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Lars-Alexander Kuehn
Nicolas Petrosky-Nadeau
Lu Zhang

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ABSTRACT

Search frictions in the labor market help explain the equity premium in the financial market. We embed the Diamond-Mortensen-Pissarides search framework into a dynamic stochastic general equilibrium model with recursive preferences. The model produces a sizeable equity premium of 4.54% per annum with a low interest rate volatility of 1.34%. The equity premium is strongly countercyclical, and forecastable with labor market tightness, a pattern we confirm in the data. Intriguingly, search frictions, combined with a small labor surplus and large job destruction flows, give rise endogenously to rare disaster risks a la Rietz (1988) and Barro (2006).

Lars-Alexander Kuehn
Carnegie Mellon University
Tepper School of Business
5000 Forbes Avenue
Pittsburgh, PA 15206
kuehn@cmu.edu

Nicolas Petrosky-Nadeau
Carnegie Mellon University
Tepper School of Business
5000 Forbes Avenue
Pittsburgh, PA 15206
npn@andrew.cmu.edu

Lu Zhang
Fisher College of Business
The Ohio State University
2100 Neil Avenue
Columbus, OH 43210
and NBER
zhanglu@fisher.osu.edu

1 Introduction

Modern asset pricing research has been successful in specifying preferences and cash flow dynamics to explain the equity premium, its volatility, and its cyclical variation in the endowment economy (e.g., Campbell and Cochrane (1999); Bansal and Yaron (2004); Barro (2006)). Explaining the equity premium in the production economy with endogenous cash flows has proven more challenging. The prior literature has mostly treated cash flows as dividends. However, in the data labor income accounts for about two thirds, while dividends account for only a small fraction of aggregate disposable income. As such, an equilibrium asset pricing model should take the labor market seriously.

We study aggregate asset prices by incorporating search frictions in the labor market (e.g., Diamond (1982); Mortensen (1982); Pissarides (1985, 2000)) into a dynamic stochastic general equilibrium economy with recursive preferences. A representative household pools incomes from its employed and unemployed workers, and decides on optimal consumption and asset allocation. The unemployed workers search for job vacancies posted by a representative firm. The rate at which a job vacancy is filled is affected by the congestion in the labor market. The degree of congestion is measured by labor market tightness, defined as the ratio of the number of job vacancies over the number of unemployed workers. Deviating from Walrasian equilibrium, search frictions create rents to be divided between the firm and the employed workers through the wage rate, which is in turn determined by the outcome of a generalized Nash bargaining process.

We find that search frictions are important for explaining the equity premium. With reasonable parameter values, the search economy reproduces an equity premium of 4.54% per annum and a low interest rate volatility of 1.34%. The equity risk premium is strongly countercyclical in the model: the vacancy-unemployment ratio forecasts stock market excess returns with a negative slope, a pattern that we confirm in the data. The model also produces an average stock market volatility of 11.07%. However, the average interest rate is 4.11%, which is high relative to 0.59% in the data. The model is also broadly consistent with the business cycle moments of the labor market.

Intriguingly, the search economy shows rare but deep disasters. In the stationary distribution from the model's simulations, the unemployment rate is positively skewed with a long right tail. The mean unemployment rate is 9.23%. The 2.5 percentile is nearby, 6.34%, but the 97.5 percentile is far away, 19.97%. As such, output and consumption are both negatively skewed with a long left tail, giving rise to disaster risks emphasized by Rietz (1988) and Barro (2006). Applying Barro and Ursúa's (2008) peak-to-trough measurement on the simulated data, we find that the consumption and GDP disasters in the model have the same average magnitude, about 20%, as that in the data. The consumption disaster probability is 3.22% in the model, which is close to 3.63% in the data. However, the GDP disaster probability is 4.94%, which is higher than 3.69% in the data.

We show via comparative statics that three key ingredients of the search economy, when combined, are capable of producing a high and countercyclical risk premium. First, we calibrate the value of unemployment activities to be (relatively) high, implying a small labor surplus (output minus wages). Intuitively, a high value of unemployment activities implies that wages are less elastic to labor productivity. By dampening the procyclical covariation of wages, a small labor surplus magnifies the procyclical covariation (risk) of dividends, increasing the equity premium. Also, the quantitative impact of the inelastic wages is especially important in bad times, when the labor surplus is even smaller (because of lower productivity). This mechanism turbocharges the risk and risk premium, making the equity premium and the stock market volatility strongly countercyclical.

Second, job destruction flows are large in the data. In contrast, swings in cyclical investment flows have little impact on the proportionally large capital stock in baseline production economies with capital as the only input. The rate of capital depreciation is around 12% per annum, whereas the job destruction rate can be as high as 60% (e.g., Davis, Faberman, and Haltiwanger (2006)). Third, the cost of vacancy posting is high relative to the small labor surplus. The high vacancy cost prevents the firm from hiring enough workers to offset large job destruction flows, giving rise to rare economic disasters. In contrast, such disasters are absent in baseline production economies.

Our work expands the disaster risks literature. Rietz (1988) argues that rare disasters help explain the equity premium puzzle. Barro (2006, 2009), Barro and Ursúa (2008), and Barro and Jin (2011) examine long-term data that include many disasters from a diverse set of countries (see also Reinhart and Rogoff (2009)). Gabaix (2009) argues that incorporating rare disasters helps improve asset pricing properties of real business cycle models. We show that search frictions give rise to disaster risks endogenously in production economies. The result that disaster risks can arise naturally in theory lends credence to the disaster risks explanation of the equity premium. We also advance this literature by starting to inquire about the origin and the internal mechanism of disasters.

Gourio (2010) embeds rare disasters exogenously into a production economy, and argues that disaster risks can explain a large and time-varying equity premium. However, Gourio defines dividends as levered output and treats the claim to the levered output as equity. The return on capital (the stock return in production economies) still has a low risk premium and a small volatility. In contrast, by riding on search frictions, our economy features a sizeable and time-varying equity premium.

Our model's success in explaining a sizeable and time-varying equity premium is surprising, and contrasts with prior studies on asset pricing in production economies. In these economies, often with capital as the only productive input, the amount of endogenous risk is too small, giving rise to a negligible and time-invariant equity premium. For example, Rouwenhorst (1995) shows that the standard production economy fails to explain the equity premium because of consumption smoothing.

With internal habit, Jermann (1998) uses capital adjustment costs, and Boldrin, Christiano, and Fisher (2001) use cross-sector immobility to restrict consumption smoothing to reproduce a high equity premium. Alas, both models struggle with excessively high interest rate volatilities. Using recursive preferences to curb interest rate volatility, Kaltenbrunner and Lochstoer (2010) show that a production economy with capital adjustment costs still fails to reproduce a high equity premium (see also Campanale, Castro, and Clementi (2010)). We show that introducing search frictions seems to overcome many difficulties encountered in baseline production economies.

Danthine and Donaldson (2002) and Favilukis and Lin (2011) show that staggered wage contracting is important for asset prices. Instead of sticky wages, we use the standard search framework with period-by-period Nash bargaining to break the link between wages and the marginal product of labor. Merz and Yashiv (2007) and Bazdresch, Belo, and Lin (2009) show that labor adjustment costs help match aggregate stock market valuation and the cross section of returns, respectively. Guvenen (2009) examines asset prices in an equilibrium model with limited stock market participation and heterogeneous agents. We differ by examining aggregate asset prices in a search economy.

Our work is built on the recent literature on labor market search. Shimer (2005) argues that the unemployment volatility in the baseline search model is too low relative to that in the data. Hagedorn and Manovskii (2008) adopt small labor surplus and Gertler and Trigari (2009) use staggered wage contracting to explain the Shimer puzzle. We instead study aggregate asset prices.

The rest of the paper is organized as follows. Section 2 constructs the model. Section 3 describes the model's solution. Section 4 discusses quantitative results. Finally, Section 5 concludes.

2 The Model

We embed the standard Diamond-Mortensen-Pissarides search model into a dynamic stochastic general equilibrium economy with recursive preferences.

2.1 Search and Matching

The model is populated by a representative household and a representative firm that uses labor as the single productive input. Following Merz (1995), we use the representative family construct, which implies perfect consumption insurance. In particular, the household has a continuum (of mass one) of members who are, at any point in time, either employed or unemployed. The fractions of employed and unemployed workers are representative of the population at large. The household pools the income of all the workers together before choosing per capita consumption and asset holdings.

The representative firm posts a number of job vacancies, V_t, to attract unemployed workers,

U_t, at the unit cost of κ. Vacancies are filled via a constant returns to scale matching function, $G(U_t, V_t)$. We specify the matching function as:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}}, \tag{1}$$

in which $\iota > 0$ is a constant parameter. This matching function, originated from Den Haan, Ramey, and Watson (2000), has the desirable property that matching probabilities fall between zero and one.

Specifically, define $\theta_t \equiv V_t/U_t$ as the vacancy-unemployment (V/U) ratio. The probability for an unemployed worker to find a job per unit of time (the job finding rate), $f(\theta_t)$, is:

$$f(\theta_t) \equiv \frac{G(U_t, V_t)}{U_t} = \frac{1}{\left(1 + \theta_t^{-\iota}\right)^{1/\iota}}, \tag{2}$$

and the probability for a vacancy to be filled per unit of time (the vacancy filling rate), $q(\theta_t)$, is:

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^\iota)^{1/\iota}}. \tag{3}$$

It follows that $f(\theta_t) = \theta_t q(\theta_t)$ and $\partial q(\theta_t)/\partial \theta_t < 0$, meaning that an increase in the relative scarcity of unemployed workers relative to job vacancies makes it more difficult for a firm to fill a vacancy. As such, θ_t is a measure of labor market tightness from the perspective of the firm.

Once matched, jobs are destroyed at an exogenous and constant rate of s per period. Total employment, N_t, then evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t. \tag{4}$$

Because the size of the population is normalized to be unity, $U_t = 1 - N_t$. As such, N_t and U_t can also be interpreted as the rates of employment and unemployment, respectively.

2.2 The Representative Firm

The firm takes aggregate labor productivity, X_t, as given. The law of motion of $x_t \equiv \log(X_t)$ is:

$$x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}, \tag{5}$$

in which $0 < \rho < 1$ is the persistence, $\sigma > 0$ is the conditional volatility, and ϵ_{t+1} is an independently and identically distributed (i.i.d.) standard normal shock.

The firm uses labor to produce with a constant returns to scale production technology,

$$Y_t = X_t N_t, \tag{6}$$

in which Y_t is output. The dividend (net payout) to the firm's shareholders is given by:

$$D_t = X_t N_t - W_t N_t - \kappa V_t, \tag{7}$$

in which W_t is the wage rate (to be determined later in Section 2.4).

Let $M_{t+\tau}$ be the representative household's stochastic discount factor from period t to $t+\tau$. Taking the matching probability, $q(\theta_t)$, and the wage rate, W_t, as given, the firm posts an optimal number of job vacancies to maximize the cum-dividend market value of equity, denoted S_t:

$$S_t \equiv \max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^{\infty}} E_t \left[\sum_{\tau=0}^{\infty} M_{t+\tau} (X_{t+\tau} N_{t+\tau} - W_{t+\tau} N_{t+\tau} - \kappa V_{t+\tau}) \right], \tag{8}$$

subject to the employment accumulation equation (4) and a nonnegativity constraint on vacancies:

$$V_t \geq 0. \tag{9}$$

Because $q(\theta_t) > 0$, this constraint is equivalent to $q(\theta_t) V_t \geq 0$. As such, the only source of job destruction in the model is the exogenous separation of employed workers from the firm.

Let μ_t denote the Lagrange multiplier on the employment accumulation equation (4), and λ_t the multiplier on the nonnegativity constraint $q(\theta_t) V_t \geq 0$. The first-order conditions with respect to V_t and N_{t+1} in maximizing the equity value of equity are given by, respectively:

$$\mu_t = \frac{\kappa}{q(\theta_t)} - \lambda_t, \quad (10)$$

$$\mu_t = E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}\right]\right]. \quad (11)$$

Combining the two first-order conditions yields the intertemporal job creation condition:

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left(\frac{\kappa}{q(\theta_{t+1})} - \lambda_{t+1}\right)\right]\right]. \quad (12)$$

The optimal vacancy policy also satisfies the Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \lambda_t \geq 0, \quad (13)$$

$$\lambda_t q(\theta_t)V_t = 0. \quad (14)$$

Intuitively, equation (10) says that the marginal cost of vacancy posting, $\kappa/q(\theta_t) - \lambda_t$, equals the marginal value of employment, μ_t. In particular, $\kappa/q(\theta_t) - \lambda_t$ is the marginal cost of vacancy posting, taking into account the matching probability and the nonnegativity constraint. When the firm posts vacancies, with $V_t > 0$ and $\lambda_t = 0$, equation (10) says that the unit cost of vacancy posting, κ, is equal to μ_t, conditional on the probability of a successful match, $q(\theta_t)$. On the other hand, when the nonnegativity constraint is binding, with $V_t = 0$ and $\lambda_t > 0$, $\theta_t = V_t/U_t = 0$. Equation (3) also implies that $q(\theta_t) = (1 + \theta_t^\iota)^{-1/\iota} = 1$. As such, equation (10) reduces to $\mu_t = \kappa - \lambda_t$.

