ON THE MODELING OF THE INCOME DISTRIBUTION BUSINESS CYCLE DYNAMICS

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Abstract

Empirically, the income share is procyclical for the low-income groups and acyclical for the top 5%. We find that business cycle models should consider overlapping generations and elastic labor supply in order to replicate this finding.

JEL Code: C68, D31, E32.

Keywords: income distribution, business cycle, overlapping generations.

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1 Introduction

Castañeda et al. (1998) document that the US income distribution is highly, but not perfectly procyclical for the low income quintiles, countercyclical for the top 60-95%, and acyclical for the top 5%. They also present a dynamic general equilibrium model with infinitely-lived agents and unemployment risk that is able to replicate the movements of the lower income quintiles. During a boom, the number of unemployed workers decreases. As a consequence, the relative income share of the lower income quintiles rises at the expense of the higher income quintiles. However, the income shares are almost perfectly correlated with output, either positively or negatively. Therefore, they also fail to replicate the income dynamics of the very rich that is acyclical.

In this paper, we present a simple business cycle model with overlapping generations and elastic labor supply in order to improve upon the modeling of the cyclical income distribution dynamics. We consider households with different productivity types. In addition, individual productivity is also age-dependent and subject to an idiosyncratic shock so that we are able to match both the observed income and wealth heterogeneity. The latter feature, of course, is important for the study of the factor income distribution dynamics. Aggregate uncertainty is introduced in the form of a shock on aggregate production technology as in Castañeda et al. (1998) .

In our model, the almost perfect correlation of the lower income quintiles with output is reduced as the high-productivity agents have a more elastic labor supply than their low-productivity contemporanies.[1] In addition, the share of the top 5% of the income earners is almost acyclical as i) many of the income-rich agents are wealth-rich retired agents and ii) the wealth-rich workers also have a less elastic labor supply than the wealth-poor workers. During an economic expansion, both wages and pensions increase. Pensions are tied to the current wage rate. However, workers increase their labor supply, which is not possible for retired workers. Therefore, the income share of workers increases and is procyclical.

The paper is organized as follows. In Section 2, we describe our model. Our results are presented in Section 3. Section 4 concludes. The computation is explained in more detail in the Appendix.

2 The model

Three different sectors are depicted: households, firms, and the government. Households differ with regard to their individual productivity and are also subject to idiosyncratic productivity risk. They maximize discounted life-time utility with regard to their intertemporal consumption, capital, and labor supply. Firms are competitive and maximize profits. The government provides pensions which it finances with a tax on wage income.

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[1] Heer and Maussner (2007) show that this need not be the case in the presence of progressive income taxation.

2.1 Households

Households live 70 periods. Periods are equal to one year. Households are born at age 1 (corresponding to real life-time age 20). Each generation is of measure 1/70. The first 45 periods, they are working, the last 35 periods, they are retired and receive pensions. Households maximize expected life-time utility at age 1 in period t:

$$E_t \sum_{s=1}^{70} \beta^{s-1} u(c^s_{t+s-1}, l^s_{t+s-1}), \tag{1}$$

where s denotes age. Instantaneous utility is a function of both consumption c and leisure l:

$$u(c,l) = \frac{\left(c^{\gamma} l^{1-\gamma}\right)^{1-\eta} - 1}{1-\eta}.$$

The total time endowment is equal to one and allocated between leisure l and work n, $n + l = 1$.

The worker's labor productivity $e(s, \epsilon, z) = \epsilon z e^{\bar{y}_s}$ depends on the agent's permanent efficiency type $\epsilon \in \mathcal{E} = \{\epsilon_1, \epsilon_2\}$, his idiosyncratic stochastic productivity $z \in \mathcal{Z} = \{z_1, z_2\}$, and his age $s \in \mathcal{S}$. This modeling of labor productivity has often been applied in DGE (dynamic general equilibrium) analysis for the following reasons: i) Differences in the permanent efficiency type ϵ help to generate the wage heterogeneity that is observed empirically. In our case, two different efficiency types are enough to achieve this aim. ii) Workers will build up precautionary savings if they face idiosyncratic productivity risk z. Therefore, the wealth distribution becomes more heterogenous in better accordance with reality. iii) The age-dependent component $\bar{y}_s$ helps to explain differences in the age-income distribution that is important to explain the movement of the cross-section factor shares.

