A Critique of the Literature on the US Financial Debt Crisis

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Abstract

A healthy financial system encourages the efficient allocation of capital and risk. The collapse of the house price bubble led to the financial crisis that started in 2007. There is a large empirical literature concerning the relation between asset price bubbles and financial crises. I evaluate the key studies with the respect to the following questions. To what extent do the empirical relations in the existing literature help to identify asset price bubbles ex-ante or ex-post? Do the empirical studies have theoretical foundations? On the basis of that critique, I explain why the application of stochastic optimal control (SOC)/dynamic risk management is a much more effective approach to determine the optimal degree of leverage, the optimum and excessive risk and the probability of a debt crisis. The theoretically founded early warning signals of a crisis are shown to be superior, in general, to those empirical relations in the literature. Moreover the SOC analysis provides a theoretical explanation of the extent that the empirical measures in the literature can be useful.

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1. The Congressional Oversight Panel (COP) Special Report on Regulatory Reform

The COP report provides an excellent guide concerning lessons to be learned from the US financial crisis and is a lucid discussion of the following problems or shortcomings of the financial system: The excessive leverage and unregulated shadow financial system are sources of systemic risk. Many financial institutions carry a "dangerous" amount of leverage. Systemic risk is not identified or regulated until the crisis is imminent. The report contains several parts: Lessons from the past; Shortcomings of the present system; Leverage, Capital requirements; Current state of the regulatory system; Critical problems and recommendations for improvement.

There is a large descriptive literature on the financial crisis. I ignore the studies concerning the reform and regulation of financial markets. I focus upon and critique several representative key articles in the literature that concern the aims of the COP Report. These key articles reflect different approaches and contain relevant references. The relevant linkages are that (i) asset prices, debt ratios/leverage affect the vulnerability of specific sectors, such as housing, to shocks, (ii) this vulnerability is transmitted to the larger financial sector through leverage and interrelationships, and (iii) the real economy is then affected. I state/paraphrase/quote and evaluate the key studies with the respect to the following questions. To what extent do the empirical relations in the existing literature help to identify asset price bubbles ex-ante or ex-post? Do the empirical studies have theoretical foundations?

On the basis of that critique, I explain why the application of stochastic optimal control (SOC)/dynamic risk management is a much more effective approach to determine the optimal degree of leverage, the optimum and excessive risk and the probability of a debt crisis. The theoretically founded early warning signals of a crisis are shown to be superior, in general, to those empirical relations in the literature. Moreover the SOC analysis provides a theoretical explanation of the extent that the empirical measures in the literature can be useful.

1.1. Background

Financial crises are not new. From 1792-1933, they occurred roughly every 15-20 years. As the US emerged from the Great Depression, the new financial regulation – including FDIC, securities regulation and banking supervision – effectively protected the system. For the next 50 years, economic growth returned without financial crises. There have always been voices predicting financial crises, but the economics profession ignored them because they lacked theoretical foundations and testable quantitative propositions. These voices of financial disaster were like those who have been predicting earthquake disasters. See Seth Stein, "Disaster Deferred", chapter 2 for a discussion of alarmist earthquake predictions.

The period from 1933 to the 1980s was tranquil. The Savings and Loan Crisis of 1980s did not produce systemic risk. The situation changed in the 1980s and 1990s. Deregulation and the growth of unregulated parallel markets were accompanied by the nearly unrestricted marketing of increasingly complex financial instruments. Alan Greenspan (2002) explained his view on the issue of regulation and disclosure in the over the counter derivatives market as follows.

"By design, this market, presumed to involve dealings among sophisticated professionals, has been largely exempt from government regulation. In part, this exemption reflects the view that professionals do not require the investor protections commonly afforded to markets in which retail investors participate. But regulation is not only unnecessary in these markets, it is potentially damaging, because regulation presupposes disclosure and forced disclosure of proprietary information can undercut innovations in financial markets just as it would in real estate markets".

The attempt by the CFTC to regulate OTC-traded derivatives in 1997-98 was blocked by Fed chairman Greenspan and Treasury Secretary Rubin allegedly on the grounds that such regulation could precipitate a financial crisis. Moreover, Congress in 2000 prohibited regulation of most derivatives.

The financial crisis that began to take hold in 2007 has exposed significant weaknesses in the nation's financial architecture and in the regulatory system designed to ensure its safety, stability and performance. As asset prices deflated, so too did the theory that increasingly guided American financial regulation over the previous three decades – that private markets and private financial institutions could largely be trusted to regulate themselves.

This philosophy characterized the economics profession. Krugman, in his influential article “How Did Economists get it So Wrong”, argues the following. “Few economists saw our current crisis coming…More important was the profession’s blindness to the very possibility of catastrophic failures in a market economy. During the golden years, financial economists came to believe…that stocks and other assets were always priced just right….Meanwhile macroeconomists were divided in their views. But the main division was between those who insisted that free-market economies never go astray and those who believed that economies may stray now and then but any major deviations from the path of prosperity could and would be corrected by the all-powerful Fed.”

Given the dominant macroeconomic philosophy of the academic economics profession, the 2007-08 crisis took the Fed by surprise. The Fed did not perceive the housing price bubble. Greenspan said (2004) that the rise in home values was “not enough in our judgment to raise major concerns”. Bernanke said (2005) that a housing bubble was “a pretty unlikely possibility”. Moreover he said (2007) that the Fed does “not expect significant spillovers from the subprime market to the rest of the economy”.

The dominant macroeconomic economic and finance theories were unable to explain the empirical phenomena. In the 1980s there was a large literature on “rational bubbles”. The irrelevance of this literature is attested to by the fact that it is ignored in the articles that describe and analyze the housing and financial crisis of 2007-08.

Alan Greenspan in his testimony before the House Committee on Oversight and Government Reform October 2008 said: “Those of us who had looked to the self-interest of lending institutions to protect shareholders’ equity, myself included, are in a state of shocked disbelief.” Paul Volcker in April 2008 said that the problem was that we have moved from a commercial bank-centered, highly regulated financial system to one where much of the financial intermediation takes place in markets beyond effective official oversight and supervision. “The sheer complexity, opaqueness, and systemic risks embedded in the new markets – complexities and risk little understood even by most of those with management responsibilities – has enormously complicated both official and private responses to this mother of all crises…".

The design of my Critique of the literature is as follows. Part 1.2 explains how the high leverage in the financial sector transmitted shocks from the housing mortgage sector to the broad financial sector. Part 1.3 gives a specific example of how the "Quants" chose an incredibly high leverage and explains its consequences. Part 2 concerns the actual market anticipation of housing prices. Part 3 discusses an ex-post mean reverting approach to detect the housing price bubble. Part 4 discusses the Bank for International Settlements and the International Monetary Fund studies to detect financial market bubbles and the link between asset prices and financial crises. Part 5 concludes with an evaluation of the limitations of the existing literature. This leads into part 6 that is an exposition of the Stochastic Optimal Control (SOC) approach. I focus upon the following questions: What are the theoretical foundations and actual performance in predicting bubbles compared to the previous discussed studies? How should one interpret the empirical relations featured in the literature?

1.2. Leveraging

It is now widely believed that "'excessive" leveraging, an "excessive" debt ratio, at key financial institutions helped convert the initial subprime turmoil in 2007 into a full blown financial crisis of 2008. Leverage is the ratio of debt L(t)/net worth X(t) , alternatively called the debt ratio, and denoted f(t) = L(t)/X(t) . Although leverage is a valuable financial tool, "excessive" leverage poses a significant risk to the financial system. For an institution that is highly leveraged, changes in asset values highly magnify changes in net worth. To maintain the same debt ratio when asset values fall either the institution must raise more capital or it must liquidate assets.

The relations are seen through equations (i) – (iii). In (i) net worth X(t) is equal to the value of assets A(t) less debt L(t). Equation (ii) is just a way of expressing the debt ratio. Equation (iii) relates the debt ratio f(t) = L(t)/X(t) to the ratio A(t)/X(t) of assets/net worth. Equation (iv) states that the percent change in net worth dX(t)/X(t) is equal to the leverage (1+f(t)) times dA(t)/A(t) the percent change in the value of assets.

(i) $X(t) = A(t) - L(t)$.

(ii) $L(t)/X(t) = f(t) = 1/[(A(t)/L(t) - 1]$.

(iii) $A(t)/X(t) = 1 + f(t)$.

(iv) $dX(t)/X(t) = (1+ f(t))\, dA(t)/A(t)$.

