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Safety First Consumption

Abstract

This paper develops a model of safety first consumption behavior in which the likelihood of survival to the next period depends on current consumption levels. Below a threshold asset level, individuals follow a decumulation path, and above that level they follow an accumulation path. Saving rates then vary discontinuously with asset level, generating a poverty trap and divergence in incomes. Reduction of risk raises saving rates. A more equitable distribution of assets can be consistent with greater aggregate savings and growth because of declining marginal propensity to save over some asset intervals.

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Michael Sattinger
Department of Economics
University at Albany
Albany, NY 12222, USA
Email m.sattinger@albany.edu
JEL: E21,O12,O16
Phone 518 442 4761, Fax 518 442 4736
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1 Introduction

In an early paper on the economics of uncertainty, A.D. Roy (1952, p. 432) argued, "If survival is always taken for granted, the rules of behaviour applicable in an uncertain and ruthless world cannot be discovered." He proposed a principle of safety first which asserted that an individual will seek to reduce as far as possible the likelihood of a catastrophe. More generally, safety first behavior can be defined as applying when the threat of a catastrophic event affects decision making. Safety first behavior is not myopic or irrational but instead arises from optimizing behavior. This paper analyzes safety first behavior in a dynamic context in which current consumption affects the likelihood of survival to the next period. Safety first behavior can generate discontinuous saving rates in contrast to solutions of standard dynamic control problems. At low incomes, safety first behavior holds down saving rates relative to standard models, providing an explanation for the transition of developing countries to higher growth rates.

A.D. Roy applied the safety first principle to the holding of assets in an analysis using the Chebychev inequality and involving means and standard deviations.[1] Strictly speaking, "safety first" suggests no trade-offs between reduction of risk and maximization of utility. However, the safety first problem has been formulated in the literature so that a minimal level of risk serves as a constraint on optimizing behavior. Telser (1955-1956) and Kataoka (1963) develop safety first criteria for the holding of assets. Pyle and Turnovsky (1970) relate safety first criteria to expected utility maximization (see also Levy and Sarnat, 1972). The safety first principle has also been applied to agricultural problems (Van Kooten, Young and Krautkraemer, 1997; Thompson and Wilson, 1994; Sadoulet, Seiichi and de Janvry, 1994).

This paper does not analyze consumption behavior as maximization of an objective subject to a constraint on the likelihood of a catastrophe. Instead, when set in a dynamic context, the individual faces a trade-off between survival to the next period and long-run survival. The

[1] Roy cites an earlier application of safety first by H. Cramér (1930) to insurance.

likelihood of survival depends on the level of consumption. Consuming more in the current period raises the likelihood of surviving to the next period, but reduces the likelihood of survival in future periods by reducing future incomes. Optimal safety first behavior, in the model developed in this paper, balances present and future income and present and future survival.

The major innovation of this paper is that uncertain survival generates alternative paths that an individual with low assets could follow. The accumulation path, with positive savings, leads to higher asset levels with greater likelihood of long run survival, but risks a higher likelihood of failing to survive to the next period. The decumulation path, with negative savings, leads to lower asset levels with little likelihood of long run survival, but with better chances of surviving to the next period. These paths can be calculated by solving backwards from their alternative eventual states of a high asset level or a zero asset level. At the threshold asset level where they yield an equal expected present value, the individual is indifferent between following the two paths. At a slightly lower asset level, the individual would be better off following the decumulation path, with a negative saving rate. At a slightly higher asset level, the individual would choose the accumulation path, with a positive saving rate. As a consequence of the two paths, the saving rate as a function of the asset level jumps from a negative level (when assets are below the threshold) to a positive rate (when assets are above the threshold).

The dichotomous saving behavior arising from the two paths explains a number of phenomena in developing economies. First, an economy could have a large proportion of individuals in a low income trap caused by asset levels below the threshold. The group of individuals with assets below the threshold level would not experience growth in assets or income, and incomes of individuals in the economy would diverge. If the proportion of individuals with assets below the threshold level is large, aggregate savings in the economy would be negligible. Safety first behavior then provides an explanation for the low equilibrium trap that is said to affect some developing economies (Leibenstein, 1954; Nelson, 1956; Hayami, 2001, p. 40, pp. 130-132). Several authors propose credit market imperfections as explanations for poverty traps. In Alghion and Bolton (1997) and Banerjee and Newman (1993), credit constraints affect individuals' occupational choices. In Piketty (1997) and Galor and Zeira (1993), credit market imperfections generate multiple accumulation paths or steady states depending on initial conditions.

Second, the safety first model developed in this paper suggests reasons for differences in aggregate saving rates among economies and strategies for raising aggregate savings. The innovative feature of the safety first result is that the saving rate increases discontinuously when assets move above the threshold level. A change in the economy that moves a significant proportion of the population above the threshold level (or moves the threshold level downwards) would then bring about a large increase in the aggregate saving rate, perhaps launching the economy into growth take-off.

The existence of dichotomous saving behavior distinguishes the safety first model from standard models of consumption under uncertainty (Aiyagari, 1994; Deaton, 1991; Kimball, 1990; Ljunqvist and Sargent 2000; Rothschild and Stiglitz, 1971; Zeldes; 1989; see also the survey by Carroll, 2001). In these models, uncertainty raises precautionary savings instead of reducing savings as in the safety first model. Standard approaches do not generate saving behavior that switches from dissaving to saving, and they often seek invariant saving behavior that does not depend on asset levels. In contrast, the safety first model explains why saving behavior at low asset levels differs from behavior at high asset levels.

