

Income Tax Incidence with Positive Population Growth

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Abstract

This paper derives the incidence of linear taxes on capital and labor in a competitive equilibrium in balanced growth. The paper further considers a tax on consumption and a tax credit. Tax incidence is determined using an analytic expression for the saving rate out of income net of all taxes and credits. Results for zero population growth do not extend to positive population growth, where the incidence of a tax on interest income is positive and a tax on consumption reduces the interest rate.

1 Introduction

This paper develops the implications of competitive equilibrium in an aggregate economy for the incidence of linear taxes on capital and labor income.¹ Consequences of a tax on consumption and a tax credit for saving are also considered. The paper shows that results generated by a steady state model with zero population growth cannot be extended to positive population growth.

The results support arguments that a positive tax on interest income can be incorporated into an optimal tax system. Andrew Abel (2007) has argued that combining a tax on interest income with an equal tax credit for saving can efficiently raise tax revenue. In the steady state case with zero population growth, this paper also shows that setting the tax on interest income equal to

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¹Tax incidence analysis seeks to determine the burden of a tax from the change in the price of an item or, in the context of factor markets, the change in the factor prices (e.g., wage or interest rate). Standard references include Fullerton and Metcalf (2002), Keller (1980), Kotlikoff and Summers (1987), Salanié (2003) and Sørensen (2004).

the tax credit for saving and raising both will generate increased tax revenues with no loss in production. If a tax credit is already in place, eliminating the tax on interest income would be distortionary and inefficient. This paper also shows that recipients of interest income can be taxed at positive rates indirectly using a tax on consumption.

The approach taken here differs from the tax efficiency literature and reaches different conclusions. Chamley (1986) and Judd (1985) analyze taxes in the context of models that reach steady state equilibrium. Following modern practice, these models are based on optimal dynamic behavior of a fixed population of representative consumers. A major point of this paper is that these fixed population models of a macroeconomy do not provide a valid basis for tax policy.

The methodology of this paper is to consider alternative balanced growth equilibria of an economy generated by competitive equilibrium for alternative parameter values. The competitive equilibrium of the economy is determined by factor market equilibrium and goods market equilibrium. Factor market equilibrium arises from the neoclassical property that factor prices (wages and the interest rate in the model developed here) are determined by marginal productivities. Government tax revenues are used to provide transfers to individuals rather than financing a separate category of government goods. Goods market equilibrium (demands for goods add up to production) therefore holds whenever national saving (consisting of personal and government saving) equals national investment (generated by the expansion of capital needed for balanced growth).

The major innovation in the methodology of this paper arises from the simple form of the condition for goods market equilibrium, with an analytic expression for the personal saving rate out of income net of all taxes and credits. In the steady state case with zero population growth, goods market equilibrium requires that the saving rate must be identically zero, leading immediately to one set of results. With positive population growth and positive government debt, goods market equilibrium implies that the saving rate will be positive, generating results that are qualitatively different from the steady state conclusions. As a consequence, the steady state results cannot be extended to the general case with positive population growth.

Conditions equivalent to goods market equilibrium have been incorporated into earlier analysis of taxation but the implications for tax efficiency and incidence have not been completely elaborated. Feldstein (1974) imposed equilibrium in capital markets in his analysis of the long-run incidence of a capital income tax (see the discussion by Fullerton and Metcalf, 2002, p. 1833). Boadway (1979) also argued that the consequences of a capital income tax cannot be completely characterized within a steady state model. Salanié (2003, p. 141) extended the Kaldor model (1955-1956), which incorporates a condition on national saving and investment without government, to analyze consequences of tax policies (see also Sattinger, 2005). Capital market equilibrium (implied by goods market equilibrium) is also incorporated into overlapping generations models with demographic growth, perhaps in the form of a production budget constraint (Salanié, 2003, Section 6.2). In a computational paper that generates an efficient positive tax on capital income, Conesa, Kitao and Krueger (2009, p.

31, equation 14) incorporate a condition on equilibrium in the capital market. They attribute the positive tax on capital income to the problem of taxing older people more, in the absence of age-dependent taxes.

The next section develops the model that provides an analytic basis for determining tax incidence. Section 3 then provides the results on tax incidence, differentiating between the general case and the exceptional case when there is no population growth. Section 4 considers tax credits for saving and relates the results to Abel (2007). Section 5 presents general conclusions for optimal income tax policy, involving both equity and efficiency considerations.

2 Model

2.1 Outline

The economy consists of the government, firms, and individuals. Perfectly competitive firms combine labor and capital and produce homogeneous output determined by an aggregate neoclassical production function. Government imposes linear taxes on capital and labor income and on consumption, allows tax credits, expands national debt, and uses tax and debt expansion revenues to pay for transfers and interest on the debt. The population of individuals expands at a constant rate, and individuals with an instantaneously separable utility function determine labor supply and saving optimally over time.

2.2 Production

Let K_t and L_t be the amounts of capital and labor in the economy in period t . If there is no ambiguity, the subscript for the time period will be dropped. Let $f[K, L]$ be the amount of output in the economy using K and L , where the production function f is a continuous function of the factors K and L and is homogeneous of degree one with declining marginal products of capital and labor. With perfectly competitive firms and perfectly competitive markets for capital and labor:

$$r = \frac{\partial f[K, L]}{\partial K} = f_1[K, L] \quad (1)$$

$$w = \frac{\partial f[K, L]}{\partial L} = f_2[K, L] \quad (2)$$

where r is the rate of return on capital (interest rate) and w is the wage rate for labor. Since f_K is homogeneous of degree zero, it is a function of the ratio of capital to labor. Let $\kappa = K/L$. Under the assumption of declining marginal products, f_K will be a decreasing function of $\kappa = K/L$ and f_L will be an increasing function. By the perfectly competitive assumptions, firms will not earn economic profits and all returns on capital will consist of interest payments to owners of capital.