The COP reported that, on the basis of recent estimates just prior to the crisis, investment banks and securities firms, hedge funds, depository institutions, and the government sponsored mortgage enterprises – primarily Fanny Mae and Freddie Mac - held assets worth $23 trillion on a base of $1.9 trillion in net worth, yielding an overall average leverage of $A/X = 12$. The leverage ratio varied widely as seen below.

Broker-dealers and hedge funds	27
Government sponsored enterprises	17
Commercial banks	9.8
Savings Banks	6.9
Average	12

Consider the average, where $A(t) = \$23$ trillion, $X(t) = \$1.9$ trillion, $L(t) = \$21.1$ trillion, then leverage $f = 11.1$. From equation (iv), a 3% decline in asset values would reduce net worth by $dX(t)/X(t) = (1+11.1)(0.03) = 36\%$. The loss of net worth is equal to (0.36)($1.9 trillion) = $0.69 trillion. To maintain the same leverage $f = 11$, the institutions must either raise capital to offset the decline in asset values $dX = dA < 0$, or it must sell off assets to reduce its debt by the same proportion $dL(t)/L(t) = dX(t)/X(t)$, derived from equation (ii). A 3% decline in asset value would require the sale of (0.03)(21.1 trillion) = $630 billion in assets to repay the debt.

Both actions have adverse consequences for the economy. Firms in the financial sector, the financial intermediaries, are interrelated as debtors-creditors. Banks lend short term to hedge funds who invest in longer term assets and who may also buy credit default swaps. Firms that lost $690 billion in net worth would have difficulty in raising capital to restore net worth, without drastic declines in share prices. Similarly, the attempt by group G_1 to sell $630 billion in assets to repay loans will have serious repercussions in the financial markets. The prices of these assets will fall, and the leverage story repeats for other sectors. Institutions G_j who hold these assets will find that the value of their portfolio has declined, reducing their net worth. In some cases, there are triggers. When the net worth of a Fund G_j falls below a certain amount ("break the buck") the fund must dissolve and sell its assets. These may include AAA assets. In turn the sale of AAA assets affects group G_k. Investors in this group thought they were holding very safe assets, but to their dismay they suffer capital losses. The conclusion is that in a highly interrelated system, "high leverage" can be very dangerous. What seems like a small shock in one market can affect via leverage the whole financial sector. The Fed seemed oblivious to this systemic risk phenomenon.

1.2. The Incredible Leverage of Atlas Capital Funding

The story of the Atlas Capital Fund is an excellent example of leveraging discussed above. This is based upon a paper given by Jichuan Yang, one of the principals of Atlas, given at an Applied Mathematics Colloquium at Brown University September 2009. See also the paper by Ren Cheng (former Chief Investment Officer at Fidelity) at the same Colloquium. A group of talented financial engineers: mathematicians, physicists specializing in mathematical finance, decided to establish a Fund in 2003 with $12 *billion* of assets, and $10 *million* of capital, - a leverage of 1200. This Fund was called the Atlas Capital Fund, due to its huge size. The fund portfolio would contain thousands of individual bonds, loans and other types of financial securities. These had longer term maturities, such as 8 years. The liabilities were commercial paper and mid-term notes with maturities ranging from 30 days to 5 years. Atlas would borrow short term and lend longer term to the Hedge Funds. The Funds were set up not to hedge risk but to seek maximum return and they were not in fear of taking risk. Atlas would make its profits from the difference between the lending rate charged to the hedge funds and the cost of short term borrowing. The latter could be reduced to a minimum if Atlas received a AAA rating. This was remarkable goal since most global banks are rated no higher than AA.

Since the portfolio had a much longer maturity than the loans, a major risk to Atlas would be the variable short term borrowing rate. When the 30-day loan matured, Atlas would roll over the 30-day loan at the current rate. If there were difficulties in rolling over, Atlas would have to find banks to give Atlas "emergency" loans to pay off the 30-day debt. These standby banks are called "liquidity providers".

The "financial engineers" built a model to evaluate the risk, which they used to convince the rating agencies to give them the AAA rating. A higher rating lowers the cost of borrowing. The model would simulate the movement of the $12 billion of individual assets as well as their correlated behavior. These assets raged from bonds, loans, to more complicated structured securities backed by all kinds of collateral. The mismatch of the timing of cash flows of assets and liabilities, the price movements , the rating changes, the defaults and recovery had to be "accurately" modeled, calculated and simulated. For each potential future price movement, the model would calculate the loss and return. After tens of thousands of such simulations, the financial engineers would get the expected loss and expected return by certain types of averaging the individual outcomes. These simulations did in fact convince the rating agencies to give Atlas a AAA rating and hence a low cost of borrowing.

At the beginning Alas was extremely profitable. Stock holders received 100% of their money back in the first year of operations. This was due to the leverage of $12 billion of assets/$10 million of capital = 1200. The Fed was most accommodating with its low interest policy. Moreover, Chairman Alan Greenspan was the champion of financial innovation and was fighting off regulatory reform on all fronts. About three years after Atlas started its operations, the US financial industry went into one of its worst crises. The cascading effects of leverage discussed above then occurred. Atlas was blamed as being one of the main culprits for causing the crisis. Jichuan Yang, one of the principals of Atlas, wrote in 2009: "Today, if someone tells me that all these things can be simulated by an elegant mathematical model with any realistic accuracy, I would be tempted to say that he's probably an overconfident idiot".

2. Market Anticipations of the Housing – Mortgage Debt Crisis

It has been commonly asserted that root of the problem lies with the subprime mortgage market. That is not quite accurate since, although the subprime market was the trigger for the crisis, any one link in the highly leveraged financial intermediaries could have precipitated the crisis, as explained in section (1.2) above. I now turn to the market anticipations of housing prices: the methods used and why they were so erroneous.

Gerardi et al explore whether market participants could have or should have anticipated the large increase in foreclosures that occurred in 2007. They decompose the change in foreclosures into two components: the sensitivity of foreclosures to a change in housing prices times the change in housing prices. The authors conclude that investment analysts had a good sense of the sensitivity of foreclosures to a change in housing prices, but missed drastically the expected change in housing prices. The authors do not analyze whether housing was overvalued in 2005-06 or whether the housing price change was to some extent predictable.

The authors looked at the records of market participants from 2004-06 to understand why the investment community did not anticipate the subprime mortgage crisis. Five basic themes emerge. The first is that the subprime market was viewed as a great success story in 2005. Second, mortgages were viewed as lower risk because of their more stable prepayment behavior. Third, analysts used sophisticated tools but the sample space did not contain episodes of falling prices. Fourth, pessimistic feelings and predictions were subjective and not based upon quantitative analysis. Fifth, analysts were remarkably optimistic about Housing Price Appreciation (HPA).

Analysts who looked at past data on housing prices, such as the four-quarter appreciation, could construct the histogram below. This is taken from Stein (2010). In the aggregate, housing prices never declined from year to year during the period 1980q1-2007q4. The mean appreciation was 5.4% pa with a standard deviation of 2.94% pa. The optimism could be understood if one asks: on the basis of this sample of 111 observations, what is the probability that housing prices will decline? Given the mean and standard deviation, there was only a 3% chance that prices would fall.

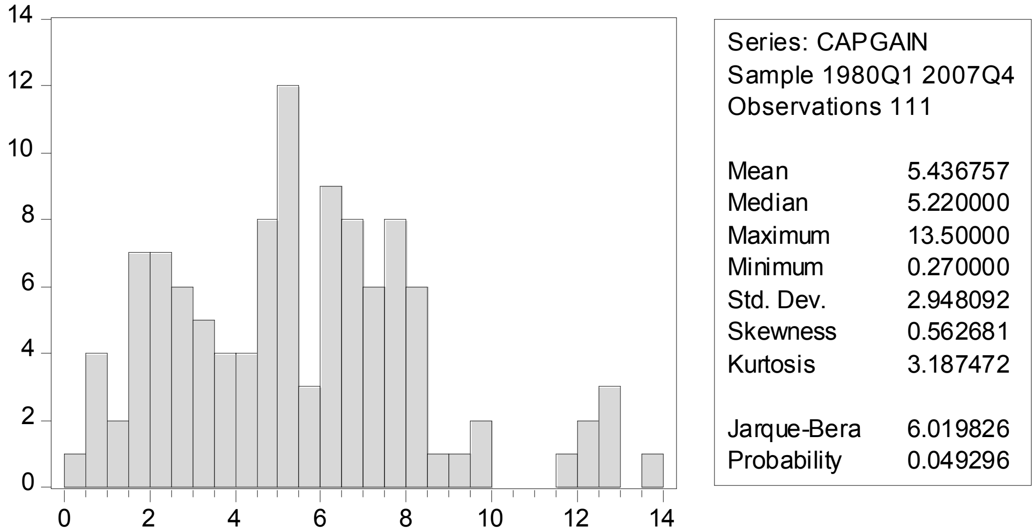

Figure 1. Histogram and statistics of CAPGAINS = Housing Price Appreciation HPA, the change from previous 4-quarter appreciation of US housing prices, percent/year on horizontal axis. Frequency is on the vertical axis. Source of data: Office of Federal Housing Price Oversight.

