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Abstract

A vast literature analyzes the real effects of price-adjustment costs assuming that quantity adjustments are costless. In this paper, we analyze whether the presence of quantity-adjustments costs, which presumably are significant, change the traditional results on the impact of inflation. In particular, recent findings suggest that quantity-adjustment costs may remove the linkage between output and inflation. We show that this is not the case when inflation is anticipated. On the contrary, quantity-adjustment costs may significantly amplify the consequences of price-adjustment costs.

Keywords: Output-inflation relationship, menu costs, quantity-adjustment costs

JEL Classification: E31

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1 Introduction

It is by now well established that small price-adjustment costs, the so-called menu costs, may cause nominal changes to have large real effects. This result is derived assuming that quantities can be adjusted costlessly. Empirical evidence shows that menu costs are not trivial (see Levy et al. 1997, 1999), but to our knowledge, there exists no empirical investigation of the size of quantity-adjustment costs. Nevertheless, one might expect such costs to be even larger than the price-adjustment costs. For instance, in a downturn it is likely to be more expensive to fire workers than to pay the menu costs and lower prices. This raises the question of whether the existence of non-trivial quantity-adjustment costs invalidates the above menu-cost result. Andersen (1994 ch. 5, 1995) address this issue and show, among other things, that following a nominal disturbance, quantity-adjustment costs larger than price-adjustment costs are sufficient to keep the output at a fixed level. However, since Andersen considers "Knightian" uncertainty (i.e., a shock occurs although the agents are completely sure that it will not happen), the fixed level of output is identical to what would be produced under complete certainty. Hence, Andersen's result indicates that output is independent of inflation.

In this paper, we consider the other extreme where there is no uncertainty, but a fully anticipated, constant rate of inflation. This is similar to Sheshinski and Weiss (1977), Kuran (1986), Naish (1986), Danziger (1988), Konieczny (1990), and Bénabou and Konieczny (1994) who have analyzed the output-inflation relationship with price-adjustment costs, but without quantity-adjustment costs. We consider the case where quantity-adjustment costs are sufficiently high that the output is kept unchanged, and for tractibility, we assume that there is no discounting, a constant elasticity of demand, and a constant unit cost of production. We show that in this case the main results obtained in this strand of literature remain valid even with quantity-adjustment costs: the higher the rate of inflation, the higher is the initial real price and the lower is the terminal real price (Sheshinski and Weiss, 1977). Furthermore, the higher the rate of inflation, the lower is the average output (Kuran, 1986; Naish, 1986).[1]

We also consider the quantitative importance of quantity-adjustment costs by

[1] In our framework, output is contant and therefore always equal to the average output, whereas without quantity-adjustment costs, output varies with the real price.

examining the size of the output loss caused by inflation with and without quantity-adjustment costs. For realistic values of the menu cost, the loss of output due to inflation is several times larger with quantity-adjustment costs than without. Thus, far from invalidating the previous finding of a negative output-inflation relationship, the introduction of quantity-adjustment costs amplifies the negative consequences of price-adjustment costs.

2 The Model

We consider a monopolistic firm that produces a non-storable good and faces the time-invariant demand function $z_t^{-\alpha}$, where z_t is the real price of the good at time t and $\alpha > 1$. The real cost of producing one unit of the good is a constant $k > 0$. The firm faces both price- and quantity-adjustments costs, implying that the firm neither wants to adjust its nominal price nor its output continuously. In fact, we assume that the quantity-adjustment costs are so high that the firm chooses to always keep its output at a constant level denoted by Q.[2] As a consequence, the firm's sales will in general differ from its output, with the firm producing excess output if $Q > z_t^{-\alpha}$, and the consumers being rationed if $Q < z_t^{-\alpha}$.[3]

Let Z be the real price at which sales equal production, that is $Z = Q^{-1/\alpha}$. The real profit of the firm at time t is therefore $\Pi(z_t, Q) \equiv z_t \min\left(z_t^{-\alpha}, Q\right) - kQ$.