Other features of the safety first model, including survival uncertainty, are in common with previous models of consumption in developing countries that do not generate dichotomous behavior (see Gersovitz, 1988, for a survey of saving behavior in developing countries and alternatives to standard models). At low incomes, expenditures on nutrition and health can affect health status and likelihood of survival. Then consumption not only yields utility but affects current and future productivity and survival. Cutler, Deaton and Lleras-Muney (2005) cite evidence by McKeown (1976), Fogel (1997, 2004) and Costa and Steckel (1997) that increased nutrition reduces mortality. However, they dispute the causal relation between income and reduced mortality as the primary explanation for the observed correlation, especially in more developed economies. Behrman and Deolalikar, in a review of health and nutrition in developing countries, include consumption expenditures as a determinant of health status (1988, pp. 640-641). Glomm and Palumbo (1993) consider optimal intertemporal consumption when consumption related to nutrition augments a health capital stock and borrowing is ruled out. Gersovitz (1983) analyzes a two-period model in which consumption affects likelihood of survival. Two threshold effects arise, one at the subsistence level of consumption in the first period and another at the consumption level above which survival is unaffected by greater consumption. The model developed in this paper does not rely on these threshold effects to generate saving rates that are discontinuous with respect to assets. Gersovitz also concludes that the saving rate will depend on income and specifies conditions for the average propensity to save to rise with income.

Ray and Streufert (1993) consider dynamic equilibria when undernourishment has intertemporal effects. They relate unemployment to the initial distribution of land. Chatterjee and Ravikumar (1999) analyze the consequences of minimum consumption levels in a nonstochastic environment. With utility depending on the difference between consumption and the minimum consumption level, the intertemporal elasticity of substitution is increasing in household wealth. Using the optimal dynamic consumption paths implied by minimum consumption levels, Chatterjee and Ravikumar analyze the evolution paths of household consumption and wealth. They conclude that there will be a transition phase during which inequality in consumption and wealth increase.

In an overlapping generations model, Chakraborty and Das (2005) show that endogenous mortality risk generates a link between health status and wealth in succeeding generations. When individuals can reduce their mortality risk through private health investment, poorer individuals choose a higher rate of time preference, making fewer investments that increase future income and leaving smaller bequests. Mortality risk then generates persistence of health and economic status from one generation to another and poverty traps.

The next section formalizes the safety first consumption problem using optimal control theory. Section 3 demonstrates the dichotomous saving behavior in the safety first model. In Section 4, factors that affect aggregate savings are considered. Section 5 compares results from the safety first model with standard models of consumption behavior.

2 Dynamic Safety First Problem

Let $rA_t + y$ be the individual's income in period t, where y is a constant income per period, A_t is the individual's assets in period t, and r is a constant rate of return on assets. At the beginning of a period, the individual allocates resources between saving and consumption. The individual's consumption in period t is then

$$C_t = (1 - s_t)(rA_t + y) \tag{1}$$

where s_t is the saving rate chosen by the individual in period t. The individual receives instantaneous utility in period t given by the logarithm of consumption:

$$U[C_t] = Log[C_t] = Log[(1 - s_t)(rA_t + y)] \tag{2}$$

This utility function yields a coefficient of relative risk aversion of 1.

Survival uncertainty can arise from uncertain consumption levels relative to requirements, random consumption requirements, or exogenous outcomes unrelated to consumption. For example, the income that is not saved may be used to plant crops for consumption, but the harvest could vary with weather conditions. In other activities, the individual's productivity in converting resources into consumption could vary. There could be losses from accidents or pests that reduce consumption. The individual could face additional consumption requirements from medical emergencies or repairs. A change in the relative price of food could alter the real consumption the individual is able to realize. As a consequence of these sources of uncertainty, the individual's survival to the next period will depend on the individual's consumption. Let $G[A_t, s_t]$ be the probability that an individual survives to the next period given that the individual's assets are

A_t and the individual has decided to save s_t of income. To incorporate alternative sources of uncertainty regarding survival, assume

$$G[A_t, s_t] = \frac{g_0 C_t}{C_t + k} = \frac{g_0(1 - s_t)(rA_t + y)}{(1 - s_t)(rA_t + y) + k} \tag{3}$$

where $0 < g_0 \leq 1$ and k is a positive constant. Then the probability of survival is a continuous function of consumption and is positive for all positive consumption levels. There is no minimum consumption level (besides zero) that the individual must achieve in order to survive.

Following standard dynamic programming formulations, the individual's value function in period t (conditional on surviving to period t) is

$$V[A_t] = Max(s_t)\,\{U[C_t] + \beta G[A_t, s_t]V[A_{t+1}]\} \tag{4}$$

where $\beta < 1$ is the discount factor and the maximization is taken over alternative saving rates, s_t.[2] In (4),

$$A_{t+1} = A_t + s_t(rA_t + y) \tag{5}$$

In choosing s_t, the individual balances current utility from consumption, survival to following periods, and utility in following periods.

In addition to (4) and (5), two additional relations can be used to determine solutions in successive periods. First, maximization of the right-hand side of (4) yields a first-order condition on s_t :

$$\frac{\partial U[C_t]}{\partial s_t} + \beta G[A_t, s_t]\frac{dV[A_{t+1}]}{dA_{t+1}}(rA_t + y) + \beta V[A_{t+1}]\frac{\partial G[A_t, s_t]}{\partial s_t} = 0 \tag{6}$$

Second, differentiation of $V[A_t]$ with respect to its argument yields the Benveniste-Scheinkman formula (Benveniste and Scheinkman, 1979):

$$\frac{dV}{dA_t} = \frac{\partial U[C_t]}{\partial A_t} + \beta V[A_{t+1}]\frac{\partial G}{\partial A_t} + \beta G[A_t, s_t]\frac{dV[A_{t+1}]}{dA_{t+1}}\frac{\partial A_{t+1}}{\partial A_t} \tag{7}$$

[2] Methods of optimal control theory applicable to saving decisions are described by Kamien and Schwartz (1987), Seierstad and Sydsaeter (1987) and Whittle (1982, 1996), among others.

Sequential values of the state variable A_t and the control variable s_t can be found by solving backwards from future levels of the value function V. Suppose numerical values for A_{t+1}, $V[A_{t+1}]$ and $V'[A_{t+1}] = dV[A_{t+1}]/dA_{t+1}$ are known. Then s_t, $V[A_t]$, and $V'[A_t]$ can be found from (6), (4) and (7), with A_t determined from s_t in (5). Backward solution of (4) through (7) for the optimal consumption path requires differentiability and concavity of the value function $V[A_t]$ in the relevant interval for assets. These features of $V[A_t]$ can be established by using A_{t+1} as the control variable instead of s_t (Ljunqvist and Sargent, 2000, p. 31). Then setting $q_t = A_{t+1}$ in (4) establishes differentiability since $U[C_t]$ and $G[A_t, s_t]$ are differentiable functions of A_t on the right hand side of (4). Concavity can be established by differentiating (7) with respect to A_t, again using $q_t = A_{t+1}$ as the control variable. The second order condition for a saving rate s_t to maximize $U[C_t] + \beta G[A_t, s_t]V[A_{t+1}]$ is that the left hand side of (6) be a declining function of s_t. This condition holds given the concavity of $V[A_t]$.

