CESifo *Working Paper Series*

INFLATION TARGETING VERSUS NOMINAL INCOME TARGETING

Helmut Frisch
Sylvia Staudinger

Working Paper No. 301

June 2000

CESifo
Poschingerstr. 5
81679 Munich
Germany
Phone: +49 (89) 9224-1410/1425
Fax: +49 (89) 9224-1409
http://www.CESifo.de

CESifo *Working Paper No. 301*
June 2000

INFLATION TARGETING VERSUS NOMINAL INCOME TARGETING

Abstract

In this paper we analyze flexible inflation targeting and nominal income targeting as two different monetary strategies in a simple dynamic macromodel. Furthermore we analyze inflation targeting in a two-period time-lag version of the model. The key results of our paper are: First, for both targeting regimes optimal monetary policy response leads to a shock-dependent feedback rule. Second, a demand shock is completely offset by both monetary strategies. Third, in case of a supply shock there is a significant difference between the two different targeting regimes. Under inflation targeting the policy makers face a trade-off between inflation and output stabilization. This trade-off depends on the weight Φ the policy makers attached to output stabilization relative to inflation stabilization in the loss function. In contrast, under nominal income targeting policy makers face a constant trade-off between inflation and real output growth: An increase in inflation leads to a fall in real output growth by an equal amount. Finally we analyze inflation targeting in a two-period time-lag version of the model. The qualitative results about the trade-off between inflation and output growth remain the same as in the basic model without time lag.

Keywords: Inflation targeting, nominal income targeting, optimal monetary policy

JEL Classification: E52, E58

Helmut Frisch
Technische Universität Wien
Argentinierstr. 8
1040 Wien
Austria

Sylvia Staudinger
Technische Universität Wien
Argentinierstr. 8
1040 Wien
Austria
email: sstaudin@pop.tuwien.ac.at

1. Introduction

Flexible inflation targeting and nominal income targeting have two desirable features as a monetary policy strategy. Both policies take into account movements in inflation and output, the two strategic variables of the central bank, and the smoothing out fluctuations in nominal income.

Today the focus in the literature is on flexible inflation targeting (Svensson 1997a, 1997b). This monetary strategy has also been adopted by the central banks of New Zealand, the United Kingdom, Sweden and some other countries in the last decade (Leiderman and Svensson 1995). Nominal income targeting typically viewed as an alternative to flexible inflation targeting has no practical application but has a long tradition in the realm of academic discussion (see for instance Taylor (1985), Frankel and Funke (1993), Hall and Mankiw (1994), McCallum and Nelson (1999)).

Two new arguments, however, provide support for further research of nominal income targeting. First the ECB announced a reference value for M3 growth of 4.5 percent calculated as a sum of an inflation target and a forecasted trend growth rate of real output of 2.5 percent. This nominal income target is corrected with a small adjustment for the decline in the velocity of money. The growth rate of nominal income is the sum of the rate of growth of money supply and the change in velocity. It is also the sum of the rate of inflation and the rate of growth of real GDP. Both nominal income growth targeting and the M3 reference value of ECB should generate similar results provided that there are no large shocks in the velocity. Second, in the light of the apparent overprediction inflation and underprediction of real output growth in the US economy, McCallum (1997), Orphanides (1999) and Rudebusch (1999) suggest that monetary policy should focus on nominal income growth and not rely on uncertain estimates of the output gap.

The aim of this paper is to work out the differences between the solution if monetary policy adopts a flexible inflation target with the alternative solution of nominal income targeting. We focus on the optimal policy reaction function of the central bank to supply and demand shocks which differ according to the different monetary policies. We explore the performance of both rules in the framework of a New Keynesian Model, a dynamic macroeconomic model that nests a forward looking Phillips curve relation and a forward looking IS-curve into a sticky price model. This model was recently summarized by Clarida, Gali and Gertler (1999) and relys on contributions by Taylor (1980), Calvo (1983), Rotemberg (1987, 1996), Roberts (1995). Section 2 describes this model. This model is in contrast to Svensson´s model, that uses a backward looking specification of both the Phillips curve and the IS-curve.

Under flexible inflation targeting, the central bank aims at two objectives, hitting the inflation and output gap targets. A specific feature of this monetary strategy is that it takes the weight the central bank attaches to output stabilization relative to inflation as given. This preference parameter determines how aggressive or accommodative is the response of the central bank to deviations of inflation from its target value. Under nominal income targeting, by contrast, the central bank only follows the objective of keeping the rate of growth of nominal income on target.

In section 3 we investigate a flexible inflation target regime. We examine the finding of Clarida et al. (1999) that in the case of a supply shock policy makers face a trade-off between inflation and output and that the preference parameter determines the amount of accommodation to the shock. In section 4 we analyze nominal income targeting and we find that in this case a constant trade-off of minus one between inflation and output exists. As main result we obtain that under nominal income targeting, when inflation goes up real output growth goes down by an equal amount.

We further demonstrate in section 4 that nominal income targeting implies a dynamically stable solution in the simple forward-looking, rational expectations model of section 2. This is in contrast to the Ball (1997) and Svensson (1997a) instability result that nominal income targeting generates a dynamical unstable path of output and inflation. Our findings support McCallum´s conjecture (1997) that the Ball-Svensson instability result is likely to depend on the specification of the Phillipscurve relationship. A forward-looking Phillips curve leads to a stable solution, a backward-looking Phillips curve as in Ball or Svensson produces the unstable result. The crucial issue whether the Ball and Svensson or the McCallum results are empirically more relevant is raised by Rudebusch (1999) and is not investigationed here.

In section 5 we introduce a two period control-lag similar to Svensson (1997a) into the basic model of section 2 and analyze the performance of inflation targeting in this modified frame work. We find that the results are qualitatively the same as in section 3.

