ON THE TREE-CUTTING PROBLEM UNDER INTEREST RATE AND FOREST VALUE UNCERTAINTY

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Abstract

The current literature on optimal forest rotation makes the unrealistic assumption of constant interest rate though harvesting decisions of forest stands are typically subject to long time horizons. We apply the Wicksellian single rotation framework to cover the unexplored case of variable and stochastic interest rate. By modelling the stochastic interest rate according to the Cox-Ingersoll-Ross model and the forest value as a geometric Brownian motion we provide an explicit solution for the Wicksellian single rotation problem and show that increased interest rate volatility increases the optimal exercise threshold of the irreversible harvesting opportunity and thereby prolongs the optimal rotation period. Numerical illustration indicates that the optimal threshold becomes higher at an increasing rate.

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1 Introduction

In forest economics the Faustmannian framework has been the most often used starting point in the analyses of optimal rotation period of forest stands. Under the assumption of constant timber price, constant total cost of clear-cutting and replanting as well as constant interest rate, perfect capital markets and perfect foresight the basic model leads to a constant rotation period for an even age stand, which maximizes the present value of forest stand over an infinite time horizon (see e.g. Samuelson 1976). The representative rotation age depends on timber price, total cost of clear-cutting and replanting, nature of forest growth as well as the interest rate. The perfect foresight assumption has been relaxed in studies focusing on the implications of stochastic timber prices (see e.g. Insley 2002), risk of forest fire (see e.g. Reed 1984) and stochastic forest growth on optimal rotation age (see e.g. Clarke and Reed 1989,1990, and Willassen 1998 and Alvarez 2001b). In the case of forest fire risk modelled as a Poisson process the rotation age will become shorter due to the higher effective discount rate, while under timber price and forest growth risk usually the reverse happens; higher risk will tend to lengthen the rotation period.

To our knowledge all the research has, however, used the assumption of constant interest rate, which is problematic because forest rotation periods are long and interest rates fluctuate over time. In this paper we analyze an important, but unexplored, issue of the impact of variable and stochastic interest rate on optimal forest rotation when forest value is also stochastic by using a Wicksellian framework of a single rotation. We model the stochastic interest rate as a parametrized mean reverting process (the so-called Cox-Ingersoll-Ross model of the interest rate) and the forest value as a simpler form of geometric Brownian motion and provide an explicit solution for two-dimensional path-dependent optimal stopping problem. We show that higher interest rate volatility increases the optimal exercise threshold of the harvesting opportunity and therefore prolongs the optimal rotation period. Numerical illustration indicates that the optimal threshold becomes higher at an increasing rate.

We proceed as follows: Section 2 presents a solvable model for optimal forest rotation in the presence of interest rate and forest value uncertainties and finally there is a brief conclusion.

2 Optimal Forest Rotation: A Solvable Model

Consider the following (path-dependent) Wicksellian optimal rotation problem

$$V(x,r) = \sup_{\tau} \mathbf{E}_{(x,r)} \left[e^{-\int_0^{\tau} r_s ds} X_{\tau} \right], \tag{2.1}$$

where the underlying forest value and interest rate processes (X_t, r_t) evolve according to the dynamics described by the following stochastic differential equations

$$dr_t = (a - br_t)dt + c\sqrt{r_t}dW_t, \quad r_0 = r \tag{2.2}$$

and

$$dX_t = \mu X_t dt + \sigma X_t d\hat{W}_t, \quad X_0 = x, \tag{2.3}$$

where $a, b, c, \sigma, \mu \in \mathbb{R}_+$ are known exogenously given constants and W_t and $\hat{W}_t$ are two stochastically independent Wiener processes (under the objective probability measure $\mathbb{P}$). According to (2.2) the interest rate follows a mean reverting process and according to (2.3) the forest value follows a geometric Brownian motion. It is worth emphasizing that the interest rate model (2.2) is known in financial economics as the Cox-Ingersoll-Ross-model of the interest rate which can be supported theoretically and lies in conformity with empirics (cf. Björk 1998, chapter 17, and Cochrane 2001, chapters 19, 20).

