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SOCIAL SECURITY, RETIREMENT AGE AND OPTIMAL INCOME TAXATION

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Abstract

It is often argued that implicit taxation on continued activity of elderly workers is responsible for the widely observed trend towards early retirement. In a world of laissez-faire or of first-best efficiency, there would be no such implicit taxation. The point of this paper is that when first-best redistributive instruments are not available, because some variables are not observable, the optimal policy does imply a distortion of the retirement decision. Consequently, the inducement of early retirement may be part of the optimal tax-transfer policy. We consider a model in which individuals differ in their productivity and their capacity to work long and choose both their weekly labor supply and their age of retirement. We characterize the optimal non linear tax-transfer that maximizes a utilitarian welfare function when weekly earnings and the length of active life are observable while individuals' productivity and health status are not observable.

JEL Classification: H55, H23, E62.

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1 Introduction

A trend towards early retirement is currently observed in most European countries. Participation rates for men aged 60 to 64, which were above 70% in the early sixties, have fallen to 57% in Sweden and to below 20% in Belgium, France Italy and the Netherlands. Similarly, the average labor participation in the age group 55–64 has declined and now ranges from 24 per cent in Belgium, to 88 per cent in Iceland, with the bulk of countries closer to Belgium than to Iceland. Early retirement *per se* is of course a blessing for a society which values consumption of leisure. However, it also puts pressure on the financing of health care and pension schemes. This problem is made worse by growing longevity. In the European Union life expectancy at age 65 has increased by more than one year per decade since 1950. As a consequence, instead of 45-50 years of work and 5-10 years of retirement of half a century ago, a young worker can now expect to work for 30-35 years and retire for 15-20 years.

The effective retirement age varies across individuals and depends on features such as wealth, productivity and health. In addition, retirement decisions are likely to be affected by the pension system. When there is no pension system, (utility maximizing) people retire when the marginal utility of inactivity is equal to their marginal productivity at work. People in poor health and with low productivity will retire earlier than people in good health and with high productivity. The same tradeoff arises under an optimal retirement system in a first-best (full information) setting. When there is a pension system, this tradeoff may or may not be affected, depending on the design of the benefit formula. In a first-best (full information) setting, an optimal retirement system would imply the same tradeoff. Such a pension system can be referred to as neutral or actuarially fair.

In reality, pension systems are typically not neutral and they distort the retirement decision. As it has been shown by a number of authors, notably

Gruber and Wise [1999] and Blondal and Scarpetta [1998] the observed age of retirement is likely to be distorted downwards in a number of countries. The main explanation for this distortion appears to be the incentive structure implied by social protection programs aimed at elderly workers: pension plans but also unemployment insurance, disability insurance and early retirement schemes. The authors show that prolonged activity for elderly workers is subject to an implicit tax which includes both the payroll marginal tax and forgone benefits. Consequently, social protection systems are far from being actuarially fair at the margin in countries such as Belgium, France, Germany or the Netherlands where people retire relatively early. On the other hand, in Japan, Sweden and the US the implicit tax is much lower so that the system tends to be rather neutral and people retire much later.

These results are essentially positive. Nevertheless, they are often, at least implicitly, given a normative connotation and used to advocate reforms tending to remove the bias in the benefit formulas. This raises the question of whether a bias in the benefit formula in favor of early retirement is necessarily the sign of a bad policy. We show in this paper that this implicit tax on postponed retirement is not necessarily due to bad design but can be due to the desire by public authorities of using social security for redistribution when non distortionary tools are not available.

To address this issue we determine the social security benefits, payroll taxation and retirement age policy that a utilitarian social planner ought to conduct. We consider a setting with heterogeneous individuals differing in two unobservable characteristics: level of productivity and health status. We use an optimal income tax approach to design a non linear tax-transfer function depending upon two variables: the weekly income and the retirement age[1]. We show that in a setting of asymmetric information, a distortion towards early retirement is desirable for some individuals. More precisely, the

[1] See Maderner-Rochet (1995) who also deal with this problem in another setting.

optimal policy in the two type case induces highly productive and healthy workers to retire efficiently (namely when their labor disutility is marginally equal to their productivity) while less productive and less healthy workers are induced to retire earlier. We also show that the tradeoff between weekly labor supply and retirement age (for a given lifetime income) may or may not be distorted. When a distortion is called for, its sign depends on whether the "dominant" source of heterogeneity is health or productivity. When individuals differ mainly (or exclusively) in productivity, the distortion goes against weekly labor supply. When health differentials are dominant, the distortion goes against retirement age.

The two dimensions of heterogeneity are a crucial ingredient of our analysis. Indeed, when designing a redistributive social security system, it is important to take into account the wide variability in the capacity to work – a variability that is likely to widen as life expectancy increases. The practical issue is how to care for elderly workers who are in poor health without, at the same time, opening the door of retirement to those who would like to stop working but are quite capable of continuing. In fact, a reform of social security ought to include a close connection between pensions systems and the system of disability insurance as well as the determination of a more flexible retirement age together with actuarial adjustment of yearly benefits. The ideal outcome would then be to have early retirees because of poor health receive relatively generous benefits while early retirees unwilling to continue working would receive actuarially low pensions.

There exists a theoretical literature dealing with various aspects of the issue of social security, disability insurance and retirement. It focuses on long-term labor contracts encompassing retirement rules [Lazear, 1979] and the implicit inducement to retirement of existing public and private pension plans [Crawford-Lilien, 1981, Fabel, 1994]. This literature is mainly positive; it analyzes retirement behavior in order to explain the observed evolutions in retirement practice. This paper uses a normative approach which is intended to provide a benchmark against which the positive results can be assessed. In that respect, it is in the vein of Diamond and Mirrlees (1986) who derive disability contingent retirement rules.

The rest of the paper is organized as follows. Section 2 presents the basic model while the laissez-faire and the first-best solutions are studied in Section3. In section 4 the optimal second best policy is studied. We characterize the optimal (incentive compatible) utilitarian allocation and the implementing income tax and social security benefit functions. To keep the presentation simple we focus on an economy with two types of individuals. Section 5 provides a number of numerical examples which illustrate the analytical results and provide some results for a three-type setting.

