Impact of the Enterprise Zone

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Abstract

Enterprise zones are place-oriented policies that are a tool of regional development, and refer to geographically targeted areas chosen for development that are designated on the basis of unemployment, poverty, population, age of housing stock, and other criteria. Firms that locate in the area and create jobs are given tax credits, abatements and exemptions. The underlying assumption is that firms and employees in the zone area benefit because of a reduction in the price of capital and/or labour, and there is expanded investment and employment generation through deregulation.

The objective of this work is to answer what is the impact of the enterprise zone (EZ) on the rest of the economy and labour. I develop a theoretical model. In the model I point to the cause of unemployment in the EZ. I show the relationship between the reservation wage and unemployment rate, following Jones (1989). I then show the general equilibrium response to the tax abatement provided in EZs, in a generalised framework incorporating capital mobility, following Harberger (1962). The analytical framework developed indicates that the capital

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and employment impact of the tax cut on capital in the EZ depends on three sets of parameters:

- Relative factor intensities of firms in the two areas.
- The elasticity of substitution between capital and labour in firms in the areas.
- The price elasticities of demand for goods Z and Y produced by EZ firms and non-EZ firms respectively

The analysis also indicates that it is impossible to isolate the incidence of the tax cut given in the EZ just to the EZ alone.

JEL Classification: R12, R13, R38, R58

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Impact of the Enterprise Zone

Introduction and Motivation

Enterprise zones are place-oriented policies, they are geographically targeted areas chosen for development. Firms that locate in these areas are given tax credits, exemptions, and abatements for making investments and creating jobs.

The enterprise zone concept originated in Great Britain in the late 1970s. Sir Geoffrey Howe, the Chancellor of Exchequer in the British government at the time, argued that excessive regulation and bureaucracy had led to the decline of Great Britain's economy. Howe advocated tax cuts to promote entrepreneurship that would create jobs. Stuart Butler, a British-trained economist, was responsible for introducing the enterprise zone concept in the United States.[1]

The enterprise zone programme has been one of the most controversial topics in the literature and in policy. The United States and a number of countries around the world – China, the Middle East, and countries including India have been adopting some form of such place oriented policy – *enterprise zones* or *special economic zones* or *growth centres*. A big advantage of the 'zone' concept being that it can be tried out experimentally in a small area. If it works, well, it can be extended to other parts of the country. If it does not, then we know that it doesn't work. It thus avoids the risks and costs of trying a programme in a full-fledged manner without knowing its effects.

Further, place-oriented policies such as enterprise zones focus on the area's development rather than argue that people should be mobile.

[1]In 1989, Housing and Urban Development (HUD) Secretary Jack Kemp said that he actually took the enterprise zone idea from a US program Operation Bootstrap which spurred postwar development in Puerto Rico by cutting taxes and fostering industry.

Place-oriented policies such as these recognise that there are likely to be a large number of people that are immobile because of the social, psychological or economic costs of moving. These people are most likely to be the ones with lower skills.

Therefore place-oriented policies such as enterprise zones deserve in-depth consideration for their impacts on labour and the rest of the economy. In this paper, I develop an analytical framework to analyse the impact of the enterprise zone on the economy that adopts it, and on labour.

II. Literature Review

There is a vast body of policy and empirical literature that evaluates enterprise zones. These studies have evaluated enterprise zones in the various states of the United States -- Indiana, Illinois, Ohio, Kentucky, New Jersey and California (*see* Rubin & Armstrong, 1989; Erickson & Friedman, 1989; Seyfried, 1990; Elling & Sheldon, 1991; Redfield & McDonald, 1991; Papke, 1994; Landers, 1996; Sridhar, 1996; 2000; 2001; Dowall, 1996; Boarnet & Bogart, 1996). *See* Sridhar (1998) for details of these various studies. My objective is not to summarise these studies, I provide a more suggestive and critical review.

Few of these studies provide an analytical framework to understand the impact of enterprise zones on their economies. Ge (1995) and Seyfried (1990) develop an analytical model for examining the effect of the enterprise zone for their various direct and indirect impacts on the regional economies. Both concentrate on the production sector for analysing the effect of enterprise zones, and focus on cost minimisation for firms when they locate in the enterprise zone. They ignore the effects on labour.

It is important to take into account some physical and distress characteristics pertaining to labour in the area, because most enterprise zone programmes, especially those in the U.S., specify explicit distress criteria that are used for zone designation. For instance, in the state of Illinois, enterprise zones are required to document size and distress criteria

for Enterprise Zone designation. The size criteria in Illinois pertain to geographical area of the zone. It specifies that an enterprise zone must be a minimum of one-half square mile and may be up to 10 square miles, excluding lakes and waterways. A proposed Enterprise Zone in Illinois must satisfy at least one of four criteria concerning poverty, unemployment, low income or population loss (*see* Sridhar, 1996).[2]

I find that none of the existing studies consider the efficiency implications of enterprise zones or construct an analytical framework to study their effects on labour as well.

I answer the following questions: are enterprise zones efficient if they are adopted by high-unemployment areas? What are the impacts of the zone on the rest of the economy? The work here addresses this gap in the literature and develops a framework to analyse the adoption of urban development programmes such as enterprise zones by high-unemployment areas and examines their efficiency. It also evaluates the impact of the zone on the rest of the economy, applying some of the standard literature on analysing the effect of tax incentives.

III. Objectives of Model

[2] The distress criteria specified by the Illinois Department of Commerce and Community Affairs (DCCA) for zone designation are as follows:
a. The poverty criterion is met if the poverty rate for each census tract that contains any part of the Enterprise Zone is at least 20 percent, as of the 1990 federal census.
b. The low income criterion is met if at least 70 percent of the households in the zone have incomes equal to or less than 80 percent of the median household income of the community in which the zone is located.
c. The unemployment criterion is met if the zone has an annual average unemployment rate of at least 120 percent of the state's average unemployment rate for the 12-month period ending the prior June 30.
d. The population loss criterion is met if the Enterprise Zone suffered a population decrease of 20 percent or more between 1980 and 1990 as determined by the 1990 federal census.

The objectives of the model to evaluate the enterprise zone are as follows:

- To describe the consequences of disequilibrium in the labour market of the high-unemployment area before it is designated as EZ, and describe the cause of unemployment in the EZ.

- To show that the area's unemployment rate determines the reservation wage along with a host of other factors affecting the benefits and costs of remaining unemployed.

- To examine whether economic rent accruing from jobs to job searchers in the EZ is higher.

