$

where c_1 is a scaling parameter.

Of course criminal losses are influenced by R. Therefore, we must include R in the mathematical expression for criminal losses, so that it becomes $L(S(t), R(t))pQ(V(t))$, where $L(S, R)$ satisfies

$$L(S, R) > 0 \quad \text{for all } R > 0 \text{ and } S \geq 0, \; L(S, 0) = 0, \tag{32a}$$

$$L_S(S, R) < 0 \quad \text{for all } R > 0, \;\; L_{SS}(S, R) > 0 \quad \text{for all } R > 0, \tag{32b}$$

$$L_R(S, R) > 0, \tag{32c}$$

$$L(0, R) \leq 1. \tag{32d}$$

Equation (32a) implies that criminal losses are positive as long as the risk is positive. On the other hand, there are no criminal losses in the case there is no risk of being robbed.

Equation (32b) shows that, as in section 2, losses from criminal activities are smaller the larger the stock of security capital, where additional equipment has less effect if this stock is already large.

Equation (32c) states that criminal losses increase with increasing risk, for a given stock of security capital.

Equation (32d) is analogous to (6), so that it means that the value of the stolen products will never exceed the value that the firm can obtain by selling these goods on the market.

A functional relationship that satisfies (32a)–(32c) is, e.g.,

$$L(S, R) = ge^{-bS/R}, \quad b > 0 \text{ and } g > 0. \tag{32e}$$

The rest of the model is the same as in section 2 so that we have the following problem:

$$\underset{I(t)\,V(t)}{\text{maximize}} \int_0^\infty e^{-rt}[pQ(V(t))\{1 - L(S(t), R(t))\} - wV(t) - C(I(t))]dt \tag{33}$$

$$\text{subject to} \quad \dot{S}(t) = I(t) - \delta S(t), \qquad S(0) = S_0 > 0, \tag{34}$$

$$\dot{R}(t) = c_1\{pQ(V(t)) - \gamma(S(t))\}, \quad R(0) = R_0 > 0, \tag{35}$$

$$V(t) \geq 0. \tag{36}$$

3.2. Mathematical analysis and economic interpretations

The problem to be solved is the optimal control problem described by (33)–(36). To obtain the optimality conditions, we use Pontryagin's maximum principle. The current value Hamiltonian is

$$H = pQ(V)\{1 - L(S,R)\} - wV - C(I) + \lambda_1(I - \delta S) + \lambda_2 c_1\{pQ(V) - \gamma(S)\}, \quad (37)$$

in which $\lambda_i = \lambda_i(t)$ are the costate variables ($i = 1, 2$).

Due to the fact that V must be non-negative, the Lagrangian is given by ($\eta = \eta(t)$ is the dynamic Lagrange multiplier)

$$\mathcal{L} = H + \eta V. \quad (38)$$

For the moment, we assume that R is always positive.

The necessary optimality conditions are (see [2, theorem 7.4])

$$pQ'(V)\{1 - L(S,R) + \lambda_2 c_1\} = w - \eta, \quad (39)$$

$$C'(I) = \lambda_1, \quad (40)$$

$$\dot{\lambda}_1 = (r + \delta)\lambda_1 + pQ(V)L_S(S,R) + \lambda_2 c_1 \gamma'(S), \quad (41)$$

$$\dot{\lambda}_2 = r\lambda_2 + pQ(V)L_R(S,R), \quad (42)$$

$$\eta \geq 0,\ \eta V = 0. \quad (43)$$

3.2.1. Stability analysis

For the moment, let us assume the existence of a steady state. To get insight into the dynamics of the optimal solution of our model, a stability analysis has to be carried out. By linearizing the nonlinear canonical system around the steady state (S^*, R^*, λ_1^*, λ_2^*, V^*, I^*), we obtain the Jacobi matrix evaluated at the steady state. The steady state satisfies

$$I - \delta S = 0, \quad (44)$$

$$pQ(V) - (S) = 0, \quad (45)$$

$$(r + \delta)\lambda_1 + pQ(V)L_S(S,R) + \lambda_2 c_1 \gamma'(S) = 0, \quad (46)$$

$$r\lambda_2 + pQ(V)L_R(S,R) = 0, \quad (47)$$

$$pQ'(V)\{1 - L(S,R) + \lambda_2 c_1\} = w, \quad (48)$$

$$C'(I) = \lambda_1 \Rightarrow \frac{dI}{d\lambda_1} = \frac{1}{C''} > 0. \quad (49)$$

