Konrad, Kai A.

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Kai A. Konrad

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Wissenschaftszentrum Berlin für Sozialforschung gGmbH
Reichpietschufer 50
10785 Berlin
Germany
www.wzb.eu

Affiliation of the authors other than WZB:

Kai A. Konrad
WZB and Max Planck Institute for Tax Law and Public Finance
MPI for Tax Law and Public Finance, Marstallplatz 1, 80539 Munich, Germany

Abstract

Strategic Aspects of Fighting in Alliances

Kai A. Konrad*

This paper surveys some of the strategic aspects that emerge if players fight in an alliance against an enemy. The survey includes the free-rider problem and the hold-up problem that emerges in the baseline model, the role of supermodularity in alliance members' effort contributions, the role of budget constraints, the role of information transfer inside the alliance, and the role of in-group favoritism.

Keywords: Alliances, contests, conflict, in-group favoritism

JEL classification: D72, D74

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But the strong man is strongest when alone.
(Friedrich Schiller, Wilhelm Tell)

1 Introduction

Players who have a common goal often form alliances. And in many instances there are multiple alliances that are rivals to each other, with the success of one alliance ruling out the success of other alliances. Military alliances, political parties, R&D alliances and team competition in sports are some of the most salient examples for this type of competition. It has been pointed out by economists for a long time that such alliances have to bridge two potentially important disadvantages compared to stand-alone players: a free-rider problem and a hold-up problem.

The free-rider problem results because alliance members, when contributing effort to the success of the alliance as a whole, make a contribution to a group-public good. If one alliance member contributes more effort to the common goal of the alliance, this player bears the cost of this effort, but the effort typically benefits other members of the alliance as well. In the absence of enforceable contracts, the contributions to this public good are made voluntarily. The principle that governs contributions hence is simple: each member of an alliance makes a contribution to the alliance public good such that a marginal increase in the player's contribution has a marginal cost for this player equal to this player's own benefit from the increased winning probability of the alliance that is caused by it. Olson and Zeckhauser (1966) early on highlighted this free-rider problem in alliances. An implication of this general insight is that the members of an alliance who contribute the lion's share to the success of the alliance are members who have higher than average total benefits if the alliance achieves its goals, and/or members who have a cost advantage in making contributions to the alliance.[1] Considering the expenditure of the different members of the North Atlantic Treaty Organization (NATO) they found patterns that are in line with these predictions. The military expenditure as a percentage of GNP (in 1964) was highest for the United States, with 9.0 percentage points of GNP, whereas Luxemburg, being the smallest member of NATO, expended a share of only 1.7 percentage points.[2] Their analysis led to a considerable amount of further theoretical work that partially refined some of the arguments and looked at various modifications.[3] They also inspired a considerable stock of further empirical analyses on this topic.

Alliances may also suffer from a hold-up problem. If the alliance is victorious, this may cause rivalry inside the alliance, depending on the goal. A defence alliance may aim at the preservation of peace and territorial protection for its member states, such that all member states participate in the benefits of

[1] For a formal analysis of the model of voluntary contributions to a group public good see the seminal work by Bergstrom, Blume and Varian (1986) and by Esteban and Ray (2001).

[2] The ranking in economic size and the ranking in military expenditure as a percentage of the GNP are not perfectly correlated, but their Table 1 (p. 267) shows the two rankings follow each other fairly closely.

[3] These include, for instance, Baik, Kim, and Na (2001), Baik and Lee (1998), Davis and Reilly (1999), Esteban and Ray (2001), Katz, Nitzan and Rosenberg (1990), Lee (1995), Nitzan (1991a), Nitzan (1991b), Nitzan and Ueda (2008), Rapoport and Amaldoss (1997), Sandler (1993) and Ursprung (1990).

achieving this goal in a non-rival and non-exclusive way; in particular, they need not struggle internally about how to distribute the peace and security benefits. This, however, is not true for all types of alliances. Alliances in an actual military conflict may win something that is not a group-public good. If an alliance defeats its rival enemy, the members of the alliance may have to divide the spoils of victory between its members. In this case a second strategic problem emerges inside alliances. The players who have formerly been members of the alliance may start quarrelling about obtaining a larger share in these spoils, and this quarrel may easily end up in a fight. The Second World War is an illustrative example. During the war a number of military powers including the United States, the UK, France and the Soviet Union formed the Great Alliance with the defeat of Nazi Germany and its allies as its common objective. However, the spoils of this victory were not a pure public good. Instead, it became clear even while Germany was not finally defeated that the Great Alliance would split into at least two major groups that would fight about how to divide the spoils of victory between them. This fight is known as the Cold War. It took about forty years, and the arms races that took place between the rivals dissipated an enormous amount of resources that could have been used peacefully, instead.