< Figure 1 >

The upper curve in Figure 1 illustrates the real profit as function of the real price if production could be adjusted costlessly. In this case the real profit is $z_t^{1-\alpha} - kz_t^{-\alpha}$, which is maximized for the real price $\hat{Z} \equiv \frac{\alpha k}{\alpha-1}$ and the quantity $\hat{Q} \equiv \left(\frac{\alpha k}{\alpha-1}\right)^{-\alpha}$. The lower curve shows the real profit when the firm fixes its output level at Q. Profits are identical only at the real price Z, which yields the maximum of $\Pi(z_t, Q)$ for the constant output level Q, and at the real price k which yields a real profit of zero in both cases. Otherwise profits are always lower with quantity-adjustment costs. The profit becomes zero for a firm with quantity-adjustment costs when $z_t = (kQ)^{\frac{1}{1-\alpha}}$,

[2] A sufficient condition is that the quantity-adjustment cost is at least as high as the price-adjustment cost.

[3] "Excess" output may be interpreted to mean that the firm must continue to pay for its quasi-fixed factors although it only produces up to the level of demand.

whereas the profit without quantity-adjustment costs and a variable output level is positive as long as $z_t > k$.

The general price level increases at the constant rate $\mu > 0$. Due to the price-adjustment cost, the firm keeps its nominal price unchanged for a fixed period of time denoted by T, and then increases it to a new level. The real price at the beginning of a period with a constant nominal price is denoted by S. After $\tau \geq 0$ of the period has elapsed, the real price has been reduced to $z_\tau = Se^{-\mu\tau}$, and as τ tends to T, the real price converges to $s \equiv Se^{-\mu T}$. The length of time from the beginning of the period until the demand equals the firm's output is denoted by T_z, that is, $Z = Se^{-\mu T_z} \Leftrightarrow T_Z = \frac{1}{\mu}\ln\frac{S}{Z}$.[4]

The firm's average real profit over a period with a constant nominal price is given by

$$V \equiv \frac{1}{T}\left[\int_0^T \Pi\left(Se^{-\mu\tau}, Q\right) d\tau - c\right],$$

where $c > 0$ is the price-adjustment cost. Substituting for $\Pi(\cdot)$, we have

$$V = \frac{1}{T}\left[\int_0^{T_Z} \left(Se^{-\mu\tau}\right)^{1-\alpha} d\tau + \int_{T_Z}^{T} Se^{-\mu\tau} Q d\tau - c\right] - kQ,$$

where the first integral is the revenue in the first part of the period in which the firm produces more output than it can sell, and the second integral is the revenue in the second part of the period where the firm rations its customers. Solving the integrals and substituting for T, T_Z, and Q yield

$$V = \frac{1}{\ln\frac{S}{s}}\left(\frac{\alpha Z^{1-\alpha} - S^{1-\alpha}}{\alpha - 1} - sZ^{-\alpha} - \mu c\right) - kZ^{-\alpha}. \quad (1)$$

The firm chooses S, s, and Z in order to maximize V. The first-order conditions are

$$\frac{\partial V}{\partial S} = \frac{1}{S\ln\frac{S}{s}}\left(-V + S^{1-\alpha} - kZ^{-\alpha}\right) = 0, \quad (2)$$

$$\frac{\partial V}{\partial s} = \frac{1}{s\ln\frac{S}{s}}\left[V - (s-k)Z^{-\alpha}\right] = 0, \quad (3)$$

$$\frac{\partial V}{\partial Z} = \alpha Z^{-\alpha-1}\left(k - \frac{Z-s}{\ln\frac{S}{s}}\right) = 0. \quad (4)$$

[4]Here we presume that $Z \in (s, S)$, or equivalently, that $T_Z \in (0, T)$. Theorem 1 shows that our solution satisfies this condition.

The first two conditions are standard and state that the profit in the beginning and in the end of a period with a constant nominal price equals the average profit over the period. The third condition is new and due to the produced quantity being held constant in the period. This condition states that the increase in costs from an increase in output (which occurs throughout the period) has to equal the increase in revenue (which occurs only when the customers are rationed).

