Money in a model of prior production and imperfectly directed search

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Abstract

This paper considers the effect of monetary policy and inflation on retail markets. It analyzes a model in which: goods are dated and produced prior to being retailed, buyers direct their search on the basis of price and general quality and, buyers' match specific tastes are their private information. Sellers set the same price for all buyers but some do not value the good highly enough to purchase it. The market economy is typically inefficient as a social planner would have the good consumed. The Friedman rule represents optimal policy as long as there is free-entry of sellers. When the upper bound on the number of participating sellers binds sufficiently, moderate levels of inflation can be welfare improving.

1 Introduction

This paper provides an analysis of how inflation affects the choices made by participants in retail markets. Goods are produced prior to being sold in a market with matching frictions. So, sellers have to make inventory decisions based on expected market conditions. At the same time, customers have to choose their cash holding based on expected trading opportunities. When customers' realized preferences for goods are their private information, sellers charge everyone the same price. Consequently, goods may not change hands even when there are positive gains to trade. The aim, here, is to identify the extent to which monetary policy can address this source of inefficiency.

The framework is based on recent work on search based models of money (e.g. Lagos and Wright [2005]). It incorporates two markets that individuals cycle through each period. One market is characterized by centralized trade and the other is a search market in which the opportunity to trade is probabilistic. Following Rocheteau and Wright [2005] (henceforth RW) there is ex ante heterogeneity in that some people, called sellers, can produce the good traded in the search market but have no interest in consuming it. Buyers, on the other hand, cannot produce the search market good but they do get utility from its consumption. This absence of a double coincidence of wants in the search market makes money essential. Buyers acquire money in the centralized market in which everyone can produce and consume. All individuals (buyers and sellers) have the same quasi-linear preferences for centralized market goods. This means that all buyers leave the market with the same cash holdings and all sellers leave with none. Individuals cycle through these markets forever.

As the focus of the paper is on how inflation affects the production decision of suppliers to frictional markets, a point of departure from the existing literature is that sellers are required to produce goods prior to market entry. Goods cannot be altered once made. Of course this requirement would have little relevance if suppliers could store goods indefinitely.[1] Either because they rot (e.g. groceries), they are subject to fashion (e.g. clothes) or they are subject to technological obsolescence (e.g. cars), many of the goods we actually buy are dated. To capture this notion in its simplest form I assume that unconsumed goods perish at the end of the search market.

From a theoretical standpoint the use of prior production has some distinct advantages over the more common approach in which production occurs at the point of sale. First, it provides a more natural way to endogenize the number of participating sellers. RW uses an arbitrary entry cost while here the cost of making the good represents the cost of market entry. (The reason why all sellers might not enter is that goods are perishable and they may not find a trading partner.) Second, it introduces an additional margin for potential hold-up problems. Choosing the quality of the good prior to market entry represents an investment by sellers who may not receive its marginal private or social benefit. Third, for buyers to direct their search requires some degree of commitment from the sellers as to the terms of trade. Here, that commitment is a natural consequence of the environment. Once a good of a given quality is made there is complete commitment to it.

Two main variants of the model are considered. In the baseline model preferences of the buyers for the search good are fixed and identical. In the extended model, potential buyers' preferences are match specific. Although other market structures are discussed, both the baseline and the extended

[1] The interaction of non-perishable good purchases, inventories and inflation is an interesting topic that is not the focus of this paper.

model incorporate what RW calls competitive search.[2] Specifically, buyers are able to direct their search according to the quality of the good for sale and its price. In the baseline model therefore, there are 4 decision margins: buyers decide how much cash to bring, sellers decide on whether or not to participate, the quality of the good to produce, and its price. The extended model introduces a fifth decision margin: buyers set a reservation preference level required to purchase the good. The paper considers the extent to which monetary policy can bring about efficiency on each or any of these margins.

A frequently voiced criticism of the random search approach to markets is: "I don't shop like that! If I want a car, I go to the dealership; if I want shoes, I go to the shoe shop. Where is the marketing?" In contrast, competitive (or directed) search may have overcorrected. There are distinct markets for every conceivable specification of goods. Introducing match specific preferences provides a notion of imperfectly directed search - even though the shoe shop may be easy to find, they may not have my size or a style I like.[3]

In the baseline model, the Friedman rule, that equates the gross rate of money growth to the common discount factor, is shown to bring about efficiency on all 4 decision margins. Of course, goods do go to waste but

[2] There is some confusion as to the distinction between directed and competitive search. Historically (e.g. Moen [1997]), competitive search was a continuous time construct in which matching occurs sequentially in submarkets indexed by those features of goods (or workers) that are known publicly. Directed search, on the other hand, has been associated with either a one-shot or a discrete time environment in which one side of the market meets the other in batches as in Acemoglu and Shimer [1999]. RW conflates these structures by introducing submarkets in discrete time with one-on-one meetings.

[3] This paper uses the term "imperfectly directed" search to capture the notion that buyers may not know everything about a good before they go shopping for it. By contrast, Menzio [2007] has a notion of "partially directed" search in which (equivalently) sellers cannot commit to prices but use them to signal the quality of their goods.

no social planner who is subject to the same search frictions as the market economy could do any better than monetary policy that follows the Friedman rule. When the rate of money growth exceeds the discount factor, money becomes costly to hold and this cost drives a “tax wedge” between the utility of buyers and the cost incurred by sellers.

In the extended model, a buyer’s true preference towards the good carried by any seller he meets is his private information. Sellers cannot, therefore, make the price contingent on the buyer’s type; they post a single price. Of course, they can only recoup their cost of production when that price is strictly positive. As long as the distribution of buyers’ preference shocks attaches positive probability to every neighborhood of zero (i.e. there is a chance that the buyer hardly likes the good) then buyers will sometimes reject goods in favor of holding on to their cash. Still, since the good will rot if not consumed, a social planner would have the seller hand the good over to the buyer no matter how little the latter likes it. Monetary policy cannot, therefore, be fully efficient. It will be shown, however, that as long as there is free-entry of sellers, the Friedman rule represents optimal monetary policy. Even though increasing inflation away from the Friedman rule makes buyers less picky and actually increases seller participation, the sellers reduce the quality of their output such that every one weakly prefers the Friedman rule.

To check the robustness of this result, the analysis considers what happens if each of the decision margins are shut down. Making the quality of the good exogenous does not affect the optimality of the Friedman rule. Increasing inflation away from the Friedman rule in this case, still has buyers being less picky but now sellers reduce participation which reduces welfare more than the improved transaction rate increases it. When the free-entry margin is shut down, however, inflation causes buyers to become less picky and sellers produce lower quality goods. The net effect of changes in inflation at the Friedman rule is ambiguous. If the upper bound on market entry for sellers binds sufficiently, increases in inflation away from the Friedman rule can improve welfare.

2 Literature

Other papers that have incorporated a requirement on sellers to produce prior to market entry are Jafarey and Masters [2003] and Dutu and Julien [2008]. Both of these papers consider indivisible money. Jafarey and Masters [2003] use a random search environment with match specific preference shocks to address the relationship between prices and output under various sources of economic growth. Dutu and Julien [2008] analyze a directed search framework and is concerned with the existence of a monetary equilibrium. The baseline model of the current paper is essentially Dutu and Julien [2008] with divisible money. The extended model of the current paper is essentially Jafarey and Masters [2003] with divisible money and competitive search.

A principal concern of the current paper like that of RW is the efficacy of monetary policy. In that paper, production only occurs at the point of exchange. They consider 3 distinct market structures for the determination of the terms of trade in the search market. The current paper only looks at (what they refer to as) competitive search in detail. Ex post bargaining does not work in the current baseline environment as buyers would bring no money. A Walrasian type market with participants randomly chosen from the buyers and sellers is discussed briefly below. That model only predicts nontrivial outcomes when the number of sellers exceeds the number of buyers. In which case, sellers enter until the number of participants equals the number of buyers. But then, the price level is indeterminate leading a to a continuum of equilibria. RW does not consider match specific preferences but the version of their model with random search and ex post bargaining does find that, like the extended model of this paper, market equilibrium is inefficient even though the Friedman rule is optimal policy. The reasons are quite different. In RW there is a hold-up problem due to the ex post division of the match surplus. Here it is the private information over preferences that prevents prices being made contingent on type.

