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CESifo Working Papers

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CESifo Working Paper No. 536

August 2001

An electronic version of the paper may be downloaded
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THE ROLE OF PRICES ON EXCLUDABLE PUBLIC GOODS

Abstract

When a poublic good ist excludable it is possible to charge individuals for using the good. We study the role of prices on excludable public goods using an extension of the Stiglitz-Stern version of the Mirrlees optimal income tax model. Our discussion includes both the case where the public good is a final consumer good and the case where it is an intermediate good.

We demonstrate that for a public consumer good charging a positive price may be desirable, but only under certain conditions. However, charging a lower than optimal price may be less efficient than setting a zero price. Conditions are identified under which consumers should be rationed in their demand rather than adjusting demand to price. We also conclude that producers using an intermediate public good as input should *not* be charged a positive price.

KEYWORDS: excludable public goods, public sector pricing, information constrained Pareto efficiency.

JEL Classification: H41, H21, H24.

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1. Introduction

In many countries there is growing concern about the size of public budgets and a tendency to look for alternative sources of funding. Since excludable public goods are by definition such that one can charge a price or fee for their use, it is part of this process that even the well established case for free provision of public goods is being challenged in the case of excludable public goods. In this paper we shall address the provision of such goods.

There are many important examples of excludable public goods. Information is in general a public good and often it can be made excludable. Patent rights is one way to exclude. For information that is only of value in the fairly short term, like meteorological and hydrological forecasts, it would probably be easy to exclude people and only let those who subscribe to the service have access to the information. The services provided by the Central Bureau of Statistics is another example of information that can easily be made excludable. Radio and television broadcasts, many services provided on the internet and non-congested roads, public beaches, national parks and museums are some other examples of excludable public goods.

There are several questions that are of interest to study. First, if there is public provision, should it then be financed by an income tax or should it be partly, or entirely, financed by a fee (a price).[2] If the price instrument is indeed used, will this lead to more or less provision of the public good as compared to the situation without exclusion. Second, since it is possible to have market provision of excludable public goods by private firms, there is the important issue of whether public or private provision is preferable. Third, some excludable public goods can be regarded as final consumer goods. However, goods, like statistics or other information, have more the character of an intermediate good. It is conceivable that the rules for provision would depend on whether the excludable public good is a final consumer good or an intermediate good. We will not attempt to answer all these questions. We will focus on public provision. However, we will consider both the case where the public good is a final consumer good and the case where it is an intermediate public good.

Empirically there are examples both of publicly provided excludable public goods being financed from general tax revenue and goods partly financed by prices. Weather forecasts are in many countries publicly provided and financed out of general tax revenue. Many forms of statistics are publicly provided and financed out of general tax revenue. However, there are examples, like Sweden, where recently substantial charges have been introduced for users to get access to Central Bureau of Statistics data.

There is a vast literature on public and/or private provision of nonexcludable public goods. However, public provision of excludable public goods has not been much studied. Of course, the first best rule does not depend on whether the public good is excludable or not. In a second best setting there might be a difference since there is one more instrument, a price on the public good, available when the public good is excludable. Fraser (1996) studied provision of excludable public goods. However, in his model the incomes are exogenous and there are no distortions from the income tax. There are several papers that have studied market provision of excludable public goods by private firms. Oakland (1974) studied a model where private firms operating under conditions of perfect competition provide an excludable public good. Brito and Oakland (1980) and Burns and Walsh (1981) consider a situation where a natural monopoly provide an excludable public good.

We believe it is important to acknowledge that taxes are not only used in order to finance public goods but also to achieve redistribution. We therefore use an extension of the Stiglitz-Stern version of the Mirrlees optimal income tax model.[3] We will assume there are two types of persons, high-skilled and low-skilled, and that it is desireable to redistribute from the high-skilled to the low-skilled. In sections 2 and 3 we consider the case where the public good is a final consumer good. In section 2 we model individual behavior. In section 3 we formulate the optimal taxation-public provision problem. The focus of that section is on the conditions under which non-zero prices would be part of the optimal provision scheme. In section 4 we consider the case where the public good is an intermediate good. Section 5 concludes.

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[2] One benefit of using prices to finance an excludable public good is that data will be generated that might be useful to derive the users' evaluation of the public good. In the analysis below we will not take this aspect into account. Hence, we will assume that individuals' preferences are known.

[3] See Mirrlees (1971), Stern (1982) and Stiglitz (1982).

2. Consumer behavior

There is one private good, one excludable public good and leisure.[4] When the public good is excludable we can charge individuals for using the public good. Depending on the technology for excluding individuals the payment scheme might take different forms. Here we assume individuals can be charged a nonnegative price q per unit they consume. Let g^i, $i = 1,2$ denote the quantity of the public good consumed by a person of type i and let c^i be the consumption of the private good. We normalize the price of the private good to one. Let g denote the quantity of the publicly provided public good. L^i, H^i, w^i, Y^i and B^i denote leisure, hours of work, the wage rate, the before-tax and after-tax income of an individual of type i. $Z = L^i + H^i$ denotes the fixed time endowment. Since $Y^i = w^i H^i$, we can express H^i by Y^i / w^i.

The utility maximisation problem for an individual will take the form: Max $U(g^i, c^i, Y^i / w^i)$ s.t. $c^i + qg^i = B^i$ and $g_i \leq g$. We solve this problem step-wise. First, we condition on Y^i / w^i and neglect the condition $g_i \leq g$. Suppressing w^i and instead indexing the demand functions we derive the notional conditional demand functions $\tilde{c}^i(q, B^i, Y^i)$ and $\tilde{g}^i(q, B^i, Y^i)$. The effective demand is given by:

$g^i(q, B^i, Y^i, g) = g$ if $\tilde{g}^i(q, B^i, Y^i) \geq g$

$g^i(q, B^i, Y^i, g) = \tilde{g}^i(q, B^i, Y^i)$ if $\tilde{g}^i(q, B^i, Y^i) \leq g$

We note that $g_g^i = 1$ if $\tilde{g}^i > g$ and that $g_g^i = 0$ if $\tilde{g}^i \leq g$. Also, if $\tilde{g}^i > g$, then $g_q^i = g_B^i = g_Y^i = g_q^{hi} = 0$, where g^{hi} denotes Hicksian demand. This implies that the Slutsky decomposition is still valid and can be used without any reservation about the

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case of rationing. We also note that Roy's identity holds even if a person is rationed.[5] We also assume that $\tilde{g}^i(0, B^i, Y^i)$ is finite.

Sticking $g^i(q, B^i, Y^i, g)$ back into the direct utility function we obtain the conditional indirect utility function $V(B^i, Y^i / w^i, q, g)$. If $\tilde{g}^i \leq g$, $V_g^i = 0$. We may note that $V_B = U_c = U_B$. It is also helpful to note that we can write $V(B^i, Y^i / w^i, q, g) = U(B^i - qg^i, Y^i / w^i, g^i)$. If the individual is rationed ($g^i = g$), $V(B^i, Y^i / w^i, q, g) = U(B^i - qg, Y^i / w^i, g)$. Then $V_g = U_g - U_c q$ and $\dfrac{V_g}{V_B} = \dfrac{U_g}{U_c} - q = MRS_{gB} - q > 0$ where

$MRS_{gB} = \dfrac{U_g}{U_c} = \dfrac{U_g}{U_B}$ is the marginal valuation of g in terms of private income. In general V^g / V^B equals the individual's valuation of g in terms of B according to the direct utility function minus the cost of acquiring g. If the individual is unconstrained, the valuation according to the market and the direct utility function will of course be the same.

