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Gains from diversification: a regret theory approach

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Abstract

In this paper we analyze a regret-averse individual best choice in a two risky assets portfolio. We extend previous literature and contribute new results by considering a model with two assets. We get the conditions for the regret-averse investor to diversify the portfolio. We additionally compare the behavior of the regret-averse investor with the behavior of its risk-averse counterpart. We characterize the conditions under which both types of agents behavior coincide.

1   Introduction

Bernoulli (1738) is one of the first to study the portfolio problem with two risky assets. Samuelson (1967) provide a thorough discussion of this problem for a risk-averse investor[1] when the random returns of the assets are independent and identically distributed (i.i.d.). Brumelle (1974) relax Samuelson´s analysis relaxing the i.i.d. condition. Hadar and Russell (1971,1974) prove that investors prefer diversification when two assets have the same distribution.

Many empirical studies show inconsistencies in expected utility theory predictions. We can name the common consequence effect and common ratio effect as one of these inconsistencies, see for example Starmer (2000). These evidences questioned whether the expected utility theory could be representative of economic behavior. Thus, many authors develop new alternative theories.

One of these alternative models of choice under uncertainty is regret theory. The seminal papers by Bell (1982) and Loomes and Sugden (1982, 1987) provide the analytical of regret theory. According to this theory, decision makers considers the feels of regret. The decision maker compares the received outcomes with the best choice under the same state of nature.

Braun and Muermann (2004) study insurance decisions under regret theory. This approach explains insurance choices that risk aversion alone would not explain. In particular, regret aversion prevents an individual from extreme decisions.

Muermann et.al.(2006) apply this analysis to a risk-free and a risky assets portfolio model. They show that regret decision maker would not prefer extremes choices. They find that regret-averse investors would hold more stocks when the equity premium is low, but they would hold less stocks when the equity premium is high.

Mulaudzi et.al.(2008) study the optimal shares of regret-averse banks between loans and treasuries. They show that, similar to previous results, regret-averse banks would always choose optimal weights away from extremes. They show if risk-averse banks select risky portfolios, its regret-averse counterpart would elect a less risky portfolio.

Michenaud and Solnik (2008) apply regret theory to find the best currency hedging choice. Their results were the opposite than traditional expected utility or prospect theories would predict. A regret-averse investor would take a currency exposure despite the absence of risk premium.

Laciana and Weber (2008) present a parametrization of an expected utility model with

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[1]We recall that risk averse decision makers decides according to an increasing and concave utility function.

a regret correcting term. They provide an upper and a lower bound for the regret model parameter k that could be useful for empirical applications in future research.

Wong (2011) applies this analysis to study banks interest spread choices. This work shows that regret-averse banks could be less prudent than risk-averse banks.

Our model is similar of that by Muermann et.al.(2006) and Mulaudzi et.al.(2008). However, we move one step further. We generalized their approach considering a model with two stochastic assets. Therefore, their findings are a special case of our generalized portfolio model.

We show that when the two assets returns are i.i.d. both individuals would choose similar shares. We relax the assumption of i.i.d.and find out necessary and sufficient conditions under which a regret-averse investor would prefer diversification. Finally, we show that a regret-averse investor would prefer to diversify when the two assets returns are negatively dependent and have the same mean.

The results on this paper may be used to extend a series of existing models of decision making under uncertainty, see for example Dodonova and Khoroshilov (2009). In the next section we present some previous results. In section 3, we develop the two risky portfolio model. We finish the paper with the conclusions.

