Technical Paper

7/RT/96

December 1996

Market Risk:

An introduction to the concept & analytics

of Value-at-risk.

by

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Abstract

In recent years the concept of Value-at-risk has achieved prominence among risk managers for the purpose of market risk measurement and control. Spurred by the increasing complexity and volume of trade in derivatives, and by the numerous headline cases of institutions sustaining enormous losses from their derivatives activities, risk managers have acknowledged the need for a unified risk measurement and management strategy.

Furthermore, the regulatory authorities, recognising the systemic threat posed by the growth and complexity of derivatives trading, moved swiftly to address this problem. As a result, the European Union approved EC/93/6, "The Capital Adequacy Directive", which mandates financial institutions to quantify and measure risk on an aggregate basis and to set aside capital to cover potential losses which might accrue from their market positions.

More recently, the Basle committee of the BIS published an amendment to the "Capital Accord" which makes provision for the use of proprietary in-house models to be employed instead of the original framework. The proposed basis of these in-house models is the value-at-risk framework.

In this paper, we present an introductory exposition to the concept of Value-at-risk describing, among other things, the methods commonly employed in its calculation, and a brief critique of each.

Introduction

While most financial institutions are particularly proficient at measuring returns and constructing benchmarks to evaluate performance, it is argued that this expertise does not extend to the measurement of risk[1]. However, it is a universally accepted precept of modern financial economics that efficient portfolios can yield higher returns only at the expense of higher risk. Performance analysis based solely on realised returns belies this very fundamental economic principle and is, therefore, incomplete. In addition, several other factors may be identified as motivating recent levels of interest in market risk. Foremost among these is the increased variety, complexity and volume of trade in financial instruments and derivatives. A rough indication of the volume of trade in derivative instruments is provided by the underlying value on which outstanding derivative contracts are based. According to the most recent study of derivative markets activity conducted by the Bank for International Settlements (BIS) at end-March 1995 this figure stood at US$40,637 billion with estimated daily global turnover of US$880 billion[2]. Secondly, in recent years the financial community has witnessed several high profile financial disasters where the institutions involved sustained enormous losses resulting from their derivatives trading, the most notable examples being Metallgesellschaft (with losses of over $1 billion), Sumitomo Corporation (estimated losses of $1.8 billion), Barings Bank (losses of $1.3 billion), Kashima Oil (losses over $1 billion) and Orange county (with realised losses of $1.69 billion). Finally, regulatory authorities, recognising the systemic threat posed by the growth in derivatives trading, have moved swiftly to address the problem. At the same time the Basle committee of the BIS presented the "Capital Accord", the European Union approved directive EC/93/6, "The Capital Adequacy Directive" (CAD) which came into force in January 1996. With few exceptions the CAD and the Capital Accord are exactly the same; both require financial institutions to quantify and report market exposures on an aggregate basis and set out frameworks for applying capital charges to the market risks incurred by the market activities of banks and investment companies. From a supervisory perspective the motivations for enforcing this requirement are well founded; consolidation of exposure on an institution-wide basis reduces the possibility

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[1] See JP Morgan (1994) for discussion.
[2] See BIS(1996) for detailed analysis.

of contagion effects[3] and double gearing[4]. This ensures that the financial institution has an adequate capital base to support the level of business being conducted and to act as a cushion against potentially disastrous losses. Further to industry consultations, the Basle committee has recently published an amendment to the Capital Accord entitled, "Amendment to the capital accord to incorporate market risks". One of the main proposals of the new document is to permit the use of proprietary in-house models for measuring risks as an alternative to the standardised measurement framework originally proposed. The foundation of the proposed alternative is based on the "value-at-risk" framework.

Value-at-risk (VAR) was originally identified by the Group of Thirty study of derivatives trading as a useful market risk management tool. It provides in a single figure a measure of an institutions exposure to market movements. The basic notions underpinning VAR are as old as the theory of statistics itself. The major innovation, however, was the re-statement of complex statistical ideas in a non-technical manner. Its swift introduction and widespread acceptance owes much to the chairman of JP Morgan who insisted on seeing, each day, a one page summary of his bank's aggregate risk exposure. In 1994, that bank published comprehensive details of the methodology they used and offered extensive data free to anyone who wished to use it. In the meantime, many other financial institutions have adopted their own VAR measurement systems and many have made them available to their clients. The purpose of this paper is to define market risk with a specific emphasis on the concept of "value-at-risk".