In general, the threat of nonsurvival reduces the saving rate compared to the level in a corresponding dynamic control problem with certain survival for two reasons. First, a survival likelihood of $G[A_t, s_t]$ that is less than one reduces the future returns to saving. Second, when current consumption affects the likelihood of survival, the negative term $\beta V[A_{t+1}]\partial G[A_t, s_t]/\partial s_t$ in (6) raises the current costs of saving and leads to a lower saving rate.

An alternative to the utility function used here would be to set $U[C_t] = (C_t)^\gamma/\gamma$, $\gamma \neq 0$, $\gamma < 1$, which has a coefficient of relative risk aversion of $1 - \gamma$. The results would be qualitatively the same, with $\partial U[C_t]/\partial s_t$ in (6) given by $-(1 - s_t)^{\gamma-1}(rA_t + y)^\gamma$ instead of $-(1 - s_t)^{-1}$. The use of $U[C_t] = Log[C_t]$ simplifies the analysis without qualitatively changing the result. With utility from non-survival normalized to 0, the individual's utility is nonnegative and does not prefer death whenever $Log[C_t] \geq 0$, which occurs even at asset level $A_t = 0$ if $y \geq 1$.

3 Discontinuous Increase in Saving Rate

A basic feature of safety first behavior in a dynamic context is that the saving rate can vary discontinuously with the asset level since the optimal path for the individual could follow either the decumulation (dissaving) or accumulation paths. As a result, the individual has optimal saving rates that differ by a finite amount for two asset levels that are arbitrarily close together. The optimal saving rate changes from a negative amount to a positive amount when assets are exogenously increased beyond a certain level. This jump in saving behavior occurs because the individual would be on different paths (accumulaton or decumulation) at slightly different levels of assets.

At each asset level, the individual could engage in accumulation of assets by saving ($s_t > 0$) or in decumulation by dissaving ($s_t < 0$).[3] For each of these two paths, a candidate value function $V[A_t]$ contingent on the path choice can be determined by solving backwards from later states. Let $V_a[A_t]$, and $V_d[A_t]$ be the candidate value functions at asset level A_t contingent on choosing the accumulation or decumulation paths, respectively. The next two sections construct the accumulation and decumulation paths. To allow specific calculations in an example, assume $\beta = .97$, $r = .2$, $k = 5$, $y = 1$ and $g_0 = 1$. These parameters were selected to show that individuals at low asset levels will dissave in spite of a large return to saving.

3.1 Accumulation Path

With accumulation, the individual eventually ends up with very high asset levels. At such asset levels, the risk of not surviving approaches a constant, g_0. The value function at such asset levels can be approximated using the solution to a standard consumption-saving problem without risk of not surviving. The conditions determining the solution are given by (4), (6) and (7) with G identically equal to g_0, using standard methods (see Sargent, 1987, pp. 31-33). The optimal saving

[3] The individual could also continue with the same level of assets by setting $s_t = 0$ and consuming all income. This alternative is checked against the accumulation and decumulation solutions in the next section.

rate for this problem, s^*, is a time-invariant proportion of income $rA + y$ given by

$$s^* = \beta g_0 - (1 - \beta g_0)/r \tag{8}$$

This saving rate will be positive and less than one when $\beta g_0 < 1$ and $1 + r > 1/(\beta g_0)$. With the parameter values assumed above, $s^* = .82$.

Consider a high asset level A_c. With the saving rate given by s^*, consumption in period $t + i$ is given by $C_{t+i} = C_t(1 + rs^*)^i$ and income in period $t + i$ is $rA_{t+i} + y = (rA_t + y)(1 + rs^*)^i$. Then the value function at A_c can be approximated by

$$\sum_{i=0}^{\infty} Log[C_{t+i}]\beta G[A_{t+i}, s^*]$$

Setting $A_c = 1,000,000,000$, $V_a[A_c] = 480.05$. The derivative $V_a'[A_c]$ can be calculated as $V_a[A_c + 1] - V_a[A_c]$. Then A_c, $V_a[A_c]$ and $V_a'[A_c]$ provide a start point for calculating A, $V_a[A]$ and $V_a'[A]$ along the accumulation path at sequentially earlier periods using (4), (6) and (7) in Section 2.

3.2 Decumulation Path

If an individual follows a path of decumulation, the end state will occur at an asset level of zero since the individual would be unable to borrow. Let $V_z[A_t]$ be the expected present discounted value of future consumption for an individual contingent on a zero saving rate. Then $V_z[A_t]$ can be found by setting $A_{t+1} = A_t$ and $s_t = 0$ in (4), so that

$$V_z[A_t] = \frac{U[rA_t + y]}{1 - \beta G[A_t, 0]} \tag{9}$$

The end point for the decumulation path has $A_t = 0$ and $V_d[0] = V_z[0]$. The derivative $V_d'[0]$ can be calculated as $(V_d[.0001] - V_d[0])/.0001$ by assuming that the individual uses the assets of .0001 for added consumption in the period before reaching the end state. Solving backwards from $A_d = 0$ using (4), (6) and (7) yields the decumulation path at sequentially earlier periods with higher asset values.

Along either the accumulation or decumulation paths, the saving rate will be an increasing function of assets if the following condition holds.

Condition 1 *The derivative of the left hand side of 6 with respect to A_t is positive for all positive asset levels.*

If Condition 1 holds, an increase in assets raises the saving rate at which (6) is satisfied. To see this, consider a graph of the left hand side of (6) versus the saving rate, s_t. This is a declining function of s_t because the second order condition holds from Section 2. The optimal s_t occurs where the left hand side cuts the horizontal axis (at 0), satisfying the first order condition in (6). When an increase in A_t raises the left hand side, the point of intersection moves to the right, so that the saving rate is an increasing function of assets.

