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INPUT DEMAND AND THE SHORT- AND  
LONG-RUN EMPLOYMENT THRESHOLDS  
An Empirical Analysis for the German  
Manufacturing Sector

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**Abstract**

The concept of the “employment threshold” plays an important role in the public discussion of unemployment. The employment threshold is defined as that growth rate of output which is necessary to keep employment constant despite the continuous rise in labour productivity. It is related to the Okun coefficient which describes the relationship between the changes of output and unemployment. Many contributions to this debate give the impression that the employment threshold is more or less a structural characteristic which remains constant over time. In this paper we derive short- and long-run employment thresholds from an input demand system and show empirically that they depend on factor prices. A moderate wage policy leads to a reduction of the output growth which is necessary for an increase in employment.

Keywords: Okun’s Law, employment threshold, productivity

JEL Classification: D24, E24, L60

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# 1 Introduction

During the last two decades unemployment has been one of the most serious economic problems in many European countries. A key issue in the long lasting debate about this problem is the role of wages in determining the level of employment. Whereas some proponents of demand side policy argue that higher wages induce an increase in output demand and hence in employment, the opponents stress that wage increases will lead to a substitution of labour by capital and to higher output prices which will erode international competitiveness.

In the following study we address some of these problems within an unifying framework of cost minimization and price setting behaviour which is modeled as a variable mark-up over marginal costs. Since we are using a restricted cost function with capital as a quasi-fixed input we are able to distinguish between the short-run and long-run effects of input prices on labour demand. Within this framework we derive short- and long-run “employment thresholds” which play an important role at least in the German debate about the employment problem. The employment threshold is defined as that output growth rate which is necessary for a constant employment level. In contrast to many previous studies which treat the employment threshold as a stable coefficient, we explicitly relate this variable to factor price changes, capital accumulation and time varying technical progress.

This approach has also some relevance for Okun’s Law which is used by many applied macroeconomists as the link between changes in output and unemployment. Based on our empirical findings we argue that neglecting the effect of factor price changes implies a serious misspecification so that one can’t hope to get a stable relationship between the two variables.

The paper is organized as follows: Section 2 presents the theoretical foundations for the derivation of the employment thresholds from an cost minimization approach. Section 3 explains the econometric specification. In Section 4 the empirical results are presented and discussed. Based on the most important results some conclusions are drawn in Section 5.

## 2 Theoretical foundations

### 2.1 The model

In the following we assume that a representative firm uses the inputs labour ( $L$ ), intermediate products ( $M$ ) and capital ( $K$ ) to produce the output ( $Y$ ). While labour (measured in “worker-hours” which denotes the product of the number of employed persons and the amount of time they work) and intermediate products (material, energy) are variable inputs which can be changed without adjustment costs, due to convex adjustment costs or external restrictions capital is treated as a quasi-fixed input. Quasi-fixed inputs are predetermined in the short-run, but can be varied in the long-run (typically in some variant of a flexible accelerator).

If firms minimize their short-run production costs at given input prices and given levels of output and capital there exists under weak assumptions with regard to production technology (see e.g. McFadden 1978 or Chambers 1988) a variable cost function:

$$CV = CV(q_m, q_l, K, Y, t) \quad (1)$$

$CV$  indicates the minimum variable cost of producing the output ( $Y$ ) at given prices ( $q_m, q_l$ ) of the variable production factors intermediate inputs ( $M$ ) and labour ( $L$ ) and at the given level of capital stock ( $K$ ). The time index  $t$  represents the state of the technology. In order to be able to represent all economically relevant information of the underlying technology the variable cost function must meet certain regularity conditions (Lau 1978, McFadden 1978):  $CV$  must be decreasing in  $K$  and increasing in  $q_l, q_m$  and  $Y$ . Moreover,  $CV$  has to be convex in  $K$  and concave and linearly homogeneous in  $q_l$  and  $q_m$ .

The short-run demand functions for the variable factors are derived via Shepards' Lemma:

$$L(q_m, q_l, K, Y, t) = \frac{\partial CV}{\partial q_l} \quad (2)$$

$$M(q_m, q_l, K, Y, t) = \frac{\partial CV}{\partial q_m} \quad (3)$$

The model is supplemented by taking a supply relation into consideration:

$$P = \frac{\mathbb{1}CV}{\mathbb{1}Y} * \Theta, \quad (4)$$

whereby  $P$  represents the output price and  $\Theta$  the mark-up over the marginal costs. The price equation given in (4) can be seen as a reduced form, since the determinants of the mark-up are not derived from a structural oligopoly model; instead the mark-up is specified as a function of exogenous variables. The empirical specification is discussed in Section 3.