2.3 Government

The government imposes linear taxes at the rate of t_r , t_w , and t_c on interest (capital) income, on labor and transfer income and on consumption, respectively, and gives a linear tax credit of t_s on savings. In a given balanced growth equilibrium, the tax rates are constant over time. Let D_{it} be national debt held by individual i in period t , and assume the government pays the individual rD_{it} interest on that debt. Let $D_t = \sum D_{it}$ be aggregate national debt in period t , and let S_t be aggregate individual savings. Assume the population is $P[t]$ in period t and grows at the rate ρ . The government expands the national debt at the rate ρ in balanced growth. The government uses tax and debt expansion revenues to provide transfers to individuals in the economy. Assume that an individual's transfer R_i does not change over time and that the average transfer for all individuals does not change over time as the population grows. Let $R_t = \sum R_{it}$. Then $R_{t+1} = (1 + \rho)R_t$. Alternatively, one could assume that the government uses revenues for government goods and services. This alternative would require working out how government goods and services are produced, how they enter individual utility functions, and how the expenditure eventually generates income for individuals. The assumption that government revenues are used for transfers greatly simplifies the model without changing any essential properties of the model or the results. Taxes collected by the government in a period are:

$$T_t = t_r r(K_t + D_t) + t_w(wL_t + R_t) + t_c C_t - t_s S_t \quad (3)$$

where C_t is aggregate consumption in the economy.

With the government expanding national debt by the amount ρD_t in a given period to continue in balanced growth, the government budget constraint is:

$$T_t + \rho D_t = R_t + rD_t \quad (4)$$

where the left side is sources of funds and the right side is uses of funds.

2.4 Individuals

Individuals in the economy supply labor, own wealth in the form of capital and government debt, and determine consumption levels in each period. Over time, individuals enter the economy, accumulate wealth starting from zero, and live forever. Utility functions are assumed to be time separable and identical for each individual, but individuals differ by wealth holdings and transfers. Let $W_{i,t}$, $L_{i,t}$ and $C_{i,t}$ be individual i 's wealth, labor supply and consumption in period t . Let the individual's utility function in period t be given by

$$U_{it} = (C_{it})^{\gamma_1} (H - L_{it})^{\gamma_2}, \quad \gamma_1 > 0, \quad \gamma_2 > 0, \quad \gamma_1 + \gamma_2 < 1 \quad (5)$$

where H is the total amount of time available for work and $H - L_{i,t}$ is individual i 's leisure. Assume that γ_1 and γ_2 are positive and $\gamma_1 + \gamma_2 < 1$. This utility function allows endogenous determination of labor supply and, as shown below,

aggregation of both consumption and labor supply.² Savings can also be written as the change in wealth:

$$S_{it} = W_{it+1} - W_{it} \quad (6)$$

The individual's after tax income is

$$r(1 - t_r)W_{it} + (1 - t_w)(wL_{it} + R_i) - t_c C_{it} + t_s S_{it} \quad (7)$$

Now consider the individual's optimal dynamic behavior. Let $V_{it}[W_{it}]$ be individual i 's value function at period t , where W_{it} is the state variable. Setting the control variable as W_{it+1} , wealth in the next period, the value function can be expressed as

$$V_{it}[W_{it}] = \max_{W_{it+1}} \{U_{it} + \beta V_{it+1}[W_{it+1}]\} \quad (8)$$

$$= \max_{W_{it+1}} \{(C_{it})^{\gamma_1} (H - L_{it})^{\gamma_2} + \beta V_{it+1}[W_{it+1}]\} \quad (9)$$

where β is the individual's discount factor (the same for all individuals). Consumption can be written as income after taxes minus savings:

$$C_{it} = r(1 - t_r)W_{it} + (1 - t_w)(wL_{it} + R_i) - t_c C_{it} + t_s (W_{it+1} - W_{it}) - (W_{it+1} - W_{it})$$

Solving this expression for C_{it} , consumption is a function of labor supply and the control variable W_{t+1} :

$$C_{it} = \frac{1}{1 + t_c} (r(1 - t_r)W_{it} + (1 - t_w)(wL_{it} + R_i) - (1 - t_s)(W_{it+1} - W_{it})) \quad (10)$$

Since $V_{it}[W_{it+1}]$ does not depend on C_{it} or L_{it} , the optimal labor supply in each period can be found as a function of the control variable by maximizing the right hand side of 8 with respect to L_{it} , yielding the condition

$$\gamma_1 \frac{U_{it}}{C_{it}} \left(\frac{1 - t_w}{1 + t_c} w \right) - \gamma_2 \frac{U_{it}}{H - L_{it}} = 0 \quad (11)$$

Substituting C_{it} from 10 yields the solution

$$L_{it} = H - \frac{\gamma_2 ((1 - t_w)(wH + R_{it}) + r(1 - t_r)W_{it} - (1 - t_s)(W_{it+1} - W_{it}))}{(\gamma_1 + \gamma_2)w(1 - t_w)} \quad (12)$$

²The utility function incorporates the separability between consumption and leisure that allows aggregation of consumption and labor. See Deaton, 1979, 1992, p. 37, Auerbach and Hines, 2002, p. 1372, and Salanié, 2003, p. 124, for discussions of aggregation of individual behavior. Note that if labor supply is exogenous, capital income taxation would be inefficient as a direct consequence of Henry George's single tax principle.

For positive levels of labor supply, the optimal dynamic solution satisfies two conditions. The first is that the control variable W_{it+1} must maximize the right hand side of 8, yielding:

$$\frac{\partial U_{it}}{\partial W_{it+1}} + \beta \frac{\partial V_{it+1}}{\partial W_{it+1}} = 0 \quad (13)$$

The second is the Benveniste-Scheinkman condition obtained by differentiating $V_{it}[W_{it}]$ with respect to W_{it} :

$$\frac{\partial V_{it}}{\partial W_{it}} = \frac{\partial U_{it}}{\partial W_{it}} \quad (14)$$

Applying this condition in the next period and substituting the result into 13 yields the Euler equation:

$$\frac{\partial U_{it}}{\partial W_{it+1}} + \beta \frac{\partial U_{it+1}}{\partial W_{it+1}} = 0 \quad (15)$$