In the subsequent analysis it will be important how leisure or labor supply will affect the marginal valuation of g^i and the demand $\tilde{g}^i$. Let us consider an increase in leisure L^i corresponding to a decrease in labor supply Y^i / w^i. We can have the various

combinations: a) $\dfrac{\partial}{\partial L^i} MRS_{gB}^i = \dfrac{\partial}{\partial L^i} \dfrac{U_g^i}{U_c^i} > 0 \Leftrightarrow \tilde{g}_L^i > 0$, b) $\dfrac{\partial}{\partial L^i} MRS_{gB}^i = \dfrac{\partial}{\partial L^i} \dfrac{U_g^i}{U_c^i} = 0 \Leftrightarrow$

$\tilde{g}_L^i = 0$, c) $\dfrac{\partial}{\partial L^i} MRS_{gB}^i = \dfrac{\partial}{\partial L^i} \dfrac{U_g^i}{U_c^i} < 0 \Leftrightarrow \tilde{g}_L^i < 0$. More leisure can have a positive effect on

the marginal valuation of g and induce a higher demand, or the effect can be zero or negative. For example more leisure can increase the marginal benefit of television programmes and can make watching television more attractive. The case with $\tilde{g}_L = 0$ occurs if the direct utility function is such that leisure is weakly separable from the goods.

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[5] If the person is rationed $g^i = g$ and $c^i = B^i - qg$. The indirect utility function is defined by $V(B, Y / w, q, g) \equiv U(B - qg, Y / w, g)$. Differentiating we obtain $\partial V / \partial q = -g(\partial U / \partial c)$ and

3. The optimal taxation-public provision problem

Our concern will be with (information constrained) Pareto efficient policy. We assume a linear production technology and denote the producer price of the public good by p. Let N^1 and N^2 denote the number of type one and type two individuals, respectively. We assume $w^2 > w^1$, and that the social planner wants to redistribute from the high-skilled to the low-skilled type. There is no reason to assume there are systematic differences in preferences between high skill and low skill persons. Differences in demand for the public good depend only on differences in leisure (work effort) and income. The optimization problem, which defines Pareto efficient taxation and provision of the public good, takes the form:

$$\underset{B^1,Y^1,B^2,Y^2,q,g}{Max} \quad V^1(B^1,Y^1,q,g) \tag{1}$$

s.t.

$$V^2(B^2,Y^2,q,g) \geq \bar{V}^2 \tag{2}$$

$$V^2(B^2,Y^2,q,g) \geq V^m(B^1,Y^1,q,g) \tag{3}$$

$$N^2(Y^2 - B^2) + N^1(Y^1 - B^1) + q(N^1g^1 + N^2g^2) - pg \geq 0 \tag{4}$$

The constraint (2) assigns a minimum utility level to individuals of type two. The constraint (3) is a self-selection constraint imposing that the vector of taxes, prices and level of g is such that type two persons do not gain by mimicking type one persons.[6] We use an index m to denote the "mimicker", i.e. $V^m()$ is the utility of type two persons evaluated at the income point intended for type one persons. In the analysis below we will also assume that q must be nonnegative. The government budget constraint is expressed by (4). It requires that revenue from income taxes and from charging a price on the public good is sufficient to finance the public good provision.

$\partial V / \partial B = \partial U / \partial c)$. It follows that $-(\partial V / \partial q)/(\partial V / \partial B) = g$.

[6] We could also have included a self-selection constraint that an individual of type one should not mimic a type two person. However, one can show that at most one of the self-selection constraints is binding. We

Before deriving the first order conditions for this optimization problem we state the following proposition:

Proposition 1: An optimum is such that $g \leq \max\{\tilde{g}^1, \tilde{g}^2\}$.

Proof: We construct a proof by contradiction. Suppose there exists an optimum such that $g > \max\{\tilde{g}^1, \tilde{g}^2\}$. Decrease the level of g down to $\max\{\tilde{g}^1, \tilde{g}^2\}$. This will not harm any of persons one or two. It will not improve the situation of the mimicker, so the self-selection constraint will not be affected. Take the resources being released and give to the actual person 2. In this way we have achieved a Pareto improvement. Hence, the initial situation with $g \leq \max\{\tilde{g}^1, \tilde{g}^2\}$ can not have been an optimum.

The Lagrangean of the optimization problem above can be written as:

$$\Lambda = V^1(B^1, Y^1, q, g) + \beta\left[V^2(B^2, Y^2, q, g) - \overline{V}^2\right] + \rho\left[V^2(B^2, Y^2, q, g) - V^m(B^1, Y^1, q, g)\right]$$

$$+ \mu\left[N^2(Y^2 - B^2) + N^1(Y^1 - B^1) - pg + q(N^1 g^1 + N^2 g^2)\right]$$

The Lagrangean should be differentiated w.r.t. B^1, Y^1, B^2, Y^2, g and q. We note that the problem formally has strong similarities with both the optimal commodity tax problem and the publicly provided private goods problem, but it is clearly a separate problem. The F.O.Cs. are the following:

$$\frac{\partial \Lambda}{\partial B^1} = V_B^1 - \rho V_B^m - \mu N^1 + \mu q N^1 g_B^1 = 0 \tag{5}$$

$$\frac{\partial \Lambda}{\partial Y^1} = V_y^1 - \rho V_Y^m + \mu N^1 + \mu q N^1 g_y^1 = 0 \tag{6}$$

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make the usual assumption that the binding self-selection constraint is that the high-skill person should not mimic the low-skill person.

$$\frac{\partial \Lambda}{\partial B^2} = \beta V_B^2 + \rho V_B^2 - \mu N^2 + \mu q N^2 g_B^2 = 0 \tag{7}$$

$$\frac{\partial \Lambda}{\partial Y^2} = \beta V_y^2 + \rho V_y^2 + \mu N^2 + \mu q N^2 g_y^2 = 0 \tag{8}$$

$$\frac{\partial \Lambda}{\partial g} = V_g^1 + \beta V_g^2 + \rho V_g^2 - \rho V_g^m - \mu p + \mu q (N^1 g_g^1 + N^2 g_g^2) = 0 \tag{9}$$

$$\frac{\partial \Lambda}{\partial q} = V_q^1 + \beta V_q^2 + \rho V_q^2 - \rho V_q^m + \mu(N^1 g^1 + N^2 g^2) + \mu q (N^1 g_q^1 + N^2 g_q^2) \tag{10}$$

The requirement that q is nonnegative implies that there might not exist a meaningful solution to $\partial \Lambda / \partial q = 0$. However, considering a case where the optimal value of q is determined by an interior solution to $\partial \Lambda / \partial q = 0$ we get:

$$\frac{\partial \Lambda}{\partial q} = V_q^1 + \beta V_q^2 + \rho V_q^2 - \rho V_q^m + \mu(N^1 g^1 + N^2 g^2) + \mu q (N^1 g_q^1 + N^2 g_q^2) = 0 \tag{11}$$

We note that eqs. (5) – (8) and (11) formally look the same as in Edwards et al. (1995). However, note that there is only a *formal* similarity. In the present context individuals may be rationed and q must be nonnegative. This also means that we have to be careful when interpreting the first order conditions.