The intertemporal job creation condition (12) is intuitive. The marginal cost of vacancy posting at period t should equal the marginal value of employment, μ_t, which in turn equals marginal benefit of vacancy posting at period $t+1$, discounted to period t with the stochastic discount factor, M_{t+1}. The marginal benefit at period $t+1$ includes the marginal product of labor, X_{t+1}, net of the wage rate, W_{t+1}, as well as the marginal value of employment, μ_{t+1}, which in turn equals the marginal cost of vacancy posting at $t+1$, net of separation.

Define the stock return as $R_{t+1} = S_{t+1}/(S_t - D_t)$ (recall S_t is the cum-dividend equity value).

The constant returns to scale assumption implies that (see Appendix A for a detailed derivation):

$$R_{t+1} = \frac{X_{t+1} - W_{t+1} + (1-s)\left(\kappa/q(\theta_{t+1}) - \lambda_{t+1}\right)}{\kappa/q(\theta_t) - \lambda_t}. \tag{15}$$

As such, the stock return is the ratio of the marginal benefit of vacancy posting at period $t+1$ over the marginal cost of vacancy posting at period t.

2.3 The Representative Household

The representative household maximizes utility, denoted J_t, over consumption using recursive preferences. The household can buy risky shares issued by the representative firm as well as a risk-free bond. Let C_t denote consumption and χ_t denote the fraction of the household's wealth invested in the risky shares. The recursive utility function is given by:

$$J_t = \max_{\{C_t, \chi_t\}} \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left(E_t\left[J_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{16}$$

in which β is time discount factor, ψ is the elasticity of intertemporal substitution, and γ is relative risk aversion. This utility separates ψ from γ, helping the model to produce a high equity premium and a low interest rate volatility simultaneously.

Tradeable assets consist of risky shares and a risk-free bond. Let R_{t+1}^f denote the risk-free interest rate (known at the beginning of period t), and let $R_{t+1}^{\Pi} = \chi_t R_{t+1} + (1-\chi_t)R_{t+1}^f$ be the return on wealth. In addition, let Π_t denote the household's financial wealth, b the value of unemployment activities, T_t the taxes raised by the government to pay for the unemployment benefits in lump-sum rebates. We can write the representative household's budget constraint as:

$$\frac{\Pi_{t+1}}{R_{t+1}^{\Pi}} = \Pi_t - C_t + W_t N_t + U_t b - T_t. \tag{17}$$

In particular, the household's dividend income, D_t, is included in the financial wealth, Π_t. (In equilibrium, $\Pi_t = S_t$, the cum-dividend market value of equity, as shown in Section 2.5 later.) Finally, the government balances its budget, meaning that $T_t = U_t b$.

The household's first order condition with respect to the fraction of wealth invested in the risky asset, χ_t, gives rise to the fundamental equation of asset pricing:

$$1 = E_t[M_{t+1}R_{t+1}]. \tag{18}$$

In particular, the stochastic discount factor, M_{t+1}, is given by:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{J_{t+1}}{E_t[J_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}. \tag{19}$$

Finally, the risk-free rate is given by $R^f_{t+1} = 1/E_t[M_{t+1}]$.

2.4 Wage Determination

The wage rate is determined endogenously by applying the sharing rule per the outcome of a generalized Nash bargaining process between the employed workers and the firm. Let $0 < \eta < 1$ be the workers' relative bargaining weight. The wage rate is given by (see Appendix B for details):

$$W_t = \eta(X_t + \kappa\theta_t) + (1-\eta)b. \tag{20}$$

The wage rate is increasing in labor productivity, X_t, and in labor market tightness, θ_t. Intuitively, the more productive the workers are, and the fewer unemployed workers seeking jobs relative to the number of vacancies, the higher the wage rate will be for the employed workers. Also, the value of unemployment activities, b, and the workers' bargaining weight, η, affect the elasticity of wage with respect to productivity. The lower η is, and the higher b is, the more the wage will be tied with the constant unemployment value, inducing a lower wage elasticity to productivity.

2.5 Competitive Equilibrium

In equilibrium, the financial markets clear. The risk-free asset is in zero net supply, and the household holds all the shares of the representative firm, $\chi_t = 1$. As such, the return on wealth equals the return on the firm, $R^{\Pi}_{t+1} = R_{t+1}$, and the household's wealth equals the cum-dividend equity value of the firm, $\Pi_t = S_t$. The goods market clearing condition implies the aggregate resource constraint:

$$C_t + \kappa V_t = X_t N_t. \tag{21}$$

The competitive equilibrium in the search economy consists of vacancy posting, $V_t^\star \geq 0$; multiplier, $\lambda_t^\star \geq 0$; consumption, $C_t^\star$; and indirect utility, $J_t^\star$; such that (i) $V_t^\star$ and $\lambda_t^\star$ satisfy the intertemporal job creation condition (12) and the Kuhn-Tucker conditions (13) and (14), while taking the stochastic discount factor in equation (19) and the wage equation (20) as given; (ii) $C_t^\star$ and $J_t^\star$ satisfy the intertemporal consumption-portfolio choice condition (18), in which the stock return is given by equation (15); and (iii) the goods market clears as in equation (21).

3 Calibration, Computation, and the Model's Solution

We calibrate the model in Section 3.1, discuss our nonlinear solution methods in Section 3.2, and describe the basic properties of the model's solution in Section 3.3.

3.1 Calibration

We calibrate the model in monthly frequency. Table 1 lists the parameter values in the benchmark calibration. Following Gertler and Trigari (2009), we set the time discount factor, β, to be $0.99^{1/3}$, and the persistence of the (log) aggregate productivity, ρ, to be $0.95^{1/3} = 0.983$. We choose the conditional volatility of the aggregate productivity, σ, to be 0.0077 to target the standard deviation of consumption growth in the data. Following Bansal and Yaron (2004), we set the relative risk aversion, γ, to be 10, and the elasticity of intertemporal substitution, ψ, to be 1.5.

Den Haan, Ramey, and Watson (2000) estimate the average monthly job filling rate to be $\bar{q} = 0.71$ and the average monthly job finding rate to be $\bar{f} = 0.45$ in the United States. The constant returns to scale property of the matching function implies that the long-run average labor market tightness is roughly $\bar{\theta} = \bar{f}/\bar{q} = 0.634$. This estimate helps pin down the elasticity parameter in the matching function, ι. Specifically, evaluating equation (3) at the long run average yields

Table 1 : Parameter Values in the Benchmark Calibration at the Monthly Frequency

Notation	Parameter	Value
β	Time discount factor	$0.99^{1/3}$
γ	Relative risk aversion	10
ψ	The elasticity of intertemporal substitution	1.5
ρ	Aggregate productivity persistence	0.983
σ	Conditional volatility of productivity shocks	0.0077
η	Workers' bargaining weight	0.052
b	The value of unemployment activities	0.85
s	Job separation rate	0.05
ι	Elasticity of the matching function	1.290
κ	The unit cost of vacancy posting	0.975

$0.71 = (1 + 0.634^{\iota})^{-1/\iota}$, or $\iota = 1.29$, which is close to Den Haan et al.'s parameter value.

The average rate of unemployment for the United States over the 1920–2009 period is roughly 7%. However, important flows in and out of nonparticipation in the labor force as well as discouraged workers not accounted for in the pool of individuals seeking employment suggest that the unemployment rate should be calibrated somewhat higher. As such, we adopt the target average unemployment rate of $\bar{U} = 10\%$, which lies within the range between 7% in Gertler and Trigari (2009) and 12% in Krause and Lubik (2007). This target pins down the monthly job separation rate, s. In particular, the steady state labor market flows condition, $s(1 - \bar{U}) = \bar{f}\bar{U}$, sets $s = 0.05$. This value of s, which is also used in Andolfatto (1996), is close to the estimate of 0.053 from Clark (1990). This value is also within the range of estimates from Davis, Faberman, and Haltiwanger (2006).

The calibration of the value of unemployment activities, b, is controversial. On the one hand, Shimer (2005) pins down $b = 0.4$ by assuming that the only benefit for an unemployed worker is the government unemployment insurance. Several subsequent studies such as Hall (2005) and Gertler and Trigari (2009) adopt such a conservative value for b, while exploring alternative wage specifications that allow more stickiness in adjusting to labor productivity than the equilibrium wage from the generalized Nash bargaining process.

On the other hand, Hagedorn and Manovskii (2008) argue that in a perfect competitive labor market, b should equal the value of employment. In particular, the value of unemployment activities measures not only unemployment insurance, but also the total value of leisure, home production, and self-employment. The long-run average marginal product of labor in the model is unity, to which b should be close. Hagedorn and Manovskii choose a high value of 0.955 for b.

We choose a value of 0.85 for b. This calibration is not as extreme as 0.955 in Hagedorn and Manovskii (2008). For the workers' bargaining weight, η, we set it to be 0.052 as in Hagedorn and Manovskii. With all the other parameters calibrated, we pin down the vacancy cost parameter, $\kappa = 0.975$, so that the average unemployment rate in the simulated economy is close to the target of 10%.

3.2 Computation

Although analytically transparent, solving the model numerically is challenging, for several reasons. First, the search economy is not Pareto optimal because the competitive equilibrium does not correspond to the social planner's solution. The firm in the decentralized economy does not take into account the congestion effect of posting a new vacancy on the labor market when maximizing the equity value, whereas the social planner does so when maximizing social welfare. As such, we must solve for the competitive equilibrium directly. Second, because of the occasionally binding constraint (9), standard perturbation methods cannot be used. As such, we solve for the competitive equilibrium using a nonlinear projection algorithm, while applying the Christiano and Fisher (2000) idea of parameterized expectations to handle the nonnegativity constraint.

Third, because of the nonlinearity in the model and our focus on nonlinearity-sensitive asset pricing moments, we solve the model on a large number of grid points to ensure accuracy. Also, we apply the idea of homotopy to visit the parameter space in which the model exhibits strong nonlinearity. Because of the nonlinearity in many economically interesting parameterizations, we can only update the parameter values very slowly to ensure the convergence of the nonlinear solution algorithm.

Specifically, the state space of the model consists of employment and productivity, (N_t, x_t).

The goal is to solve for the optimal vacancy function: $V_t^\star = V(N_t, x_t)$, the multiplier function: $\lambda_t^\star = \lambda(N_t, x_t)$, and an indirect utility function: $J_t^\star = J(N_t, x_t)$ from two functional equations:

$$\begin{aligned} J(N_t, x_t) &= \left[(1-\beta)C(N_t, x_t)^{1-\frac{1}{\psi}} + \beta\left(E_t\left[J(N_{t+1}, x_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} & (22) \\ \frac{\kappa}{q(\theta_t)} - \lambda(N_t, x_t) &= E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left(\frac{\kappa}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1})\right)\right]\right]. & (23) \end{aligned}$$

$V(N_t, x_t)$ and $\lambda(N_t, x_t)$ must also satisfy the Kuhn-Tucker condition: $V_t \geq 0, \lambda_t \geq 0$, and $\lambda_t V_t = 0$.