Using the expressions for utility, consumption and labor supply in 5, 10 and 12, respectively, and rearranging yields the solvable form of the Euler equation:

$$\begin{aligned} & \frac{1}{\beta(1+r(1-t_r)/(1-t_s))} \\ = & \left(\frac{(1-t_w)(wH+R_i) + (1-t_r)rW_{it+1} - (1-t_s)(W_{it+2} - W_{it+1})}{(1-t_w)(wH+R_i) + (1-t_r)rW_{it} - (1-t_s)(W_{it+1} - W_{it})} \right)^{\gamma^{1+\gamma^2-1}} \end{aligned} \quad (16)$$

Let

$$\eta = \left(\beta \left(1 + r \frac{1-t_r}{1-t_s} \right) \right)^{1/(1-\gamma^1-\gamma^2)} \quad (17)$$

The solution can be found by expressing the condition that income net of income taxes grows by a factor η each period:

$$(1-t_w)(wH+R_i) + r(1-t_r)W_{it+1} = \eta((1-t_w)(wH+R_i) + r(1-t_r)W_{it})$$

Rearranging and solving yields:

$$W_{it+1} - W_{it} = \frac{\eta - 1}{r(1-t_r)} ((1-t_w)(wH+R_i) + r(1-t_r)W_{it}) \quad (18)$$

so that $(1-t_w)(wH+R_i) + (1-t_r)rW_{it} - (1-t_s)(W_{it+1} - W_{it})$ also grows by the constant factor η in each period.

With this solution, it is possible to work out the pattern of saving, wealth accumulation and labor supply over time for an individual. For reasonable parameter values, the individual accumulates wealth and eventually labor supply

goes to zero. Since most individuals in the economy will have positive labor supply, the solution in 18 will be used to characterize individual behavior.³

These results can now be used to obtain aggregate expressions for the economy's labor supply and private savings. Substituting savings from the right hand side of 18 into 12 yields

$$L_{it} = H - \frac{\gamma 2(r(1-t_r) - (\eta-1)(1-t_s))((1-t_w)(wH + R_i) + r(1-t_r)W_{it})}{(\gamma 1 + \gamma 2)r(1-t_r)w(1-t_w)} \quad (19)$$

Summing over all individuals yields

$$L_t = \sum_i L_{it} = HP[t] - \frac{\gamma 2(r(1-t_r) - (\eta-1)(1-t_s))((1-t_w)(wHP[t] + R_t) + r(1-t_r)W_t)}{(\gamma 1 + \gamma 2)r(1-t_r)w(1-t_w)} \quad (20)$$

Following the same steps, aggregate consumption can be expressed as:

$$C_t = \frac{\gamma 1(r(1-tr) - (\eta-1)(1-t_s))((1-t_w)(wHP[t] + R_t) + r(1-t_r)W_t)}{(\gamma 1 + \gamma 2)(1+t_c)(1-t_r)} \quad (21)$$

2.5 General Equilibrium

General equilibrium in the model considered here is determined by competitive equilibrium with optimizing agents, given the tax, transfer and debt policies of the government. For expositional purposes, the competitive equilibrium conditions are divided into a factor market equilibrium condition and a goods market equilibrium condition. The factor market equilibrium condition arises from the neoclassical result that factor prices equal factors' marginal products when factor markets are competitive. Since the production function is homogeneous of degree one, the derivatives are homogeneous of degree zero so that

$$r = f_1(K, L) = f_1(\kappa, 1) \quad (22)$$

When this condition holds, the wage rate w will also equal its marginal product $f_2(\kappa, 1)$. The conditions for factor market equilibrium therefore reduce to a single inverse relation between the interest rate r and the capital to labor ratio $\kappa = K/L$.

Goods market equilibrium arises when the demand for goods (individual consumption plus firm investment) equals production. In the context of the

³For the parameters used in Figure 1 below, labor supply goes to zero after 200 periods, 1.9 percent of the population supplies zero labor, and they own 10.9 percent of the wealth. The present discounted value of individual wealth in year n approaches zero as n increases indefinitely whenever $(\beta(1+r(1-tr))\gamma^1)^{1/(1-\gamma^1)} < 1$, satisfying the transversality condition.

model developed here, this occurs when individual consumption equals production minus investment, given by the expansion of capital needed for balanced growth (with a constant interest rate):

$$C_t = f[K_t, L_t] - \rho K_t. \quad (23)$$

Definition 1 *Balanced growth equilibrium arises when firms maximize profits, satisfying the condition for factor market equilibrium in 1, individuals maximize utility in 5 over time and satisfy the Euler equation in 16 and optimal labor supply in 12, the government satisfies its budget constraint in 4, and the condition for goods market equilibrium in 23 is satisfied, with these conditions continuing to hold indefinitely with a constant population growth rate ρ .*

Let NSI be national savings (personal savings plus government savings) minus national investment per unit of labor:

$$NSI = s(f[\kappa, 1] + \rho D/L) - \rho D/L - \rho \kappa \quad (24)$$

Theorem 2 *In a balanced growth equilibrium, i. aggregate personal savings in a period are*

$$S_t = s(f(K_t, L_t) + \rho D_t) \quad (25)$$

where s , the saving rate out of income net of all taxes and credits, is given by

$$s = \frac{(\gamma 1 + \gamma 2)(1 + t_c)(\eta - 1)}{(\gamma 1 + \gamma 2)(1 + t_c)(\eta - 1) + \gamma 1(r(1 - t_r) - (\eta - 1)(1 - t_s))} \quad (26)$$

ii. the goods market is in equilibrium when

$$NSI = 0 \quad (27)$$

Proof. i. First consider the saving rate s . From 18, aggregating personal savings over all individuals yields

$$\frac{\eta - 1}{r(1 - t_r)} ((1 - t_w)(wHP[t] + R_t) + r(1 - t_r)(K_t + D_t)) \quad (28)$$

where aggregate wealth is given by capital plus national debt, $K_t + D_t$. Aggregate income net of all taxes and credits in a period is

$$(1 - t_w)(wL_t + R_t) + r(1 - t_r)(K_t + D_t) - t_c C_t + t_s S_t \quad (29)$$

Substituting aggregate consumption from 21, aggregate labor from 20, and aggregate savings from 28, aggregate income net of all taxes and credits simplifies to