Having characterized the underlying stochastic dynamics in (2.2) and (2.3) and the corresponding optimal single rotation problem (2.1) we can now state the following

Lemma 2.1. *The path-dependent optimal rotation problem (2.1) can be re-expressed as an ordinary path-independent optimal stopping problem*

$$V(x,r) = xe^{Ar} \sup_{\tau} \mathbf{E}_r \left[e^{(\mu + aA)\tau - A\hat{r}_{\tau}} \right], \tag{2.4}$$

where

$$A = \frac{b}{c^2} - \sqrt{\frac{b^2}{c^4} + \frac{2}{c^2}} < 0$$

denotes the negative root of the quadratic equation $c^2z^2 - 2bz - 2 = 0$ *and*

$$d\hat{r}_t = \big(a - (b - Ac^2)\hat{r}_t\big)\,dt + c\sqrt{\hat{r}_t}dW_t, \quad \hat{r}_0 = r. \tag{2.5}$$

Proof. See Appendix A. □

Lemma 2.1 is important in the sense that under the assumptions we have made concerning the stochastic processes modelling the interest rate and the forest value, the path-dependent single rotation problem can actually be transformed into an ordinary path-independent optimal stopping problem. Our main new result is now summarized in the following

Theorem 2.2. *Assume that the absence of speculative bubbles condition* $\mu + aA < 0$ *is satisfied. Then the value of the single rotation problem (2.1) reads as*

$$V(x,r) = xe^{Ar}\psi(r)\sup_{y\geq r}\left[\frac{e^{-Ay}}{\psi(y)}\right] = \begin{cases} x, & r \geq r^* \\ xe^{A(r-r^*)}\frac{\psi(r)}{\psi(r^*)}, & r < r^* \end{cases}$$

where the increasing fundamental solution

$$\psi(r) = \int_0^1 e^{2(b-Ac^2)rt/c^2}t^{\rho-1}(1-t)^{2a/c^2-\rho-1}dt$$

is known as Kummer's confluent hypergeometric function (see e.g. Abramowitz and Stegun 1968, pp. 503–535) and

$$\rho = \frac{\mu + aA}{Ac^2 - b} > 0.$$

Moreover, the optimal exercise threshold r^* *is the unique root of the ordinary first order condition* $\psi'(r^*) = -A\psi(r^*)$*. Especially,* $r^* > \mu$ *for all* $c > 0$ *and* $r^* = \mu$ *when* $c = 0$*.*

Proof. See Appendix B. □

Theorem 2.2 demonstrates that the path-dependent optimal rotation problem (2.4) is explicitly solvable whenever the absence of speculative bubbles condition $aA + \mu < 0$ is satisfied. It is worth observing that since

$$\frac{\partial A}{\partial c} = \frac{cA^2}{b - c^2A} > 0$$

and $A \downarrow -1/b$ as $c \downarrow 0$ we find that the absence of speculative bubbles condition can be satisfied only if the inequality $\mu < a/b$, stating that the expected percentage growth rate of the revenues has to be smaller than the long run steady state interest rate, holds. If this is indeed the case, then there is a critical volatility c^*, satisfying the condition

$$\mu c^{*2} = ab - \sqrt{a^2b^2 + 2a^2c^{*2}},$$

above which the the absence of speculative bubbles condition $aA + \mu < 0$ is violated and, therefore, above which the value of the optimal policy becomes unbounded. We find that the condition $\mu + aA < 0$ is strengthened by the higher volatility described by the parameter c. This increases the required exercise premium and, thus, prolongs the optimal rotation period.

In Figure 1 we illustrate the optimal rotation threshold under the assumption that $b = 0.1, a = 0.045b$, and $\mu = 0.03$ (implying that the critical volatility above which the absence of speculative bubbles condition $aA + \mu < 0$ is violated is $c^* = 12.25\%$). As

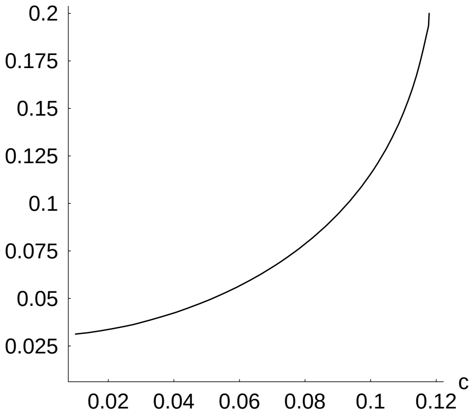

Figure 1: The optimal rotation threshold as a function of c

one can immediately observe from Figure 1, increased volatility not only increases the optimal threshold, but does it at an increasing rate. Thus, close to the critical levels where the absence of speculative bubbles condition is compromised, a small increase in the volatility coefficient results into a relatively large increase in the required exercise premium so that the optimal rotation period will increase more than the volatility.

3 Conclusions

To our knowledge all the research about the determination of optimal forest rotations has used the assumption of constant interest rate, which is problematic because forest rotation periods are long and interest rates fluctuate over time. In this paper we have used the Wicksellian single rotation framework to study an unexplored issue of forest rotation under variable and stochastic interest rate when forest value is also stochastic. We have modelled the stochastic interest rate as a parametrized mean reverting process by using the Cox-Ingersoll-Ross model of interest rate - which is well-known in financial economics, can be supported theoretically and lies in conformity with empirics - and the forest value as a simpler form of geometric Brownian motion. We have provided an explicit solution for two-dimensional path-dependent optimal stopping problem and shown that higher interest rate volatility increases the optimal threshold and therefore prolongs the optimal rotation period. Numerical illustration indicates that the optimal threshold is a strictly convex function of the volatility coefficient of the underlying interest rate process. Thus our illustrations indicate that the optimal exercise threshold becomes higher at an increasing rate as the interest rate volatility increases.