Historians document that the struggle about the spoils of victory among the players who had been members of the winning alliance is a rather common phenomenon. O'Connor (1969) documents this outcome for the Napoleonic wars: "The conflicting ambitions of the allies were subdued so long as the military danger was paramount; when the enemy weakened perceptibly, the concept of victory embraced by each member of the alliance either changed or became more distinctive. [...] A quarrel over the spoils need not await the end of hostilities, and the form of victory envisioned by the participants may vary accordingly." (O'Connor, 1969, p. 369]. Similar outcomes are reported about the aftermath of the First World War: "The British also disagreed with the French over economic policy. To be sure, the two Allies cooperated on economic matters during most of the War, and they were the principal sponsors of the Paris Economic Revolutions. However, their cooperation dissolved as victory became certain and reparation and indemnity replaced other wartime planning. Thereafter, they became the principal competitors for shares of compensation from Germany." (Bunselmeyer, 1975, p. 15).[4] For the Second World War: "The fact that victory was finally in sight in 1944 thus had a double and contradictory effect on the alliance. On the one hand, the removal of mortal danger made them less inclined to subordinate individual aims to the need for hanging together and hence a greater willingness to disregard the susceptibilities of allies. On the other hand, the imminence of victory and the obvious desperation of the Germans suggested that this was a poor time to allow divergent views of policy and strategy to break up a winning coalition and thereby risk all that had already been attained at huge cost in lives and treasure." (Weinberg, 1994, p. 736).

If former members of a victorious alliance break up and fight with each other about how to share the spoils of victory, this should reduce the value that members of an alliance attribute to the victory of their alliance. This, in turn creates a hold-up problem as regards their decision about how much effort to invest or contribute to increase the probability that their alliance is victorious.

[4] See also Kent (1989).

The first who formally analyzed this hold-up problem in the context of alliances were Katz and Tokatlidu (1996) and Wärneryd (1998). Later Esteban and Sákovics (2003) considered a two-stage contest in which players A and B are teamed up in an alliance in a contest against player C. Should C win, C receives the full prize and the game ends. Should the alliance win, the alliance members have to fight about who of them receives the prize, in a contest that follows similar rules as the contest in the first stage. They compare this game with a symmetric, one-stage contest among the three players A, B and C, and find that, under rather general conditions, players A and B are better-off in the symmetric, one-stage contest. The combined strategic disadvantages of the hold-up problem and the free-riding problem inside the alliance make the formation of the alliance fairly unattractive. Their result points at what could be called an "alliance puzzle": the formation of an alliance actually weakens the members of the alliance, in comparison to a situation in which they act as stand-alone players.

The insights of Esteban and Sákovics (2003) contrast with the major role that alliances play in military conflict, in politics, in business and in daily life. This suggests that there must be other aspects and considerations in the formation of alliances that more than offset these strategic disadvantages. This contribution is a study of some of these offsetting effects, and of strategic environments in which the strategic disadvantages of alliance formation do not play a major role. In particular, the next sections consider (1) supermodularity in alliance members' efforts, (2) budget constraints that are sufficiently tight to remove the strategic problems of free-riding and the hold-up problem, (3) the potentially beneficial role of a threat of internal conflict as an incentive device for overcoming the free-riding problem in making contributions to the alliance effort, (4) possible benefits from information transfers among members of an alliance, (5) the role of multiple fronts if alliance members are resource constrained and can mutually support each other by resource transfers, and (6) evolutionary forces that generate in-group favoritism and spiteful behavior towards the out-group.

There are evidently many strategic aspects in the context of alliances that are not considered here. For instance, the formation and the break-up of alliances typically take place voluntarily and in the absence of property rights or binding contracts. This requires alliances to be self-enforcing. The strategic considerations (1)-(5) are important for explaining why the formation of an alliance can be mutually beneficial and self-enforcing. But they do not directly address the problem of who forms an alliance with whom, or the dynamic process of formations and dissolutions and re-arrangements of alliances. Political scientists have studied the sequential process of the formation and re-grouping of countries in different alliance networks over time. Economists have studied the process of endogenous alliance formation from a theory perspective. Their analysis of coalition formation shows that much depends on the assumptions about the rules that apply if members of an alliance disagree or find it advantageous to leave a given alliance, and join another alliance, on how far-sighted they are etc. (see, e.g., Bloch, Sánchez-Pagés and Soubeyran, 2006, and Bloch, 2011, also for further references).

2 The baseline model

Almost all considerations (1) - (5) can be discussed as disgressions from a single baseline model, which is essentially the framework considered by Esteban and Sákovics (2003). This generic game in which the alliance paradox is analyzed is as follows. There is a set of three players, $N = \{A, B, C\}$. All three players compete for a prize of a given size that is normalized to a monetary value $v = 1$. Player C is a stand-alone player who chooses an effort $x_C \geq 0$ that has a cost $C(x_i) = x_i$. Players A and B are members of an alliance. They also choose efforts $x_A \geq 0$ and $x_B \geq 0$, and have costs $C(x_A) = x_A$ and $C(x_B) = x_B$, respectively.[5] All effort choices take place independently and simultaneously. A contest success function determines the allocation of the prize as a random function of the effort choices. In general terms, the probability for the alliance to win is $p(x_A, x_B, x_C)$, and the probability for C to win is $1-$ $p(x_A, x_B, x_C)$, where p is a probability and will be specified in more detail. In the baseline framework, $p(x_A, x_B, x_C) = p(x_A + x_B, x_C)$; that is, the effort choices of alliance members simply add up to the total effort of the alliance, making their efforts perfect substitutes. Should C win the prize, the game ends and the payoffs of players are $\pi_C = 1 - x_C$ for player C, $\pi_A = -x_A$ for player A and $\pi_B = -x_B$ for player B. If the alliance wins, then the alliance members must determine how to allocate the prize between them. Taking the behavior of victorious war alliances as the role model, they start another fight about the prize. Players A and B expend intra-alliance efforts $y_A \geq 0$ and $y_B \geq 0$, and a contest success function $q(y_A, y_B)$ that may be of similar nature as p determines the win probability of A under these circumstances. Accordingly, the payoffs of members of a winning alliance in this subgame are $1 - y_i - x_i$ for the winner and $-y_j - x_j$ for the loser. All this is common knowledge among the players.