2. The Model

We use a simple dynamic equilibrium model which nests a forward looking Phillips curve relationship (2.1) and a forward looking IS-curve (2.2) with nominal rigidities into a sticky price macroeconomic model. The model relies on contributions by Taylor (1980), Calvo (1983), Rotemberg (1987), Roberts (1995) and McCallum (1997) and is known as New Keynesian model in the literature. Clarida, Gali and Gertler (1999) emphasize that the consensus view of this model provides a useful framework for analyzing monetary policy.

$$\pi_t = E_t\pi_{t+1} + ax_t + \varepsilon_t \tag{2.1}$$

$$x_t = E_t x_{t+1} - b\left(i_t - E_t\pi_{t+1}\right) + \eta_t \tag{2.2}$$

The Phillips curve (2.1) relates inflation positively to the output gap x_t ($x_t = y_t - y^n$) of the current period and the IS-curve (2.2) relates output negatively to the real interest rate. Looking at (2.1) $E_t\pi_{t+1}$ is the expected inflation rate of the next period based on the information available in period t and the term ε_t denotes a supply shock.

In (2.2) $E_t x_{t+1}$denotes the expected output gap of the next period based on the information available in period t and η_t denotes a demand shock, not correlated with the supply shock.

$$\varepsilon_t = \theta\varepsilon_{t-1} + \xi_t \tag{2.3}$$

$$\eta_t = \rho\eta_{t-1} + \upsilon_t \tag{2.4}$$

Both disturbances follow a first order autoregressive process with $0 \leq \theta, \rho \leq 1$. ξ_t, υ_t are i.i.d. random variables with zero mean and variance σ_ξ^2 and σ_υ^2. For the special case of $\theta = \rho = 0$ the shocks are purely transitory, whereas $\theta = \rho = 1$ means that both shocks follow a random walk process.

The central bank objective function is

$$E_t \sum_{j=0}^{\infty} \frac{1}{2}\delta^j L_{t+j} \tag{2.5}$$

where L_{t+j} is a standard quadratic loss function

$$L_{t+j} = (\pi_{t+j} - \pi^*)^2 + \Phi x_{t+j}^2. \tag{2.6}$$

The loss function of the central bank increases with deviations from natural output and increases also if inflation deviates from an exogenously given inflation target π^*. Since the target for the output gap is zero there is no incentive to generate an inflation bias. The parameter Φ measures the weight policymakers attached to output stabilization relative to inflation stabilization such that $0 < \Phi < \infty$. The case $\Phi = 0$ coincides with a regime that Svensson (1997a) describes as strict inflation targeting, whereas $\Phi > 0$ describes flexible inflation targeting. The instrument of the monetary authority is the nominal interest rate i_t and the central bank controls the nominal rate to affect output and inflation.

Using the IS-curve (2.2) in (2.1) gives

$$\pi_t = E_t\pi_{t+1} + a\left(E_t x_{t+1} + \eta_t - b\left(i_t - E_t\pi_{t+1}\right)\right) + \varepsilon_t. \tag{2.7}$$

Equations (2.2) and (2.7) show the relationship between output and inflation and the control variable i_t. An increase in the nominal interest rate unambiguously reduces both output and inflation, whereas higher expected output and/or expected inflation lead to an increase in both current output and inflation.

3. Optimal Monetary Policy

In this section we discuss the optimal discretionary policy of the central bank. As Clarida, Gali and Gertler (1999) argue discretion is the scenario which fits best reality, since no central bank will make any binding commitments over the future course of its policy. Bernanke and Mishkin (1997) and Bernanke et al. (1999, Ch. 1 and Ch. 2) suggest that inflation targeting should be only a framework for monetary policy which allows for the exercise of "constrained discretion", but should not be interpreted as rule in the classical Friedman sense. Without commitment the central bank takes expectations as given in the optimization problem and chooses the nominal interest rate which minimizes the loss function. The private sector forms its expectations rationally conditional on the central banks optimal policy rule.

Since i_t affects only x_t and π_t the optimization problem reduces to the following simple optimization problem for period t.

$$\min_{i_t} \frac{1}{2}\left(\Phi x_t^2 + (\pi_t - \pi^*)^2\right) \tag{3.1}$$

subject to (2.2) and (2.7).

Differentiating (3.1) with respect to i_t we obtain the following optimality condition:

$$x_t = -\frac{a}{\Phi}(\pi_t - \pi^*) \tag{3.2}$$

The first order condition can be interpreted as follows: If inflation is above its target contract demand below the natural output by increasing the interest rate. If inflation is below its target expand demand above capacity by decreasing the interest rate. The coefficient of proportionality depends positively on a, the coefficient of the output gap in the Phillips curve and inversely on the weight Φ attached to output stabilization in the objective function. The smaller is Φ, the stronger is the demand contraction initiated by the central bank if inflation deviates from the target and vice versa. If the central bank gives no weight to output stabilization($\Phi = 0$) then inflation should always attain its target level.

From the optimality condition we obtain the following reaction function for the nominal interest rate i_t

$$bi_t = \left(a\left(\Phi + a^2\right)^{-1} + b\right) E_t\pi_{t+1} + a\left(\Phi + a^2\right)^{-1}\varepsilon_t - a\left(\Phi + a^2\right)^{-1}\pi^* + \eta_t + E_t x_{t+1} \tag{3.3}$$

The optimal reaction for the interest rate requires an increase in the nominal rate if a negative supply (note that a positive realization of ε_t indicates an upward shift of the Phillipscurve) or a positive demand shock occurs. It is clear that the coefficient on expected inflation exceeds unity, implying that the real interest rate must rise in response to higher expected inflation. An increase in the conditional output forecast also requires a rise in the interest rate. Finally, the optimal interest rate depends negatively on the inflation target. This reaction function for the nominal rate closely resembles the Taylor rule. Whereas Taylor (1993) characterizes the nominal interest rate as function of lagged inflation and output, in our forward looking model the interest rate depends on expected future rather then on lagged values of inflation and output.