On the basis of equations (4) and (5) the short -run demand elasticities can be easily calculated; for example, the own price elasticity of labour demand is given by  $e_{L,q_l}$  :

$$e_{L,q_l} = \frac{\mathbb{1}L}{\mathbb{1}q_l} \frac{q_l}{L} = \frac{\mathbb{1}^2 CV}{\mathbb{1}q_l^2} \frac{q_l}{\mathbb{1} CV / \mathbb{1}q_l} \quad (5)$$

The elasticities  $e_{M,q_m}$ ,  $e_{L,q_m}$ ,  $e_{M,q_l}$ ,  $e_{L,Y}$  and  $e_{M,Y}$  can be determined in an analogous manner.

## 2.2 Employment Thresholds

For studying the impact of output changes on labour demand, instead of the demand elasticities of labour the concept of the “employment threshold” is frequently used in empirical economic research and in the economic-policy discussion. The employment threshold is meant to indicate the output growth rate at which employment remains constant. The impression is frequently given that there exists a constant employment threshold for a longer period of time (e.g. Siebert 1999). For the empirical determination of the employment threshold, simple regressions of the growth rate of employment and the growth rate of average labour productivity on the growth rate of output are estimated (Okun’s Law and Verdoorn’s Law) (see Hof 1994, Weeber 1997). What is completely neglected is that employment is also greatly affected by factor prices. To eliminate this deficiency we derive the employment threshold from the theoretically well founded labour-demand function (2). For this we take the total differential of the labour demand function with respect to time:

$$\frac{dL}{dt} = \frac{\mathbb{1}^2 CV}{\mathbb{1}q_l^2} \frac{dq_l}{dt} + \frac{\mathbb{1}^2 CV}{\mathbb{1}q_l \mathbb{1}K} \frac{dK}{dt} + \frac{\mathbb{1}^2 CV}{\mathbb{1}q_l \mathbb{1}q_m} \frac{dq_m}{dt} + \frac{\mathbb{1}^2 CV}{\mathbb{1}q_l \mathbb{1}Y} \frac{dY}{dt} + \frac{\mathbb{1}^2 CV}{\mathbb{1}q_l \mathbb{1}t} \quad (6)$$

In order to calculate the employment threshold, we set  $\frac{dL}{dt}$  in equation (6) to zero and get after some manipulations the short-run employment threshold  $w_Y^s$ .

$$w_Y^s = -(\mathbf{e}_{L,q_l} w_{q_l} + \mathbf{e}_{L,q_m} w_{q_m} + \mathbf{e}_{L,K} w_K + \mathbf{e}_{L,t}) / \mathbf{e}_{L,Y}, \quad (7)$$

whereby  $w$  represents the growth rate for the individual variables,  $\mathbf{e}_{L,t} = \frac{\eta^2 CV}{\eta_{q_l} \eta_t} \frac{1}{L}$  is the growth rate of employment due to autonomous technological progress and the other elasticities are defined in equation (5).  $w_Y^s$  indicates the short-run employment threshold, i.e. the output growth that is necessary so that labour input remains constant at given changes of factor prices and capital stock. Since the labour demand function is homogenous of degree zero in the factor prices, we have:

$$w_Y^s = -(\mathbf{e}_{L,q_l} (w_{q_l} - w_{q_m}) + \mathbf{e}_{L,K} w_K + \mathbf{e}_{L,t}) / \mathbf{e}_{L,Y} \quad (8)$$

This equation shows that the short-run employment threshold depends on the growth rate of relative factor prices, capital accumulation and technical change. The employment thresholds determined on the basis of Qkun's Law or similar concepts and their assumed stability over longer periods of time are thus based on theoretically unsatisfactory assumptions. Blinder (1997) has even stated that Qkun's Law is anti-theoretical.

The short-run demand elasticity and the employment thresholds do not take into consideration the long-run substitution possibilities between the variable inputs and the quasi-fixed inputs. The long-run demand elasticities can also be derived from the variable cost function. In the following we will present, as an example, the derivation of the long-run own price elasticity of labour demand (for the more general case with more than one quasi-fixed production factor, see Brown and Christensen (1981)).

For the derivation of long-run factor demand, the long-run optimal capital stock is needed as a first step. This can be calculated from the following first-order condition

$$\frac{\eta CV(q_m, q_l, K^*, Y, t)}{\eta K} \Big|_{K^*} + q_k = 0 \quad (9)$$

The condition states that the shadow price of capital (the reduction of variable costs due to an additional unit of capital) must correspond in optimum to its one-period user cost  $q_k$ .