From (48), we obtain that

$$\frac{\partial V}{\partial S} = \frac{p(Q')^2 L_S}{wQ''} > 0, \quad (50a)$$

$$\frac{\partial V}{\partial R} = \frac{p(Q')^2 L_R}{wQ''} < 0, \tag{50b}$$

$$\frac{\partial V}{\partial \lambda_2} = \frac{-p(Q')^2 c_1}{wQ''} > 0. \tag{50c}$$

If S is large, then a smaller amount of the finished products will be stolen. Therefore, it is attractive for the firm to raise production with S (50a). If R is large, the opposite is true: now there is a big risk that the firm will face severe criminal losses and this makes production less attractive (50b). If λ_2 is not too low, then the firm is not bothered that much by an increased risk of being robbed (for instance, because security equipment is large). Therefore, in this respect it does not hurt to increase production (50c).

Analytical computations do not lead to clear conclusions. Therefore, we proceed with numerical analysis, in which we consider specific scenarios. Let us study the case where $Q(V)$ is given by (26) and $L(S, R)$ by (33e). Furthermore, we specify

$$C(I) = \alpha I + \frac{d}{2} I^h, \tag{51}$$

$$\gamma(S) = \gamma S^f. \tag{52}$$

3.3. Numerical analysis

Fixing parameters by $a = 7/10, f = 4/5, \delta = 1/10, r = 1/25, p = 1, h = 2, g = 1$ $\alpha = 1, \gamma = 1, \omega = 1/10, c_1 = 1/10, b = 1/5, g = 1/2$, we find a steady state $(I, V, S, R) = (1.30, 18.8, 13.0, 0.527)$ with a two-dimensional stable manifold and a two-dimensional unstable manifold. The corresponding eigenvalues are real; hence, we have a so-called saddle node. Each initial value in R and S uniquely defines a trajectory on the stable manifold, which is a two-dimensional surface in the four-dimensional state–control space (S, R, I, V). Illustrating dynamic behavior on the stable manifold, we project the stable manifold orthogonally on the (S, R) and (I, V) plane.

Figure 2 covers the $\dot{S} = 0$ isocline on the stable manifold, more precisely, its projection on the (S, R) and (I, V) plane. The $\dot{S} = 0$ isocline divides the stable manifold

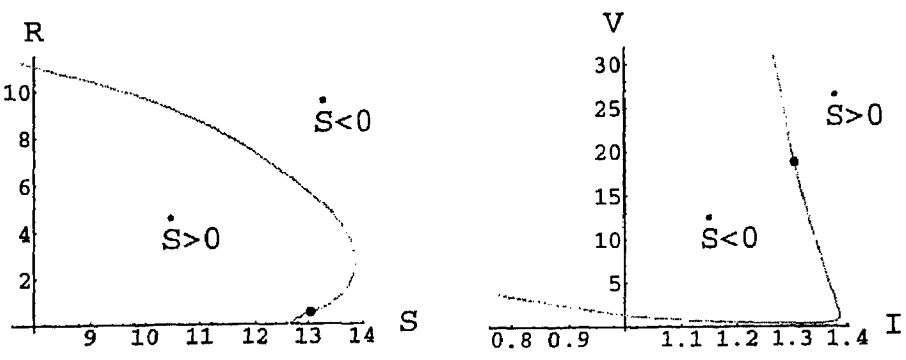

Figure 2. Projection of the intersection of the $\dot{S} = 0$ isoclinic surface and the stable manifold onto the (S, R) plane and (V, R) plane, respectively.