2 Previous results

In this section we will make a brief review of previous results. To distinguish these previous findings with the new ones, we will denote *Propositions* to the existing results and *Theorems* or *Corollaries* to the new ones.[2]

We define a general utility function, which also considers risk aversion, based on the work by Braun and Muermann (2004), as follows:

$$u(x) =: v(x) - kg\left[v(x^{\text{max}}) - v(x)\right] \tag{2.1}$$

where v is a standard Bernoulli utility function $v'(x) > 0$ and $v''(x) < 0$ for all $x \geq 0$. The ex-post optimal level of x is identify by x^{max}.[3] The function g is the regret function, with the following properties: $g(0) = 0$, $g'(x) > 0$ and $g''(x) > 0$ for all $x \geq 0$. The parameter $k \geq 0$ measures the weight of the regret term respect to the first risk aversion term. We note that if $k = 0$ then the utility function becomes $u(x) = v(x)$. As $v(x)$ is a typical Bernoulli utility function, then we are under the theory of a risk-averse decision maker.

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[2]We refer to the cited papers for the proofs of the *Propositions*.

[3]This term will be define later in Section 3.

This problem consists in finding the best weights, $\alpha_k \in [0,1]$ with $k \geq 0$, of an initial wealth w_0,(which we normalize, without loss of generality, to one), to invest in two assets with random returns X and Y.[4]

Therefore the final wealth, W, can be written as follows:

$$W(\alpha_k) = 1 + \alpha_k X + (1 - \alpha_k)Y. \tag{2.2}$$

When $0 < \alpha_k < 1$ a portfolio is diversify. On the other hand, when $\alpha_k = 0$ or $\alpha_k = 1$, a portfolio is specialize.

Therefore, the investor optimization problem is to maximize:

$$\max_{\alpha_k \in [0,1]} Eu\left[1 + \alpha_k X + (1 - \alpha_k)Y\right]. \tag{2.3}$$

This is a general model that includes the risk aversion case. Therefore, we can note by α_0 to the portfolio weights of a risk-averse investor.

Samuelson (1967) prove that diversification always "pays" when the random variables are i.i.d. Besides, he finds that a risk-averse investor would choose equal amount shares of the initial wealth in each asset.

Proposition 2.1 *Let X and Y be two i.i.d. random variables and an investor with a concave utility function, u. Then the maximum of $Eu\left(\alpha_0 X + (1 - \alpha_0)Y\right)$ is $\alpha_0 = \frac{1}{2}$, where E is the expectation operator.*

This proposition is generalize to exchangeable random variables by Marshall and Olkin (1979). We note that i.i.d. random variables are exchangeable, but the opposite is not true. Pellerey and Semeraro (2005) relax the exchangeable assumption, as the following proposition shows.

Proposition 2.2 *Let X and Y be two random variables and an investor with a concave utility function, u. The maximal $Eu\left(\alpha_0 X + (1 - \alpha_0)Y\right)$ is $\alpha_0 = \frac{1}{2}$ if and only if*

$$E\left(X/\left(X + Y\right)\right) = E\left(Y/\left(X + Y\right)\right) \tag{2.4}$$

almost surely.

Brumelle (1974) shows that risk-averse investor may not benefit from diversification. In the next Proposition she shows the necessary and sufficient conditions to assure gains from diversification.

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[4]Hence, we do not consider the case of short selling

Proposition 2.3 *If X and Y be two random variables, then an investor with a concave utility function, u, will invest a positive amount in each asset if and only if*

$$E\left(Xu'\left(X\right)\right) < E\left(Yu'\left(X\right)\right) \ \ and \ E\left(Yu'\left(Y\right)\right) < E\left(Xu'\left(Y\right)\right). \tag{2.5}$$

Hadar and Russell (1974) show that if the marginal distributions are identical diversification is optimal.

Proposition 2.4 *Let X and Y be two random variables with the same marginal distributions. Consider a risk-averse investor with utility function, u. The maximal of $Eu\left(\alpha_0 X + (1 - \alpha_0)Y\right)$ is at an interior point, i.e. $0 < \alpha_0^* < 1$.*

Hadar et.al. (1977) relax the previous result. When the asset´s have same mean and range diversification is still optimal. We show in the next Proposition this result.