- To characterise the effect of the tax abatement on capital given in the EZ, on the EZ and the rest of the economy, taking into account full capital mobility.

IV. Assumptions of Model

The model that follows is based on certain assumptions.

- There are two areas in the economy: one designated as EZ and the other being the rest of the economy, which I refer to as the non-EZ area (or area Y). The non-EZ area may be considered as all areas that do not have an EZ program or other programmes that abate taxes.

EZ firms produce good Z, and non-EZ area firms produce good Y. In reality this assumption is consistent with the fact that certain areas specialise in the production of primarily certain goods. Rubin and Zorn (1985) show the comparative cost advantage different states in the United States have in different SIC category industries. In India, a typology of cities and states is emerging whereby each specialises in the output of a certain good or service. Thus, based on comparative advantage, we could easily classify areas without loss of generality.

Goods Y and Z are produced by two factors of production, capital (K) and labour (L). In taking into account capital and labour, all conventional factors of production – land, labour, capital and organisation -- are implicitly taken into account in the model; land can be considered a special kind of capital (real capital), and organisational skills, a special case of labour.

L_Z and L_Y refer respectively to labour employed in the EZ and the non-EZ area firms. K_Z and K_Y refer to capital employed in the EZ and non-EZ firms respectively. P_Z and P_Y are the prices of the final goods Z and Y respectively. P_{KZ} and P_{LZ} refer to the factor prices of capital and labour respectively in the EZ. Similarly, P_{KY} and P_{LY} refer to factor prices and refer to the price paid by users of capital and the price of labour (wage) in the non-EZ area. Because EZ areas are, in reality, blighted, it is assumed that f_{LZ} (the marginal product of labour in the EZ) < f_{LY} (the marginal product of labour in the non-EZ area). This is due to differences in the use of capital across the areas. Specifically, $K_Z/L_Z < K_Y/L_Y$, and for this reason, the marginal product of labour in the EZ is lower than in the other areas.

Because $f_{LZ} < f_{LY}$, the labour market is not in equilibrium when the EZ is designated. The model elaborates on this. The capital market is in equilibrium so that $P_{KZ}=P_{KY}=P_K$. Consumers supply the factors in fixed amounts.

- It is assumed that production is subject to constant returns to scale (CRTS) technology because CRTS has many interesting properties. The assumption of CRTS means that the average as well as the marginal products of the factors are dependent only on the ratio in which they are combined (Chiang, 1984; Krauss and Johnson, 1974), which, here is the capital-labour ratio.

In empirical work, studies have found that a majority of the two-digit SIC industries they studied were subject to CRTS. An interesting old example is Moroney (1967), in which, based upon estimates of production functions in 2-digit SIC manufacturing industries in the United States, he concluded that a majority of these industries were subject to CRTS. More recent examples relate to plant-level data. Bailey *et al*., (1992) used plant level data and found that the plants they examined were characterised by constant returns to scale. Griliches and Ringstad

(1971) argued that essentially constant returns were needed to rationalise the observed large dispersion of establishment sizes within a given industry (Basu and Fernald, 1997). Basu and Fernald (1997) concluded that a typical 2-digit industry in the United States appears to have constant or slightly decreasing returns to scale.

Thus when we look at empirical work, it appears that CRTS may not be an unrealistic assumption to make.

- Firms in the two areas are unequally factor intensive, or $K_Y/L_Y > K_Z/L_Z$ (Y relatively capital intense) at any given feasible factor-price ratio. This assumption, along with CRTS, creates a production-possibility frontier that is uniformly concave to the origin.

- The model allows for unemployment to exist.[3] This is consistent with the way in which enterprise zones are designated on the basis of certain distress criteria that includes (high) unemployment. L_{ZU} is the number unemployed in the EZ. The total labour force in the EZ area is $L_Z+L_{ZU} =N_Z$. The model allows for unemployment to exist in the non-EZ area too (let's say, the natural rate of unemployment), but it is less than that in the EZ so that it does not qualify for EZ designation. In the non-EZ area, the total labour force N_Y is equal to $L_Y + L_{YU}$. L_{YU} is the number unemployed in the non-EZ area. The total labour force in the economy is $N_Z+N_Y = N$.

- Factors are paid according to the value of their marginal products in equilibrium. Taken along with the assumption that $f_{LZ} < f_{LY}$, this assumption means that wages in the EZ in the initial equilibrium are lower than in the non-EZ area.

- The government provides a subsidy on the use of capital as well as labour in the EZ. This is equivalent to a refund of taxes paid on capital and labour to firms in the EZ.

- The prices of goods are defined such that the value of all goods in the original equilibrium is 1.

[3] The section on disequilibrium in the labour market elaborates on why unemployment comes to exist in the model. In short, unemployment exists because of high reservation wages of labour in the EZ area when compared to market wages.

V. The Model

Disequilibrium in the Labour Market

The disequilibrium in the labour market of the EZ is due to its high unemployment rate, which, by definition, means that a large proportion of its labour force is unemployed.

Butler (1981), one of the pioneers of the EZ concept in the United States, argued that minimum wage legislation is the primary cause of unemployment in the United States and so relaxing this minimum wage constraint in the areas designated as EZs would alleviate their unemployment. However, this still does not explain why unemployment is concentrated in *certain* areas that get designated as EZs. Clearly then, the literature on EZs does not provide a model of unemployment in the EZs, i.e., does not explain the cause of unemployment in EZs.

This model makes an attempt to provide that explanation. Here I elaborate on the reasons for unemployment in the EZ and unemployment rate of the area.

a Model of Unemployment

The model of unemployment that is developed here draws from the neoclassical assumption of high reservation wages. This is the explanation that individuals are unemployed if their reservation wages exceed market wages. Such an explanation appears reasonable for individuals in the EZ because of their high reservation wages relative to the market wage prevailing in the EZ.

The high reservation wages in the EZ is realistic because of the benefits of remaining unemployed in the United States (*see* Feldstein, 1978). The income of the unemployed is high because they are most

probably recipients of unemployment compensation and probably other non-market income such as welfare payments. These individuals are unlikely to find gainful employment given their skills. This causes them to remain unemployed. The literature on job search indicates that high reservation wages relative to market wages cause unemployment, especially when unemployment benefits are generous. Feldstein (1978) shows how a combination of a high marginal tax on earnings and no tax on unemployment compensation makes the private cost of unemployment small and causes an individual to remain unemployed.