In each period t, an equal measure of 1-year old workers of productivity types $e(1, \epsilon_i, z_j)$, $i = 1, 2$, $j = 1, 2$, is born. During working age, $s = 1, \ldots, 44$, the process for idiosyncratic productivity z_s is a Markov chain:

$$\pi(z'|z) = Prob\left\{z_{s+1} = z'|z_s = z\right\} = \begin{pmatrix} \pi^z_{11} & \pi^z_{12} \\ \pi^z_{21} & \pi^z_{22} \end{pmatrix}. \tag{2}$$

Depending on his efficiency type ϵ, the agent receives pensions $b_t(\epsilon) = \epsilon \bar{b}_t$ in old age that are financed by a social security tax $\tau_{w,t}$ on the young workers' wage income.

Let k, w, and r denote the individual capital stock, the wage rate and the interest rate, respectively. The working agent of age s faces the following budget constraint in period t:

$$k^{s+1}_{t+1} = (1 + r_t)k^s_t + (1 - \tau_{w,t})w_t e(s, \epsilon, z)n^s_t - c^s_t, \quad s = 1, \ldots, 45. \tag{3}$$

The budget constraint of the retired worker is given by

$$k_{t+1}^{s+1} = (1+r_t)k_t^s + b_t(\epsilon) - c_t^s, \quad s = 46, \ldots, 70. \tag{4}$$

Agents are born without capital at age 1, $k_t^1 \equiv 0$, and do not work in old age, $l_t^s = 1$ for $s \geq 46$. In addition, we impose a borrowing constraint with $k_t^s \geq 0$.

2.2 Firms

Firms are competitive and produce output using capital K and labor N. Production Y_t is characterized by constant returns to scale and assumed to be Cobb-Douglas:

$$Y_t = A_t F(K_t, N_t) = A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{5}$$

The aggregate technology level $A_t \in \{A_1, A_2\}$ follows a 2-state Markov process:

$$\pi(A'|A) = Prob\{A_{t+1} = A'|A_t = A\} = \begin{pmatrix} \pi_{11}^A & \pi_{12}^A \\ \pi_{21}^A & \pi_{22}^A \end{pmatrix}. \tag{6}$$

In a factor market equilibrium, factors are rewarded with their marginal product:

$$\begin{aligned} w_t &= (1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha}, & (7) \\ r_t &= \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} - \delta. & (8) \end{aligned}$$

Capital K depreciates at rate δ.

2.3 Government

The government provides pensions to the retired agents. Pensions are proportional to the current-period wage rate with the replacement ratio being denoted by ζ. In addition, we distinguish the two cases that pensions are either lump-sum or depend on the permanent efficiency type ϵ:

$$b_t = \begin{cases} \zeta w_t \bar{n} & \text{lump-sum,} \\ \zeta \epsilon w_t \bar{n} & \text{efficiency-dependent.} \end{cases} \tag{9}$$

$\bar{n}$ denotes the average labor supply in the economy in the non-stochastic steady state (with $A \equiv 1$). Therefore, pensions of the retired agents do not increase if the contemporary workers increase their labor supply.

2.4 Stationary equilibrium

In the stationary equilibrium, individual behavior is consistent with the aggregate behavior of the economy, households maximize intertemporal utility, firms maximize profits, and factor and goods' markets are in equilibrium. Let $f_t(k,s,\epsilon,z)$ denote the distribution of individual wealth k, age s, the efficiency type ϵ, and idiosyncratic productivity z in the period t.

A *stationary equilibrium* for a government policy $\{\zeta\}$ and initial measures $f_0(k,s,\epsilon,z)$ in period 0 corresponds to a price system, an allocation, and a sequence of aggregate productivity indicators $\{A_t\}$ that satisfy the following conditions:

1. Households maximize the intertemporal utility (1) subject to the budget constraint (3) or (4), and the dynamics of the idiosyncratic productivity level z, (11). This gives rise to the following first-order conditions:

$$\begin{aligned}
\frac{1-\gamma}{\gamma}\frac{c_t^s}{1-n_t^s} &= (1-\tau_{w,t})w_t e(s,\epsilon,z), \\
\left(c_t^s\right)^{\gamma(1-\sigma)-1}\left(1-n_t^s\right)^{(1-\gamma)(1-\sigma)} &= \beta E_t\left\{[1+r_{t+1}]\left(c_{t+1}^{s+1}\right)^{\gamma(1-\sigma)-1}\left(1-n_{t+1}^{s+1}\right)^{(1-\gamma)(1-\sigma)}\right\}.
\end{aligned}$$

   Individual labor supply $n_t(k,s,\epsilon,z)$, consumption $c_t(k,s,\epsilon,z)$, and optimal next period capital stock $k_t'(k,s,\epsilon,z)$ in period t are functions of the individual state variables $\{k,s,\epsilon,z\}$ and also depend on the period t.