The best estimates of the analysts were that the rates of housing price appreciation CAPGAIN or HPA in 2005 - 2006 of 10 to 11 % per annum would be unlikely to be repeated but that it would revert to its longer term average. A Citi report in December 2005 stated that "…the risk of a national decline in home prices appears remote. The annual HPA has never been negative in the United States going back at least to 1992." *Therefore no mortgage crisis was anticipated.*

There was no economic theory or analysis in this approach. It was simply a VaR value at risk implication from a sample based upon relatively recent data. There was no consideration of what would happen if the probability distribution/histogram would change. More fundamentally, no consideration was given to the economic determinants of the probability distribution. This was the fatal error.

3. Mean Reverting Ratios used in prediction: Moody's Model

Another approach was taken in evaluating and predicting changes in housing prices. The issue revolves around the challenge of assessing if the actual market value of a financial variable is consistent with its underlying or "fundamental" value. If the market value deviates from its "fundamental" value, then a reversion is anticipated. This type of analysis was successfully used in the evaluation of real exchange rates. A dynamic model was developed where the longer run time varying equilibrium real exchange rate, called the Natural Real Exchange rate NATREX, was a function of specific time varying "fundamentals". The dynamic processes specified just how the actual real exchange rate converges to the NATREX. See Stein (2006) for a theoretical exposition of the NATREX model and it application to the Euro and currencies of the Central and Eastern European Countries. Many authors successfully applied the NATREX model to explain the movements of real exchange rates of countries in Europe, Asia, China, Latin America, Canada, and Africa.

Thus the procedure was to specify an equation for the "equilibrium" value of the asset based upon specific fundamentals" and then an equation for the adjustment of the actual asset price to the "equilibrium" value. In the case of housing, the basic statistic is the Fiserv Case-Shiller repeat purchase house price index P(t). Moody's Economy (2008) for example developed an econometric model of the housing market to identify the forces determining P(t) and evaluate to what extent it can be explained by "fundamentals" and to what degree they are the result of temporal factors. The dynamics are mean reversion to the level associated by the fundamentals. Several approaches are taken. In one, the dependent variable is the ratio of housing prices to household income. In another the dependent variable is the ratio of housing prices to apartment rents. The logic is that owning a house and renting an apartment are substitutes, though not perfect. In these approaches the hypothesis is that housing prices will be mean reverting.

Moody's model has two equations. One is that the equilibrium housing price P*(t) is related to fundamentals Z(t), which can be household income, household wealth, age distribution and other variables. The second equation is actual change in price dP(t) equation, which contains serial correlation terms, a mean reversion term and other factors. They used the estimates from these two equations to predict housing price changes. This approach is a significant advance from the VaR approach described in part 2 above.

One can get a feeling of the "overpricing" of houses or the housing bubble in the following way. I constructed a ratio PRICEINC of housing prices P(t) to disposable income Y(t). This is almost identical to Shiller's Ratio of Median Houses Price to Median Income.

The latter came from FRED data set of the Federal Reserve Bank of St. Louis. The housing price index was based upon the 4-quarter appreciation of US housing prices reported by the Office of Federal Enterprise Oversight, labeled CAPGAIN in the figure 2. The housing price P(t) was derived from an equation P(t) = P(t-1)[1+ CAPGAIN], where the initial value P(1980q1) = 1. The ratio of housing price/disposable income PRICEINC = P(t)/Y(t). In figure 2, both variables are normalized, with a mean of zero and standard deviation of one.

It is seen that the ratio PRICEINC = P(t)/Y(t) was very stable, almost constant from 1980 to 2000. Then there was a housing bubble, the CAPGAIN or price appreciation shot up from 2000 to 2005. As a result the ratio of housing prices to disposable income rose drastically, by more than two standard deviations from 2000 to 2007. The great deviation of the price/income ratio from its long term mean would suggest that there was a housing price "bubble" and that housing prices were greatly overvalued. A housing crisis would be predicted, where the ratio P(t)/Y(t) would return to the long term mean, which is the zero line.

Figure 2. Housing price/disposable income P(t)/Y(t) =PRICEINC, normalized; CAPGAIN = HPA, Housing price appreciation quarter over previous fourth quarter, normalized.

4.1. The BIS study of Asset Prices and Financial Instability

The Bank for International Settlements study by Borio and Lowe (2002) presents empirical evidence that it is possible to identify financial imbalances and that sustained rapid credit growth combined with large increases in asset prices appears to increase the probability of an episode of financial instability, banking system crises. They write that the existing literature provides little insight into key questions that are of concern to central banks and supervisory authorities. (i) When should credit growth be judged "too fast"? (ii) What is the cumulative effect of an extended period of strong credit growth? (iii) Are lending booms more likely to end in problems for the real economy if they occur simultaneously with other imbalances in either the financial sector or in the real economy?

The aim of their paper is to investigate the usefulness of credit, asset prices and investment as predictors/Early Warning Signals (EWS) of future problems in the financial system. Specifically they are interested in two questions. (a) Can useful indicators be constructed using only information available to policymakers at the time the policy decisions are made? (b) Can signals be made more accurate by jointly considering asset prices, credit and investment?

Their work builds on that of Kaminsky and Reinhart (K-R) and of Bordo et al (2001). They ask to what extent the occurrence of a boom in asset prices, credit or investment provides a useful signal that a financial crisis is imminent. Like K-R, the BIS study defines a threshold value of each relevant indicator series. When the indicator takes a value that exceeds the threshold value, they define this as a "boom" and it is said to signal an impending crisis. The BIS study differs from that of K-R in three respects. The BIS study: (i) focuses upon cumulative processes rather than just one year; (ii) only uses ex-ante information. (iii) considers combinations of indicators. The sample consists of annual data 1960-99 for 34 countries including all of the G-10.

They define a credit boom as a period where the ratio of credit/GDP deviates from its trend by a specific amount called the "credit gap" Similarly they define an asset price boom as a period in which real asset prices deviate from their trends by specified amounts. This is defined as the "asset price gap". The BIS study concludes that the best EWS is a combination of a credit gap of 4 percentage points and an asset gap of 40 percent.

The BIS study, like that of K-R is a search for empirical relations and neither is based upon an analytical structure. For example, why are the arbitrary asset and price gaps relevant? They use a cross country empirical analysis, but how well do their empirical measures work for specific countries? Can their approach shed light upon the 2007-08 housing crisis that led to a financial crisis?

They make suggestions for further research. (1) Such work should pay greater attention to conceptual paradigms and be more closely tailored to the needs of policymakers: length of horizons in identifying cumulative processes, the use of ex-ante information, balancing type I/II errors. (2) The definition of financial strains should be examined more carefully. (3) The analytical research concerning the interaction between financial imbalances and the real economy.

4.2. International Monetary Fund: Vulnerabilities to Housing Market Corrections

The International Monetary Fund WEO report April 2008 Box 3.1 can be viewed as a follow up to the BIS study. The IMF study reflects the state of knowledge concerning Assessing Vulnerabilities to Housing Market Corrections. The study asks: Which countries are most likely to experience further slowdown in housing prices and residential investment? Like the BIS study the WEO study is essentially an empirical approach. Vulnerability to a housing market correction is assessed based on two different indicators. First: the extent to which the increase in house prices in recent years cannot be explained by fundamentals. Second: the size of the increase in the residential investment/GDP ratio experienced during the past ten years.