To understand this effect, consider a situation in which each job searcher faces a distribution of wage offers with w^r (his reservation wage) < E(w) (his expected wage) < w^s (the maximum wage given his skills). This may be graphically shown in Figure 1. In figure 1, the horizontal axis is the wage rate w(s) (wage given skills). The vertical axis is a frequency (refers to the number) of job offers available at the various wage rates. In the middle of the curve is shown the highest frequency and it decreases as we move away from the mean/median/mode. The curve that is thus obtained in the figure is the probability density function of the random variable called wage. It is normally distributed with mean at E(w). Given that the person finds a job, his/her expected wage is the weighted average of the job offers in the w(r) to w(s) range. This average wage is denoted by E(w) in figure 1, which shows that the mean is also the median and the mode.

We know that the individual would reject any job offer that offers $w < w^r$. Thus the area under the curve between w^r and w^s represents the probability of the job searcher finding an acceptable job in any period (*see* Ehrenberg and Smith, 1994). The higher this probability, the lower the expected duration of unemployment. The benefits (such as unemployment compensation) of remaining unemployed essentially decrease the area under this curve by increasing w^r (moves it further to the right) and lengthens the duration of unemployment.

The unemployed in high-unemployment areas place a high value on the importance of having a job when compared to those in low-unemployment areas, as Bartik (1991) argues (*see* also Theodossiou, 1992). They are willing to take up a job that offers income higher than their current income. But they are less mobile due to psychological ties to the area and costs of relocation. Therefore such individuals (who have

high reservation wages relative to low market wages) tend to be concentrated in the EZ area.

The market wage is low because profit-maximising employers are willing to pay a wage only according to the skill of the workers. We know that by assumption 2 of the model that $f_{LZ} < f_{LY}$, i.e., the marginal product of labour in the EZ is less than in the non-EZ area. The literature on enterprise zones points to the blight in these areas, which, according to this model, is due to the low capital-labour ratio in the EZ. The low capital-labour ratio leads to low productivity in the EZ for those employed and potential low productivity for those who are unemployed. So the EZ labour force is eligible only for a lower market wage.

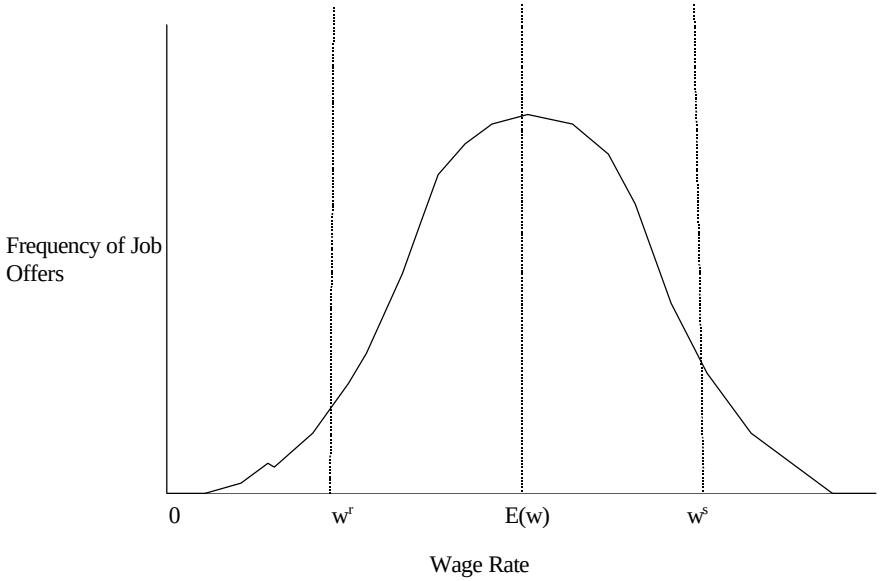

Figure 1: The Distribution of Wage Offers

This model offers an explanation as to why individuals with high reservation wages relative to low potential market wages tend to be concentrated in the EZ area. So prior to the designation of the area as an EZ, profit-maximising employers do not have incentives to hire workers with low skills. It is likely too that these individuals are recipients of unemployment compensation, and are unlikely to find gainful

employment given their skills, although they do place a high value on the importance of having a job.

So formally, the unemployment status (US_{ij}) of an individual i living in the jth area is determined by the difference between reservation and market wage.

$US_{ij} = f(w^r_{ij} - w_{ij})$----------[1a]

w^r_{ij} is the reservation wage and w_{ij} is the market wage of the ith individual living in the jth area. The unemployment rate of the jth area may be summed over the unemployment status of i individuals living in the jth area. It may be written as follows:

$$U_j = \frac{\sum_i US_{ij}}{Population}$$

So, substituting for US_{ij} from [1a],

$$U_j = \frac{\sum_i f(w^r{}_{ij} - w_{ij})}{Population}$$ ----------[2a]

Thus the aggregate unemployment rate in the jth area is dependent on the extent to which reservation wages of individuals are higher than market wages in the aggregate. Specifically, [1a] and [2a] show that the unemployment rate of the EZ is high if the reservation wages of individuals are high relative to market wage in the area. Thus the high unemployment rate in the EZ is due to the low productivity of labour and their relatively higher reservation wages compared to the market wage.

Thus under conditions of high unemployment rate, EZ designation of the area acts as an important place-oriented policy to improve the blighted area in which tax abatements are provided to arrest the decline of the area. Tax abatements provide firms with incentives for investment. In the section on general equilibrium I show how the tax abatement in the EZ induces in-migration of capital into the EZ due to the shifting of resources to the EZ area from the other area. I elaborate on the general equilibrium effects of tax abatement provided to firms in the EZ, first explaining the relationship between the reservation wage and

the local unemployment rate and the effect of new employment on economic rents in the EZ.

Next, I explain the relationship between the reservation wage and unemployment rate.

The Reservation Wage

It is useful to recognise in a model that allows for unemployment that the standard environment in which job search is modeled involves the search for a job from a known distribution of job offers. Various models of job search (Jones, 1989; Addison & Siebert, 1979; Ehrenberg and Smith, 1993) show that the unemployed individual's decision to work is determined by the various costs and benefits of unemployment.