1. Market Risk

Market risk can be defined as the risk to an institution's financial condition resulting from adverse movements in the level or volatility of market prices. The process of market risk management is, therefore, an endeavour to measure and monitor risk in a unified manner. By implication, this necessitates the aggregation of market risks across all categories of assets and derivatives in a firm's trading book. One method of

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[3] The spreading of losses to other parts of the same group.
[4] Where the same capital is used by a parent and its subsidiary to support both businesses, for example.

accomplishing this task is achieved through the concept of Value-at-risk (VAR). VAR is an attempt to summarise the total market risk associated with a firm's trading book in a single monetary figure. VAR is defined as "the maximum possible loss with a known confidence interval over an orderly liquidation period" Wilson (1993, p.40). VAR seeks to "translate all instruments into units of risk or potential loss based on certain parameters" Chew (1994, p.65). While the concept of VAR is firmly grounded in probability theory various methods may be employed in practice.

To be useful, VAR models must accurately capture the risk profile of a portfolio insofar as it must describe how a portfolio will react when shocked. For certain categories of traditional assets this process is relatively straightforward since many are only exposed to price or rate risk. Take, for example, a spot foreign exchange transaction, the dealer is only exposed to the risk that the relevant exchange rate changes. This is also true for equity positions. The risk associated with bonds is slightly different since the relationship between the spot rate and the bond's price is non-linear. Unlike those associated with traditional assets, the risks attendant on derivatives trading are inextricably more complex. Hence, the inclusion of derivative products in the trading books complicates the computation of VAR. A report compiled by the Group of thirty (1993, p.44) identifies six distinct types of risks associated with derivatives summarised below[5]:

- *Absolute price or rate (or delta) risk*: the exposure to a change in the value of a transaction or portfolio corresponding to a given change in the price of the underlying asset.

- *Discount rate (or rho) risk*: the exposure to a change in value of a transaction or portfolio corresponding to a change in the rate used for discounting future cash flows.

- *Convexity (or gamma) risk*: the risk that arises when the relationship between the price of the underlying asset and the value of the transaction or portfolio is not linear. The greater the non-linearity (i.e., convexity) the greater the risk.

- *Basis (or correlation) risk*: the exposure of a transaction or portfolio to differences in the price performance of the derivatives it contains and their hedges.

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[5] For a more detailed discussion of option risk see Meegan (1995) and Browne *et al*.(1994).

- *Volatility (or vega risk):* the exposure in the value of a transaction or portfolio associated with a change in the volatility of the price of the underlying. This type of risk is typically associated with options.

- *Time decay (or theta) risk*: the exposure to a change in the value of a transaction or portfolio arising from the passage of time. Once again this risk is typically associated with options.

Clearly, this very brief discussion highlights the complexity involved in quantifying market risks particularly when the instrument under consideration is a derivative. The process of market risk quantification becomes inextricably more complex when one is considering a portfolio comprising traditional assets and derivatives. Not alone must one consider the risks associated with particular classes of assets but also the interdependence between individual positions.

2. Correlation Method

The correlation method, otherwise known as the variance/covariance method, is essentially a parametric approach in which an estimate of VAR is derived from the underlying variances and covariances of the constituents of a portfolio. It should be noted that the correlation approach to VAR is not, by any means, a new or revolutionary concept. In portfolio theory it is the basis of standard Markowitz mean-variance analysis. To estimate VAR certain statistical assumptions are made about the distribution of returns which allow us to express risk in monetary rather than standard deviation terms. Several variations of the correlation method exist, however, in this section we will confine our discussion to the portfolio-normal, asset-normal and delta-normal approaches. Thereafter we will turn our attentions to the interpretation of parametric VAR estimates and the validity of the statistical assumptions upon which many estimates are based.