We can solve this game by backward induction, starting with stage 2, for different types of contest success functions p and q. Figure 1 reports the equilibrium results for three cases.

The first line in Figure 1 reports the case in which winning is determined by pure luck ($p = q \equiv 1/2$). In this case the disadvantage of alliance players is smallest, and is essentially induced by the specific choice of the exogenous win-probabilities.

The second line in Figure 1 considers the case in which winning or losing follows the Tullock (1980) lottery contest success function, i.e., $p = (x_A + x_B)/(x_A + x_B + x_C)$, where $x_A + x_B + x_C > 0$, and $p = 1/2$ otherwise, and $q = y_A/(y_A + y_B)$ where $y_A + y_B > 0$, and $q = 1/2$ otherwise. In this case the internal fight between former alliance members A and B is well-known to reduce the value that each of them attributes to winning the contest from 1/2 (which each would attribute to winning if they just shared the prize equally and peacefully in this case) to 1/4. Accordingly, the stage-1 contest between the alliance and player C is highly asymmetric as regards the values attributed to winning. This also makes the equilibrium contributions and the equilibrium payoffs very asymmetric.

The third line in Figure 1 considers the case in which, at both stages, the

[5] Esteban and Sákovics (2003) assume convex cost functions. This has the advantage of making interior equilibrium outcomes more likely, but also introduces notation which we do not need in what follows.

	$x_A + x_B$	x_C	win probability of the alliance $p(x_A, x_B, x_C)$	$\mathrm{E}y_A (= \mathrm{E}y_B)$	win probability in the intra-alliance contest $q(y_A, y_B)$	$\mathrm{E}\pi_A (= \mathrm{E}\pi_B)$	$\mathrm{E}\pi_C$
lottery	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$
lottery contest	$\frac{1}{25}$	$\frac{4}{25}$	$\frac{x_A + x_B}{x_A + x_B + x_C}$	$\frac{1}{4}$	$\frac{y_A}{y_A + y_B}$	$\frac{3}{100}$	$\frac{16}{25}$
all-pay auction without noise	0	0	1 if $x_A + x_B > x_C$ 0 if $x_A + x_B \leq x_C$	$\frac{1}{2}$	1 if $y_A > y_B$ $\frac{1}{2}$ if $y_A = y_B$ 0 if $y_A < y_B$	0	1

Figure 1: Equilibrium strategies and payoffs for the alliance game in the baseline model for three different contest success functions. In case of multiple equilibria, these values refer to the symmetric equilibrium.

party which expends the higher effort wins the contest. The contest success functions are those of an all-pay auction without noise in this case.[6] Here the hold-up problem is at a maximum: players A and B know that they will dissipate the full value of the prize in their internal fight, should their alliance win against player C.[7] Hence, even if they win, they win nothing. Each of them attributes a value of zero to the victory of the alliance, whereas player C attributes the full value of the prize to winning against the alliance, as this player receives this prize without any further fighting. In this extremely asymmetric contest between the alliance and C, the players A and B do not have an incentive to expend positive effort in stage 1, and this explains the outcome in the third line of Figure 1.

In what follows we will sometimes focus on the lottery contest, sometimes on the all-pay auction without noise.

3 Complementarities in fighting power

A first modification from the baseline model is discussed by Skaperdas (1998). He points out that alliance members' effort choices need not be perfect substitutes, and that the impact of efforts x_A and x_B may simply be much higher than the sum of these efforts. The reasons behind this can be manifold. The alliance may make the sum of their efforts more powerful due to ways to use the efforts that are unavailable in the absence of an alliance. Or the two efforts of alliance members may be complementary to each other, such that one player's effort x_A makes the effort of player x_B more valuable.

[6] Note that different tie-breaking rules are applied in the inter-alliance contest and in the intra-alliance contest in this case, in order to avoid an openness problem for the equilibrium in the inter-alliance conflict.

[7] This equilibrium outcome is well known from work by Hillman and Riley (1989), and Baye, Kovenock and de Vries (1996).

Note that this effect needs to be considerable if the contest between former alliance members dissipates a large share of the prize. In particular, if this contest is described by a symmetric all-pay auction without noise, the incentive for players A and B to contribute a positive effort in stage 1 is zero, regardless of how strong the synergies between A's and B's efforts are.