2 The model

Most of our analysis is based on a reduced form specification. We start by presenting the underlying micro model and show how it leads to the specification we use. This detour is necessary to grasp the proper interpretation of our setting. Consider an individual who has preferences over consumption c and labor l which can be expressed by an instantaneous utility function $U(t)$ assumed to be additively separable:

$$U(t) = u(c(t)) - r(t)V(l(t))$$

where u and V fulfill the usual assumptions and $r(t)$ denotes the instantaneous increasing intensity of labor disutility. Let date 0 denote entrance to the labor force, h, the maximum life-span and z, the retirement age (length of working life). For simplicity we shall often refer to l as "weekly" labor supply. Though somewhat abusive, this terminology is also useful to avoid confusion with z which represents another dimension of (lifetime) labor supply. Assuming the interest rate and the discount factor both equal to 0, lifetime utility can be written as:

$$U = \int_{0}^{h} u\left(c\left(t\right)\right)dt - \int_{0}^{z} r(t)V\left(l\left(t\right)\right)dt. \tag{1}$$

Assuming a constant weekly productivity w over time, the lifetime budget constraint is:

$$\int_{0}^{h} c(t)dt = \int_{0}^{z} \left[wl(t) - \tau(wl(t))\right]dt + (h-z)p(z) \tag{2}$$

where $\tau(wl(t))$ is an instantaneous non linear tax depending on labor income and $p(z)$ the instantaneous level of pension which may depend on the individual's retirement age (via the benefit formula). The total (lifetime) retirement benefits are given by $(h-z)p(z)$. For the sake of simplicity, we impose that $l\left(t\right) = l$ is a time invariant choice[2]. Separability, concavity of the instantaneous utility functions, perfect capital markets and certain lifetimes imply that each individual will set his level of consumption equal in all periods. Denoting $y = wl$, one can rewrite the budget constraint as follows:

$$hc = zwl - T(y,z)$$

where

$$T(y,z) = z\tau(wl) - (h-z)p(z), \tag{3}$$

is the difference between total tax payments and total retirement benefits. The function $T(y,z)$ represents the *net* social security cum income tax paid by an individual. Alternatively, we can think of $-T(y,z)$ as the net transfer an individual receives from the social security system. Differentiating T yields the implicit tax on retirement that have estimated Gruber and Wise [1999]. To see this note that

$$\frac{\delta T(y,z)}{\delta z} = \tau(wl) + p(z) - (h-z)p'(z). \tag{4}$$

[2]Without such a restriction, $l(t)$ would be decreasing over time.

In words, an additional year of work may imply a double cost: the payroll tax $\tau(wl)$ and foregone benefits if $p(z) > 0$. On the other hand, postponing retirement may imply higher per period benefits during retirement. This positive effect (negative cost) is captured by the third term on the RHS of (4). In the rest of the paper, we use this reduced tax function $T(y, z)$.

With c and l constant over time, lifetime utility is given by

$$U = h\,u\,(c) - V\,(l)\,R\,(z)\,, \tag{5}$$

where

$$R\,(z) = \int_0^z r(t)dt.$$

The function $R\,(z)$ denotes the disutility for a working life of length z; we have $R'\,(z) = r\,(z) > 0$ and $R''\,(z) = r'(z) > 0$. Labor disutility, regarding the length of working life z, can be interpreted in terms of an indicator of health. Healthy individuals accordingly would have a lower $R(z)$ than individuals whose poor health makes it harder to work beyond a certain age. There is another term in the labor disutility which concerns the length of work week, $V(l)$. We assume that this function $V(l)$ is the same for all. In other words, there is no heterogeneity in this respect. This simplifying assumptions is motivated by the fact that we want to focus on the retirement decision rather than on the determination of weekly labor supply.

Each individual is characterized by two parameters: his productivity level w_i and his disutility for the retirement age $R_j\,(z) = R(z;\alpha_j)$ with $\delta R/\delta\alpha_j > 0$. There are two levels of productivity w_h and w_l with $w_h \geqslant w_l$. Similarly, the health status parameter takes two values with $\alpha_h \geqslant \alpha_l$ so that $R_h\,(z) \geqslant R_l\,(z)$ for every z. Note that the subscript h when associated with w, refers to the "good" (high productivity) type, while h associated with α is the "bad" (high disutility of remaining in the labor force) type. We denote a type of individual with subscripts (i, j), i denoting the productivity index and j the age of retirement disutility index.

3 The *laissez-faire* economy and the first best

3.1 The *laissez faire*

In a *laissez-faire* economy, deleting the subscripts referring to individuals types, every agent solves the following problem:

$$\max_{l,z} h\, u\left(\frac{wlz}{h}\right) - V(l)\, R(z) \tag{6}$$

The first order conditions with respect to l and z are respectively:

$$u'(c)\, wz - V'(l)\ R(z) = 0 \tag{7}$$

$$u'(c)\, wl - V(l)\ R'(z) = 0. \tag{8}$$

With (7) and (8) one obtains the usual equality between marginal rates of substitution between work and consumption and the corresponding relative price:

$$MRS_{cl} = \frac{V'(l)R(z)}{u'(c)} = wz \tag{9}$$

$$MRS_{cz} = \frac{V(l)R'(z)}{u'(c)} = wl \tag{10}$$

where MRS_{ab} stands for the marginal rate of substitution between a and b. Combining (7) and (8), the tradeoff between l and z is determined by:

$$MRS_{lz} = \frac{V(l)R'(z)}{V'(l)R(z)} = \frac{l}{z} \tag{11}$$

To interpret condition (11) observe that the maximization of (6) requires the minimization of "effort" as described by the following dual problem.

$$\begin{aligned} \min_{l,z} \quad & E = V(l)R(z) \\ \text{s.t.} \quad & I = wlz, \end{aligned} \tag{12}$$

where I represents lifetime earnings and E denotes aggregate effort (or utility cost). Figure 1 represents problem (12) in the $(z,\ l)$ space.

The curve with equation $l = I/wz$ represents all the combinations of (z, l) that yield a given level of lifetime income I. Note that the slope of

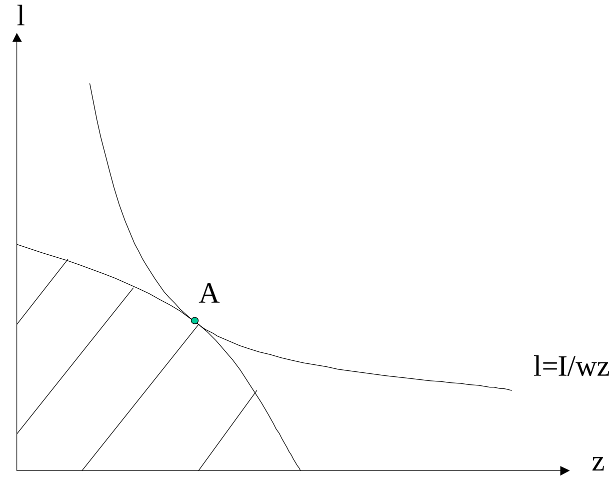

Figure 1: The effort minimization problem

this curve is given by $-I/wz^2 = -l/z$. To maximize utility it is necessary that this income level be produced so as to minimize the lifetime disutility of labor (effort). The shaded area represents the (z, l) combinations that generate a level of effort lower than or equal to a fixed level $\overline{E}$. The optimal (non distorted) (z, l) choice is given by the point A satisfying

$$\frac{R'(z)/R(z)}{V'(l)/V(l)} = \frac{l}{z}$$

that is, where the marginal rate of substitution between l and z is equal to the slope of the lifetime earnings curve; this is of course exactly equivalent to condition (11). Rearranging the terms, one obtains:

$$\varepsilon_V(l) \equiv \frac{lV'(l)}{V(l)} = \frac{zR'(z)}{R(z)} \equiv \varepsilon_R(z) \tag{13}$$

which corresponds to an equality between the elasticities of disutility for the work week $\varepsilon_v(l)$ and that for the retirement age $\varepsilon_R(z)$. For simplicity, we shall refer to the first one as the "work week elasticity" and to the second one as the "retirement elasticity".