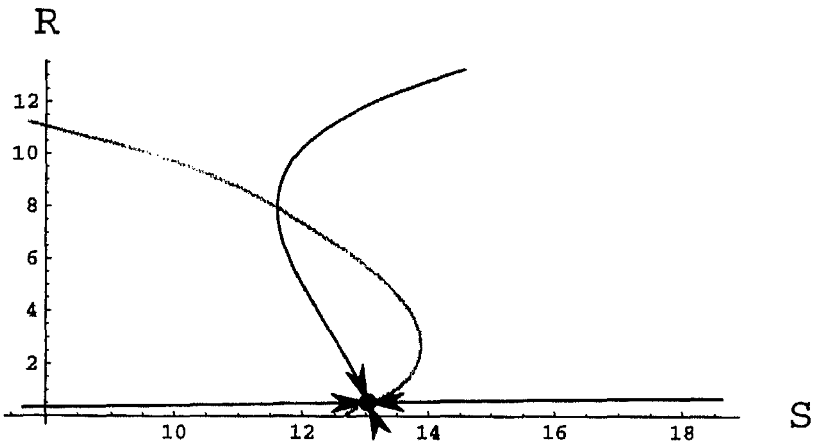

Figure 3. The stable eigenvectors and four sample trajectories on the stable manifold projected onto the (S, R) plane.

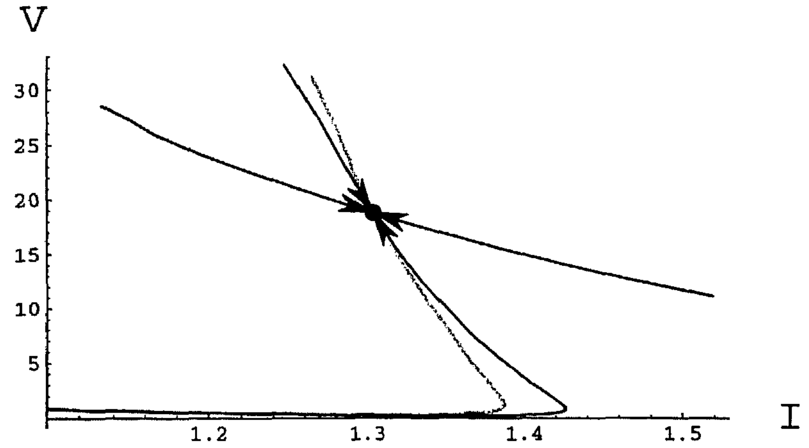

Figure 4. The stable eigenvectors and four sample trajectories on the stable manifold projected onto the (I, V) plane.

into a region where security investment I is high enough to increase the security capital stock and into a region where investment I is too low to keep the security capital stock constant. However, investment I slows down dilapidation of security equipment.

Figures 3 and 4 show the projections of the stable eigenvectors (depicted by arrows) and the projections of four sample trajectories of the stable manifold.

3.3.1. High initial risk of being robbed

Let us investigate the situation when the risk of being robbed, R, is very high. Generally, the firm has two possible policies to manage this situation. On the one hand, the firm can choose a low level of input in production V, which means that there is almost nothing to steal and therefore the firm becomes more and more unattractive for robbery. On the other hand, the firm protects its output by security precaution S. Figure 5 shows four trajectories, where on the first three trajectories the firm starts with a high risk of being robbed and a relatively large amount of security equipment.

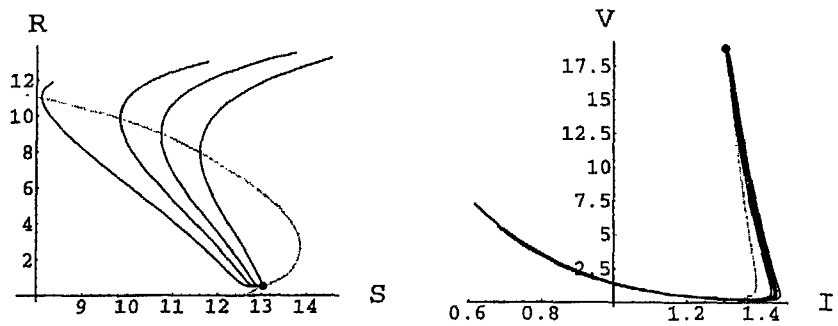

Figure 5. High initial risk of being robbed.

The firm starts out with a medium production level V because it makes the best use of the high security standard. However, it is too expensive to maintain this large security capital stock, which is why the firm decides to choose a very low investment level I. As the security capital stock decreases relative quickly, the firm encounters a high risk of being robbed with a very low production level V and, moreover, the firm tries to slow down the depreciation of the security equipment by increasing security investment I.

Continuing over time, the firm reaches combinations of security standard S and risk of being robbed R (in this area, the fourth trajectory starts), where security investment I is high enough to increase security standard, but in the course of which the firm still cuts back production level V. When the risk of being robbed, R, decreases more and more, the firm attains a point from which it boosts production level V dramatically up to the equilibrium value. At this stage, growing security precautions deter potential robbers, but the rise of S slows down because, on the one hand, the firm reduces security investment I slightly and, on the other hand, a high level of security equipment raises depreciation.