Using the optimal interest rate in (2.7) yields the quasi-reduced form for the inflation rate:

$$\pi_t = \left(\Phi + a^2\right)^{-1}\Phi E_t\pi_{t+1} + \left(\Phi + a^2\right)^{-1}\Phi\varepsilon_t + aa\left(\Phi + a^2\right)^{-1}\pi^*. \tag{3.4}$$

To determine π_t we employ the technique of undetermined coefficients where the bubble-free solution is obtained via a minimal-state-variable procedure described by McCallum (1983). Since the relevant state variables in (3.4) are ε_t and π^* ,it is apparent that π_t will be of the form

$$\pi_t = b_0\varepsilon_t + b_1\pi^*. \tag{3.5}$$

For the expected inflation rate $E_t\pi_{t+1}$ we get

$$E_t\pi_{t+1} = b_0\theta\varepsilon_t + b_1\pi^*$$

Substituting the expression for $E_t\pi_{t+1}$ into (3.4) and comparing with (3.5) yields the following expressions for the undetermined coefficients b_i :

$b_0 = \left(\Phi + a^2\right)^{-1}\Phi\left(b_0\theta + 1\right)$

$b_1 = \left(\Phi + a^2\right)^{-1}\left(\Phi b_1 + a^2\right).$

Solving b_i, in terms of the parameters of the model we obtain:

$b_0 = \left(\Phi\left(1-\theta\right) + a^2\right)^{-1}\Phi$

$b_1 = 1.$

The complete solution for our model is thus

$$\pi_t = \left(\Phi\left(1-\theta\right) + a^2\right)^{-1}\Phi\varepsilon_t + \pi^* \tag{3.6}$$

$$x_t = -\left(\Phi\left(1-\theta\right) + a^2\right)^{-1}a\varepsilon_t \tag{3.7}$$

$$i_t = b^{-1}\left(\Phi\left(1-\theta\right) + a^2\right)^{-1}\left(a + b\Phi\theta\right)\varepsilon_t + \pi^* + b^{-1}\eta_t \tag{3.8}$$

$$E_t\pi_{t+1} = \left(\Phi\left(1-\theta\right) + a^2\right)^{-1}\Phi\theta\varepsilon_t + \pi^* \tag{3.9}$$

$$E_tx_{t+1} = -\left(\Phi\left(1-\theta\right) + a^2\right)^{-1}a\theta\varepsilon_t \tag{3.10}$$

Equations (3.6) and (3.7) exhibit that in the case of a supply shock a trade-off between inflation and the output gap exists. It is easily seen that the trade-off depends on Φ, the weight attached to output stabilization in the objective function. A large value of Φ implies a strong preference for output stabilization at the expense of high inflation. A small value of Φ, by contrast, implies a low value of inflation at the expense of greater output losses.

The two polar cases occur when $\Phi = 0$ and $\Phi = \infty$. In the first case $\Phi = 0$, i.e. "strict inflation targeting" (Svensson 1997a, 1997b) the supply shock does not affect inflation ($\pi_t = \pi^*$) and the demand contraction reaches with $\partial x_t/\partial\varepsilon_t = -a^{-1}$ its maximum value. The second case, $\Phi = \infty$, implies a monetary policy of "strict output targeting". Output is unchanged but the inflation effect of the shock is maximized. In this case $\partial\pi_t/\partial\varepsilon_t = \left(1-\theta\right)^{-1}$ and $\partial x_t/\partial\varepsilon_t = 0$.

From the interest rate rule by equation (3.8) it is apparent that optimal monetary policy requires a rise of the nominal interest rate in response to a negative supply shock. The higher the degree of persistence of a supply shock (the larger θ) the stronger the reaction of the interest rate. The optimal adjustment of the nominal rate depends on the parameter Φ and calls for an increase in the nominal interest rate, $\partial i_t/\partial\varepsilon_t = b^{-1}\left(\Phi\left(1-\theta\right) + a^2\right)^{-1}\left(a + b\Phi\theta\right) > 0$. The same condition holds for the real interest rate, from (3.8) and (3.9) follows that $r_t = i_t - E_t\pi_{t+1} = b^{-1}\left(\Phi\left(1-\theta\right)+a^2\right)^{-1} a\varepsilon_t + b^{-1}\eta_t$ and $\partial r_t/\partial\varepsilon_t > 0$.

The optimal reaction of the nominal rate implies an increase in the real rate, except in the case of $\Phi = \infty$. For $\Phi < \infty$ the nominal interest rate adjusts more then expected inflation, so the real short term rate moves in the same direction as the nominal rate. According to Clarida et al.(1998a, 1998b) this policy has been adopted by the Federal Reserve since the Volcker-Greenspan era to control inflation. Since the early 1980s the Fed systematically raised real and nominal short-term rates in response to higher expected inflation. The Bundesbank adopted a similar policy during the same period.

In rational expectations models inflationary expectations are endogenous and react to changes in ε_t. Equation (3.9) shows that expected future inflation rises with a high weight Φ. The private sector expects a higher future inflation if the central bank places a high weight to output stabilization. In the case of strict inflation targeting, $\Phi = 0$ the private agents expect a sufficient contraction in demand (below capacity) by an adequate rise in the interest rate, so that inflationary expectations will remain on target. Furthermore, expected inflation will increase more if the degree of persistence is higher (a larger θ). If the shock is transitory ($\theta = 0$) inflationary expectations are always on target.

In the case of a demand shock, on the other hand, a trade-off between inflation and output does not exist. From (3.6) and (3.7) follows that a demand shock is perfectly eliminated by the optimal monetary policy. Equation (3.8) shows that the interest rate has to be raised by $b^{-1}\eta_t$ to perfectly offset the demand shock. In terms of Figure 3.1, the optimal monetary policy response implies that the IS-curve does not shift from its original position, since the net effect of the two opposing forces (the demand shock and the interest rate reaction) is zero.

The same analysis can be conducted by means of a simple diagram. Figure 3.1. is a graphical representation of three equations. The locus FF represents the Phillips curve (2.1), RR the line of the optimal monetary response, according to (3.2) and equation (2.2) is the IS-curve. Assume that a negative supply shock occurs, i.e. $\varepsilon_t > 0$. The impact effect and the revision of the expected inflation $E_t\pi_{t+1}$ (3.9) shifts the FF_0-curve to FF_1. The resulting inflation π_t is above π^*. The optimal policy response requires to contract demand below the natural rate ($x_t < 0$) by raising the interest rate. If the latter exceeds the rise in expected inflation, the real interest rate increases. The IS-curve shifts to position A (in Figure 3.1) causing the required fall in demand.

The trade-off between inflation and output depends mainly on the parameter Φ. If Φ is large, the optimal policy line RR is "steep" and the solution implies a strong output stabilization at the expense of a high inflation. This represents an accommodative policy. If Φ decreases the RR-curve rotates to the left and becomes "flat" implying a stronger anti-inflationary policy at the expense of higher output losses. This would represent a non-accommodative policy. Not only the RR-curve depends on Φ but so does the position of the FF-curve. Small values of Φ make the upward shift of the FF-curve less pronounced, larger values of Φ cause a pronounced upward shift of the FF-curve.