If all inputs are at their long-run equilibrium, the long-run total costs can be described as follows as the sum of the variable costs and the capital costs:

$$CT(K^*) = CV(q_m, q_l, K^*, Y, t) + q_k K^*, \quad (10)$$

whereby  $K^*$  indicates the long-run optimal level of  $K$  which is calculated using equation (9).

The partial derivative of  $CT$  with respect to the wage rate indicates the long-run labour demand function

$$L^* = \frac{\mathbb{1}CT(K^*)}{\mathbb{1}q_l} = \frac{\mathbb{1}CV(K^*)}{\mathbb{1}q_l} + \left[ \frac{\mathbb{1}CV(K^*)}{\mathbb{1}K^*} \frac{\mathbb{1}K^*}{\mathbb{1}q_l} + q_k \frac{\mathbb{1}K^*}{\mathbb{1}q_l} \right], \quad (11)$$

whereby the expression in brackets is zero according to equation (9). The difference between the short-run (2) and the long-run labour demand (11) is that in the first case the historically given value  $K$  is used and in the second case the long-run optimal value  $K^*$ .

The long-run reaction of factor demand on exogenous changes of factor prices is obtained from the second partial derivative of  $CT$ :

$$\frac{\mathbb{1}L^*}{\mathbb{1}q_l} = \frac{\mathbb{1}^2CT}{\mathbb{1}q_l^2} = \frac{\mathbb{1}^2CV}{\mathbb{1}q_l \mathbb{1}q_l} + \frac{\mathbb{1}^2CV}{\mathbb{1}q_l \mathbb{1}K^*} \frac{\mathbb{1}K^*}{\mathbb{1}q_l} + \frac{\mathbb{1}K^*}{\mathbb{1}q_l} \left( \frac{\mathbb{1}^2CV}{\mathbb{1}K^* \mathbb{1}q_l} + \frac{\mathbb{1}^2CV}{\mathbb{1}K^{*2}} \frac{\mathbb{1}K^*}{\mathbb{1}q_l} \right) + \left[ \frac{\mathbb{1}CV}{\mathbb{1}K^*} \frac{\mathbb{1}^2K^*}{\mathbb{1}q_l^2} + q_k \frac{\mathbb{1}^2K^*}{\mathbb{1}q_l^2} \right], \quad (12)$$

whereby again the expression in brackets is zero on the basis of (9). Equation (12) also requires a calculation of the expression  $\mathbb{1}K^* / \mathbb{1}q_l$ . For many functional forms of the cost function, especially also for the translog specification, there is no analytical solution of equation (9) for  $K^*$  and thus also no analytical solution for the required partial derivative in (12).

With the application of the total differential and the implicit function theorem on equation (9), the necessary partial derivative is received taking into consideration  $dY = dt = dq_k = dq_m = 0, :$

$$\frac{\mathbb{1}K^*}{\mathbb{1}q_l} = - \frac{\frac{\mathbb{1}^2CV}{\mathbb{1}K^* \mathbb{1}q_l}}{\frac{\mathbb{1}^2CV}{\mathbb{1}K^{*2}}} \quad (13)$$

Thus also the expression in parentheses in equation (12) is zero. With equation (13) the calculation of equation (12) for any functional form of the cost function is possible. All derivatives in equations (11) and (12) are calculated at the point  $K = K^*$ . Thus, from (12) we have the following expression of the long-run own-price elasticity of labour demand

$$\mathbf{h}_{L,q_l} = \left( \frac{\mathfrak{I}^2 CV}{\mathfrak{I}q_l^2} + \frac{\mathfrak{I}^2 CV}{\mathfrak{I}q_l \mathfrak{I}K^*} \frac{\mathfrak{I}K^*}{\mathfrak{I}q_l} \right) \frac{q_l}{\mathfrak{I}CT/\mathfrak{I}q_l}, \quad (14)$$

whereby  $\mathfrak{I}K^*/\mathfrak{I}q_l$  is substituted from equation (13). The long-run elasticities  $\mathbf{h}_{M,q_m}$ ,  $\mathbf{h}_{L,q_m}$ ,  $\mathbf{h}_{M,q_l}$ ,  $\mathbf{h}_{L,y}$  and  $\mathbf{h}_{M,y}$  are calculated analogously. For the long-run elasticity of labour demand with respect to the capital user price we have:

$$\mathbf{h}_{L,q_k} = \left( \frac{\mathfrak{I}^2 CV}{\mathfrak{I}q_l \mathfrak{I}K^*} \frac{\mathfrak{I}K^*}{\mathfrak{I}q_k} \right) \frac{q_k}{\mathfrak{I}CT/\mathfrak{I}q_l} \quad (15)$$

In comparison to (14) the first term in parentheses is eliminated, since the partial derivative of  $CV$  with respect to the capital user cost is zero. Analogously  $\mathbf{h}_{M,q_k}$  is determined. The long-run elasticities of capital demand are obtained directly from the application of the implicit function theorem on the first-order conditions for the optimal capital use in (9).