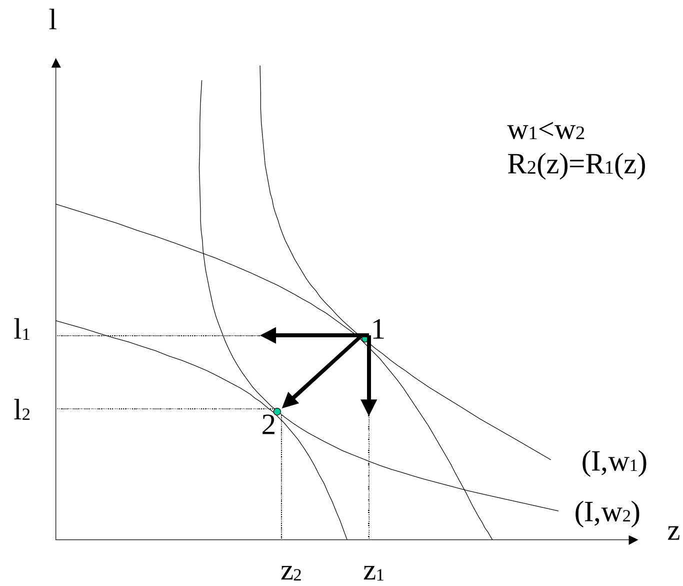

Figure 2: Choices of (z, l) for the two productivity types

We assume the two following monotonicity properties:

- *Assumption 1*: $\varepsilon_V(l)$ and $\varepsilon_{R_j}(z)$ $(j = h, l)$ are non decreasing functions (of l and z respectively).[3]

- *Assumption 2*: For every z one has $\varepsilon_{R_l}(z) \leqslant \varepsilon_{R_h}(z)$. In words, for any given age of retirement, the retirement elasticity of the more disabled individual is greater than or equal to the retirement elasticity of the more healthy individual.

These two assumptions allow us to compare the two optimal choices of l and z for the same aggregate earnings I with two individuals differing respectively in their productivity and their preferences for retirement.

[3] A necessary and sufficient condition for this is that:

(i) $1 + \dfrac{lV'(l)}{V(l)} - \dfrac{lV''(l)}{V(l)} > 0$ for every l ;

(ii) $1 + \dfrac{zR'(z)}{R(z)} - \dfrac{zR''(z)}{R(z)} > 0$ for every z.

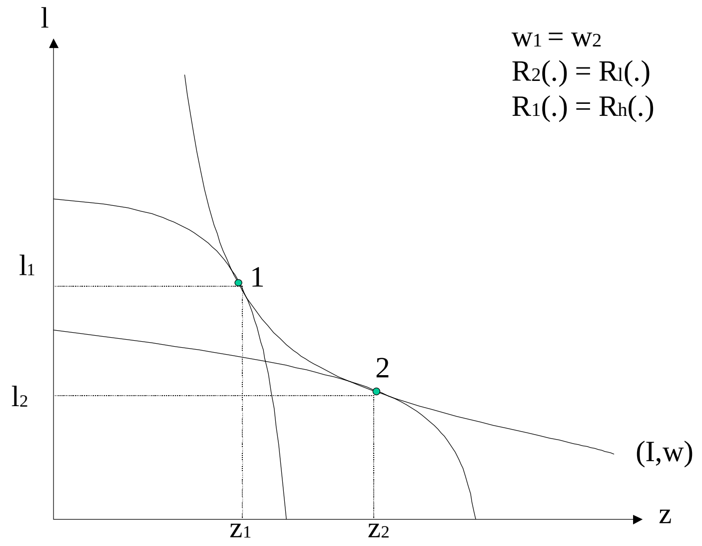

Figure 3: Choices of (z, l) for the 2 health types

Let us start with the case where individuals differ solely in their productivity. In figure 2, the more able individual (individual 2) chooses both a lower z and a lower l. This is the case if $\varepsilon_R(z)$ and $\varepsilon_V(l)$ are *strictly* increasing functions of z and l. For the special cases where R or V are isoelastic functions, the choice of either l (horizontal arrow) or z (vertical arrow) are the same for the two individuals (see equation (13)). To sum up, *Assumption 1* ensures that for a given level of lifetime earnings, the (l, z) choice of an individual with a higher ability lies south west of the point chosen by a less able individual.

We now turn to the case where individuals solely differ in their health status, where *Assumption 2* becomes relevant. Figure 3 shows that the individual who has a greater disutility for the retirement age (individual 2) will choose a higher l and a lower z than the other individual if the marginal rate of substitution between l and z is higher for this individual, that is, if $\varepsilon_{R_h}(z) > \varepsilon_{R_l}(z)$. In the extreme example where $\varepsilon_{R_h}(z) = \varepsilon_{R_l}(z)$, the two iso-effort curves will be parallel in the z, l space so that for the same aggregate earnings, they will choose the same pair z, l.

3.2 The social optimum

The above market solution can be contrasted with the first best social optimum which obtains when the social planner observes w_i and R_j. We consider a utilitarian social welfare function given by $\sum_{ij} f_{ij} U_{ij}$, where f_{ij} is the proportion of type ij's individuals, and $U_{ij} = h\, u(c_{ij}) - V(l_{ij})\, R_j(z_{ij})$ is the lifetime utility of ij individuals.[4] Welfare maximization is subject to the resource constraint that aggregate consumption cannot exceed aggregate production. This problem is given by:

$$\max_{c_{ij}, \ell_{ij}, z_{ij}} \quad \sum_{i,j} f_{ij} \left[h\, u(c_{ij}) - V(l_{ij})\, R_j(z_{ij})\right] - \mu \sum_{i} f_{ij} \left(h c_{ij} - w_i l_{ij} z_{ij}\right), \tag{14}$$

where μ is the Lagrange multiplier associated to the resource constraint. The first order conditions for every i, j are:

$$u'(c_{ij}) - \mu = 0 \tag{15}$$

$$V'(l_{ij})\; R_j(z_{ij}) - \mu\, w_i\, z_{ij} = 0 \tag{16}$$

$$V'(l_{ij})\; R_j'(z_{ij}) - \mu\, w_i\, l_{ij} = 0 \tag{17}$$

Combining these expression we find for every type i, j the non distorted tradeoffs described by equations (9), (10) and (11). In addition, (15) requires identical consumption levels for all individuals.[5]

With the reduced form utility function (5), the time dimension is implicit. In the laissez-faire solution there is implicitly saving during the

[4] We consider the utilitarian case to keep the expressions as simple as possible. Our analysis can easily be generalized to the case where social welfare is a weighted sum of individual utilities. This would not affect our main results. However, some specific results may change and some assumptions on the weights may be necessary; see footnote 10.