In Figure 3.2 we represent the "extreme" case $\Phi = 0$; a policy of "strict" inflation targeting. In this case RR becomes parallel to the abscissa.. The FF-curve shifts

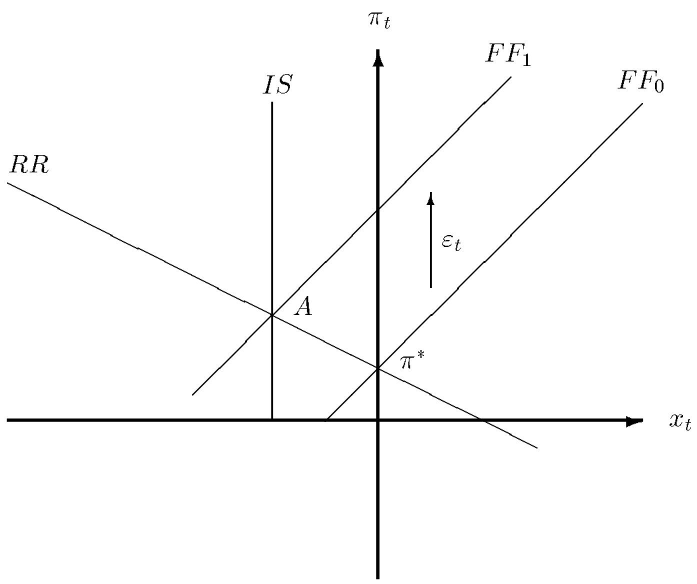

Figure 3.1:

upward due to the impact effect; there is no additional effect through a change in the expected inflation: $E_t\pi_{t+1} = 0$. By a substantial rise of the nominal interest rate the central bank engineers an output contraction which perfectly eliminates the deviation of the actual inflation from its target. The output contraction is maximized $\left(\partial x_t/\partial\varepsilon_t = -a^{-1}\right)$. The shift of the IS-curve is caused by an increase in the real interest rate.

The second "extreme" case is given by $\Phi = \infty$. The central bank pursues a "strict output targeting" and is not concerned about inflationary developments. The change in the relevant macroeconomic variables due to a supply shock is now:

$$\begin{aligned} &\partial\pi_t/\partial\varepsilon_t = (1-\theta)^{-1}, && \partial E_t\pi_{t+1}/\partial\varepsilon_t = (1-\theta)^{-1}\theta, \\ &\partial x_t/\partial\varepsilon_t = 0, && \partial i_t/\partial\varepsilon_t = (1-\theta)^{-1}\theta. \end{aligned}$$

In this case the optimal policy line RR coincides with the ordinate π_t of Figure 3.1. The FF-line shifts upward to its upper bound, since the expected inflation is maximized. The policy response is an increase of the interest rate to the same extent as the rise in the expected rate of inflation. The real interest rate remains constant and the IS-curve does not change relative to its initial position.

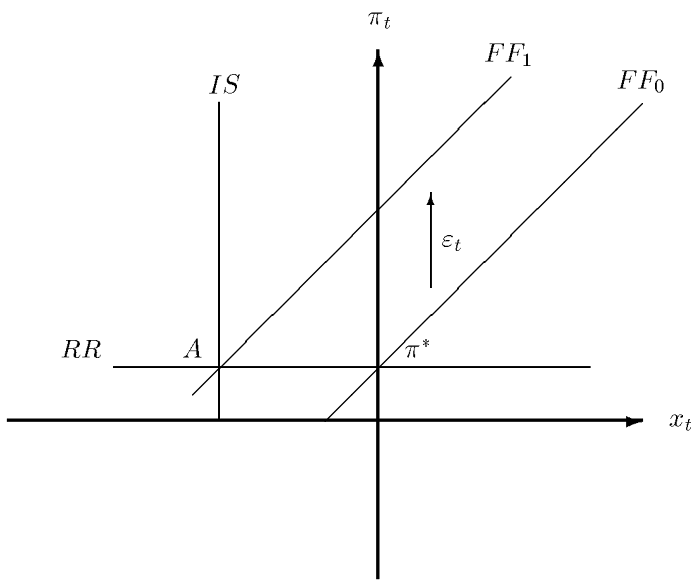

Figure 3.2:

4. Nominal income targeting

In this section we study optimal policy if the central bank adopts a nominal income target. This monetary rule has two favorable features for monetary policy: it aims at smoothing out fluctuations in nominal income and provides a long-run nominal anchor for monetary policy.

Since monetary policy cannot influence the real economy in the long-run it should focus on stabilizing nominal income growth. We conduct out analysis in the context of the model which was introduced in the previous section. The Phillips curve (2.7) and the IS-curve (2.2) are unchanged. Instead of (2.6) we employ the loss function

$$L_{t+j} = \left(x_t - x_{t-1} + \pi_t - (\Delta x + \pi)^*\right)^2. \tag{4.1}$$

The loss of the central bank is an increasing function of the deviations of the nominal income growth from exogenously given target $(\Delta x + \pi)^*$.

Since i_t affects only x_t and π_t the optimization problem is simply to minimize the loss for period t.

$$\min_{i_t} \frac{1}{2} \left(x_t - x_{t-1} + \pi_t - (\Delta x + \pi)^*\right)^2 . \tag{4.2}$$

subject to (2.2) and (2.7).

Differentiating (4.2) with respect to i_t one obtains the following first order condition:

$$x_t - x_{t-1} + \pi_t - (\Delta x + \pi)^* = 0 \tag{4.3}$$

This condition means that the nominal interest rate has to be chosen such that the nominal income growth reaches its target in every period. Note, that (4.3) requires that the real output growth falls by one percent for each percentage point that the rate of inflation rises.