3 The Impact of Inflation

If there were no costs of price-adjustment, the nominal price would be adjusted continuously at the rate of inflation. Whether or not there are costs of adjusting output, the real price would always be at its profit-maximizing level, $\hat{Z}$, and the output correspondingly at $\hat{Q}$. We now prove the following theorem, which characterizes the firm's optimal strategy:

Theorem 1 (***i***) *$s < \hat{Z} < Z < S$ and $Q < \hat{Q}$.*
(***ii***) *$\frac{dS}{d\mu} > 0$, $\frac{ds}{d\mu} < 0$, $\frac{dZ}{d\mu} > 0$, and $\frac{dQ}{d\mu} < 0$.*

Proof. See the appendix.

It is quite intuitive that the real price exceeds its profit-maximizing level in the beginning of a period with a constant nominal price, i.e., that $\hat{Z} < S$, and that the real price is less than its profit-maximizing level in the end of a period, i.e., that $s < \hat{Z}$. Furthermore, the higher the inflation rate, the higher is the initial real price and the lower is the terminal real price. These results are identical to what is found in models with only price-adjustment costs (see Sheshinski and Weiss, 1977).

With quantity-adjustment costs, we also find that the fixed output level, Q, is below the profit-maximizing output level if there were no price-adjustment costs, $\hat{Q}$, and that the fixed output level decreases with the rate of inflation. Thus, the presence of quantity-adjustment costs does not alter the conclusion that with a constant-elasticity demand function and a constant per-unit cost of production, inflation reduces output (see Kuran, 1986; Naish, 1986).[5] When output is fixed, the lowering of the terminal real price reduces profits to a large extent in the end of a period since the firm is unable to satisfy the extra demand. In isolation this

[5] It is straightforward to show that in the case of deflation, $\mu < 0$, the output is also less than $\hat{Q}$ and decreases with deflation (i.e., $dQ/d\mu > 0$ for $\mu < 0$).

effect tends to make the firm increase output. However, the increase in the initial real price has a large detrimental effect on the fixed output level because the firm cannot accommodate the reduction in demand in the start of a period by reducing production below the fixed level. Therefore, it is optimal to reduce the fixed output level, and since this effect dominates, output falls with inflation.

Part of the produced output does not reach the consumers when the real price exceeds Z, and it is therefore also of considerable interest to consider the average quantity sold to the consumers. This is defined as

$$Y \equiv \frac{1}{T}\left[\int_0^{T_Z} \left(Se^{-\mu\tau}\right)^{-\alpha} d\tau + Q\left(T - T_Z\right)\right],$$

where the first term in the bracket is the output sold in the first part of a period where the firm sells less than it produces, whereas the second term is the output sold in the second part of a period where the firm sells its entire production.

Integrating the above equation and substituting $T_Z = \frac{1}{\mu}\ln\frac{S}{Z}$ yield

$$Y = Q\frac{1-\left(\frac{S}{Z}\right)^{-\alpha}}{\alpha\mu T} + Q\left(1 - \frac{\ln\frac{S}{Z}}{\mu T}\right).$$

Substituting $S/Z = (S/s)^{1/\alpha}$, which is derived from conditions (2) and (3), and $\mu T = \ln(S/s)$ then yield

$$Y \equiv \frac{Q}{\alpha}\left(\alpha - 1 + \frac{1-\frac{s}{S}}{\ln\frac{S}{s}}\right).$$

We can now establish

Theorem 2 $\frac{dY/Y}{d\mu/\mu} < \frac{dQ/Q}{d\mu/\mu} < 0.$

Proof. We only need to show that the last term inside the parenthesis in the expression for Y is decreasing in μ. Since, we know from Theorem 1 that S/s is increasing in μ, this is equivalent to showing that this last term is decreasing in S/s. However, this is true since its derivative with respect to S/s has the same sign as $1 - \frac{S}{s} + \ln\frac{S}{s}$, and the latter decreases in $\frac{S}{s}$ and approaches 0 for $\frac{S}{s} \to 1$. □

Thus, not only does the output sold decrease with the rate of inflation, as does the output produced, the negative effect of the rate of inflation on the output sold is proportionally larger than on the output itself.

4 The Loss of Output with and without Quantity-Adjustment Costs

In this section we study the quantitative importance of quantity-adjustment costs in the presence of inflation. We compare the loss of output relative to the frictionless output level both with and without quantity-adjustment costs. The relative loss of output with both price- and quantity-adjustment costs is $1 - Q/\hat{Q}$, and similarly, the relative loss of output sold to consumers is $1 - Y/\hat{Q}$.