Analogously to (8) by the total derivative of the long-run labour demand function we obtain the long-run employment threshold  $w_y^l$ :

$$w_y^l = - \left( \mathbf{h}_{L,q_l} w_{q_l} + \mathbf{h}_{L,q_m} w_{q_m} + \mathbf{h}_{L,q_k} w_{q_k} + \mathbf{h}_{L,t} \right) / \mathbf{h}_{L,y}. \quad (16)$$

with the long-run elasticities from (14) and (15) as well as  $\mathbf{h}_{L,t} = \frac{\mathfrak{I}^2 CT(K^*)}{\mathfrak{I}q_l \mathfrak{I}t} \frac{1}{L}$ . The long-run employment threshold indicates the necessary output change as a function of factor price changes and technical progress in order that employment remains constant even when taking additional long-run substitution possibilities (capital) into consideration. Equations (8) and (16) allow to calculate the impact of hypothetical factor price changes on the short-run and long-run employment threshold in a straightforward manner.



### 3 Empirical specifications and data

In order to calculate the demand elasticities and employment thresholds discussed in Section 2 empirically, a parametric variable cost function must be specified. In the following we use the translog function, which, for example, is the basis of the studies by Brown/Christensen (1981) and Berndt/Hesse (1986) and which presents a flexible functional form (see e.g. Chambers 1988, Ch.5 or Morrison 1992, Ch.7.2).

For our model with the variable inputs of labour and intermediate products and the quasi-fixed input capital, the variable translog cost function is given by

$$\begin{aligned} \ln CV = & \mathbf{a}_0 + \mathbf{a}_Y \ln Y + \mathbf{a}_l \ln q_l + \mathbf{a}_m \ln q_m + \mathbf{a}_k \ln K + \mathbf{a}_t t + 0.5 \mathbf{g}_{YY} (\ln Y)^2 + 0.5 \mathbf{g}_{ll} (\ln q_l)^2 \\ & + 0.5 \mathbf{g}_{mm} (\ln q_m)^2 + 0.5 \mathbf{g}_{kk} (\ln K)^2 + 0.5 \mathbf{g}_{tt} t^2 + \mathbf{g}_{Yl} \ln Y \ln q_l + \mathbf{g}_{Ym} \ln Y \ln q_m + \mathbf{g}_{Yk} \ln Y \ln K \\ & + \mathbf{g}_{Yt} t \ln Y + \mathbf{g}_{lm} \ln q_l \ln q_m + \mathbf{g}_{lk} \ln q_l \ln K + \mathbf{g}_{lt} t \ln q_l + \mathbf{g}_{mk} \ln q_m \ln K + \mathbf{g}_{mt} t \ln q_m + \mathbf{g}_{kt} t \ln K \end{aligned} \quad (17)$$

The necessary condition of linear homogeneity in the input prices implies (see e.g. Berndt/Hesse 1986)

$$\begin{aligned} \mathbf{a}_l + \mathbf{a}_m = 1 & & \mathbf{g}_{lt} + \mathbf{g}_{mt} = 0 \\ \mathbf{g}_{Yl} + \mathbf{g}_{Ym} = 0 & & \mathbf{g}_{ll} + \mathbf{g}_{lm} = 0 \\ \mathbf{g}_{lk} + \mathbf{g}_{mk} = 0 & & \mathbf{g}_{lm} + \mathbf{g}_{mm} = 0 \end{aligned} \quad (18)$$

In order that the underlying dual production function displays constant return to scale, the following necessary and sufficient conditions must be fulfilled (see Brown/Christensen 1981)

$$\begin{aligned} \mathbf{a}_Y + \mathbf{a}_k = 1 \quad \text{a)} & & \mathbf{g}_{Ym} + \mathbf{g}_{mk} = 0 \quad \text{d)} \\ \mathbf{g}_{YY} + \mathbf{g}_{Yk} = 0 \quad \text{b)} & & \mathbf{g}_{Yk} + \mathbf{g}_{kk} = 0 \quad \text{e)} \\ \mathbf{g}_{Yl} + \mathbf{g}_{lk} = 0 \quad \text{c)} & & \mathbf{g}_{Yt} + \mathbf{g}_{kt} = 0 \quad \text{f)} \end{aligned} \quad (19)$$

In using the translog function, the factor demand functions are mostly estimated in the form of cost shares (e.g. for the input labour:  $L^* q_l / CV = \partial \ln CV / \partial \ln q_l$ ) together with the cost function CV.