The model here follows Jones (1989) closely. As in Jones (1989), let Θ be the instantaneous probability of receiving a job offer and F(w) be the distribution of job offers. From the viewpoint of the individual job searcher, the decision to work is a function of the benefits and costs (b-c) of remaining unemployed. b is the benefit level while remaining unemployed, and c is a measure of the costs of being unemployed. I assume that job searchers try to maximise their utility by maximising the income they receive from a job. Such utility-maximising behaviour leads to the equation:

$$rV = b - c + \Theta \int_0^{\infty} \max\{0, W(x) - V\} dF(x) \text{ ----------[1c]}$$

In [1c], r is the interest rate, V is the present discounted value of being unemployed, (b-c) refers to the net benefits of being unemployed. The latter part of the equation represents the capital gains derived from the income from a job, which is the maximum of the net income from a job (which is net of the value of being unemployed, V) if employment is found or zero. In such a situation, the optimal job search strategy displays the reservation wage property with the critical value being the reservation wage w^r (also *see* Zuckerman, 1984). The reservation wage, or the lowest wage at which the unemployed are willing to supply positive labour (accept a new job), obeys:

$rV = rW(w^r) = w^r$----------[2c]

[2c] shows that the reservation wage w^r equals imputed search income.[4] Substituting for rV from [2c] in [1c],

$$w^r = b - c + (\Theta / r) \int_{w^r}^{\infty} \max(x - w^r) dF(x) \text{ ----------[3c]}$$

Thus the reservation wage, as in [3c], equals the net benefits (b-c) while unemployed, and a factor that depends on the expected wage in next employment adjusted for the arrival rate of job offers. Then it is possible to approximate [3c] in some linear fashion as in Jones (1989):

$$w_i^r = \alpha_0 + \alpha_1(b_i - c_i) + \alpha_2\mu_i + \alpha_3\Theta_i + e_i \text{ ----------[4c]}$$

In [4c], $b_i - c_i$ is the difference between benefits and costs of remaining unemployed for the ith individual, μ_i is the expected wage in next employment, Θ_i is the arrival rate of job offers as defined earlier, and e_i is a random error term. Θ_i is a function of the various individual-specific characteristics as well as regional labour market conditions that determine the arrival rate of job offers. So

$$\Theta_i = \sum_j a_{3j} X_{ij}$$

The unemployment rate of the area is an important indicator of the regional labour market conditions that determines the arrival rate of job offers for individual i and so of his/her reservation wage. So substituting for Θ_i in [4c],

[4] Alternatively, rather than measuring the reservation wage as a single point that I actually measure in the empirical work, the reservation wage can be identified as a locus of points at various hours of work, consistent with neoclassical labour theory. This is because the reservation wage could be declining with the hypothetical hours of work that is offered at the "new job". One could expect that the reservation wage increases with additional hours of work, to compensate for leisure lost. In the literature, there are few instances in which the hours of work as well as the reservation wage are taken into account in job search. Blau (1991) develops a model which predicts that a low-earnings job might be accepted if the hours were low also, and a high-earnings job might be rejected if the hours were also high. Empirically also, he finds that workers are clearly not indifferent between alternative combinations of weekly hours and earnings.

$w_i^r = \alpha_0 + \alpha_1(b_i - c_i) + \alpha_2\mu_i + \alpha_3 U_{ij} + e_i$----------[5c]

U_{ij} is the unemployment rate of the jth area (j=Z,Y) in which the ith individual lives.

[5c] is estimable. Jones (1989), Sridhar (1996) and Haurin and Sridhar (2003) estimate this relationship.

Moreover, a hypothesis can also be formulated with regard to the relationship between the reservation wage and the area's unemployment rate. The actual experience of job searchers in high-unemployment areas shows that they are willing to accept lower reservation wages for some reasons: it is more likely than not that they are risk-averse. Although it is reasonable to expect that unemployed job searchers even in high-unemployment areas frequently begin their search with a high reservation wage, as time passes on, they are likely to lower their reservation wage for reasons of family or financial hardship (*see* Theodossiou, 1992). Thus the reservation wage of unemployed searchers can be considered a gradually declining function of time spent in unemployment, which can be considered long in high-unemployment areas because they are risk-averse, or have family/psychological ties to the area or are not willing to bear the costs of relocation. The testable hypothesis that comes out of this is that unemployment rate of the area has a negative impact on the reservation wage of individuals residing in the area.

Economic Rent

As in standard labour economic theory, we can define economic rent b_{ij} as the extent to which actual wages are above the reservation wage. That is,

$b_{ij} = w_{ij} - w^r_{ij}$, j=z,y

w_{ij} and w^r_{ij} are respectively the wage and reservation wage of the ith individual in the jth area.[5]

[5] The important point here is that if hours are flexible and chosen by the individual, they will continue to increase work hours until, at the margin, the reservation wage = market wage. In this case, the economic rent is zero. An alternative assumption is that hours of work are not flexible. Under this

If the hypothesised relationship between the reservation wage and the area's unemployment rate were true, the reservation wage (w^r_{ij}) would be low in the EZ because of its high unemployment rate. Notice that the cause of unemployment for individuals in the high-unemployment area is high reservation wages, but this is relative to the market wage. This is not inconsistent with the relatively low reservation wages in the high-unemployment area when compared to the low-unemployment area (this refers to the relationship between the reservation wage and unemployment rate of the area).

However, wages (w_z) in the EZ are also low (compared to the non-EZ area) due to low capital-labour ratio in the EZ (see assumption 6 of model). If a similar hypothesis were to hold in the non-EZ area, it would have higher reservation wage (w^r_y) than the EZ. However wages in the non-EZ area (w_y) are high by assumption 6. So it is difficult to conclude whether the EZ or non-EZ area will benefit from higher economic rents. Thus, it is an open question that can be tested in the empirical work as to whether economic rents would be higher in high or low unemployment areas. Remembering, however, that the relationship between the reservation wage and the unemployment rate is hypothesised to be negative, meaning that reservation wages can be expected to be lower in high-unemployment areas.

In the section on general equilibrium I show how the tax abatement in the EZ induces in-migration of capital into the EZ due to the shifting of resources to the EZ area from outside. I elaborate on the general equilibrium effects of tax abatement provided to firms in the EZ, and the effect of new employment on economic rents in the EZ.

The General Equilibrium Response to the Tax Abatement

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assumption, there is an economic rent. Given that the number of hours are fixed per week at 40 and the individual's reservation wage is less than the wage, there is economic rent. Here I rely on the fixed hours hypothesis as a possibility, and so economic rents do accrue to individuals. This corresponds to the assumption of the reservation wage for a full-time job (40 hours a week) in the empirical work, where the reservation wage is measured as the lowest wage one is willing to take home as pay for a full-time job (as in the PSID).