2. Firms maximize profits satisfying (7) and (8).

3. Aggregate variables are equal to the sum of the individual variables:

$$\begin{aligned}
N_t &= \sum_{s=1}^{45}\sum_{\epsilon,z}\int_k e(s,\epsilon,z)n_t(k,s,\epsilon,z)\ f_t(k,s,\epsilon,z)\ dk, \\
K_t &= \sum_{s=1}^{70}\sum_{\epsilon,z}\int_k k\ f_t(k,s,\epsilon,z)\ dk, \\
C_t &= \sum_{s=1}^{70}\sum_{\epsilon,z}\int_k c_t(k,s,\epsilon,z)\ f_t(k,s,\epsilon,z)\ dk, \\
B_t &= \sum_{s=46}^{70}\sum_{\epsilon,z}\int_k b_t(\epsilon)\ f_t(k,s,\epsilon,z)\ dk,
\end{aligned}$$

   where C_t and B_t denote aggregate consumption and pensions, respectively.

4. The government budget is balanced:

$$B_t = \tau_{w,t} w_t N_t.$$

In particular, the contribution rate $\tau_{w,t}$ adjusts in each period.

5. The goods' market clears:

$$C_t + K_{t+1} - (1-\delta)K_t = Y_t.$$

6. The cross-sectional measure f_t evolves as

$$f_{t+1}(\mathcal{K} \times \mathcal{S} \times \mathcal{E} \times \mathcal{Z}) = \int \sum_{s,\epsilon,z} P_t\left((k,s,\epsilon,z), \mathcal{K} \times \mathcal{S} \times \mathcal{E} \times \mathcal{Z}\right) \; f_t(k,s,\epsilon,z)\, dk$$

with

$$P_t\left((k,s,\epsilon,z), \mathcal{K} \times \mathcal{S} \times \mathcal{E} \times \mathcal{Z}\right) = \begin{cases} \sum_{z' \in \mathcal{Z}} \pi(z'|z) & \text{if } k'_t(k,s,\epsilon,z) \in \mathcal{K}, \\ & \epsilon \in \mathcal{E}, s+1 \in \mathcal{S}, \\ & \\ 0 & \textit{else}, \end{cases}$$

and for the newborns

$$f_{t+1}(\mathcal{K} \times 1 \times \mathcal{E} \times \mathcal{Z}) = \begin{cases} \sum_{\epsilon \in \mathcal{E}, z \in \mathcal{Z}} \frac{1}{4 \cdot 70} & \text{if } 0 \in \mathcal{K} \\ & \\ 0 & \textit{else}. \end{cases}$$

2.5 Calibration and computation

We choose the parameter values $\beta = 0.99$, $\eta = 2.0$, $\gamma = 0.28$, $\alpha = 0.35$, $\delta = 0.08$ that are standard in the business cycle literature.[2] The Markov process (6) of aggregate technology level is calibrated so that the average duration of one cycle is equal to 6 years:

$$\pi(A'|A) = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix}. \tag{10}$$

Aggregate technology is chosen so that the mean $\bar{A}$ is equal to one and the annual standard deviation of output is approximately equal to 2% implying $\{A_1, A_2\} = \{0.98, 1.02\}$.

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[2] See, for example, Heer and Maussner (2005).