Making use of Roy's identity [7] and the Slutsky equation; we can rewrite (10) as

$$\frac{\partial \Lambda}{\partial q} = -V_B^1 g^1 - \beta V_B^2 g^2 - \rho V_B^2 g^2 + \rho V_B^m g^m + \rho V_B^m g^1 - \rho V_B^m g^1 + \mu N^1 g^1$$
$$+ \mu N^2 g^2 + \mu q N^1 g_q^{h1} + \mu q N^2 g_q^{h2} + \mu q N^1 (-g^1) g_B^1 + \mu q N^2 (-g^2) g_B^2 \tag{12}$$

We then make use of (5) and (7) to eliminate terms:

[7] From Roy's identity we know that $V_q = -V_B g^i$. Finally, we also use the Slutsky decomposition $g_q^i = g_q^{hi} - g^i g_B^i$, where we use the superscript h to denote the Hicksian demand.

$$\frac{\partial \Lambda}{\partial q} = \rho V_B^m (g^m - g^1) + \mu q (N^1 g_q^{h1} + N^2 g_q^{h2}) \tag{13}$$

If $\partial \Lambda / \partial q = 0$ we get

$$q\left[N^1 g_q^{h1} + N^2 g_q^{h2}\right] = \mu * (g^1 - g^m) \tag{14}$$

where $\quad \mu* = \dfrac{\rho V_B^m}{\mu} > 0$.

The expression in brackets on the left-hand side of (14) is the sum of substitution effects, which is non-positive.

Equation (14) expresses a trade-off between two effects. Consider the case in which the demand for g is decreasing in labour and accordingly increasing in leisure ($g_L > 0$). If type one is not rationed ($g^1 < g$), the mimicker, enjoying more leisure, will be the larger buyer of g ($g^1 < g^m$), and the price q can be used to relax the self-selection constraint. By increasing q and lowering the income tax to make person one equally well off, the mimicker, incurring a larger real income loss from the price increase, will be made worse off and the self-selection constraint (3) is being relaxed. This effect is reflected by the right hand side of (14). It has to be traded off against the effect on the left hand side which is the the distortionary effect of pricing the public good in a way which drives the consumption below the available amount.

We shall now identify and characterise the respective optima of the various regimes that can materialise. The impact of leisure on the demand for the public good is crucial in characterising different regimes. For the case of a zero or negative impact we can show the following result:

Proposition 2: Nothing can be gained by a positive price on the excludable public good if $\tilde{g}_L^i \leq 0$.

Proof : To prove this proposition we will consider each of the cases *i.* $\tilde{g}_L^i = 0$; and *ii.* $\tilde{g}_L^i < 0$.

i. $\tilde{g}_L^i = 0$. In this case $\tilde{g}^1 = \tilde{g}^m < \tilde{g}^2$. From proposition 1 we know that $g \leq \tilde{g}^2$. Suppose first that $\tilde{g}^1 < g \leq \tilde{g}^2$, i.e., that the type one individual is unconstrained. This implies that the Hicksian substitution effect is negative. It follows from eq. (14) that $q = 0$. Suppose next that $g < \tilde{g}^1 < \tilde{g}^2$. Since both types are constrained, eq. (14) is consistent with an infinity of possible values of q. Hence, q is indeterminate. However, *q=0* is one optimum, and nothing can be gained by having a positive q. To see this, consider an initial situation with *q>0*. Decreasing q to zero and simultaneously decreasing B^1 and B^2 by the amounts qg the situation for both actual persons and the mimicker are unchanged.

ii. $\tilde{g}_L^i < 0$. In this case $\tilde{g}^m < \tilde{g}^1$ and $\tilde{g}^m < \tilde{g}^2$. However, we do not know how $\tilde{g}^2$ relate to $\tilde{g}^1$. We consider the various possibilities. a) $\tilde{g}^m < \tilde{g}^2 < g \leq \tilde{g}^1$. Eq. (14) then implies a negative q. However, since this is not feasible q has to be truncated to zero. b) $\tilde{g}^m < \tilde{g}^1 < g \leq \tilde{g}^2$. Eq. (14) implies a negative q, which must be truncated to 0. c) $\tilde{g}^m < g < \tilde{g}^2 < \tilde{g}^1$. Both actual persons are constrained, so eq. (14) is indeterminate. However, suppose the optimum is such that *q>0*. Decrease q to zero and decrease B^1 and B^2 with the amount qg. The situation for the actual persons will be unchanged. However, the mimicker will be hurt, so the self-selection constraint will slacken and there is room for a Pareto improvement. d) $g < \tilde{g}^m < \tilde{g}^2 < \tilde{g}^1$. In this case both actual persons and the mimicker are constrained. q is indeterminate by reasoning similar to that under i. We deduce that nothing can be gained by having a positive price.

It follows from Proposition 2 that only if $\tilde{g}_L^i > 0$ can the economic situation be improved by using the price instrument. However, as will be shown below this is a necessary, not a sufficient condition.

Proposition 3 :
Assume $\tilde{g}_L^i > 0$. Then there is no gain (or loss) from introducing a positive price as long as both types of persons remain rationed.

Proof : For q such that $g^i \leq g$ is strictly binding for both types of persons, the mimicker is also rationed since $\tilde{g}_L^i > 0$, the compensated price derivatives are zero, and it follows immediately from (13) that $\partial \Lambda / \partial q = 0$.

Proposition 4 :
Assume $\tilde{g}_L^i > 0$ and that both persons are constrained up to price $\bar{q}$. Then, marginally increasing the price so that the quantity constraint of one type ($g^i \leq g$) is just being relaxed, will make the allocation strictly less efficient.

Proof :
Suppose it is the quantity constraint for type i that is being relaxed first as the price increases. Then, from eq. (13) we see that the change in welfare is given by
$\partial \Lambda / \partial q = \mu q N^i g_q^{hi} < 0$.

It is of interest to compare this result with a corresponding result from the optimal commodity tax literature. Had the good in question been an ordinary private good x with $\partial x / \partial L > 0$ it would have been optimal to introduce a small tax on the good. This is because in the optimal commodity tax case, at a zero tax the size of the deadweight loss from a marginal tax would be of second order, whereas the benefits from the slackening of the self-selection constraint would be a first order effect. For a publicly provided excludable public good it is the other way around.

Since the marginal cost of providing one more unit of g to a person is zero as long as the person is not constrained, the price in our model resembles a tax. However, in our model, if $\tilde{g}_L > 0$, there would be a range of the price where there is neither a gain nor a loss of having a positive price. This holds as long as both persons are constrained. Then a further increase in q that results in the quantity constraint being relaxed for one type of person leads to a *decrease* in welfare due to the deadweight-losses created by the price. This is quite different from the optimal commodity tax result and is due to the fact that we, as q is marginally increased above $\bar{q}$, come from a regime where the individuals are constrained. Looking at eq. (13) we see that at the price $\bar{q}$, where one type of person just

ceases to be constrained, $g^1 - g^m = 0$ and, initially, there is no gain in terms of slackening the self-selection constraint. However, as we increase the price above $\overline{q}$ the compensated price effects take on negative finite values, implying finite deadweight-losses from the price increase and a total decrease in welfare. As the price is increased further there will eventually be a difference between g^1 and g^m such that $g^m > g^1$ and an increase in q hurts the mimicker more then person one. It is then possible that the gains from slackening the self-selection constraint outweighs the deadweight losses.