In this study we use the input coefficients as the dependent variables:

$$\frac{L}{Y} = \frac{\mathcal{I}CV}{\mathcal{I}q_l} \frac{1}{Y} = \frac{\mathcal{I} \ln CV}{\mathcal{I} \ln q_l} \frac{CV}{q_l} \frac{1}{Y} \quad (20)$$

$$\frac{M}{Y} = \frac{\mathcal{I}CV}{\mathcal{I}q_m} \frac{1}{Y} = \frac{\mathcal{I} \ln CV}{\mathcal{I} \ln q_m} \frac{CV}{q_m} \frac{1}{Y} \quad (21)$$

For the empirical implementation of the price function we assume that the output price is determined by means of a variable mark-up over the short-run marginal costs. The main discussion in the literature is whether the price mark-up rate exhibits a procyclical or a countercyclical behaviour (see e.g. Rotemberg/Woodford 1991). In this paper we assume that the mark-up rate may depend on the growth rate of output.

For this reason we specify the price equation as

$$P = (b_0 + b_1 w_Y) \frac{\mathcal{I}CV}{\mathcal{I}Y} \quad (22)$$

with  $w_Y$  as the growth rate of real output.

The investigation is based on the West German manufacturing sector for the period from 1968 to 1995. Where no other reference is given, all data are from National Accounts. As measures for real output and intermediate product we use the productions value and intermediate input in 1991 prices. The implicit price deflators serve as prices for output and intermediate input. The total labour volume is constructed by multiplying the actual annual working hours (source: Görzig et al. 1997) with the number of all employed persons<sup>1</sup>. The nominal costs of employees correspond to income from dependent employment. If this amount is divided by the annual labour volume of employees, we obtain the nominal hourly wage rate.

According to the concept of National Accounts, the purchases of new capital goods are recorded for the purchaser (owner concept). Since leasing and other forms of hiring of capital goods has become more widespread in the business sector since the 1970s, we employ the user concept for the production factor capital. The nominal user costs of capital and the gross capital stock in 1991 prices are based on calculations of the ifo Institute (for the methodology, see *Gerstenber-*

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<sup>1</sup> It is assumed that the annual hours worked by dependent employees and by self-employed as well as by helping family members do not differ.

ger/Heinze/Hummel/Vogler-Ludwig 1989). For the gross capital stock, values at the beginning of the year are used.

## 4 Results

### 4.1 Short- and long-run factor demand

The system to be estimated consists of the cost function (17), the short-run input demand functions (20), (21) and the price equation (22). Since the cost function in (17) contains a lot of parameters to be estimated and we only have observations for 27 years,<sup>2</sup> the linear homogeneity of the variable costs functions in the factor prices (18) were imposed on the estimates. On the other hand, we do not force constant returns to scale onto the estimates in order not to unnecessarily restrict the technology from the very outset.

The model was estimated with the non-linear, three-stage least squares method. In order to take the potential endogeneity of the output variables into consideration, we have used all factor prices, the trend terms and the lagged values of the endogenous variables and of the output as instrumental variables. Subsequently, we tested for constant returns to scale. To do this the restrictions in (19) were examined both individually and simultaneously with the Wald test. The hypothesis of constant returns to scale (Wald test statistics = 177 at 5 degrees of freedom) was rejected.<sup>3</sup> It became apparent, however, that restriction (19f) is completely insignificant. Since in addition also the individual parameters of this restriction proved to be insignificant (t-values: -0.56 and -0.56), these parameters were set to zero for the final estimation in order to achieve as economical a specification as possible.

The cost function must satisfy certain regularity conditions (see Section 2) in order to be able to describe the technology and firms cost-minimizing behaviour. They are all fulfilled here since the cost function in our estimates for each year proved to be increasing and concave in the factor prices and decreasing and convex in the quasi-fixed factor capital.

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<sup>2</sup> Since the growth rate of output enters into the price comparison, one observation is lost.

<sup>3</sup> On the basis of the restrictions in (18) we only receive five linearly independent restrictions in (19), since in the case of linear homogeneity in input prices, the restrictions (19c) and (19d) are identical.

The estimated parameter and the accompanying t-values as well as the summary statistics are displayed in Tables 1 and 2. Nearly all parameters are highly significant. Thus, simple cost or production functions like Cobb Douglas or CES functions are not suitable for depicting all economically relevant aspects of technology. The adjustment of the model to the data is very good on the whole (see Table 2): In every equation  $R^2$  amounts to more than 98 percent, and there is no sign of autocorrelation of the first degree, which is particularly important for the chosen instruments (lagged values of endogenous variables). In addition a comparison of the t-values in columns three and four of Table 1 indicate that no significant heteroscedasticity problems occur.