[5] When social welfare puts different weights on the individual utilities, the marginal social valuations of consumption, rather than the actual consumption levels, must be the same for all individuals.

working period: $(w_i l_{ij} - c_{ij}) z_{ij}$ which is used to finance consumption during retirement: $(h - z_{ij}) c_{ij}$; recall that we have assumed a zero interest rate. Consequently, the first best allocation $c_{ij}^*, l_{ij}^*, z_{ij}^*$ can be decentralized through a social security scheme with a non distortionary (lump-sum) contribution $\left(w_i l_{ij}^* - c_{ij}^*\right) z_{ij}^*$ and (lump-sum) social security benefits equal to $\left(h - z_{ij}^*\right) c_{ij}^*$. The number of hours l_{ij} and the age of retirement z_{ij} would be chosen optimally according to (15) and (16) coinciding with the first-best tradeoffs.

The first-best solution and its decentralization have been derived under the assumption that individual types w_i and R_j are observable. When there is asymmetric information, first-best lump sum transfers are (generally) not feasible; redistribution then has to rely on potentially distortionary taxes and transfers based on observable variables. In the remainder of the paper, we adopt an information structure that is inspired by the optimal taxation literature. Specifically, we assume that productivities, w_i, labor supply l_{ij} and health status R_j are not observable, while (weekly) before tax income $y_{ij} = l_{ij} w_i$ is observable. And added feature of our analysis compared to conventional optimal tax models is that the retirement age z_{ij} is also observable. Taxes and transfers can then be based both on y and on z. We use a non linear tax function $T(y, z)$ which, as shown by (3), accounts for income taxation, payroll taxes and retirement benefits.

4 Second best taxation

4.1 Implementation

Let us first examine how an individual's choices are affected by a non-linear income tax schedule $T(y, z)$. The first-order condition of this modified individual problem are crucial for understanding the implementation of the optimal tax policy derived below. The individual's problem now becomes

$$\max u(c_{ij}) - V(\frac{y_{ij}}{w_i})R_j(z_{ij})$$
$$\text{s.t.} \quad c_{ij} = y_{ij}z_{ij} - T(y_{ij}, z_{ij})$$

From the first order conditions, one obtains:

$$MRS_{cl}^{ij} = w_i z_{ij}\left(1 - \frac{1}{z_{ij}}\frac{\partial T(y_{ij}, z_{ij})}{\partial y_{ij}}\right) \tag{18}$$

$$MRS_{cz}^{ij} = y_{ij}\left(1 - \frac{1}{y_{ij}}\frac{\partial T(y_{ij}, z_{ij})}{\partial z_{ij}}\right) \tag{19}$$

and the implicit relation between l and z being:

$$MRS_{lz}^{ij} = \frac{l_{ij}}{z_{ij}}\left[1 - \theta_{ij}\right] \tag{20}$$

where

$$\theta_{ij} = \frac{T_z^{ij}/y_{ij} - T_y^{ij}/z_{ij}}{1 - T_y^{ij}/z_{ij}} \tag{21}$$

θ_{ij} is the marginal tax rate of z with respect to l.

Distortions in the (l, c) and (z, c) choices are assessed by comparing (18) and (19) to their laissez-faire counterparts (9) and (10). Not surprisingly, a positive marginal tax on either l or z implies a downward distortion on the corresponding variable.

Let us now turn to the tradeoff between z and l. Comparing (20) to its laissez-faire and first-best counterpart, (11) shows that when θ_{ij} is equal to zero, there is no distortion (in the tradeoff between z and l). This is true in particular when $T(y_{ij}, z_{ij}) = T(y_{ij}z_{ij})$, so that the tax depends only on total lifetime income. Furthermore, if θ_{ij} is negative z is encouraged with respect to l, while a positive θ_{ij} implies a distortion in favor of l.[6]

An alternative view on these distortions consists in saying that the choice between z and l is distorted downwards if individuals who retire earlier pay less taxes, *for a given level of before tax lifetime income* I, that is, when:[7]

$$\left.\frac{dT(y_{ij}, z_{ij})}{dz_{ij}}\right|_I = \frac{\partial T(y_{ij}, z_{ij})}{\partial z_{ij}} - \frac{y_{ij}}{z_{ij}}\frac{\partial T(y_{ij}, z_{ij})}{\partial y_{ij}} > 0 \tag{22}$$

[6] The distortions mentioned here are substitution effects, for given levels of I.

[7] In the same way, there will be a upwards distortion of the (z, l) choice when:

Using (21) it appears that (22) amounts to $\theta_{ij} > 0$.[8] Consequently the two alternative ways to define the distortions are effectively equivalent.

4.2 The second best optimum

To determine the second best optimum, we concentrate on settings with two types only (each of which being characterized by a specific value for the two parameter of heterogeneity). We assume that the correlation between the two characteristics is non positive[9]. Figure 4 illustrates three possible cases with the arrow representing the direction of the binding incentive constraint; see below.

We will first present the general case where the two types effectively differ in the two dimensions (represented by the diagonal arrow in the graphic). Then we consider two subcases where heterogeneity is only in one dimension. Subcase (a) will refer to the case where both individuals have the same preference over the age of retirement but differ in their productivity. Subcase (b) will refer to the case where both agents differ in their preference for the age of retirement but have the same productivity.

Formally, the economy is composed of two agents 2 ($= hl$) and 1 ($= lh$) being characterized respectively by a pair ($w_2 = w_h, R_2(z) = R_l(z)$) and ($w_1 = w_l, R_1(z) = R_h(z)$), with strict inequalities $w_2 > w_1$ and $R_2(z) < R_1(z)$. In case (a), the two agents will have, $R_1(z) = R_2(z) = R_h(z)$ and in case (b) $w_1 = w_2 = w_h$.