Proposition 5 : If $\tilde{g}^i_L > 0$ the optimum can be of one of the following types : i. Persons of type one are not rationed in their demand for the excludable public good and a strictly positive price on the good is desirable. Persons of type two may or may not be rationed. ii. Both types of persons are rationed in their demand for the excludable public good and q assumes any value in the interval $\{0, \overline{q}\}$ where $\overline{q}$ is the price at which the rationing constraint ceases to bind.

Proof : It is not possible to have an optimum at which only type one is rationed because then, from (13), $\partial\Lambda/\partial q = \mu q N^2 g_q^{h2} < 0$. Either type one is not rationed or both types are rationed. If type one is not rationed it follows from (14) that q is positive since the right hand side is then negative and the compensated price derivative appearing on the left hand side is also negative. If both types are rationed, the first order condition that $\partial\Lambda/\partial q = 0$ holds for the given price interval according to proposition 3.

We note that under the conditions of the model the price is within certain limits arbitrary. However, there is also an administrative cost of collecting a price which in practice will rule out indifference between a zero and a non-zero price.

Combining proposition 3 and proposition 5 we note that a regime with a price that prevents rationing must be characterised by a price which is set discretely above the price at which the rationing constraint ceases to bind. Let us see how this may happen. Consider the case described in proposition 3. Departing from the price $\overline{q}$ and then increasing the price will at first reduce efficiency as stated by proposition 3. But as q is

increased and the *g*-consumption of person 1 declines a discrepancy between g^1 and g^m will occur. This discrepancy makes it possible to use the price *q* as a device for relaxing the self-selection constraint. On the other hand increasing *q* will discourage the consumption of *g* and distort the allocation. In order to obtain an efficiency gain from increasing *q* above $\bar{q}$, the marginal benefits from softening the self-selection constraint must not only outweigh the cost of further distortions beyond some point, but the benefits must also outweigh the latter by a margin which offsets the initial loss from increasing *q*. In this sense one may argue that there is a relatively weak case for regime i of proposition 5.

We may also note that the two effects discussed above are not entirely independent. One way that an increase in *q* may strongly discourage the consumption of *g* by person one and thus create a discrepancy between g^1 and g^m is through a strong substitution effect, but this will also add to the consumption distortion and thus create a countervailing deadweight loss.

It follows from our results that the welfare level can follow three different types of paths as *q* is being increased. These are depicted in the figures 1a-c below. We illustrate how the welfare of individual one changes with the level of *q*, keeping the utility of person 2 constant. As *q* is increased beyond a certain level efficiency declines and may never rise again (fig. 1a). For other properties of the economy efficiency may start picking up again at some level of *q*, but may never fully recover, and keeping *q* sufficiently low is optimal (fig. 1b). Only in the last case (fig 1c) does optimality require a strictly positive price.
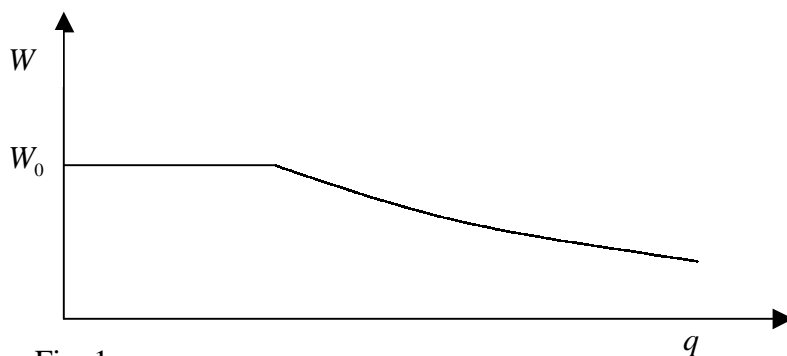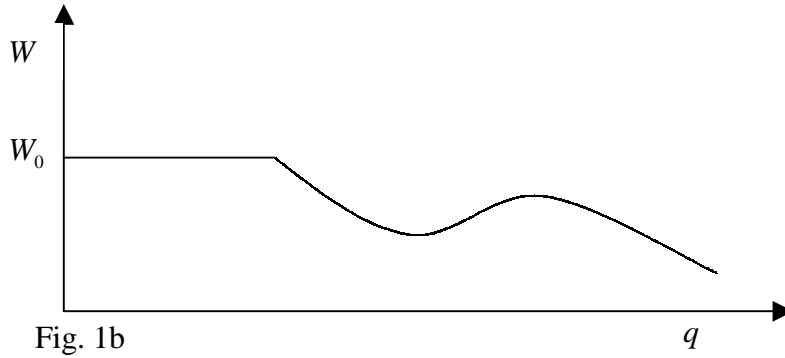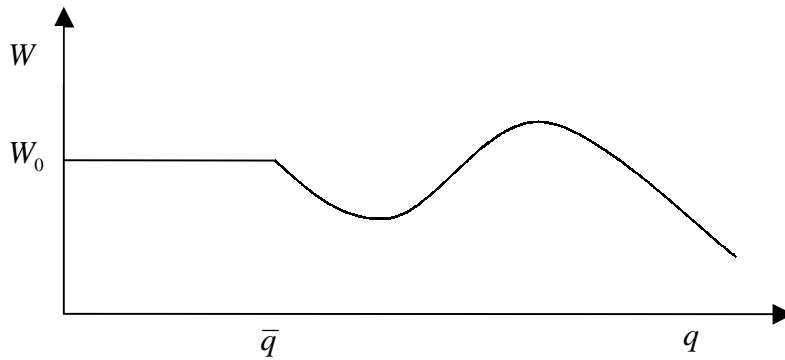

Fig. 1a

Fig. 1b

Fig. 1c

 In a mathematical appendix we have derived expressions that may help us clarify further the conditions that are conducive to the outcome illustrated in the figure 1c. Two aspects need to be considered. Firstly, it is important that the first interval in which the utility is decreasing produces a sufficiently strong decline in g^1 without lowering utility too much. It is crucial that g^1 declines to get to a position where the slackening of the self-selection constraint starts to dominate without sacrificing too much utility on the way. Secondly, it is essential to get an interval in which the utility is increasing at a sufficient rate. In order to get a clue about the relative changes in g^1 and V^1 we consider the case in which $g^m - g^1 = 0$, and obtain the following expression due to (A23) in the appendix

$$\frac{dg^1}{dV^1}\bigg/V_B^1 = -\frac{1}{\dfrac{V_Y^m}{V_B^m}-\dfrac{V_Y^1}{V_B^1}}\,g_Y^{h1}+g_B^1+g_q^{h1}N^1\,\frac{1+qg_Y^1+\dfrac{V_Y^m}{V_B^m}\left(1-qg_B^1\right)}{\left(\dfrac{V_Y^m}{V_B^m}-\dfrac{V_Y^1}{V_B^1}\right)q\left(N^1 g_q^{h1}+N^2 g_q^{h2}\right)} \tag{15}$$

We note that the change in g^1 is large relative to the change in utility if there is a strong income effect (g_B^1 is large) and the labour effect is strong (- g_Y^1 and - g_Y^{h1} are large). We also note that if the more skilful type is rationed ($g_q^{h2}=0$) the substitution term of the less skilful, g_q^{h1}, vanishes and is immaterial for the change in g^1 relative to the change in utility. However, if $g_q^{h2}<0$, the change in g^1 is larger relative to the change in utility the stronger the substitution effect (g_q^{h2}) of the more skilful and the *weaker* the substitution effect of the less skilful (g_q^{h1}). The denominator is positive as person one has a steeper indifference curve than the mimicker as implied by standard assumptions. An increase in the disposable income of person one when the gross income is increased, is hampered by the fact that the utility level must not be increased above the level actually enjoyed by person two (the self-selection constraint). This impediment is stronger the flatter is the relevant indifference curve of person two as a mimicker, i.e. the smaller is the absolute value of $\dfrac{V_Y^m}{V_B^m}$ compared to that of $\dfrac{V_Y^1}{V_B^1}$.