2.1. Portfolio-normal and asset-normal approaches

The portfolio-normal method which is the simplest method is calculated as a multiple of the standard deviation of the aggregate portfolio's return:

$$VAR_p = \alpha s_p \sqrt{t} \qquad \textbf{Eq.1}$$

$$\Delta P \sim N(m_p, s_p) \qquad \textbf{Eq.2}$$

where s_p is the standard deviation of the entire portfolio's return in a unit period, a is the constant that gives the one-tailed confidence interval for the normal distribution and t is the orderly liquidation period. Clearly, the portfolio-normal method is a simplification of the problem since it considers the aggregate portfolio return as opposed to the component asset return. In effect this specification reduces the dimensionality and hence the complexity of the problem. Consider a time series of portfolio returns running over the period 1.........T which consists of N assets. With the portfolio-normal method instead of considering an NxT matrix of returns one merely considers a Tx1 vector.

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\begin{bmatrix} P_t \\ . \\ . \\ P_2 \\ P_1 \end{bmatrix} = \begin{bmatrix} r_{1t} & r_{2t} & . & . & r_{Nt} \\ . & . & . & . & . \\ . & . & . & . & . \\ r_{12} & r_{22} & . & . & r_{N2} \\ r_{11} & r_{12} & . & . & r_{N1} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ w_N \end{bmatrix}
$$

or, $\textbf{\textit{P}} = \textbf{\textit{r}}\,\textbf{\textit{w}}$ **Eq.3**

where P is the vector of portfolio returns over T observations, r the matrix of asset returns and w the vector of weights of each asset in the portfolio. Implicitly, this formulation assumes that asset weights remain constant. Given this simplification one can show that the portfolio-normal and asset-normal approaches are in effect equivalent[6]. The portfolio-normal approach will give an accurate reflection of the risk on a portfolio only when the weights remain constant or nearly constant over time. The alternative, known as the asset-normal approach considers the variance/covariance matrix of the individual assets, i.e. the right hand side of equation 3.

Essentially, the asset-normal approach disaggregates the overall portfolio return into the returns on the component assets. Thereafter, one determines the value-at-risk by considering the variance/covariance matrix of the respective returns. The asset-normal approach assumes that the Nx1 vector of returns for the individual assets is jointly normal:

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[6] See Appendix A.

$$\Delta P \sim N(\mathbf{m}, \Sigma) \qquad\qquad \textbf{Eq.4}$$

where Σ is the NxN covariance matrix of returns. In this case the variance on the portfolio is:

$$\mathbf{s}_p^2 = \mathbf{w}'\Sigma\mathbf{w} \qquad\qquad \textbf{Eq.5}$$

When the number of assets comprising the portfolio, N, is large the asset-normal approach can become cumbersome since one must estimate $N/2(N+1)$ variances and covariances. For example, suppose N=100 assets, this requires estimation of 5050 parameters of which 4950 are covariances.

The portfolio normal is clearly a simplification of the true relationships. Therefore, Wilson (1994 p.79) argues that the portfolio-normal approach might be used as a rough estimate of VAR at the business unit level: "For example, consider the calculation of VAR of an equity trading unit, achieved by dividing monthly income over the past three to five years by the market value of its equity holdings. Based on this time series one could estimate the volatility of returns per dollar invested in the equity portfolio to estimate the capital needed to support each dollar worth of open equity positions." In contrast, the asset normal is a more rigorous approach to calculating VAR since the resulting estimate of risk takes account of the effects of changing portfolio weights in the period over which the variances and covariances are estimated. By comparison with the portfolio-normal approach the asset-normal approach appears to require the estimation of an excessive number of parameters. This difference is illusory as the calculation of these parameters is implicit in the portfolio-normal approach[7].

2.2. The delta-normal approach

Clearly as the number of positions comprising a portfolio becomes large the asset-normal approach becomes rather unwieldy. In this case it may be more practical to concentrate on the risk factors which drive the prices of particular categories of assets rather than on the prices of the individual asset themselves. For example, one would model the risk of a bond not as the standard deviation of the bond's price but as the standard deviation of the appropriate spot rate, obtained from the term structure,

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[7] The equivalence of the two approaches in the case of constant weights in established in Appendix A.

multiplied by its sensitivity to that spot rate, i.e., its modified duration. Hence, the delta-normal approach is based on a factor delta decomposition of the portfolio.