The traditional projection method would approximate $V(N_t, x_t)$ and $\lambda(N_t, x_t)$ to solve the two functional equations, while obeying the Kuhn-Tucker condition. As pointed out by Christiano and Fisher (2000), with the occasionally binding constraint, this approach is tricky and cumbersome. Instead, we adapt their method of parameterized expectations by approximating:

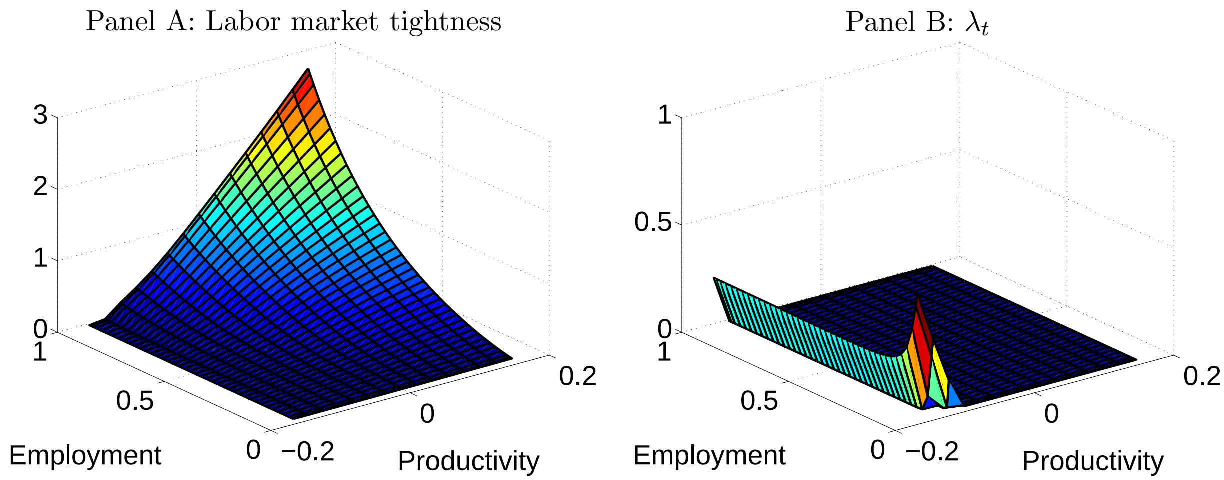
$$\mathcal{E}_t \equiv \mathcal{E}(N_t, x_t) = E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left(\frac{\kappa}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1})\right)\right]\right]. \quad (24)$$

We exploit a convenient mapping from the conditional expectation function to policy and multiplier functions, so as to eliminate the need to parameterize the multiplier function separately. In particular, after obtaining the parameterized $\mathcal{E}_t$, we first calculate:

$$\tilde{q}(\theta_t) = \frac{\kappa}{\mathcal{E}_t}. \quad (25)$$

If $\tilde{q}(\theta_t) < 1$, the nonnegativity constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\kappa/\mathcal{E}_t)$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\cdot)$ defined in equation (3), and $V_t = \theta_t(1 - N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the nonnegativity constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa - \mathcal{E}_t$. This approach is convenient in practice because it enforces the Kuhn-Tucker conditions automatically, thereby eliminating the need of parameterizing the multiplier function. (Appendix C contains additional details of our numerical implementation.)

Figure 1 : Labor Market Tightness and the Multiplier of the Occasionally Binding Constraint on Vacancy (λ_t)



3.3 Basic Properties of the Model's Solution

The business cycle dynamics of endogenous quantities, from the labor market variables to aggregate consumption, and by extension most asset pricing implications, are driven by the equilibrium value of labor market tightness. Panel A of Figure 1 plots the ratio of optimal job vacancies to unemployment against aggregate employment and labor productivity. We see that labor market tightness is increasing in both states. The labor market is tighter from the perspective of the firm when there are fewer unemployed workers searching for jobs (employment is high), and when the demand for workers is high (productivity is high). More generally, increases in productivity lead firms to post more vacancies to create more jobs. The rise in employment and wages, and therefore household income, increases aggregate consumption. The magnitude and persistence of these changes all depend on the elasticity of labor market tightness to changes in productivity.

From Panel B, we see that the multiplier of the occasionally binding constraint on vacancy, λ_t, is countercyclical: λ_t equals zero for most values of productivity, but turns positive as productivity approaches its lowest level. The multiplier is also convex in employment: λ_t is flat across most values of employment, but rises with an increasing speed as it approaches the lowest level. Intuitively, when the constraint is binding, $V_t = 0, \theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa - \mathcal{E}_t$, in which $\mathcal{E}_t$ is the expectation in equation (24). As the economy approaches the low-employment-low-productivity states, $\mathcal{E}_t$ drops precipitously. This nonlinearity is a natural result of the stochastic discount factor, M_{t+1}. As consumption approaches zero, marginal utility blows up, causing $\mathcal{E}_t$ to fall and λ_t to rise rapidly.

4 Quantitative Results

We present basic business cycle and asset pricing moments in Section 4.1 and labor market moments in Section 4.2. In Section 4.3, we examine the linkage between the labor market and the financial market by using labor market tightness to forecast stock market excess returns. In Section 4.4, we quantify the model's endogenous rare disaster risks that are important for asset prices. We study the model's implications for long run risks and uncertainty shocks in Section 4.5, and cyclical dividend dynamics in Section 4.6. Finally, Section 4.7 reports several comparative static experiments.

4.1 Basic Business Cycle and Financial Moments

Panel A of Table 2 reports the standard deviation and autocorrelations of (log) consumption growth and (log) output growth, as well as unconditional financial moments in the data. Consumption is annual real personal consumption expenditures, and output is annual real gross domestic product from 1929 to 2010 from the National Income and Product Accounts (NIPA) at Bureau of Economic Analysis. The annual consumption growth in the data has a volatility of 3.04%, and a first-order autocorrelation of 0.38. The autocorrelation drops to 0.08 at the two-year horizon, and turns negative, −0.21, at the three-year horizon. The annual output growth has a volatility of 4.93% and a high first-order autocorrelation of 0.54. The autocorrelation drops to 0.18 at the two-year horizon, and turns negative afterward: −0.18 at the three-year horizon and −0.23 at the five-year horizon.

We obtain monthly series of the value-weighted market returns including all NYSE, Amex, and Nasdaq stocks, one-month Treasury bill rates, and inflation rates (the rates of change in Consumer Price Index) from Center for Research in Security Prices (CRSP). The sample is from January 1926 to December 2010 (1,020 months). The mean of real interest rates (one-month Treasury bill rates

Table 2 : Basic Business Cycle and Financial Moments

In Panel A, consumption is annual real personal consumption expenditures (series PCECCA), and output is annual real gross domestic product (series GDPCA) from 1929 to 2010 (82 annual observations) from NIPA (Table 1.1.6) at Bureau of Economic Analysis. σ^C is the volatility of log consumption growth, and σ^Y is the volatility of log output growth. Both volatilities are in percent. $\rho^C(\tau)$ and $\rho^Y(\tau)$, for $\tau = 1, 2, 3$, and 5, are the τ-th order autocorrelations of log consumption growth and log output growth, respectively. We obtain monthly series from January 1926 to December 2010 (1,020 monthly observations) for the value-weighted market index returns including dividends, one-month Treasury bill rates, and the rates of change in Consumer Price Index (inflation rates) from CRSP. $E[R - R^f]$ is the average (in annualized percent) of the value-weighted market returns in excess of the one-month Treasury bill rates, adjusted for the long-term market leverage rate of 0.32 reported by Frank and Goyal (2008). (The leverage-adjusted average $E[R - R^f]$ is the unadjusted average times 0.68.) $E[R^f]$ and σ^{R^f} are the mean and volatility, both of which are in annualized percent, of real interest rates, defined as the one-month Treasury bill rates in excess of the inflation rates. σ^R is the volatility (in annualized percent) of the leverage-weighted average of the value-weighted market returns in excess of the inflation rates and the real interest rates. In Panel B, we simulate 1,000 artificial samples, each of which has 1,020 monthly observations, from the model in Section 2. In each artificial sample, we calculate the mean market excess return, $E[R - R^f]$, the volatility of the market return, σ^R, as well as the mean, $E[R^f]$, and volatility, σ^{R^f}, of the real interest rate. All these moments are in annualized percent. We time-aggregate the first 984 monthly observations of consumption and output into 82 annual observations in each sample, and calculate the annual volatilities and autocorrelations of log consumption growth and log output growth. We report the mean and the 5 and 95 percentiles across the 1,000 simulations. The p-values are the percentages with which a given model moment is larger than its data moment.

	Panel A: Data	Panel B: Model			
		Mean	5%	95%	p-value
σ^C	3.036	3.596	1.975	7.143	0.465
$\rho^C(1)$	0.383	0.182	−0.046	0.465	0.089
$\rho^C(2)$	0.081	−0.145	−0.346	0.091	0.055
$\rho^C(3)$	−0.206	−0.126	−0.341	0.112	0.717
$\rho^C(5)$	0.062	−0.071	−0.279	0.139	0.151
σ^Y	4.933	4.147	2.559	7.421	0.171
$\rho^Y(1)$	0.543	0.182	−0.032	0.450	0.019
$\rho^Y(2)$	0.178	−0.137	−0.325	0.086	0.018
$\rho^Y(3)$	−0.179	−0.121	−0.326	0.114	0.674
$\rho^Y(5)$	−0.227	−0.071	−0.284	0.133	0.891
$E[R - R^f]$	5.066	4.538	3.765	5.292	0.124
$E[R^f]$	0.588	4.113	3.684	4.424	1.000
σ^R	12.942	11.074	10.059	12.085	0.001
σ^{R^f}	1.872	1.337	0.832	2.165	0.100

minus inflation rates) is 0.59% per annum, and the annualized volatility is 1.87%.

The equity premium (the average of the value-weighted market returns in excess of one-month Treasury bill rates) in the 1926–2010 sample is 7.45% per annum. Because we do not model financial leverage, we adjust the equity premium in the data for leverage before matching with the equity premium implied from the model. Frank and Goyal (2008) report that the aggregate market leverage ratio of U.S. corporations is fairly stable around 0.32. As such, we calculate the leverage-adjusted equity premium as $(1 - 0.32) \times 7.45\% = 5.07\%$ per annum. The annualized volatility of the market returns in excess of inflation rates is 18.95%. Adjusting for leverage (taking the leverage-weighted average of real market returns and real interest rates) yields an annualized volatility of 12.94%.

Panel B of Table 2 reports the model moments. From the initial condition of zero for log productivity, x_t, and 0.90 for employment, N_t, we first simulate the economy for 6,000 monthly periods to reach its stationary distribution. We then repeatedly simulate 1,000 artificial samples, each with 1,020 months. On each artificial sample, we calculate the annualized monthly averages of the equity premium and the real interest rate, as well as the annualized monthly volatilities of the market returns and the real interest rate. We also take the first 984 monthly observations of consumption and output, and time-aggregate them into 82 annual observations. (We add up 12 monthly observations within a given year, and treat the sum as the year's annual observation.) For each data moment, we report the average as well as the 5 and 95 percentiles across the 1,000 simulations. The p-values are the frequencies with which a given model moment is larger than its data counterpart.

The model predicts a consumption growth volatility of 3.60% per annum, which is somewhat higher than 3.04% in the data. However, this data moment lies within the 90% confidence interval of the model's bootstrapped distribution with a bootstrapped p-value of 0.47. The model also implies a positive first-order autocorrelation of 0.18 for consumption growth, but is lower than 0.38 in the data. At longer horizons, consumption growth in the model are all negatively autocorrelated. All the autocorrelations in the data are within 90% confidence interval of the model.

The output growth volatility implied by the model is 4.15% per annum, which is somewhat lower than 4.93% in the data. The model implies a positive first-order autocorrelation of 0.18 for the output growth, and is lower than 0.54 in the data. The model also implies a negative second-order autocorrelation of −0.14, but this autocorrelation is 0.18 in the data. Both correlations in the data are outside the 90% confidence interval of the model's bootstrapped distribution. However, at longer horizons, the autocorrelations are negative in the model, consistent with the data.

The model's performance in matching financial moments seems fair. The equity premium in the model is 4.54% per annum, which is not too far from the leverage-adjusted equity premium of 5.07% in the data. In particular, this data moment lies within the 90% confidence interval of the model's bootstrapped distribution with a p-value of 0.12. The volatility of the stock market return in the model is 11.07% per annum, which is close to the leverage-adjusted market volatility of 12.94% in the data. The volatility of the interest rate in the model is 1.34%, which is close to 1.87% in the data. However, the model implies an average interest rate of 4.11% per annum, which is too high relative to 0.59% in the data. On balance, however, the model's fit seems like progress in the asset pricing literature with production (see the references discussed in Section 1).

4.2 Labor Market Moments

To evaluate the model's fit for labor market moments, we first document these moments per Hagedorn and Manovskii (2008, Table 3) using an updated sample (see also Shimer (2005)). We obtain seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from the Bureau of Labor Statistics (BLS), and seasonally adjusted help wanted advertising index from the Conference Board. The sample is from January 1951 to June 2006.[1] We take quarterly averages of the monthly series to obtain 222 quarterly observations. The average labor productivity is seasonally adjusted real average output per person in the nonfarm business sector from BLS.

[1]The sample ends in June 2006 because the Conference Board switched from help wanted advertising index to help wanted online index around that time. The two indexes are not directly comparable. As such, we follow the standard practice in the labor search literature in using the longer time series before the switch.

Hagedorn and Manovskii (2008) report all variables in log deviations from the Hodrick-Prescott (1997, HP) trend with a smoothing parameter of 1,600. In contrast, we detrend all variables as the HP-filtered cyclical component of proportional deviations from the mean (with the same smoothing parameter).[2] We do not take logs because vacancies can be zero in the model's simulations when the nonnegativity constraint on vacancy is binding. In the data, the two detrending methods yield quantitatively similar results, which are in turn close to Hagedorn and Manovskii's. In particular, from Panel A of Table 3, the standard deviation of the V/U ratio is 0.26. The ratio is procyclical with a positive correlation of 0.30 with labor productivity. Finally, vacancy and unemployment have a negative correlation of -0.91, indicating a downward-sloping Beveridge curve.