$$\frac{(\gamma 1 + \gamma 2)(1 + t_c)(\eta - 1) + \gamma 1(r(1 - t_r) - (\eta - 1)(1 - t_s))}{(\gamma 1 + \gamma 2)r(1 - t_r)(1 + t_c)} \quad (30)$$

times

$$((1 - t_w)(wHP[t] + R_t) + r(1 - t_r)(K_t + D_t)) \quad (31)$$

Note that the factor $(1 - t_w)(wHP[t] + R_t) + r(1 - t_r)(K_t + D_t)$ appears in both aggregate personal savings and income net of all taxes and credits. Then the saving rate s in 26 can be derived as the ratio of aggregate personal savings to income net of all taxes and credits. Next, income net of all taxes and credits can be constructed as the economy's output plus transfers plus interest on the debt minus net taxes, or $f(K_t, L_t) + R_t + rD_t - T_t$. From the government's budget constraint in 4, $R_t - T_t = (\rho - r)D_t$. Aggregate personal savings can then be expressed as:

$$\begin{aligned}
S_t &= s((1 - t_w)(wL_t + R_t) + r(1 - t_r)(K_t + D_t) - t_c C_t + t_s S_t) \\
&= s(wL_t + rK_t + R_t + rD_t - T_t) \\
&= s(f(K_t, L_t) + R_t + rD_t - T_t) \\
&= s(f(K_t, L_t) + \rho D_t)
\end{aligned} \tag{32}$$

ii. Goods market equilibrium occurs when 23 holds. The amount individuals choose to spend on consumption is given by income net of all taxes and credits (essentially disposable income) minus saving:

$$\begin{aligned}
C_t &= f[K_t, L_t] + R_t + rD_t - T_t - S_t \\
&= f[K_t, L_t] + \rho D_t - S_t \\
&= (1 - s)(f[K_t, L_t] + \rho D_t)
\end{aligned} \tag{33}$$

Setting $(1 - s)(f[K_t, L_t] + \rho D_t)$ equal to $f[K_t, L_t] - \rho K_t$, rearranging and dividing by L_t (and dropping time subscripts) yields $NSI = s(f[\kappa, 1] + \rho D/L) - \rho D/L - \rho \kappa = 0$. ■

The division of competitive equilibrium conditions into factor market and goods market equilibrium facilitates the representation of the equilibrium solution (given by r and κ) as the intersection between two relations. Factor market equilibrium (hereafter *FME*) in 22 determines a downward sloping relation between r and κ (with r on the vertical axis and κ on the horizontal axis). The condition $NSI = 0$ implied by goods market equilibrium determines a second relation between r and κ . The *FME* relation does not depend on any government tax or debt parameters and is unaffected by any changes. Tax incidence can then be studied by examining how the relation determined by goods market equilibrium (hereafter *GME*) shifts when parameters change. The slope of the *GME* relation can be determined as follows.

The slope of this relation (with r on the vertical axis) depends on the effects of r and κ on NSI . Consider first the effect of the interest rate. The effect depends only on what happens to the saving rate s in 26 since r does not appear elsewhere in NSI . From 26, the saving rate depends on $r(1 - t_r)/(\eta - 1)$, where η is also a function of the interest rate net of taxes. Using a standard approximation that is valid when $r(1 - t_r)/(1 - t_s)$ is small relative to 1,

$$\left(1 + r \frac{1 - t_r}{1 - t_s}\right)^{1/(1 - \gamma_1 - \gamma_2)} \approx 1 + \frac{r(1 - t_r)/(1 - t_s)}{1 - \gamma_1 - \gamma_2} \tag{34}$$

Then

$$\begin{aligned}\eta - 1 &= \left(\beta \left(1 + r \frac{1 - t_r}{1 - t_s} \right) \right)^{1/(1-\gamma_1-\gamma_2)} - 1 \\ &\approx \beta^{1/(1-\gamma_1-\gamma_2)} \frac{r(1-t_r)/(1-t_s)}{1-\gamma_1-\gamma_2} - (1 - \beta^{1/(1-\gamma_1-\gamma_2)})\end{aligned}$$

and

$$\frac{\eta - 1}{r(1-t_r)/(1-t_s)} \approx \frac{\beta^{1/(1-\gamma_1-\gamma_2)}}{1-\gamma_1-\gamma_2} - \frac{1 - \beta^{1/(1-\gamma_1-\gamma_2)}}{r(1-t_r)/(1-t_s)} \quad (35)$$

In this expression, $\beta < 1$ so $1 - \beta^{1/(1-\gamma_1-\gamma_2)} > 0$. Thus an increase in $r(1-t_r)$ raises $(\eta-1)/r(1-t_r)$, lowers $r(1-t_r)/(\eta-1)$ and increases the saving rate as expected.⁴ It follows that an increase in r , holding t_r fixed, will also increase s and NSI . Next, consider how κ affects NSI .

Theorem 3 *An increase in κ , holding r and other variables constant, reduces NSI whenever*

$$w > (r - \rho)D/L \quad (36)$$

Proof. First, assume $\rho > 0$. Differentiation of NSI with respect to κ yields

$$\frac{\partial NSI}{\partial \kappa} = s \frac{\partial f[\kappa, 1]}{\partial \kappa} - \rho$$

This derivative will be negative whenever

$$\frac{\partial f[\kappa, 1]}{\partial \kappa} - \rho/s = f_1[\kappa, 1] - \rho/s < 0$$

Applying the condition for GME in 27, the first order conditions 1 and 2, and Euler's law,

$$\begin{aligned}f_1[\kappa, 1] - \rho/s &= f_1[\kappa, 1] - \frac{f[\kappa, 1] + \rho D/L}{\kappa + D/L} \\ &= \frac{-(f[\kappa, 1] - \kappa f_1[\kappa, 1]) + (f_1[\kappa, 1] - \rho)D/L}{\kappa + D/L} \\ &= \frac{-w + (r - \rho)D/L}{\kappa + D/L}\end{aligned}$$

Then $\partial NSI/\partial \kappa < 0$ whenever $w > (r - \rho)D/L$. This argument does not hold if $\rho = 0$. Then GME requires that $s = 0$, so that the ratio ρ/s would be indeterminate. However, from GME and 26,

$$\lim_{\rho \rightarrow 0} \frac{\rho}{s} = \frac{f[\kappa, 1]}{\kappa + D/L}$$

⁴ Adding n additional terms for the series expansion for $(1+r(1-t_r)/(1-t_s))^{1/(1-\gamma_1-\gamma_2)}$ does not change the results as long as $1/(1-\gamma_1-\gamma_2) - n$ is positive.