Harberger (1962) considers the effect (and the ultimate incidence) of a sector-specific corporation income tax, taking into account its general equilibrium effects on the entire economy. His analysis has become a standard framework for analysing different kinds of taxes in the literature. I adopt his analytical framework here to analyse the effect of the property tax abatement provided to firms in EZs that is equivalent to a tax cut on the use of capital in the EZ.

In addition to a subsidy to capital, most state enterprise zone programs (including Ohio) also include a subsidy for labour. So I also include in the model a subsidy to labour. An equal subsidy for labour and capital is the same as a subsidy for the good produced by the sector. Therefore I decompose the subsidy to capital and labour into a subsidy for one input (i.e., the tax abatement which is a subsidy to capital), and a subsidy for the output (produced by the EZ sector). The subsidy for the output results in a reduction in the price of the good produced by the EZ sector, which becomes (P_Z - C_Z), where C_Z is the subsidy on the output.

The tax abatement, according to Harberger's analysis, is equivalent to a tax cut which will have the immediate effect of creating a wedge between the price of capital between the EZ and the rest of the economy. Users of capital in the EZ pay a lesser price (P_K - A_{KZ}) for the use of capital, where A_{KZ} is the tax abated on capital invested in the EZ. The tax abatement has the effect of lowering the price of good Z that EZ firms produce because investors in the EZ can now produce and supply higher output at the same cost as before, due to the savings induced by the abatement. Investors in the non-EZ area firms still continue to pay the price on capital, $P_{K,}$ which includes the tax (there are no taxes abated in the non-EZ). It is likely at this point that firms in the non-EZ area will increase their supply of good Y by an amount that is sufficient to lower the price of Y on par with that of Z. Whether or not the non-EZ area firms are able to do this depends on the **demand elasticity for the good** (Z) produced by the EZ firms. The percentage change in the demand for Z as a result of the tax abatement, is formally derived below.

The Goods Market: Demand

In a two-good economy, one would expect the demand for goods to have some price elasticity and that Z and Y would be substitutes. To see this, let us characterise the demand equation for Z as follows, following Harberger:

$$Z=f\left(\frac{P_z-C_z}{P_Y}\right)$$ ----------[1d]

[1d] shows that the demand for Z is a function of the price of Z and Y. Totally differentiating the demand for Z,

$$dZ=\frac{\P f}{\P\left(\frac{P_z-C_z}{P_Y}\right)}\frac{P_Y dP_z-P_Y dC_z-(P_z-C_z)dP_Y}{P_Y{}^2}$$ ----------[2d]

In order to express the change in percentage form, we divide [2d] by Z:

$$\frac{dZ}{Z}=\frac{\P Z}{\P\frac{P_z-C_z}{P_Y}}\frac{1}{Z}\left[\frac{dP_z}{P_Y}-\frac{dC_z}{P_Y}-\frac{(P_z-C_z)}{P_Y{}^2}dP_Y\right]$$ ----------[3d]

By assumption [8], the price of goods (P_Z - C_Z) and P_Y, are equal to 1, and [3d] can be written as:

$$\frac{dZ}{Z}=\frac{\P Z}{\P\frac{P_z-C_z}{P_Y}}\frac{\frac{P_z-C_z}{P_Y}}{Z}\left[dP_z-dC_z-dP_Y\right]$$ ----------[4d]

In [4d], it may be noted that $\frac{\P Z}{\P\frac{P_z-C_z}{P_Y}}\frac{\frac{P_z-C_z}{P_Y}}{Z}$ is the demand elasticity for Z in terms of the relative prices $\frac{P_z-C_z}{P_Y}$. So in simplified form, [4d] may be written finally as:

$$\frac{dZ}{Z}=e(dP_z-dC_z-dP_Y)$$ ----------[5d]

where $e = \dfrac{\P Z}{\P \dfrac{P_z - C_z}{P_Y}} \dfrac{\dfrac{P_z - C_z}{P_Y}}{Z}$.

Equation [5d] shows that the percentage change in the demand for Z depends on elasticity of demand for Z in terms of relative prices. It should be noted that the elasticity appearing in [5d] is compensated because the marginal propensity to consume of households and the government are assumed to be equal so that the income effect in consumer demand exactly offsets that in government demand (*see* Myles, 1995). Thus only the substitution effect is present.

If $|\varepsilon| > 0$, the demand for Z (produced by the EZ firms) increases in response to a decrease in its price.

The Goods Market: Supply

For market clearance the percentage change in demand for Z must equal the percentage change in supply of Z. To determine the percentage change in the supply of Z, we totally differentiate the production function $Z = f(K_Z, L_Z)$:

$$dZ = \frac{\P f}{\P K_z} dK_z + \frac{\P f}{\P L_z} dL_z \text{ ----------[6d]}$$

Expressing [6d] in percentage change form, and dividing the LHS and RHS by Z and f respectively,

$$\left.\frac{dZ}{Z}\right|_{\text{Supply}} = \frac{\dfrac{\P f}{\P K_z}}{f} dK_z + \frac{\dfrac{\P f}{\P L_z}}{f} dL_z \text{ ----------[7d]}$$

Multiplying and dividing through by K_Z and L_Z,

$$\left.\frac{dZ}{Z}\right|_{\text{Supply}} = \frac{\frac{\partial f}{\partial K_z}K_z}{f}\frac{dK_z}{K_z} + \frac{\frac{\partial f}{\partial L_z}L_z}{f}\frac{dL_z}{L_z} \text{ ----------[8d]}$$

[8d] can be written as:

$$\left.\frac{dZ}{Z}\right|_{\text{Supply}} = q_{\text{KZ}}\frac{dK_z}{K_z} + q_{LZ}\frac{dL_z}{L_z} \text{ ----------[9d]}$$

In [9d], it may be noted that q_{KZ} is the share of capital income in the value-added, q_{LZ} is the share of labour income, in the value added for firms in the EZ.

[9d] shows that the percentage change in the supply of output by EZ firms depends on the shares of labour and capital income in the value-added for EZ firms.

The Factor Market: Firms' demand for factors

Because of a shift in demand for good Z, increased profits accrue to EZ firms that produce Z. So the non-EZ area capital will have an incentive to flow into the EZ, in order to equalise returns to factors in both the areas. What happens then depends on **relative factor intensities**, and the firms' demand for factors in the two areas.