3.3 Choice Between Accumulation and Decumulation Paths

This section describes the optimal saving behavior for an individual facing the particular safety first consumption problem that has been developed in the previous section. At each asset level, the individual optimally chooses the saving rate (and the subsequent path of assets and consumption) that yields the greatest value function. Figure 1 shows the candidate value functions for the accumulation and decumulation paths and the problem facing the individual. The two intersecting lines are the accumulation and decumulation paths calculated in Sections 3.2 and 3.3. The two paths intersect at $A_e = 133.08$, shown by the vertical dashed line in Figure 1. At asset levels greater than A_e, the accumulation path yields the higher value function. Then the individual chooses a positive saving rate and has greater assets in the next period. At higher asset levels, the value function for accumulation continues to be higher, so the individual moves in the direction of the arrow towards higher assets, consumption, and levels of the value function. At asset levels lower than A_e, the decumulation path yields the higher value function. The individual then chooses a negative saving rate and follows the decumulation path towards lower assets, consumption and value function levels, as shown by the downward sloping arrow. Just where the two paths intersect, the individual would be indifferent between the accumulation and decumulation paths since both yield the same level of the value function. The individual would not choose $s_t = 0$ at A_e since $V_z[A_e]$ is less than either $V_a[A_e]$ or $V_d[A_e]$. The value function for the safety first problem consists

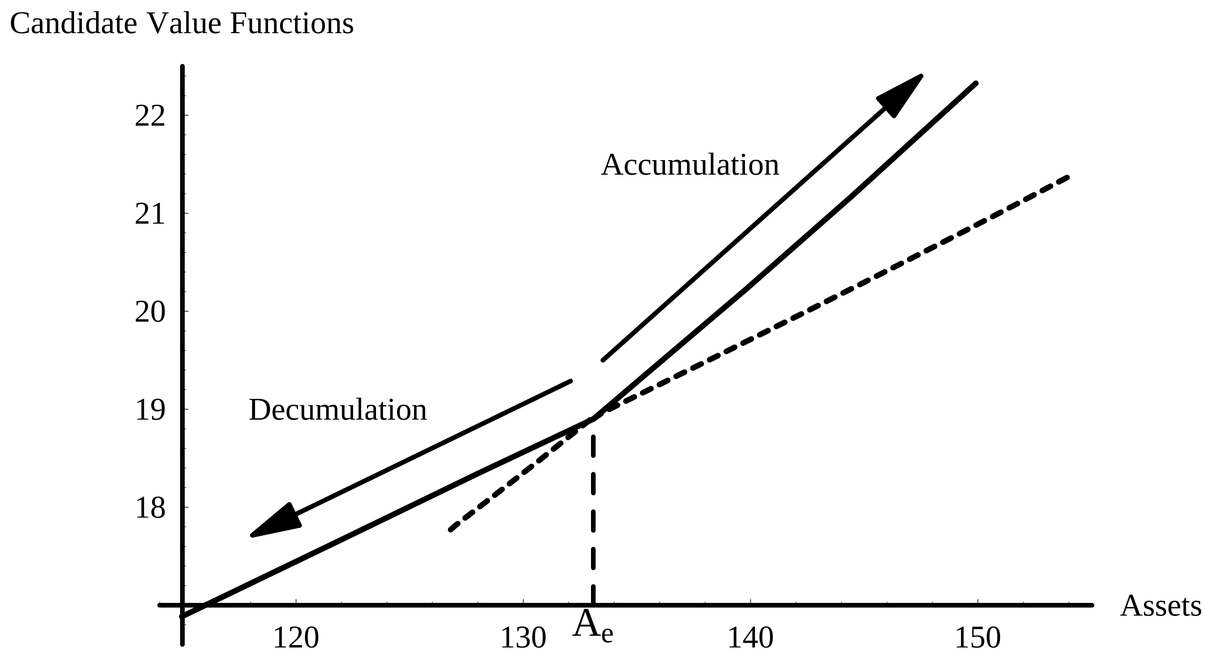

Figure 1: Value Functions Along Accumulation and Decumulation Paths

of the upper envelope of the candidate value functions for the accumulation and decumulation paths.

The consequences of the individual's choice of path are shown in Figure 2, which demonstrates the major point of this paper. At asset levels above A_e, shown by the dashed line, the optimal saving rate is positive. At asset levels below A_e, the optimal saving rate is negative. The saving rate jumps from a negative to a positive level at A_e.

Note that the discontinuity in the saving rate occurs with respect to asset level and not with respect to time. In the absence of any exogenous change in circumstances, the individual never switches from one path to another. If the individual is currently on one path (either accumulation or decumulation), the individual would continue on the same path in all future periods. Nevertheless, the discontinuity with respect to assets means that an individual's saving rate can increase by a significant amount in response to a relatively small exogenous change that either

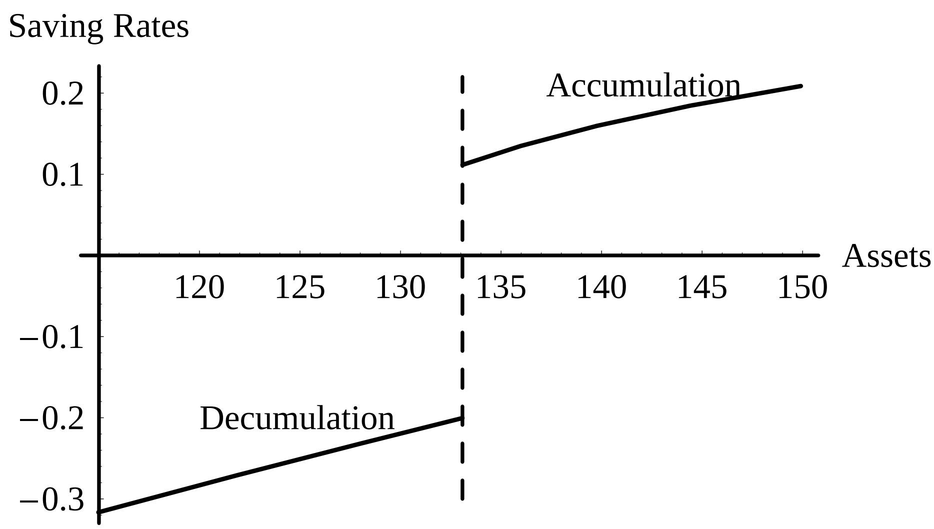

Figure 2: Saving Rates Along Accumulation and Decumulation Paths

moves the individual's assets from below to above the path intersection at A_e or that moves the path intersection below the individual's current level of assets.