To evaluate the model's fit with the labor market moments, we simulate 1,000 artificial samples, each with 666 months. We take the quarterly averages of the monthly unemployment, U, vacancy, V, and labor productivity, X, to obtain 222 quarterly observations for each series. We then apply the exactly same procedures as in Panel A of Table 3 on the artificial data, and report the cross-simulation averages (as well as standard deviations) for the model moments.

Panel B reports the model's quantitative performance. The standard deviations of U and V in the model are 0.14 and 0.10, which are close to those in the data. The model implies a standard deviation of 0.16 for the V/U ratio, which is lower than 0.26 in the data. The bootstrapped standard deviation of this model moment is 0.03. As such, the data moment is more than three standard deviations away from the model moment. This result is consistent with the Shimer (2005) critique that the standard search model fails to explain the volatility of labor market tightness in the data.

The model also generates a Beveridge curve with a negative U-V correlation of -0.57. However, its magnitude is lower than -0.91 in the data. The correlation between the V/U ratio and labor productivity is 0.99 in the model, which is higher than 0.30 in the data. In addition, the model implies an average unemployment rate of 9.23% in simulations. (Because of the model's nonlinear-

[2]Specifically, for any variable Z, the HP-filtered cyclical component of proportional deviations from the mean is calculated as $(Z-\bar{Z})/\bar{Z}-\mathrm{HP}[(Z-\bar{Z})/\bar{Z}]$, in which $\bar{Z}$ is the mean of Z, and $\mathrm{HP}[(Z-\bar{Z})/\bar{Z}]$ is the HP trend of $(Z-\bar{Z})/\bar{Z}$.

Table 3 : Labor Market Moments

In Panel A, seasonally adjusted monthly unemployment (U, thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics. The seasonally adjusted help wanted advertising index, V, is from the Conference Board. The series are monthly from January 1951 to June 2006 (666 months). Both U and V are converted to 222 quarterly averages of monthly series. The average labor productivity, X, is seasonally adjusted real average output per person in the nonfarm business sector from the Bureau of Labor Statistics. All variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. In Panel B, we simulate 1,000 artificial samples, each of which has 666 monthly observations. We take the quarterly averages of monthly U, V, and X to convert to 222 quarterly observations. We implement the exactly same empirical procedures as in Panel A on these quarterly series, and report the cross-simulation averages and standard deviations (in parentheses) for all the model moments.

	U	V	V/U	X	
		Panel A: Data			
Standard deviation	0.119	0.134	0.255	0.012	
Quarterly autocorrelation	0.902	0.922	0.889	0.761	
Correlation matrix		−0.913	−0.801	−0.224	U
			0.865	0.388	V
				0.299	V/U
		Panel B: Model			
Standard deviation	0.140	0.098	0.155	0.016	
	(0.069)	(0.019)	(0.030)	(0.002)	
Quarterly autocorrelation	0.849	0.647	0.791	0.773	
	(0.051)	(0.061)	(0.038)	(0.040)	
Correlation matrix		−0.571	−0.696	−0.662	U
		(0.109)	(0.131)	(0.141)	
			0.920	0.948	V
			(0.049)	(0.025)	
				0.994	V/U
				(0.015)	

ity, the long-run average unemployment rate in simulations is somewhat lower than the calibration target of 10%, which holds only roughly based on deterministic steady state relations.)

4.3 The Linkage between the Labor Market and the Financial Market

A large literature in financial economics shows that the equity risk premium is time-varying (and countercyclical) in the data (e.g., Lettau and Ludvigson (2001)). In the labor market, as vacancy is procyclical and unemployment is countercyclical, the V/U ratio is strongly procyclical (e.g., Shimer (2005)). As such, the V/U ratio should forecast stock market excess returns with a negative slope at business cycle frequencies. Panel A of Table 4 documents such predictability in the data.

Table 4 : Long-Horizon Regressions of Market Excess Returns on Labor Market Tightness

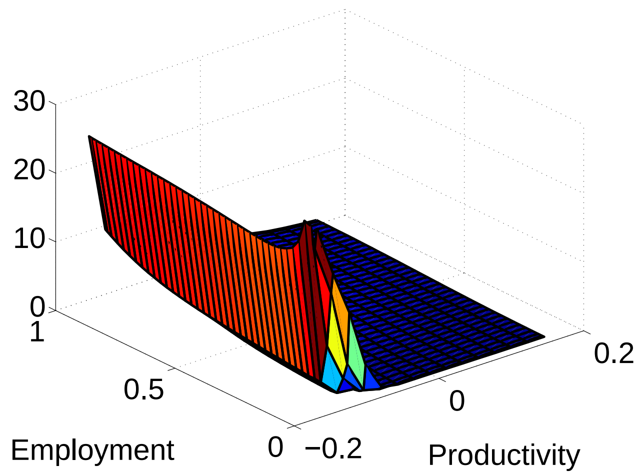
Panel A reports long-horizon regressions of log excess returns on the value-weighted market index from CRSP, $\sum_{h=1}^{H} R_{t+3+h} - R_{t+3+h}^f$, in which H is the forecast horizon in months. The regressors are two-month lagged values of the V/U ratio. We report the ordinary least squares estimate of the slopes (Slope), the Newey-West corrected t-statistics (t_{NW}), and the adjusted R^2s in percent. The seasonally adjusted monthly unemployment (U, thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics, and the seasonally adjusted help wanted advertising index (V) is from the Conference Board. The sample is from January 1951 to June 2006 (666 monthly observations). We multiply the V/U series by 50 so that its average is close to that in the model. In Panel B, we simulate 1,000 artificial samples, each of which has 666 monthly observations. On each artificial sample, we implement the exactly same empirical procedures as in Panel A, and report the cross-simulation averages and standard deviations (in parentheses) for all the model moments.

	Forecast horizon (H) in months					
	1	3	6	12	24	36
	Panel A: Data					
Slope	−1.425	−4.203	−7.298	−10.312	−9.015	−10.156
t_{NW}	−2.575	−2.552	−2.264	−1.704	−0.970	−0.861
Adjusted R^2	0.950	2.598	3.782	3.672	1.533	1.405
	Panel B: Model					
Slope	−0.807	−2.378	−4.610	−8.657	−15.378	−20.707
	(0.428)	(1.244)	(2.379)	(4.416)	(7.573)	(9.799)
t_{NW}	−2.228	−2.341	−2.475	−2.776	−3.504	−4.138
	(0.800)	(0.852)	(0.925)	(1.111)	(1.508)	(1.846)
Adjusted R^2	0.692	2.035	3.915	7.323	13.000	17.568
	(0.463)	(1.321)	(2.489)	(4.596)	(7.890)	(10.331)

Specifically, we perform monthly long-horizon regressions of log excess returns on the CRSP value-weighted market returns, $\sum_{h=1}^{H} R_{t+3+h} - R_{t+3+h}^f$, in which $H = 1, 3, 6, 12, 24$, and 36 is the forecast horizon in months. When $H > 1$, we use overlapping monthly observations of H-period holding returns. The regressors are *two*-month lagged values of the V/U ratio. The BLS takes less than one week to release monthly employment and unemployment data, and the Conference Board takes about one month to release monthly help wanted advertising index data.[3] We impose the two-month lag between the V/U ratio and market excess returns to guard against look-ahead bias in predictive regressions. To make the regression slopes comparable to those in the model, we also scale up the V/U series in the data by a factor of 50 to make its average close to that in the model. This scaling is necessary because the vacancy and unemployment series in the data have different units.

[3]We verify this practice through a private correspondence with the Conference Board staff.

Figure 2 : The Equity Risk Premium in Annual Percent, $E_t[R_{t+1} - R^f_{t+1}]$



Panel A of Table 4 shows that the V/U ratio is a reliable forecaster of market excess returns. At the one-month horizon, the slope is -1.43, which is more than 2.5 standard errors from zero. (The standard errors are adjusted for heteroscedasticity and autocorrelations of 12 lags per Newey and West (1987)). The adjusted R^2 is close to 1%. The slopes are significant at the three-month and six-month horizons but insignificant afterward. The adjusted R^2s peak at 3.78% at the six-month horizon, and decline to 3.67% at the one-year horizon and further to 1.41% at the three-year horizon.

To see how the model can explain this predictability, we plot in Figure 2 the equity risk premium in annual percent on the state space of employment and productivity. We observe that the risk premium is strongly countercyclical: it is low in good times when both employment and productivity are high, but high in bad times when both employment and productivity are low. As shown in Panel A of Figure 1, the labor market tightness, V_t/U_t, is strongly procyclical: it is low in bad times (the high-unemployment-low-productivity states), but high in good times (the low-unemployment-high-productivity states). The joint cyclical properties of the equity premium and the V/U ratio imply that V/U should forecast market excess returns with a negative slope in the model.

Panel B of Table 4 reports the model's quantitative fit for the predictive regressions. Consistent with the data, the model predicts that the V/U ratio forecasts market excess returns with a negative slope. In particular, at the one-month horizon, the predictive slope is -0.81 with a t-statistic of -2.23. At the six-month horizon, the slope is -4.61 with a t-statistic of -2.38. However, the model implies stronger predictive power for the V/U ratio than that in the data. Both the t-statistic of the slope and the adjusted R^2 increase monotonically with the forecast horizon. In contrast, both statistics peak at the six-month horizon but decline afterward in the data.

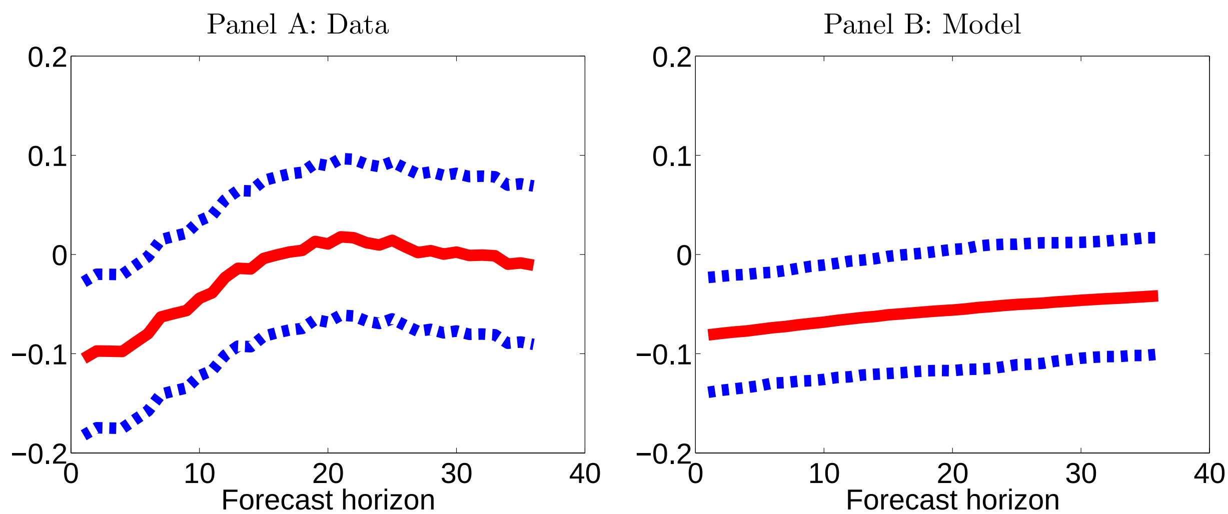
Panel A of Figure 3 plots the cross-correlations and their two standard-error bounds between the V/U ratio, V_t/U_t, and future market excess returns, $R_{t+H} - R^f_{t+H}$, for $H = 1, 2, \ldots, 36$ months in the data. No overlapping observations are used. The panel shows that the correlations are significantly negative for forecast horizons up to six months, consistent with the predictive regressions in Table 4. Panel B reports the cross-correlations and their two cross-simulation standard-deviation bounds from the model's bootstrapped distribution. Consistent with the data, the model predicts significantly negative cross-correlations between V_t/U_t and future market excess returns for short horizons. However, although the cross-correlations are insignificant at long horizons, the correlations decay more slowly than those in the data, consistent with Panel B of Table 4.

4.4 Endogenous Rare Disasters

The search economy gives rise endogenously to rare disaster risks à la Rietz (1988) and Barro (2006). We simulate 1,006,000 monthly periods from the model, discard the first 6,000 periods, and treat the remaining one million months as the model's stationary distribution. Figure 4 reports the empirical cumulative distribution functions for unemployment, output, consumption, and dividend. From Panel A, unemployment is positively skewed with a long right tail. As the population moments, the mean unemployment rate is 9.23%, the median is 8.05%, and the skewness is 7.46. The 2.5 percentile of unemployment is close to the median, 6.34%, whereas the 97.5 percentile is far away, 19.97%. As a mirror image, the employment rate is negatively skewed with a long left tail. As a result, output, consumption, and dividend all show infrequent but deep disasters (Panels B–D). With small probabilities, the economy falls off the cliff in simulations.