Following Harberger, changes in factor demands can be specified in terms of their direct elasticities of substitution. So I define elasticities of substitution, as in Tresch (1981):

$$S_z = \frac{d\log\left(\frac{K_z}{L_z}\right)}{d\log\left(\frac{f_{LZ}}{f_{KZ}}\right)} \text{ ----------[10d]}$$

$$S_Y = \frac{d \log \left(\frac{K_Y}{L_Y} \right)}{d \log \left(\frac{f_{LY}}{f_{KY}} \right)}$$ ----------[11d]

S_Z and S_Y are respectively the direct elasticity of substitution between capital and labour, of firms in the EZ and non-EZ areas. With the marginal products in the EZ and non-EZ area firms equal to their respective price ratios, we can write:

$$d \log \left(\frac{f_{LZ}}{f_{KZ}} \right) = d \log \left(\frac{P_K - A_{KZ}}{P_{LZ}} \right)$$ ----------[12d]

$$d \log \left(\frac{f_{LY}}{f_{KY}} \right) = d \log \left(\frac{P_K}{P_{LY}} \right)$$ ----------[13d]

Then, substituting for the ratio of marginal products from [12d] in [10d], we have:

$$d \log \left(\frac{K_Z}{L_Z} \right) = S_Z d \log \left(\frac{P_K - A_{KZ}}{P_{LZ}} \right)$$ ----------[14d]

Similarly,

$$d \log \left(\frac{K_Y}{L_Y} \right) = S_Y d \log \left(\frac{P_K}{P_{LY}} \right)$$ ----------[15d]

Consider the left-hand side of equation [14d],

$$d \log \left(\frac{K_Z}{L_Z} \right) = \frac{1}{\frac{K_Z}{L_Z}} d \left(\frac{K_Z}{L_Z} \right) = \frac{1}{\frac{K_Z}{L_Z}} \left[\frac{L_Z dK_Z - K_Z dL_Z}{{L_Z}^2} \right]$$

$$= \left(\frac{dK_Z}{K_Z}\right) - \left(\frac{dL_Z}{L_Z}\right)$$ ----------[16d]

Similarly for the price ratios,

$$d\log\left(\frac{P_K - A_{KZ}}{P_{LZ}}\right) = \frac{dP_K - dA_{KZ}}{P_K - A_{KZ}} - \frac{dP_{LZ}}{P_{LZ}}$$

Because $P_K = P_{LZ} = 1$,

$$d\log\left(\frac{P_K - A_{KZ}}{P_{LZ}}\right) = \frac{dP_K - dA_{KZ}}{1 - A_{KZ}} - dP_{LZ}$$ ----------[17d]

Substituting [16d] and [17d] into [14d] gives us:

$$\left(\frac{dK_Z}{K_Z}\right) - \left(\frac{dL_Z}{L_Z}\right) = S_Z\left(\frac{dP_K - dA_{KZ}}{1 - A_{KZ}} - dP_{LZ}\right)$$ ----------[18d]

When we do this similarly for Y, we get:

$$\left(\frac{dK_Y}{K_Y}\right) - \left(\frac{dL_Y}{L_Y}\right) = S_Y(dP_K - dP_{LY})$$ ----------[19d]

Equations [18d] and [19d] show that the percentage change in the demand for capital over labour depends on the elasticity of substitution between the factors as well as change in their prices. Thus the relative growth of demand for capital and labour will differ. While factor markets continue to equate factor prices with values of marginal products, equilibrium is attained only when factor prices (wages and returns to capital) are equalised across the areas.

It may be noted here that reservation wages (w^r_z) are lower in the EZ area because of its high unemployment rate (recall the hypothesised relationship between the area's unemployment rate and w^r_z (the reservation wage)). As capital continues to flow into the EZ, demand for labour increases, as described above. This increases employment (L_Z) in the EZ. However, it has to be noted that first-order effects always dominate. This means that firms that locate eventually in the EZ are

more capital-intensive, although they also have increased demand for labour.[6]

With increasing employment in the EZ, there is a high probability that local residents get the jobs created by the new capital. This is because new jobs in the EZ are assumed to be allocated to unemployed on a random selection process. When the selection process for jobs is random, zone residents (even though they have few skills) have equal chance of being selected from the pool of unemployed, by employers who are now subsidised.

The assumption of a random selection process in the EZ when firms create jobs is supported by empirical evidence. Empirical work reports that on average, about 50 percent of jobs created in the EZ go to zone residents. In a collection of ten case studies of state-designated enterprise zones, the U.S.Department of HUD (1986) observed that 70 percent of jobs in the Bridgeport EZ in CT, 70 percent of jobs created in the Chicago EZ in IL, 46 percent of those created in the Macon EZ in MO, 19 percent in Michigan City, IN, 30 percent in the Tampa zone (FL) and 5 percent of the York zone (in PA) were held by zone residents. In the Louisville (Kentucky) zone, it was found that 31 percent of the jobs created were held by persons who were either lower income or zone residents. Erickson and Friedman (1989), based on a survey of local enterprise zone coordinators conducted by the U.S. Department of HUD, found that the mean share of jobs held by zone residents was over 61 percent with a median of over 68 percent. More recently, Immergluck's (1997) data from the Chicago EZ indicated that the barriers between EZ residents and jobs are dependent on some factors. He found that local employment was much higher in Latino parts of the zone and in African American neighborhoods where there were more public sector jobs, very small firms, and few manufacturers.

Thus, when a substantial portion of the jobs that are created go to zone residents the unemployment rate of the EZ decreases. With increasing use of capital and increase in the capital-labour ratio, labour productivity and industrial output rise. Some explanations for rising industrial output (apart from the increasing use of capital) could be due to

[6] We should note that we are comparing firms with high capital-labour ratios to ones with low ratios.

improved managerial and organisational capabilities and enhanced labour efficiency through on-the-job training programs in the EZ area.

Thus at the optimum, the unemployment rate in the EZ is lower than in the initial equilibrium, and that in the non-EZ areas is the same as before. This is because of increasing demand for labour due to rise in the capital-labour ratio and rising productivity of labour in the EZ. We have to note that the non-EZ areas were operating at a natural rate of unemployment initially. Tobin (1972) estimated that a 5-6 percent natural rate of unemployment has been associated with more than 20 percent excess capacity in the capital stock. Thus the EZ acts only as a tool to direct the excess capacity of capital in the full-employment areas to high unemployment areas and serves to reduce their unemployment rate. It should be noted that the model says that capital moves across areas; this does not necessarily imply migration of firms. But the movement of capital causes an improvement in the total economy's overall well-being, although a change in the price of capital initiated in one area due to the existence of the EZ, is transmitted to other areas.