The discontinuity in saving rates with respect to assets does not arise from any discontinuity in functional forms. In particular, the probability of survival is a continuous function of asset levels and saving rates at all positive asset levels. Instead, the discontinuity arises from the individual's discrete choice between the accumulation and decumulation paths.

With the structure of the safety first model developed above, the major results can be explained in more basic terms. The first result is that under certain conditions, the individual saves at higher asset levels and dissaves at lower asset levels. In the standard consumption model with certain survival (generated by (4), (6) and (7) with $G[A_t, s_t]$ identically one), saving generates a loss in consumption and utility in the current period, and an increase in assets, income, consumption and utility in the future. The positive rate of return on assets creates a greater increase in assets in the future than is saved in the current period, but the individual prefers a current change in utility to an equal change in utility in the future because of time preferences. The individual decides whether to save or dissave based on a comparison between increase in assets and the rate at which future utility is discounted because of time preferences. In the model generated by (4), (6) and (7) with $G[A_t, s_t]$ identically one, the individual chooses a positive saving rate whenever $1 + r > 1/\beta$, which can be derived from (8). In this expression, $1/\beta$ can be termed the time preference factor and measures the amount of utility one period in the future that would exactly compensate the individual for the loss of one unit of utility in the current period.

In the safety first model, survival depends on consumption and the result departs to varying degrees from the standard consumption problem. However, when assets are high, survival will be almost certain and the saving rate will be positive and close to the saving rate in the standard consumption problem when $1 + r > 1/\beta$.

Now consider how the problem changes when assets are low and the individual faces a substantial risk of not surviving to the next period. The present value of future contributions to utility is reduced because the individual is less likely to survive to benefit from those contributions. The time preference factor $1/(\beta G[A_t, s_t])$ rises as the likelihood of survival $G[A_t, s_t]$ declines. If the likelihood of survival falls enough, the time preference factor will exceed the return on assets, $1+r$, and the individual will be better off dissaving.

The discontinuity in saving rates proceeds from a different phenomenon. With positive saving at high asset levels and dissaving at low asset levels, the common expectation would be that the saving rate would gradually change from positive to negative as assets decline. But this is not the case because of the nature of dynamic optimization. Instead of a solution that can be worked out at each asset level without reference to solutions at other asset levels, the solution in a dynamic optimization problem is found by working backwards from future optimal solutions. In the safety first problem, there are two potential end points from which optimal solutions can be calculated backwards. Along the accumulation path, the combinations of asset levels and positive saving rates can be found by working backwards from a very high asset level, as calculated above in Section 3.1. That is, at each asset level, it is possible to calculate the asset level and saving rate in the previous period that would have yielded the current asset level. Similarly, the decumulation path can be calculated by solving backward from the lowest asset level. Since these two paths are calculated separately and from different end points, there is no reason for the saving rates to be equal at the asset level where the accumulation and decumulation paths intersect (and where the value functions are equal). Then the saving rates for the two paths will differ by a finite amount at that asset level, generating the discontinuity in saving behavior.

In the safety first model, risk arises from the possibility that the individual will not survive from one period to another rather than from stochastic income. Incorporating stochastic income into the model introduces the complication that an individual could move between saving and dissaving depending on the outcome from the stochastic process in a particular period. Then the value function for levels of assets where the individual dissaves must be determined simultaneously with the value function for levels of assets where the individual saves. This simultaneous calculation is in contrast with the separate calculation of the candidate value functions for accumulation and decumulation in the model developed here. The individual would still save at high asset levels and dissave at low asset levels, but the discontinuity in saving rates would be absent. The saving rate could change rapidly with asset level, generating substantially the same outcomes for poverty traps and effects of policies that arise with discontinuity in the safety first model. Although the value function with stochastic income can be computed numerically, it may not be concave so properties based on concavity could not be established.

A stochastic element could instead be introduced through consumption. In this alternative, the individual plans a consumption level but actual consumption differs from the planned level by a factor θ that varies between 0 and 1. This assumption is consistent with an agricultural worker who commits resources to a crop but faces an uncertain harvest because of weather, pests or prices. If actual consumption falls below a minimum level, the individual fails to survive. The probability of not surviving can then be calculated from the interval of values of θ that yield consumption below the minimum. With this stochastic factor in consumption, assets in the next period are known with certainty and the accumulation and decumulation paths can again be calculated separately as in the model developed here.

3.4 General Results for the Safety First Consumption Problem

The intersection between the accumulation and decumulation paths calculated in the previous section will not occur for some parameter values in the safety first consumption problem. This section first considers conditions when an intersection will not occur, so that there would be no discontinuity in safety rates. Then the section presents formal results for the safety first consumption problem.

Consider first the possibility that the individual dissaves over the entire asset range. At high asset levels, $G[A_t, s_t]$ approaches g_0. With a constant likelihood of survival, the optimal saving rate would be given by s^* in (8).

Condition 2 *In (8),* $r < (1 - \beta g_0)/(\beta g_0)$ *so that* s^* *is negative.*

If Conditions 1 and 2 hold, the saving rate will be negative at high asset levels and will be lower (and negative) at lower asset levels. Then the individual would always dissave and there would be no jump in saving rates.

Now consider the possibility that the individual engages in positive saving at all positive asset levels. The following condition, combined with Condition 1, yields this case.

Condition 3 *The left hand side of (6) is positive at* $A_t = 0$ *and* $s_t = 0$.