What kind of policies should a firm select when there is a high initial risk of being robbed while it owns an extremely large security capital stock?

As figure 6 shows, the firm starts with a combination in production V and investment in security I, similar to the above policies where the initial risk is the same but the initial security level is lower. Due to the deterrent effect of a high security level, the risk of being robbed decreases rapidly, and the boost in production level occurs at a higher security standard S and allows less investment in security I compared to the above policies. Certainly, less investment in security finally results in declining

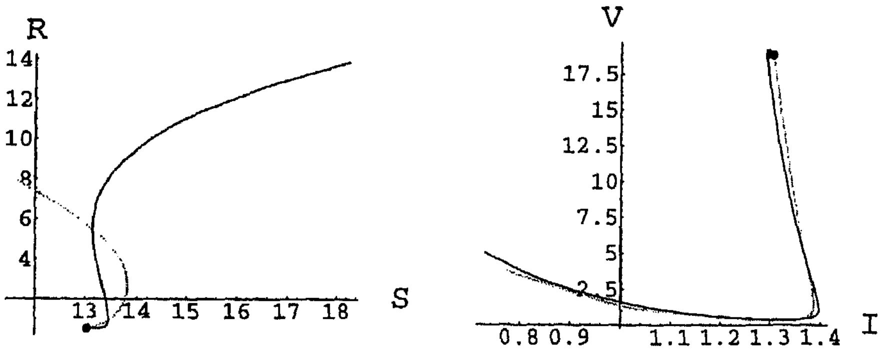

Figure 6. High initial risk of being robbed combined with a huge amount of initial security equipment.

security equipment S, and the firm responds by a minor cutting back of production level V.

3.3.2. Medium initial risk of being robbed

If the risk of being robbed R is modest and the security level S is high (see figure 7), the firm starts out with a relatively low production level V and a medium investment level in security I. If the initial risk of being robbed is significantly higher

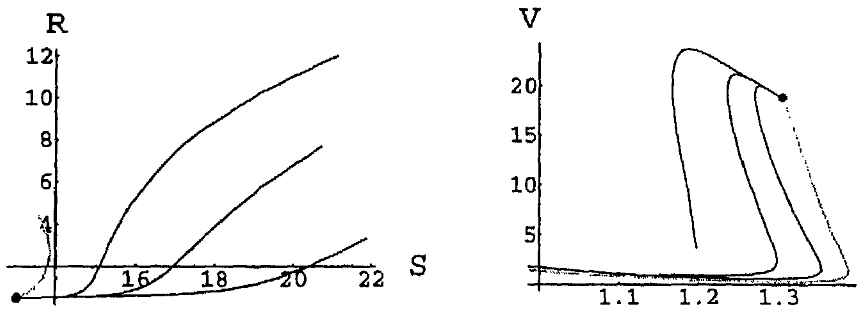

Figure 7. Medium initial risk of being robbed combined with a high amount of initial security equipment.

than the risk's equilibrium value, the firm first reduces risk by keeping the production level low, and a high security level allows the firm to maintain only parts of the security equipment. If the risk of being robbed is close to the risk's equilibrium value, the firm continues with a tremendous stimulation of production and a minor cutback in security investment I. Since security equipment decreases, the risk of being robbed has a tendency to increase. Finally, the firm controls this tendency by both decreasing production and slowing down the decline of security equipment by increasing investment in security.

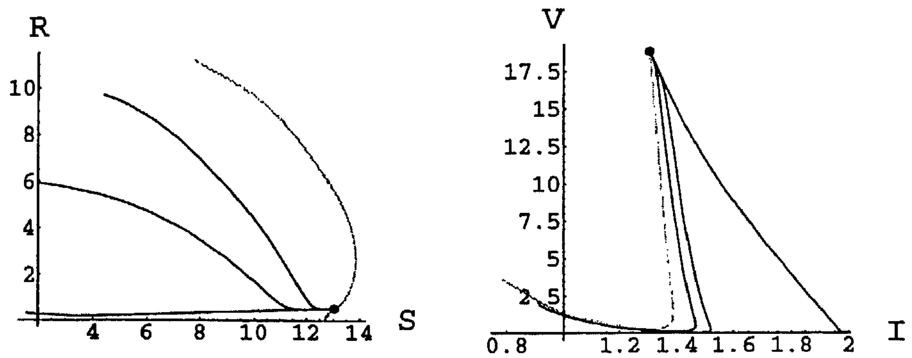

Figure 8. Medium initial risk of being robbed combined with a low amount of initial security equipment.