Market Clearance

For market clearance, the goods and factor markets have to remain in balance. The following has to be true for the goods market to be in equilibrium:

$$\left.\frac{dZ}{Z}\right|_{Demand} = \left.\frac{dZ}{Z}\right|_{Supply} \text{ ----------[20d]}$$

Because capital and labour are in fixed supply, the amounts of their in-migration into the EZ must be equal to the amount of their out-migration from the non-EZ region so that:

$dK_Z = -dK_Y$ ----------[21d]

$dL_Z = -dL_Y$----------[22d]

The above conditions show that what capital and labour the EZ gains must be equal to the amounts of the factors lost by the non-EZ area.

Comparative Statics: Change in Capital and Labour in the EZ

In order to obtain the change in the capital invested (dKz) and labour (dLz) in the EZ at the equilibrium as in Harberger's model, I follow the procedure described in Myles (1995) most closely. After some manipulation, we end up with a system of three simultaneous equations that can be solved simultaneously for dK_z, dL_z, and dP_K. The simultaneous system that was used to solve for dK_z, dL_z, and dP_K is shown below in matrix form.

$$\begin{bmatrix} \frac{f_{kz}}{K_z} & \frac{f_{Lz}}{L_z} & -e(f_{kz}-f_{kz}) \\ \frac{1}{K_z} & -\frac{1}{L_z} & -\frac{S_Z}{(1-A_{kz})} \\ -\frac{1}{K_Y} & \frac{1}{L_Y} & -S_Y \end{bmatrix} \begin{bmatrix} dK_Z \\ dL_Z \\ dP_K \end{bmatrix} = \begin{bmatrix} -edA_{KZ} f_{kz} \\ -\frac{S_Z}{(1-A_{kz})} - S_Z dP_{LZ} \\ -S_Y dP_{LY} \end{bmatrix}$$

The simultaneous system, when solved, gives the solution for dK_z, dL_z, and dP_K. The exact expressions obtained for dK_z and dL_z (which are of greater interest because one of the important goals of EZs is to promote employment and investment growth in the designated areas) are quite cumbersome and so I do not report them here. It is reported in the Appendix. It is sufficient to note that the change in capital invested and labour, dK_z and dL_z, depend on the relative magnitude of certain parameters in equilibrium. At any rate the framework provided above indicates that the effect of the tax abatement on dK_z, and dL_z is testable.

More generally, the Harberger analysis, when applied to property tax abatements in EZs, indicates that the capital and employment impact of the tax cut on capital in the EZ depends on three sets of parameters:

- Relative factor intensities of firms in the two areas.
- The elasticity of substitution between capital and labour in firms in the areas.

- The price elasticities of demand for goods Z and Y produced by EZ firms and non-EZ firms respectively.

The analysis also indicates that it is impossible to isolate the incidence of the tax cut given in the EZ just to the EZ alone. Because competitive factor markets equalise returns to capital everywhere in the economy, if investors in the EZ enjoyed an increase in the return to capital, investors everywhere will experience the same increase as well. Moreover because goods and factor markets are interdependent, the changes could get transmitted to consumers in the form of changes in goods prices.

Thus Harberger's analysis describes how the migration of capital occurs in response to changes in the rate of return to capital across areas. The general equilibrium response to the tax abatement is shown in figure 2. Figure 2 shows that the tax abatement (tax cut) on capital leads to in-migration of capital into the EZ. Depending on the elasticity of substitution between capital and labour (S_Z), the new capital leads to increase in productivity, wages and employment. This affects the unemployment rate. The unemployment rate finally determines the reservation wage. The difference between wages and reservation wages determines economic rent.

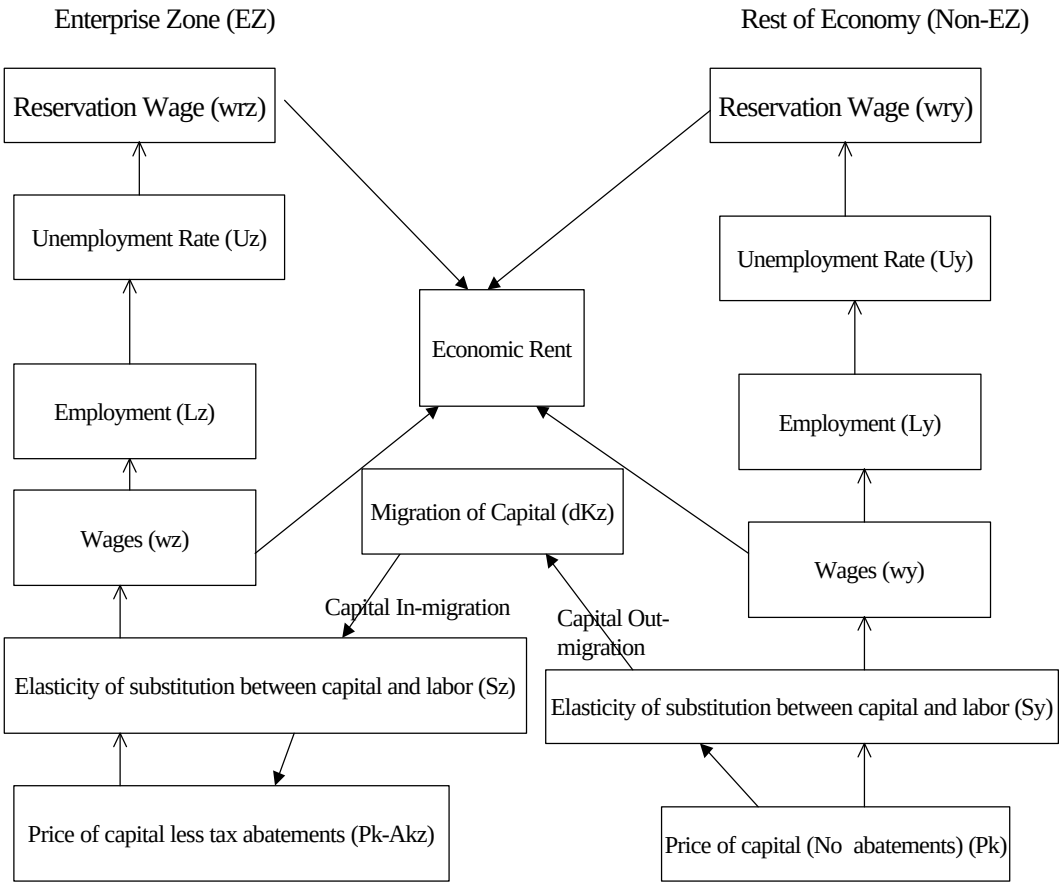
More specifically, the testable hypotheses that emerge from the theoretical model are:

- The relationship between the reservation wage and unemployment rate

- The determinants of area unemployment rate being the EZ (or the existence of other tax incentive programs), the duration of the EZ program's existence and labour in-migration into the area. It may be noted that the context for this test is laid out in figure 2. A reduction in the price of capital through tax abatements (that occurs only in EZ-designated or tax incentive areas) increases the capital-labour ratio in the EZ and increases labour productivity, raising wages and employment and affects the unemployment rate. Increasing wages encourage labour in-migration into the zone, which affects the unemployment rate by changing the allocation of employment created by zone firms. Thus, zone designation, the duration of the zone's existence, and population in-migration into the area affect its unemployment rate according to the model.