To examine the effect on the utility of type one as q is being increased we can make use of the following expression due to (A20).

$$\frac{\partial V^1}{\partial q}\bigg/\mu = \frac{\left[1+\dfrac{V_y^1}{V_B^1}\left(1-qg_B^1\right)+qg_y^1\right]}{\dfrac{V_y^m}{V_B^m}-\dfrac{V_y^1}{V_B^1}}\,N^1\left(g^m-g^1\right)+q\left(N^1 g_q^{h1}+N^2 g_q^{h2}\right) \tag{16}$$

Considering the expression above as consisting of two main terms, the former term becomes positive when q increases and g^1 falls below g^m. We note that like in (15) the denominator is positive as person one has a steeper indifference curve than the mimicker by standard assumptions. The latter main term is negative and reflects the distortion due to an increase in q. In order to achieve a welfare enhancement this effect should not be too strong while the former main term should be strongly positive. This effect is due to the slackening of the self-selection constraint. As we see the magnitude of the effect will depend on the fraction on the right hand side of (16). The bracketed numerator can be interpreted as the marginal public revenue from increasing the gross income of person one and adjusting the disposable income so that the individual stays equally well off. The term consists of a one unit increase in gross income, 1, minus the increase in the disposable income required to compensate for the corresponding increase in labour effort and allowing for the fact that increased expenditure will also raise revenue to the extent that it is spent on g, $\frac{V_y^1}{V_B^1}\left(1 - qg_B^1\right)$. There is also an additional effect to the extent that an increase in labour effort (Y) induces further consumption of g also raising revenue if a price is charged, qg_Y^1.

On the whole we note that strong income and labour effects and weak substitution effects on the demand for g^1 as well as comparatively flat indifference curves for person two as a mimicker are conducive to the outcome in Fig. 1c.

We next consider the rule for the optimal quantity of public good provison. To accomplish this we rewrite eq. (9) as:

$$V_g^1 + (\beta + \rho)V_g^2 - \rho V_g^m - \mu p + \mu q(N^1 g_g^1 + N^2 g_g^2) = 0 \tag{17}$$

Adding and subtracting $\rho V_B^m\left(\frac{V_g^1}{V_B^1}\right)$, we obtain:

$$(V_B^1 - \rho V_B^m)\frac{V_g^1}{V_B^1} + (\beta + \rho)V_B^2\frac{V_g^2}{V_B^2} + \rho V_B^m\left(\frac{V_g^1}{V_B^1} - \frac{V_g^m}{V_B^m}\right) = \mu p - \mu q(N^1 g_g^1 + N^2 g_g^2) \tag{18}$$

Using eqs. (6) and (8) we rewrite this as:

$$\mu N^1 \frac{V_g^1}{V_B^1} + \mu N^2 \frac{V_g^2}{V_B^2} + \rho V_B^m \left(\frac{V_g^1}{V_B^1} - \frac{V_g^m}{V_B^m} \right) = \mu p - \mu q (N^1 g_g^1 + N^2 g_g^2) + \mu q \left(N^1 g_B^1 \frac{V_g^1}{V_B^1} + N^2 g_B^2 \frac{V_g^2}{V_B^2} \right)$$

(19)

For an individual with consumption $\tilde{g}^i \le g$, g_g^i and V_g^i are zero. For an individual where the constraint is binding it will be true that g_B^1 will be zero. Hence, the term $g_B^1 \frac{V_g^1}{V_B^1}$ will always vanish. We can therefore rewrite the expression as:

$$\mu N^1 \frac{V_g^1}{V_B^1} + \mu N^2 \frac{V_g^2}{V_B^2} + \rho V_B^m \left(\frac{V_g^1}{V_B^1} - \frac{V_g^m}{V_B^m} \right) = \mu p - \mu q (N^1 g_g^1 + N^2 g_g^2)$$

(20)

From section 2 we know that $\frac{V_g^i}{V_B^i} = MRS_{gB}^i - q$ which is zero if the person is not rationed.

Inserting this result and dividing through by μ and rearranging we obtain:

$$N^1 MRS_{gB}^1 - N^1 q + N^2 MRS_{gB}^2 - N^2 q = MRT_{gB} + \mu^* (MRS_{gB}^m - MRS_{gB}^1) - q(N^1 g_g^1 + N^2 g_g^2)$$

(21)

We now consider the interpretation of (21) under various assumptions.

Proposition 6. If $\tilde{g}_L^i = 0$ the Samuelson rule $\sum MRS = MRT$ applies.

Proof: The fact that $\tilde{g}_L^i = 0$ implies that $\tilde{g}^1 = \tilde{g}^m < \tilde{g}^2$. It also implies that $MRS_{gB}^1 = MRS_{gB}^m$. Hence, the self-selection term in eq. (21) vanishes. From proposition 1 we know that $g \le \tilde{g}^2$. Suppose first that $\tilde{g}^1 < g \le \tilde{g}^2$, i.e., that the type one individual is not rationed. This implies that the Hicksian substitution effect is negative. It follows from eq. (14) that $q = 0$. Hence, the q-terms in (21) vanishes. Suppose next that $g < \tilde{g}^1 < \tilde{g}^2$, that is, both types are rationed. Then $g_g^1 = g_g^2 = 1$ and the q-terms cancel out. This implies that we are left with the Samuelson rule.

Proposition 7: If $\tilde{g}^i_L < 0$ the rule for provision of the public good takes the form:

$$N^1 MRS^1_{gB} + N^2 MRS^2_{gB} = MRT_{gB} + \mu^* (MRS^m_{gB} - MRS^1_{gB})$$

Proof: If $\tilde{g}^i_L < 0$ it will be true that $\tilde{g}^m < \tilde{g}^1$, $\tilde{g}^m < \tilde{g}^2$ and $(MRS^m_{gB} - MRS^1_{gB}) < 0$. If at least one of the actual persons is not rationed it follows from eq. (14) that q is zero. If both actual persons are rationed $g^1_g = g^2_g = 1$ and the q-terms will cancel out. Hence, the q-terms will always disappear from the expression. However, the self-selection term remains.