The first part attempts to assess the "Overvaluation in Housing Prices". The sample is a cross section of countries. For each country, house price growth is modeled as a function of an "affordability ratio" - the lagged ratio of house prices/disposable income, growth in disposable income per capita, short-term interest rates, credit growth, changes in equity prices and working age population. The unexplained increase in housing prices – house price gaps - could be interpreted as a measure of overvaluation and therefore used to identify which countries may be particularly prone to a correction in house prices. The Figure on WEO page 113 plots house price gaps 1997-2007 to countries. Ireland, Netherlands, UK and Australia are in the high end, and US is in the low range in terms of vulnerability to housing correction.

The second part of the study concerns the ratio of residential investment/output that is a measure of the direct exposure of the economy to a weakening housing market. Residential investment does not normally account for a very large share of the economy. Average for advanced economies was 6.5%. Ireland and Spain are at the high end and US at 4% is below average. They use arbitrary measures to gauge a country's vulnerability to a decline in housing construction. Was residential investment/GDP significantly above the historical trend? Since 2006, the decline in the ratio brought it back to trend in the US. Countries that look particularly vulnerable to further housing price correction are Ireland, UK, Netherlands and France.

The limitations of the IMF/ WEO study can be seen from the vantage of 2009. The *London Summit report* March 2009 provided a plan for recovery. Its recommendations were partially fulfilled with the establishment of the EU/European Systemic Risk Board, devoted to the monitoring of systemic risk. The report emphasizes that the crisis and in particular the real *estate downturn was not predictable since "traditional* macro warning signals were absent", and that "*the lack* of *precise warning signals seems to be present in all crises including the current one"* – (my italics).

In summary both the BIS and he WEO studies leave unanswered questions: (i) What are theoretically based fundamentals? (ii) Are they good predictors by country? Did the house price gap explain the recent US experience?

5. Conclusions

Several questions are the focus of this Critique of the literature on the US financial crisis. To what extent do the empirical relations in the existing literature help to identify asset price bubbles ex-ante or ex-post? Do the empirical studies have theoretical foundations? What is an excessive leverage or debt ratio that increases the probability of a debt crisis?

The key studies have limitations. As a rule, the housing price bubble was not predicted ex-ante. The most useful warning signal was the rapid rise in the ratio of housing prices/disposable income or the ratio of housing prices/rents. On a macro level, there were empirical studies of whether either credit growth or asset prices was "excessive". The criteria for "excessive" were arbitrary. There was no concept of optimality as a benchmark in measuring "excessive" asset growth or asset price. On either the macro or the micro level, there were no analytic foundations of whether the price ratio or the asset price growth deviated from "fundamentals" to justify alarm.

In section 2 above, I showed how the financial engineering approach by the "Quants", where price anticipations were based upon recent frequency distributions, led to a severe underestimation of risk. They assumed that the observed prices or price changes are samples from a distribution with a constant mean μ and finite variance σ^2. They used the central limit theorem that states that the sample mean approaches the normal distribution with mean μ and variance σ^2/n as the sample size increases. Therefore they could use the VaR value at risk to estimate probability of losses.

Their fatal error was to assume that the probability density function of prices or price changes is relatively constant and independent of the behavior of market participants. They viewed the distribution function of price changes like mortality tables, which are not affected by those who study them.

The "Quants" failed to show understanding of the economics underlying financial crises: what produced the price movements, how the market participants acted upon these price movements in a way that led to further price movements and what price movements are or are not sustainable. The financial engineering by the "Quants", for example of Atlas Fund, led one of the principals in retrospect to call the approach "idiotic".

The approach that I now discuss concerns my recent work, which applies the techniques of stochastic optimal control to derive an optimal debt ratio or leverage that "optimally" balances risk and expected return in a world where the future is unpredictable. I explain: What are the consequences of a debt ratio that deviates in either direction from the derived optimal ratio? Why did the observed leverage deviate form the optimal? What are early warning signals of a debt crisis? How can the more successful empirical studies be explained theoretically? The answers to these questions have significant implications for both internal monitoring by firms or for "regulation". Regulation is the subject of a large literature, but it is not discussed in this paper.
The techniques of analysis are developed in Fleming and Stein (2006). The exposition in the text below is a development of Stein (2010). My exposition here attempts to be more intuitive, and focuses upon the ideas and results relevant for economics.

6. Stochastic Optimal Control (SOC)/Dynamic risk management

6.1. Performance Criterion Function

The financial crisis was precipitated by the mortgage crisis and spread through the financial sector. At one end of the financial chain are the mortgagors/debtors who borrow from financial intermediaries – banks, hedge funds, government sponsored enterprises. The latter are creditors of the mortgagors, but who ultimately are debtors to institutional investors at the other end. For example FNMA borrows in the world bond market and uses the funds to purchase packages of mortgages. If the mortgagors fail to meet their debt payments, the effects are felt all along the line. The stability of the financial intermediaries and the value of the traded derivatives CDO, CDS, ultimately depend upon the ability of the mortgagors to service their debts. For this reason I focus upon the mortgagors.

One must have a performance criterion to answer the question: what is an optimal leverage in a stochastic environment. The techniques of analysis are drawn from the mathematical literature Stochastic Optimal Control, which is just optimal dynamic risk management. As my criterion of performance, I could either consider maximizing the expected net worth of the mortgagors or of the consolidated industry of mortgagors and financial intermediaries. Net worth of a sector X is equal to assets (capital) K less debt L, equation (1). The only difference is that in the first case, debt L(t) is that of the mortgagors and in the second case it is of the financial intermediaries.

The mathematics will be the same in both cases. Let X_1 be the net worth of the mortgagors who have capital K and debt L_1. Thus $X_1(t) = K(t) – L_1(t)$. Let X_2 be the net worth of the financial intermediaries. Their net worth is $X_2(t) = L_1(t) – L_2(t)$, since their assets are the liabilities of the mortgagors. The net worth of the consolidated mortgagors and financial intermediaries is $X(t) = X_1(t) + X_2(t) = K(t) – L_2(t)$.

(1) $X(t) = K(t) – L(t)$.

The performance criterion that I have chosen is the maximization of W(T) the expectation E(.) of the logarithm of net worth at some later time T from the present, equation (2). This is a sensible and very risk averse objective criterion because, in the deterministic case, if net worth X(T) = 0, the value of W(T) is minus infinity.

(2) $W(T) = \max E \ln X(T)$.

The stochastic optimal control problem is to select debt ratios $f(t) = L(t)/X(t)$ during the period (0,T) that will maximize W(T) in equation (2). This ratio is precisely the optimal leverage, and will vary over time. The solution of the stochastic optimal control/dynamic risk management problem tells us what is an optimal and what is an "excessive" leverage. Since W(T) is a positively sloped concave function, both expected return and risk are taken into account. Bankruptcy X = 0 is severely penalized. Low values of net worth close to zero may not be likely, but they have large negative utility weights. Hence the criterion function reflects strong risk aversion.

6.2. Dynamics of net worth

State variable net worth X(t) varies over time. The optimization of W(T) must be subject to how net worth varies. Whereas the choice of criterion function (1) is not controversial, there are several choices for the dynamic process of net worth. Each one has a different implication for the optimal leverage or debt ratio. Some assumed processes, such as discussed in part 2, led to bubbles and are unsustainable. This point will be discussed below in detail.

The dynamics of net worth start with equation (3). I focus upon the housing market as the example, but the analysis is quite general. The change in net worth is the change in capital dK(t) less the change in debt dL(t). Capital K(t), equal to the value of houses, is the product of the price P(t) of the asset (Housing price index) and the Q(t) the physical quantity. Hence K(t) = P(t)Q(t).

(3) $dX(t) = dK(t) - dL(t)$.

The change in capital in equation (4) is the sum of two terms. The first P(t)dQ(t) is simply I(t) investment in housing. The second is the total capital gain or loss, equal to the product of the value of housing K(t) times the price change dP(t)/P(t).

(4) $dK(t) = d[P(t)Q(t)] = P(t)dQ(t) + Q(t)dP(t) = I(t) + K(t)\, dP(t)/P(t)$.

The change in the debt dL(t) equation (5) has two broad components. The first term i(t)L(t) is the interest payments on the existing debt, where i(t) is the interest rate. The second set of terms is expenditures less income. Income is assumed to be derived from capital, as would be the case if the housing generated rents. This is equation (6), where $\beta(t)$ is the ratio of income Y(t) to K(t) capital. Expenditures are investment I(t) plus consumption C(t).

The debt grows when interest owed on the existing mortgages plus the excess of expenditure less income is positive. An example is that households borrowed and refinanced their mortgages to allow them to spend in excess of their income. Their anticipation was that, at some future date T, the value of the house exceeded their debt. If at date T, the value of capital K(T) exceed the debt L(T), the mortgagor had a "free lunch". If at date T the value of the house is less than the debt, the mortgagor has negative equity and faces foreclosure.