The highly significant parameters  $b_0$  and  $b_1$  describe the price setting behaviour of the firms. The estimates show that rising marginal costs lead enterprises to increase their prices. In case of perfect competition,  $b_0$  must be one, since this parameter measures the average markup over the marginal costs. This null hypothesis, however, is clearly rejected (Wald test statistic = 7.78 with one degree of freedom). Parameter  $b_1$  measures the cyclical variability of the markup. The parameter is positive which means that the mark-ups display a procyclical behaviour.

For 1990 the short- and long-run elasticities of factor demand are given in Table 3. The short-run elasticities of factor demand with respect to the wage rate is -0.17. The long-run absolute elasticity of labour with respect to the wage rate is approximately double that of the short-run. Since the short-run cross price elasticities of the variable inputs do not differ from the long-run, this effect must take place by means of an adjustment of the quasi-fixed factor to wage changes. As seen in Table 3, the long-run elasticity of capital demand with respect to the wage rate is approximately 0.32. Thus, the long-run substitution of labour by capital makes a major contribution to the higher long-run elasticity of labour demand. Wage increases thus lead in the long run to a stronger decline in employment than the short-run reaction of labour demand indicates.

**Table 1: Parameter estimations**

Parameter <sup>1</sup>	Estimation	t-value	t-value (White)
$a_0$	4.9904	1.42	1.29
$a_Y$	1.3824	8.34	7.47
$a_l$	1.3890	18.05	18.64
$a_k$	-1.5507	-1.54	-1.40
$a_t$	-0.0144	-8.53	-8.51
$g_{tt}$	0.0004	4.90	4.65
$g_{YY}$	0.1201	3.51	4.55
$g_{ll}$	0.1566	21.87	27.29
$g_{kk}$	0.3615	2.38	2.21
$g_{Yl}$	-0.0047	-0.43	-0.42
$g_{Yk}$	-0.1715	-4.06	-4.41
$g_{mk}$	0.1311	9.81	7.86
$g_{mt}$	0.0028	11.12	8.91
$b_0$	1.0829	43.20	36.43
$b_1$	0.2350	5.08	5.19

**Table 2: Summary statistics**

Equation	$R^2$	Durbin-Watson
$CV$	0.9997	1.2695
$A/Y$	0.9965	1.1162
$M/Y$	0.9850	1.5771
$P$	0.9967	1.4471

<sup>1)</sup> For parameters  $a_m$ ,  $g_{lm}$ ,  $g_{mm}$ ,  $g_{Ym}$ ,  $g_{lk}$ ,  $g_{lt}$  see the restrictions in (30)

<sup>2)</sup> Calculated with the help of a heteroscedasticity – consistent variance/covariance matrix

In contrast, the long-run elasticities of demand for intermediate input hardly differs from that of the short run, which is not surprising given the low cross-price elasticities between intermediate input and capital. In the long run, all three production factors are Allen substitutes, whereby the substitutional relationship between intermediate input and capital is insignificantly small. All-in-all, intermediate input show the least price elasticity. Similar results were obtained by Falk and Koebel (1997) for the West German manufacturing industry.

The short-run elasticities of the variable inputs with respect to output is in both cases approximately one. In the short-run we thus have a homothetic technology in the variable production factors, since in the case of output change, *ceteris paribus*, the factor input ratio does not change. In the long run, however, we cannot assume a homothetic technology. The long-run elasticity of labour demand with respect to output falls to approximately 0.62, whereby the elasticity of intermediate input demand hardly changes. Also the elasticity of capital demand with respect to output is considerably below one. The long-run output elasticities thus show that there are increasing returns to scale in the manufacturing industry.