Proposition 2.5 *Let X and Y be two random variables with same mean and same range and an investor with a concave utility function, u. The maximal expected utility of $Eu\left(\alpha_0 X + (1 - \alpha_0)Y\right)$ is at an interior point .*

So far, we have reviewed some results for risk-averse investors. Now, we present some previous results of portfolio selection under regret theory. Muermann et.al.(2006) and Mulaudzi et.al.(2008) study preferences for diversification when the choices consist in one-safe and one-risky asset. In their model, the risky asset has random returns identify by R and the risk-free asset has returns equal to r_f. From the first order condition we get the following result.[5]

Proposition 2.6   *1. If $ER = r_f$ then $\alpha_k^* > 0$ for all $k > 0$ and $\alpha_0^* = 0$.*

2. If $ER - r_f = \frac{Cov[-R, v'(1+R)]}{Ev'(1+R)}$ then $\alpha_k^* < 1$ for all $k > 0$ and $\alpha_0^* = 1$.

In the first case, a regret-averse investor would invest some amount in the risky asset. However, a risk-averse decision maker would invest all the initial wealth in the risk-free asset. On the other hand, in the second case, a regret-averse investors would invest some amount of her money in the risk-free asset. However, a risk-averse investors would specialized, investing all the initial wealth in the risky asset.

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[5]With our notation this means $X = R$ and $Y = r_f$.

3   The two risky assets model

In this section we develop a one-period portfolio selection model for two risky assets. The investor uses the same two-attribute regret utility function, $u(x)$, as in Eq.(2.1). However, instead of deciding between a risky assets and a risk-free assets, as in the work by Muermann et.al.(2006) and Mulaudzi et.al.(2008), now we consider two random assets. Let X and Y be random variables that reflects the returns of these two risky assets. Therefore, we can write the final wealth as:

$$W(\alpha_k) = 1 + \alpha_k X + (1 - \alpha_k)Y. \tag{3.1}$$

Therefore, the regret investor utility function is define as follows,

$$u\left(W(\alpha_k)\right) = v(W(\alpha_k)) - kg\left[v(W^{\max}) - v(W(\alpha_k))\right]. \tag{3.2}$$

The term $W^{\max}$ is the ex post optimal final wealth if the investor had chosen the optimal choice for each state of the world. We note that $W^{\max}$ is a random variable independent of α_k. In particular, is define as follows:

$$W^{\max} = \begin{cases} 1 + X & \text{if} \quad Y \le X \\ 1 + Y & \text{if} \quad Y \ge X \end{cases}. \tag{3.3}$$

If the realized returns of X is larger than Y then the best choice the investor would had chosen is $\alpha_k = 1$. On the other hand, the investor would had wanted to invest all his wealth in Y, when the realized returns of Y are larger than X.

The problem is to find the allocation, α_k, that maximizes the following expected utility function:

$$V(\alpha_k) =: Eu\left(1 + \alpha_k X + (1 - \alpha_k)Y\right). \tag{3.4}$$

The first order condition to maximize Eq.(3.4) is:

$$\begin{aligned} V'(\alpha_k) &= \frac{dEu(W(\alpha_k))}{d\alpha_k} \\ &= E(X - Y)v'(W(\alpha_k)) + kE(X - Y)v'(W(\alpha_k))g'(v(W^{\max}) - v(W(\alpha_k))) = 0 \end{aligned} \tag{3.5}$$

and the second order condition is:

$$
\begin{aligned}
V''(\alpha_k) &= \frac{d^2 E u\left(W(\alpha_k)\right)}{d\alpha_k^2} \\
&= \frac{d^2 E v(W(\alpha_k))}{d\alpha_k^2} - kE(X-Y)^2 v'(W(\alpha_k))^2 g''(v(W^{\max}) \\
&\quad - v(W(\alpha_k))) + kE(X-Y)^2 v''(W(\alpha_k)) g'(v(W^{\max} - v(W(\alpha_k)))). \quad (3.6)
\end{aligned}
$$

The second derivative of $V(\alpha_k)$ is negative since $k \geq 0$, v is strictly concave and g is strictly convex. Therefore, we have a unique global optimum, α_k^*. However, we cannot assure is an interior optimum (diversification is prefer) or is a binding solution (specialization is prefer). As V is a concave function, $\alpha_k^* = 0$ is optimal if and only if $V'(0) \leq 0$. Similarly, $\alpha_k^* = 1$ is optimal if and only if $V'(1) \geq 0$. We will later discuss the conditions to have an interior optimum.