Then the optimal saving rate at $A_t = 0$ will be positive (because the left hand side is a declining function of s_t). In some interval of assets above $A_t = 0$, the saving rate will continue to be positive since the increase in assets shifts the left hand side of (6) up. If Condition 1 holds, the saving rate will be higher at greater asset levels and will therefore be positive at all positive asset levels. Then no jump in saving rates would occur.

The third case arises when the accumulation and decumulation paths do not intersect but instead meet at an asset level at which the optimal saving rate is zero. The asset level at which this occurs could be determined from (4), (6) and (7), setting $s_t = 0$ and $A_{t+1} = A_t$.[4] Let A^* be an asset level at which these conditions are satisfied and let $V_0[A^*]$ be determined from (9). The following condition yields cases where setting $s_t = 0$ would be optimal.

Condition 4 *Asset level A^* satisfies (4), (6) and (7) with $s_t = 0$ and $V_0[A^*] = V_a[A^*] = V_d[A^*]$.*

When condition 4 holds, the accumulation and decumulation paths meet at A^*, with the optimal saving rate for both paths equal to zero so that there is no jump in saving rates. In the example that generates Figure 1, the accumulation and decumulation paths meet at $A_e = 133.08$, but $V_0[A_e] = 18.57 < V_a[A_e] = V_d[A_e] = 18.92$, so condition 4 does not hold for that example.

The following proposition summarizes these results.

Proposition 5 *In the safety first consumption problem, there will be no discontinuous jump in saving rates with respect to assets if conditions 1 and 2 hold, or if conditions 1 and 3 hold, or if condition 4 holds.*

Ruling out cases where the accumulation and decumulation paths do not intersect yields the following theorem, the main result of this paper.

Theorem 6 *Consider an individual facing a safety first consumption problem with utility given by $U[C_t]$ in (2) and survival probability given by $G[A_t, s_t]$ in (3). Suppose Condition 1 holds and suppose accumulation and decumulation paths intersect at asset level A_e. Then*

1. *at A_e, the values of A_{t+1} and $V[A_{t+1}]$ are greater for the accumulation path*
2. *at A_e, the accumulation path is steeper than the decumulation path*
3. *the intersection between the accumulation and decumulation paths is unique*
4. *at asset levels above A_e, the individual will always choose to accumulate assets*
5. *at asset levels below A_e, the individual will always choose to decumulate assets by dissaving*
6. *at A_e, the individual will be indifferent between following the accumulation path and choosing a positive saving rate, or following the decumulation path and choosing a negative saving rate, but would not choose a zero saving rate if $V_z[A_e]$ in (9) is less than $V_a[A_e]$ and $V_d[A_e]$*
7. *the saving rate is discontinuous with respect to assets at A_e*

Proof. 1. For the accumulation path, savings are positive so that the next value of assets, A_{t+1}, is greater than the current value of assets, A_e. For the decumulation path, $s_t < 0$ and $A_{t+1} < A_e$.

[4] First, 4 and 7 can be solved algebraically for $V[A]$ and $V'[A]$, which can then be substituted into 6 to yield a single equation in the unknown asset level.

Then A_{t+1} is greater along the accumulation path. In (4), $V[A_e]$ is the same for both paths, C and $U[C]$ are smaller on the decumulation path at A_e, and $G[A_e, s]$ is greater for the decumulation path. Then $V[A_{t+1}]$ must be greater for the accumulation path by calculation from (6). 2. Using q_t as the control variable, (7) becomes

$$\frac{dV}{dA_t} = \frac{\partial U}{\partial A_t} + \beta V[q_t]\frac{\partial G}{\partial A_t}$$

where $C_t = rA_t + y + A_t - q_t$. Then

$$\frac{\partial U}{\partial A_t} = \frac{1+r}{rA_t + y + A_t - q_t}$$

$$\frac{\partial G}{\partial A_t} = \frac{g_0 k(1+r)}{(rA_t + y + A_t - q_t + k)^2}$$

Along the accumulation path, q_t is greater. Then $\partial U/\partial A_t$ and $\partial G/\partial A_t$ are greater at A_e on the accumulation path. Since $V[q_t]$ is also greater at A_e from 1., dV/dA_t is greater along the accumulation path. Then the accumulation path (as shown in Figure 1) is steeper than the decumulation path at the point of intersection. 3. Suppose, contrary to 3., that the two paths intersect at an asset level above A_e, and suppose the first such intersection is at A_{2nd}. Then the decumulation path would intersect the accumulation path from below and would have a steeper slope, contradicting 2. Therefore a second intersection to the right of A_e could not occur. Similarly, if the first intersection below A_e is at A_{2nd}, the decumulation path would cut the accumulation path from below (as assets increased past A_{2nd}), again contradicting 2. These contradictions imply that a second intersection would not occur. 4. At asset levels above A_e, $V_a[A]$ starts out greater than $V_d[A]$ and continues to be greater than $V_d[A]$ at higher asset levels since an intersection between the two paths could not occur. The individual would always choose the path yielding the higher value function, which would be the candidate value function for the accumulation path as assets increased from A_e. 5. Similarly, at asset levels below A_e, $V_d[A]$ would be greater than $V_a[A]$, so the individual would continue to choose the decumulation path. 6. At A_e, $V_a[A_e] = V_d[A_e]$ because the two paths intersect. The individual would then achieve the same expected present value whether choosing to follow the accumulation path or the decumulation path. Assuming $V_z[A_e] < V_a[A_e] = V_d[A_e]$, the individual would be worse off choosing a saving rate of zero and would not choose to stay at the same asset level. 7. Consider the saving rate as assets approach A_e from above. Since $s = 0$ is not a solution to the safety first consumption problem at A_e, the saving rate along the accumulation path will have as a limit a positive saving rate. Similarly, considering the saving rate as assets approach A_e from below, the saving rate along the decumulation path will have as a limit a negative saving rate. Then the saving rate chosen by the individual will be discontinuous with respect to assets at A_e. ■

In deciding the level of consumption, the individual faces two types of intertemporal trade-offs. As in standard dynamic models of consumption and saving, the individual can achieve greater consumption in the future by reducing current consumption, accumulating assets at a more rapid rate. In the dynamic safety first model, the individual additionally faces a trade-off between survival to the next period and long-term survival. Greater current consumption, while raising the likelihood of surviving to the next period, extends the time that the individual bears the threat of not surviving by reducing the rate of increase in income. These trade-offs operate with different effects at different asset levels, leading to behavior that varies with asset level.