If there are no quantity-adjustment costs, the firm's sales always equal its output. Therefore, the firm's average real profit over a period with a constant nominal price is

$$\tilde{V} \equiv \frac{1}{\tilde{T}} \left\{ \int_0^{\tilde{T}} \left[\left(\tilde{S} e^{-\mu\tau} \right)^{1-\alpha} - k \left(\tilde{S} e^{-\mu\tau} \right)^{-\alpha} \right] d\tau - c \right\},$$

where $\tilde{T}$ is the length of the period in which the nominal price is kept unchanged, and $\tilde{S}$ is the real price at the beginning of the period. Let $\tilde{s}$ be defined as the real price when τ approaches $\tilde{T}$, that is, $\tilde{s} \equiv \tilde{S} e^{-\mu \tilde{T}}$. Integrating and substituting for $\tilde{T}$ in the equation above yield

$$\tilde{V} \equiv \frac{1}{\ln \frac{\tilde{S}}{\tilde{s}}} \left[\frac{\tilde{s}^{1-\alpha} - \tilde{S}^{1-\alpha}}{\alpha - 1} - \frac{k \left(\tilde{s}^{-\alpha} - \tilde{S}^{-\alpha} \right)}{\alpha} - \mu c \right].$$

The firm chooses $\tilde{S}$ and $\tilde{s}$ to maximize $\tilde{V}$. The first-order conditions are

$$\frac{\partial \tilde{V}}{\partial \tilde{S}} = \frac{1}{\tilde{S} \ln \frac{\tilde{S}}{\tilde{s}}} \left[-\tilde{V} + \tilde{S}^{-\alpha} \left(\tilde{S} - k \right) \right] = 0, \tag{5}$$

$$\frac{\partial \tilde{V}}{\partial \tilde{s}} = \frac{1}{\tilde{s} \ln \frac{\tilde{S}}{\tilde{s}}} \left[\tilde{V} - \tilde{s}^{-\alpha} \left(\tilde{s} - k \right) \right] = 0. \tag{6}$$

The average quantity the firm produces and sells is

$$\tilde{Q} \equiv \frac{1}{\tilde{T}} \left[\int_0^{\tilde{T}} \left(\tilde{S} e^{-\mu\tau} \right)^{-\alpha} d\tau \right] = \frac{\tilde{s}^{-\alpha} - \tilde{S}^{-\alpha}}{\alpha \ln \frac{\tilde{S}}{\tilde{s}}},$$

where the last equality follows from integrating and substituting for $\tilde{T}$. The relative loss in average output (and sales) in this case equals $1 - \tilde{Q}/\hat{Q}$.

We now simulate the two models using conditions (2)-(4) for the model with both price- and quantity-adjustment costs, and conditions (5) and (6) for the model without quantity-adjustment costs, and thereafter using the solutions to obtain the relative output losses. Inspection of the expressions for the relative output losses shows that these are uniquely determined from knowledge of the demand elasticity, α, and of $\mu\psi$, where $\psi \equiv c/(\hat{Z}\hat{Q})$ is the menu cost as a proportion of the firm's frictionless revenue.

In Figure 2 we illustrate the results for $\alpha = 5$ and $\psi = 0.7\%$, which is the menu-cost estimate given by Levy et al. (1997). The lower curve shows the relative loss of average output (= sales) without quantity-adjustment costs (QAC); the upper solid curve shows the relative loss of output with quantity-adjustment costs; and the upper dashed curve shows the relative average loss of sales with quantity-adjustment costs. The figure confirms the conclusions of Kuran (1986) and Naish (1986): the loss of output (and sales) without quantity-adjustment costs is non-neglible, although not large for moderate inflation rates, and increases with inflation. Thus, the loss is 0.9% for an inflation rate of 5%, and 2.5% for an inflation rate of 25%. It is also clear from the figure that the loss of output and sales is several times higher in the presence of quantity-adjustment costs: the loss of output is 5.9% (and of sales is 6.2%) for an inflation rate of 5%, and the loss of output is 13.2% (and of sales is 13.9%) for an inflation rate of 25%. Thus, the quantity-adjustment costs significantly enlarge the negative consequences of price-adjustment costs.