**Table 3: Elasticities of factor demand (1990)**

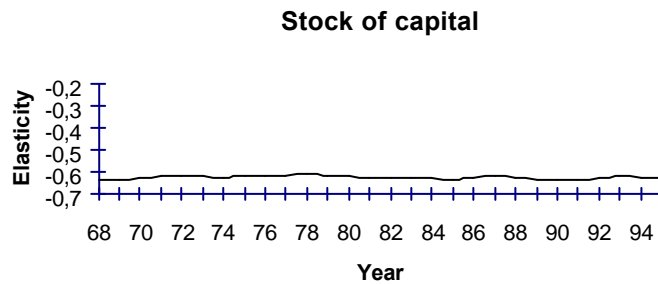
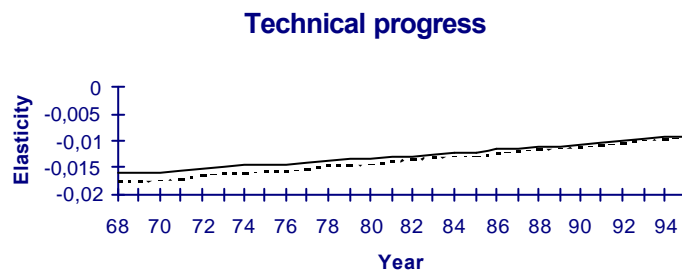
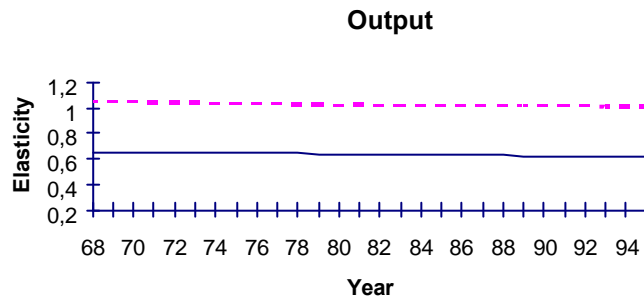
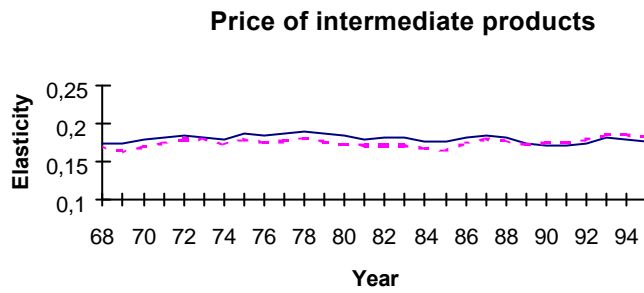
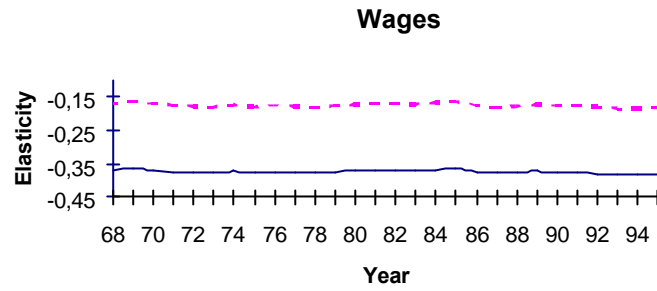
	Short run			Long run		
	Labour	Intermediate input	Capital	Labour	Intermediate input	Capital
$q_l$	-0.174	0.073	-	-0.375	0.071	0.318
$q_m$	0.174	-0.073	-	0.170	-0.072	0.005
$q_k$	-	-	-	0.205	0.001	-0.323
$Y$	1.020	1.043	-	0.622	1.040	0.627
$K$	-0.635	-0.003	1	—	—	—

In Figure 1 the time path of all estimated demand-elasticities is displayed. With the exception of the elasticity with respect to time the estimated short- and long-run elasticities proved to be very stable for the whole period. The negative effect of time on labour demand declined by half over the estimation period. This is attributable to the productivity slowdown, since at a lower growth rate of the total factor productivity, the reduction in labour demand due to autonomous technical progress gets smaller.

## 4.2 Employment thresholds

In Section 2 it was shown that the employment threshold and the input-elasticities are closely related. Based on the estimated parameters of our empirical model we have calculated the

**Fig. 1: Elasticities of labour demand with respect to wages, intermediate input prices, output, technical progress and capital stock**



-----short run    \_\_\_\_\_long run

short-run employment threshold  $w_Y^s$  (equation (8)) for each year in our sample at the realized historical growth rates of factor prices and the capital stock (Figure 2).

If the elasticities used in equation (8) are constant over time, equation (8) can be interpreted as a regression equation, whereby the estimated coefficients reflect the effects of factor prices and the capital stock on the output growth which is necessary for a constant labour volume. The estimated coefficient for the intercept shows the influence of technical progress on  $w_Y^s$ . If the elasticities are not constant over time and exhibit high volatility it would not be reasonable to run a regression with constant coefficients. As Fig. 1 shows the elasticities are not constants but show a very stable behaviour. The only exception is the effect of technical progress which can however be modeled as a linear function of time.