The problem of the government is directly obtained from (14) to which we add the incentive compatibility constraint that agent 2 does not want to

$$\left.\frac{dT_{ij}(y_{ij}, z_{ij})}{dy_{ij}}\right|_I = \frac{dT_{ij}(y_{ij}, z_{ij})}{dy_{ij}} - \frac{z_{ij}}{y_{ij}}\frac{dT_{ij}(y_{ij}, z_{ij})}{dz_{ij}} > 0$$

[8] As long as $1 - T_y^{ij}/z_{ij} > 0$, a condition which necessarily holds at an interior solution; see (18).

[9] We exclude the strict positive correlation case for which little can be said except when one difference overwhelmingly dominates the other.

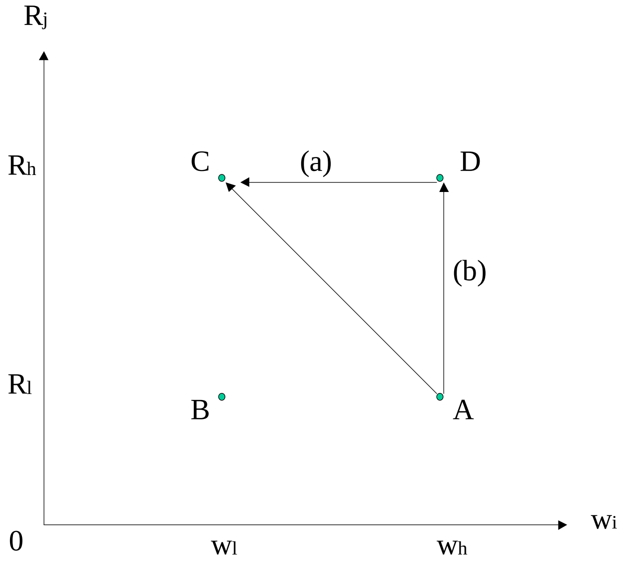

Figure 4: The configuration of types

mimic agent 1.[10] This yields the following problem:

$$\max_{c_i, y_i, z_i} \sum_i f_i U_i + \mu \left(\sum_i f_i \left(y_i z_i - h c_i \right) \right)$$
$$+ \lambda \left(h u(c_2) - V(\frac{y_2}{w_2}) R_2(z_2) - h u(c_1) + V(\frac{y_1}{w_2}) R_2(z_1) \right)$$

where μ and λ denote the multipliers associated with the revenue and the incentive compatibility constraints.[11] First order conditions with respect to

[10] In the utilitarian case (and with the considered configuration of types) this constraint is necessarily binding. However when, the social welfare function is a weighted sum of individual utilities and when the weight of the (able and/or healthy) type 2 is sufficiently large this may not be true anymore. Though formally possible, this does not appear to be a particular relevant case to consider. For the rest, as long as the constraint from 2 to 1 is binding, all our result in this section go through for a weighted social welfare function.

[11] When contrasting this with the first-best problem recall that index 2 stands for hl, while 1 stands for lh. Also note that we now optimize with respect to y (observable variable) rather than l.

c_i, y_i and z_i are:

$$f_1 u'(c_1) - \mu f_1 - \lambda u'(c_1) = 0 \tag{23}$$

$$f_2 u'(c_2) - \mu f_2 + \lambda u'(c_2) = 0 \tag{24}$$

$$-\frac{f_1}{w_1} V'(l_1) R_1(z_1) + \mu f_1 z_1 + \frac{\lambda}{w_2} V'(\bar{l}_2) R_2(z_1) = 0 \tag{25}$$

$$-\frac{f_2}{w_2} V'(l_2) R_2(z_2) + \mu f_2 z_2 - \frac{\lambda}{w_2} V'(l_2) R_2(z_2) = 0 \tag{26}$$

$$-f_1 V(l_1) R_1'(z_1) + \mu f_1 y_1 + \lambda V(\bar{l}_2) R_2'(z_1) = 0 \tag{27}$$

$$-f_2 V(l_2) R_2'(z_2) + \mu f_2 y_2 - \lambda V(l_2) R_2'(z_2) = 0 \tag{28}$$

where the upper bar denotes the choice of the mimicker, so that $\bar{l}_2 = y_1/w_2$, i.e., the quantity of labor type 2 must supply to earn y_1.

Combining (24), (26) and (28), one obtains non distorted tradeoffs for type 2; marginal rates of substitution for this individual continue to be given by (9)–(11). From (18) (19) and (20) this implies that marginal tax rates with respect to y and z are zero so that we also have $\theta_2 = 0$. This is the usual no distortion at the top property.

We now turn to individual 1's and study successively his tradeoffs in the $(l_1, c_1), (z_1, c_1)$ and (l_1, z_1) planes. Equation (23) and (26) yield:

$$MRS^1_{cl} = \left[\frac{1 - \dfrac{\lambda}{f_1}}{1 - \dfrac{\lambda}{f_1} \dfrac{w_1}{w_2} \dfrac{\overline{MRS}^2_{cl}}{MRS^1_{cl}}} \right] w_1 z_1, \tag{29}$$

where $\overline{MRS}^2$ denotes individual 2's marginal rate of substitution when mimicking individual 1. We have $MRS^1_{cl} > \overline{MRS}^2_{cl}$ because $\overline{l_2} < l_1$ and $R_2(z_1) < R_1(z_1)$. Consequently, (29) implies $MRS^1_{cl} < w_1 z_1$ so that there is a *marginal* downward distortion in the work week. In other words, by equation (18), the marginal tax on weekly income, y, is positive. This property does not come as a surprise and it is in line with the standard property obtained in the optimal income tax literature.

Now combining equation (23) and (27) one obtains:

$$MRS_{cz}^{1} = \left[\frac{1 - \dfrac{\lambda}{f_1}}{1 - \dfrac{\lambda}{f_1} \dfrac{w_1}{w_2} \dfrac{\overline{MRS}_{cz}^{2}}{MRS_{cz}^{1}}} \right] y_1 \tag{30}$$

where $MRS_{cz}^{1} > \overline{MRS}_{cz}^{2}$. Consequently, one has $MRS_{cz}^{1} < y_1$ so that there is a *marginal* downward distortion on z_1. That is, for a given weekly labor supply, the individual is induced to choose a lower z relative to c than he would do in a first best setting. By equation (19), the marginal tax on the retirement age is positive. Intuitively this property can be explained by the fact that type 1 individuals have steeper indifference curves at any given point in the (z, c) space than type 2 individuals. This is because type 1 individuals must be compensated more to accept to work longer than the mimicking individual (they are less healthy and have a higher weekly labor supply). This implies that, starting from the first best tradeoff, a variation $dz_1 < 0$ along with a variation $dc_1 = (MRS_{cz}^{1})dz_1$ has no (first-order) effect on the utility of type 1, but it decreases the utility of type 2 mimicking type 1. Consequently, the downward distortion in z_1 is a way to relax an otherwise binding self selection constraint.