Search based monetary models that incorporate private information include Faig and Jerez [2006] and Ennis [2008]. In both of these there is match-specific preferences along the lines of that considered here but production occurs at the point of sale. In Ennis [2008] matching is random and sellers get all of the bargaining power. In the absence of private information there would be no monetary exchange because buyers would have no incentive to carry cash. With the private information, however, informational rents accrue to the buyers restoring the possibility of monetary equilibria. In Faig and Jerez [2006] search is directed but sellers post price schedules which lead buyers to purchase goods in such a way as to reveal their type. Inflation causes price schedules to flatten out creating greater efficiency losses than would emerge without private information. By comparison, in the current paper, because production occurs prior to market entry there is no mechanism that can separate buyers by type.

3 Baseline Model

3.1 Environment

Time is discrete and continues forever. Every period of time is divided into 2 subperiods: day and night. During the day there is a centralized frictionless market for a homogeneous and perfectly divisible good. In the night, trade occurs in a decentralized market characterized by search frictions. There are two types of individual in the model: those who consume the good produced in the night market and those who can produce the night market good. Following RW they will be referred to as buyers and sellers respectively. The measure of buyers is normalized to 1 while the measure of sellers is $\bar{n}$. All the buyers enter each night market but only a subset of the sellers do. In contrast to RW, for sellers to indicate a willingness to trade at night they must produce a good. In any period t, the measure of sellers, n_t, who enter the night market will be controlled by a free-entry condition. The day good is non-storable. The night good can be held by the seller for that night but cannot be stored through the ensuing day.

The net instantaneous utility of a buyer at date t is

$$U_t^b = v(x_t) - y_t + \beta_d u(q_t),$$

where x_t is the quantity of the day good consumed, y_t is the quantity of day good produced and q_t is quality of the night good consumed. Here, $\beta_d \leq 1$ is a common discount factor between the day and the night. Both instantaneous utility functions u and v are strictly increasing and strictly concave. I also require that $u(0) = 0$, $u'(0) = \infty$ and that there exists an x^* such that $v'(x^*) = 1$. The function $v(.)$ is normalized so that $v(x^*) = x^*$.

The instantaneous utility of a seller at date t is

$$U_t^s = v(x_t) - y_t - c(q_t),$$

where q_t is now the quality of production and $c(.)$ is the associated cost function. Notice that the production occurs in the evening (i.e. at the end of the daytime subperiod) and so β_d does not apply. Sellers have to produce before they enter the night market. They cannot augment their production after the beginning of the night market which means that they are fully committed to their choice of q_t. I assume the cost function is strictly increasing, strictly convex and $c(0) = c'(0) = 0$. I also assume $\lim_{q\to\infty} c'(q) = \infty$. This implies a unique $\bar{q} > 0$ such that $\beta_d u(\bar{q}) = c(\bar{q})$ and a unique q^* in $[0, \bar{q}]$ such that $\beta_d u'(q^*) = c'(q^*)$. There is a common discount factor, $\beta_n < 1$ between night and day. I will denote the product $\beta_n\beta_d$ by β, the daily discount factor. Thus, lifetime utility of an individual type $i = b, s$ is $\sum_{t=0}^{\infty} \beta^t U_t^i$.

The night market is characterized by trading frictions. Specifically, the measures of buyers and sellers who get a trading opportunity are both equal to $\alpha(n)$ where n is the measure of sellers who enter the market. I assume that $\alpha(n) \leq \min\{1, n\}$, $\alpha(0) = 0$, $\alpha'(n) > 0$, $\alpha''(n) < 0$ and $\lim_{n\to\infty} \alpha(n) = 1$. These largely reflect the requirement that the matching probability of the buyers, $\alpha(n)$, does not exceed 1. The matching probability of the sellers (with goods in hand) is $\alpha(n)/n$ which will also be less than 1 under these restrictions. Constant returns to scale in matching is a further desirable (and commonly assumed) feature of the underlying matching technology. Here, this amounts to $\alpha(n)/n$ decreasing in n or $\alpha(n) \geq \alpha'(n)n$. Given the previous assumptions, this guaranteed if $\alpha'(0) = 1$ which will be imposed in the sequel.

3.2 Efficiency in the baseline model

The Central Planner weights all individuals equally in the welfare function. (Money provides no utility so it does not feature in the Planner's problem.) As recognized in RW, these models do not feature any transitional dynamics so the Planner can and will always choose a stationary path for consumption. Equal treatment implies that contingent on type (buyer or seller) everyone produces and consumes the same amount. The Planner is subject to the same trading frictions as the market but does not require *quid pro quo* for exchange to occur. As utility functions are strictly increasing and goods are perishable, all output brought to any trading opportunity will be consumed. Unlike in RW, however, the Planner cannot avoid output going to waste. The Planner maximizes

$$\begin{aligned} W(x,n,q;\bar{n}) &\equiv (v(x)-x)(1+\bar{n})+\beta_d\alpha(n)u(q)-nc(q) \qquad (1) \\ \text{subject to } n &\leq \bar{n}. \end{aligned}$$

The first order conditions for an internal solution are

$$\begin{aligned} x &: v'(x_p)=1 \\ q &: \beta_d\alpha(n_p)u'(q_p)-n_pc'(q_p)=0 \qquad (2) \\ n &: \beta_d\alpha'(n_p)u(q_p)-c(q_p)=0 \qquad (3) \end{aligned}$$

The choice of $x_p = x^*$ is clearly optimal (by assumption) and independent of q and n. As $W(x^*,.,.;\bar{n})$ is not necessarily concave, the existence of n_p and q_p are more problematic.

Claim 1 *The optimal choices, n_p and q_p respectively of the number of active sellers and output per seller exist in* $(0,\bar{n}]\times(0,q^*)$.

Proof. This is special case of Claim 3 below. ■

The differentiability of $W(x, n, q; \bar{n})$ and the fact that the first order conditions are necessary for an interior solution means that whenever $n_p < \bar{n}$, (n_p, q_p) solves (2) and (3). Otherwise, q_p is the unique solution to

$$\beta_d \alpha(\bar{n}) u'(q_p) - \bar{n} c'(q_p) = 0.$$

3.3 Baseline Model Market Economy

In the absence of a Central Planner, the coincidence of wants problem makes some form of money essential. Here, money is perfectly divisible and agents can hold any non-negative amount. The aggregate nominal money supply M_t grows at constant gross rate γ so that $M_{t+1} = \gamma M_t$. New money is injected (or withdrawn if $\gamma < 1$) by lump-sum transfers (taxes) in the day (centralized) market. Following RW I assume these transfers go only to buyers, but this is not essential for the results. What matters is that transfers do not depend on the choices individuals make. Also, we restrict attention to policies where $\gamma \geq \beta$. For $\gamma < \beta$ there is no equilibrium.

In the day market the price of goods is normalized to 1 at each date t, while the relative price of money is denoted ϕ_t. Let $z_t = \phi_t m_t$ be the real value of an amount of money m_t. I will focus throughout on steady-state allocations in which aggregate real variables are constant over time. This means that $\phi_{t+1} = \phi_t / \gamma$. It will be useful to use individual real money balances, z_t, as the individual's choice variable for money holding rather than m_t.

Once a seller chooses the quality, q, of the good she intends to bring to the night market and the *real* price, d, at which she intends to sell it, neither can be changed.[4] These price quality pairs (q, d) are observed by all buyers who use them a basis for directing their search. Thus, a seller's the choice of a pair, (q, d) opens a submarket for goods of quality q at price d to which other sellers and buyers may be attracted. If n represents the ratio of sellers to buyers in that submarket, then $\alpha(n)$ and $\alpha(n)/n$ will be respectively the matching probability of buyers and sellers in that submarket. Submarkets are therefore indexed by $\omega \in \Omega = \mathbb{R}^3_+$ where $\omega = (d, q, n)$.