4 Tight budget caps

An alliance between players A and B can be attractive from their perspective if players have tight budget limits, even if the contest success functions are those of an all-pay auction without noise. This case has been analyzed by Konrad and Kovenock (2009). To illustrate their point, consider the baseline framework, assuming that the contest success functions p and q are all-pay auctions without noise, as in the third line in Figure 1. Let the prize be normalized to 1, and assume that players face budget constraints, with $x_i \in [0, m_i]$ for $i \in \{A, B, C\}$, and $y_i \in [0, m_i]$, for $i \in \{A, B\}$.[8] They show that

$$m_A = m_B \equiv m \in (\frac{m_C}{2}, \frac{1}{2} - \frac{m_C}{2}) \tag{1}$$

is a sufficient condition for the existence of a subgame perfect equilibrium in which A and B benefit from forming an alliance. Their payoffs increase from zero in a contest among three stand-alone players to $1 - m_C - 2m$. At the same time, the formation of the alliance reduces the payoff of player C to zero. To illustrate, consider, for instance, $m_A = m_B = 0.1$, and $m_C = 0.15$. In this example, should C win in stage 1, this player takes home the full prize $v = 1$. This determines C's valuation of winning in stage 1. If the alliance wins, they have to fight about the prize. As both have the same budget, and as this budget is very small, they both expend the whole budget in stage 2, i.e., $y_A = y_B = 0.1$, and each of them wins with probability 1/2. As a result, each of them attributes a value to the outcome in which the alliance wins that is equal to $\frac{1}{2} - m = 0.4$. Both the continuation value for C and the continuation values for A and B are higher than the respective budgets of the players. Accordingly, the players A and B are also budget constrained in stage 1.

In the absence of an alliance, the fact that $m_C = 0.15 > 0.1 = m$ implies that player C can always outcompete both stand-alone players A and B. And while $x_C = m + \epsilon$ is not the equilibrium outcome, it determines C's payoff in the equilibrium in absence of an alliance, as $1 - m = 0.9$. And the equilibrium payoffs of players A and B are zero in this case.

If A and B form an alliance, they can jointly outcompete C, as their joint resources exceed the joint resources of player C. This potentially shifts the rent from player C to the players A and B, should they manage to make clever joint bids. Konrad and Kovenock (2009) show that they can use a stochastic coordination mechanism (only observable to them) that makes their joint contributions follow the equilibrium bidding strategies of a single player with budget $2m$ who plays against C.

[8] Note that this assumes that players face the same budget constraint in the inter-alliance contest as in the intra-alliance contest, and that their effort choices in the inter-alliance contest do not affect their budgets in the intra-alliance contest. Of course, many variations of these assumptions that are also plausible are possible, and can lead to different results.

Note that, for this mechanism to work, it is important that the budgets of A and B that are available for fighting internally against each other, do not depend on the players' effort choices in the fight between player C and the alliance of A and B.

5 Overcoming free-riding by a threat of internal conflict

In the framework by Esteban and Sákovics (2003), the free-rider problem and the hold-up problem inside an alliance add or even compound. There is hope, however, that the two problems interact in a more constructive way. Konrad and Leininger (2011) analyzed an inter-alliance conflict in which the hold-up problem of possible internal fighting can be used to incentivize the members of an alliance and thereby overcome the free-rider problem of making contributions to the total fighting effort of the alliance.

A modified and simplified version of their mechanism can be described as follows. In the center of the structure they consider is an alliance. This group consists of n players, numbered by $1, ...n$, and for simplicity – unlike Konrad and Leininger (2011) – let us consider a symmetric set of players. The players contribute efforts x_i to the total alliance effort X that is the sum of these contributions. The group fights against an outside enemy that may consist of a group or a stand-alone player, and the group wins this fight with a probability $p(X, Y)$, where Y is the aggregate effort generated by the enemy, and p is a contest success function, as in the baseline model. For the sake of simplicity and in order to stay close to the baseline model, assume that the enemy is a stand-alone player. If the alliance loses, the prize goes to the enemy and the game is over. If the alliance wins, then they have to find a solution for the division of this gain among themselves.

A simplified version of the subgame considered by Konrad and Leininger (2011) that follows if the alliance was victorious and that yields similar overall results as in their framework is a Nash division game.[9] This subgame is described as follows: each of the n former members of the victorious alliance chooses a share $\alpha_i \geq 0$. If all shares α_i sum up to an amount that is less or equal to 1, then each player receives his share in the prize, and what is not claimed by the players is costlessly disposed. If, however, the sum of shares claimed exceeds 1, then the whole prize is lost for the members of the alliance. The Nash division subgame has multiple equilibria. Any set of $\alpha_i < 1$ that sum up to 1 also constitutes an equilibrium of this subgame, and in each of these equilibria the full prize is peacefully allocated to members of the alliance. Let us denote a representative of this class of equilibria as $\boldsymbol{\alpha}^* = (\alpha_1^*, ..., \alpha_n^*)$. However, a whole further set of equilibria exists which has the property that $\Sigma_{j \neq i} \alpha_j > 1$ for all $i = 1, ...n$.[10] One element of this set is $\boldsymbol{\alpha}^0 = (1, 1, ..., 1)$.[11] All players

[9] This simplification has been suggested to me by James Fearon.