To interpret this result it is useful to recall (4) which relates $\partial T/\partial z$ to the implicit tax that the pension system imposes on continued labor force participation. It thus appears that it is optimal to adopt a retirement system with a benefit formula which induces early retirement for the low ability (and high disutility) individual.

Combining equations (29) and (30), we find:

$$MRS_{lz}^{1} = \left[\frac{1 - \dfrac{\lambda}{f_1} \dfrac{w_1}{w_2} \dfrac{\overline{MRS}_{cl}^{2}}{MRS_{cl}^{1}}}{1 - \dfrac{\lambda}{f_1} \dfrac{\overline{MRS}_{cz}^{2}}{MRS_{cz}^{1}}} \right] \frac{l_1}{z_1}, \tag{31}$$

which, as shown in Appendix A implies

$$MRS_{lz}^1 = \frac{R_1'(z_1)/R_1(z_1)}{V'(l_1)/V(l_1)} \gtreqless \frac{l_1}{z_1} \Leftrightarrow \frac{\varepsilon_V(l_1)}{\varepsilon_V(\overline{l_2})} \gtreqless \frac{\varepsilon_{R_1}(z_1)}{\varepsilon_{R_2}(z_1)}. \tag{32}$$

Regarding the tradeoff between l and z we thus obtain an ambiguous result. Whether the (z, l) is distorted upward or downward (i.e., whether θ_1 is negative or positive) depends upon the relative differences in characteristics. If the ratio between week labor elasticities of individual 1 and the mimicker and is larger than the one between retirement elasticities, the (z, l) choice is upward distorted ($\theta_1 < 0$). Otherwise, it is downward distorted ($\theta_1 > 0$). In order to better understand the role of the relative differences in the two characteristics, two extreme cases are now considered.

4.2.1 Subcase (a): $R_1(z) = R_2(z)$

In this case, individuals differ only in productivity. Consequently, we have $\varepsilon_{R_1}(z_1) = \varepsilon_{R_2}(z_1)$ and (32) simplifies to:

$$MRS_{lz}^1 = \frac{R_1'(z_1)/R_1(z_1)}{V'(l_1)/V(l_1)} \gtreqless \frac{l_1}{z_1} \Leftrightarrow \varepsilon_V(\overline{l_2}) \lesseqgtr \varepsilon_V(l_1). \tag{33}$$

First notice that when V is isoelastic, the (z_1, l_1) choice is not distorted. As discussed earlier, if V is isoelastic, z is fixed and equal for both individuals. The weekly income is then distorted downwards while the age of retirement is the same as in the first-best.

In the general case, when ε_V is increasing (Assumption 1), we have $\varepsilon_V(\overline{l_2}) < \varepsilon_V(l_1)$ yielding $MRS_{lz}^1 > l_1/z_1$, the marginal rate of substitution between l and z is greater in absolute value than the slope of the gross income curve. From (21) this is equivalent to $\theta_1 < 0$. Consequently, for a given I, individual 1 has, at the second best a greater z and a lower l relative to the first best choice.

This result is an interesting extension of the optimal income taxation literature. The intuition can be understood most easily by considering the slope of individual indifference curves in the (z, y) space (i.e., the space of observable variables). When the elasticity of V is increasing (in l), individual 2 prefers to have a relatively greater weekly income than individual 1 for a given gross life cycle income. Consequently, the slope of the iso effort curve (indifference curve) is lower for individual 2 in the (z, y) space; see Appendix B. To make type 1's consumption bundle less attractive to type 2 (and relax an otherwise binding incentive constraint), the optimal policy then implies a relatively higher retirement age and a lower weekly labor supply for individual 1.[12]

4.2.2 Subcase (b): $w_1 = w_2$

Individuals now differ solely in their health status and (32) reduces to

$$MRS_{lz}^1 = \frac{R_1'(z_1)/R_1(z_1)}{V'(l_1)/V(l_1)} \gtreqless \frac{l_1}{z_1} \Leftrightarrow \varepsilon_{R_2}(z_1) \gtreqless \varepsilon_{R_1}(z_1). \tag{34}$$

When R_1 and R_2 have the same elasticity (for any given level of z), there is no distortion for the (l_1, z_1) choice. A simple example of this is when $R_1(z) = \delta R_2(z)$ with $\delta > 1$. As shown previously, the (l, z) choices are the same for the 2 individuals for a given gross life cycle income.

In the general case when $\varepsilon_{R_2}(z_1) < \varepsilon_{R_1}(z_1)$ (Assumption 2) we obtain $MRS_{lz}^1 < l_1/z_1$, so that the slope of the effort frontier is lower than the slope of the gross income line. Using (21) we thus obtain $\theta_1 > 0$. Consequently, for I given, at the second best, the individual will choose a greater l and a lower z relative to the first best choice. In this special case, early retirement is encouraged.

This result is in contrast with that obtained in the previous subsection. This can be explained as follows. Individual 2 now prefers to have a (relatively) higher retirement age than individual 1 for a given gross life cycle income (the slope of the iso-effort curve is greater for individual 2 in the (z, y) space; see Appendix B). Consequently, the incentive constraint can be relaxed by setting a relatively lower retirement age and a greater weekly labor supply for individual 1.

[12] The argument presented above for the (z, c) space can easily be adapted to apply here.

5 Numerical examples

We now present some numerical examples which illustrate our results. In particular, they highlight the role of the relative differences in characteristics. In addition, we provide some results for the three types case, a setting not considered in the analytical part. We use a quasi-linear utility function which implies that there are no income effects on labor supply. This leads to crisper results and facilitates their interpretation. However, it also imposes the restriction that everyone has the same marginal utility of income. To introduce concern for redistribution we thus consider a social welfare function of the type $\sum_{ij} f_{ij}\Phi\left[U_{ij}\right]$, where Φ is a strictly concave function reflecting social preference for equity. The following specific functions are used:

$$\Phi(x) = x^{1-\rho}/(1-\rho), \quad u(c) = c,$$
$$V(l) = 1/(1-l)^{\beta}, \quad R_j(z) = 1/(1-z)^{\alpha_j}$$

with fixed parameters $\beta = 2$, $\rho = 2$. The distribution of characteristics is uniform.

For each simulation, we report the optimal allocations (c, l and z), marginal tax rates on z and l and the relative marginal tax rates θ's. Tables 1 and 2 present the results for case (a) where individuals only differ in ability w_i, and case (b) where individuals differ in their health status $R_j(z)$. Then, Table 3 presents an example with two types who differ both in ability and in health. Finally, we consider a setting with three individuals as represented by ABC on Figure 4.