The calibration of the individual productivity $e(s,\epsilon,z)$ is chosen in accordance with Krueger and Ludwig (2007). In particular, we pick $\{\epsilon_1,\epsilon_2\} = \{0.57, 1.43\}$ so that the average productivity is one and the implied variance of labor income for the new entrants at age $s = 1$ is equal to the value reported by Storesletten et al. (2004). The annual persistence of the idiosyncratic component z is chosen to be 0.98. In addition, idiosyncratic productivity has a conditional variance of 8%, implying $\{z_1, z_2\} = \{0.727, 1.273\}$, and

$$\pi(z'|z) = \left(\begin{array}{cc} 0.98 & 0.02 \\ 0.02 & 0.98 \end{array} \right). \tag{11}$$

The age-efficiency $\bar{y}_s$ profile is taken from Hansen (1993). The calibration implies an average labor supply approximately equal to $\bar{n} = 0.3$ and a Gini coefficient of income (wealth) equal to 0.42 (0.58) in good accordance with empirical observations, even though the values are lower than those of most recent studies on the empirical wealth and income distribution. In particular, for the US economy, Rodriguez et al. (2002) find a value of 0.55 (0.80) for the income Gini (wealth Gini).[3]

The replacement ratio of average pensions relative to net wage earnings is equal to $\zeta = \frac{\bar{b}_t}{(1-\tau_{w,t})w_t\bar{n}} = 30\%$, with $\bar{n} = 0.3$.

The computation is based upon the algorithm of Krusell and Smith (1998) and follows Storesletten et al. (2004). A detailed description is provided in the Appendix.

3 Results

Figure 1 describes the behavior of our economy in the non-stochastic steady state. In the upper row, we graph the average wealth and labor supply of each generaton, while the average total income of each generation and the efficiency-age profiles $e(s,\epsilon,z)$ for the four productivity types $\{\epsilon_i, z_j\}$ for $i = 1, 2$, $j = 1, 2$, are displayed in the lower row. Agents accumulate savings until retirement age $s = 45$ (corresponding to real lifetime age 65 in the Figure 1) and dissave thereafter. Total income (wage plus interest income before taxes) peaks at real lifetime age 50. Our average-age profiles accord very well with empirical observations in Rodriguez et al. (2002). Based on the 1998 data from the Survey of Consumer Finances they find that US household income, earnings, and wealth peak around ages 51-55, 51-55, and 61-65, respectively.

In order to compute the correlation of the income distribution with output, we simulate the dynamics of our economy repeatedly over 200 periods. One of these simulations is

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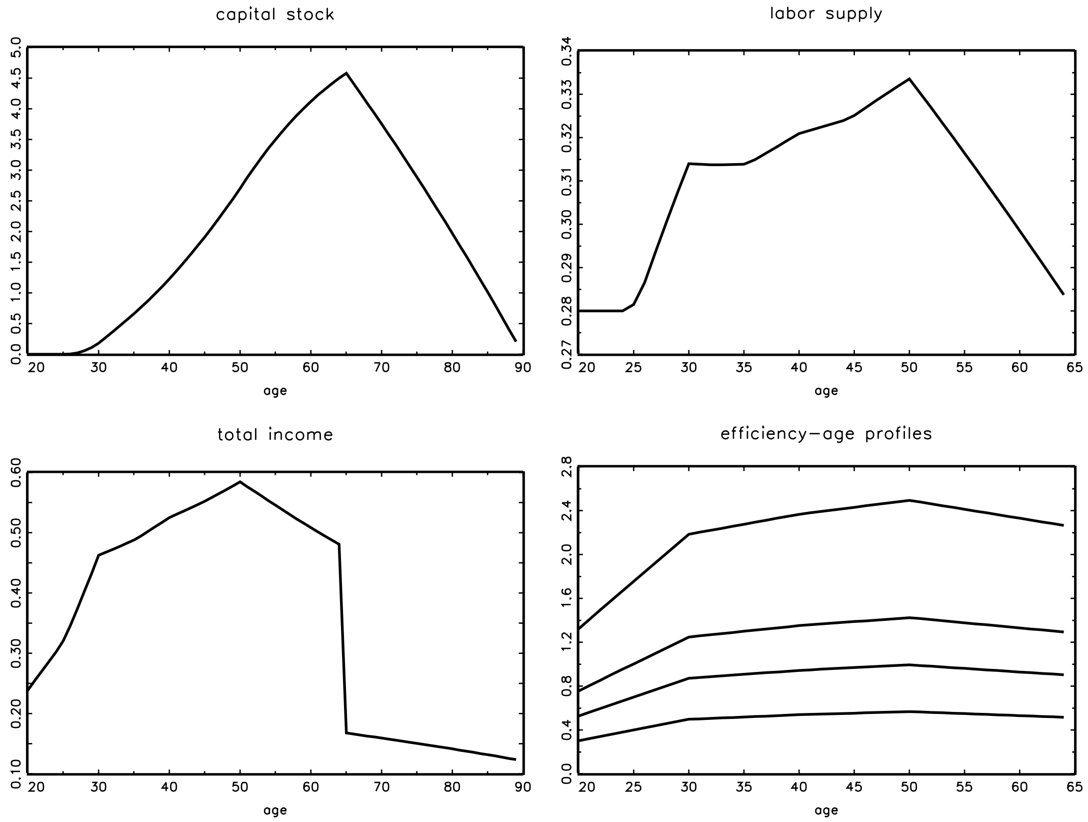
[3] In order to get an even better match of the model and empirical Gini coefficients, we should introduce entrepreneuership into the model as, for example, in Quadrini (2000). However, we kept the model as simple as possible in order to illustrate the main effects.