(5) $dL(t) = [i(t)L(t) + I(t) + C(t) – Y(t)]dt$

(6) $Y(t) = \beta(t) K(t)$.

The change in net worth dX(t) is equation (7).

(7) $dX(t) = K(t)[dP(t)/P(t) + \beta(t)dt] – i(t)L(t)dt – C(t) dt$.

A *simplifying* assumption is that consumption C(t) is proportional to X(t) net worth, where the proportion is the productivity of capital: $C(t) = \beta(t)X(t)$.

Let $f(t) = L(t)/X(t)$ be the leverage or debt ratio and $k(t) = K(t)/X(t) = (1+f(t))$ is therefore the ratio of capital (assets) to net worth. That is why I referred to either f(t) or k(t) as leverage. Then the change in net worth can be written as equation (8), which is the basic equation for the dynamics of net worth.

(8) $dX(t) = X(t)\{(1+f(t))dP(t)/P(t) + [\beta(t) – i(t)]f(t)dt\}$

The productivity of capital less the interest rate $[\beta(t) – i(t)]$ is time varying and observable. The capital gain term dP(t)/P(t) is not observable since dP(t) involves the future.

6.3. Stochastic Processes

The basic stochastic variable in equation (8) is dP(t) the change in the housing price. Equations (9) – (10) contain two ideas, inspired by Bielecki and Pliska and Platen-Rebolledo, and discussed in Fleming (1999).

The first, in equation (9)/(9a), is that there is a price trend ρ. The initial value of the price is P, which can be normalized at one. Variable y(t) in (9)/(9a) is a deviation from the trend. The second idea, expressed in equations (10)-(11), is that deviation y(t) is an ergodic mean reversion term whereby the price converges towards the trend. The speed of convergence of the deviation y(t) towards the trend is described by finite coefficient $\alpha > 0$. The stochastic term is $\sigma dw(t)$. The solution of stochastic differential equation (10) is (11). The deviation from trend converges to a distribution with a mean of zero and a variance of $\sigma^2/2\alpha$.

(9) $P(t) = P \exp (\rho t + y(t))$, $P = 1$, (9a) $y(t) = \ln P(t) – \ln P – \rho t$.

(10) $dy(t) = -\alpha y(t)dt + \sigma dw(t)$. $\infty > \alpha > 0$, $E(dw) = 0$, $E(dw)^2 = dt$.

(11) $\lim y(t) \sim N(0, \sigma^2/2\alpha)$.

The choice of price trend ρ is very important in determining the optimal leverage. I impose a constraint that the assumed price trend must not exceed the rate of interest. If this constraint is violated, as occurred during the housing price bubble, debtors were offered a "free lunch" as described above. Borrow/Refinance the house and incur a debt that grows at the rate of interest. Spend the money in any way that one chooses. Insofar as the house appreciates at a rate greater than the rate of interest, at the terminal date T the house is worth more than the value of the loan, P(T) > L(T). The debt L(T) is easily repaid by selling the house at P(T) or refinancing. On has had a free lunch. In the optimization, one must constrain the trend ρ not to exceed the rate of interest i(t). This constraint is equation (12).

(12) $\rho \leq i(t)$. No free lunch constraint

An alternative justification for equation (12) is as follows. The present value of the asset

(12a) $PV(T) = P(0) \exp [(\rho - i)t]$,

where trend ρ is the rate of appreciation or capital gain and i is the interest rate. If $(\rho - i) > 0$, the present value diverges to plus infinity. An infinite present value is not sustainable.

The Market estimated the price trend from recent experience, described in figure 2 and histogram figure 1. From 2000 to 2004, the capital gain greatly exceeded the interest rate. This assumption violates the "no free lunch" constraint, equation (12)/(12a). That is why the rates of appreciation of 10 – 14 % p.a. were unsustainable. This assumption was to have dire consequences, as discussed below.

6.4. Optimal debt ratio - leverage

The expected growth of net worth is equation (13), graphed as figure 3. It is a concave quadratic function of the control variable, the leverage or debt ratio f (t) = L(t)/X/(t). The debt ratio that maximizes the expected growth of net worth is f*(t), equation (16). This is the time varying ratio that maximizes equation (1) subject to the stochastic processes (8), (9) - (10). At the optimum debt ratio the expected growth of net worth is maximal at W*. The variance of the growth of net worth var d[ln X(t)] is equation (15). It is a quadratic function of the leverage times the variance in the price equation (10).

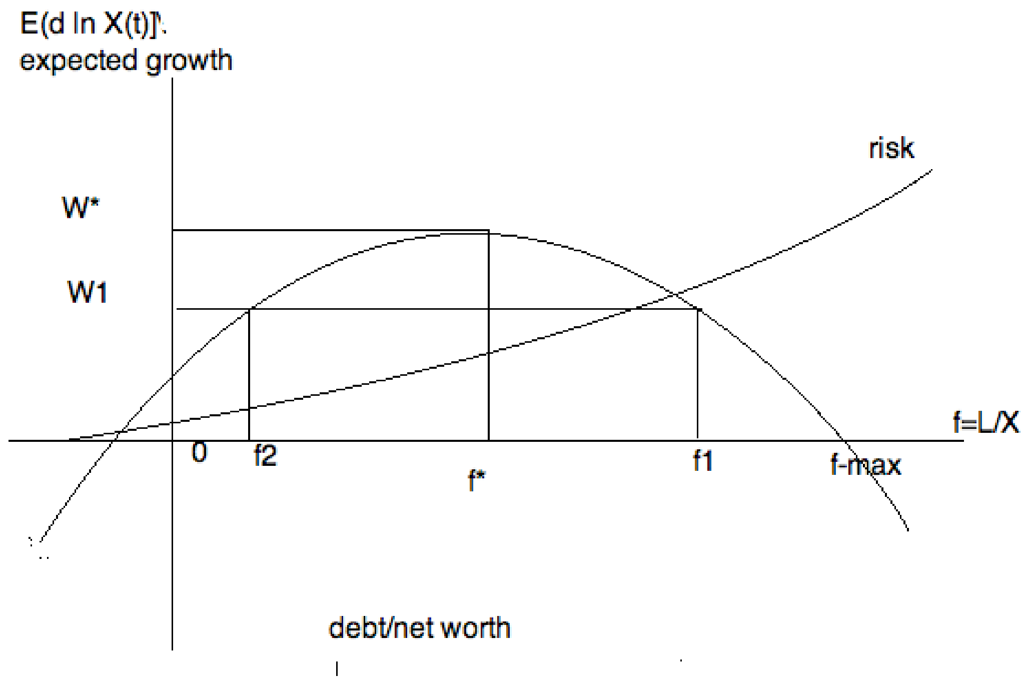

Figure 3. Expected Growth of Net Worth W(f(t)) equation (13), and variance of growth of net worth, risk, equation (15). The optimum debt ratio f*(t)/ leverage, is equation (16). As f(t) exceeds optimum f*(t), expected growth declines and risk rises. At f-max, the expected growth is zero.

The optimum debt ratio, leverage f*(t) in equation (16) is positively related to the productivity of capital $\beta(t)$ less the real rate of interest $r(t) = i(t) - \rho$, equal to the nominal rate of interest i(t) less ρ the trend of prices. The "no free lunch" constraint is that the real rate of interest must be non-negative.

Expected Growth W(f(t)) and Risk

(13) $W(f(t)) = E[d \ln(X(t)]$

$= [(1+f(t))(\rho + (1/2)\sigma^2 - \alpha y(t)] + (\beta(t) - i(t))\, f(t) - (1/2)(1+f(t))^2\sigma^2$

(14) $r(t) = i(t) - \rho \geq 0$. Real rate of interest constraint

(15) $\text{var } d[\ln X(t)] = (1+f(t))^2\sigma^2\, dt$ Risk

Optimal debt/net worth, leverage, f(t)= L(t)/X(t).

(16) $f^*(t) = \{[\beta(t) - (i(t) - \rho) - (1/2)\, \sigma^2] - \alpha y(t)\}/\, \sigma^2$

$f^*(t) = \{[\beta(t) - r(t)] - (1/2)\, \sigma^2] - \alpha y(t)\}/\, \sigma^2$

Corresponding to any debt ratio f(t) is an expected growth of net worth W(f(t)). The optimum leverage, debt ratio f*(t) maximizes the expected growth of net worth W[f*(t)] = W*(t). As the debt ratio deviates from the optimum, the expected growth of net worth declines.