< Figure 2 >

Figure 2 can also be used for other sizes of the menu cost since the different losses depend on only $\mu\psi$ for a given α. Simultaneously changing the menu cost by a factor of $\gamma > 0$ and the inflation rate by a factor of $1/\gamma$ leave the loss of output and sales unchanged in all cases. In terms of Figure 2, halving the menu cost is equivalent to rescaling the horizontal axis by doubling all inflation rates. Thus, for a menu cost equal to 0.35% and an inflation rate equal to 10%, the loss of output is 5.9% with quantity-adjustment costs, but only 0.9% without quantity-adjustment costs.

To examine whether our results are sensitive to the choice of α, we have calculated the losses of output and sales also for $\alpha = 2$ and $\alpha = 8$. As shown in Table I, although the losses of output and sales increase with α in all cases, it is true for the other values of α as well that the loss with quantity-adjustment costs is several times higher than the loss without.

Table I. Loss of Output and Sales with and without QAC for $\psi = 0.7$ and Different Values of α.

α	μ	$1 - Q/\hat{Q}$	$1 - Y/\hat{Q}$	$1 - \tilde{Q}/\hat{Q}$
2	5%	1.9%	2.8%	0.22%
	25%	4.2%	6.2%	0.64%
5	5%	5.9%	6.2%	0.9%
	25%	13.2%	13.9%	2.5%
8	5%	9.9%	10.1%	1.7%
	25%	22.2%	22.5%	4.95%

5 Concluding Remarks

This paper analyzes the impact of inflation on the price and production decisions of a firm facing both price- and quantity-adjustment costs. The results show that price-adjustment costs reduce output also if there are quantity-adjustment costs, whereas quantity-adjustment costs alone have no impact on the firm's optimal choice of output. Our simulations reveal that quantity-adjustment costs may significantly amplify the negative impact of price-adjustment costs on output.

Because of tractability, our results are derived under rather specific assumptions. One of these is the absence of discounting, the consequences of which are analyzed in Danziger (2000). There it is shown, for a general profit function, that output is lower than the frictionless level and decreases with the inflation rate when the inflation rates are low. Another assumption is that the elasticity of demand is constant, and other demand functions may lead to a different result. For instance, simulations with a linear demand show that it is possible for output to increase with inflation. However, this does not change the overall conclusion that menu costs matter even when the output is completely fixed due to the quantity-adjustment costs. Finally, it is assumed that there are only nominal changes. If there were real changes in a firm's demand or cost, the firm might find it optimal to change its output even if the cost of doing so is considerable.

Appendix

We start be proving that $s < Z < S$. Conditions (2) and (3) can be written as $V = \Pi(S, Q) = \Pi(s, Q)$. Since a solution must satisfy the second-order conditions $\partial \Pi(S, Q)/\partial S < 0$ and $\partial \Pi(s, Q)/\partial s > 0$, it follows that $s < S$. Condition (4) then show that $s < Z$.

From condition (2), we get

$$-V + \left[\left(\frac{S}{Z}\right)^{-\alpha} S - k\right] Z^{-\alpha} = 0,$$

and comparing this with condition (3) shows that $Z < S$.

To establish that $s < \hat{Z}$, we use condition (2) to get

$$V = (s - k) Z^{-\alpha}.$$

Inserting equation (1) yields

$$\frac{1}{\ln \frac{S}{s}} \left(\frac{\alpha Z^{1-\alpha} - S^{1-\alpha}}{\alpha - 1} - sZ^{-\alpha} - \mu c \right) = sZ^{-\alpha}.$$

Using condition (4) to substitute for $\ln \frac{S}{s}$ gives

$$\frac{k}{Z - s} \left(\frac{\alpha Z^{1-\alpha} - S^{1-\alpha}}{\alpha - 1} - sZ^{-\alpha} - \mu c \right) = sZ^{-\alpha}.$$

Conditions (2) and (3) imply that $S^{1-\alpha} = sZ^{-\alpha}$, from which it follows that

$$-k\mu c = Z^{-\alpha} (Z - s) \left(s - \frac{\alpha k}{\alpha - 1} \right) \quad \Rightarrow \quad s < \frac{\alpha k}{\alpha - 1} = \hat{Z}.$$