Figure 1: Non-stochastic steady state age-profiles



illustrated in Figure 2. In the lower picture, we graph the dynamics of output.[4] If the technology level jumps from A_1 to A_2 or vice versa, this is also instantaneously reflected in the movement of the production level. In the upper picture, we graph the behavior of the Gini coefficient of total income. Obviously, total income is highly procyclical. The correlation coefficient of the total income Gini coefficient with output amounts to 0.87. As a simple explanation, the high-productivity workers increase their labor supply by a higher percentage than the low-productivity workers when the wage rates increases during an economic expansion.

Table 1 takes a detailed look at the behavior of the income quintiles. In the first entry row, we display the empirical correlations of output with the 1st, 2nd, 3rd, and 4th income quintiles, and the 80-95% and 95-100% income groups for the US economy, respectively.[5] In the second row, you find the values as resulting from the simulation of the most preferred model of Castañeda et al. (1998). The last two lines display the values obtained from simulating our economy for the two cases that pensions are either proportional to the

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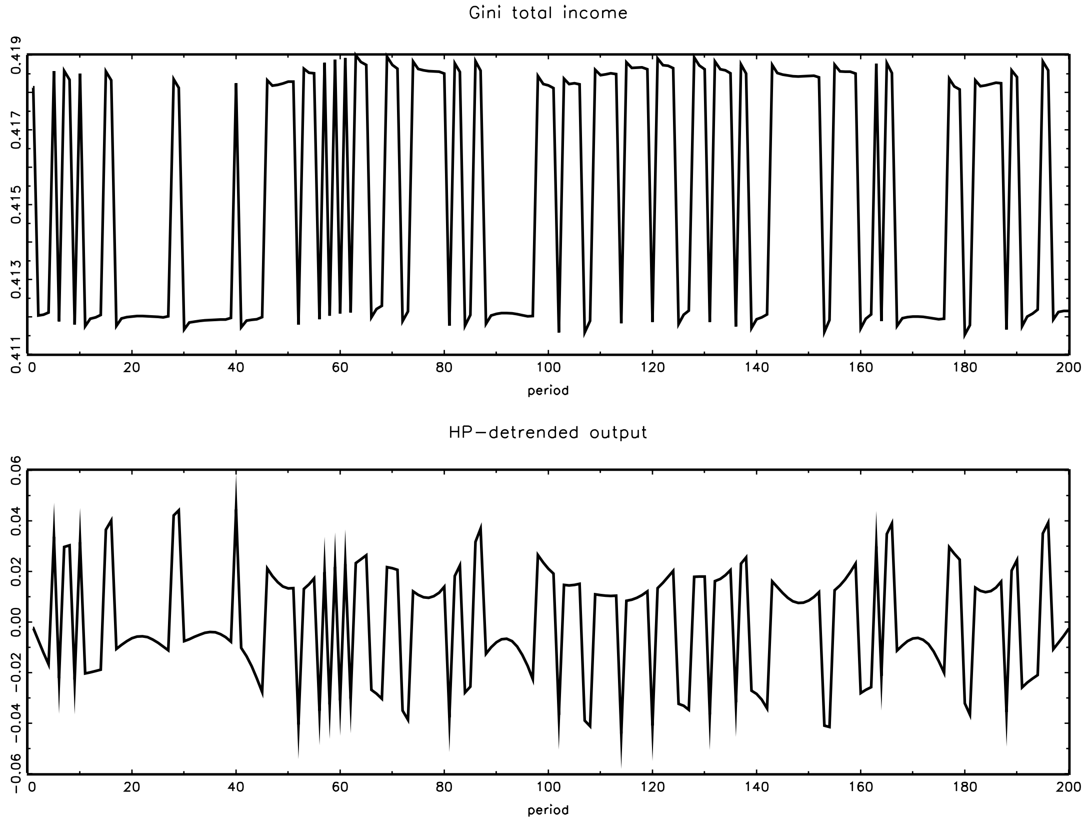
[4] Logarithmic output has been detrended using the Hodrick-Prescott filter with smoothing parameter $\lambda = 100$.