The simplest assumption is to assume that the random variables are i.i.d. We have seen that risk-averse investors will invest equal amounts of their initial wealth in this case. Does a regret-averse investor will choose this allocation? The answer is affirmative, as the following Theorem shows.

Theorem 3.1 *If X and Y are i.i.d. random variables, then an investor will invest equal amount of its wealth in each asset, i.e. $\alpha_k^* = \frac{1}{2}$ for all $k \geq 0$.*

Proof. The first order condition, Eq.(3.5), can be rewritten as:

$$
E(X-Y)v'(W(\alpha_k)) + kE(X-Y)v'(W(\alpha_k))g'(v(W^{\max}) - v(W(\alpha_k))) = 0 \quad (3.7)
$$

Since X and Y are i.i.d we would have $E\left[Xv'W\left(\alpha_k^* = \frac{1}{2}\right)\right] = E\left[Yv'W\left(\alpha_k^* = \frac{1}{2}\right)\right]$, thus $E(X-Y)v'\left(W\left(\alpha_k^* = \frac{1}{2}\right)\right) = 0$. For this reason, we have :

$$
E(X-Y)v'(W\left(\alpha_k^* = \frac{1}{2}\right))g'\left[v(W^{\max}) - v(W\left(\alpha_k^* = \frac{1}{2}\right))\right] = 0. \quad (3.8)
$$

The second order condition holds by concavity of $V(\alpha_k)$. ∎

This result shows that a regret-averse investors and a risk-averse investors would coincide in their allocation weights if the random returns are i.i.d. In this particular case regret-averse and risk averse investors would coincide in their optimal allocation.

If we relax the i.i.d condition, the analysis becomes a little bit more complex. In the next Theorem we consider independent random returns, but not identical distributed.

Theorem 3.2 *Consider the assets returns X and Y independent and not identical distributed:*

1. Then $\alpha_k^* > 0$ for all $k > 0$ if and only if $EX - EY \geq \frac{Cov[Y, v'(1+Y)]}{Ev'(1+Y)}$.

2. Then $\alpha_k^* < 1$ for all $k > 0$ if and only if $EX - EY \leq \frac{Cov[-X, v'(1+X)]}{Ev'(1+X)}$.

Proof. For the first part, since V is strictly concave function, we need to check the sign of Eq.(3.5) evaluated at $\alpha_k = 0, V'(\alpha_k)|_{\alpha_k=0}$. If it is positive, the investor would prefer to hold some portion of X. Now, using Eq.(5.1) of Lemma 5.1:

$$V'(\alpha_k)|_{\alpha_k=0} > (1 + kg'(0)) \left[Cov[X - Y, v'(1 + Y)] + E(X - Y)Ev'(1 + Y)] \right] \geq 0 \qquad (3.9)$$

Thus,

$$\alpha_k^* > 0 \text{ for all } k > 0 \text{ if and only if } EX - EY \geq \frac{Cov[Y, v'(1 + Y)]}{Ev'(1 + Y)}.$$

For the second part, similarly, we need to study the sign of Eq.(3.5) evaluated at $\alpha_k = 1, V'(\alpha)|_{\alpha_k=1}$. If it is negative, the investor would prefer to invest some portion of his wealth in asset Y. Now, again, using using Eq.(5.2) of Lemma 5.1:

$$V'(\alpha_k)|_{\alpha_k=1}$$
$$< (1 + kg'(0)) \left[Cov[X - Y, v'(1 + X)] + E(X - Y)Ev'(1 + X)] \right] \leq 0. \qquad (3.10)$$