Thus, one of the testable hypotheses that emerges from the model is that in an empirical examination of the unemployment rate, we include a dummy for tax incentive programs, the duration of the zone's existence, in addition to other controls as exogenous variables. The other control variables are skill differences (some skills have lower unemployment rates than others), and demographic characteristics (unemployment rates vary across various demographic groups) such as race, age, sex composition, and educational attainment of the area.

Figure 2: The Impact of the Enterprise Zone (Tax Incentives) with Capital Mobility



In the empirical work, one could empirically test these hypotheses that emerge from the theoretical model. The question as to whether or not $dA_{jz} > db_{jz,}$ i.e., whether the cost of the abatement (EZ program) exceeds the economic rent that accrues to the individuals employed in the EZ at equilibrium forms the basis for the benefit-cost analysis in the empirical work reported in Sridhar (2001).

VI. Predictive Power of Model

It is a widely accepted view that a model should be judged by the accuracy with which it can predict what we observe as well as the realism of its assumptions. The predictive power of the model that is presented here depends on the realism of the assumptions.

First, the assumption of disequilibrium in the EZ before EZ designation is consistent with what we observe. EZs are in reality abandoned areas with high unemployment rate (when we take into account the designation criteria in most of the state enterprise zone programs). Hence for persons in the EZ, the job search behavior characterised by low reservation wages seems realistic due to reasons of family or other psychological reasons or costs of relocation.

Further, the general equilibrium response to the tax abatement is to be expected because of the inherent mobility of capital in response to changes in its price until it is equalised across areas. Further, the effects could get transmitted to the goods market in both the areas as well. Thus the analysis indicates that it is impossible to isolate the incidence of the tax abatement to the EZ alone, which mirrors reality. A simple example of this in the model is the movement of capital. The model indicates that the migration of capital (not necessarily of firms) serves to lower the unemployment rate of the EZs, without changing that in the other areas.

Thus, when we begin with realistic assumptions regarding initial conditions in the EZ and study the implications of a tax abatement in the context of a theoretical framework, we obtain a set of hypotheses that seem plausible and testable. Given these facts, the model that is

presented here is of sufficient generality that it is applicable to most enterprise zone programmes.

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APPENDIX

Solutions For Change In Employment And Investment In Enterprise Zone.

Solution to the System of Equations

The simultaneous equations are in matrix form, and Cramer's rule is used to solve for dK_Z and dL_Z. So

$$dK_Z = \frac{|D_1|}{|D|}$$

where |D| is the determinant of the matrix and $|D_1|$ is the determinant of the matrix with its first column replaced by the d vector. So

$$dK_Z = \frac{\frac{-S_Y dP_{LY}}{L_Z}\left[\frac{-S_Z f_{LZ}}{1-A_{KZ}} - e(f_{KZ}-f_{KY})\right] - \frac{e}{L_Y(1-A_{KZ})}\left[\begin{matrix}S_Z dA_{KZ} f_{KZ} - S_Z(f_{KZ}-f_{KY})^* \\ (dP_{LZ}(1-A_{KZ}) + dA_{KZ})\end{matrix}\right]}{\frac{S_Z f_{KZ}}{(1-A_{KZ})}\left[\frac{1}{K_Y L_Z} + \frac{1}{L_Y K_Z}\right] + e(f_{KZ}-f_{KY})\left[\frac{1}{K_Y L_Z} - \frac{1}{L_Y K_Z}\right] + S_Z\left[\frac{f_{KZ}+f_{LZ}}{K_Z L_Z}\right]}$$

$$- \frac{\frac{S_Y e dA_{KZ} f_{KZ}}{L_Y} - \frac{f_{LZ} S_Y S_Z}{L_Z}\left[\frac{dP_{LZ}(1-A_{KZ}) + dA_{KZ}}{(1-A_{KZ})}\right]}{\frac{S_Z f_{KZ}}{(1-A_{KZ})}\left[\frac{1}{K_Y L_Z} + \frac{1}{L_Y K_Z}\right] + e(f_{KZ}-f_{KY})\left[\frac{1}{K_Y L_Z} - \frac{1}{L_Y K_Z}\right] + S_Z\left[\frac{f_{KZ}+f_{LZ}}{K_Z L_Z}\right]}$$

In a very similar way,

$$dL_Z = \frac{|D_2|}{|D|}$$

where |D| is the determinant of the matrix as before and $|D_2|$ is the determinant of the matrix with its second column now replaced by the d vector. The expression obtained for dL_Z turns out to be:

$$dL_Z = \frac{\begin{array}{l} \frac{-1}{K_Y(1-A_{KZ})}\left[S_Z(edA_{KZ}f_{KZ} - (dA_{KZ} + dP_{LZ}(1-A_{KZ})e(f_{KZ} - f_{KY}))\right] \\ + S_Y dP_{LY} f_{KZ}\left[\frac{-S_Z}{L_Z(1-A_{KZ})} + \frac{e(1-f_{KY})}{K_Z}\right] + \frac{S_Y S_Z f_{KZ}}{L_Z}\left[\frac{dA_{KZ} + dP_{LZ}(1-A_{KZ})}{(1-A_{KZ})}\right] \end{array}}{\frac{S_Z f_{KZ}}{(1-A_{KZ})}\left[\frac{1}{K_Y L_Z} + \frac{1}{L_Y K_Z}\right] + e(f_{KZ} - f_{KY})\left[\frac{1}{K_Y L_Z} - \frac{1}{L_Y K_Z}\right] + S_Z\left[\frac{f_{KZ} + f_{LZ}}{K_Z L_Z}\right]}$$