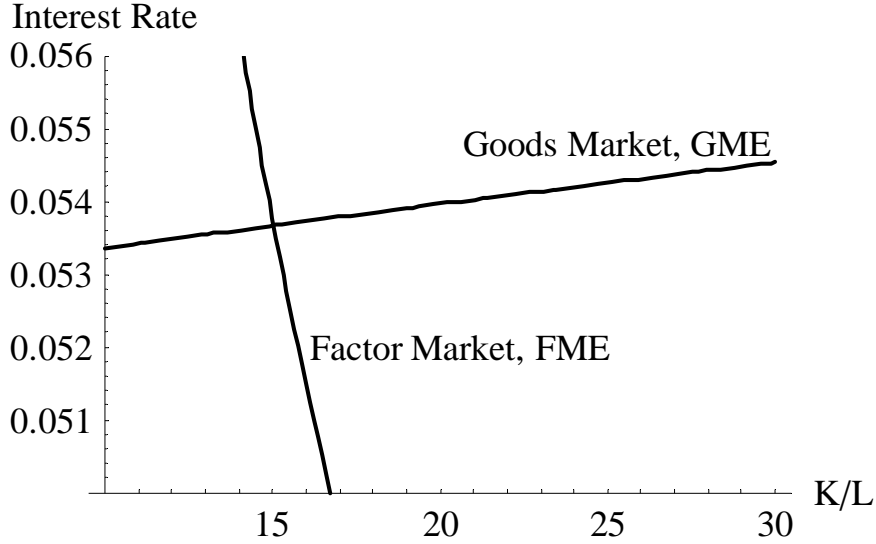


Figure 1: Conditions for Competitive Equilibrium

Then an increase in κ reduces NSI whenever $w > rD/L$, so that Theorem 3 remains valid for the case $\rho = 0$. ■

To summarize, an increase in r raises NSI (given the approximation in 35) while an increase in κ reduces NSI . For GME to continue to hold, an increase in κ must be accompanied by an increase in r . In a graph with r on the vertical axis and the capital to labor ratio κ on the horizontal axis, the GME will be upward sloping.

Figure 1 shows the curves for the two conditions and the intersection where balanced growth equilibrium occurs.⁵

Analysis proceeds by determining how a parameter shifts the GME . In Figure 1, NSI is positive to the left of the FME and negative to the right. If a change in a parameter causes NSI to be positive at the former combination of r and κ , then κ must be higher to reduce NSI to zero at the same interest rate, i.e. the GME shifts to the right. Similarly, if a parameter change reduces NSI below zero, the GME shifts to the left.

The intersection of FME and GME determines the interest rate and the capital to labor ratio but not the labor supply. When considering shifts in the GME in response to alternative parameter values, it is also informative to consider how labor supply changes. The expression for aggregate labor supply in 20 determines labor supply implicitly since aggregate wealth depends on the

⁵The assumptions for the figure are that $f(K, L) = K^{1-a}L^a$, with $a = .67$, $t_w = .3$, $t_r = .2$, $t_c = .1$, $t_s = 0$, $\rho = .02$, $\beta = .96$, $\gamma_1 = \gamma_2 = .3$, $D/L = 3$, and $H = 12$.

labor supply (holding D/L constant):

$$W_t = K_t + D_t = (K_t/L_t + D_t/L_t)L_t = (\kappa + D/L)L_t \quad (37)$$

In the analysis that follows, the ratio of debt to labor supply, D/L , will be treated as a policy variable that will remain constant when considering effects of changes in tax rates. Substituting this expression for W_t into 20 and solving for L_t yields

$$L_t = \frac{(\gamma_1 + \gamma_2)wHP[t] - \gamma_2(r(1 - t_r) - (\eta - 1)(1 - t_s))\frac{R_t + wHP[t]}{r(1 - t_r)}}{(\gamma_1 + \gamma_2)w + \gamma_2(r(1 - t_r) - (\eta - 1)(1 - t_s))\frac{\kappa + D/L}{1 - t_w}} \quad (38)$$

Next, solving for R_t , T_t , C_t and L_t simultaneously, an expression for R_t in terms of the tax rates can be substituted for R_t , so that the effects of changes on L_t can be determined. In the resulting expression for aggregate labor supply, it is necessary to determine the sign of the term $r(1 - t_r) - (\eta - 1)(1 - t_s)$. The sign can be found from the condition that the saving rate, if positive, will be less than one (otherwise it can be shown that consumption would be negative and investment would exceed production). Setting $s < 1$ in 26 and simplifying yields

$$r(1 - t_r) - (\eta - 1)(1 - t_s) > 0 \quad (39)$$

As a consequence, the partial derivatives of labor supply with respect to t_r , w and η are positive, and the partial derivatives with respect to t_w , t_c , t_s , κ , D/L and r are negative. For some combinations of changes, it is then possible to determine the effects on labor supply.

3 Tax Incidence

Tax incidence will be determined by comparing balanced growth equilibria under alternative parameter values. This differs from standard comparative statics analysis in that the economy does not move from one equilibrium to another in any specified amount of time. In making comparisons, it will be assumed that the ratio of debt to labor supply, D/L , remains the same. (Alternatively, it is possible to examine what happens when debt remains constant, so that D/L would decline if L increases.) The analysis is simplified because the tax rates on capital income and consumption and the interest rate only enter the saving rate s , and the capital to labor ratio, ratio of debt to labor, and population growth rate ρ only enter the remaining terms.

First, consider the steady state case of zero growth.

Theorem 4 *Assume that the growth rate ρ is zero in a balanced growth equilibrium and D/L is the same for alternative tax rates.*

- i. The incidence of a tax on interest income is zero.*
- ii. The incidence of a tax on labor or transfer income is 100%.*

iii. An increase in the tax on consumption lowers the after-tax purchasing power of interest income and labor and transfer income but does not change the wage or interest rate or the capital to labor ratio.

iv. An increase in D/L has no effect on the interest rate, wage rate, or capital to labor ratio.