Timing within a period is as follows. In the morning, sellers each announce q, the quality of the good they are going to produce and d, the real price at which they will part with it. On the basis of this knowledge, buyers decide how much money they will need for the night market and trade in the day market accordingly. As the terms of trade are predetermined and there is an opportunity cost to holding money (foregone daytime consumption), buyers only bring enough with them to acquire the a good in the submarket they have chosen to enter: $z = d$. So that when sellers pick d they are effectively also choosing the cash holdings of anyone they meet. In the sequel I use $\omega = (z, q, n)$. As sellers have no use for money in the night market they enter with zero cash balances.

Let $V^i(\omega)$ be the value to entering submarket ω for $i = s, b$ (respectively the seller and the buyer). We have

$$\begin{aligned} V^b(\omega) &= \alpha(n)\left[u(q) + \beta_n W^b(0)\right] + [1-\alpha(n)]\,\beta_n W^b\left(\frac{z}{\gamma}\right) \qquad (4) \\ V^s(\omega) &= \frac{\alpha(n)}{n}\beta_n W^s\left(\frac{z}{\gamma}\right) + \left[1 - \frac{\alpha(n)}{n}\right]\beta_n W^s(0) \end{aligned}$$

where $W^i(z)$, $i = s, b$ are the values to entering the day market with real

[4]The nominal price of a good with real price d is d/ϕ_t.

money holding z for sellers and buyers respectively. Thus

$$\begin{aligned} W^b(z) &= \max_{x,y,\hat{\omega}} \left\{v(x) - y + \beta_d V^b(\hat{\omega})\right\} \\ \text{subject to } \hat{z} + x &= z + T + y \end{aligned} \tag{5}$$

where T is the real transfer/tax. And,

$$\begin{aligned} W^s(z) &= \max_{x,y,\hat{\omega}} \left\{v(x) - y + \max\left[\beta_d V^s(\hat{\omega}) - c(\hat{q}), \beta W^s(0)\right]\right\} \\ \text{subject to } x &= z + y \end{aligned} \tag{6}$$

Both problems (5) and (6) and are subject to non-negativity constraints on x, y, and $\hat{z}$. In the absence of these, it should be clear that irrespective of the values of other variables, both buyers and sellers pick $x = x^* > 0$. For $y \geq 0$ to bind on buyers, requires $x^* < z - \hat{z} + T$. As all buyers get the same transfer, this will never bind in steady-state.[5] As sellers do not bring money into the night market, $y \geq 0$ requires that $x^* \geq z$. The analysis continues as if this is true. Any implied restrictions on parameters will be derived below. It means that $x = x^*$ throughout and neither x nor y appear as components of equilibrium.

Definition 2 *A symmetric, competitive search equilibrium is a submarket, $\tilde{\omega} = (\tilde{z}, \tilde{q}, \tilde{n})$ such that given all other buyers and sellers enter $\tilde{\omega}$, then $\tilde{\omega}$ solves both the individual buyer's problem, (5), and the individual sellers problem,*

[5] Suppose $\bar{z}$ is the steady state real money balances brought to the night market by buyers in period $t-1$. Then in the morning of period t, if they were successful in buying something $z_t = 0$, if not $z_t = \bar{z}/\gamma$. But,

$$T = (\gamma - 1)m_t/\phi_{t+1} = \bar{z}(\gamma - 1)/\gamma$$

So y is at least equal to x^*.

(6) subject to

$$\beta_d V^s(\hat{\omega}) - c(\hat{q}) = \beta_d V^s(\tilde{\omega}) - c(\tilde{q}) \begin{cases} = \beta W^s(0) \text{ } for \text{ } \tilde{n} \leq \bar{n} \\ \geq \beta W^s(0) \text{ } for \text{ } \tilde{n} = \bar{n} \end{cases}$$

The restriction to symmetry here means that there is a unique market to which all buyers and sellers go. This choice is motivated by RW who provide the analysis for the more general case but then restrict attention to equilibria with a unique active market. As non-negativity of y, does not bind (by assumption) it is immediate from (5) and (6), that $W^i(z) = z + W^i(0)$ for $i = b, s$ and that the amount of money brought into the night market is independent of the amount brought into the day market that morning.

To characterize equilibrium we posit the existence of an $\tilde{\omega}$ and consider the choice of a seller who considers deviation by announcing production of a good of quality q and demanding real balances z in exchange for the good. Buyers will enter the deviant's submarket in such numbers as makes them indifferent between that market and the one specified by the equilibrium. So, after accounting for the opportunity cost of cash brought into the night market,

$$\begin{gathered} \tilde{\omega} \in \arg\max_{\omega} \{\beta_d V^s(\omega) - c(q)\} \\ \text{subject to } \beta_d V^b(\omega) - z = \beta_d V^b(\tilde{\omega}) - \tilde{z} \\ \beta_d V^s(\tilde{\omega}) - c(\tilde{q}) \geq \beta W^s(0), \\ \text{and } \tilde{n} \leq \bar{n} \end{gathered} \tag{7}$$

It is now well known (see RW, Masters [2009], or Rogerson et al [2005]) that competitive search equilibrium in this environment will be isomorphic to that in the "dual" economy. In the dual here, buyers will commit to and advertise both prices and goods qualities at which they are willing to trade and sellers search accordingly. This latter economy is preferred here for expositional purposes. The buyers' problem that corresponds to Problem (7) is:[6]

$$\tilde{\omega} \in \arg\max_{\omega} \left\{ \beta_d V^b(\omega) - z \right\}$$

$$\text{subject to } \beta_d V^s(\omega) - c(q) = \beta_d V^s(\tilde{\omega}) - c(\tilde{q}) \tag{8}$$

$$\text{where free-entry implies} \begin{cases} \beta_d V^s(\tilde{\omega}) - c(\tilde{q}) = \beta W^s(0) \text{ for } \tilde{n} < \bar{n} \\ \beta_d V^s(\tilde{\omega}) - c(\tilde{q}) \geq \beta W^s(0) \text{ for } \tilde{n} = \bar{n} \end{cases}$$

Substituting in for the value functions and eliminating z using the constraint, reduces problem (8) to

$$(\tilde{n}, \tilde{q}) \in \arg \max_{(n,q)\in[0,\bar{n}]\times[0,\infty)} \left\{ \begin{array}{c} \beta_d \alpha(n) u(q) - nc(q) - \left(\frac{nc(q)}{\alpha(n)\beta}\right)[\gamma - \beta] \\ +[\gamma - \beta + \alpha(n)\beta]\left[\beta_d V^s(\tilde{\omega}) - c(\tilde{q}) - \beta W^s(0)\right] \\ +\beta W^b(0) \end{array} \right\} \tag{9}$$

The reason for recasting the problem should now be clear. Except for constant terms, problem (9) in the market economy and the Planner's problem, (1), coincide when $\gamma = \beta$. That is, the Friedman rule is efficient in this baseline economy. Of course goods carried by sellers who do not match still go to waste but a social planner subject to the same matching frictions could not do any better than the market economy does at the Friedman rule.

In general the objective function in problem (9) is not concave so that a unique solution is not assured.

Claim 3 *A solution, $(\tilde{n}, \tilde{q})$, to problem (9) exists within* $(0, \bar{n}] \times (0, q^*)$

Proof. Define $\Phi(n, q)$ as

$$\Phi(n, q) = \beta_d \alpha(n) u(q) - nc(q) - \left(\frac{nc(q)}{\alpha(n)\beta}\right)[\gamma - \beta].$$

[6] It is simple to verify that the first order conditions are identical.

Continuity of $\Phi(n,q)$ immediately implies that it achieves a maximum on $[0,\bar{n}] \times [0,q^*]$. So we have to show that (i) this maximum must also solve problem (9) (ii) it cannot exist on any boundary except $n=\bar{n}$.