From the first order condition follows the reaction function for the nominal interest rate i_t :

$$\begin{aligned} bi_t &= \eta_t + E_t x_{t+1} - x_{t-1} (1+a)^{-1} + \left(b + (1+a)^{-1}\right) E_t \pi_{t+1} \\ &\quad + (1+a)^{-1} \varepsilon_t - (1+a)^{-1} (\Delta x + \pi)^* \end{aligned} \tag{4.4}$$

The interest rate rule closely resembles the equation derived under flexible inflation targeting (3.3). The optimal reaction of the interest rate requires an increase in the nominal rate when both conditional output and forecasted inflation increase. Note, that the coefficient on expected inflation is larger than unity. In other words, the central bank has to raise the interest rate in order to increase the real interest rate. Likewise a negative supply shock (a positive realization of ε_t) and a positive demand shock call for an increase in the short term interest rate. Finally the optimal interest rate depends negatively on the target for the growth rate of nominal income.

Substituting the optimal interest rate in (2.2) gives the quasi-reduced form for output:

$$x_t = x_{t-1} (1+a)^{-1} - (1+a)^{-1} E_t \pi_{t+1} - (1+a)^{-1} \varepsilon_t + (1+a)^{-1} (\Delta x + \pi)^* \tag{4.5}$$

Using the technique of undetermined coefficients it is apparent that the solutions for π_t and x_t will be of the form

$$\pi_t = b_1 x_{t-1} + b_2 \varepsilon_t + b_3 (\Delta x + \pi)^* \tag{4.6}$$

$$x_t = c_1 x_{t-1} + c_2 \varepsilon_t + c_3 (\Delta x + \pi)^* \tag{4.7}$$

Shifting the time index one period, substituting for y_t and taking expectations we get for $E_t \pi_{t+1}$:

$$E_t \pi_{t+1} = b_1 c_1 x_{t-1} + \varepsilon_t (b_2 \theta + c_2 b_1) + (b_1 c_3 + b_3) (\Delta x + \pi)^* .$$

Substituting $E_t\pi_{t+1}$ into (2.1) resp. (4.5) and comparing with (4.6) resp.(4.7) yields the equations for the undetermined coefficients b_i and c_i :

$b_1 = c_1 (b_1 + a)$

$b_2 = b_2\theta + c_2 b_1 + 1 + ac_2$

$b_3 = b_1 c_3 + b_3 + ac_3$

$(1 + a) c_1 = 1 - b_1 c_1$

$(1 + a) c_2 = -(b_2\theta + c_2 b_1 + 1)$

$(1 + a) c_3 = -(b_1 c_3 + b_3 - 1)$.

To satisfy these expressions the coefficients have to be of the form.

$b_1 = -A \qquad c_1 = (1 + A)$

$b_2 = \left((1-\theta)^2 - a\theta\right)^{-1}(A + 1 - \theta) \qquad c_2 = -(1 - \theta + A)\left((1-\theta)^2 - a\theta\right)^{-1}$

$b_3 = 1 \qquad c_3 = 0$

with $A = \frac{1}{2}\left(a - \sqrt{a^2 + 4a}\right) < 0$ and $1 + A > 0$ (we know the relevant sign of the root $-\sqrt{a^2 + 4a}$ via a MSV procedure (see McCallum, 1983)).

Substituting the coefficients into (4.6) and (4.7) gives the solution for the model if the central bank follows a nominal income target.

$$\pi_t = -Ax_{t-1} + \left((1-\theta)^2 - a\theta\right)^{-1}(A + (1-\theta))\,\varepsilon_t + (\Delta x + \pi)^* \tag{4.8}$$

$$x_t = (1 + A)\,x_{t-1} - ((1-\theta) + A)\left((1-\theta)^2 - a\theta\right)^{-1}\varepsilon_t \tag{4.9}$$

$$i_t = \zeta_1 x_{t-1} + b^{-1}(1 + a)^{-1}\zeta_2\varepsilon_t + (\Delta x + \pi)^* + b^{-1}\eta_t \tag{4.10}$$

with

$\zeta_1 = b^{-1}(1 + a)^{-1}\left(A + a(1 + A)^2 - Ab(1 + A)(1 + a)\right)$

$\zeta_2 = \left((1 - \theta + A)\left((1-\theta)^2 - a\theta\right)^{-1}((b(1 + a) - a)(A + \theta) - (1 + a))\right) + 1$

Observe from (4.8) and (4.9) that the effects of a demand shock are fully absorbed under a monetary policy of nominal income targeting. Neither real output nor the rate of inflation are affected. The rise of the interest rate according to (4.10) completely offsets the demand shock.

A supply shock, by contrast, cannot be neutralized by a monetary policy of nominal income targeting. An ε_t -shock generates a stagflation episode. Equations (4.8) and (4.9) show that with nominal income targeting that if the rate of inflation goes up the rate of real output growth goes down by an equal amount. If the supply shock is a random walk, i.e. $\theta = 1$ the effects are: $\partial\pi_t/\partial\varepsilon_t = -a^{-1}A > 0$ and $\partial x_t/\partial\varepsilon_t = a^{-1}A < 0$; in the case of a transitory supply shock $(\theta = 0)$, the division of the shock is: $\partial\pi_t/\partial\varepsilon_t = 1 + A > 0$ and $\partial x_t/\partial\varepsilon_t = -(1 + A) < 0$.

Figure 4.1. illustrates this case. As in the previous section FF represents the Phillips curve (2.1) and equation (2.2) the IS-curve. The RR-line is the first order condition; its slope is minus one. Assume a negative supply shock occurs $(\varepsilon_t > 0)$ and the nature of the shock is random walk $(\theta = 1)$. The impact effect and the revision of the expected inflation $\left(E_t\pi_{t+1} = -a^{-1}(1 + A)A\varepsilon_t > 0\right)$ shifts the Phillips curve from

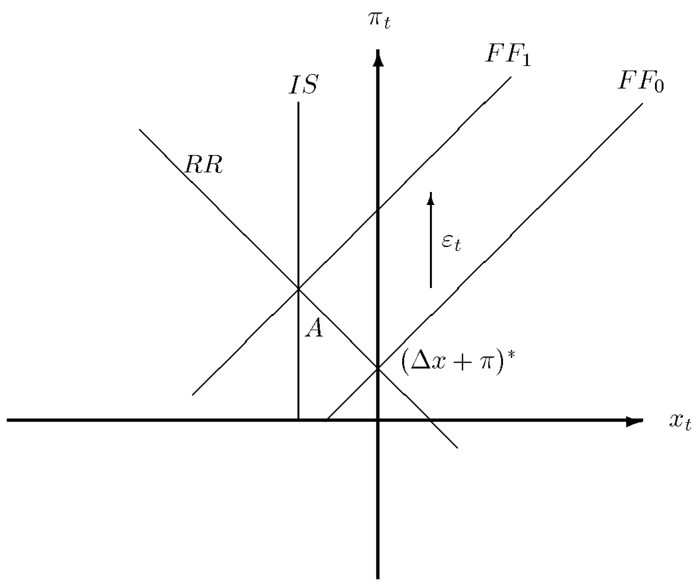

Figure 4.1:

FF_0 to FF_1. The resulting inflation is too high given the nominal income target. The optimal policy calls for a negative rate of growth of real output to compensate for the rise in inflation. This is engineered by the central bank through an appropriate rise in the nominal interest rate. The latter shifts the IS-curve to the left along the optimal policy line RR to position A.