The change in the value of a portfolio is modelled as the change in the factor times the net factor delta:

$$\Delta P = \boldsymbol{d}\Delta F \qquad\qquad \textbf{Eq.6}$$

where F is an $M \times 1$ vector of risk factors and $\boldsymbol{d}$ is an $M \times 1$ vector of net deltas with respect to each of these factors:

$$\boldsymbol{d}_i = \sum_{j=1}^{i} \boldsymbol{w}_j \, \P A_j \,/\, \P F_i \qquad\qquad \textbf{Eq.7}$$

where, $\boldsymbol{w}_j$ and A_j are the amount and price of asset j in the portfolio respectively.

Value-at-risk is then calculated as:

$$VAR_d = \boldsymbol{a}\boldsymbol{s}_d\sqrt{t} \qquad\qquad \textbf{Eq.8}$$

where, as before, $\boldsymbol{a}$ is defined as the constant that gives the appropriate one-tailed confidence interval for the standardised normal distribution, t is the orderly liquidation period, and $\boldsymbol{s}_d = \sqrt{\boldsymbol{d}\Sigma\boldsymbol{d}}$ is the standard deviation of the portfolio's return, calculated as a function of the portfolio's delta; $\boldsymbol{d}$ is an Mx1 vector of rate sensitivities, and Σ is the MxM covariance matrix of the risk factors.

Implicitly, it is assumed once again that portfolio returns are normally distributed. This result can only follow from two further assumptions; the first is that market rate innovations for the various risk factors are jointly normal:

$$\Delta F \sim N(\boldsymbol{m},\Sigma) \qquad\qquad \textbf{Eq.9}$$

The second assumption, which is the more problematic, pertains to the way in which the risk factor is related to the price. Implicitly, the only way the portfolio returns can be normally distributed is if the price function, i.e., the equation relating the price of an instrument to its risk factor, is linear. Essentially this implies that the delta's are entirely sufficient to characterise the price/rate function completely. As Wilson (op.cit) shows this necessarily implies that the price functions are reasonably approximated by a first-order Taylor series around the current market price. This is called a delta approximation.

$$\Delta P = q\Delta t + d\Delta F + o(2) \qquad \textbf{Eq.10}$$

$$\Delta P \approx q\Delta t + d\Delta F \qquad \textbf{Eq.11}$$

where q is the portfolio theta or $\partial P(F)/\partial t$, d is the portfolio Mx1 vector of delta sensitivities or $\partial P(F)/\partial F$. Using the properties of the normal distribution, it follows directly that portfolio returns are normally distributed. Clearly, a problem arises if this relationship is non-linear which is the case even for bonds.

Interpreting parametric VAR estimates

The assumption of normality of portfolio returns is widely used in VAR methodologies primarily because of its tractability. The portfolio-normal, asset-normal and delta-normal approaches provide their own measure of the one-period standard deviation (variance) of the returns to the portfolio. If the orderly liquidation period consists is deemed to be *t* periods the value-at-risk is given by

$$VAR = as\sqrt{t} \qquad \textbf{Eq.12}$$

where s is the measure of the standard deviation of aggregate portfolio returns and a is the constant that gives the one-sided confidence level for the normal distribution. The assumption of normality allows us to interpret this statistics as follows; if the 99% one-sided confidence interval for the t-statistic is 2.33 and taking *t* as 10 days we may conclude that the loss incurred by the portfolio will exceed

$$VAR = 2.33s\sqrt{10} = 7.37s \qquad \textbf{Eq.13}$$

on average, only in one 10-day period in every one-hundred 10-day periods. Equivalently, the loss on the portfolio will be less than this figure 99 times out of every 100. By an appropriate choice of a we can increase VAR to a level that will be exceeded less often. If we take a value of a equal to 3.09 the resulting VAR will only be exceeded once in every 1000 periods.