Figure 3 : Cross-Correlations between the V/U Ratio and Future Market Excess Returns

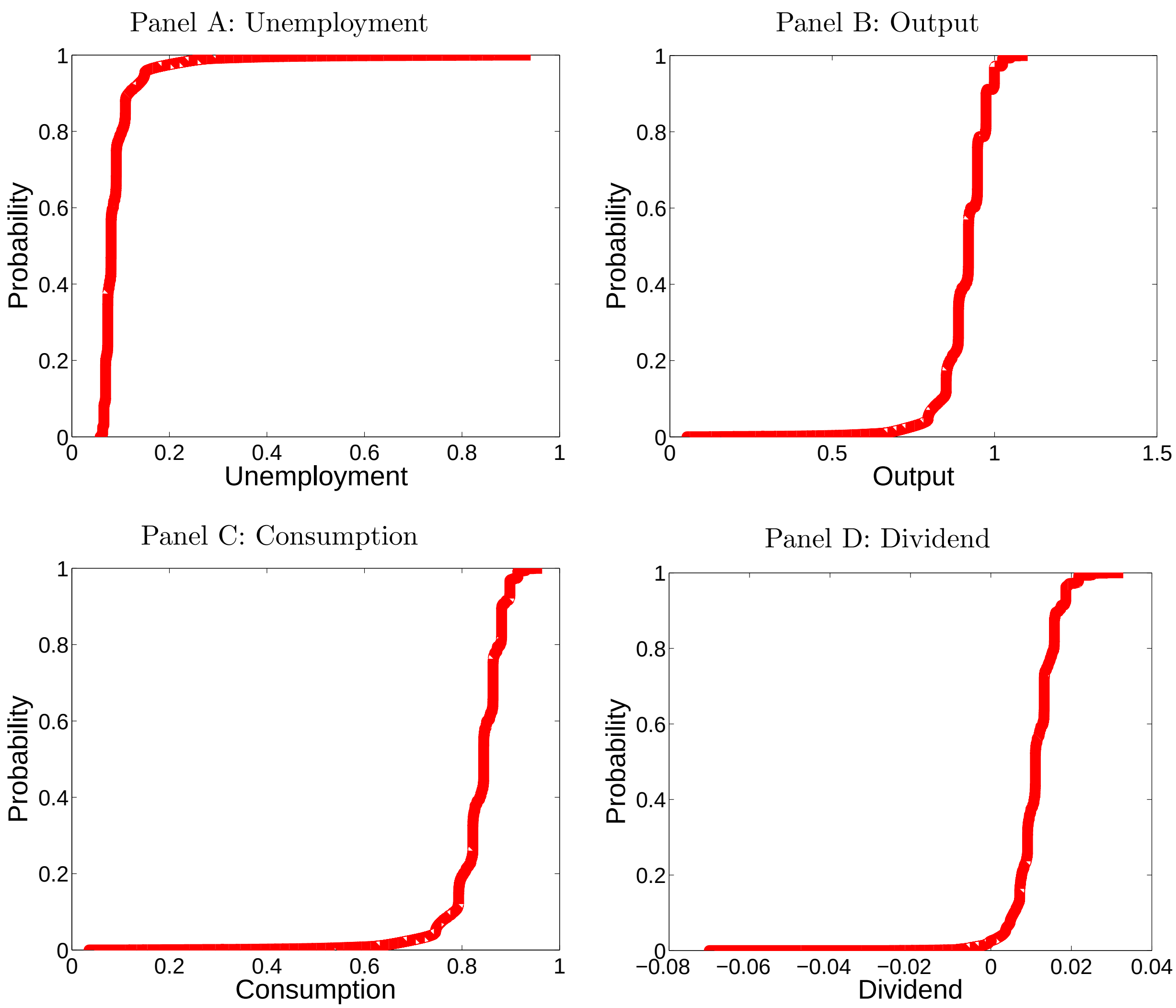
We report the cross-correlations (in red) between labor market tightness, V_t/U_t, and future market excess returns, $R_{t+H} - R^f_{t+H}$, in which $H = 1, 2, \ldots, 36$ is the forecast horizon in months, as well as their two standard-error bounds (in blue broken lines). In Panel A, V_t is the seasonally adjusted help wanted advertising index from the Conference Board, and U_t is the seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from the BLS. The sample is from January 1951 to June 2006. The market excess returns are the CRSP value-weighted market returns in excess of one-month Treasury bill rates. In Panel B, we simulate 1,000 artificial samples, each with 666 monthly observations. On each artificial sample, we calculate the cross-correlations between V_t/U_t and $R_{t+H} - R^f_{t+H}$, and plot the cross-simulation averaged correlations (in red) and their two cross-simulation standard-deviation bounds (in blue broken lines).



The disasters in macroeconomic quantities reflect in asset prices as rare upward spikes in the equity risk premium, $E_t[R_{t+1} - R^f_{t+1}]$. From Figure 5, its stationary distribution is positively skewed with a long right tail. The mean risk premium is 5.02% per annum, and its 2.5 and 97.5 percentiles are 1.63% and 8.93%, respectively. However, with small probabilities, the risk premium can reach very high levels: the maximum equity premium is close to 23% in simulations.

Do macroeconomic disasters arising endogenously from the model resemble those in the data? Barro and Ursúa (2008) apply a peak-to-trough method on samples from 1870 to 2006 to identify economic crises, defined as cumulative fractional declines in per capita consumption or GDP of at least 10%. Suppose there are two states, normalcy and disaster. The disaster probability measures the likelihood with which the economy shifts from normalcy to disaster in a given year. The number

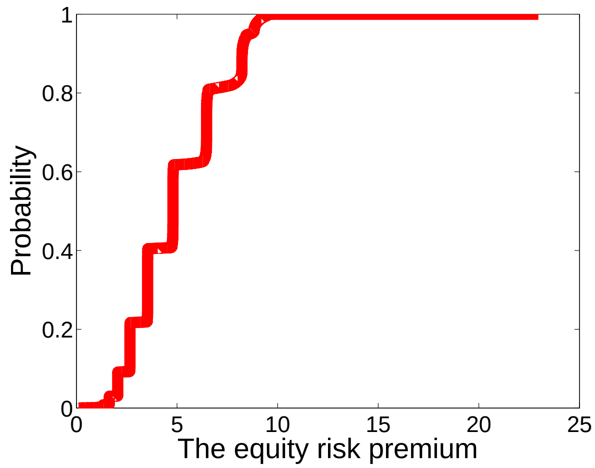
Figure 4 : Empirical Stationary Distribution of the Model: Unemployment, Output, Consumption, and Dividend



of disaster years is defined as the number of years in the interval between peak and trough for each disaster event. The number of normalcy years is the total number of years in the sample minus the number of disaster years. The disaster probability is the ratio of the number of disasters over the number of normalcy years. Barro and Ursúa estimate the disaster probability to be 3.63%, the average size 22%, and the average duration 3.6 years for consumption disasters. For GDP disasters, the disaster probability is 3.69%, the average size 21%, and the average duration 3.5 years.

To quantify the magnitude of the disasters in the model, we first simulate the economy for 6,000 monthly periods to reach the stationary distribution. We then repeatedly simulate 1,000 artificial

Figure 5 : Empirical Stationary Distribution of the Model: The Equity Risk Premium



samples, each with 1,644 months (137 years). The sample size matches the average sample size in Barro and Ursúa (2008). On each artificial sample, we time-aggregate the monthly observations of consumption and output into annual observations. We apply Barro and Ursúa's measurement on each artificial sample, and report the cross-simulation averages and the 5 and 95 percentiles for the disaster probability, size, and duration for both consumption and GDP (output) disasters.

Table 5 reports the detailed results. From Panel A (consumption disasters), the disaster probability and the average disaster size in the model are close to those in the data, but the average duration is somewhat higher in the model. The disaster probability is 3.22%, which is close to 3.63% in the data. The average size of the disasters in the model is 19.7%, which is close to 22% in the data. The average duration is 4.81 years, which is longer than 3.6 years in the data. However, the cross-simulation standard deviation of the average duration is 1.21, meaning that the data duration is about one standard deviation from the model's estimate.

From Panel B of Table 5, the average size of GDP disasters in the model, 18.84%, is close to that in the data, 21%. However, the disaster probability of 4.94% is higher than 3.69% in the data. The cross-simulation standard deviation of the GDP disaster probability is 2.01%, meaning that the probability in the data is within one standard deviation from the model's estimate. In addition, the average duration of the GDP disasters in the model is 4.47 years, which is longer than 3.5 years

Table 5 : Moments of Macroeconomic Disasters

The data moments are from Barro and Ursúa (2008). The model moments are from 1,000 simulations, each with 1,644 monthly observations. We time-aggregate these monthly observations of consumption and output into 137 annual observations. On each artificial sample, we apply Barro and Ursúa's peak-to-trough method to identify economic crises, defined as cumulative fractional declines in per capita consumption or GDP of at least 10%. We report the mean and the 5 and 95 percentiles across the 1,000 simulations. The p-values are the percentages with which a given model moment is higher than its data moment. The disaster probabilities and average size are all in percent, and the average duration is in terms of years.

	Data	Model			
		Mean	5%	95%	p-value
Panel A: Consumption disasters					
Probability	3.63	3.22	0.76	6.42	0.33
Average size	22	19.70	11.44	37.45	0.23
Average duration	3.6	4.81	3.00	7.00	0.84
Panel B: GDP disasters					
Probability	3.69	4.94	1.61	8.59	0.32
Average size	21	18.84	12.42	32.63	0.27
Average duration	3.5	4.47	3.25	6.00	0.86

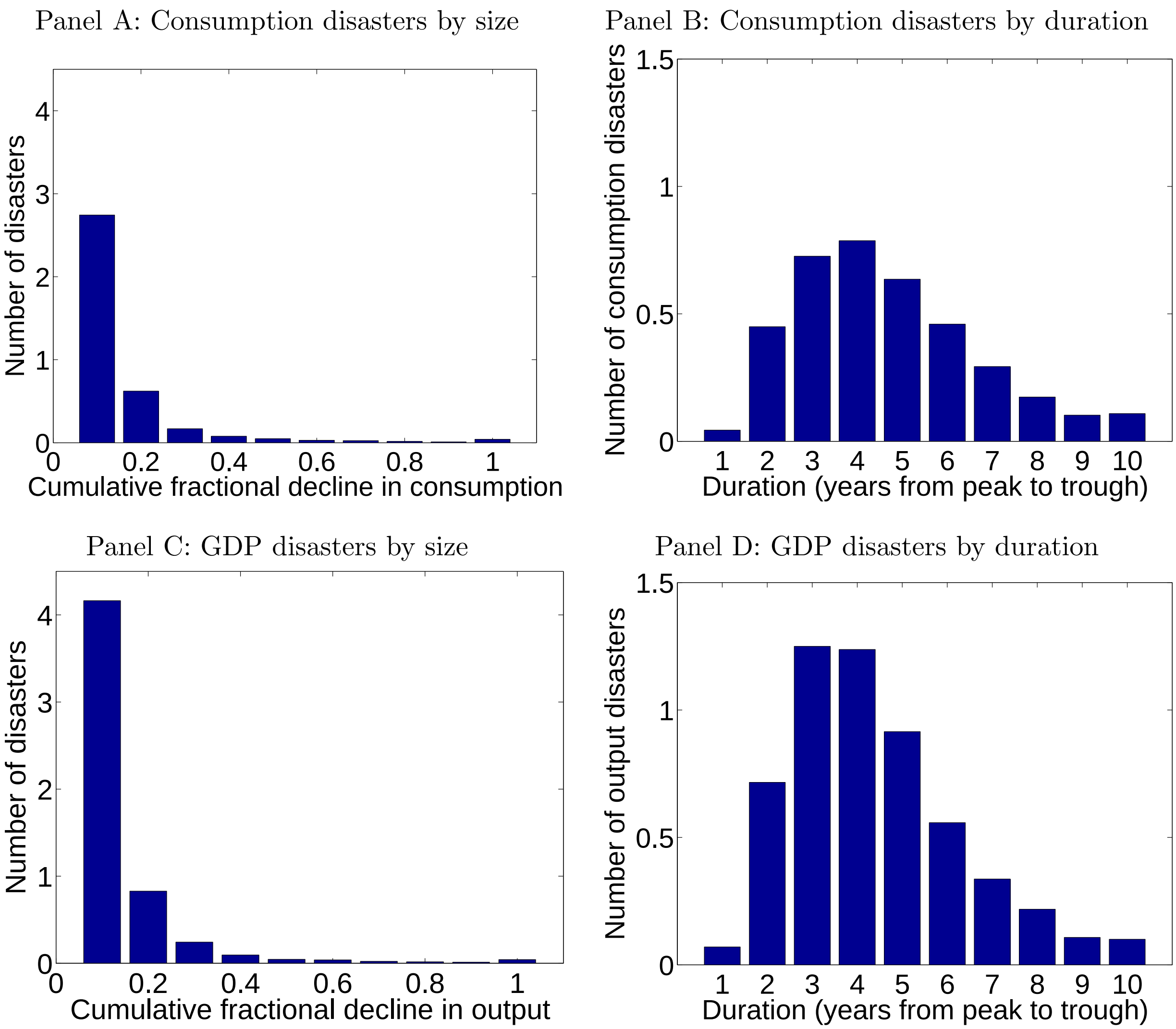
in the data. The cross-simulation standard deviation of the duration is 0.84 years, meaning that the duration in the data is slightly more than one standard deviation from the model's estimate.