One can never be certain of what is the correct trend ρ of prices, even with the constraint that it is not greater than the rate of interest. The positive value of the real rate of interest r(t) in (14) is unknown. Therefore the choice of an optimum leverage f*(t) at any time is subject to specification error.

Consider several cases. In one, the market anticipations of the price trend as described in part 2 is based upon the relatively recent experience of large capital gains. From histogram figure 1 and figure 2, the mean capital gain over the entire period 1980q1v- 2007q4 is approximately equal to the interest rate. But from 1998 to 2004, the capital gain rose to about two standard deviations above the mean. This would imply capital gains of 5.4 + 2(2.9) = 11.2% per annum. Insofar as the market estimated the trend from recent experience, the trend would have been estimated at 11% pa, significantly above the rate of interest. This overestimate of the trend leads to a selected leverage say f1 or f > f-max. For leverage greater than f-max the expected growth of net worth is negative. The risk, equation (15), rises at an increasing rate, for all positive leverage (1+f(t)).

If leverage f1 is selected, the loss of the growth of net worth is W* -W1. The excess debt $\Psi(t) = f(t) - f^*(t) = f1 - f^*$ is the difference between the actual debt f(t) = f1 and the optimal debt f*(t). The loss of expected growth is a quadratic function of the excess debt. This is equation (17).

(17) $[W^* - W(f(t)] = (1/2)\sigma^2[f(t) - f^*(t)]^2 = (1/2)\sigma^2 \Psi(t)^2$.

$\Psi(t) = f(t) - f^*(t)$ Excess debt

The excess debt ratio $[f1 - f^*(t)] > 0$ reduces expected growth from W* to W1 and increases risk. The distribution function of the expected growth shifts upwards and to the left. Insofar as there is an excess debt, the probability of losses of net worth increases.

Alternatively, suppose that there is government "regulation" to reduce risk and leverage $f2 < f^*(t)$ is imposed. Then the risk is indeed reduced, according to equation

(15), the convex risk curve in figure 2. However, the expected growth is reduced to W1 < W*. The loss of expected growth is the same as before, but now the risk is lower.

Finally, suppose that one tries to estimate the trend with the constraint (14). There is bound to be an error h > 0, which leads to a leverage ratio between (f1= f* - h) < f*(t) < (f* + h = f2). The loss of expected growth does not average out because the income loss in (17) is in the square. As the leverage ratio ranges between f1 and f2, the average loss of growth worth is again W1 < W*. Equation (17) explicitly measures the loss of expected growth due to misspecification of the trend.

6.5. Summary

The contributions of the SOC analysis above can be summarized.

(1) The optimum debt ratio or leverages maximizes the expected growth of net worth.

(2) As the debt ratio rises above the optimum, the expected growth of net worth declines and the risk rises. (3) The probability of a crisis is positively related to the excess debt, equal to the difference between the actual and optimal debt ratio, measured in standard deviations. (4) An unambiguous early warning signal EWS of a debt crisis would be that the leverage f(t) = L(t)/X(t) exceeds f-max, so that the expected growth of net worth is negative and the risk is high.

7. Early Warning Signals of the Crisis

The financial crisis was precipitated by the mortgage crisis for several reasons. First, a whole structure of financial derivatives was based upon the ultimate debtors – the mortgagors. Insofar as the mortgagors were unable to service their debts, the values of the derivatives fell. Second, the financial intermediaries whose assets and liabilities were based upon the value of derivatives were very highly leveraged. Changes in the values of their net worth were large multiples of changes in asset values. Third, the financial intermediaries were closely linked – the assets of one group were liabilities of another – as described in sections 1.1 and 1.2. A cascade was precipitated by the mortgage defaults.

The “Quants”/financial engineers ignored these points. They have had very microscopic points of view and the unfounded belief that the probability distribution of recent price changes is time invariant. Moreover they ignored the fact that other "Quants" were doing the same thing, based upon the same models. They did not consider that their collective behavior would affect the probability distribution. For these reasons, I focus upon the excess debt of the mortgagors. The whole structure of derivatives rested upon the mortgagors being able to service their debts. Hence my basic question is: Did the debt ratio of the mortgagors significantly exceed f-max in figure 3?

The application of the optimal dynamic risk management/SOC analysis is done in several steps. First, the bubble was generated by the market view that the trend of prices – the capital gains – exceeded the rate of interest. Then I show how the collapse occurred when the capital gain fell below the rate of interest. Defaults and bankruptcies occurred. Second, on the basis of the analysis in part 6, I derive estimates of the excess debt $\Psi(t) = f(t) - f^*(t)$ that lowered the expected return and raised risk. Thereby early warning signals are derived. Finally, I relate the SOC analysis to the Case-Shiller index and Moody's mean reversion approach.

7.1. The Bubble and Collapse

Demyanyk and Van Hemert (D-VH) had a data base consisting of one half of the US subprime mortgages originated during the period 2001-2006. At every mortgage age, loans originating in 2006 had a higher delinquency rate than in all the other years since 2001. They examined the relation between the probability Π of delinquency or/foreclosure/binary variable $z = \{1,0\}$, denoted as $\Pi = Pr(z)$ and sensible economic variables, vector X. They investigated to what extent a logit regression $\Pi = Pr(z) = \Phi(\beta X)$ can explain the high level of delinquencies of vintage 2006 mortgage loans. A logit model specifies that the probability that z = 1 is:

$Pr(z = 1) = \exp(X\beta)/[1+ \exp (X\beta)]$. Hence $\ln \{Pr(z=1)/Pr(z=0) \} = X\beta$.

Vector β is the estimated regression coefficients.

They estimated vector β based upon a random sample of one million first-lien subprime mortgage loans originated between 2001 and 2006. The first part to their study provides estimates of β, the vector of regression coefficients and the importance of the variables in vector X. The second part inquires why the year 2006 was so bad. The approach is based upon the equation (18). The contribution C(i) of component X_i in vector X to the probability of default in year 2006 relative to the mean is:

(18) $C(i) = (\delta\Pi/\delta X_i)\, dX_i = \Phi(\beta X_m + \beta_i\, dX_i) - \Phi(\beta X_m)$, m = mean value

The probability of delinquency when the vector X is at its mean value is $\Phi(\beta X_m)$. The added probability resulting from the change in component X_i in 2006 comes from $\beta_i dX_i$ where β_i is the regression coefficient of element X_i whose change was dX_i.

Table 1 below (based upon D-VH, table 3) displays the largest factors that made the delinquencies and foreclosures in year 2006 worse than the mean over the entire period. For year 2006, the largest contribution to delinquency and to foreclosure was the low house price appreciation. It accounted for 1.08% of the greater delinquencies and 0.61% for the greater foreclosures. The debt/income, the balloon dummy and the documentation variables are significantly smaller.

Table 1. Contribution C(i) of factors to probability of delinquency and defaults 2006, relative to mean for the period 2001-2006. Source: D-VH, table 3.

Variable X(i), see D-VH table 2 for definitions	Contribution C(i) to delinquency rate	Contribution C(i) to foreclosure rate
House price appreciation	1.08 %	0.61 %
Balloon	0.18	0.09
Documentation	0.16	0.07
Debt/income	0.15	0.04

Table 1, the sketch of the sub-prime mortgage story in part 2 and the violation of the "no free lunch" constraint in equation (12) above explain how the excess debt $\Psi(t)$ led to the crisis. The bubble started with an estimate of the price trend greater than the rate of interest. Risk was assumed to be low because of the high capital gains relative to the interest rate raised the value of the houses above the debt owed. An entire structure of financial instruments/derivatives was based upon these mortgages. The debt ratio was greater than f-max. The collapse occurred when the capital gains declined as shown in figure 2 and table 1.

7.2. *Estimates of Excess Debt, Early Warning Signal of a Crisis*

An Early Warning Signal of a debt crisis is a series of excessive debts $\Psi(t) = f(t) - f^*(t) > 0$. As shown in figure 3/eq. (19), the loss of growth from non-optimal debt ratios over a period (0,T) is

(19) $E[\ln X^*(T) - \ln X(T)] = \int^T [W^*(t) - W(t)]dt = (1/2) \int^T \sigma^2 \Psi(t)^2 dt$.