Table 1 presents the case where the two individuals are equally healthy but have different productivities, the ratio w_2/w_1increasing from 1.25 to 2. In this special case, the informational problem rests on the components of y_1 namely w_1 and l_1; the relative distortion goes against l. One observes that l decreases more than z relative to their first-best values. Naturally, there is no distortion for type 2's individuals.

TABLE 1: *Case* (a): $\alpha_1 = \alpha_2 = 2$								
		c	l	z	T'_z	T'_y	θ
(a_1): $w_1 = 200, w_2 = 250$							
Individual 1	First Best	68	0.55	0.55			
	Second Best	63	0.538	0.547	6.16	0.05	-0.038
Individual 2	First Best	74	0.57	0.57			
	Second Best	78	0.57	0.57			
(a_2): $w_1 = 200, w_2 = 300$							
Individual 1	First Best	77	0.55	0.55			
	Second Best	69	0.533	0.545	9.19	0.07	-0.049
Individual 2	First Best	88	0.59	0.59			
	Second Best	93	0.59	0.59			
(a_3): $w_1 = 200, w_2 = 400$							
Individual 1	First Best	95	0.55	0.55			
	Second Best	86	0.532	0.543	10.39	0.07	-0.045
Individual 2	First Best	118	0.61	0.61			
	Second Best	124	0.61	0.61			

Table 2 presents the case where the two types are equally productive but have different health conditions. The ratio α_1/α_2 goes from 1.18 to 2. Now l_i can be directly inferred from y_i and is thus effectively observable. The distortion is on the age of retirement of type 1. We can see that the distortion goes against z which falls more than l.

Table 3 considers two individuals corresponding to A and C on Figure 4. Whether or not z is more distorted than l depends on the relative ratios w_2/w_1 and R_1/R_2. Not surprisingly for w_2/w_1 given (and equal to 1.2), as R_1/R_2 or rather α_1/α_2 increases, the relative downward distortion first affects $l_1$and then $z_1$and we go from $\theta > 0$ to $\theta < 0$. For a specific intermediate level of the health ratio, namely $\alpha_1/\alpha_2 = 1.43$ there is no distortion in the (z, l) tradeoff and we have $\theta = 0$.

Finally Table 4 is devoted to the 3 individuals case. Returning to Figure 4, these three individuals are represented by A, B, C. With that configuration one may expect the self-selection constraint to go downwards from A to B (healthy and more productive mimicking healthy and less productive) and

TABLE 2: *Case* (b) : $w_1 = w_2 = 200$								
		c	l	z	T'_z	T'_y	θ
(b_1): $\alpha_1 = 2, \alpha_2 = 1.7$							
Individual 1	First Best	64	0.55	0.55			
	Second Best	60	0.549	0.546	4.43	0.01	0.014
Individual 2	First Best	65	0.56	0.6			
	Second Best	68	0.56	0.6			
(b_2): $\alpha_1 = 2,\ \alpha_2 = 1.4$							
Individual 1	First Best	68	0.55	0.55			
	Second Best	62	0.546	0.538	10.62	0.03	0.032
Individual 2	First Best	71	0.58	0.66			
	Second Best	75	0.58	0.66			
(b_3): $\alpha_1 = 2,\ \alpha_2 = 1$							
Individual 1	First Best	75	0.55	0.55			
	Second Best	66	0.542	0.53	15.91	0.05	0.044
Individual 2	First Best	79	0.61	0.76			
	Second Best	85	0.61	0.76			

TABLE 3: Strict negative correlation, $w_1 = 200,\ w_2 = 240$							
		c	l	z	T'_z	T'_y	θ
$\alpha_1 = 2,\ \alpha_2 = 1.7$							
Individual 1	First Best	71	0.55	0.55			
	Second Best	63	0.537	0.541	10.71	0.06	-0.015
Individual 2	First Best	77	0.58	0.62			
	Second Best	82	0.58	0.62			
$\alpha_1 = 2,\ \alpha_2 = 1.4$							
Individual 1	First Best	76	0.55	0.55			
	Second Best	67	0.535	0.535	14.58	0.07	0
Individual 2	First Best	84	0.60	0.68			
	Second Best	90	0.60	0.68			
$\alpha_1 = 2,\ \alpha_2 = 1$							
Individual 1	First Best	85	0.55	0.55			
	Second Best	74	0.534	0.531	17.24	0.08	0.012
Individual 2	First Best	94	0.63	0.77			
	Second Best	101	0.63	0.77			

TABLE 4: 3 individuals with: $w_1 = w_2 = 200, w_3 = 400$.							
		c	l	z	T'_z	T'_y	θ
$\alpha_1 = 2, \alpha_2 = \alpha_3 = 1.5$.							
Individual 1	First Best	145	0.764	0.618			
	Second Best	131	0.760	0.605	15.64	0.045	0.03
Individual 2	First Best	146	0.787	0.711			
	Second Best	140	0.769	0.702	19.29	0.123	-0.06
Individual 3	First best	168	0.83	0.76			
	Second Best	182	0.83	0.76			
$\alpha_1 = 2, \alpha_2 = \alpha_3 = 1.8$.							
Individual 1	First Best	135.2	0.764	0.618			
	Second Best	125	0.763	0.613	6.04	0.016	0.013
Individual 2	First Best	135.7	0.772	0.653			
	Second Best	128	0.755	0.645	17.31	0.106	-0.059
Individual 3	First best	158	0.81	0.71			
	Second Best	170	0.81	0.71			
$\alpha_1 = 2, \alpha_2 = \alpha_3 = 1.9$.(IC 31 binding)							
Individual 1	First Best	132	0.764	0.618			
	Second Best	124.2	0.762	0.615	4	0.015	0
Individual 2	First Best	132.3	0.768	0.635			
	Second Best	124.8	0.752	0.627	15.4	0.09	-0.04
Individual 3	First best	154	0.81	0.69			
	Second Best	166	0.81	0.69			

from B to C (healthy and less productive mimicking unhealthy and less productive). However, our results indicate that the pattern of binding incentive constraints may be more complex than this conjecture would suggest.