4 Policies Affecting Aggregate Savings

This section considers the results of reducing risk and redistributing wealth. In the safety first model, these policies have effects that are opposite to the effects predicted by standard consumption theory. As a consequence, policies that promote efficiency and equity can be consistent with growth objectives in the safety first model.

4.1 Reduction of Risk

In standard models of consumption for developed countries, individuals have a precautionary motive to save to smooth out consumption in the presence of risks of income shocks in the absence of insurance.[5] Saving accumulates assets that can be used to provide self-insurance against negative shocks to income. Policies that reduce income shocks would then also reduce precautionary savings. In the model developed here, the risks arise from failure to survive to the next period, rather than shocks to income. Since saving is determined before survival is known, assets cannot be used as self-insurance against a negative shock. Individuals instead save because accumulation raises future consumption levels as well as reducing future risks of not surviving. In contrast to standard consumption theory for developed countries, policies that reduce risk would raise the saving rate as stated in the following proposition.

Proposition 7 *Suppose the accumulation and decumulation paths for the safety first consumption problem intersect at A_t, so that $V_a[A_t] = V_d[A_t]$. Then an increase in the likelihood of survival from a higher value of g_0 in (3) reduces the asset value at which the accumulation and decumulation paths intersect and raises the saving rate at each asset level on the accumulation path.*

Proof. Since the saving rate is higher for the accumulation path at A_t, the consumption level and utility in period t are lower for the accumulation path. Then in (4), since $V_a[A_t] = V_d[A_t]$, $\beta G[A_t, s_t]V[A_{t+1}]$ must be greater for the accumulation path. The derivative

$$\partial V/\partial g_0 = \partial \beta G[A_t, s_t]V[A_{t+1}]/\partial g_0$$

must also be greater for the accumulation path, so that an increase in g_0 shifts the accumulation path up more at A_t than it shifts the decumulation path. It follows that the intersection between the two paths must occur at a lower asset level after g_0 increases. The effect of an increase in g_0 on the optimal saving rate can be determined from the first order condition for s_t in (6). Since $\beta G[A_t, s_t]V[A_{t+1}]$ in (4) is proportional to g_0, its derivative with respect to s_t in (6) will increase when g_0 increases, shifting the left hand side of the first order condition upwards. By the second order condition for the optimal saving rate, the saving rate at which the left hand side equals zero must be greater, completing the proof. ■

The first part of the proposition establishes that more individuals will engage in positive saving and the second part establishes that the saving rates for individuals on the accumulation path will

[5] Besley (1995) and Rosenzweig and Wolpin (1993) discuss precautionary savings, credit markets and insurance in developing countries.

be greater when g_0 increases. The reduction in the threat of nonsurvival then raises the aggregate saving rate instead of reducing it.

4.2 Redistribution of Wealth

In many economic models, individuals with greater wealth are also assumed to have higher saving rates.[6] Higher aggregate saving rates, in turn, are generally linked with a greater rate of growth of the economy through capital accumulation. Then a policy that generates greater equity in the distribution of assets would apparently carry a negative consequence of reducing the aggregate saving rate and the economy's rate of growth (Kuznets, 1962, pp. 7-8; Nelson, 1956, p. 897; see Gersovitz, 1988, pp. 407-409, for a discussion). However, it is incorrect to conclude that a shift of wealth from those with high saving rates to those with low saving rates would necessarily reduce aggregate savings. Such a conclusion would confuse average saving rates with marginal saving rates. Paradoxically, an individual with greater assets can have a higher average propensity to save but a lower marginal propensity to save compared to an individual with lower assets.

The following proposition identifies the condition for the marginal propensity to save out of assets to be a decreasing function of assets. To consider the general case, suppose income at asset level A is given by $rA+y$ as before, and let $S[A]$ be the optimal saving rate as a function of assets. Suppose $S[A]$ has continuous first and second derivatives on the interval A_1 to A_2. Total saving at asset level A is given by $S[A](rA+y)$, and the marginal propensity to save out of assets is

$$MPS[A] = rS[A] + \frac{dS[A]}{dA}(rA+y) = rS[A] + S'[A](rA+y) \tag{10}$$

The marginal propensity to save out of income can be calculated from the marginal propensity to save out of assets by dividing $MPS[A]$ by the derivative of income with respect to assets, $d(rA+y)/dA = r$. Since the two marginal propensities to save are proportional, the marginal propensity to save out of income will be a declining function of income whenever $MPS[A]$ is a declining function of assets.

[6] For example in Kaldor's model of income distribution (1955-1956), individuals with income from profits have a higher saving rate than individuals with earned income.

Proposition 8 *Suppose the saving rate $S[A]$ is an increasing function of assets on the interval A_1 to A_2. The marginal propensity to save out of assets, $MPS[A]$, is a declining function of assets on the interval whenever*

$$\frac{dLog[S'[A]]}{dA} < \frac{-2r}{rA+y}$$

Proof. From (10),

$$\frac{dMPS[A]}{dA} = 2rS'[A] + (rA+y)\frac{dS'[A]}{dA} \tag{11}$$

Then after rearrangement $dMPS[A]/dA$ is negative whenever

$$\frac{dS'[A]/dA}{S'[A]} = \frac{dLog[S'[A]]}{dA} < \frac{-2r}{rA+y} \tag{12}$$

completing the proof. ■

In the safety first consumption model, redistributions of assets towards individuals with lower asset levels can raise aggregate savings in two ways. First, consider individuals with asset levels just below the level where the accumulation and decumulation paths intersect. They would follow the decumulation path and would be engaged in dissaving. Redistributions of small amounts of assets to them would shift them to the accumulation path, and their increase in saving would more than compensate for the reduction in saving by individuals whose assets were reduced for the redistribution. Second, in the interval of assets for which individuals are on the accumulation path, a redistribution of assets to individuals at the lower end of the asset interval would raise aggregate savings if the condition for Proposition 8 holds. This condition is likely to hold for the safety first problem since the saving rate is a concave function of assets at lower asset levels, generating a negative second derivative of the saving rate $S[A]$.