Proof. i. Since $\rho = 0$, the saving rate in 25 must be zero to satisfy the *GME*. Then from 26, $\eta = 1$ or

$$r(1 - t_r) = (1 - t_s)(1 - \beta)/\beta \quad (40)$$

For any given tax credit t_s , the right hand side is fixed and any increase in t_r raises r sufficiently that $r(1 - t_r)$, the after tax return on wealth, remains the same. Then the tax incidence is zero.

ii. If the tax rate t_w on labor or transfer income increases, the saving rate must remain zero so 40 continues to hold. Then the capital to labor ratio is unaffected since r remains the same, and the wage rate also stays the same. Labor and transfer income recipients pay the entire tax.

iii. Since 40 will also continue to hold when t_c increases, the tax has no effect on the wage rate, interest rate, or capital to labor ratio.

iv. With $\rho = 0$, the national debt does not appear in the *GME* and has no effect on w , r , or the capital to labor ratio. National debt reduces government revenues since there are no revenues from expanding the debt. ■

In a steady state economy, only one saving rate and one after tax interest rate can arise. While a tax on interest income can raise the interest rate and reduce the capital to labor ratio, no other tax or debt policy can alter factor payments or the capital to labor ratio. This accounts for the simplicity of the results in Theorem 4. Now consider how the results differ when population growth is positive.

Theorem 5 Assume that the growth rate ρ is positive in a balanced growth equilibrium, and D/L is the same for alternative tax rates. Then:

i. An increase in the tax rate on interest income t_r will have a positive incidence on interest recipients. The interest rate will be higher and the wage rate lower. For a given shift in the *GME*, the incidence of the tax on interest income recipients will be greater when the *FME* and *GME* are steeper.

ii. An increase in the tax rate on labor and transfer income, t_w , leaves the capital to labor ratio, the interest rate and the wage rate unaffected but reduces the labor supply. The tax therefore falls completely on recipients of labor and transfer income.

iii. An increase in the tax on consumption, t_c , raises the saving rate, shifting the *GME* to the right and generating a lower interest rate and higher wage rate.

iv. An increase in the level of national debt per worker, D/L , shifts the *GME* leftward, generating a lower capital to labor ratio, a higher interest rate and a lower wage rate. As a tax, national debt falls more heavily on workers.

Proof. i. Suppose r_0 and κ_0 are the interest rate and capital to labor ratio in the balanced growth equilibrium before t_r increases by Δt_r . At κ_0 , the interest

rate at which GME holds after the tax increase (determined by the interest rate such that the saving rate s is the same at κ_0) is $r_0 + r_0\Delta t_r$ (that is, the GME shifts up by $r_0\Delta t_r$). At that interest rate, $(1 - t_r)r$ would be the same after the tax increase as before. However, at the original interest rate r_0 , the increase in t_r reduces the saving rate, generating a decline in NSI , and shifting the GME leftward. At the new equilibrium intersection of FME and GME , the capital to labor ratio κ will be lower and the increase in the interest rate (from r_0) will be less than $r_0 + r_0\Delta t_r$. Then $(1 - (t_r + \Delta t_r))r < (1 - t_r)r_0 = r_0 + r_0\Delta t_r - (t_r + \Delta t_r)r_0$ so that the incidence of the increase in t_r on interest income is positive. At the lower capital to labor ratio, wage rates will be lower. For a given shift in the GME , the intersection of the FME and GME will occur at a higher interest rate when they are steeper.

ii. The tax rate t_w does not appear in the expression for the saving rate, which stays the same for a given value of the interest rate. Since t_w does not enter elsewhere in the GME , the condition does not shift, and the solution for the capital to labor ratio stays the same. However, the labor supply declines because of the lower after tax wage, and output declines. At the same capital to labor ratio, the wage rate and interest rate stay the same, and the tax on labor and transfers falls completely on recipients.

iii. The tax on consumption, t_c , enters the saving rate but not the rest of the GME . Using 39, differentiation shows that an increase in t_c raises the saving rate s . With a higher saving rate, the GME shifts rightward, generating a higher capital to labor ratio, a lower interest rate and a higher wage rate. The tax on consumption falls more heavily on recipients of interest income, since the interest rate is lower and recipients must also pay the higher consumption tax.

iv. National debt does not enter the saving rate, so the derivative of NSI with respect to national debt per worker is $-(1 - s)\rho < 0$. Then the GME shifts to the left, generating a lower capital to labor ratio, a higher interest rate and a lower wage rate. National debt therefore raises the interest rate and lowers the wage rate. ■

Only one result carries over from the steady state case in Theorem 4 to the general case with positive population growth in Theorem 5: the tax on labor and transfer income, t_w , has no effect on r , w or κ . Otherwise, the results of Theorem 4 provide no indication of the results in the general case. The conclusion in Theorem 5, part i, that the incidence of a tax on interest income is positive conflicts with the conventional wisdom. Mankiw, Weinzierl and Yagan (2009, p. 167) argue that the incidence of a tax on interest income is zero, without limiting the result to the special case of zero population growth.

The conclusion that the tax on consumption raises κ and reduces r conflicts with expectations derived from zero population growth models. Consumption taxation as an alternative to capital income taxation has been discussed by Zodrow (2007) and Salanié (2003, Chapter 9). With $\rho = 0$, a change in t_c does not affect an individual's intertemporal choice of consumption levels and should therefore have no effect on savings or the interest rate. However, the mechanism through which t_c affects balanced growth equilibrium is not through individual intertemporal choices, as it would be when $\rho = 0$, but through goods

market equilibrium. Since t_c enters the saving rate s in 26, savings go up when t_c increases. More directly, aggregate personal saving in 28 increases because aggregate transfers, R_t depend positively on t_c . Then an increase in t_c shifts GME rightward as described in the proof. The positive effect of t_c on the capital to labor ratio demonstrates the limitations of conclusions based on steady state or representative agent models.