Hence,

$$\alpha_k^* < 1 \text{ for all } k > 0 \text{ if and only if } EX - EY \leq \frac{Cov[-X, v'(1 + X)]}{Ev'(1 + X)}. \qquad (3.11)$$

This concludes the proof. ∎

Note 3.1 *We note that Lemma 5.1 requires that $k > 0$. Otherwise $(k = 0)$, a binding solution could be optimal.*

Note 3.2 *This proposition is a generalization of Muermann et.al.(2006) and Mulaudzi et.al.(2008) first result. We note that if Y is a degenerate random variable, then $Cov[Y, v'(1+Y)] = 0$, and we would have the same assumptions as in the risk-free and risky assets model.*

Note 3.3 *Besides, since $f(x) = -x$ is a decreasing function and, by concavity of v, $v'(1 + x)$ is also a decreasing function, then we deduce that $Cov[-X, v'(1 + X)]$ is positive. Similarly, as $f(y) = y$ is increasing and $v'(1 + y)$ is decreasing then $Cov[Y, v'(1 + Y)]$ is negative.[6] However*

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[6]This inequality is know as covariance rule, see Gollier (1995). We refer to Gurland (1967) and Egozcue et.al.(2009, 2010) for the proof and further inequalities of the covariance.

this Theorem does not assures us that diversification is optimal, as in each case specialization (corner solution) can be the best choice. We need to combine these two results to get an interior solution. Since V is strictly concave, a necessary and sufficient condition for diversification to occur is that $V'(0) > 0$ and $V'(1) < 0$.

Corollary 3.1 *Let the assets returns X and Y are independent and not identical distributed then a regret-averse investor would prefer diversification if and only if*

$$\frac{Cov[Y, v'(1+Y)]}{Ev'(1+Y)} \leq EX - EY \leq \frac{Cov[-X, v'(1+X)]}{Ev'(1+X)}. \tag{3.12}$$

We note that, since $Cov[Y, v'(1+Y)]$ is nonpositive and v is increasing, then the lower bound, $\frac{Cov[Y,v'(1+Y)]}{Ev'(1+Y)}$, is negative. Therefore, these inequalities hold for random variables with the same mean.. We incorporate this fact in the following Corollary.

Corollary 3.2 *If the assets returns X and Y are independent and not identical distributed, but have the same mean, then a diversified portfolio is always optimal, (i.e. α_k^* : with $0 < \alpha_k^* < 1$).*

Proof. The proof follows immediately from the last step of the previous proof, incorporating the equal mean assumption. Consequently, using Eq.(5.1) of Lemma 5.1

$$\begin{aligned}
V'(\alpha_k)|_{\alpha_k=0} &> (1 + kg'(0))\left[Cov[X - Y, v'(1+Y)] + E(X - Y)Ev'(1+Y)\right] \\
&= (1 + kg'(0))\,Cov[-Y, v'(1+Y)] \geq 0.
\end{aligned} \tag{3.13}$$

The last inequality holds, since by: stictly concavity assumption of v, $k > 0$ and g being an increasing function then $Cov[-Y, v'(1+Y)]$ is non negative. Similarly, using Eq.(5.2) of Lemma 5.1

$$\begin{aligned}
V'(\alpha_k)|_{\alpha_k=1} &< (1 + kg'(0))\left[Cov[X - Y, v'(1+X)] + E(X - Y)Ev'(1+X)\right] \\
&= (1 + kg'(0))\,Cov[X, v'(1+X)] \leq 0.
\end{aligned} \tag{3.14}$$

Similarly, since k is positive and g is an increasing function and $Cov[X, v'(1+X)$ is non positive the last inequality holds. This finishes the entire proof of this Proposition. ■

This Corollary shows that for independent returns with the same mean diversification is optimal. We remark that these conditions are not sufficient for diversification of a risk-averse investor.