The striking difference to the case of flexible inflation targeting is:

1. The slope of the RR-curve is minus one and independent of the parameters of the model

2. If the central bank moves the IS-curve along the RR-line it follows the rule to reduce real output growth by one percent for each percentage point by which the rate of inflation goes up.

Since the solution for the state variables π_t , x_t and i_t depend on the output gap of last period x_{t-1}, the results of nominal income targeting are more complex compared to the results under inflation targeting. It is interesting to note that in both regimes a demand shock is always offset by an appropriate reaction of the nominal interest rate.

Equation (4.10) shows that optimal monetary policy calls for a rise in the nominal interest rate in response to a negative supply shock, but the influence of the supply shock is gradually reduced as long as $\theta < 1$. The sign of the lagged output coefficient is negative while the rise of the target rate of nominal income raises the interest rate by an equal amount.

An interesting question is to analyze if the solution under an nominal income target is stable or not. Svensson (1997c) and Ball (1997) use a backward-looking version of a Phillips and IS-curve with two time-lags. They show that even if (expected) nominal income is on target, both inflation and output gap are unstable. Svensson argues that this outcome is due to the time-lag structure of the model and he rejects nominal income targeting as strategy for the central bank. In Ball's version both output and inflation have infinite variances regardless of whether the central bank targets the growth rate or the level of nominal GDP. The effect of a supply shock never vanishes, even though the nature of the shock is transitory.

In contrast, McCallum (1997) considers a forward looking model similar to the one we applied and his analysis implies stability of π_t and x_t. It is obvious that the Svensson-Ball result is not general, but depends on the specification of the Phillips curve. A forward looking Phillips curve leads to a stable solution, a backward looking Phillips curve as in Svensson leads to an unstable solution.

Stability is easily shown in our model. We derived the solution for x_t in (4.9) as:

$$x_t = (1 + A)\, x_{t-1} - ((1-\theta) + A)\left((1-\theta)^2 - a\theta\right)^{-1} \varepsilon_t$$

Since $0 < (1 + A) < 1$ it follows immediately that x_t is stable. For π_t we obtained in (4.8):

$$\pi_t = -Ax_{t-1} + \left((1-\theta)^2 - a\theta\right)^{-1} (A + (1-\theta))\, \varepsilon_t + (\Delta x + \pi)^* .$$

From $-1 < A < 0$ follows $1 > -A > 0$ and, therefore, implies stability of the inflation rate.

An obvious question is what parametervalue for Φ generates the same qualitative result for flexible inflation targeting and NI targeting. Comparing (3.2) with (4.3) we get that for $\Phi = a$ the two first order conditions are equal. In that case we obtain the interesting result that for both monetary strategies the trade-off between inflation and output stabilization is minus one. In terms of our grafical representation this would mean that in both cases the slope of the RR-curve is minus one.

5. A two-period time-lag

To compare the Svensson model with our model, we start with the forward-looking model of section 2 and introduce the Svensson-Ball two period control lag. There is a one period lag between the change in the output gap and the rate of inflation (Phillips curve) and again a one period lag between the output gap and the change in the real interest rate (IS-curve).

$$\pi_t = E_t\pi_{t+1} + ax_{t-1} + \varepsilon_t \tag{5.1}$$

$$x_t = E_t x_{t+1} + \eta_t - b\left(i_{t-1} - E_{t-1}\pi_t\right) \tag{5.2}$$

From (5.1) and (5.2) it is apparent that a change in the control variable i_t affects next period output and the inflation rate two periods ahead. The central bank´s loss function is the same as in section 2 namely (2.6).

We shift (5.2) forward for one period:

$$x_{t+1} = E_{t+1}x_{t+2} + \eta_{t+1} - b\left(i_t - E_t\pi_{t+1}\right) \tag{5.3}$$

Substituting (5.3) in (5.1) and taking into account the time-lag, we obtain

$$\pi_{t+2} = E_{t+2}\pi_{t+3} + a\left(E_{t+1}x_{t+2} + \eta_{t+1} - b\left(i_t - E_t\pi_{t+1}\right)\right) + \varepsilon_{t+2}. \tag{5.4}$$

Since i_t affects only x_{t+1} and π_{t+2} the optimization problem reduces to the following simple intertemporal optimization problem and whose solution procedure is straightforward.

$$\min E_t\frac{1}{2}\left(\delta\left(\left(\pi_{t+1} - \pi^*\right)^2 + \Phi x_{t+1}^2\right) + \delta^2\left(\left(\pi_{t+2} - \pi^*\right)^2 + \Phi x_{t+2}^2\right)\right) \tag{5.5}$$

subject to (5.3) and (5.4). Differentiating (5.5) with respect to i_t and taking expectations we obtain the following optimality condition:

$$E_t x_{t+1} = -\delta a\Phi^{-1}\left(E_t\pi_{t+2} - \pi^*\right) \tag{5.6}$$

The first order condition has a straightforward interpretation: Whenever $E_t\pi_{t+2}$ (i.e. the conditional forecast of inflation two periods ahead) is above its target level, contract the expected demand (output gap) of period $t+1$ below capacity by raising the current control variable i_t. The optimal response of the interest rate depends positively on a, the parameter of the Phillips curve and inversely on Φ the relative weight on output loss. The qualitative results concerning the trade-off between inflation and output and the amount of accommodation measured by the parameter Φ extend to this model with the two-period time lag.