We focus on the health ratio α_1/α_2, with $\alpha_1 = \alpha_h$ and $\alpha_2 = \alpha_3 = \alpha_l$. Three examples are studied with the ratio α_h/α_l decreasing from 4/3 to 1. When the ratio is sufficiently large, the self-selection constraints go along the sequence ABC. Type 3 is subject to no distortion. Type 2 — less productive than 1 but equally healthy — is subject to the same distortion as in subcase (a): downward distortion on l relatively stronger than that on z. Type 3 is subject to the same distortion as in subcase (b): downward distortion on z relatively stronger than that on l. The same pattern of results holds when the health ratio starts to decrease from 12/9 to 10/9. However, when it becomes sufficiently small, the incentive compatibility constraint between type 3 and type 1 becomes binding. In other words, both individuals 2 and 3 now have to be prevented from mimicking type 1 who benefits from an attractive early age of retirement. As a consequence, the marginal tax on type 1 individuals has to compromise between two binding incentive constraints. To be more precise, the incentive constraint $3 \rightarrow 1$ pushes θ_1 to be negative. This is because the disparity between w_3 and w_1 dominates the disparity between α_3 and α_1. The incentive constraint $2 \rightarrow 1$, on the other hand, pushes θ_1 to be positive. As a consequence, the net distortion is ambiguous. For the parameter values we reported it happens be just equal to zero. Finally, for individual 2, there is a relative subsidy on z which decreases as the health ratio decreases.

6 Conclusion

During the last decades, a number of European countries, some more than others, have expanded their social security systems in ways which have discouraged labor market participation in old age and thus fostered early retirement. The question we raised at the outset is whether these disincentives to postponed activity are the result of a bad tax-transfer scheme design or the consequence of an explicit desire to achieve some redistribution in a world of asymmetric information.

To address this issue, we have studied the design of retirement contribution and benefits in a setting where an optimal non-linear income tax is also available. Individuals differ in ability and/or health status (disutility of retirement age). Given the tax-transfer policy every individual chooses weekly labor supply (not observable) and retirement age (observable). As in the traditional income taxation literature, the optimal policy implies a positive marginal tax on the low ability (and/or less healthy) individual. More interestingly, the retirement benefit formula also introduces a bias towards early retirement in this individuals life cycle labor supply decisions.

Finally, the relative distortion between weekly labor supply and retirement age (length of active life), if any, depends on the relative heterogeneity in ability and health. When heterogeneity in ability dominates, the distortion is in favor of retirement age. Put differently, while both weekly and labor supply and retirement age are distorted downwards, the tradeoff between these two variable *for a given lifetime income* is biased towards later retirement.

To sum up, we have shown in this paper that a redistributive social security scheme implies distortions which induce early retirement relative to what would be the first-best solution. Of course, this does of course not imply that the currently observed biased retirement benefit formulas are otherwise optimally designed. However, it does imply that a bias towards early retirement is not in itself sufficient to conclude that a retirement system is poorly designed. Furthermore, it follows from our results that a move towards a neutral system as is often advocated does not necessarily represent a step in the right direction.

On our research agenda there are two issues we would like to pursue as an extension of this paper. The first one would be to combine policy schemes based on self-selection with policy schemes based on explicit control of health conditions. If controls are not too expensive and sufficiently accurate they could contribute in getting a social security scheme that would be less discouraging for prolonged activity.

Another issue concerns the expected increase in longevity and changes in health conditions. When all individual characteristics regarding to life expectancy and resistance to longer careers move up in an homogeneous way, the age of retirement for each type of individuals will also increase homogeneously. However, if for some types of work and/or some types of health problems there is little improvement, then increased longevity would imply a wider range of retirement ages. This would make the use of control even more desirable.

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Appendix

A Derivation of expression (32)

Rearranging (31) yields

$$MRS^1_{lz} = \left[\frac{1 - \frac{\lambda}{n_1}\frac{w_1}{w_2}\frac{\overline{MRS}^2_{cl}}{MRS^1_{cl}}}{1 - \frac{\lambda}{n_1}\frac{\overline{MRS}^2_{cz}}{MRS^1_{cz}}} \right] \frac{l_1}{z_1}$$

Consequently,

$$MRS^1_{lz} \gtreqless \frac{l_1}{z_1} \Leftrightarrow \frac{\overline{MRS}^2_{cz}}{MRS^1_{cz}} \gtreqless \frac{w_1}{w_2}\frac{\overline{MRS}^2_{cl}}{MRS^1_{cl}}$$

Using the definition of MRS^1_{cz} and MRS^1_{cl}, the property $w_1 = w_2\bar{l}_2/l_1$and rearranging yields

$$MRS^1_{lz} \gtreqless \frac{l_1}{z_1} \Leftrightarrow \frac{\frac{\bar{l}_2 V'(l_1)}{V(l_1)}}{\frac{l_1 V'(\bar{l}_2)}{V(l_2)}} \gtreqless \frac{\frac{R'_1(z_1)}{R_1(z_1)}}{\frac{R'_2(z_1)}{R_2(z_1)}}.$$

Multiplying the numerator and the denominator of the right hand side by z_1 then yields (32).

B Marginal rates of substitution in the (z, y) space

We compare the types marginal rates of substitution between y at z at any given point (z, y). By definition, one has:

$$MRS^{ij}_{yz}(y,z) = \frac{V(\frac{y}{w_i})R'_j(z)}{\frac{1}{w_i}V'\left(\frac{y}{w_i}\right)R_j(z)}$$

Multiplying and dividing by yz yields:

$$MRS^{ij}_{yz}(y,z) = \frac{y}{z}\frac{V(\frac{y}{w_i})}{\frac{y}{w_i}V'\left(\frac{y}{w_i}\right)}\frac{zR'_j(z)}{R_j(z)} = \frac{y}{z}\frac{\varepsilon_{R_j}(z)}{\varepsilon_V\left(\frac{y}{w_i}\right)}$$

Assumption 1 implies $\varepsilon_V\left(\frac{y}{w_h}\right) \leq \varepsilon_V\left(\frac{y}{w_l}\right)$, so that

$$MRS^{hj}_{yz}(y,z) \geqslant MRS^{lj}_{yz}(y,z) \text{ for every } j = h, l.$$

Consequently, in subcase (a) we have

$$MRS^2_{yz}(y,z) \geqslant MRS^1_{yz}(y,z)$$

Similarly, Assumption 2, implies $\varepsilon_{R_l}(z) \leq \varepsilon_{R_h}(z)$, so that

$$MRS^{ih}_{yz}(y,z) \geqslant MRS^{il}_{yz}(y,z) \text{ for every } i = h, l.$$

In subcase (b) we thus have

$$MRS^2_{yz}(y,z) \leq MRS^1_{yz}(y,z).$$

In the more general case of negative correlation between the two types, one has

$$MRS^{hl}_{yz}(y,z) \gtreqqless MRS^{lh}_{yz}(y,z) \text{ if and only if } \frac{\varepsilon_{R_l}(z)}{\varepsilon_V\left(\frac{y}{w_h}\right)} \gtreqqless \frac{\varepsilon_{R_h}(z)}{\varepsilon_V\left(\frac{y}{w_l}\right)}.$$