We have relaxed the identical distributed assumption, now we shall also consider random variables that are stochastically dependent. First, we need to define the concept of dependence between random variables. A well known measure of dependence is define by Lehmann (1966), which we recall in the next definition.

Definition 3.1 *Two random variables X and Y are positive (negative) quadrant dependent if*

$$P\left(X \le x, Y \le y\right) \ge (\le)\ P\left(X \le x\right)P\left(Y \le y\right) \text{ for all } x, y. \tag{3.15}$$

Esary et.al.(1967) introduce the idea of associated random variables and its relation with quadrant dependence. The following inequalities are derive using these two ideas.

Proposition 3.1 *Let f and g be two real functions. If X and Y are positive (negative) quadrant dependent then:*

1. *$Cov[f(X), g(Y)] \ge (\le)\ 0$ if f and g are increasing (or both are decreasing).*

2. *$Cov[f(X), g(Y)] \le (\ge)\ 0$ if one function is increasing and the other decreasing.*

Readers may refer to Egozcue et.al.(2009) for the proof of this Proposition.

Now we present our final result.

Theorem 3.3 *If the assets returns X and Y are negative quadrant dependent and have the same mean then a diversify portfolio is optimal, (i.e. α_k^*: with $0 < \alpha_k^* < 1$).*

Proof. Similarly to the above propositions, we need to show that $V'(\alpha_k)|_{\alpha_k^*=0}$ is positive and $V'(\alpha_k)|_{\alpha_k^*=1}$ is negative. By the assumption of negative quadrant dependent, since $f(y) = -y$ is a decreasing function and $v'(1 + x)$ is also a decreasing function then $Cov[-Y, v'(1 + X)]$ is non positive. Similarly, as $f(y) = x$ is increasing and $v'(1+y)$ is a decreasing function, negative quadrant dependence implies that $Cov[X, v'(1 + Y)]$ is non negative. Hence, using the using Eq.(5.2) of Lemma 5.1, we have

$$V'(\alpha_k)|_{\alpha_k=0} > \left(1 + kg'(0)\right) Cov[X, v'(1 + Y)] \ge 0 \tag{3.16}$$

and similarly, we deduce that

$$V'(\alpha_k)|_{\alpha_k=1} < \left(1 + kg'(0)\right) Cov[-Y, v'(1 + X)] \le 0. \tag{3.17}$$

This concludes the proof. ∎

This Theorem shows that negative quadrant dependence and equal returns means are sufficient conditions to assure diversification for a regret-averse investor. Nevertheless, we remark that a risk-averse investor may not choose to diversify under these two conditions, see Hadar et. al. (1977).

In the following example we consider two random returns that are independent, but not identical distributed. We will see that a regret-averse investor prefer to diversify, while risk-averse investors prefer to specialize.

Example 3.1 Let $v(x) = \sqrt{x}$ and $g(x) = x^2$. Let X and Y be two independent binary random variables, which represents random returns of two assets, with joint distribution as follows:

	$X = 0$	$X = 1$	$Total$
$Y = 0$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{4}$
$Y = 1$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
$Total$	$\frac{1}{2}$	$\frac{1}{2}$	1

We can see that $E(X) = \frac{1}{2} > E(Y) = \frac{1}{4}$. As one can see both random variables are not identical distributed, therefore we are under the assumptions of Theorem 3.2 . Let $k = 2$, then

$$
\begin{aligned}
V(\alpha_2) &= Eu\left(1 + \alpha_2 X + (1 - \alpha_2)Y\right) \\
&= \frac{3}{8} + \frac{1}{8}\left[\sqrt{2 - \alpha_2} - 2\left(\sqrt{2} - \sqrt{2 - \alpha_2}\right)^2\right] \\
&\quad + \frac{3}{8}\left[\sqrt{1 + \alpha_2} - 2\left(\sqrt{2} - \sqrt{1 + \alpha_2}\right)^2\right] + \frac{1}{8}\left[\sqrt{2}\right].
\end{aligned}
$$

The maximum of $V(\alpha_2)$ is achieved when $\alpha_2^* = 0.92$, see Figure 1. Therefore there is a preference for diversification, i.e. $0 < \alpha_2^* < 1$.