[5] The estimates are reproduced from Table 4 in Castañeda et al. (1998).

Figure 2: Time series simulation



individual efficiency ϵ or lump-sum. Obviously, the model with lump-sum pensions is our preferred model (last row). In this case, the income share of the first and fourth income quintile and the top 5% group match the empirical correlations almost perfectly, while the correlations of the 2nd and 3rd income quintiles with output are too low.

In our model, the dynamics of the income distribution are mainly driven by the intertemporal substitution of labor. During an economic expansion, wages increase and labor (replacement) income is redistributed 1) from low-productivity to high productivity workers, 2) old wealth-rich to young wealth-poor workers, and 3) retired agents to working agents as the former groups increase their labor supply to a larger extent than the latter, respectively. In our economy with overlapping generations, the highest income quintile consists of the workers aged 50-60 with high productivity as these agents have the highest wage income and relatively high interest income. As these agents also hold relatively high wealth, they do not increase their labor supply as much as the younger high-productivity workers. As a consequence, the total income share of the top 5% income earners is almost acyclical.

The lowest income quintile in our economy consists of the very old retired workers (aged

Table 1: Correlation of output with income shares

	0-20%	20-40%	40-60%	60-80%	80-95%	95-100%
US	0.53	0.49	0.31	-0.29	-0.64	0.00
Castañeda et al. (1998)	0.95	0.92	0.73	-0.56	-0.90	-0.84
our model						
i) $b_t(\epsilon) = \epsilon \bar{b}_t$	-0.15	-0.07	-0.08	-0.01	0.31	0.03
ii) $b_t(\epsilon) = \bar{b}_t$	0.40	-0.47	-0.11	-0.24	0.60	0.04

Notes: Entries in rows 1 and 2 are reproduced from Table 4 in Castañeda et al. (1998). Annual logarithmic output has been detrended using the Hodrick-Prescott filter with smoothing parameter $\lambda = 100$.

80 and above) and the young workers with low productivity ϵ_1 and z_1 (aged 20-30). Since the pension income falls relative to the wage income during an economic expansion, the correlation of output with the income share of the first quintile is not close to unity as in Castañeda et al. (1998). Therefore, the introduction of overlapping generations, pensions, and elastic labor helps to improve upon the modeling of the income distribution business cycles dynamics. In fact, in our model, the correlations of the first and second income quintiles with output are even a little bit too low compared to empirical observations. The latter deficiency of our model, of course, could be improved by the introduction of unemployment risk so that the number of employed workers in the lowest income quintiles becomes more procyclical again.

4 Conclusion

Previous work by Castañeda et al. (1998) has been very successful to model the business cycle dynamics of the different income quintiles except for the top 5% group. In addition, the lower income quintiles very almost perfectly correlated with output which is at odds with empirical observations for the US economy. We presented a model with overlapping generations, pensions, and elastic labor that helps to overcome these deficiencies. We therefore conclude that a business cycle model of the income distribution should be a hybrid model including both the risk of unemployment and overlapping generations with elastic labor supply.

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5 Appendix: Computation of the model

In order to compute the model of Section 2, we use the algorithm of Krusell and Smith (1998). This algorithm has been applied to overlapping generations models by Storesletten et al. (2004), among others. Our computation follows common practice in this literature and is described by the following steps:

Algorithm: *Computation of the OLG model with individual and aggregate uncertainty*

Step 1: Compute the non-stochastic steady state with $A \equiv 1$. Store the policy functions and the steady-state distribution of $\{k, s, \epsilon, z\}$.

Step 2: Choose an initial parameterized functional form for the law of motion for the aggregate next-period capital stock $K' = g(K, A)$ and employment $N' = h(K', A')$.

Step 3: Solve the consumer's optimization problem as a function of the individual and aggregate state variables, $\{k, s, \epsilon, z; K, A\}$.