Assertion (i) follows from the strict concavity of $\Phi(n,q)$ with respect to q and the fact that $\alpha(n) < n$ for any $n > 0$ means that $\frac{\partial \Phi}{\partial q}\Big|_{q=q^*} < 0$. So for any $n \in [0,\bar{n}]$, and $q > q^*$, $\Phi(n,q)$ can always be increased by reducing q to be less than q^*.

Assertion (ii) follows because

$$
\begin{aligned}
\lim_{q\to 0} \Phi(n,q) &= 0 \text{ for all } n \geq 0 \\
\lim_{q\to 0} \frac{\partial \Phi(n,q)}{\partial q} &= \lim_{q\to 0} \left\{ \beta_d \alpha(n) u'(q) - nc'(q) - \left(\frac{nc'(q)}{\alpha(n)\beta} \right) [\gamma - \beta] \right\} > 0 \text{ for all } n > 0
\end{aligned}
$$

This means that $\Phi(n,q)$ achieves some strictly positive value for some $q > 0$ ruling out the portion of the q axis in the interval $(0,\bar{n}]$. Now, for all $n > 0$ we know that $\alpha(n) < n$ so $\frac{\partial \Phi(n,q)}{\partial q}\Big|_{q=q^*} < 0$ and $\Phi(n,q)$ is strictly concave in q. This rules out the portion of $q = q^*$ in the interval $(0,\bar{n}]$.

As $\lim_{n\to 0} n/\alpha(n) = 1/\alpha'(n) = 1$,

$$
\lim_{n\to 0} \Phi(n,q) = -c(q) \left(\frac{\gamma - \beta}{\beta} \right)
$$

so $\Phi(n,q) \leq 0$ for $n = 0$ and $q \in [0,q^*]$. This means that $\Phi(n,q)$ cannot attain a maximum on the n axis. ∎

The concavity of the objective function with respect to q means that if $\tilde{n} = \bar{n}$, the solution is unique. In that case $\tilde{q}$ solves,

$$
\alpha^2(\bar{n})\beta_d \beta u'(q) - \bar{n}\left[\gamma - \beta + \alpha(\bar{n})\beta\right] c'(q) = 0.
$$

For any interior solution $(\tilde{n},\tilde{q})$ to problem (9) we have,

$$
\begin{aligned}
\alpha^2(n)\beta_d \beta u'(q) - n\left[\gamma - \beta + \alpha(n)\beta\right] c'(q) &= 0 \qquad (10) \\
\alpha'(n)\alpha(n)\beta_d \beta u(q) - \left[(1-\eta(n))(\gamma - \beta) + \alpha(n)\beta\right] c(q) &= 0 \qquad (11)
\end{aligned}
$$

where $\eta(n) \equiv \frac{\alpha'(n)n}{\alpha(n)}$, the elasticity of the matching function with respect to the number of sellers in the submarket. A requirement for the solution of (9) to be an equilibrium is that the implied value of y, production in the day market, be non-negative for all participants. As discussed above this requires $z \leq x^*$. Given other parameters, we can always chose $v(.)$ in such a way that this requirement will not bind. Given $v(.)$, however, equilibrium is characterized by the solutions to equations (10) and (11) subject to $\gamma \leq \alpha(\tilde{n})\beta x^*/\tilde{n}c\,(\tilde{q})$ (a sufficient condition for this is $\gamma \leq \alpha(\bar{n})\beta x^*/\bar{n}c\,(q^*)$).

Rearranging equations (10) and (11), dividing (10) by (11), and multiplying through by q yields

$$e_u(q) = \left(\frac{\eta(n)\left[\gamma - \beta + \alpha(n)\beta\right]}{(1-\eta(n))(\gamma-\beta) + \alpha(n)\beta} \right) e_c(q). \tag{12}$$

Here e_u and e_c are elasticities of the utility and cost function respectively. It is straightforward to show that if $\eta(n)$ decreases monotonically so do the contents the parentheses in (12).[7] Consequently, a sufficient condition for the uniqueness of equilibrium is that the utility and cost functions be isoelastic and $\eta'(n) < 0$.[8] Under the Friedman rule, $\gamma = \beta$, equation (12) reduces to

$$e_u(q) = \eta(n)e_c(q). \tag{13}$$

[7] The elasticity of the matching function cannot increase monotonically as $\eta(0) = 1$ and $\eta(n) \in [0,1]$ for all n.

[8] Examples of functions that satisfy the requirements on $\alpha(n)$ are

$$\begin{aligned} \alpha(n) &= 1 - e^{-n} \\ \alpha(n) &= \frac{n}{1+n} \\ \alpha(n) &= \tanh(n) \end{aligned}$$

or any convex combination of any 2 of them. In all such cases $\eta(n)$ declines monotonically from 1 to 0.

As the contents the parentheses in (12) are increasing in γ, money is not super-neutral. Increasing the rate of money growth increases the wedge between the elasticities of cost and utility. If they are isoelastic and $\eta'(n) < 0$, participation by sellers, n, will fall as the rate of money growth increases. It then follows from the second order conditions that if (q, n) is an optimum, q also decreases with γ.

3.4 Alternative market economies

The environment studied so far is essentially the directed search version of RW but with ex ante production. For comparability with the literature then, we should also consider random search with ex post bargaining, and prices determined in a Walrasian fashion in the night market.

In order to use random search with ex post Nash bargaining we need to assume (as in RW and Lagos and Wright [2005]) that buyers' money holdings are revealed in bilateral meetings.[9] In any case buyers are aware that they cannot pay more for the good than they bring to the market. When goods are produced at the point of sale, buyers are incentivized to bring cash because more money gets them a nicer good. When goods are produced ex ante, no such benefit to bringing more money exists. So, for any strictly positive amount of money held by other buyers an individual can will always bring slightly less money and acquire the good. That is, any proposed non-trivial equilibrium will unravel as in Diamond [1971].

As recognized in RW, in the presence of a double coincidence of wants problem and anonymity, money continues to be essential even under price

[9] In contrast, the directed search model described above remains completely agnostic with respect to whether or not buyers' money holdings are private information - money holding is revealed in equilibrium through their search behavior.

taking behavior. Frictions are captured by introducing a probability that any buyer only gets an opportunity to trade with probability α_b and sellers get to trade with probability $\alpha_s = \alpha(n)/n$ where n is the number of sellers who produce a good.

Definition 4 *A Walrasian equilibrium is a tuple, (p_w, q_w, n_w) such that given the real price, p_w, and goods quality, q_w:*

- *buyers are at least as well off from bringing real balances p_w to the market as bringing 0 (i.e. not trading)*
- *if $n_w < \bar{n}$ sellers are indifferent between market entry and non-participation*
- *if $n_w = \bar{n}$ sellers are at least as well off from participation as non-participation*
- *the market clears*

If $\alpha_b > \alpha(\bar{n})$ then buyers necessarily outnumber sellers. Because final goods are indivisible, for any given quality of good, the price will be that which extracts all ex post surplus from buyers. But as buyers face an opportunity cost of acquiring money in the day market they will simply bring no money and the market collapses. If $\alpha_b < \alpha(\bar{n})$ the entry margin for sellers is active. Whenever there are more sellers than buyers the price of goods will fall to zero - the residual value of the good to the seller. Consequently only enough sellers will enter so as to ensure that $\alpha_b = \alpha(n)$. That is, the number of buyers and sellers entering the market will endogenously be equal. The value of α_b pins down n_w.[10] Given n_w, equilibrium requires that sellers be

[10]If, as in RW, $\alpha_b \equiv \alpha(n)$ then there will necessarily be indeterminacy of equilibrium.

indifferent between participation and non-participation. Thus,

$$\beta_d V^s(p, q, n_w) - c(q) = \beta W^s(0).$$

The implied relationship between q and p is

$$p = \frac{\gamma n_w c(q)}{\beta \alpha(n_w)} \tag{14}$$

We also require that buyers are at least as well off from bringing sufficient cash to buy the good into the night market as they are from bringing no money and consuming nothing. Thus

$$\beta_d V^b(p, q, n_w) - p \geq V^b(0, 0, n_w).$$

Substituting for V^b implies

$$\beta_d \alpha(n_w) u(q) - (1 - \alpha(n_w)) p \left(\frac{\gamma - \beta}{\gamma} \right) \geq 0 \tag{15}$$

For given $v(.)$, non-negativity of day time production for the sellers requires that $p \leq x^*$. That is

$$\frac{\gamma n_w c(q)}{\beta \alpha(n_w)} \leq x^* \tag{16}$$

Walrasian equilibria are therefore characterized as any combination (p_w, q_w, n_w) that satisfy equation (14) and inequalities (15) and (16) with $\alpha(n_w) = \alpha_b$.