When the debt ratio f(t) exceeds f-max in figure 3, the expected growth is negative and the risk is high. A crisis is likely when $\int^T \sigma^2 \Psi(t)^2 dt$ is large. The next question is: What are the appropriate measures of the actual and the optimal debt ratio to evaluate $\Psi(t)$?

In order to make alterative measures of the debt ratio and key economic variables comparable, I use normalized variables where the normalization (N) of a variable Z(t) called N(Z) = [Z(t) – mean Z]/standard deviation. *The mean of N(Z) is zero and its standard deviation is unity*.

For the actual debt ratio I use the debt burden i(t)L(t)/Y(t). There is a great heterogeneity in interest rates charged to the subprime borrowers depending upon the terms of the mortgage, so it is difficult to state exactly what corresponds to i(t) in the analysis above. I therefore use "Household Debt Service Payments as a Percent of Disposable Personal Income" (This is series TDSP in FRED.

as a measure of iL/Y the *debt burden.* This includes all household debt, not just the mortgage debt, because the capital gains led to a general rise in consumption and debt. The normalized value of the debt service N(f) or debt burden, is equation (20), which is graphed in figure 4 as DEBTSERVICE. This is measured in units of standard deviations from the mean of zero. There is a dramatic deviation above the mean from 1998 to 2006. This sharp rise coincides with the ratio of housing price index P/disposable income Y, P/Y = PRICEINC in figure 2. During this period, there is more than a two standard deviation rise in P/Y and a two standard deviation rise in iL/Y debt service/disposable income.

(20) N(f) = DEBTSERVICE = [i(t)L(t)/Y(t) – mean]/standard deviation.

As explained in connection with figure 3 there will always be a specification error in estimating the optimal debt ratio. The main reason is that the price trend ρ cannot be known with certainty, but I require that it not exceed the rate of interest. Therefore a rather flexible approach will be taken to estimate the optimal debt ratio f*(t).

The *optimum debt ratio* f* is based upon eqn. (16), with the constraint that r = ρ – i > 0. From the histogram of the capital gains in figure 1, the mean capital gain was 5.4% per annum with a standard deviation of 2.9%. It is reasonable to argue that, over a long period, the real appreciation of housing prices was not significantly different from "the mortgage rate of interest", (i–ρ) = r = 0. The optimal debt ratio from (16) should be (16a) below. The normalized optimal debt ratio is N(f*) in equation (21).

(16a) $f^*(t) = [(\beta(t) - (1/2)\sigma^2 - \alpha y(t)]/\sigma^2$.

(21) $N(f^*(t)) = [[(\beta(t) - \beta)] - \alpha y(t)]/ \sigma(\beta)$

The main term is [(β(t) – β)] the deviation of the return on capital from its mean value over the entire period. We must estimate β(t), the productivity of capital. The productivity of housing capital is the implicit net rental income/value of the home plus a convenience yield in owning one's home. Assume that the convenience yield in owning a home has been relatively constant. Approximate the return β(t) by using the ratio of rental income/disposable personal income. This ratio is not sensitive to the level of housing prices, whereas rents/value of housing is statistically negatively related to the level of housing prices.

In figure 4/eqn. (22) variable RENTRATIO is the normalized return, measured in units of standard deviation from the mean β . This ratio was relatively constant from 1994 to 2002 and then fell drastically.

(22) RENTRATIO ~ $[\beta(t) - \beta]/\sigma(\beta)$

= (rental income/disposable personal income – mean)/standard deviation.

The second variable in the optimal debt ratio equation (16a) is y(t), the deviation of the price of the asset from trend in equation (9). One cannot be sure of what is the appropriate value of the trend ρ < i, but the normalized capital gain CAPGAIN described in figure 2 gives us the clue. The mean capital gain is normalized at zero. From 1999 to 2004 it rose rapidly and was two and one half standard deviations above the mean in 2004. Therefore one can be confident that deviation y(t) from trend was positive and rising during this period.

Putting together the two components of the optimal debt ratio in equation (21), one estimates a drastic decline in the measure of the optimal debt ratio. The normalized RENTRATIO in (22) is an upper bound measure of the optimal debt ratio, equation (23) during the period 2000 – 2004.

(23) $N(f^*(t)) = [\beta(t) - \beta]/\sigma(\beta) > [[(\beta(t) - \beta)] - \alpha y(t)]/\sigma(\beta)$

Both the actual (equation (20)) and optimal (equation (23)) are graphed in normalized form in figure 4.

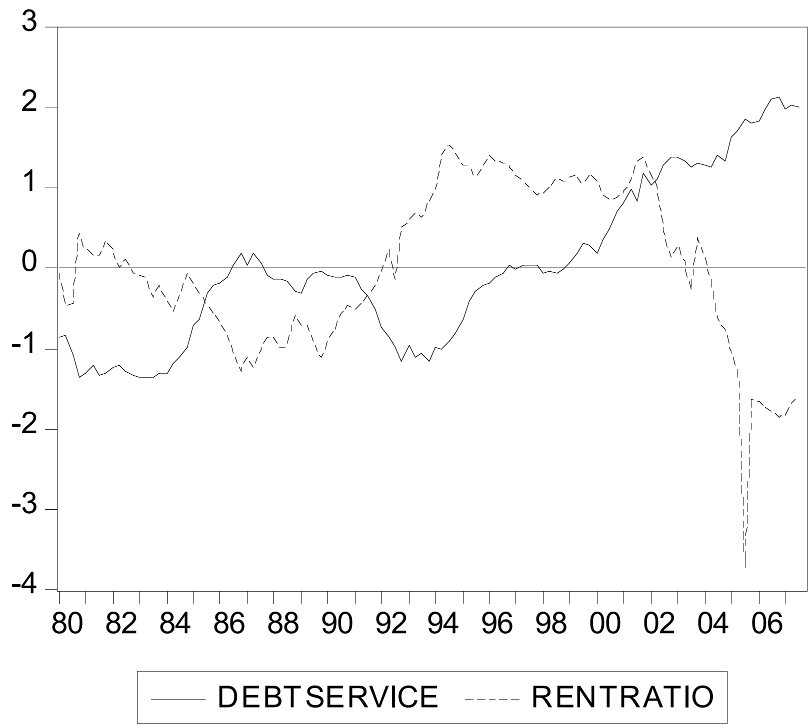

Figure 4. Early Warning Signals: Excess debt Ψ(t) = N[f(t)] – N[f*(t)].
N[f(t)] = DEBTSERVICE = (household debt service as percent of disposable income – mean)/standard deviation. N[f*(t)] = RENTRATIO = (rental income/disposable personal income – mean)/standard deviation; Sources FRED

The next question is how to estimate the excess debt Ψ(t) that corresponds to eq.17/figure 3, and is consistent with alternative estimates of the optimal debt.

I estimate excess debt Ψ(t) = (f(t) – f*(t)) by using the difference between two normalized variables N(f) – N(f*), equation (24). This difference is *measured in standard deviations*.

(24) Ψ(t) = Excess Debt ~ N[f(t)] – N[f*(t)] = DEBTSERVICE - RENTRATIO.

Excess Debt graphed in figure 5 corresponds to the difference Ψ(t) = f*(t) – f(t) on the horizontal axis in figure 3, measured in standard deviations. The probability of a decline in net worth Pr(d ln X(t) < 0) is positively related to Ψ(t) the excess debt. As the excess debt rises, the expected growth declines and the risk increases, equation (25).

(25) Pr(d ln X(t) < 0) = H(Ψ(t)), H' > 0, H(0) = W*.