[10] If this condition is fulfilled for player i, the player knows that the full prize is waisted in any case, regardless of his own choice α_i. Accordingly, the player can make a claim that is sufficiently high to support this equilibrium and to make this condition fulfilled for other players.

[11] To be precise, for $n = 2$, there is only one inefficient equilibrium, $\boldsymbol{\alpha}^0 = (1, 1)$ but a

who have been members of the alliance receive a zero payoff in any of these equilibria for which $\Sigma_{j\neq i}\alpha_j > 1$ in the distribution subgame.

The multiplicity of equilibria can now be used to implement alliance members' effort choices in the inter-alliance fight as follows. Suppose the combination of contributions to be implemented is $\mathbf{x}^* = (x_1^*, ..., x_n^*)$. Suppose further that players have the following beliefs about the equilibrium that is played in case the alliance wins the prize: they believe that one specific $\boldsymbol{\alpha}^* = (\alpha_1^*, ..., \alpha_n^*)$ is chosen if all player behave according to $\mathbf{x}^*$, and $\boldsymbol{\alpha}^0$ is selected if at least one of the alliance players deviates from $\mathbf{x}^*$. In this case, for a given aggregate effort choice Y^* of the enemy, $(\mathbf{x}^*, \boldsymbol{\alpha}^*)$ can be the outcome of selfish, individually rational interaction among the alliance members if

$$\alpha_i^* p(\Sigma_{i=1}^{i=n} x_i^*, Y^*) - x_i^* \geq 0 \text{ for all } i = 1, ...n. \tag{2}$$

The left-hand side in (2) is the expected equilibrium payoff of player i if player i makes an effort choice in accordance with $\mathbf{x}^*$ and this is followed by the peaceful sharing outcome in case the alliance is victorious.

Konrad and Leininger (2011) consider a distribution game inside the alliance that has considerably more structure and is closer to the structure of the baseline model. In their framework, one of the alliance members receives the whole winner prize and makes (unconditional) payments or gifts to the members of his group. And the members of the group are either satisfied both with their own and other members' contributions to the alliance effort, and with what they received from the prize, or they are displeased with one or several of these items. As a result of their satisfaction with these choices they either choose a non-cooperative interaction that is peaceful, or they choose to fight between them, in which case a large share of the winner prize is dissipated. This more complex structure serves a very similar purpose as a Nash division game does.

6 The role of information

An alliance may provide benefits to its members that are not accounted for in the baseline model or by the variants analyzed so far. As has been discussed by Bearce, Flanagan and Floros (2006), alliances may serve the purpose of information exchange. They consider some military alliances, including the North Atlantic Treaty Organization (NATO) and argue that such alliances may facilitate the flow of information between its members. Such an information flow may be beneficial for its members, may facilitate coordination and may help increasing the efficiency of use of their fighting resources. One of many of such aspects is addressed in a paper by Konrad (2011).

This paper considers n players and starts with assuming that these players may be partitioned arbitrarily into subsets A_1,...,A_r of alliances that consist of one or several players. Each player chooses his or her effort $x_i \in [0, m_i]$, where m_i is the player's budget limit, which is assumed to be small in comparison to the size of the prize that is again normalized to 1. A contest success function allocates the prize to one of the n players as a probabilistic function of all players' effort choices x_1,...,x_n, where $p_i(x_1, ..., x_n)$ denotes the win probability for player

continuum of efficient equilibria. For $n > 2$, the set of inefficient equilibria is a continuum as well.

i. This latter assumption removes one of the most salient features of alliances from the picture on purpose: efforts of alliance members do not add or compound in this framework, and it is not the alliance that wins the prize, but the prize is appropriated directly by one of the players in the set of players, even if this player is a member of an alliance that consists of several players. In this way a possible hold-up problem from intra-alliance fights and the free-rider problem of making contributions to total alliance effort that benefits also the other members of the alliance are both removed from the picture. The framework also does not allow for effects of supermodularity of alliance members' contributions (as in Skaperdas 1998), nor for the possibility that players can overcome budget limits by way of forming an alliance (as in Konrad and Kovenock 2009). Instead, the framework focuses on a specific aspect of information exchange: it assumes that players' budget limits m_i are their private information. These limits are random draws from a commonly known distribution $F(m)$ with support $[0, b]$, but the actual realizations of these random choices are observed only by the player itself, and by all co-players who are in the same alliance as this player.

Konrad (2011) analyses first the Bayesian Nash equilibrium in the case in which all alliances consist of single players ($r = n$) and derives a condition for $F(m)$ which makes sure that it is a dominant strategy for each player to simply expend his whole budget in the fight of all-against-all. It limits consideration to contests with p being the all-pay auction without noise. Intuitively, expending the whole military capacity available to a player is a worthwhile strategy if the likely distribution of budgets of all other players is such that the player would even like to expend more than he has, even if he anticipates that all other players will expend their full budgets. For instance, if the contest success function is the all-pay auction without noise, then a sufficient condition for $x_i = m_i$ to constitute an equilibrium is shown in Konrad (2011) to be

$$(n-1)(F(m))^{n-2}F'(m) \geq 1 \text{ for all } m \in (0, b). \tag{3}$$

The condition (3) states that, for any level m, the marginal increase in win-probability from expending one additional marginal unit more effort (left-hand side of (3)) is at least as high as the marginal cost of this additional unit of effort (right-hand side of (3)).