At the Friedman rule, we have

$$p_w = \frac{n_w c(q_w)}{\alpha(n_w)} \leq x^*$$

$$\beta_d \alpha(n_w) u(q_w) \geq 0$$

Because there is no opportunity cost of carrying money, buyers do not care about the real price of the good so any positive q_w such that p_w does not exceed x^* is consistent with equilibrium. Away from the Friedman rule (i.e. when $\gamma > \beta$) substituting for p_w into inequality (15) yields

$$\beta_d \beta \alpha^2(n_w)u(q_w) - (1 - \alpha(n_w))n_w c(q_w)(\gamma - \beta) \geq 0. \tag{17}$$

The left hand side of (17) is concave in q_w. It holds with equality at zero and some strictly positive finite value of q_w which I will label $\bar{q}_w$. Clearly, for n_w given, $\bar{q}_w$ approaches zero as γ gets large. Now let $\tilde{q}_w$ be the solution to

$$\gamma n_w c(q_w) - \beta\alpha(n_w)x^* = 0$$

Again, $\tilde{q}_w$, approaches zero as γ gets large. So, we have continuum of equilibria for every value of n_w which are indexed by q_w in the range $[0, \max\{\bar{q}_w, \tilde{q}_w\}]$.

4 Match-specific heterogeneity

This section incorporates the idea that search may be imperfectly directed in that all aspects of a good may not be known prior to seeing it. Thus, a buyer's instantaneous utility from consumption of a good of general quality q becomes $\varepsilon u(q)$ where $\varepsilon \sim G(.)$ is realized after the buyer and seller meet. The distribution $G(.)$ is assumed to be continuous on $(0, \bar{\varepsilon}]$ and differentiable with density, $g(.)$. The realized value of ε is the private information of the buyer. For this extended environment $\bar{q}$ is redefined such that $\bar{\varepsilon}u(\bar{q}) = c(\bar{q})$.

4.1 Efficiency

As utility functions are strictly increasing, goods are perishable and $\varepsilon > 0$, whether or not the planner observes the realized value of ε is moot - all output brought to any trading opportunity should be consumed. The Planner's objective function becomes

$$W(x,n,q;\bar{n}) \equiv (v(x)-x)(1+\bar{n}) + \beta_d \alpha(n)\hat{\varepsilon}u(q) - nc(q)$$

where $\hat{\varepsilon}$ is the unconditional expected value of ε. The efficient outcome is identical to that of the baseline model with $u(q)$ replaced by $\hat{\varepsilon}u(q)$.

4.2 Market Economy

Borrowing from the preceding analysis, we know that as long as sellers set the same real price, z, buyers will bring that amount of money into the night market (or nothing). Let $\omega \equiv (z,q,n)$. The value to being a buyer in the night market is then

$$V^b(\omega) = \alpha(n)\mathbb{E}_\varepsilon\left[\max\left\{\varepsilon u(q) + \beta_n W^b(0), \beta_n W^b\left(\frac{z}{\gamma}\right)\right\}\right] + (1-\alpha(n))\beta_n W^b\left(\frac{z}{\gamma}\right)$$

where $W^b(.)$ is the value function for buyers in the day market and is identical to that derived for the baseline model. This is because they choose whether to give up their cash for the good only after they meet the seller and realize the extent to which they want it. From earlier analysis we know that $W^b(z) = z + W^b(0)$ so

$$V^b(\omega) = \alpha(n)\mathbb{E}_\varepsilon\left[\max\left\{\varepsilon u(q) - \beta_n \frac{z}{\gamma}, 0\right\}\right] + \beta_n W^b\left(\frac{z}{\gamma}\right)$$

Given, ω, let $\varepsilon_R = \beta_n z/\gamma u(q)$ which represents the reservation match specific preference shock for the buyer above which she will purchase the good and below which she will not. It follows that

$$V^b(\omega) = \alpha(n)u(q)S_G(\varepsilon_R) + \beta_n W^b\left(\frac{z}{\gamma}\right)$$

where $S_G(.)$ is the "surplus function" of distribution G. Thus

$$S_G(\varepsilon_R) \equiv \int_{\varepsilon_R}^{\bar{\varepsilon}} [\varepsilon - \varepsilon_R]\, dG(\varepsilon) = \int_{\varepsilon_R}^{\bar{\varepsilon}} [1 - G(\varepsilon)]\, d\varepsilon$$

where the final equality comes from integration by parts.

The value to being a seller in the night market is then,

$$V^s(\omega) = \frac{\alpha(n)}{n}\left[1 - G(\varepsilon_R)\right]\beta_n W^s\left(\frac{z}{\gamma}\right) + \left(1 - \frac{\alpha(n)}{n}\left[1 - G(\varepsilon_R)\right]\right)\beta_n W^s(0)$$

This is because the probability that a seller gets to trade in the night market is equal to the probability, $\frac{\alpha(n)}{n}$, he meets a buyer, times the probability, $1 - G(\varepsilon_R)$, with which the buyer is wiling to give up her cash for the good he carries. As $W^s(z) = z + W^s(0)$ we obtain

$$V^s(\omega) = \frac{\alpha(n)}{n\gamma}\left[1 - G(\varepsilon_R)\right]\beta_n z + \beta_n W^s(0)$$

Equilibrium is still described by Definition 2 and can be characterized as the solution to Problem (18).

$$\begin{aligned} &\tilde{\omega} \in \arg\max_{\omega}\left\{\beta_d V^b(\omega) - z\right\} \\ &\text{subject to } \beta_d V^s(\omega) - c(q) = \beta_d V^s(\tilde{\omega}) - c(\tilde{q}) \\ &\text{where free-entry implies} \begin{cases} \beta_d V^s(\tilde{\omega}) - c(\tilde{q}) = \beta W^s(0) \text{ for } \tilde{n} < \bar{n} \\ \beta_d V^s(\tilde{\omega}) - c(\tilde{q}) \geq \beta W^s(0) \text{ for } \tilde{n} = \bar{n} \end{cases} \\ &\text{and } \gamma u(q)\varepsilon_R = \beta_n z \end{aligned} \tag{18}$$

Substituting for the value functions and eliminating z yields

$$(\tilde{n}, \tilde{q}, \tilde{\varepsilon}_R) \in \arg\max_{(n,q,\varepsilon_R)\in[0,\bar{n}]\times[0,\infty)\times[0,\bar{\varepsilon}]}\left\{\left[\alpha(n)\beta S_G(\varepsilon_R) - (\gamma - \beta)\varepsilon_R\right]u(q)/\beta_n\right\} \tag{19}$$

$$\text{subject to: } \quad \frac{\alpha(n)}{n}\beta_d[1 - G(\varepsilon_R)]\varepsilon_R u(q) - c(q) = X$$

where $X = \frac{\alpha(\tilde{n})}{\tilde{n}}\beta_d[1 - G(\tilde{\varepsilon}_R)]\tilde{\varepsilon}_R u(\tilde{q}) - c(\tilde{q})$ which is 0 under free-entry and positive when $n = \bar{n}$. Simple inspection of the constraint in problem (19) and the fact that $X \geq 0$, indicate that for any solution $\tilde{q} \leq \bar{q}$. The choice set is therefore compact and continuity of the objective function implies that a solution exists.[11] The appropriate Lagrangian is

$$\begin{aligned}\mathcal{L} &= [\alpha(n)\beta S_G(\varepsilon_R) - (\gamma - \beta)\varepsilon_R]\, u(q)/\beta_n \\ &+\mu \left\{ X - \frac{\alpha(n)}{n}\beta_d[1 - G(\varepsilon_R)]\varepsilon_R u(q) + c(q) \right\}\end{aligned}$$