Figure 8 combines a modest initial level of risk of being robbed and a high initial security level. If the risk of being robbed R is of the same size as the equilibrium value and the security level S is low, then the firm starts out with a practically zero production level V and with a huge investment level in security I. The significantly growing security capital stock compensates the effect of growing production on the risk of being robbed. The firm cuts back security investment I more and more; however, I at all times is large enough to ensure an increasing security capital stock. In comparison, by first combining a low security level S with a distinctly higher risk than in the equilibrium, the firm is reducing risk R, due to a lower than ever security standard, by a low production level V until risk R relapses to an equilibrium risk level. The firm then continues in a fashion similar to the above policies.

3.3.3. *Low initial risk of being robbed*

In principle, the firm makes the best use of low risk R by introducing a high production level V, as depicted in figures 9 and 10. The higher the initial security equipment level, the higher the initial production level and the lower the initial investment in security. After some time, large output attracts robbers. This is why the firm responds by a decrease in production level V and an increase in security investment I, which is either to increase security equipment level S when S is low or to slow down depreciation of security capital stock S when S is high. In the latter case, the firm alters its behavior when the risk level R is close to the equilibrium level. A decrease in security investment together with rising depreciation costs slow down the increase of security equipment. The security equipment, however, is now on such a high level that increasing production level V does not have a large impact on the risk level.

3.3.4. *Optimal security investment and production*

The optimal level of the variable input satisfies (in the case $V > 0$):

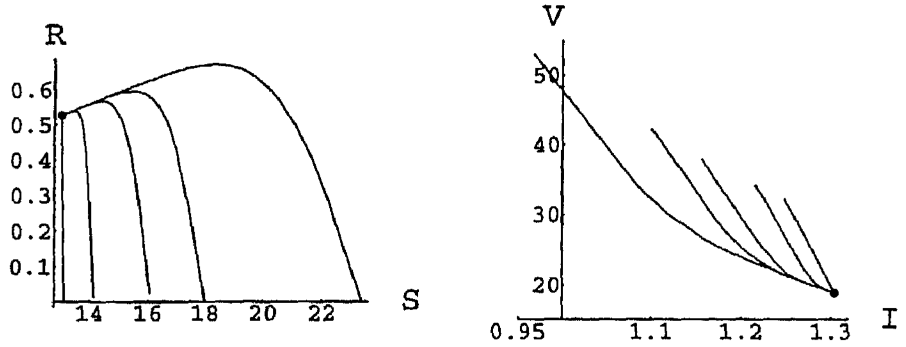

Figure 9. Low initial risk of being robbed combined with a high amount of initial security equipment.

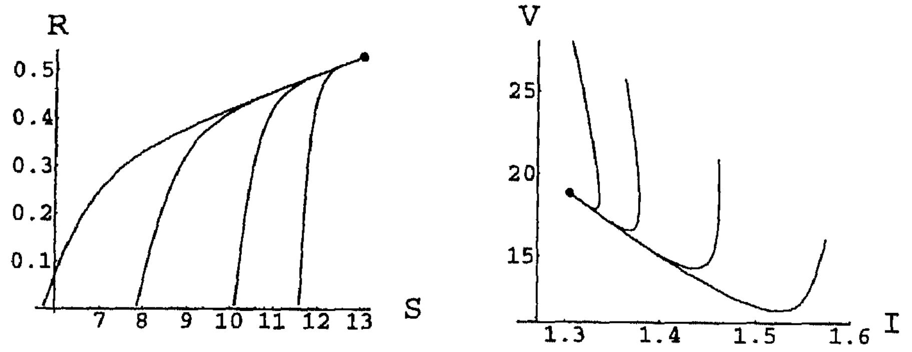

Figure 10. Low initial risk of being robbed combined with a low amount of initial security equipment.

$$pQ'(V)\{1 - L(S, R) + \lambda_2 c_1\} = w. \tag{53}$$

Comparing this relation to equation (11), we notice that the term with λ_2 is negative in the steady state. Since λ_2 is the costate variable belonging to R, which means that λ_2 is equal to the discounted contribution to the objective of an additional unit of criminal risk R, it can be expected that λ_2 will always be negative.