To many commentators the notion of value-at-risk imports a large measure of precision and certitude. This trust is unfounded and misplaced. The conclusions about VAR hinge critically on the assumption of normality about which we will be saying more later on. However, even with confidence levels of 99% or even 99.9% one would expect one failure per 100 periods or 1000 periods per institution which would

be completely unacceptable. This is a fact often glossed over by those same commentators.

2.3. Statistical assumptions

In each of the approaches examined so far we have interpreted a multiple of the estimated standard deviation of the value of the portfolio as the Value-at-risk. The particular multiples chosen are valid only if the probability density function of that value is normal. This assumption may be justified in several ways;

1. The joint distribution of the prices of the individual assets in the portfolio follow a stationary multivariate normal process. However, as many prices are known to contain random walk elements it would be very difficult to uphold such a hypothesis;

2. The prices of the assets in the portfolio are generated by a continuous diffusion process; or,

3. Each series is generated by a normal random walk:

$$y_t = y_{t-1} + e_t \qquad e_t \sim i.i.d. \, N(0, s^2) \qquad \textbf{Eq.13}$$

Where y_t is the observation at time t, e_t is the random innovation at time t which is serially uncorrelated, identically and normally distributed. We will now look at the implication of each of these assumptions.

• Serially uncorrelated - implies that current values of e are serially uncorrelated with past values, i.e., $E(e_t e_k) = r_k = 0$, $t \neq k$, e is a white noise process (i.e. a purely random disturbance term).

• Identically distributed - all e's are drawn from the same distribution.

• Normality - implies that the distribution referred to above is assumed to be normal with mean 0 and variance s^2.

Empirical research abounds, in particular, refuting the hypothesis that the market rate innovations are normal and identically distributed (see Bollerslev (1987,1990), Engle (1982,1993), Theodossiou (1993), Fell (1994), Mandelbrot (1963,1967) Friedman et al. (1982), Hsieh (1988) and Westerfield (1977a, 1977b))[8].

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[8] For a more comprehensive review of the literature and analysis of the consequences of the violation of statistical assumptions see Meegan (1995).

The principal problem in applying the correlation method relates to the manifest instability, i.e., non-stationarity of the key statistical relationships of and between financial series, namely risk, return and correlation (see *inter alia* Bollerslev (1986,1987,1990), Thoedossiou (1993) and Fell (1994)). Clearly, this is major problem as calculations based on incorrect parameter estimates of inappropriate distributional assumptions will produce inaccurate VAR estimates.

A general finding relating to the behaviour of high frequency financial data concerns the presence of volatility clustering. Mandelbrot (1963) observed that large changes tend to follow large changes while small changes tend to follow other small changes of unpredictable sign. Therefore, various authors (such as Engle (1982) and Bollerslev (1986)) have argued that market innovations are drawn from a distribution whose variance changes over time.

Secondly, the multiplicative factor used to assign a confidence interval to the computed VAR depends on the form of the hypothesised distribution. Hence, our ability to calculate potential loss with a given confidence interval hinges crucially on the validity of those distributional assumptions. However, empirical research suggests that the normal distribution is inadequate to capture the tail events in that it attributes a lower probability to the occurrence of those events than is typically observed in reality. Consequently, the assumption of normality will typically understate the level of risk. In other words, one would expect that the computed VAR numbers will be exceeded more frequently than would be predicted by the normal distribution.

3. Estimating risk using Monte Carlo simulation

To use Monte Carlo simulations to estimate risk we need a knowledge of the probability distributions of the individual assets in our portfolio. These distributions can take any form and may include diffusions, and distributions derived from random walks and non-normal probability distributions. It is likely that in these circumstances the probability distributions of the value of the portfolio cannot be derived in closed-form. We may be able to use linear approximations (delta) for valuation but the extent of non-linearities may cause some concern. The methods described here may be used to look at the error imposed by the assumption of linearity.

Suppose that we have 40 assets, some of which may be derivatives. We draw a large number (say 10,000) sets of 40 random numbers. Each set of 40 random numbers is used in conjunction with the known distributions to generate a scenario or possible value of the portfolio. We are thus left with a set of 10,000 scenarios. Value-at-risk is measured by reference to the worst 5% of these scenarios.