These expenditures $x_i = m_i$ are worthwhile for player i under this condition only in the absence of information about the actual resources expended by other players. If a player i has a small budget and knows that one or several other players can and will expend more effort, the player i may want to withdraw. This is particularly true if the contest success function is highly discriminatory, like, for instance, for the all-pay auction without noise. Two players i and j would generally benefit if they could truthfully exchange information about their budget limits m_i and m_j. If p is the contest success function of the all-pay auction, the player with the higher budget may then still want to continue to expend very much, given that there are other players who are expected to expend their whole budget. But the player i who learns that his budget falls short of the budget of j may withdraw from the competition and save his own resources, instead of burning them in a competition that is impossible to win. The information exchange is then beneficial for both players in expectation. This mutual benefit of information explains why players benefit from forming alliances in this framework, even if the only implication of being in an alliance with another player is the exchange of information about their budget limits. Based on this principle, the formation of such information alliances is always beneficial for the members of such an alliance, and the merger of two alliances to an even larger alliance is also beneficial. This suggests that, with other aspects being absent, a process of alliance formation comes to an end in this framework only if the smallest number of alliances that is possible has been reached, i.e., if a further merger between existing alliances is no longer feasible.

7 Multiple fronts and resource transfers

A further important reason for why the formation of an alliance can be advantageous has been illustrated by Kovenock and Roberson (2011). Their analysis is in the context of Colonel Blotto games and departs from the baseline model along several dimensions: First, players have exogenously given amounts of military resources. These resources are used up in the military contest, because they have no use other than in the military competition. Second, players fight against each other at several fronts simultaneously, with a competition for one prize at each front. Their main concern is therefore not about the choice of the total quantity of effort, but about how to use the given capacity of effort along different fronts. They consider alliances which allow players to transfer military resources between them inside the alliance.

A main result of their analysis is that alliances may increase alliance members' payoffs, compared to a situation in which they stand alone. Even the player who transfers military resources to another player and is left with a lower stock of military resources himself may gain from the transfer.

To give a simplified example of the type of situation that has a similar flavor, consider a military conflict between three players that may be considered as countries A, B and C. Let country C have common frontiers with countries A and B, as in Figure 2, and let C fight two wars simultaneously, one at its common frontier with country A, another at the common frontier with country B. By assumption, in each of the two conflicts a prize of size 1 is at stake which is appropriated by the winner of the respective conflict. Consider next the respective conflicts and the rules that apply more specifically. Each country has a given endowment of military resources, denoted as m_A, m_B and m_C. Recall that the military resources have no value or use other than in the military conflict. Let $x_{ij} \geq 0$ denote the military resources used by country i in the direct conflict with country j. By definition, $x_{AB} = x_{BA} = 0$, as these countries have no common frontier. Let country C's probability of winning the respective conflict at its frontiers with A and B be defined by the lottery contest; i.e., $p_{Cj}(x_{Cj}, x_{jC}) = x_{Cj}/(x_{Cj} + x_{jC})$, if $x_{Cj} + x_{jC} > 0$, and equal to 1/2 otherwise. Of course, the respective win probabilities for A and for B are $p_A = 1 - p_{CA}$ and $p_B = 1 - p_{CB}$.[12]

In the absence of an alliance between A and B we require that $x_{AC} \in [0, m_A]$, $x_{BC} \in [0, m_B]$, and $x_{CA} + x_{CB} \in [0, m_C]$. Consider now the equilibrium choices of efforts. As p_A and p_B are increasing in x_{AC} and x_{BC}, we can safely assume

[12] Here the example departs from Kovenock and Roberson (2008), as they consider a Colonell-Blotto game in which the conflict at each front is determined by the rules of an all-pay auction without noise. Also, they consider a framework with many players and battle fronts, allowing for much richer combinations of resource transfers.

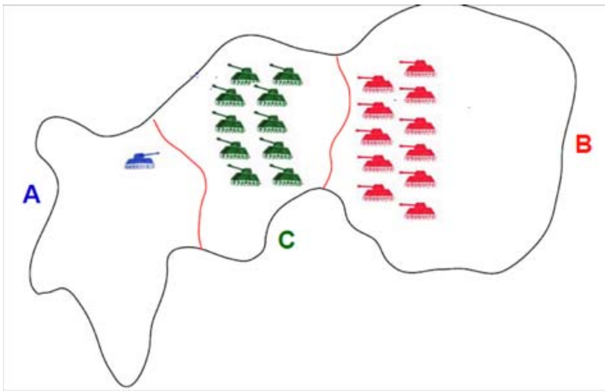

Figure 2: Two-front war with very unequal alliance partners: the initial distribution of military capacity.