So again, as in the model of section 2, it will hold that whenever it is optimal to have V positive, the amount of variable input will be fixed such that marginal revenue net from criminal losses equals the unit cost of the variable input. The difference however, is that criminal losses now consist of two effects. First, we have the direct effect implying that if there is more production, then there are also more goods that can be stolen. This effect is represented by $-pQ'(V)L(S, R)$. Second, more production means that more criminals become interested in robbing the firm so that the criminal risk R increases. This is the indirect effect, which is quantified as $pQ'(V)\lambda_2 c_1$.

From here, we assume that the optimal trajectory finally enters the steady state; this turned out to occur in the numerical analysis in section 3.3. Now, from (42) and (47), we can obtain that λ_2 is equal to

$$\lambda_2(t) = -\int_t^\infty e^{-r(s-t)} pQ(V(s))L_R(S(s), R(s))ds. \tag{54}$$

As said before, λ_2 measures the discounted contribution to the objective of an extra unit of R. If R is increased by one unit at time t, then from t onwards criminal losses increase with pQL_R at each time period. Equation (54) confirms this statement.

From (40), (41) and (46), we obtain that the level of security investment always satisfies the following equation:

$$-\int_t^\infty e^{-(r+\delta)(s-t)}\{pQ(V(s))L_S(S(s), R(s)) + \lambda_2(s)c_1\gamma'(S(s))\}ds - C'(I(t)) = 0. \tag{55}$$

As could be concluded from equation (30) in section 2, equation (55) also implies that the level of security investments is such that marginal security investment expenses (C') balance the discounted decrease in criminal costs over the whole planning period, caused by an extra unit of security investment at time t. But, compared to (30), here $-\lambda_2 c_1 \gamma'$ is added to the criminal cost reduction, reflecting the fact that an extra unit of security investment increases the stock of security capital S, which in turn decreases the criminal risk R now and in the future with $c_1\gamma'$. This leads to a positive contribution to the objective of $-\lambda_2 c_1 \gamma'$. The cost reduction associated with this positive contribution is equal to (see (54))

$$-\lambda_2(s)c_1\gamma'(S(s)) = c_1\gamma'(S(s)) \int_s^\infty e^{-r(w-s)} pQ(V(w))L_R(S(w), R(w))dw. \tag{56}$$

Of course, this reduction of the criminal costs is also discounted with rate r and corrected for the depreciation factor δ.

We conclude from (55) that the net present value of marginal security investment is zero.

4. Conclusions and extensions

In this paper, we consider two models in which losses from criminal activities influence the economic behavior of a firm. In the first model, the firm has the possibility to reduce these losses by building up a security capital stock. A necessary condition for saddle point convergence is derived and is shown to hold for a specific scenario. In this case, the firm fixes its investment in security equipment such that the discounted stream of reductions of criminal losses due to owning an additional unit of security equipment is equal to marginal security investment expenses.

The second model is more advanced in that it takes the reputation of the firm in the criminal world into account. A firm that produces many goods while not taking many safety precautions faces a high risk of being robbed. In this model, we introduce a variable that represents the firm's "reputation in the criminal world", and which has an increasing effect on criminal losses. Unfortunately, analytical calculations lead to ambiguous stability results. Therefore, we apply a numerical analysis. Taking different initial values (we combine high, medium, and low risk of being robbed with a low or high stock of security, respectively), it turns out that the dynamic system always converges to the same saddle point. Finally, based on the assumption of saddle point convergence, we are able to show how introducing this reputation variable affects the level of investment in security equipment.

In general, analogies can be detected between the topics of "economics of crime" and "pollution control". To see this, in the present case R and S can be interpreted as pollution and abatement capital stock. Then, according to (35), pollution increases by production and decreases due to the use of abatement capital. In the objective, the loss function can be interpreted as that part of the production that is wasted due to the presence of pollution.

An interesting extension of the models in this paper is to include a productive capital stock. In this way, the production process will occur more prominently in the analysis. Moreover, the possibility of robbing the machines of this firm can be taken into account, so that criminal losses will no longer depend solely on production.

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