Proposition 8: If $\tilde{g}^i_L > 0$ the following regimes are possible

i) Persons of type one and the mimicker are not rationed ($q > \bar{q}$ as defined in Propostion 5), and persons of type two are rationed. The public provision rule has the form $N^2 MRS^2_{gB} = MRT_{gB}$.

ii) Persons of type one are not rationed ($q \geq \bar{q}$), while the mimicker and persons of type two are rationed. The public provision rule has the form

$$N^2 MRS^2_{gB} = MRT_{gB} + \mu^* (MRS^m_{gB} - q).$$

iii) Persons of both types and consequently the mimicker are rationed ($q < \bar{q}$). The public provision rule has the form

$$N^1 MRS^1_{gB} + N^2 MRS^2_{gB} = MRT_{gB} + \mu^* (MRS^m_{gB} - MRS^1_{gB})$$

iv) No type is actually rationed ($q > \bar{q}$). Only the mimicker is rationed. The public provision rule has the form

$$N^1 MRS^1_{gB} + N^2 MRS^2_{gB} = MRT_{gB} + \mu^* (MRS^m_{gB} - MRS^1_{gB}).$$

The last case is a special one. Since $g \leq \max\{\tilde{g}^1, \tilde{g}^2\}$, this case emerges only if we have the very special coincidence that $\tilde{g}^1 = \tilde{g}^2 = g$. Then there can only be a loss from increasing g. However, it is possible to reduce g by one unit, which will imply that both types of persons and the mimicker have to reduce consumption by one unit, and the

production cost is reduced[8]. Optimality requires that there is no net from such a cutback as prescribed by the optimality condition above.

The intuition for the results above is pretty straightforward in most cases. There is a gain from deviating from the Samuelson rule if by doing so one succeeds in relaxing the self-selection constraint. This will happen if the low-ability person and the mimicker behave differently and can be discriminated between. If the mimicker, enjoying more leisure, has a lower marginal valuation of g as in Proposition 7, the public good should be oversupplied compared to the Samuelson rule. Suppose that departing from the Samuelson optimum, an additional unit of g is supplied. The high- and low-ability persons are charged through their tax liabilities to be left equally well off. Then the mimicker is made worse off as his lower valuation of g does not compensate for the tax increase. Mimicking is being deterred. However, if the mimicker's valuation exceeds that of the low-ability person the mimicker will gain from a tax-paid increase in g, but will be made worse off if g is decreased. Hence g should be undersupplied relative to the Samuelson rule as prescribed by Proposition 8 ii-iv.

If they have the same valuation of *g*, as in Proposition 6, a change in the supply of g and compensating tax changes will not discriminate between the two. Nothing can be gained by deviating from the Samuelson rule as stated by Proposition 6.

A special case arises when the low-ability type is not rationed. Then an increase in the provision of *g* will not affect the consumption of this type and the sum of marginal benefits of increasing *g* is equal to the sum taken solely across type two individuals. If the low ability type and the mimicker are both left unaffected by an increase in *g*, no discrimination of the mimicker can be achieved, we have a special case of the Samuelson rule as in case i of Proposition 8. If relaxing the mimicking constraint is possible case ii gives us a special case iii in Proposition 8.

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[8] Mathematically the g-function is not differentiable at g as a kink arises when the demand function hits the rationing constraint.

4. Excludable intermediate public good

We shall consider the same two-type, asymmetric information model as above with the following qualifications. An excludable public good is used as an intermediate good in the production sector, but there is no public good in the consumption bundle.

There are two consumer goods. We assume that each good is being provided by producers each producing a fixed quantity normalised to unity. Let c_i be the labour cost in efficiency units per unit output of commodity i. This unit cost is assumed to be a function of the amount of the public good being used in the production of that commodity. The idea is that the use of the public good makes production more efficient and economizes on the use of labour.[9]

Assuming that each producer has a fixed output is a simplifying assumption. If a producer could make any acquired amount of g available at no further cost to all parts of the production, there would be economies of scale in production. Concentrating production would economise on the cost of acquiring the public good. However, there may be other disadvantages from having large production units, but with the cost of acquiring g still being one determinant of the optimum size. We do not want to model these various factors, which would take us far beyond the focus of our discussion. We observe that there are sectors with many producers making use of public goods in their production, and we want to consider such a setting without modeling too many details of the production structure.

The producer will minimize the unit cost of production including the cost of acquiring g_i, $c_i(g_i) + g_i q$, implying that

$$-c_i' = q . \tag{22}$$

The cost saving at the margin is equated to the price, or the producer may be rationed such that

$$-c_i'(g_i) > q \text{ and } g_i = g, \ i = 1,2 \tag{23}$$

Neglecting the public input we would have the standard two-type, non-linear income tax model with two consumption commodities. (See for instance Edwards et al.

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[9] An alternative perspective would be to assume that the effect of g is to raise the quality of the goods being produced rather than to lower their production cost. Essentially this is the same thing because producing higher quality at the same cost can be perceived as producing a larger amount at the same cost, which is equivalent to a lower cost per unit.

(1994)). We know that in general efficiency can be improved (a Pareto improvement can be achieved) by supplementing the income tax with commodity taxes (Christiansen (1984), Edwards et al. (1994)). Let us assume that such commodity taxes are in place, and let t_i denote the unit tax imposed on commodity i, and let p_i be the consumer price. Under competitive, free-entry conditions the equilibrium market price equals the producers' marginal and unit cost which consist of the production cost c_i (in terms of efficiency labour units), the cost of buying the required input of the public good $(g_i q)$ and the tax.

$$p_i = c_i(g_i) + g_i q + t_i \tag{24}$$

The Pareto efficiency problem can then be formulated as

$$\max V^1\left(B^1, Y^1, p_1, p_2\right)$$

w.r.t.

$$B^1, Y^1, B^2, Y^2, g, t_1, t_2, q$$

s.t

$$V^2\left(B^2, Y^2, p_1, p_2\right) \geq \overline{V}^2 \tag{25}$$

$$V^2\left(B^2, Y^2, p_1, p_2\right) \geq V^m\left(B^1, Y^2, p_1, p_2\right) \tag{26}$$

$$N^1\left(Y^1 - B^1\right) + N^2\left(Y^2 - B^2\right) + t_1 x_1 + t_2 x_2 + g_1 q x_1 + g_2 q x_2 - g \geq 0 \tag{27}$$

$$p_i = c_i(g_i) + g_i q + t_i \quad i = 1,2 \tag{28}$$

$$-c_i'(g_i) = q \text{ and } g_i \leq g, \ i = 1,2 \tag{29}$$
$$\text{or } -c_i'(g_i) > q \text{ and } g_i = g, \ i = 1,2.$$

A necessary condition for a Pareto efficient allocation is that it is impossible to use the instruments of the government in such a way that both types of persons are kept equally well off while the government revenue is increased. If that were possible the additional revenue could be used to increase the disposable income of the high-skilled person. The self-selection constraint would not be violated and a Pareto improvement would be obtained.