4. Non-parametric methods - Historical Simulation

Among the alternative approaches typically employed in VAR calculations, historical simulation, otherwise known as backtesting, is conceptually the simplest. At its most basic level, historical simulation involves revaluing an arbitrary portfolio using actual historical price and rate series as they prevailed over a suitably long period and calculating the resulting return series. This series will give a good indication of the behaviour of the portfolio under typical market conditions. An examination of the extreme values or outliers will give a good indication of VAR. In order to determine the distribution of returns it may be useful to generate a histogram. However, Allen (1994) contends that a non-parametric approach should be adopted in favour of a standard deviation based approach since the actual distribution will typically deviate from the normal. "Using this technique, it is possible to calculate the 99% confidence interval without assuming that price changes are normally distributed, by computing the loss which was not exceeded on 99% of occasions." Jackson (1995 p.180). Ideally, non-parametric methods should be used in parallel with the parametric methods described earlier. While the resulting VAR estimates will not be the same, this need not give rise to concern. Particular attention should, however, be paid if the respective VAR estimates are diverging.

4.1. Assumptions and implications.
The historical simulation method is essentially a non-parametric approach to quantifying VAR since it makes minimal assumptions regarding the distributional properties or the rate/price generating stochastic process. It simply assumes that future market price/rate changes are drawn from the same distribution as those which generated the historical return series; essentially one assumes that the past is

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representative of the future. With good reason, practitioners may find this assumption particularly unpalatable.

One of the main advantages of the historical simulation approach is the absence of arbitrary statistical and distributional assumptions which were discussed previously. Hence, model risk is not inadvertently introduced into the calculation of VAR through model misspecification. Allen (1994) summarises the main advantages of the historical simulation approach over parametric approaches:

1. As actual changes in underlying prices are used this approach is more apt to capture outlier events that occurred over the historical period, rather than relying on a standard deviation approach that could potentially understate risk by not properly capturing the tail events.

2. For each set of returns we are capturing the actual covariation between risk factors.

3. This method also overcomes problems inherent in the variance/covariance approach due to convexity.

"Since we use the same data for the simulations as for calculating the risk factor volatilities and correlations, we can compare these results directly. The deviation between them will give a measure of the error introduced in the correlation-based technique due to violation of its underlying assumptions and convexity." Allen (op.cit. p.78)

5. Stress Testing

The methods discussed hitherto are largely only appropriate under normal or stable market conditions. However, in stressed financial markets many previously stable relationships, particularly correlation, break down. Indeed, certain practitioners would argue that correlation is inherently unstable even in non-stressed financial markets. Therein lie the main problems associated with both the correlation and historical simulation approaches since both rely to varying extents on the premise that the past is indicative of the future and that parameters observed or estimated from historical data will prevail in the future. Hence, when key relationships do break down the value of these methods will be, at best, dubious. Recognising the limitations of the type of model outlined above the G30 report entitled "Risk management guidelines for

derivatives" identifies the necessity to identify the impact of severe or unusual market conditions on the trading book;

"Analysing stress situations, including combinations of market events that could affect the banking organisation, is also an important aspect of risk measurement. Sound risk measurement practices include identifying possible events or changes in market behaviour that could have unfavourable effects on the institution and assessing the ability of the institution to withstand them. These analyses should consider not only the likelihood of adverse events, reflecting their probability, but also 'worst case' scenarios." G30 (1994 p.8)

The objective of stress testing, therefore, is to identify and measure the impact or consequences of unusual or non-recurrent market movements on the behaviour of the trading book. By definition, stress testing is a scenario based method for identifying market risk, since the events we are concerned with are non-recurrent and, hence, cannot be modelled on the basis of the approaches outlined previously. Broadly speaking, stress testing involves the identification of a range of unusual market rate movements whose occurrence, although unlikely, is still possible. The trading book is then revalued on the basis of these scenarios. One of the key outcomes of stress testing is the identification of hidden risks in the trading books which may not have emerged from a VAR analysis.