Figure 3 shows that Proposition 8 holds for the example used to generate Figure 1.[7] Up to an asset level of about 400, the marginal propensity to save is a declining function of income and assets even though the average saving rate is increasing. At higher asset levels than shown in the figure, the marginal propensity to save rises until it reaches the optimal level for certain survival, .82. Since the marginal propensity to save is highest at the lower boundary of the interval,

[7] Figure 3 shows the marginal propensity to save out of income to facilitate comparison with the average propensity to save out of income.

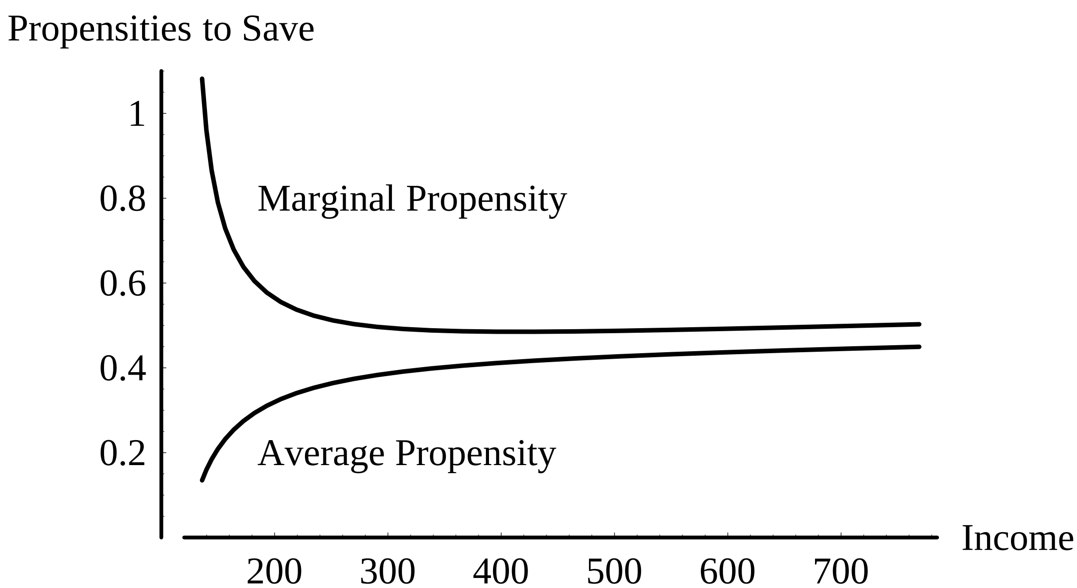

Figure 3: Marginal and Average Propensities to Save out of Income

aggregate savings would be increased by redistributing wealth from individuals with higher asset levels to individuals with asset levels near the bottom of the interval. It is unknown whether an interval of declining marginal propensity to save always occurs in the general case considered in Section 3. However, an increase in aggregate saving would always occur by transferring wealth from individuals at higher asset levels to an individual with assets just below the threshold between decumulation and accumulation.

5 Conclusions

In dynamic models of optimal consumption behavior under certainty, the individual typically consumes a constant proportion of income. Such a policy is invariant to asset level and yields a constant marginal propensity to save. Even at low income levels, consumers choose the same accumulation path. There is no jump in the saving rate when individuals shift from one optimal path to another in response to an increase in assets. Perfect foresight and certainty equivalent models adapt the model with certainty to fluctuations in income without significantly changing the results. Without these simplifying assumptions, uncertainty from random incomes generates a precautionary motive for saving. Rothschild and Stiglitz (1971) describe conditions under which an increase in uncertainty raises the saving rate. Carroll and Kimball (1996) describe a class of utility functions that yield a concave consumption function with stochastic income. Numerical solutions of stochastic optimal control models obtained by calculating backwards from high income levels also yield a concave consumption function, with monotonically increasing marginal propensity to save (Carroll, 2001, p. 30). Dynan, Skinner and Zeldes (2004), using data from the U.S., find a strong, positive relation between average saving rates and lifetime incomes and a weaker but positive relation between the marginal propensity to save and lifetime incomes. With these standard models, there is no low equilibrium trap, the saving rate does not jump discontinuously with changes in assets, and the marginal propensity to save is either constant or increases monotonically. Inequality in wealth arises from different initial endowments, higher saving rates for individuals with greater assets, and random incomes. The only policy implication from these models for the objective of raising aggregate savings is to redistribute assets to the wealthy. Such a policy would have a negligible effect on aggregate savings but would raise inequality in wealth.

In contrast, the safety first model yields a low equilibrium trap, in which individuals with low income do not rise above some level. The economic status of the population in a developing economy would then diverge, depending on whether individuals' assets were greater or less than the threshold for the accumulation path. Because the saving rate increases discontinuously when assets go above the threshold level, the aggregate saving rate for an economy can increase substantially if a significant proportion of the population is shifted to the accumulation path. The safety first model suggests policies that would achieve such an increase in the aggregate saving rate. Reduction of risk would raise the aggregate saving rate rather than reducing the precautionary motive for saving as in standard models of consumption. The safety first model suggests a strong relationship between health, as reflected in the likelihood of survival, and the aggregate saving rate in developing economies.

This paper extends the analysis of types of risk that individuals face. Responses to changes in risks vary depending on the nature of risk. In past analyses of consumption, risk takes the form of stochastic variation in income. In the paper developed here, risk arises in the possibility of not surviving to the next period. Risk is further differentiated by the time pattern of likelihood of survival to future periods. These results suggest that conclusions generated by models with a particular form of risk (i.e. stochastic income) cannot be generalized to other forms of risk. The safety first model analyzed here provides a theory of consumption behavior that is more appropriate to low income developing countries than standard models of consumption because it emphasizes risk of survival rather than stochastic income. It explains phenomena occurring in developing countries and reverses policy conclusions drawn from the standard theory.

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