To derive such necessary conditions let us do the following exercise. We consider changes in q and g. At the same time we keep the gross and net incomes of both types of persons unchanged and adjust the commodity taxes in such a way that the consumer prices of both goods remain the same. Then obviously both types of persons stay equally well off, the mimicking constraint remains satisfied and demand for the consumer goods is left unchanged. Pegging the consumer prices at fixed values $\bar{p}_1$ and $\bar{p}_2$, the commodity taxes are restricted by

$$c_i(g_i) + g_i q + t_i = \bar{p}_i \quad i = 1,2 \tag{30}$$

The necessary adjustments in response to changes in q and g are given by

$$\frac{dt_i}{dq} = -(c_i' + q)\frac{dg_i}{dq} - g_i \tag{31}$$

$$\frac{dt_i}{dg} = -c_i' - q \tag{32}$$

Let us then explore how the government revenue is affected by changing q and g subject to the constraint imposed above. Taking net and gross incomes and hence the income tax revenue as fixed we focus on the revenue from g and commodity taxes

$$R = t_1 x_1 + t_2 x_2 + q g_1 x_1 + q g_2 x_2 - g \tag{33}$$

We differentiate with respect to q and then insert the expressions for commodity tax changes derived above to obtain

$$\frac{dR}{dq} = x_1 \frac{dt_1}{dq} + x_2 \frac{dt_2}{dq} + g_1 x_1 + g_2 x_2 + q x_1 \frac{dg_1}{dq} + q x_2 \frac{dg_2}{dq}$$

$$= x_1 (-c_1' - q) \frac{dg_1}{dq} - g_1 x_1 + x_2 (-c_2' - q) \frac{dg_2}{dq}$$

$$- g_2 x_2 + g_1 x_1 + g_2 x_2 + q x_1 \frac{dg_1}{dq} + q x_2 \frac{dg_2}{dq}$$

$$= -c_1' \frac{dg_1}{dq} - c_2' \frac{dg_2}{dq} = 0$$

implying that

$$- \frac{dg_i}{dq} = 0 \text{ or } c_i' = 0 \tag{34}$$

Either the producer is not rationed and the price q is zero or the producer is rationed.

The optimum amount of g is characterised by the first order condition

$$\frac{dR}{dg} = x_1 \frac{dt_1}{dg_1} \frac{dg_1}{dg} + x_2 \frac{dt_2}{dg_2} \frac{dg_2}{dg} + q x_1 \frac{dg_1}{dg} + q x_2 \frac{dg_2}{dg} - 1$$

$$= x_1 (-c_1' - q) \frac{dg_1}{dg} + x_2 (-c_2' - q) \frac{dg_1}{dg} + q x_1 \frac{dg_1}{dg} + q x_2 \frac{dg_2}{dg} - 1$$

$$= -c_1' x_1 \frac{dg_1}{dg} - c_2' x_2 \frac{dg_2}{dg} - 1 = 0,$$

which is equivalent to

$$- c_1' x_1 \frac{dg_1}{dg} - c_2' x_2 \frac{dg_2}{dg} = 1 \tag{35}$$

We know that q=0 or producers are rationed. If q=0 and there is no rationing also $c_i' = 0$

and (35) does not hold. Producers in both sectors are rationed, or one producer is rationed

and one producers is at a satiation point where $c_i' = 0$. For a rationed producer $\frac{dg_i}{dg} = 1$.

We can conclude that

$$- c_1' x_1 - c_2' x_2 = 1 \tag{36}$$

which holds with at least one rationed producer.

We note that the gain from using an extra unit of g in the production of good i is a cost saving per unit equal to $-c_i'$. The total cost saving at an output level x_i is then $-c_i'x_i$. The sum of marginal benefits is then equal to the left hand side of (36). The right hand side is the marginal cost of providing g, so (36) is the Samuelson rule for an intermediate public good. We can conclude that the Samuelson rule is valid.

Proposition 9: Producers using the intermediate public good as input should face a zero price. Producers in both sectors are rationed (g_i=g) or there is rationing in one sector and satiation with respect to the public good in the other. The optimum supply of the intermediate public good is the amount which yields production efficiency in the first best sense.

If for some reason the desirable commodity taxes are not available there may be a case for achieving some of the same effect through a policy that violates production efficiency. Suppose it is desirable to increase the price of a commodity, and imposing a tax is not feasible. Then by making production in that sector less efficient, the price can be increased, but at the expense of production efficiency. However, we shall refrain from further analysis of such a regime.

5. Concluding remarks

We have used the Stiglitz-Stern model to study the respective conditions under which it is useful or harmful to set a positive price on a publicly provided excludable public good. As in the optimal commodity tax literature we find that the potential role of a price is to discourage the high-skilled person from mimicking. Therefore, whether the price instrument should be used or not hinges on how the valuation of the public good depends on the amount of leisure which is the only feature of consumption distinguishing the high-skilled mimicker from the low-skilled persons. We first consider the case where the excludable public god is a final consumer good. We find that only if the demand for the excludable public good increases with the amount of leisure can it be gainful to use the price instrument. The reason why it may be gainful in this case is that the mimicker has a larger consumption of the excludable public good and is hurt more than a low-skilled

person by a price on the public good. As such this is not a surprising result. However, the positive leisure impact is not a sufficient condition for this outcome. Only if the demand for the excludable public good increases in leisure and the optimum is such that the low-skilled person is not rationed is it desirable to set a positive price on the public good. A finding which is somewhat surprising on intuitive grounds is that even when a strictly positive price is optimal, introducing a positive but too low price may be harmful compared to charging no price at all (even when we neglect the administrative cost of charging a price). We also characterize how the Samuelson rule is modified depending on how the demand for the public good varies with the amount of leisure.

We have studied the case where the excludable public good is an intermediate good in a framework where optimal commodity taxes on the consumer goods are available. Given the optimal use of these instruments we find that nothing can be gained by using the quantity of the public good or the price on the public good for deterring mimicking. The rule for determining the quantity of the public good implies production efficiency.

When a public good is excludable private firms could provide the good. Such provision has been studied in Brito and Oakland (1980) and Burns and Walsh (1981). They study the case where there is a natural monopoly. Oakland (1974) studied the case where the provision is from firms operating under conditions of perfect competition. Both forms of provision involve inefficiencies. In the monopoly case we know that the quantity in general tends to be too low. In the case with perfect competition the fact that there must be a large number of firms means that the public good will not be produced under conditions that take full advantage of the fact that it is a public good. However, also the public provision scheme involves inefficiencies. It would therefore be of interest to compare public provision and provision from private firms. However, we leave that for future research.

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Appendix

In order to investigate the effects of changing q we need to explore how the key variables are affected. Combining (2) and (3) we can rewrite the binding self-selection constraint as

$$V^m(B^1, Y^1, q, g) = \bar{V}^2 \tag{A1}$$

The utility of person one is

$$V^1(B^1, Y^1, q, g) = V^1 \tag{A2}$$

Differentiating these two equations keeping g constant and using Roy's identity we get

$$-V_B^m g^m dq + V_B^m dB^1 + V_Y^m dY^1 = 0 \tag{A3}$$

$$-V_B^1 g^1 dq + V_B^1 dB^1 + V_Y^1 dY^1 = dV^1 \tag{A4}$$

Further manipulations yield

$$dB^1 = -\frac{V_Y^m}{V_B^m}dY^1 + g^m dq \tag{A5}$$

and

$$-g^1 dq + dB^1 + \frac{V_Y^1}{V_B^1}dY^1 = \frac{dV^1}{V_B^1} \tag{A6}$$

Plugging (A5) into (A6), we get

$$-g^1 dq + g^m dq - \frac{V_Y^m}{V_B^m}dY^1 + \frac{V_Y^1}{V_B^1}dY^1 = \frac{dV^1}{V_B^1} \tag{A7}$$

and moreover

$$dY^1 = \left(\frac{V_Y^m}{V_B^m} - \frac{V_Y^1}{V_B^1}\right)^{-1}\left(g^m - g^1\right)dq + \left(\frac{V_Y^m}{V_B^m} - \frac{V_Y^1}{V_B^1}\right)^{-1}\frac{dV^1}{V_B^1} \tag{A8}$$

All the expressions in parentheses are positive by the assumptions that have been made. When q is being increased and we are in the interval in which V^1 is declining, it follows that Y^1 increases.