that $x_{AC} = m_A$ and $x_{BC} = m_B$. Moreover, $x_{CA}+x_{CB} = m_C$ in the equilibrium will also hold, given that military effort within the given capacity limits has no opportunity cost. The problem therefore reduces to the optimal division of m_C between the two fronts. The (interior) solution for an optimum requires that military effort has the same marginal impact in both fronts, that is

$$\frac{\partial p_{CA}}{\partial x_{CA}} = \frac{m_A}{(m_A + x_{CA})^2} = \frac{m_B}{(m_B + (m_C - x_{CA}))^2} = \frac{\partial p_{CB}}{\partial x_{CB}} \tag{4}$$

Consider now a possible alliance between A and B, and assume that the alliance allows them to reallocate military capacity between them, prior to the two actual conflicts. For instance, in Figure 2, assume that country B may consider to ship a number of tanks from country B to country A, hence, enhancing the military capacity in A from m_A to $m_A + D$, and reducing the military capacity in country B from m_B to $m_B - D$. As it fits with the non-cooperative framework we consider, let this be an unconditional transfer of resources; let us rule out any transfers or side payments between the two countries in particular, both prior to, and past to the actual conflicts. We ask whether such a resource transfer can be desirable from the perspective of country B. The answer is less obvious than it may seem. The resource transfer reduces the fighting power of B. Considered in isolation, this is bad for country B. If country C were to divide its military capacity between the two fronts in the same way as in the absence of the transfer, country B would simply reduce its win probability. However, the transfer makes country A a stronger military power. This is observed by country C, and country C may react to this fact and shift more of its overall military capacity from the front with country B to the front with country A. This relocation of military resources by country C is advantageous for player B. Whether or not an unconditional resource transfer from B to A can improve B's payoff then depends on whether the effect of directly weakening own military power is stronger or weaker than the strategic effect that is caused by the relocation of military resources away from the front between B and C.

The answer about the relative size of the direct effect and the strategic effect generally depends on the type of contest success functions that apply for the two

m_A	m_B	m_C	x_{CB}	x_{CA}	π_B	π_A
1	11	10	5.9	4.1	0.65076	0.19621
2	10	10	5.2	4.8	0.65782	0.29419

Figure 3: Payoffs in the numerical example on two-front war.

fronts, and on the military endowments which are at the command of the three countries. A simple example can illustrate, however, that the strategic effect can be stronger than the direct effect of weakening own military power. Figure 3 considers a specific numerical example. The first row considers a given initial distribution of military capacities, the division of m_C between the two fronts as an optimal reply by C for this given distribution, and the resulting payoffs for A and B. The second line considers a reallocation of military capacity between A and B, the optimal division of m_C as a reply to this changed allocation, and the resulting payoffs. Note that the payoffs of both countries B and A are increased due to the shift of one unit of military capacity from country B to country A. This shows that such a transfer can be self-enforcing in a fully non-cooperative framework.

One of the properties of this example is that the transfer is made from a comparatively well-endowed country to a country with very little military resources. Intuitively, the alliance allows the countries to reallocate their fighting capacity to where it generates higher gains. This, in isolation, increases the sum of their payoffs. Moreover, reallocation of one unit has a very large marginal impact on the military threat that C encounters at the border with country A. This is why C relocates a considerable amount of resources away from the border with B and to the border with A, and this makes such a transfer beneficial for country B.

8 In-group favoritism and out-group spite

Much of the historical evidence on the possible break-up of victorious alliances is seemingly in line with the ideas about the hold-up problem of military and other alliances. Research in psychology, however, pointed at some implications of the formation of an alliance and the existence of an enemy that are not captured in the baseline model. There is considerable evidence that members of a group develop what is called in-group favoritism: members of a group such as an alliance tend to choose actions that benefit the whole group, even if these actions are seemingly not based on narrowly defined selfish material interest (Brewer 1979). They also showed that what is needed in terms of institutional setup, joint history etc. to generate such in-group behavior is extremely little (Tajfel and Turner 1979). This is referred to as the minimal group paradigm. And there is also evidence that in-group favoritism becomes stronger if there is a conflict between this group and another group, referred to as the out-group (e.g., Sherif et al. 1961).

Konrad and Morath (2011) take this observation as the starting point to consider whether in-group altruism and spiteful behavior vis-a-vis the out-group and its members can be rationalized on the basis of evolutionary arguments. They consider a framework in which two symmetric groups, A and B interact in a static game. Each group consists of n members. The two groups fight in a standard Tullock (1980) contest for a prize that is allocated to the members of the winning group and yields a value of 1 to each member of the winning group.[13] Each individual i provides some non-negative effort x_i to the total effort of the own group. Let X_K denote the sum of efforts of members of group $K \in \{A, B\}$, and X_{-K} the sum of efforts in the respective other group. Then the monetary payoff of member i of group K is

$$\pi_i(x_i, \mathbf{x}_{-i}^E) = \frac{X_K}{X_K + X_{-K}} - x_i, \tag{5}$$

if $X_A + X_B > 0$, and equal to $1/2$ if $X_A + X_B = 0$. In (5), $\mathbf{x}_{-i}^E$ is the vector of effort choices of all players other than player i, assuming that all of them choose the effort that is equal to the evolutionarily stable strategy. This problem constitutes the "state game" of an evolutionary game in which each single generation of players interacts in one such state game, but where there is a series of non-overlapping generations and a series of such state games.