Whether our conclusions remain valid in the Faustmann's ongoing rotation framework is an open question beyond the scope of this paper. Given the close connection of impulse control problems and optimal stopping theory (see Alvarez 2001b) we are tempted to conjecture that our conclusions will likely remain qualitatively valid in the Faustmann framework as well. But the verification of this conjecture is an open issue for further research.

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A Proof of Lemma 2.1

Since

$$X_t = x\exp((\mu-\sigma^2/2)t+\sigma\hat{W}_t)$$

we find by applying Itô's theorem to the mapping $r \mapsto e^{zr}$ that

$$e^{-\frac{1}{2}(z^2c^2-2zb)\int_0^t r_s ds} = e^{z(r-r_t)+zat}M_t,$$

where

$$M_t = \exp\left(\int_0^t zc\sqrt{r_s}dW_s - \frac{1}{2}\int_0^t z^2c^2r_s ds\right)$$

is a positive exponential martingale. Thus, choosing $z = A$ implies that the discount factor can be re-expressed as

$$e^{-\int_0^t r_s ds} = e^{A(r-r_t)+Aat}M_t.$$

Therefore, we find that the present value of the forest stand can be expressed as

$$e^{-\int_0^t r_s ds}X_t = xe^{A(r-r_t)+Aat+\mu t}\hat{M}_t M_t,$$

where $\hat{M}_t = e^{\sigma\hat{W}_t-\frac{1}{2}\sigma^2 t}$ is a positive exponential martingale. Consequently, we find that the path-dependent optimal rotation problem (2.1) can be re-expressed as an ordinary path-independent optimal stopping problem

$$V(x,r) = xe^{Ar}\sup_\tau \mathbf{E}_r\left[e^{(\mu+aA)\tau-Ar_\tau}\hat{M}_\tau M_\tau\right]. \tag{A.1}$$

Defining the equivalent measure $\mathbb{Q}$ through the likelihood-ratio $\frac{d\mathbb{Q}}{d\mathbb{P}} = \hat{M}_t M_t$ we can now re-express (A.1) as

$$V(x,r) = xe^{Ar}\sup_\tau \mathbf{E}_r^{\mathbb{Q}}\left[e^{(\mu+aA)\tau-Ar_\tau}\right], \tag{A.2}$$

where the interest rate process r_t evolves under $\mathbb{Q}$ according to the dynamics described by the stochastic differential equation

$$dr_t = \big(a - (b - Ac^2)r_t\big)\,dt + c\sqrt{r_t}d\tilde{W}_t, \quad r_0 = r,$$

where $\tilde{W}_t$ is a standard Brownian motion under the equivalent measure $\mathbb{Q}$. However, given the strong uniqueness of a solution for the stochastic differential equation above (cf. Øksendal, 1998, p. 66) we finally find that the rotation problem (2.1) can be rewritten in the path-independent form (2.4) defined under the objective measure $\mathbb{P}$.

B Proof of Theorem 2.2

Proof. Since

$$L(r) = \mathbf{E}_r\left[e^{(\mu+aA)\tau - A\hat{r}_\tau}\right]$$

is an ordinary path-independent optimal stopping problem of a linear diffusion and, therefore, can be solved by relying on ordinary variational inequalities, the alleged result is a direct implication of Theorem 3 in Alvarez 2001 a. It is, therefore, sufficient to determine the increasing fundamental solution of the ordinary second order differential equation

$$\frac{1}{2}c^2ru''(r) + (a - (b - c^2A)r)u'(r) + (\mu + aA)u(r) = 0.$$

Making the transformation $u(r) = v(\theta r)$,where $\theta \in \mathbb{R}$ is an unknown constant, and defining the variable $y = \theta r$ then yields that

$$yv''(y) + \left(\frac{2a}{c^2} - \frac{2(b - Ac^2)}{c^2\theta}y\right)v'(y) + \frac{2(\mu + aA)}{\theta c^2}v(y) = 0.$$

Choosing $\theta = 2(b - Ac^2)/c^2$, then finally implies that the differential equation can equivalently be expressed as

$$yv''(y) + \left(\frac{2a}{c^2} - y\right)v'(y) - \frac{2(\mu + aA)}{Ac^2 - b}v(y) = 0,$$

which is Kummer's differential equation. $\square$

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