Let us then explore the effect on g^1. Letting $e^1(q,Y^1,V^1)$ be the expenditure function, we can write the demand for g^1 as

$$\tilde{g}^1\left(B^1,Y^1,q\right) = \tilde{g}^1\left(e^1(q,Y^1,V^1),Y^1,q\right) = g^{h1}(q,Y^1,V^1). \tag{A9}$$

Differentiating the function we get

$$dg^1 = g_q^{h1}dq + g_Y^{h1}dY^1 + g_B^1 e_V^1 dV^1 \tag{A10}$$

Using the fact that $e_V^1 = 1/V_B^1$ and inserting the expression for dY^1 from (A8), we get

$$dg^1 = g_q^{h1}dq - g_Y^{h1}\frac{1}{\dfrac{V_Y^m}{V_B^m} - \dfrac{V_Y^1}{V_B^1}}\left(g^m - g^1\right)dq - \frac{1}{\dfrac{V_Y^m}{V_B^m} - \dfrac{V_Y^1}{V_B^1}}g_Y^{h1}\frac{dV^1}{V_B^1} + g_B^1\frac{dV^1}{V_B^1} \tag{A11}$$

According to (A10) there are three effects on the *g*-consumption. There is a negative substitution effect when *q* increases. There is an effect from the increased labour supply (increase in Y^1) which is also negative if the compensated effect of increasing Y^1 (like the uncompensated effect) is to discourage the demand for g. Finally, there is, under the same assumption, a negative effect from the loss of real income (utility).

Let us then consider in further detail how the utility of type one changes when q is being increased. We can invoke the Envelope Theorem by which the effect can be derived from the Lagrange function. Assuming that, for any value of q, an optimal policy in other respects is being pursued, we know by the Envelope Theorem that

$$\frac{\partial V^1}{\partial q} = \frac{\partial \Lambda}{\partial q},$$

(A12)

which has been expressed by (13). To eliminate Lagrange multipliers we use (6) and (7) to solve for ρ and μ to get :

$$\rho V_B^m = V_B^1 \left[1 - \frac{\mu}{V_B^1} N^1 \left(1 - q g_B^1 \right) \right] = \mu \left[\frac{V_B^1}{\mu} - N^1 \left(1 - q g_B^1 \right) \right]$$

(A13)

$$\mu = \frac{V_B^1}{N^1} \frac{\left(\dfrac{V_y^m}{V_B^m} - \dfrac{V_y^1}{V_B^1} \right)}{1 + q g_y^1 + \dfrac{V_y^m}{V_B^m} \left(1 - q g_B^1 \right)}$$

(A14)

which yields

$$\frac{V_B^1}{N^1 \mu} = \frac{1 + q g_y^1 + \dfrac{V_y^m}{V_B^m} \left(1 - q g_B^1 \right)}{\dfrac{V_y^m}{V_B^m} - \dfrac{V_y^1}{V_B^1}}$$

(A15)

implying that

$$\frac{V_B^1}{N^1 \mu} - \left(1 - q g_B^1 \right) = \frac{1 + q g_y^1 + \dfrac{V_y^m}{V_B^m} \left(1 - q g_B^1 \right) - \dfrac{V_y^m}{V_B^m} \left(1 - q g_B^1 \right) + \dfrac{V_y^1}{V_B^1} \left(1 - q g_B^1 \right)}{\dfrac{V_y^m}{V_B^m} - \dfrac{V_y^1}{V_B^1}}$$

(A16)

$$= \frac{1 + qg_y^1 + \dfrac{V_y^1}{V_B^1}\left(1 - qg_B^1\right)}{\dfrac{V_y^m}{V_B^m} - \dfrac{V_y^1}{V_B^1}}$$

Employing (13) and (A13) we get

$$\frac{\partial \Lambda}{\partial q} = \mu \left[\frac{V_B^1}{\mu} - N^1\left(1 - qg_B^1\right)\right]\left(g^m - g^1\right) + q\mu\left(N^1 g_q^{h1} + N^2 g_q^{h2}\right), \qquad (A17)$$

which we can rewrite as

$$\frac{\partial \Lambda}{\partial q}\bigg/\mu = N^1\left[\frac{V_B^1}{N^1\mu} - \left(1 - qg_B^1\right)\right]\left(g^m - g^1\right) + q\left(N^1 g_q^{h1} + N^2 g_q^{h2}\right). \qquad (A18)$$

Making use of (A16) we get

$$\frac{\partial \Lambda}{\partial q}\bigg/\mu = \frac{\left[1 + \dfrac{V_y^1}{V_B^1}\left(1 - qg_B^1\right) + qg_y^1\right]}{\dfrac{V_y^m}{V_B^m} - \dfrac{V_y^1}{V_B^1}} N^1\left(g^m - g^1\right) + q\left(N^1 g_q^{h1} + N^2 g_q^{h2}\right). \qquad (A20)$$

Starting from (A12) and multiplying both sides of (A20) by μ taken from (A14) we have that

$$\frac{dV^1}{V_B^1} = \frac{1 + qg_Y^1 + \dfrac{V_Y^1}{V_B^1}\left(1 - qg_B^1\right)}{1 + qg_Y^1 + \dfrac{V_Y^m}{V_B^1}\left(1 - qg_B^1\right)}\left(g^m - g^1\right)dq + \frac{1}{N^1}\frac{\dfrac{V_Y^m}{V_B^m} - \dfrac{V_Y^1}{V_B^1}}{1 + qg_Y^1 + \dfrac{V_y^m}{V_B^m}\left(1 - qg_B^1\right)}q\left(N^1 g_q^{h1} + N^2 g_q^{h2}\right)dq$$

$$(A21)$$

Let us then consider the ratio between the change in g^1 and the change in utility denominated in income units (dV^1/V_B^1). Combining the equation above and (A11), we get

$$\frac{dg^1}{dV^1/V_B^1} = g_q^{h1} \frac{V_B^1}{dV^1} dq - g_Y^{h1} \frac{1}{\dfrac{V_Y^m}{V_B^m} - \dfrac{V_Y^1}{V_B^1}} \frac{V_B^1\left(g^m - g^1\right)}{dV^1} dq - \frac{1}{\dfrac{V_Y^m}{V_B^m} - \dfrac{V_Y^1}{V_B^1}} g_Y^{h1} + g_B^1 \ . \tag{A22}$$

Consider the case in which $g^m - g^1 = 0$. Then

$$\frac{dg^1}{dV^1 \Big/ V_B^1} = -\frac{1}{\dfrac{V_Y^m}{V_B^m} - \dfrac{V_Y^1}{V_B^1}} g_Y^{h1} + g_B^1 + g_q^{h1} N^1 \frac{1 + q g_Y^1 + \dfrac{V_Y^m}{V_B^m}\left(1 - q g_B^1\right)}{\left(\dfrac{V_Y^m}{V_B^m} - \dfrac{V_Y^1}{V_B^1}\right) q\left(N^1 g_q^{h1} + N^2 g_q^{h2}\right)} \ . \tag{A23}$$