Taking first-order conditions and eliminating μ implies

$$\left(\frac{\varepsilon_R[1 - G(\varepsilon_R)]}{S_G(\varepsilon_R)}\right) u'(q) = \left(\frac{\beta_n n\eta(n)}{[\alpha(n)\beta S_G(\varepsilon_R)(\gamma - \beta) - \varepsilon_R(1 - \eta(n))]}\right) c'(q) \tag{20}$$

and

$$\frac{\varepsilon_R[1 - G(\varepsilon_R)]}{S_G(\varepsilon_R)\,[1 - G(\varepsilon_R) - g(\varepsilon_R)\varepsilon_R]} = \frac{\alpha(n)\beta\eta(n)}{(1 - \eta(n))\,[\gamma - \beta + \alpha(n)\beta(1 - G(\varepsilon_R))]} \tag{21}$$

It is immediate from equation (20) that in any non-trivial equilibrium, $\varepsilon_R > 0$. A Social Planner, however, could choose $n = \tilde{n}$ and $q = \tilde{q}$ and achieve a higher degree of welfare by setting $\varepsilon_R = 0$ – the market equilibrium cannot be efficient.

In a labor market context with a similar source of private information, Guerrieri [2008] shows that steady-state allocations are constrained efficient. The source of inefficiency here by comparison comes from the lack of long-term trading relationships and the need to provide incentives for sellers to produce.

[11] In this model, depending on the form of $G(.)$, the possibility that $\tilde{n}$ and $\tilde{q}$, and hence $\tilde{z}$, are all zero cannot be ruled out.

4.3 Monetary Policy

The objective of monetary policy is to bring about the best social outcome achievable within the market economy through the manipulation of γ, the gross growth rate of the money supply. Thus we view $(\tilde{\omega}, \tilde{\varepsilon}_R)$ as a function of γ. The the contribution to welfare from night market activity is[12]

$$\tilde{W}_m(\gamma) \equiv \alpha(\tilde{n})\beta_d[1 - G(\tilde{\varepsilon}_R)]\mathbb{E}_{\{\varepsilon|\varepsilon \geq \tilde{\varepsilon}_R\}}\varepsilon u(\tilde{q}) - \tilde{n}c(\tilde{q})$$

This is because the number of exchanges is equal to the measure of buyers, 1, times the probability they meet a buyer, $\alpha(n)$, times the probability they will find the good attractive enough to give up their cash for it. And, $\mathbb{E}_{\{\varepsilon|\varepsilon \geq \varepsilon_R\}}\varepsilon u(q)$ represents the expected utility of the buyer when trade occurs. As

$$\mathbb{E}_{\{\varepsilon|\varepsilon \geq \varepsilon_R\}}\varepsilon = \int_{\varepsilon_R}^{\bar{\varepsilon}} \frac{\varepsilon dG(\varepsilon)}{1 - G(\varepsilon_R)}$$

we have

$$\tilde{W}_m(\gamma) = \alpha(\tilde{n})\beta_d \left[S_G(\tilde{\varepsilon}_R) + (1 - G(\tilde{\varepsilon}_R))\tilde{\varepsilon}_R\right] u(\tilde{q}) - \tilde{n}c(\tilde{q}). \tag{22}$$

Using the constraint in problem (19) we obtain

$$\tilde{W}_m(\gamma) = \alpha(\tilde{n})\beta_d S_G(\tilde{\varepsilon}_R)u(\tilde{q}) + \tilde{n}X \tag{23}$$

The optimal policy problem is to maximize $\tilde{W}_m(\gamma)$ over γ given $\tilde{n}$, $\tilde{q}$ and $\tilde{\varepsilon}_R$ are determined by the constraint in problem (19), equation (20) and equation (21). Under free-entry, $X = 0$ so when $\gamma = \beta$, the optimal policy problem is identical to maximizing the solution to problem (19) with respect to γ. By the envelope theorem, then

$$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} = 0.$$

[12] Since $x = x^*$ at every period, the day market contribution to welfare is not affected by monetary policy.

While the second order condition has not been verified, in all of the examples that follow, under free-entry moving away from the Friedman rule lowers $\tilde{W}_m(\gamma)$. The Friedman rule therefore implements optimal monetary policy under free-entry and inflation has only second-order welfare effects.

This result reflects the universality of the Friedman rule. In this model, buyers get to choose how much money to bring with them and make purchasing decisions based on private information which is not fully revealed in equilibrium. Sellers get to choose whether to enter the market or not and if they choose to enter, the quality of the good to bring with them. Still, optimal monetary policy follows a simple rule that sets the gross rate of money growth equal to the discount factor of the population. New to this extension of the baseline line environment however is the buyers' purchasing decision. We know that a social planner would prefer that goods change hands in every meeting but because prices cannot be made contingent on a buyer's realized preference for the good, some trades do not occur. What the policy result tells us is that when all other decisions in the economy are being made optimally, the Friedman rule causes buyers to reject goods in a way that is undistorted by the opportunity cost of holding money.

When the population of participating sellers is fixed or the upper bound on the number of sellers binds (so that $n = \bar{n}$), equation (23) still applies. But, X is not fixed with respect to γ. In such cases, the Friedman rule may fail to implement optimal monetary policy. Further discussion of this possibility is provided in the next section.

5 Discussion

5.1 Interior solution (free-entry)

As long as the upper bound on n does not bind, there is an interior solution to problem (19) and equilibrium is characterized by equations (20), (21) and the free-entry condition,

$$\alpha(n)\beta_d[1 - G(\varepsilon_R)]\varepsilon_R u(q) - nc(q) = 0. \tag{24}$$

Dividing equation (20) by (24) yields

$$\alpha(n)\beta S_G(\varepsilon_R)\left[e_u(q) - \eta(n)e_c(q)\right] = \varepsilon_R(\gamma - \beta)(1 - \eta(n))e_u(q) \tag{25}$$

Which means that at the Friedman rule, the elasticities equation (13) still holds. In addition at the Friedman rule (21) implies

$$\frac{\eta(n)}{1-\eta(n)} = \frac{\varepsilon_R[1 - G(\varepsilon_R)]^2}{S_G(\varepsilon_R)\left[1 - G(\varepsilon_R) - g(\varepsilon_R)\varepsilon_R\right]}$$

Beyond this, equations (20), (21) and (24) have not been amenable to general analysis. Equation (25), however, points to an obvious simplification. If $u(.)$ and $c(.)$ have constant elasticities e_u and e_c respectively then (25) is an equation in n and ε_R only. And, if $G(.)$ is uniform on $[0, 1]$, then (21), (24) and (25) become

$$2\varepsilon_R(1-\eta(n))[\gamma - \beta + \alpha(n)\beta(1-\varepsilon_R)] - \alpha(n)\beta\eta(n)(1-\varepsilon_R)(1-2\varepsilon_R) = 0 \tag{26}$$

$$\alpha(n)\beta_d[1 - \varepsilon_R]\varepsilon_R u(q) - nc(q) = 0 \tag{27}$$

$$\alpha(n)\beta\eta(n)(1-\varepsilon_R)^2[e_u - \eta(n)e_c] - 2\varepsilon_R\,(\gamma - \beta)\,(1 - \eta(n))e_u = 0 \tag{28}$$