The result that the level of debt per worker affects the interest rate when population growth is positive conflicts with conclusions generated from zero population growth. Debt levels have no effect on the interest rate with zero population growth because, with no expansion in debt, the GME is unaffected. In contrast, when $\rho > 0$, greater debt requires higher personal saving and a higher interest rate, so that debt is not neutral (P. Weil, 1987).

Another major difference arising from positive population growth is that tax parameters affect the pattern of wealth accumulation, income and consumption over individual lifetimes. In contrast, with $\rho = 0$, the growth rate of wealth, income and consumption are invariant to parameter changes. When $\rho > 0$, an increase in t_r reduces $(1 - t_r)r$ and the rate of accumulation of individual wealth, given by η in 17. Individuals experience higher income when younger (from higher transfer incomes) but achieve lower growth rates of income and consumption and therefore lower incomes and consumption when older. An increase in t_c , by shifting GME to the right, lowers the interest rate and thereby reduces η . As a result, individuals face higher incomes and consumption levels when young and lower incomes and consumption levels when older. An increase in debt per worker has the opposite effect, raising the interest rate and the rate of individual wealth accumulation.

In the balanced growth model with positive population growth developed here, changes in parameter values have significant distributional consequences. In contrast, models based on representative agents or a fixed population can only have very limited distributional consequences, perhaps encouraging policy makers to disregard distributional consequences in evaluating alternative tax policies. With positive population growth, both efficiency and distribution must be considered. The following table summarizes the distributional consequences when $\rho > 0$.

Table 1: Distributional Consequences with $\rho > 0$

Parameters	Interest	Income	Labor Income	Transfers
t_r	-		-	+
t_w	0		-	+
t_c	-		+	+
D/L	+		-	?

An advantage of analyzing distributional consequences of alternative tax policies is that the results lead to formation of combinations of policies that would have minimal distributional effects on the economy. The following theorem, which only arises with positive population growth, describes one possible combination.

Theorem 6 *Assume that the growth rate ρ is positive in a balanced growth*

equilibrium and D/L is the same for alternative tax rates. Then there exists a combination of increases in tax rates on interest income and consumption that leaves the ratio of capital to labor, the saving rate s , the wage rate and the interest rate unchanged.

Proof. By Theorem 5, part i, an increase in t_r reduces the capital to labor ratio while by part iii, an increase in t_c increases the capital to labor ratio. Then there exists a combination of positive increases in t_r and t_c that leaves the capital to labor ratio unchanged. Since the capital to labor ratio would be the same after the tax increases, the marginal products of capital and labor would be the same (from 1 and 2) and the interest rate and wage rate would be unchanged. From the *GME*, s must remain the same since neither κ nor D/L change. ■

The incidence of the tax combination on interest rate recipients would be positive both because t_r rises and because they will pay more consumption taxes. The combination would indirectly fall on labor income recipients through their increased payment of consumption taxes. Transfer recipients would receive a net increase from the increased level of tax payments. It can be shown that η and the accumulation rate of personal assets would decline. The combination of greater transfer payments and lower rates of asset accumulation would alter patterns of income over lifetimes by increasing income at younger ages and reducing incomes at older ages.

This theorem suggests that it may be possible to find combinations of changes in tax policies that generate more tax revenues while causing no additional distortions in the economy. In the case that generates Figure 1, an increase in the tax on interest income from .2 to .202 could be combined with an increase in the tax on consumption from .1 to .219 to yield a solution with the same interest and wage rate and the same capital to labor ratio. However, labor supply would decline from 3.92 to 3.66, with consequent reductions in production and consumption and increases in tax revenue and transfers. By reducing the tax on labor from .3 to .224, labor supply can be returned to its former level with no net change in wage rate, interest rate, production or consumption, but with a decline in η and an increase in taxes and transfers. This example demonstrates that equally efficient tax combinations (in terms of production) can be found that have substantially different distributional consequences.

4 Tax Credits

Andrew Abel (2007) has argued that a constant tax rate on capital income is non-distortionary when producers can deduct capital purchases from capital income taxes.⁶ This result appears to conflict with previous results by Chamley (1986) and Judd (1985) that efficiency requires $t_r = 0$. Abel's analysis is set in the context of corporate investment decisions and specifies in more detail corporate tax policies including depreciation. With different assumptions, the model developed here generates differences in the efficiency conclusions when

⁶ Abel cites as predecessors Hall and Jorgenson (1971), Lucas (1990) and Samuelson (1964).

population growth is positive. Nevertheless, the results are fundamentally in agreement that taxation of capital income can be incorporated into optimal tax policy.

There are several differences between the results for tax credits discussed here and Abel's development. In Abel's model, the tax credit goes to new purchasers of capital instead of savers, but the question of who receives the credit has no consequences for tax incidence. The tax credit in Abel's model is limited to capital purchases while the tax credit considered here applies to all saving, including acquisition of government debt. However, the return on government debt is determined by the return on purchases of real assets, $r(1 - t_r)/(1 - t_s)$. If the government yields a tax credit on savings used to purchase government assets different from t_s (including no tax credit), there will be a compensating differential in the interest rate such that the consumer is indifferent and the government budget is unaffected. There are other differences between the models (including technological change, which would cause positive investment at zero population growth, and depreciation) but these are not germane.

Although seemingly contradictory, Abel's results are consistent with Chamley and Judd because different combinations of taxes and credits can have the same consequences. Specifically, consider a uniform tax on all incomes and an equal tax credit rate, with $t_r = t_w = t_s = t_u$ and $t_c = 0$. This combination has exactly the same consequences as a consumption tax with $t_c = t_u/(1 - t_u)$ and $t_r = t_w = t_s = 0$ (of course there could be practical differences in the administration of these different combinations of taxes).⁷ This equivalence can be established by comparing the outcomes for the two systems (including η , the saving rate s , GME and labor supply). The first combination, with positive taxation of interest income, generates the same outcomes as the second combination, with zero capital taxation, and raises the same tax revenues, consistent with Abel's results.

As in the tax incidence results in the previous section, Abel's results depend on whether population growth is positive. The following theorem reproduces Abel's results using the model developed here in the case where $\rho = 0$.