Figure 6 reports the frequency distributions of consumption and GDP disasters by size and duration based on 1,000 simulations of the model. This figure is the model's counterpart to Figures 1 and 2 in Barro and Ursúa (2008). We see that the size and duration distributions for both consumption and GDP disasters display largely similar patterns as those in the data. In particular, the size distributions seem to follow a power-law density as emphasized by Barro and Jin (2011).

4.5 Endogenous Long Run Risks and Endogenous Uncertainty Risks

In this subsection, we explore the model's implications for long run risks and uncertainty shocks. Bansal and Yaron (2004) propose long-run consumption risks as a mechanism for explaining aggre-

Figure 6 : Distributions of Consumption and GDP Disasters by Size and Duration



gate asset prices. Bansal and Yaron specify the following exogenous consumption growth process:

$$z_{t+1} = .979 z_t + .044 \sigma_t e_{t+1}, \tag{26}$$

$$g_{t+1} = .0015 + z_t + \sigma_t \eta_{t+1}, \tag{27}$$

$$\sigma^2_{t+1} = .0078^2 + .987(\sigma^2_t - \sigma^2) + .23 \times 10^{-5} w_{t+1}, \tag{28}$$

in which g_{t+1} is the consumption growth, z_t is the expected consumption growth, σ_t is the conditional volatility of g_{t+1}, and $e_{t+1}, u_{t+1}, \eta_{t+1}$, and w_{t+1}, are i.i.d. standard normal shocks, which are mutually uncorrelated. Bansal and Yaron argue that the stochastic slow-moving component,

z_t, of the consumption growth is crucial for explaining the level of the equity premium, and that the mean-reverting stochastic volatility helps explain the time-variation in the risk premium.

Kaltenbrunner and Lochstoer (2010) lend support to Bansal and Yaron's (2004) long-run risks argument by showing that these risks can arise endogenously via consumption smoothing. Within a production economy with capital as the only productive input, Kaltenbrunner and Lochstoer (Table 6) show that the (monthly) consumption growth follows:

$$z_{t+1} = .986 z_t + .093 \sigma e_{t+1}, \tag{29}$$

$$g_{t+1} = .0013 + z_t + \sigma \eta_{t+1}, \tag{30}$$

with transitory productivity shocks. With permanent productivity shocks, the z_t process follows:

$$z_{t+1} = .990 z_t + .247 \sigma e_{t+1}. \tag{31}$$

However, their economies fail to generate heteroscedasticity in shocks to expected and realized consumption growth (see their footnote 15). Also, the permanent shocks model produces an equity premium very close to zero. Although the transitory shocks model offers a high equity premium, its calibration includes a time discount factor that is larger than unity.

We ask how consumption dynamics in the search economy compare with those in the Kaltenbrunner and Lochstoer (2010) economy and with those calibrated in Bansal and Yaron (2004). This question is interesting because different parameterizations of the consumption process specified in equations (26)–(28) can be consistent with simple moments of consumption growth such as volatility and autocorrelations (see Table 2). Yet, different parameterizations can imply very different economic mechanisms for the equity risk premium and its time-variation.

We simulate the search economy for 6,000 monthly periods to reach the stationary distribution, and then simulate one million monthly periods. We calculate the expected consumption growth and the conditional volatility of the realized consumption growth on the employment-productivity grid, and use the solutions to simulate these moments. Fitting the consumption growth process specified by Bansal and Yaron (2004) on the simulated data, we obtain:

$$z_{t+1} = .700 z_t + .570 \sigma_t e_{t+1}, \tag{32}$$

$$g_{t+1} = z_t + \sigma_t \eta_{t+1}, \tag{33}$$

$$\sigma_{t+1}^2 = .0063^2 + .908(\sigma_t^2 - \sigma^2) + 1.93 \times 10^{-5} w_{t+1}. \tag{34}$$

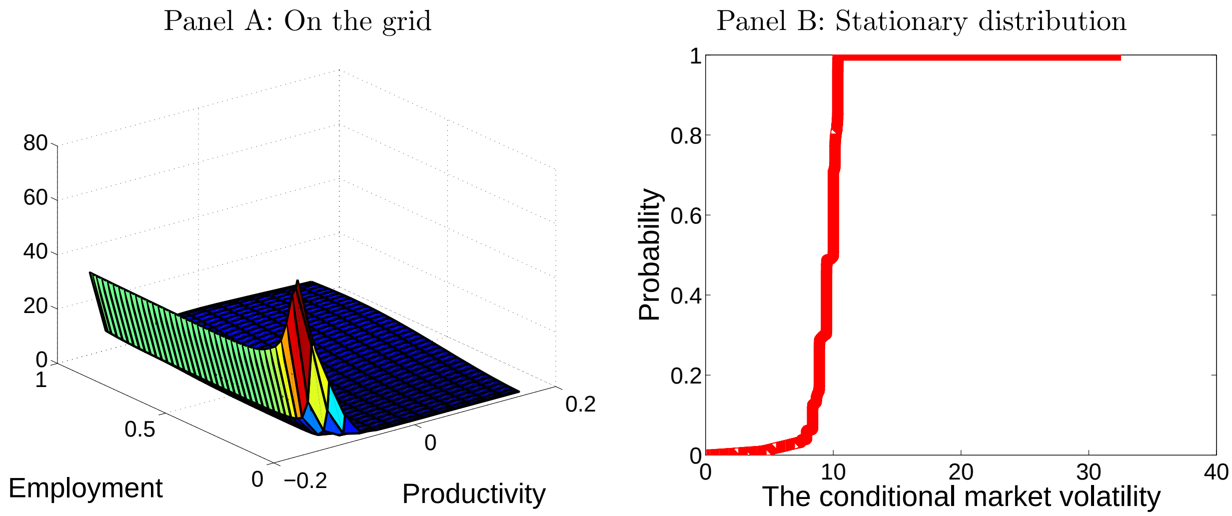
In addition, the unconditional correlation between e_{t+1} and η_{t+1} is 0.26, and the correlations between e_{t+1} and w_{t+1} and between η_{t+1} and w_{t+1} are close to zero.

Although the consumption growth is not i.i.d. in our economy, the persistence in the expected consumption growth is only 0.70, which is lower than that in Kaltenbrunner and Lochstoer (2010) and that calibrated in Bansal and Yaron (2004). However, the expected consumption growth is more volatile in our economy. The conditional volatility of the expected consumption growth is 57% of the conditional volatility of the realized consumption growth. This percentage is substantially higher than 9.3% and 24.7% in Kaltenbrunner and Lochstoer as well as 4.4% in Bansal and Yaron. For the stochastic variance, its persistence is 0.908 in our economy, which is lower than 0.987 in Bansal and Yaron. However, the volatility of our stochastic variance is more than eight times of theirs.

We interpret these results as saying that disasters risks play a more important role than long-run risks (in the sense of extreme persistence of expected consumption growth) in our economy. Because the economy occasionally falls into disasters, shocks to both expected consumption growth and the conditional variance of consumption growth are magnified. Disasters also give rise to lower persistence for the expected consumption growth and the conditional variance.

To further characterize the endogenous uncertainty risks in our economy, we plot the conditional market volatility, σ_t^R, on the employment-productivity grid. From Panel A of Figure 7, the market volatility is strongly countercyclical: it is low in good times and high in bad times. Panel B reports the empirical cumulative distribution of the market volatility in simulations. We simulate 1,006,000

Figure 7 : The Conditional Market Volatility in Annual Percent, σ_t^R, on the Grid and Empirical Stationary Distribution in Simulations



monthly periods, discard the first 6,000 periods, and treat the remaining one million periods as from the stationary distribution. The conditional volatility hovers around its median about 10% per annum. However, with small probabilities, the market volatility can jump to more than 30%.

4.6 Procyclical Dividend Dynamics

Kaltenbrunner and Lochstoer (2010) show that dividend is countercyclical in the standard production economy with capital as the only productive input (see also Jermann (1998)). Intuitively, dividend roughly equals profits minus investment, and profits in turn roughly equal output minus wages. Without labor market frictions, the wage rate equals the marginal product of labor, meaning that profits are proportional to (and as procyclical as) output. Because investment is more procylical than output (and profits), dividend, as profits minus investment, has to be countercyclical.

The dividend countercyclicality in the baseline production economy is counterfactual. Dividend in the model corresponds with net payout (dividend plus stock repurchases minus new equity issues) in the data. Following Jermann and Quadrini (2010), we measure net payout using aggregate data from the Flow of Funds Accounts of the Federal Reserve Board.[4] The sample is quarterly from the

[4]Specifically, we calculate net payout as net dividends of nonfarm, nonfinancial business (Table F.102, line 3) plus net dividends of farm business (Table F.7, line 24) minus net increase in corporate equities of nonfinancial business (Table F.101, line 35) minus proprietors' net investment of nonfinancial business (Table F.101, line 39).

fourth quarter of 1951 to the fourth quarter of 2010. We obtain quarterly real GDP (NIPA Table 1.1.6) and quarterly implicit price deflator for GDP (NIPA Table 1.1.9) used to deflate net payout. We detrend real net payout and real GDP as HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. We do not take logs because net payout can be negative in the data. Consistent with Jermann and Quadrini, we find that the cyclical components of real net payout and real GDP have a positive correlation of 0.55.

The search economy avoids the pitfall of counterfactual dividend dynamics. Intuitively, the wage rate from the generalized Nash bargaining process is no longer equal to the marginal product of labor. Because the value of unemployment activities, b, is positive, wages are less procyclical than productivity. As such, profits are more procyclical than output. In effect, the relatively inelastic wage rate works as operating leverage to magnify the procyclicality (and volatility) of profits. This turbocharged procyclicality of profits is sufficient to overcome the procyclicality of total vacancy costs, κV_t, so as to produce procyclical dividend dynamics.

Quantitatively, the model also succeeds in replicating the procyclicality of dividend. We first simulate the economy for 6,000 monthly periods to reach the stationary distribution, and then repeatedly simulate 1,000 artificial samples, each with 711 months (237 quarters). The sample size matches the quarterly series from the fourth quarter of 1951 to the fourth quarter of 2010. On each artificial sample, we time-aggregate monthly observations of dividend and output into quarterly observations. After detrending the quarterly series as HP-filtered propertional deviations from the mean, we calculate the correlation between the cyclical components of dividend and output. We find that this correlation to be 0.53 in the model, with a cross-simulation standard deviation of 0.14, thereby replicating the correlation of 0.55 in the data.

We also compare the wage dynamics in the model to those in the data. Following Hagedorn and Manovskii (2008), we measure wages as labor share times labor productivity from BLS. The sample is quarterly from the first quarter of 1947 to the four quarter of 2010 (256 quarters). We take logs and HP-detrend the series with a smoothing parameter of 1,600. We find that the wage elasticity to labor productivity is 0.46, close to Hagedorn and Manovskii's estimate, meaning that a one percentage point increase in labor productivity delivers a 0.46 percentage increase in real wages. In addition, we measure (quarterly) output as real GDP (NIPA Table 1.1.6). We find that the correlation between the log wage growth and the log output growth is 0.48, and that the ratio of the wage growth volatility over the output growth volatility is 0.70.

To see the model's performance, we first simulate the economy for 6,000 monthly periods to reach the stationary distribution, and then repeatedly simulate 1,000 artificial samples, each with 768 months (256 quarters). On each artificial sample, we take quarterly averages of monthly wages and labor productivity to obtain quarterly series. Implementing the same empirical procedure used on the real data, we find that the wage elasticity to productivity is 0.56 in the model, which is not far from 0.46 in the data. However, the correlation between the log wage growth and the log output growth in the model is 0.90, which is too high relative to 0.48 in the data. At the same time, the relative volatility of the wage growth is 0.36, which is too low relative to 0.70 in the data.