The system is block recursive. Equations (26) and (28) give n and ε_R. Then, q can be obtained from (27). To obtain comparative statics at the Friedman rule we differentiate the system (26) and (28) and then impose $\gamma = \beta$. Under this restriction, (26) and (28) reduce to $\varepsilon_R = \eta(n)/2$ and $e_u = \eta(n)e_c$ respectively which means $(\tilde{n}, \tilde{q}, \tilde{\varepsilon}_R)$ is unique and

$$\begin{aligned} \left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} &= -\frac{\eta(\tilde{n})(2+\eta(\tilde{n}))(1-\eta(\tilde{n}))}{\alpha(\tilde{n})\beta(\eta(\tilde{n})-2)^2} \\ \left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta} &= -\frac{4(1-\eta(\tilde{n}))\eta^2(\tilde{n})}{\alpha(\tilde{n})\beta\eta'(\tilde{n})(\eta(\tilde{n})-2)^2} \end{aligned}$$

Under the maintained assumption that $\eta'(n) < 0$, at the Friedman rule $\tilde{\varepsilon}_R$ is decreasing and $\tilde{n}$ is increasing in γ. This means that increased inflation lowers the reservation utility at which buyers will part with their money. Essentially, because holding on to money is costly due to inflation, buyers are less picky about what to purchase. We also see increased seller participation. Recall that, under free-entry, equilibrium welfare in the market economy is

$$\tilde{W}_m(\gamma) = \alpha(\tilde{n})\beta_d S_G(\tilde{\varepsilon}_R)u(\tilde{q}) = \alpha(\tilde{n})\beta_d(1-\tilde{\varepsilon}_R)^2 u(\tilde{q})/2$$

Both the effect on $\tilde{\varepsilon}_R$ and $\tilde{n}$ taken alone should therefore increase overall welfare. However, it is straightforward to verify that the effect of increased inflation on $\tilde{q}$ is sufficiently negative that it exactly offsets the welfare gains through $\tilde{\varepsilon}_R$ and $\tilde{n}$.

5.2 Exogenous q

Given the foregoing, an obvious question is: if we shut down the quality margin will increased inflation close to the Friedman rule improve welfare? If $q = \bar{q}$ is exogenous, sellers simply enter with a good of quality $\bar{q}$ or stay home. Of course the price, z, is still endogenous along with n and ε_R. Under uniform $G(.)$, It is straightforward to show that equilibrium conditions are

$$\begin{aligned} 2\varepsilon_R(1-\eta(n))[\gamma-\beta+\alpha(n)\beta(1-\varepsilon_R)]-\alpha(n)\beta\eta(n)(1-\varepsilon_R)(1-2\varepsilon_R) &= 0 \\ \alpha(n)\beta_d[1-\varepsilon_R]\varepsilon_R u(\bar{q})-nc(\bar{q}) &= 0 \end{aligned}$$

Which imply

$$\begin{aligned} \left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} &= \frac{\eta^2(\tilde{n})(1-\eta(\tilde{n}))}{\alpha(\tilde{n})\beta\left[2\eta'(\tilde{n})\tilde{n}-\eta(\tilde{n})(2-\eta(\tilde{n}))\right]}<0 \\ \left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta} &= -\frac{4\tilde{n}(1-\eta(\tilde{n}))\eta(\tilde{n})}{\alpha(\tilde{n})\beta(2-\eta(\tilde{n}))\left[2\eta'(\tilde{n})\tilde{n}-\eta(\tilde{n})(2-\eta(\tilde{n}))\right]}<0 \end{aligned}$$

But here

$$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta}=\alpha'(\tilde{n})(1-\tilde{\varepsilon}_R)^2\left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta}-2\alpha(\tilde{n})(1-\tilde{\varepsilon}_R)\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta}=0$$

that is, with q held fixed, inflation has the opposite effect on sellers propensity to enter the market. When the quality of the good is a choice variable, sellers reduce the quality of their goods due to the lower value of money but increase participation because the chance of trading increases. Here, although buyers get less picky, sellers reduce participation because money is less valued and they cannot adjust the quality of the good to be sold.

5.3 Exogenous n

When there is no free-entry, competition for buyers still prevails across potential markets. However, as we seek a symmetric (single market) equilibrium, $\tilde{n}=\bar{n}$. The goods quality, q, the reservation utility, ε_R and the price, z remain endogenous. The equilibrium conditions are therefore (20) and (21) with $n=\bar{n}$. Restricting attention to uniform $G(.)$, at the Friedman rule, these equations reduce to $\varepsilon_R=\eta(n)/2$ and

$$\alpha(\bar{n})\beta_d[2-\eta(\bar{n})]u'(q)-4\bar{n}c'(q)=0$$

Differentiating the system and evaluating at $\gamma = \beta$ yields

$$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} = \frac{-\eta(\bar{n})(1-\eta(\bar{n}))}{\alpha(\bar{n})\beta(2-\eta(\bar{n}))} < 0$$

$$\left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta} = \frac{-4(1-\eta(\bar{n}))(3-\eta(\bar{n}))u'(\tilde{q})c'(\tilde{q})}{\alpha(\bar{n})\beta(2-\eta(\bar{n}))^2\left[u'(\tilde{q})c''(\tilde{q})-u''(\tilde{q})c'(\tilde{q})\right]} < 0.$$

Again, inflation makes buyers less picky even though sellers reduce the quality of their good.

Under uniform G, with $n = \bar{n}$,

$$\tilde{W}_m(\gamma) = \frac{1}{2}\alpha(\bar{n})\beta_d\left[1-\tilde{\varepsilon}_R^2\right]u(\tilde{q}) - \bar{n}c(\tilde{q}). \tag{29}$$

So that at the Friedman rule, after substituting for $\frac{d\tilde{\varepsilon}_R}{d\gamma}$ and $\frac{d\tilde{q}}{d\gamma}$,

$$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} = \frac{\eta(\bar{n})(1-\eta(\bar{n}))}{\beta_n(2-\eta(\bar{n}))}\left[\eta(\bar{n})u(\tilde{q}) - \frac{(3-\eta(\bar{n}))c'(\tilde{q})\left(u'(\tilde{q})\right)^2}{u'(\tilde{q})c''(\tilde{q})-u''(\tilde{q})c'(\tilde{q})}\right].$$

The sign of which is ambiguous. Simulations indicate that when $\bar{n}$ is significantly lower than the number of sellers that would participate under free-entry, welfare increases with inflation.[13] When the upper bound on n binds, sellers can extract positive rents from those buyers who get trading opportunities. As the cost of holding money increases, those rents are diminished.

5.4 Exogenous n and q

If $n = \bar{n}$ and $q = \bar{q}$, equilibrium is characterized by the first-order conditions of problem (19) with respect to n and ε_R. An immediate implication is that

[13] For example if $\alpha(n) = 1 - e^{-n}$, $u(q) = 4q^{\frac{1}{2}}$, $c(q) = q^2$, $\beta_d = 0.95$, and $\beta_n = 0.96$ then at the Friedman rule, $\gamma = \beta$, with free entry, $\tilde{n} = 2.337$, $\tilde{q} = 0.296$, $\tilde{z} = 0.258$, $\tilde{\varepsilon}_R = 0.125$.

If we now set $\bar{n} = 0.2337$, a tenth of its free entry value, we have $\tilde{n} = 0.2337$ (the upper bound on n binds), $\tilde{q} = 0.381$, $\tilde{z} = 1.042$, $\tilde{\varepsilon}_R = 0.444$. And, the elasticity of welfare with respect to γ (at the Friedman rule), is 0.119.

$\tilde{\varepsilon}_R$ solves equation (21) with $n = \bar{n}$. If G is uniform on $(0, 1]$,

$$\frac{d\tilde{\varepsilon}_R}{d\gamma} = \frac{-2\tilde{\varepsilon}_R(1-\eta(\bar{n}))}{\alpha(\bar{n})\beta\left[\eta(\bar{n}) + 2(1-2\tilde{\varepsilon}_R)\right] + 2(1-\eta(\bar{n}))(\gamma-\beta)} < 0. \tag{30}$$

When the population of sellers and the quality of goods are fixed, inflation makes buyers less picky. Which means, from equation (29), that welfare increases with inflation. The effect of inflation on on the real price of night market goods, z, can be positive or negative depending on the magnitudes of $\alpha(\bar{n})$ and $\eta(\bar{n})$. What equation (30) tells us is that z can never fall sufficiently that buyers get picker as inflation increases.