Theorem 7 *Suppose $\rho = 0$ in the balanced growth equilibrium and suppose $t_r = t_s = t_u$. At a higher tax rate t_u ,*

- i. The growth factor η remains unaffected at $\eta = 1$ and the saving rate s remains unaffected at $s = 0$.*
- ii. The interest rate is unaffected at $r = (1 - \beta)/\beta$*
- iii. The wage rate w and the capital to labor ratio κ are unaffected.*
- iv. The labor supply is unaffected.*
- v. Tax revenues net of credits rise with no loss in production.*

Proof. i. As in Theorem 4, $\eta = 1$ and $s = 0$ when $\rho = 0$. ii. From 40, with $t_r = t_s$, $r = (1 - \beta)/\beta$. iii. Since the GME does not shift, k , w and r stay the same. iv. In the expression for labor supply in 38, after substituting the

⁷Salanié, 2003, p. 188, discusses equivalent tax combinations. Abel (2007, p. 24) proposes combining a tax on consumption with a subsidy of labor income to generate a tax on leisure.

solution for transfers in terms of tax parameters, t_r and t_s cancel out so labor supply stays the same. v. With the labor supply and the capital to labor ratio unchanged, production stays the same. Since aggregate saving is zero, aggregate tax credits do not rise when t_s goes up, but tax revenues go up from a higher t_r . ■

Since there are no savings in the absence of population growth, one may expect that t_s would have no effect. However, the tax credit affects marginal saving and investment and lowers the interest rate. Now consider how positive population growth affects the results with tax credits.

Theorem 8 *Suppose $\rho > 0$ in the balanced growth model and suppose $t_r = t_s = t_u$. At a higher tax rate t_u ,*

- i. *The growth factor η will stay the same and the saving rate s will increase.*
- ii. *The GME shifts to the right, raising κ and w and lowering r .*

Proof. i. By inspection, η stays the same since $1 - t_r$ and $1 - t_s$ cancel out. In 26, the terms $\gamma 1(r(1 - t_r) - (\eta - 1)(1 - t_s))$ decrease proportionately to $1 - t_u$ as t_u increases, raising s .

ii. The increase in s raises NSI above zero at the former interest rate, shifting the GME rightward. The balanced growth equilibrium with $t_r = t_s = t_u$ higher will have a lower r , higher w and higher κ . ■

When $\rho > 0$, setting $t_s = t_r > 0$ does not return the economy to the same solution as with zero taxes, as in Abel's model, because of the increase in the capital to labor ratio. Using the expression for aggregate labor supply in terms of parameters, an increase in t_r and t_s , with $t_r = t_s$, raises labor supply. Since κ also increases, capital and production are higher. The essential difference in the models is that Abel bases his conclusions on the absence of changes in the First Order Conditions for an individual's dynamic optimization problem. He argues that the FOC's are the same as an efficient solution (with only lump-sum taxes) when there is a tax on capital income combined with a tax credit on new investments. Under the restriction to the case where $\rho = 0$, this result holds for the model developed in this section. Then η is the same and the FOC's for the individual's dynamic optimization problem are the same whether or not $t_r = t_s = t_u$ are positive or zero. Although this result does not carry over to the case with positive population growth, as demonstrated in the previous theorem, the changes generate an increase in labor supply instead of a decrease that would reduce production.

Consider the effects of changing t_s by itself, holding other tax rates unchanged. An increase in t_s raises η at any given interest rate and raises $(\eta - 1)(1 - t_s)$ if the approximation in 35 holds. Then s increases, so that NSI is positive at the previous combination of r and κ . An increase in t_s essentially shifts the GME down, just as an increase in t_r would shift it up. The increase in t_s then raises κ , lowers r and raises w .

Despite the conclusion that taxation of interest income combined with tax credits will in general be distortionary to some extent when $\rho > 0$, Abel's results

are fundamentally correct regarding how to carry out taxation of interest income recipients. Either of the two equivalent policies (uniform income tax with a tax credit or indirect taxation based on consumption) can raise substantial tax revenues while causing less redistribution of tax burdens to labor and transfer recipients. Furthermore, if tax credits are already in place in an economy, a policy to reduce taxation of interest income to zero could itself be distortionary.

5 Conclusions

This paper develops a model of the competitive general equilibrium of an economy in balanced growth to determine income tax incidence. The paper shows that the results for zero population growth, which have been the basis for previous results on tax incidence and efficiency, do not extend to the general case with positive population growth. Specifically, conventional wisdom asserts that the incidence of a tax on interest income is zero. In contrast, when population growth is positive, the incidence of the tax is positive. Also, the conventional wisdom is that a tax on consumption cannot change saving because the intertemporal consumption decision is unaffected. While this result is valid for zero population growth, it no longer holds when population growth is positive.

A major tenet of the conventional wisdom is that the tax rate on interest income should be zero for efficiency reasons.⁸ There is no valid basis for this conclusion. The problem is not so much that the conclusion is derived from steady state equilibrium without population growth, but that it focuses on the taxation of interest income in isolation rather than in combination with other tax policies. If the tax credit for saving is zero, then the efficient tax rate on interest income would be zero, as established in the efficiency literature for the steady state case. As Abel (2007) has shown, a positive tax on interest income can be combined with a tax credit for saving without affecting the interest rate or capital to labor ratio. Then if the tax credit is positive, reducing the tax rate on interest income to zero would be inefficient and distortionary. In the steady state case, setting the tax on interest income equal to the tax credit for saving and raising both would raise tax revenue without affecting production. With no production losses, it would be inefficient to tax anything else. Only consideration of equity and compassion for impoverished rentiers would prevent tax authorities from deriving all tax revenue from interest income.

In the model developed here, with positive population growth, individuals enter the economy and remain indefinitely, saving over the course of their lifetimes. It is possible to construct models with birth, death and overlapping generations that reproduce the results of an economy in steady state equilibrium with no population growth. However, if endogenous bequests are included, the results will differ. Overlapping generations models require a different analysis that will be developed in future work. In that context, the analysis would

⁸Mankiw, Weinzierl and Yagan (2009, p. 167), in their review of optimal taxation, label Lesson 7 as "Capital Income Ought To Be Untaxed, At Least in Expectation." They further comment that ". . . its strong underlying logic has made it the benchmark."

involve estate and pension taxes as well as tax deferrals.

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