Now, if we assume that $k = 0$, we are in the risk aversion case. It is easy to see that $V(\alpha_0)$ is a strictly increasing function:

$$
\begin{aligned}
V(\alpha_0) &= Eu\left(1 + \alpha_0 X + (1 - \alpha_0)Y\right) \\
&= \frac{3}{8} + \frac{1}{8}\sqrt{2 - \alpha_0} + \frac{3}{8}\sqrt{1 + \alpha_0} + \frac{1}{8}\left[\sqrt{2}\right].
\end{aligned}
$$

Thus, the optimal weight is to invest all the initial wealth in X, i.e. $\alpha_0^* = 1$, see Figure 2.
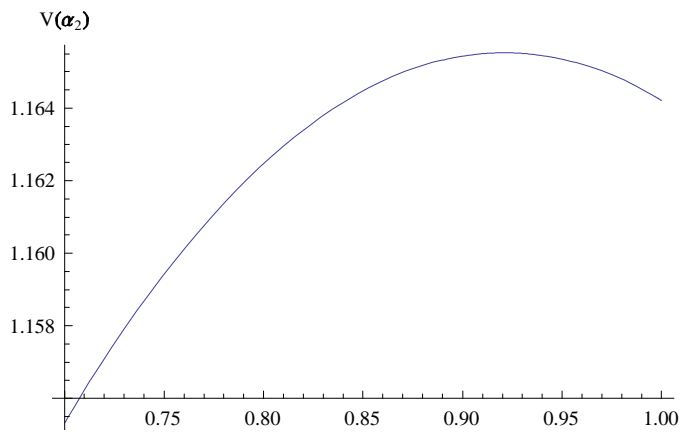

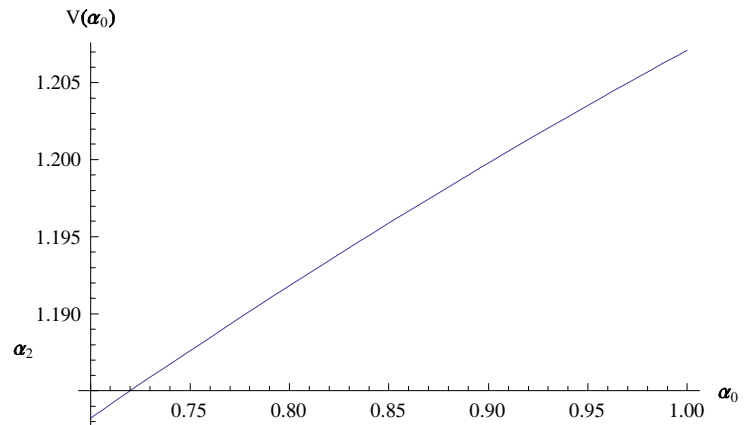Figure 1: Expected utility for a regret-averse investor.

Figure 2: Expected utility for a risk-averse investor

$$(3.18)$$

4 Concluding remarks

In this paper, we generalize the model of a regret-averse investor in a two risky assets portfolio selection problem. We get properties for this investor would gains from diversification or specialization. We show that if the random returns are i.i.d. complete diversification is optimal. We find out that under certain circumstances regret-averse investor would prefer diversification, while a risk-averse investor would choose to specialize. We study the diversification conditions for two different distributed random variables.

We study this problem when the random returns are quadrant. We find the difference between the expected return of both assets play a crucial role to find out whether there are gains from diversification. When random returns have the same mean and are negative quadrant dependent then diversification continues to be optimal.