5.5 Degenerate $G(.)$

If we let $G(.)$ be represented by a mass point at $\bar{\varepsilon}$, the economy reverts back to that of the baseline model. The fraction, ε_R, of utility derived from the consumption of a good that would be just acceptable to buyers ex post, however, is still well defined. From (21) we know that $\varepsilon_R < \bar{\varepsilon}$ so $S_G(\varepsilon_R) = 1 - \varepsilon_R$, $G(\varepsilon_R) = 0$ and $g(\varepsilon_R) = 0$. Equation (21) now implies

$$\varepsilon_R = \frac{\alpha(n)\beta\eta(n)}{(1-\eta(n))(\gamma-\beta) + \alpha(n)\beta}$$

so that under the Friedman rule $\varepsilon_R = \eta(n)$. This is because the baseline model achieves efficiency at the Friedman rule and efficiency requires that the (proportional) marginal contribution to the matching rate of buyers should be equal to ratio of the value of matching to the value of being unmatched. Inflation creates a distortionary cost of holding money which drives a wedge between $\eta(n)$ and ε_R.

Shutting down either free-entry or fixing the quality of goods in the baseline model does not affect the efficiency of the Friedman rule. If all decision margins (except how much cash to bring to the night market) are shut down, directed search ensures that the night market is still active in that buyers pay positive amounts of money for goods. However, monetary policy cannot affect welfare because all of the variables that matter to the Social Planner are fixed.

6 Alternative market structures

6.1 Lucky bags

The most straightforward institutional arrangement that would return the environment to essentially that of the baseline model is to make buyers pay for the good before observing it. In this arrangement, the general quality of the good is still universally known but sellers require payment before they allow the buyer to realize his match-specific taste shock. Hotwire.com currently uses a similar approach to sell hotel rooms.

6.2 Market makers

A common device used to motivate competitive search (see Moen [1997]) are "market makers" who announce the set of submarkets that will be open that night. Faig and Huangfu [2007] point out that in such an environment there is nothing to prevent market makers from charging entry fees. Moreover, because entry fees are paid for sure, they reduce the need for buyers to carry idle cash balances. Their paper addresses this issue in the RW model. It incorporates competitive entry of market makers, so that their profits are driven to zero and entry fees are simply transfers from buyers to sellers. In the competitive search equilibrium with market makers, they are shown to completely eliminate the transaction price. Furthermore, the market maker equilibrium is always at least as efficient as the competitive search equilibrium without market makers. In the current paper, because they reduce transactions prices, entry fees can also help to address the inefficiency generated by private information.

To reduce the extent of exposition, attention is restricted to configurations in which the free-entry margin is active (i.e. $n < \bar{n}$). We know that the Friedman rule represents optimal policy in this case so any further welfare gains will be attributable to the entrance fees. Let Z_i, $i = b, s$ be the real entrance fees for buyers and sellers respectively. A typical element of Ω, the set of all potential submarkets, is now $\omega = (Z_b, Z_s, z, q, n)$. Here, z represents the total real cash balances carried by buyers into the night market. So the transaction price is $z - Z_{b.}$ For a buyer who chooses to enter submarket ω,

$$\begin{aligned} V^b(\omega) = \alpha(n)\mathbb{E}_\varepsilon \left[\max\{\varepsilon u(q) + \beta_n W^b(0), \beta_n W^b\left(\frac{z - Z_b}{\gamma}\right)\} \right] \\ + (1 - \alpha(n))\beta_n W^b\left(\frac{z - Z_b}{\gamma}\right) \end{aligned}$$

This implies

$$V^b(\omega) = [\alpha(n) S_G(\varepsilon_R) - \varepsilon_R]\, u(q) + \beta_n W^b(0)$$

where $\varepsilon_R = \beta_n (z - Z_b)/\gamma u(q)$.

For a seller in submarket ω,

$$\begin{aligned} V^s(\omega) = \frac{\alpha(n)}{n} [1 - G(\varepsilon_R)]\, \beta_n W^s\left(\frac{z - Z_b - Z_s}{\gamma}\right) \\ + \left(1 - \frac{\alpha(n)}{n} [1 - G(\varepsilon_R)]\right) \beta_n W^s\left(\frac{-Z_s}{\gamma}\right) \end{aligned}$$

After recognizing that competition between market makers drives their profit to zero we have $Z_s = -Z_b/n$. So that

$$V^s(\omega) = \frac{\beta_n}{n\gamma} [\alpha(n)(1 - G(\varepsilon_R))\, (z - Z_b) + Z_b] + \beta_n W^s(0)$$

Equilibrium is characterized by the solution to

$$\begin{aligned} \tilde{\omega} \in \arg\max_{\omega} \left\{ \beta_d V^b(\omega) - z \right\} \\ \text{subject to: } \beta_d V^s(\omega) - c(q) = \beta W^s(0) \\ \gamma u(q)\varepsilon_R = \beta_n (z - Z_b) \\ \varepsilon_R \geq 0 \end{aligned} \tag{31}$$

Problem (31) corresponds to problem (18) in the extended model. The difference simply reflects the focus on the free-entry of sellers and that the non-negativity of ε_R might now bind. After substituting for the value functions and eliminating Z_b, the appropriate Lagrangian is

$$\begin{aligned} \mathcal{L} = {} & \beta_d \left[\alpha(n) S_G(\varepsilon_R) - \varepsilon_R\right] u(q) - z \\ & + \mu \beta_d \left[\beta_n z - \gamma u(q) \varepsilon_R \left\{1 - \alpha(n)(1 - G(\varepsilon_R))\right\}\right] - \mu \gamma n c(q) + \lambda \varepsilon_R \end{aligned}$$

where μ and λ are the co-state variables.

The first-order condition for z implies $\mu = \beta^{-1}$. Which means the first-order condition for ε_R reduces to

$$\frac{u(q)}{\beta_n} \left\{ -\gamma \varepsilon_R \alpha(n) g(\varepsilon_R) - \left[1 - \alpha(n)(1 - G(\varepsilon_R))\right] (\gamma - \beta) \right\} + \lambda = 0. \tag{32}$$

Suppose the solution for $\tilde{\varepsilon}_R > 0$. Then, the contents of the curly brackets in equation (32) are strictly negative which implies $\lambda > 0$. Complementary slackness, however, requires that $\lambda \varepsilon_R = 0$ – the only viable solution is $\tilde{\varepsilon}_R = 0$. This means that $Z_b = z$ and the transaction price are zero. The remaining equilibrium conditions are,

$$\begin{aligned} \beta z - \gamma n c(q) &= 0 \\ \alpha'(n) \beta_d \hat{\varepsilon} u(q) - \frac{\gamma}{\beta} c(q) &= 0 \\ \alpha(n) \beta_d \hat{\varepsilon} u'(q) - \frac{\gamma}{\beta} n c'(q) &= 0. \end{aligned}$$

These clearly coincide with the Planner's optimality conditions at the Friedman rule. For values of γ close to β, inflation only has second-order welfare effects. Moreover, away from the Friedman rule welfare in the market maker model is higher than that under the baseline economy. This is because (similarly to Faig and Huangfu [2007]) buyers get $W^s(0)$ in both regimes and the objective functions in both problems (7) and (31) are buyer's welfare. The difference between the problems is that in problem (7), Z_b and Z_s are constrained to be zero.

7 Conclusion

This paper provides insight into the effect of monetary policy and inflation on participants in retail markets. It provides an analysis of a model that has three features that add realism to those considered in the literature. These are that goods are produced prior to being retailed, that goods are dated, and that buyers cannot base their choice over where to shop on every aspect of the good they wish to purchase. As the realized preference for a good is a buyer's private information, a market economy is necessarily inefficient. Sellers need to set prices that allow them to recoup their costs but those prices can turn out to be too high for buyers who just do not value the good highly enough. In this context, the Friedman rule represents optimal policy as long as there is free-entry of sellers. If there is an upper bound on the number of sellers which binds sufficiently, moderate levels of inflation can be welfare improving.

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