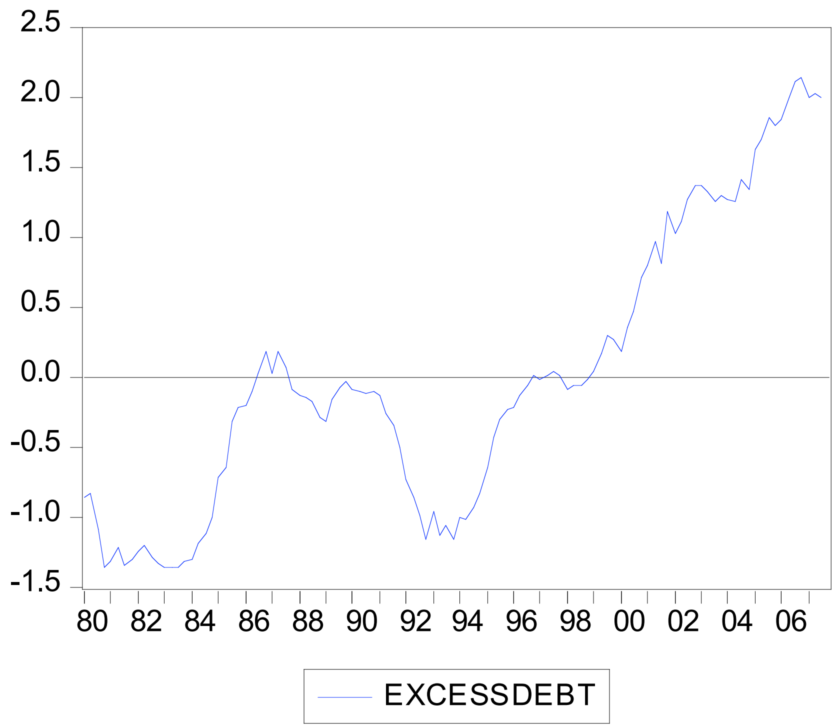

Figure 5. Excess Debt = Debt service – rent ratio. Normalized.

Assume that over the entire period 1980 – 2007 the debt ratio was not excessive. During the period 2000-2004, the high capital gains and low interest rates induced rises in housing prices relative to disposable income and led to rises in the debt ratio. Figure 2 shows this relation clearly.

By 2005-06 the ratio of housing price/disposable income was about three standard deviations above the long-term mean. See PRICEINC in figure 2. This drastic rise alarmed several economists who believed that the housing market was drastically overvalued. As indicated in part 2 above, they were in a minority. It certainly had a negligible effect upon the market for derivatives and the optimism of the "Quants".

The advantages of using excess debt Ψ(t)in figure 5 as an Early Warning Signal compared to just the ratio of housing price/disposable income are that Ψ(t) focuses upon the fundamental determinants of the optimal debt ratio as well as upon the actual ratio. The probability of declines in net worth and a crisis are directly related to the excess debt. Moreover, the use of normalized variables indicates the magnitude of the excess debt in terms of standard deviations, and more meaningful estimates can be made of the probability of a crisis.

Based upon figure 5, early warning signals were given as early as 2002. By 2005, the excess debt was two standard deviations above the mean. Hence the debt ratio was in the region of f-max in figure 3. The actual debt was induced by capital gains in excess of the interest rate. The debt could only be serviced from capital gains. This situation is unsustainable. When the capital gains fell below the interest rate, the debts could not be serviced. A crisis was inevitable.

8. Conclusions

Given the dominant macroeconomic paradigms and the efficient market philosophy of the economics profession, the 2007-08 crisis took the Fed and the academics by surprise. The Fed did not perceive the housing price bubble. Greenspan said (2004) that the rise in home values was "not enough in our judgment to raise major concerns". Bernanke said (2005) that a housing bubble was "a pretty unlikely possibility". In (2007) he said that the Fed does "not expect significant spillovers from the subprime market to the rest of the economy".

Peter Clark (2009) wrote that "no measure of underlying or fundamental value will provide consistently accurate predictions of emerging bubbles, but the prior question is whether it is useful to even contemplate the exercise of assessing market values. In light of the huge costs of the housing and credit bubble, the answer must be in the affirmative". Fed Vice Chairman Kohn indicated that the Fed's thinking may have changed. He wrote (2009, quoted by Clark): "As researchers, we need to be honest about our very limited ability to assess the 'fundamental value' of an asset or to predict its price. But the housing and credit bubbles have had a substantial cost. Research on asset prices…should help to identify risks and inform decisions about the costs and benefits from a possible regulatory or monetary policy decision attempting to deal with a potential asset price bubble."

The widespread reaction to the crisis was to suggest arbitrary regulations designed to lower leverage and risk in the financial system. The proposals lacked an economic rationale about the desirable function of financial markets to allocate saving to investment and the optimal way to manage risk.

The main questions are: What methods can detect bubbles? What is their empirical performance? What are the theoretical foundations of the empirical measures? As was explained above, the measures in the literature lack theoretical foundations and their empirical performance as early warning signals were ambiguous. I restate several questions posed in part 5 and explain how each one is answered by the Stochastic Optimal Control (SOC) analysis.

First: What is an optimum risk in a world where the future development of asset prices is unpredictable? What is an excessive risk? The SOC answers this by deriving an optimum debt/net worth or leverage that balances expected return and risk. The optimum debt ratio maximizes the expected logarithm of net worth at a later time subject to a stochastic process on asset prices. The optimum ratio of capital (i.e., assets)/net worth follows directly from the optimum leverage. The optimum leverage and capital requirements are time varying insofar as the underlying fundamentals are time varying.

The danger from "overvaluation" of housing prices is that the debt used to finance the purchase is excessive. Figure 6 graphs the ratio of housing prices/disposable income PRICEINC and the debt service DEBTSERVICE, which is interest payments/disposable income. They are significantly positively related. The SOC focuses upon the debt, which can cause a crisis.

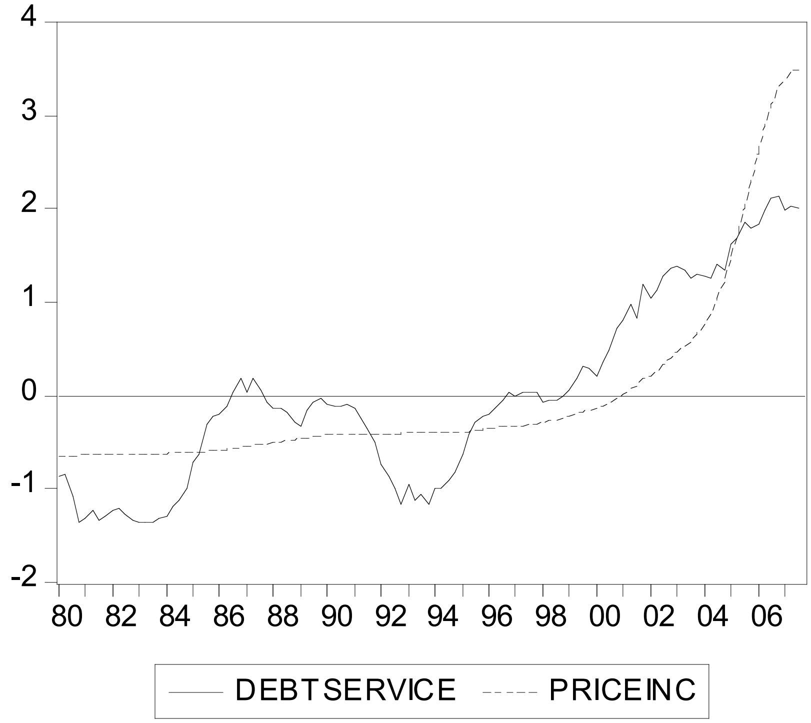

Figure 6. PRICEINC = Ratio of housing prices/disposable income. DEBTSERVICE = Debt service/disposable income. Both variables are normalized.

Second: how should one formulate and model the expected trend of asset prices to avoid bubbles and subsequent crashes? The major failing of the market was to anticipate a trend of housing prices that was based upon the probability distribution over the recent past. This was a period where the asset prices were growing at a rate greater than the rate of interest. Loans could only be serviced from the capital gains. This probability distribution was unsustainable. The SOC analysis constrains the trend of asset prices to be less than or equal to the rate of interest. Thereby a "no free lunch" constraint is imposed in the optimization.

Third: what are early warning signals of a crisis? The SOC analysis derives an "excess debt" defined as the difference between the actual and optimal debt ratio. The optimal ratio depends upon the productivity of capital less the real interest rate, the variance of the capital gain and the deviation of asset prices from a trend, which does not exceed the interest rate. The optimal debt ratio/ leverage is objectively measured.

As the debt ratio exceeds the optimal ratio, the expected growth of net worth declines and the risk rises. Since the probability of losses and bankruptcy is directly related to the excess debt, the excess debt is an early warning signal of a crisis.

Empirically, measures of actual, optimal and excess debt are expressed in normalized form, where the mean is zero and the standard deviation is unity. Probability measures can be associated with excess debt, and the probability of a crisis is more clearly defined. This theoretically derived approach is a more useful warning signal than is the arbitrary “stress testing”.

There are several unresolved issues that are left for further research. First, given that the Federal Reserve may be concerned with asset market bubbles, how should its monetary policy be conducted? Second, what is an optimal system of regulation to avoid subsequent crises?

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