This general model can be use in many economic and financial applications, such as: optimal capital structure or optimal insurance. The extension to n random assets is far beyond the scope of this paper. It remains as a task for future research

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5   Appendix

To establish the results, we require the following Lemma.

Lemma 5.1 *Let $V(\alpha_k) = Eu\left(1 + \alpha_k X + (1 - \alpha_k)Y\right)$, with $\alpha_k \in [0,1]$ and $k > 0$ then*

$$V'(\alpha_k)|_{\alpha_k=0} > (1 + kg'(0)) \left[Cov[X - Y, v'(1 + Y)] + E(X - Y) Ev'(1 + Y) \right] \qquad (5.1)$$

and

$$V'(\alpha_k)|_{\alpha_k=1} < (1 + kg'(0)) \left[Cov[X - Y, v'(1 + X)] + E(X - Y)Ev'(1 + X) \right]. \qquad (5.2)$$

Proof. We prove the first inequality. From the first order derivative, Eq. (3.5), we have:

$$V'(\alpha_k)|_{\alpha_k=0}$$

$$= E(X - Y) v'(1 + Y) + kE(X - Y)v'(1 + Y)g'(v(W^{\max}) - v(1 + Y))$$

$$= E(X - Y) v'(1 + Y) + kE(X - Y)v'(1 + Y)g'(v(1 + Y) - v(1 + Y)))1\{X < Y\}$$

$$\quad + kE(X - Y)v'(1 + Y)g'(v(1 + X) - v(1 + Y)))1\{X \geq Y\}$$

$$> E(X - Y) v'(1 + Y) + kE(X - Y)v'(1 + Y)g'(0)1\{X < Y\} + kE(X - Y)v'(1 + Y)g'(0)1\{X \geq Y\}$$

$$= E(X - Y) v'(1 + Y) + kE(X - Y)v'(1 + Y)g'(0)$$

$$= (1 + kg'(0)) E(X - Y) v'(1 + Y)$$

$$= (1 + kg'(0)) \left[Cov[X - Y, v'(1 + Y)] + E(X - Y) Ev'(1 + Y) \right].$$

The strict inequality can be deduce as follows. Note that we can write the following equality,

$$E(X - Y)v'(1+Y)g'(v(1+X) - v(1+Y)))1\{X \geq Y\} = E(X - Y)v'(1+Y)g'(v(1+X) - v(1+Y)))1\{X > Y\}.$$
$$(5.3)$$

Since $k > 0$, v and g are strictly increasing functions, hence we deduce the following strict inequality,

$$kE(X - Y)v'(1+Y)g'(v(1+X) - v(1+Y)))1\{X > Y\} > kE(X - Y)v'(1+Y)g'(0)1\{X > Y\}.$$
$$(5.4)$$

This completes the proof of the first part.

The second inequality can be proved with the same argument, as follows.

$$V'(\alpha_k)|_{\alpha_k=1}$$

$$= E(X - Y) v'(1 + X) + kE(X - Y)v'(1 + X)g'(v(W^{\max}) - v(1 + X))$$

$$= E(X - Y) v'(1 + X) + kE(X - Y)v'(1 + X)g'(v(1 + Y) - v(1 + X)))1\{X < Y\}$$

$$\quad + kE(X - Y)v'(1 + X)g'(v(1 + X) - v(1 + X)))1\{X \geq Y\}$$

$$< E(X - Y) v'(1 + X) + kE(X - Y)v'(1 + X)g'(0)1\{X < Y\} + kE(X - Y)v'(1 + X)g'(0)1\{X \geq Y\}$$

$$= E(X - Y) v'(1 + X) + kE(X - Y)v'(1 + X)g'(0)$$

$$= (1 + kg'(0)) E(X - Y) v'(1 + X)$$

$$= (1 + kg'(0)) \left[Cov[X - Y, v'(1 + X)] + E(X - Y)Ev'(1 + X) \right].$$

This finishes the entire proof of Lemma 5.1. ∎