**Fig. 2: Short-run employment threshold**

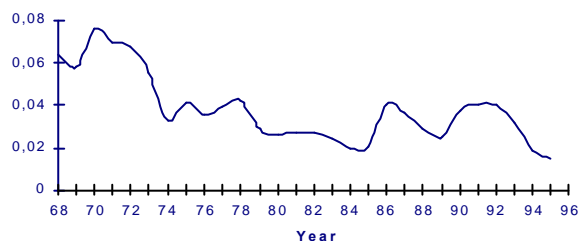


Table 4 shows two versions of OLS estimates. The estimates in version I take into account as explanatory variables only the growth rates of the relative factor prices and the capital stock, in addition to a constant term. The very low Durbin-Watson Test statistic points to a potential misspecification. For this reason and for capturing the absolutely declining effect of technical progress in the estimation of version II we have additionally taken a time trend into consideration. This improved the Durbin-Watson statistic considerably and all variables are highly significant. These four variables suffice to explain nearly completely the variation of  $w_Y^s$ . The corrected  $R^2$  amounts to nearly 99 percent. Therefore the relationship between the employment threshold and its determinants can be depicted as a linear relationship. The influence of technical progress on the employment threshold has declined over time. At the end of the 1960s an output growth of 2 percent was necessary in order that the productivity growth, ceteris paribus, do not lead to a reduction of labour demand whereas in the mid 1990's only a 1 percent increase was necessary. If the wage rate increases by 1 percent or if the intermediate product prices fall by 1 percent, the output must rise, ceteris paribus, by an additional



0.2 percentage points so that employment remains constant. This only takes into account the short-run substitution possibilities between the labour and the intermediate product input.

**Table 4: OLS estimates for short-run employment threshold  $w_Y^s$**

Parameter	Version I		Version II	
	Estimate	t-value	Estimate	t-value
Constant	0.011	10.02	0.020	12.69
$\Delta \ln(q_l/q_m)$	0.173	11.13	0.174	17.75
$\Delta \ln K$	0.704	19.05	0.579	18.91
T			-0.0004	-6.24
$\bar{R}^2$	0.968		0.987	
Durbin-Watson	0.684		1.179	

On the basis of the long-run labour-demand function, we can, in turn, calculate a hypothetical output threshold for labour volume at a given historical development of factor prices but now including capital user costs instead of the capital stock (see equation 16). Since all long-run elasticities except for  $\mathbf{h}_{L,t}$  show little change over time (see Figure 1), the effect of factor prices on this hypothetical employment threshold can be easily calculated. From equation (16) it follows that the long-run effect of factor price changes on the employment threshold is equal to the ratio of the respective long-run price elasticity to the long-run output elasticity of labour demand. Thus, a one percent wage increase requires, ceteris paribus, in the long run an output growth of approximately 0.6 percent so that labour input remains constant. In the long run, therefore, an output growth that is three times as high as the short-run growth is necessary. This is for two reasons. Firstly, the long-run labour-demand elasticity is approximately double that of the short-run labour-demand elasticity. Secondly, labour demand reacts less to output increases in the long run than in the short run so that a high output growth is necessary in order to stabilize employment.

## 5 Summary and conclusions

In this study we have analyzed the labour demand of the manufacturing industry on the basis of a variable translog cost function. We found that there are significant and important substitution effects and that thus wage increases lead to a reduction of labour demand, at a given output level. The short-run and long-run elasticities of labour demand with respect to the wage rate amounts to -0.17 and -0.37, respectively. Wage increases thus lead to a lower increase in the nominal wage bill than the corresponding percentage increase in hourly wage rates. As our study shows, firms also raise their output prices, since wage increases lead to a rise in marginal costs. Thus the increase in the real wage bill and the associated demand effect is lower than the increase in the nominal wage bill. In addition, price increases reduce the international competitiveness of domestic industry, which causes a reduction of exports and an increase in import demand. If the firms were not able to transfer the cost increases to their prices, their real profits would decline and thus also the consumption and investment demand that is dependent on profits. For all these reasons it seems highly unlikely to us that a one-percent wage increase would raise overall demand in the long term by the same amount as the employment threshold (0.6 percent) increases. Wage increases thus induce a reduction in employment.

As this discussion shows, the concept of “employment threshold” may be an useful tool for analyzing the employment problem. But this is only the case if the concept is correctly applied. The widespread belief that employment thresholds are something like a natural constant is very misleading. The employment threshold depends crucially on the time path of wages and the prices of intermediate products as well as on capital accumulation in the short run and user cost of capital in the long run. It evolves in a complicated way over time when some of its determinants are changed. There is no simple coefficient which can capture all these aspects.

There is one additional complication when we are interested not only in total labour volume, which is the object of this study, but also in the number of employed persons. This variable is typically thought to be a quasi-fixed input since hiring and firing is associated with severe restrictions and high adjustment costs. The employment threshold for the number of workers may be an even more complicated dynamic function of the determinants of employment. The analysis of this problem within a cost-minimizing framework is left for future research.

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