Being scenario based, stress testing does not rely on imposed or posited statistical assumptions. In particular, the methods used are correlation independent since we wish to identify the behaviour of the trading book under stressed market conditions when these statistical relationships break down.

In the following sections we will briefly discuss three methods commonly used in stress testing; scenario analysis, the factor push method, and maximum loss optimisation. While the factor push and maximum loss optimisation methods appear to differ substantially from one another, they do in fact share a common mathematical link. Subject to certain criteria regarding the nature of the price function being satisfied,

there exists an equivalence between the two methods. We will return to this point later.

5.1 Scenario Analysis

Scenario analysis involves the identification of periods of extreme market movements or excessive volatility. The financial institution then revalues its current portfolio using these historical scenarios and measures the difference between the current market value and its value under the hypothesised scenario. The difference between the two would give an indication of the behaviour of the portfolio under extreme market conditions. A typical scenario might be the stock market crash recorded on Black Monday 1987.

5.2 Factor Push Method

The factor push method is a relatively simple method for determining the effect of stressed financial markets on the behaviour of the trading book. Given the current status of each component of the trading book, i.e., short or long positions, one pushes the relevant price or rate - the risk factors - in the direction which would result in a loss on the trading book. In practice, it is usual to set a boundary on the size of the hypothesised market movements. For example, some companies may shift each rate by two or four standard deviations. Others simply add 1% along the entire length of the yield curve. VAR is then simply calculated by multiplying the percentage change in each price or rate by the vector of net factor sensitivities. This yields a measurement of the percentage change in the value of the current trading book if the hypothesised market prices or rates were to prevail.

5.3 Maximum Loss Optimisation

The procedure for implementing a maximum loss optimisation method is relatively straightforward. One simply defines the maximum range over which the risk factors may vary and, similar to the delta-normal approach, a vector of net factor sensitivities for the identified risk factors. Using numerical optimisation procedures one conducts an exhaustive search for the combination of price changes which maximise portfolio loss.

Mathematically, maximum loss optimisation is a constrained optimisation problem which, in contrast with the factors push method, permits interior solutions. Several important issues are raised here;

1. The problem is constrained - by specifying or pre-determining the permissible variation range, one imposes constraints on the optimisation routine (i.e. the range within which the value that the risk factor assumes may vary);

2. An optimisation problem - using numerical optimisation procedures one conducts an exhaustive search for the combination of permissible price or rate changes which maximise portfolio loss. Mathematically MLO may, therefore, be stated as follows[9]:

$$Maximise \; -\Delta P = c\Delta F$$
$$Subject \; to: B_{L_u} \leq \Delta F_i \leq B_{U_i}$$

Where the price functions of the assets comprising a portfolio are well approximated by first-order Taylor series, the factor push method will, in fact, produce exactly the same results as the more computationally intensive MLO procedure. The factor-push method implicitly assumes that the maximum loss occurs at the boundary of the predetermined variation range - a boundary solution[10]. In the case of equity, exchange rate and interest rate positions this assertion is valid. However, if the portfolio incorporates the characteristics of traded options, one could conceive of a situation where the maximum loss occurred within the pre-defined range - an interior solution. "Consider, for example, a butterfly spread position on which losses occur when rates are stable, not when they reach their extremes. Because we are pushing each of the market rates by n standard deviations, we are actually pushing the position further into the money. In this case the maximum loss is found closer to, rather than further from, the current market prices." Wilson (1994, p.80)

Clearly, under such circumstances the MLO procedure is the more appropriate method since all possible combinations of risk factor moves within the pre-defined variation range are examined.

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[9] Alternately, this is exactly equivalent to; $Min \; \Delta P = c\Delta F$

[10] In mathematical terms the constraint is said to be binding.

In summary, the choice between the MLO method and the Factor-push method will depend entirely on the nature of the payoff. If the investment strategy payoff is linear in the risk factor then the factor-push method will produce exactly the same result as the more computationally intensive MLO. However, if the investment strategy payoff incorporates the characteristics of traded options MLO may be more appropriate method.