From the optimality condition follows for i_t

$$bi_t = E_t x_{t+2} + \left(\delta a^2 + \Phi\right)^{-1}\delta a E_t\pi_{t+3} + \rho\eta_t + bE_t\pi_{t+1} + \delta a\left(\delta a^2 + \Phi\right)^{-1}\theta^2\varepsilon_t - \left(\delta a^2 + \Phi\right)^{-1}\delta a\pi^* \tag{5.7}$$

The optimal interest rate response depends on the conditional output and inflation forecast. In addition the **central bank** is aiming at $\theta^2\varepsilon_t$ the expected output disturbance in $t+2$ (generated by the supply shock ε_t in t) and at $\rho\eta_t$, the expected demand disturbance in $t+1$. Using the optimal interest rate in (5.4) resp.(5.3) yields the quasi-reduced form for the inflation rate, (resp. for the output gap):

$$\pi_{t+2} = E_{t+2}\pi_{t+3} + aE_{t+1}x_{t+2} - aE_t x_{t+2} + av_{t+1} - a\delta a\left(\delta a^2+\Phi\right)^{-1} E_t\pi_{t+3} + \left(\delta a^2+\Phi\right)^{-1}\Phi\theta^2\varepsilon_t + a^2\delta\left(\delta a^2+\Phi\right)^{-1}\pi^* + \theta\xi_{t+1} + \xi_{t+2}. \qquad (5.8)$$

$$x_{t+1} = E_{t+1}x_{t+2} - E_t x_{t+2} + v_{t+1} - \left(\delta a^2+\Phi\right)^{-1}\delta a E_t\pi_{t+3} - \delta a\left(\delta a^2+\Phi\right)^{-1}\theta^2\varepsilon_t + \left(\delta a^2+\Phi\right)^{-1}\delta a\pi^*$$

To determine π_{t+2} and x_{t+1} we employ once again the McCallum (1983) technique of undetermined coefficients where the bubble-free solution is obtained via a minimal-state-variable procedure. Since the relevant state variables in (5.8) are $\varepsilon_t, \xi_{t+1}, \xi_{t+2}, v_{t+1}, v_{t+2}$ (which enters the solution via $E_{t+2}\pi_{t+3}$) and π^*, it is apparent that π_t will be of the form

$$\pi_{t+2} = b_0\varepsilon_t + b_1\xi_{t+1} + b_2\xi_{t+2} + b_3v_{t+1} + b_4v_{t+2} + b_5\pi^*. \qquad (5.9)$$

$$x_{t+1} = c_0\varepsilon_t + c_1\xi_{t+1} + c_2v_{t+1} + c_3\pi^*. \qquad (5.10)$$

For the expected inflation rates and expected output we obtain

$$E_t\pi_{t+3} = b_0\theta\varepsilon_t + b_5\pi^*$$

$$E_{t+2}\pi_{t+3} = b_0\left(\theta\varepsilon_t + \xi_{t+1}\right) + b_1\xi_{t+2} + b_3v_{t+2} + b_5\pi^*$$

$$E_t x_{t+2} = c_0\theta\varepsilon_t + c_3\pi^*$$

$$E_{t+1}x_{t+2} = c_0\left(\theta\varepsilon_t + \xi_{t+1}\right) + c_3\pi^*$$

Substituting the expectations into (5.8) and comparing with (5.9) yields the equations for the undetermined coefficients b_i and c_i :

$c_0 = -\left(a^2\delta + \Phi - \theta\Phi\right)^{-1}\delta a\theta^2$
$c_1 = -\left(a^2\delta + \Phi - \theta\Phi\right)^{-1}\delta a\theta^2$
$c_2 = 1$
$c_3 = 0$
$b_0 = \Phi\left(a^2\delta + \Phi - \theta\Phi\right)^{-1}\theta^2$
$b_1 = \theta\left(a^2\delta + \Phi - \theta\Phi\right)^{-1}\left(a^2\delta(1-\theta) + \Phi\right)$
$b_2 = \left(a^2\delta + \Phi - \theta\Phi\right)^{-1}\left(a^2\delta\left(1+\theta-\theta^2\right) + \Phi\right)$
$b_3 = a$
$b_4 = a$
$b_5 = 1$

$$\pi_{t+2} = \left(a^2\delta + \Phi(1-\theta)\right)^{-1}\left(\Phi\theta^2\varepsilon_t + \theta\left(a^2\delta(1-\theta)+\Phi\right)\xi_{t+1} + \left(a^2\delta\left(1+\theta-\theta^2\right)+\Phi\right)\xi_{t+2}\right) + av_{t+1} + av_{t+2} + \pi^* \qquad (5.11)$$

$$x_{t+1} = -\left(a^2\delta + \Phi\left(1-\theta\right)\right)^{-1}\delta a\theta^2\left(\varepsilon_t + \xi_{t+1}\right) + \upsilon_{t+1} \quad (5.12)$$

$$\begin{aligned}\pi_{t+2} - E_t\pi_{t+2} &= \left(a^2\delta + \Phi\left(1-\theta\right)\right)^{-1}\left(\theta\left(a^2\delta\left(1-\theta\right)+\Phi\right)\xi_{t+1} + \left(a^2\delta\left(1+\theta-\theta^2\right)+\Phi\right)\xi_{t+2}\right)\\ &\quad + a\upsilon_{t+1} + a\upsilon_{t+2}\end{aligned}$$

$$x_{t+1} - E_t x_{t+1} = -\left(a^2\delta + \Phi\left(1-\theta\right)\right)^{-1}\delta a\theta^2\xi_{t+1} + \upsilon_{t+1}$$

The actual rate of inflation deviates from the conditional forecast at a two year horizon. Since the central bank sets the interest rate in t, it cannot respond to supply and demand shocks occurring in $t+1$ and $t+2$. For this reason it cannot offset the deviation $\pi_{t+2} - E_t\pi_{t+2}$ of inflation in period $t+2$. Nevertheless, it can influence the deviation of the two year forecast from its target rate.

$$E_t\pi_{t+2} - \pi^* = \left(a^2\delta + \Phi\left(1-\theta\right)\right)^{-1}\Phi\theta^2\varepsilon_t$$

The deviation depends on Φ, the weight on output loss in the objective function. If $\Phi = 0$ (strict inflation targeting) the forecast is always on target. If Φ increases (indicating a greater preference for output stabilization) inflationary expectations and, therefore, the deviation from the target π^* increase. For $\Phi = \infty$ (strict output targeting) the deviation is maximized: $E_t\pi_{t+2} - \pi^* = \left(\left(1-\theta\right)\right)^{-1}\theta^2\varepsilon_t$.