4.7 Comparative Statics

To shed further light on the economic mechanisms underlying the risk premium in the model, we conduct several comparative statics by varying the model's key parameters. Table 6 reports four experiments: (i) the value of unemployment activities, b, changed from 0.85 in the benchmark calibration to 0.4; (ii) the job separation rate, s, from 0.05 to 0.035; (iii) the vacancy cost, κ, from 0.975 to 0.2; and (iv) the workers' bargaining power, η, from 0.052 to 0.10. In each experiment, except for the parameter in question, all the other parameters are the same as in the benchmark calibration.

The Value of Unemployment Activities

In the first experiment, the value of unemployment activities, $b = 0.4$, is set to be the parameter value in Shimer (2005). Because unemployment is less valuable to workers, the unemployment rate drops to 5.17%. A lower b also means that the wage rate is more sensitive to shocks. As such,

Table 6 : Comparative Statics

We report four experiments: (i) $b = .4$ is for the value of unemployment activities set to 0.4; (ii) $s = .035$ is for the job separation rate set to 0.035; (iii) $\kappa = .2$ is for the vacancy cost set to 0.2; and (iv) $\eta = .1$ is for the workers' bargaining power set to 0.1. In each case, all the other parameters are identical to those in the benchmark calibration. See the caption of Table 2 for the description of Panel A. See the caption of Table 3 for the description of Panel B: σ^U, σ^V, and $\sigma^{V/U}$ denote the standard deviations of unemployment, vacancy, and the vacancy-unemployment ratio, respectively. $\rho^{U,V}$ is the correlation between unemployment and vacancy. See the caption of Table 4 for the description of Panel C: (1) and (12) denote for forecast horizons of one and 12 months, respectively. See the caption of Table 5 for the description of Panel D: (C) and (Y) denote consumption and GDP disasters, respectively. In Panel E, $\rho^{D,Y}$ is the correlation between the cyclical components of quarterly dividends and output, and $e^{W,X}$ is the wage elasticity to productivity. The moments from the benchmark calibration and from the data are also reported.

	Data	Benchmark	$b = .4$	$s = .035$	$\kappa = .2$	$\eta = .15$
	Panel A: Basic business cycle and financial moments					
σ^C	3.036	3.596	1.781	2.153	1.405	4.640
$\rho^C(1)$	0.383	0.182	0.138	0.142	0.134	0.242
$\rho^C(3)$	−0.206	−0.126	−0.093	−0.105	−0.094	−0.133
σ^Y	4.933	4.147	2.047	2.637	2.100	4.942
$\rho^Y(1)$	0.543	0.182	0.137	0.140	0.133	0.235
$\rho^Y(3)$	−0.179	−0.121	−0.093	−0.103	−0.093	−0.130
$E[R - R^f]$	5.066	4.538	0.020	0.014	−0.782	3.668
$E[R^f]$	0.588	4.113	4.032	4.044	4.028	3.751
σ^R	12.942	11.074	3.537	11.339	15.903	7.830
σ^{R^f}	1.872	1.337	0.132	0.630	0.157	1.440
	Panel B: Labor market moments					
σ^U	0.119	0.140	0.002	0.087	0.011	0.145
σ^V	0.134	0.098	0.022	0.081	0.090	0.095
$\sigma^{V/U}$	0.255	0.155	0.024	0.120	0.097	0.158
$\rho^{U,V}$	−0.913	−0.571	−0.947	−0.658	−0.833	−0.508
	Panel C: Forecasting market excess returns with the V/U ratio					
Slope(1)	−1.425	−0.807	−0.140	−0.387	−0.111	−0.725
Slope(12)	−10.312	−8.657	−1.550	−4.259	−1.218	−7.716
$t_{NW}(1)$	−2.575	−2.228	−0.874	−0.971	−0.786	−1.858
$t_{NW}(12)$	−1.704	−2.776	−1.123	−1.239	−1.021	−2.280
	Panel D: Moments of macroeconomic disasters					
Probability(C)	3.63	3.224	0.526	1.183	0.149	5.859
Size(C)	22	19.705	11.786	14.857	11.437	20.861
Duration(C)	3.6	4.809	6.034	5.535	6.582	4.545
Probability(Y)	3.69	4.939	0.769	2.084	0.322	6.937
Size(Y)	21	18.841	12.930	15.215	14.785	20.697
Duration(Y)	3.5	4.474	5.769	5.087	6.049	4.367
	Panel E: Procyclical dividend dynamics					
$\rho^{D,Y}$	0.546	0.527	0.999	0.832	0.996	0.408
$e^{W,X}$	0.463	0.557	0.645	0.593	0.500	0.711

the wage elasticity to productivity increases to 0.65 from 0.56 in the benchmark economy. As a result of the higher wage elasticity to productivity, profits and vacancies are less sensitive to shocks. Employment and output are also less sensitive, giving rise to a low consumption growth volatility of 1.78% per annum and a low output growth volatility of 2.05% (Panel A of Table 6).

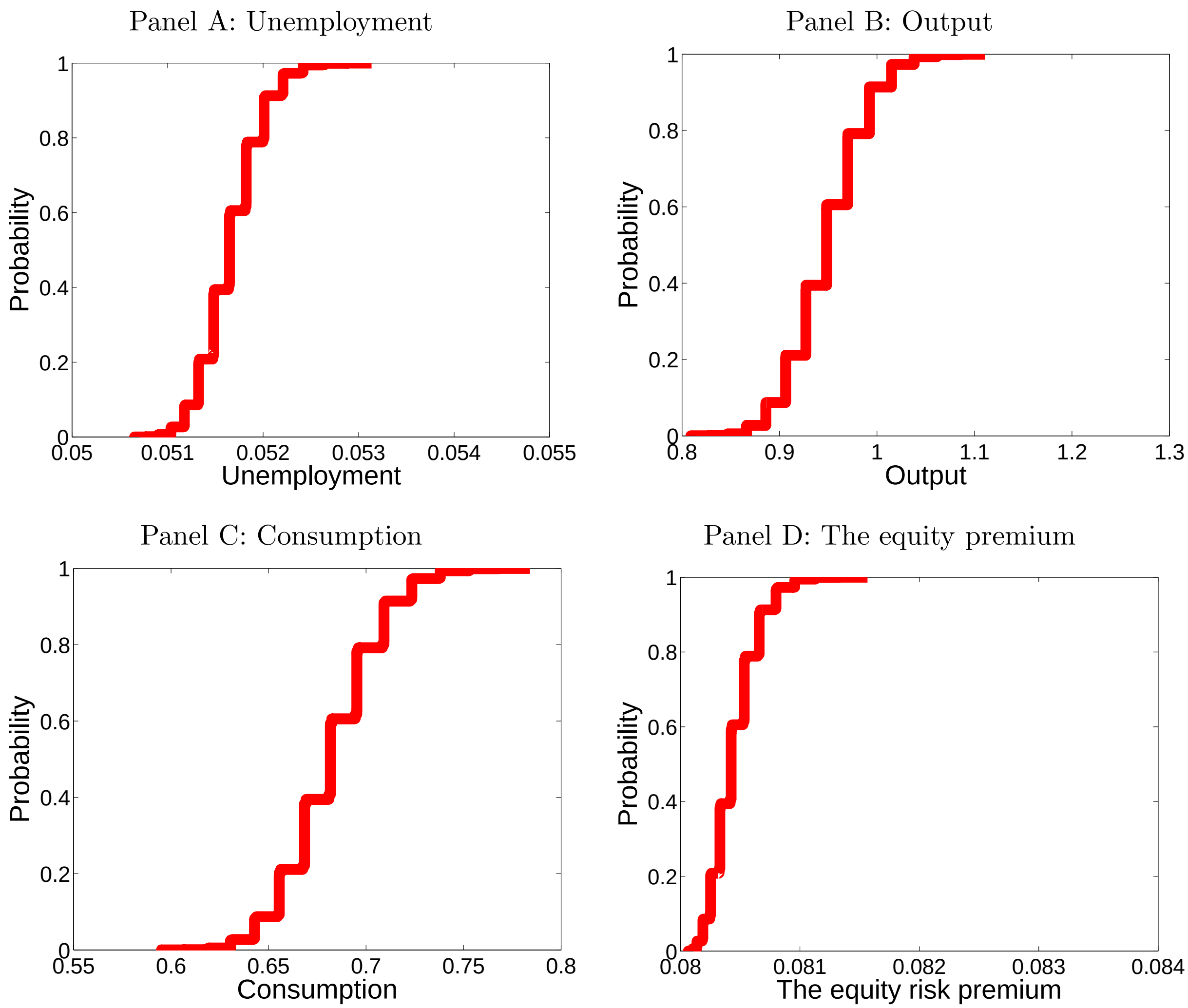
The low-b economy shows essentially no disaster risks. From the stationary distributions reported in Figure 8, the unemployment rate varies within a narrow range close to 5%. Neither output nor consumption has a long left tail in its empirical cumulative distribution. Using Barro and Ursúa's (2008) peak-to-trough measurement, Panel D of Table 6 shows that the disaster probabilities are less than 0.8%, and the average size of the disasters are less than 13%. Because of the lack of disaster risks, the equity premium drops to slightly above zero, and is largely time-invariant: the V/U ratio shows no predictive power for market excess returns. The market volatility also drops.

Consistent with Shimer (2005), the standard deviation of the V/U ratio in the low-b model is only 0.02, which is an order of magnitude smaller than 0.26 in the data. As such, a high value of unemployment activities helps alleviate simultaneously the Shimer puzzle and the equity premium puzzle. Intuitively, by dampening the procyclical covariation of wages with productivity, a high value of b magnifies the procyclical variation of profits and vacancies, so as to increase the volatility of the V/U ratio. At the same time, the high value of b also turbocharges the procyclical variation of dividend, so as to raise the equity risk premium and the stock return volatility.

The Job Separation Rate

In the second experiment, we reduce the job separation rate, s, from 0.05 to 0.035. Because the productive input in the economy is destructed at a lower rate, the disasters in the low-s economy are less extreme and less frequent than those in the benchmark economy. The consumption and GDP disaster probabilities are 1.18% and 2.08%, respectively, which are more than halved relative to those in the benchmark economy. The average magnitudes of consumption and GDP disasters are also lower: 14.86% and 15.22% versus 19.71% and 18.84%, respectively. The low-s economy

Figure 8 : Empirical Cumulative Distribution Functions, the Model with a Low Value of Unemployment Activities, $b = 0.40$



also has a lower consumption growth volatility than the benchmark economy: 2.15 versus 3.60% per annum. Because of the lower consumption volatility and the dampened disaster risks, the equity risk premium is close to zero and largely time-invariant in the low-s economy.

The Vacancy Cost

In the third experiment, we reduce the vacancy cost parameter, κ, from 0.975 in the benchmark economy to 0.2, which is close to the value in Shimer (2005). Lower vacancy costs mean that the firm is more capable of smoothing the impact of exogenous productivity shocks by varying vacancies. As such, the output growth volatility falls to 2.1% per annum, and the consumption growth volatility to 1.41%. Both consumption and GDP disaster probabilities are close to zero. As a result, the equity premium is close to zero, even slightly negative. However, lower vacancy costs imply that both vacancies and the marginal value of posting vacancies are more sensitive to shocks. From the definition of the stock return in equation (15), the heightened sensitivity of the marginal value translates to a higher market volatility.

The Workers' Bargaining Weight

In the final experiment, we increase the workers' bargaining weight, η, from 0.052 to 0.10. A higher η makes wages more procyclical and profits less procyclical. Dividend is also less procyclical. This effect weakens operating leverage, and lowers the equity risk premium. However, the high-η economy shows more volatile consumption growth and output growth than the benchmark economy. Because wages are more procyclical and profits are less procyclical, vacancies are less responsive to shocks. As such, unemployment (and thus employment) are more responsive to shocks, making output more responsive to shocks as well. From the resource constraint in equation (21), consumption must then absorb more shocks to become more volatile.

5 Conclusion

We study aggregate asset pricing by embedding the standard Diamond-Mortensen-Pissarides search model of the labor market into a dynamic stochastic general equilibrium economy. We find that search frictions in the labor market help explain the equity premium in the financial market. With reasonable parameter values, the model reproduces a sizeable equity risk premium with a low interest rate volatility. The equity premium is also strongly countercyclical, and is forecastable by labor market tightness, a prediction that we confirm in the data. Intriguingly, search frictions create endogenously rare disaster risks as emphasized by Rietz (1988) and Barro (2006).