Conclusions

Value-at-risk has become the industry standard as a measure of market risk. It brings a level of sophistication and rigour to the measurement of market risk not present previously. Moreover, it expresses risk in an accessible form measured in monetary terms. However, expressing risk as a single monetary figure lends it an air of certainty and irrefutability. Such an interpretation is misguided, VAR is not definitive nor is it an accounting identity. These criticisms are not so much levelled at the concept of VAR as at its frequent interpretation. As we showed, with a 99% confidence interval one would expect that computed VAR to be exceeded once in every 100 periods. This is a perspective not often presented in the literature. Used correctly and if well understood, VAR is a key risk measurement tool which provides an invaluable framework within which to monitor the risk on a portfolio. In this paper we present a brief exposition of the concept of value-at-risk and the various methods employed in its calculation. We also highlight the main inadequacies associated with the respective methods. Bearing in mind the previous discussion, the main conclusion is that no single method is adequate in isolation. As we have shown, the parametric approaches may inadvertently introduce model risk into the VAR estimate through inappropriate statistical assumptions. To alleviate this, it is suggested that the non-parametric methods such as historical simulation be run side-by-side with the parametric approach. We argue that while differences between the two methodologies are to be expected, particular attention should be paid when the respective estimates of VAR begin to diverge. Finally, we argue that both of these methods are only relevant under typical market conditions. In periods of turbulence these methods will not be relevant. To overcome this it is of paramount importance that stress testing be conducted periodically, to identify the effects of stressed markets on the behaviour of the trading book.

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Appendix A: The equivalence of the Portfolio and Asset normal measures of variance and Value-at-risk.

Let x_{it} be the price of asset i (i = 1, 2, 3, ..., M) in period t (t = 1, 2, 3, ..., T). Let w_i be the weight of asset i in the portfolio. It is assumed that these weights are constant.

The value of the portfolio in period t is $\sum_{t=1}^{M} w_i x_{it}$. The average value of the portfolio over the period t =1 to t = T is $\dfrac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{M} w_i x_{it}$. Thus the portfolio normal estimate of the variance is given by;

$$s_p^2 \;=\; \frac{1}{T}\sum_{s=1}^{T}\left[\left(\sum_{i=1}^{M} w_i x_{is} - \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{M} w_j x_{jt}\right)\right]^2$$

$$=\; \frac{1}{T}\sum_{s=1}^{T}\left[\sum_{i=1}^{M} w_i\left(x_{is} - \frac{1}{T}\sum_{t=1}^{T} x_{it}\right)\right]^2$$

$$=\; \frac{1}{T}\sum_{s=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M} w_i\left(x_{is} - \frac{1}{T}\sum_{t=1}^{T} x_{it}\right) w_j\left(x_{jt} - \frac{1}{T}\sum_{t=1}^{T} x_{jt}\right)$$

Changing the order of summation gives;

$$=\; \sum_{i=1}^{M}\sum_{j=1}^{M} w_i\left(\frac{1}{T}\sum_{s=1}^{T}\left(x_{is} - \frac{1}{T}\sum_{i=1}^{T} x_{it}\right)\left(x_{js} - \frac{1}{T}\sum_{j=1}^{T} x_{jt}\right)\right) w_j$$

Note that $\dfrac{1}{T}\sum_{s=1}^{T}\left(x_{is} - \dfrac{1}{T}\sum_{i=1}^{T} x_{it}\right)\left(x_{js} - \dfrac{1}{T}\sum_{j=1}^{T} x_{jt}\right) = s_{ij}$ is the estimate of the covariance matrix of the prices of the assets. If we write

$$\Sigma = \begin{bmatrix} s_{11} & s_{12} & . & s_{1M} \\ s_{21} & s_{22} & . & s_{2M} \\ . & . & . & . \\ s_{M1} & s_{M2} & . & s_{MM} \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ . \\ . \\ . \\ w_M \end{bmatrix}$$

$$s_p^2 = w' \sum w$$

$$= s_A^2$$

which is the asset normal measure of variance. We have proven that when the weights of the various assets in the portfolio are constant the portfolio estimate of variance and thus the Value-at-risk is the same as the asset-normal.

$$s_p^2 = w' \sum w$$

$$= s_A^2$$