Now, we derive $\frac{dS}{d\mu}$, $\frac{ds}{d\mu}$, and $\frac{dZ}{d\mu}$. Total differentiation of conditions (2)-(4) and use of Cramer's rule yield

$$\begin{aligned}
\frac{dS}{d\mu} &= -\frac{1}{D} \frac{cZ^{-\alpha}}{Z \ln \frac{S}{s}} [Z - \alpha (s - k)], \\
\frac{ds}{d\mu} &= -\frac{1}{D} \frac{cS^{-\alpha}}{Z \ln \frac{S}{s}} [Z - \alpha (Z - k)], \\
\frac{dZ}{d\mu} &= -\frac{1}{D} \frac{cS^{-\alpha}}{s \ln \frac{S}{s}} [s - \alpha (s - k)],
\end{aligned}$$

where D is the Hessian determinant, which is negative due to the second-order conditions. It follows from $s < \hat{Z}$ that $\frac{dZ}{d\mu} > 0$. For $\mu \to 0$ it follows from the first-order conditions that $Z \to \hat{Z}$, implying that $Z > \hat{Z}$. It then follows that $\frac{dS}{d\mu} > 0$ and $\frac{ds}{d\mu} < 0$. Finally, $Z > \hat{Z}$ and $dZ/d\mu > 0$ imply that $Q < \hat{Q}$ and $dQ/d\mu < 0$. □

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Figure 1. Real Profit with and without Quantity-Adjustment Costs

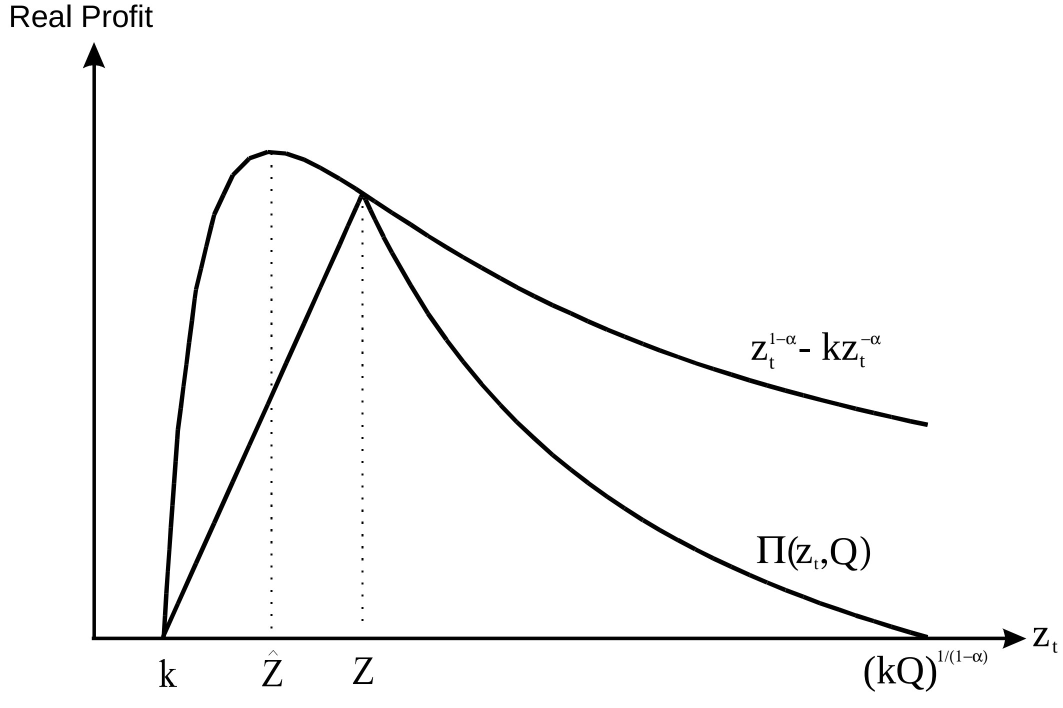

Figure 2. Loss of Output and Sales with and without Quantity-Adjustment Costs (QAC) for α=5 and ψ=0.7%