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A The Stock Return Equation

We prove equation (15) following an analogous proof in Liu, Whited, and Zhang (2009) in the context of the q-theory of investment. Rewrite the equity value maximization problem as:

$$S_t = \max_{\{V_{t+\tau}, N_{t+\tau+1}\}} E_t \left[\sum_{\tau=0}^{\infty} M_{t+\tau} \left[\begin{array}{c} X_{t+\tau}N_{t+\tau} - W_{t+\tau}N_{t+\tau} - \kappa V_{t+\tau} \\ -\mu_{t+\tau}[N_{t+\tau+1} - (1-s)N_{t+\tau} \\ -V_{t+\tau}q(\theta_{t+\tau})] + \lambda_{t+\tau}q(\theta_{t+\tau})V_{t+\tau} \end{array} \right] \right], \tag{A.1}$$

in which μ_t is the Lagrange multiplier on the employment accumulation equation, and λ_t is the Lagrange multiplier on the irreversibility constraint on job creation. The first order conditions are equations (10) and (11), and the Kuhn-Tucker condition is equation (14).

Define dividends as $D_t = X_tN_t - W_tN_t - \kappa V_t$ and the ex-dividend equity value as $P_t = S_t - D_t$. Expanding S_t yields:

$$\begin{aligned} P_t + X_tN_t - W_tN_t - \kappa V_t = S_t = X_tN_t - W_tN_t - \kappa V_t - \mu_t\left[N_{t+1} - (1-s)N_t - V_tq(\theta_t)\right] + \lambda_t q(\theta_t)V_t \\ +E_tM_{t+1}\left[X_{t+1}N_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1} - \mu_{t+1}\left[N_{t+2} - (1-s)N_{t+1} - V_{t+1}q(\theta_{t+1})\right]\right. \\ \left. +\lambda_{t+1}q(\theta_{t+1})V_{t+1}\right] + \ldots \end{aligned} \tag{A.2}$$

Recursively substituting equations (10) and (11) yields: $P_t + X_tN_t - W_tN_t - \kappa V_t = X_tN_t - W_tN_t + \mu_t(1-s)N_t$. Using equation (10) to simplify further: $P_t = \kappa V_t + \mu_t(1-s)N_t = \mu_t[(1-s)N_t + q(\theta_t)V_t] + \lambda_t q(\theta_t)V_t = \mu_t N_{t+1}$, in which the last equality follows from the Kuhn-Tucker condition (14).

To show equation (15), we expand the stock returns:

$$\begin{aligned} R_{t+1} &= \frac{S_{t+1}}{S_t - D_t} = \frac{\mu_{t+1}N_{t+2} + X_{t+1}N_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1}}{\mu_t N_{t+1}} \\ &= \frac{X_{t+1} - W_{t+1} - \kappa\frac{V_{t+1}}{N_{t+1}} + \mu_{t+1}\left[(1-s) + q(\theta_{t+1})\frac{V_{t+1}}{N_{t+1}}\right]}{\mu_t} \\ &= \frac{X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}}{\mu_t} + \frac{1}{\mu_t N_{t+1}}\left[\mu_{t+1}q(\theta_{t+1})V_{t+1} - \kappa V_{t+1}\right] \\ &= \frac{X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}}{\mu_t}, \end{aligned} \tag{A.3}$$

in which the last equality follows because $\mu_{t+1}q(\theta_{t+1})V_{t+1} - \kappa V_{t+1} = -\lambda_{t+1}q(\theta_{t+1})V_{t+1} = 0$ from the Kuhn-Tucker condition. ■

B Wage Determination under Nash Bargaining

Let $0 < \eta < 1$ denote the relative bargaining weight of the worker, J_{Nt} the marginal value of an employed worker to the representative family, J_{Ut} the marginal value of an unemployed worker to the representative family, ϕ_t the marginal utility of the representative family, S_{Nt} the marginal value of an employed worker to the representative firm, and S_{Vt} the marginal value of an unemployed worker to the representative firm. Let $\Lambda_t \equiv (J_{Nt} - J_{Ut})/\phi_t + S_{Nt} - S_{Vt}$ be the total surplus from the Nash bargain. The wage equation (20) is determined via the Nash worker-firm bargain:

$$\max_{\{W_t\}} \quad \left(\frac{J_{Nt} - J_{Ut}}{\phi_t}\right)^{\eta} (S_{Nt} - S_{Vt})^{1-\eta}, \tag{B.1}$$

The outcome of maximizing equation (B.1) is the surplus-sharing rule:

$$\frac{J_{Nt} - J_{Ut}}{\phi_t} = \eta\Lambda_t = \eta\left(\frac{J_{Nt} - J_{Ut}}{\phi_t} + S_{Nt} - S_{Vt}\right). \tag{B.2}$$

As such, the worker receives a fraction of η of the total surplus from the wage bargain. In what follows, we derive the wage equation (20) from the sharing rule in equation (B.2).

B.1 Workers

Let ϕ_t denote the Lagrange multiplier for the household's budget constraint (17). The household's maximization problem is given by:

$$J_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} - \phi_t\left(\frac{\Pi_{t+1}}{R_{t+1}^{\Pi}} - \Pi_t + C_t - W_t N_t - U_t b + T_t\right), \tag{B.3}$$

The first-order condition for consumption yields:

$$\phi_t = (1-\beta)C_t^{-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1}, \tag{B.4}$$

which gives the marginal utility of consumption.

Recalling $N_{t+1} = (1-s)N_t + f(\theta_t)U_t$ and $U_{t+1} = sN_t + (1-f(\theta_t))U_t$, we differentiate J_t in equation (B.3) with respect to N_t:

$$\begin{aligned} J_{Nt} \quad &= \quad \phi_t W_t + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\times\frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}[(1-s)J_{Nt+1} + sJ_{Ut+1}]\right]. \end{aligned} \tag{B.5}$$

Dividing both sides by ϕ_t:

$$\frac{J_{Nt}}{\phi_t} = W_t + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}} \left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}[(1-s)J_{Nt+1} + sJ_{Ut+1}]\right]. \tag{B.6}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned} \frac{J_{Nt}}{\phi_t} &= W_t + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} \left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] \\ &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right]. \end{aligned} \tag{B.7}$$

Similarly, differentiating J_t in equation (B.3) with respect to U_t yields:

$$\begin{aligned} J_{Ut} &= \phi_t b + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\times\frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}[f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}]\right]. \end{aligned} \tag{B.8}$$

Dividing both sides by ϕ_t:

$$\frac{J_{Ut}}{\phi_t} = b + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}} \left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}[f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}]\right]. \tag{B.9}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned} \frac{J_{Ut}}{\phi_t} &= b + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} \left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] \\ &= b + E_t\left[M_{t+1}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right]. \end{aligned} \tag{B.10}$$

B.2 The Firm

We start by rewriting the infinite-horizon value-maximization problem of the firm recursively as:

$$S_t = X_t N_t - W_t N_t - \kappa V_t + \lambda_t q(\theta_t) V_t + E_t[M_{t+1} S_{t+1}], \tag{B.11}$$

subject to $N_{t+1} = (1-s)N_t + q(\theta_t)V_t$. The first-order condition with respect to V_t says:

$$S_{Vt} = -\kappa + \lambda_t q(\theta_t) + E_t[M_{t+1} S_{Nt+1} q(\theta_t)] = 0 \tag{B.12}$$

Equivalently,

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t[M_{t+1} S_{Nt+1}] \tag{B.13}$$

In addition, differentiating S_t with respect to N_t yields:

$$S_{Nt} = X_t - W_t + E_t[M_{t+1}(1-s)S_{Nt+1}]. \tag{B.14}$$

Combining the last two equations yields the intertemporal job creation condition in equation (12).

B.3 The Wage Equation

From equations (B.7), (B.10), and (B.14), the total surplus of the worker-firm relationship is:

$$\begin{aligned}
\Lambda_t &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] - b \\
&\quad - E_t\left[M_{t+1}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] + X_t - W_t + E_t[M_{t+1}(1-s)S_{Nt+1}] \\
&= X_t - b + (1-s)E_t\left[M_{t+1}\left(\frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}} + S_{Nt+1}\right)\right] - f(\theta_t)E_t\left[M_{t+1}\frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}}\right] \\
&= X_t - b + (1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] - \eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right],
\end{aligned} \tag{B.15}$$

in which the last equality follows from the definition of Λ_t and the surplus sharing rule (B.2).

The surplus sharing rule implies $S_{Nt} = (1-\eta)\Lambda_t$, which, combined with equation (B.14), yields:

$$(1-\eta)\Lambda_t = X_t - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right]. \tag{B.16}$$

Combining equations (B.15) and (B.16) yields:

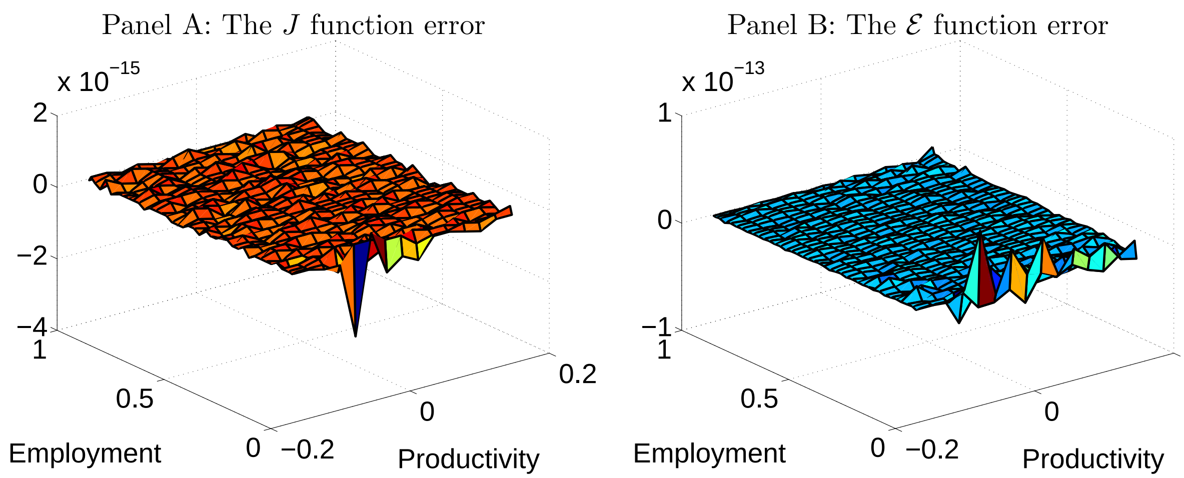
$$\begin{aligned}
X_t - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] &= (1-\eta)(X_t - b) + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\
&\quad - (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\
X_t - W_t &= (1-\eta)(X_t - b) - (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\
W_t &= \eta X_t + (1-\eta)b + (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right]
\end{aligned}$$

Using equations (B.2) and (B.13) to simplify further:

$$W_t = \eta X_t + (1-\eta)b + \eta f(\theta_t)E_t\left[M_{t+1}S_{Nt+1}\right] \tag{B.17}$$

$$W_t = \eta X_t + (1-\eta)b + \eta f(\theta_t)\left(\frac{\kappa}{q(\theta_t)} - \lambda_t\right) \tag{B.18}$$

Figure C.1 : Errors in the J and $\mathcal{E}$ Functional Equations



Using the Kuhn-Tucker condition, when $V_t > 0$, then $\lambda_t = 0$, and equation (B.18) reduces to the wage equation (20) because $f(\theta_t) = \theta_t q(\theta_t)$. On the other hand, when the irreversibility constraint is binding, $\lambda_t > 0$, but $V_t = 0$ means $\theta_t = 0$ and $f(\theta_t) = 0$. Equation (B.18) reduces to $W_t = \eta X_t + (1-\eta)b$. Because $\theta_t = 0$, the wage equation (20) continues to hold.

C Additional Details of the Nonlinear Solution

We approximate the x_t process in equation (5) based on the discrete state space method of Rouwenhorst (1995) with 15 grid points. (Kopecky and Suen (2010) show that the Rouwenhorst (1995) method is more reliable and accurate than other methods in approximating highly persistent first-order autoregressive processes.) This grid is large enough to cover the values of x_t within four unconditional standard deviations from its unconditional mean of zero. We set the minimum value of N_t to be 0.0355 and the maximum value to be 0.99. This range is large enough so that N_t never hits one of the boundaries in simulations. We use cubic splines with 40 basis functions on the N space to approximate $\mathcal{E}(N_t, x_t)$ on each grid point of x_t. We use extensively the approximation took kit in the CompEcon Toolbox of Miranda and Fackler (2002). To obtain an initial guess for the projection algorithm, we use the social planner's solution via value function iteration.

Figure C.1 reports the error in the J functional equation (22), defined as $J(N_t, x_t)^{1-\frac{1}{\psi}} - (1-\beta)C(N_t, x_t)^{1-\frac{1}{\psi}} - \beta\left(E_t\left[J(N_{t+1}, x_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}$, and the error in the $\mathcal{E}$ functional equation (24), defined as $\mathcal{E}(N_t, x_t) - E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left(\frac{\kappa}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1})\right)\right]\right]$. These errors, in the magnitude no higher than 10^{-13}, are extremely small. As such, our nonlinear algorithm does an accurate job in characterizing the competitive equilibrium in the search economy.