Step 4: Simulate the dynamics of the distribution function.

Step 5: Use the time path for the distribution to estimate the law of motion for K' and N'.

Step 6: Iterate until the parameters converge.

Step 7: Test the goodness of fit for the functional form using, for example, R^2. If the fit is satisfactory, stop, otherwise choose a different functional form for $g(.)$ and/or $h(.)$.

In the first step, the non-stochastic steady state allocation is computed with standard methods. In particular, we discretize the individual state space using a grid over the individual asset space k of 50 points and interpolate linearly between points. The policy functions $k'(k, s, \epsilon, z)$ and $n(k, s, \epsilon, z)$ are computed from the first-order conditions of the household starting in the last period of life, $s = 90$. Special care has to be taken of the corner solutions with $n = 0$ and $k' = 0$.[6] The optimal policy functions are stored in order to use them as an initial guess for the policy functions in step 3. Similarly, we save the non-stochastic steady state distribution and use it as initial distribution for the simulation of the stochastic economy in step 4.

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[6] In addition, we use our own routine for the solution of non-linear equations that is able to handle returns in the form of missing values if, for example, consumption is negative and utility is not defined. For details, please see Chapters 7 and 8 in Heer and Maussner (2005).

In the second step, we postulate the following laws of motion for the next-period capital stock and employment:

$$\begin{aligned} K' &= exp(\theta_0 + \theta_1 \ln(K) + \theta_2 \mathbf{1}_{A'=A_1} + \theta_3 \mathbf{1}_{A'=A_1} \ln(K)), \\ N' &= exp(\kappa_0 + \kappa_1 \ln(K') + \kappa_2 \mathbf{1}_{A'=A_1} + \kappa_3 \mathbf{1}_{A'=A_1} \ln(K')). \end{aligned}$$

Notice in particular that next-period employment is a function of next-period capital stock K' and next-period aggregate productivity A' only. Therefore, employment N is not an aggregate state variable. As an initialization, we set $\theta_2 = \theta_3 = \kappa_1 = \kappa_2 = \kappa_3 = 0$. We choose $\theta_1 = 0.9$ and compute θ_0 and κ_0 so that $K' = K$ and $N' = N$ correspond to their non-stochastic steady state values, respectively.

As our solution, we find the following laws of motion:

$$\begin{aligned} K' &= exp(0.0610 + 0.0126 \ln(K) + 0.9076 \ \mathbf{1}_{A'=A_1} - 0.0043 \ \mathbf{1}_{A'=A_1} \ln(K)), \\ N' &= exp(-1.265 + 0.0179 \ln(K') - 0.1751 \ \mathbf{1}_{A'=A_1} + 0.0064 \ \mathbf{1}_{A'=A_1} \ln(K')). \end{aligned}$$

In step 3, we compute the individual policy function as functions of the individual and aggregate state variables for given law of motion for K' and N'. For this reason, we choose a rather loose grid for the aggregate capital stock K as the curvature of the policy function with respect to this argument is rather low. We find that 7 points are sufficient. Furthermore, we choose 80% and 120% of the non-stochastic steady state aggregate capital stock as the lower and upper boundary for this interval. In our simulations, the aggregate capital stock always remains within these boundaries.

Starting with the non-stochastic steady state distribution as our initial distribution $f_0(k, s, \epsilon, z)$, we simulate the dynamics of the economy. We use a pseudo-random number generator in order to simulate the economy over 200 periods repeatedly. Given the distribution in period t, $f_t(k, s, \epsilon, z)$, we can compute the next-period distribution, $f_{t+1}(k, s, \epsilon, z)$, with the help of the policy functions $k'(k, s, \epsilon, z; K, A)$ and $n(k, s, \epsilon, z; K, A)$. In addition, we can compute aggregate production and the income shares of the different quintiles. For a more detailed description of this step, please see Heer and Maussner (2005), Chapter 6.

We update the parameters by estimating the law of motions for the simulated time series with the help of OLS. We stop the algorithm as soon as the maximum change of the θ_i and κ_j is below 0.001. In our last iteration, the R^2 in the two regressions of the law of motion exceeds 0.999, respectively. Therefore, we can be confident that our postulated laws of motion $g(.)$ and $h(.)$ are satisfactory. The computation takes some 20 hours on a Intel Pentium(R) M, 319 MHz machine. The computer programs are available from the author upon request.

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