Searching for evolutionarily stable strategies in this context, first a set of possible types of players has to be defined. We may define this type space as the interval $[0, \infty)$, where a player of type t is a player who chooses effort $x_i \equiv t$. Applying Schaffer's (1988) concept of evolutionary stability, a population that consists of players of type t^* is a monomorphic equilibrium in evolutionarily stable strategies if t^* is a solution to

$$\max_{x_i} [\pi_i(x_i, \mathbf{x}_{-i}^E) - \pi_{-i}(x_i, \mathbf{x}_{-i}^E)] \tag{6}$$

for $\mathbf{x}_{-i}^E = (t^*, ..., t^*)$ being the vector of the efforts t^* chosen by all $2n-1$ other players. This notion of evolutionary stability compares the monetary payoff of a player who has a possibly different type x_i and who interacts with $2n-1$ players who all are of the same, equilibrium type t^*, with the expected monetary payoff of each of these $2n-1$ players if they interact with $2n-2$ players of type t^* and one player of type x_i. A strategy t^* is evolutionarily stable according to this condition, if there is no other (mutant) strategy that, if used by a single player in an otherwise homogenous population of type t^*, yields a higher payoff to this mutant strategy than to the average non-mutant player.

The Schaffer (1988) definition of evolutionary stability in finite populations as in (6) has an interesting feature: A player who adopts a mutant strategy can do well in this population not only if this mutant strategy yields a higher payoff to this player, but also if this mutant strategy yields a lower average payoff to the other, non-mutant players. A player can therefore improve his evolutionary fitness by adopting actions that harm others, even if these actions do not benefit the player directly, and a player harms himself if he adopts actions that benefit others, even if these actions do not harm the player directly (in terms of monetary payoff). Applied to the inter-alliance conflict, this feature

[13] Their framework therefore includes cases in which the prize is a public good for the members of the group as well as the cases in which the prize is a private good and they have to fight internally about its distribution.

has interesting implications. Consider a player i who contributes more effort to his own alliance. Apart from the direct implications for the player's monetary payoff, this affects both the payoffs of other group members, and the payoffs of members of the out-group. Higher effort by i benefits the members of the in-group, as this higher effort by i increases the win probability for the players in i's alliance. Also, higher effort by i reduces the win probability for the rival alliance; hence, it harms the members of the out-group.

As shown in Konrad and Morath (2011), the evolutionarily stable strategies involve higher effort than the effort emerging in a Nash equilibrium with payoff functions as in (5). Intuitively, let us start at a situation that is an interior Nash equilibrium for players who maximize their material payoffs (5). Consider player i who thinks about increasing his effort just above the equilibrium effort level. This increase has a zero first-order effect for the player's own material payoff – due to the fact that this increase happens at Nash equilibrium levels. However, the increase has two other first-order effects. The additional effort benefits co-players from the same group as i, and it harms players from the rival group, as it makes the own group more likely to win, and reduces the win probability of the other group. From an evolutionary point of view, the first effect is a disadvantage for i, the second effect is an advantage for i. Overall, starting from a Nash equilibrium, the beneficial effect for i from an increase in own effort dominates.

They also consider the evolutionarily stable preferences that can implement these higher efforts as mutually optimal replies to each other for individuals who maximize these preferences by their choices of effort. They find that a combination of in-group altruism and spiteful preferences vis-a-vis members of the out-group is suitable for inducing such effort choices as Nash equilibrium choices. And an interpretation of these findings is in line with the in-group favoritism found by psychologists.

9 Conclusions

Alliances are very common, despite possible free-riding inside the alliance and a possible hold-up problem that is due to fighting and rivalry inside the alliance. This suggests that there are benefits of alliance formation that counterbalance these strategic disadvantages, or reasons why these strategic disadvantages do not have strong effects. This paper highlights some of the reasons that have been put forward why the strategic disadvantages may be less strong than in a benchmark model of alliances. The survey also highlighted a number of potential benefits of fighting inside an alliance. Among these is the possible synergies between alliance players' efforts. What they expend individually may have a total impact that is higher than the sum of their individual efforts if the alliance gives them means to use their military capacity in a superior fashion. On the general level, there are many reasons for why this may be the case. Also, even if the strategic disadvantage from a possible hold-up problem exists, it need not dominate the benefits from joint action if players' budgets are small in comparison to the stakes they are fighting about. One mechanism that overcomes both problems simultaneously is based on the idea that internal rivalry and fighting may be triggered by a lack of contributions to the total alliance effort. The threat of internal conflict may then fully eliminate the strategic disadvantages.

Information exchange inside the alliance has been identified as another possible benefit from alliance formation that is absent in the baseline model. The survey also highlighted one instant in which this may be due to multiple fronts and the possibility to optimally use a given military capacity inside an alliance. Finally, the paper touched upon the literature on the role of group spirit, the in-group favoritism and the negative attitudes for members of other groups, particularly if the own group competes with these outgroups. In a fight between groups, in-group favoritism and out-group spite can be based on evolutionary grounds and may allow players to overcome the free-riding problem and induce them to make high effort.

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