6. Conclusions

In this paper we analyze two alternative monetary policies within a simple dynamic macromodel that nests a forward looking Phillipscurve relation and a forward looking IS-curve with a sticky price model. In this model we explore the effects of supply and demand shocks that follow an AR(1)-process. We then contrast the performance of flexible inflation targeting with nominal income targeting. Furthermore we analyze inflation targeting in a model with a two-period time-lag of Ball Svensson type.

The following conclusions summarize this analysis: First, for both targeting regimes optimal monetary policy response leads to a shock-dependent feedback rule. The optimal reaction implies for both strategies an increase in the nominal interest rate. This rise has to be larger than the increase of expected inflation so that the real short run rate moves in the same direction as the nominal rate.

Second, if the shock is a demand disturbance flexible inflation targeting and nominal income targeting produce the same outcome: A demand shock is completely offset by both monetary strategies.

Third, if the shock is a supply disturbance there is a significant difference between the performance of the two different monetary policies. If the central bank follows inflation targeting the policy makers face a trade-off between inflation and output stabilization. This trade-off depends on the preference parameter Φ attached to output stabilization relative to inflation stabilization. A large value of Φ implies an accommodative monetary policy, i.e. strong output stabilization at the expense of high inflation, a small value implies low inflation at the expense of high output losses, i.e. a non-accommodative policy.

If the central bank follows nominal income targeting policy makers face a constant trade-off between inflation and real output growth. With nominal income targeting an increase in inflation leads to a fall in real output growth by an equal amount.

Finally we analyze inflation targeting in a two-period time-lag version of the model. The qualitative results about the trade-off between inflation and output growth are the same as in the basic model without time lag. A demand shock can be compensated by an appropriate adjustment of the nominal interest rate, whereas a supply shock can only partly be compensated. Since the interest rate affects inflation two periods later, demand shocks occurring in $t + 1$ and $t + 2$ can not be offset.

References

[1] Ball Laurence (1997), Efficient Rules for Monetary Policies, Reserve Bank of New Zealand Discussion Paper G97/3.

[2] Bean Charles R. (1983), Targeting Nominal Income: An Appraisal, The Economic Journal 93, 806-819.

[3] Bernanke B., Laubach T., Mishkin F. & Posen A. (1999), Inflation Targeting, Princeton University Press.

[4] Bernanke B. & Mishkin F. S. (1997), Inflation Targeting: A new Framework for Monetary Policy?, Journal of Economic Perspectives 11(2), 97-116.

[5] Calvo Guillermo A. (1983), Staggered Contracts in a Utility-Maximizing framework, Journal of Monetary Economics 12, 383-398.

[6] Clarida R., Gali J. & Gertler M. (1998a), Monetary Policy Rules and Macroeconomic Stability: Evidence and some Theory, CEPR Discussion Paper 1908.

[7] Clarida R., Gali J. & Gertler M. (1998b), Monetary Rules in Practice: Some International Evidence, European Economic Review 42, 1033-1068.

[8] Clarida R., Gali J. & Gertler M. (1999), The Science of Monetary Policy: A new Keynesian Perspective, CEPR Discussion Paper 2139.

[9] Clark P., Goodhart C. & Huang H. (1999), Optimal monetary policy rules in a rational expectations model of the Phillips curve, Journal of Monetary Economics 43, 497-520.

[10] Frankel J.A. & Funke N. (1993), A Two-Country Analysis of International Targeting of Nominal GNP, Revista di Politica Economica 83, 69-106.

[11] Gali J. & Gertler M. (1999), Inflation Dynamics: A structural Econometric Analysis, CEPR Discussion Paper 2246.

[12] Hall R. & Mankiw N.G. (1994), Nominal Income Targeting, in: N.G. Mankiw (ed.), Monetary Policy, NBER.

[13] Leiderman L. & Svensson L. (1995), Inflation Targets, CEPR, London.

[14] McCallum Bennett T. (1983), On the Non-Uniquess in Rational Expectations Models: An Attempt at Perspective, Journal of Monetary Economics 11, 139-168.

[15] McCallum Bennett T. (1997), The Alleged Instability of Nominal Income Targeting, Reserve Bank of New Zealand Paper G97/6.

[16] McCallum B.T. & Nelson E. (1999), Nominal income targeting in an open-economx optimizing model, Journal of Monetary Economics 43, 553-578.

[17] Mishkin Frederic S. (1999), International experiences with different monetary policy regimes, Journal of Monetary Economics 43, 579-605.

[18] Roberts John M. (1995), New Keynesian Economics and the Phillips curve, Journal of Money, Credit and Banking 27, 975-984.

[19] Rotemberg Julio J. (1982), Sticky Prices in the United States, Journal of Political Economy 60, 1187-1211.

[20] Rudebusch G.D. & Svensson L.E.O. (1999), Policy Rules for Inflation Targeting, in J.B. Taylor (ed.), Monetary Policy Rules, University of Chicago Press.

[21] Svensson Lars E.O. (1997a), Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets, European Economic Review 41, 1111-1146.

[22] Svensson Lars E.O. (1997b), Optimal Inflation Targets, Conservative Central Banks and Linear Inflation Contracts, American Economic Review 87, 98-114.

[23] Svensson Lars E.O. (1997c), Inflation Targeting: Some Extensions, NBER Working Paper 5962.

[24] Svensson Lars E.O. (1999a), Inflation Targeting as a Monetary Policy Rule, Journal of Monetary Economics 43, 607-654.

[25] Svensson Lars E.O. (1999b), Monetary policy issues for the Eurosystem, CEPR Discussion Paper 2197.

[26] Taylor John B. (1979), Staggered Contracts in a Macro model, American Economic Review 69, 108-113.

[27] Taylor John B.(1993), Discretion versus policy rules in practice, Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

[28] Tobin John (1980), Stabilization policy ten years after, Brooking Papers on Economic Activity 1, 19-72.

[29] Walsh Carl (1998), Monetary Theory and Policy, MIT Press.