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LIFE INSURANCE, PRECAUTIONARY SAVING AND CONTINGENT BEQUEST

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LIFE INSURANCE, PRECAUTIONARY SAVING AND CONTINGENT BEQUEST

Abstract

Purchasing life insurance is for the welfare of young children, particularly preteens, who are liquidity constrained. In this paper, we present a life cycle model of life insurance that takes into account the ages of these young beneficiaries. We show that, as the child ages, the need for protection is reduced and, consequently, the size of contingent bequest may shrink. The demand for life insurance is positively related to the number, age differentials, living standards, and the time needed to reach adulthood. Also, the breadwinner's lifetime uncertainty and the unfairness of the insurance market encourage precautionary saving

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1. Introduction

According to Belth (1985), life insurance purchase depends on how much and how long the needs of the beneficiary are. Young children, particularly preteens, who are liquidity constrained certainly need the breadwinner's support until they reach independence. We argue that a major factor in purchasing life insurance is the welfare of these young dependents. In this paper, we present a life cycle model of life insurance purchase that reflects this need.

The standard model of the demand for life insurance, e.g. Fischer (1973), assumes that the breadwinner maximizes his expected utility over an uncertain lifetime by choosing the level of consumption and life insurance.¹ The demand for life insurance thus obtained is dictated by the bequest function. Shorrocks (1979) pointed out that such a model is unsatisfactory because, among other things, the purchase of insurance is independent of the number or the circumstances of the beneficiaries. Subsequently, Lewis (1989) presented a model of the demand for life insurance that maximizes the beneficiary's utility, not the breadwinner's own utility. He argued that such an approach simulates the actual calculation of household insurance purchase. We contend that life insurance purchase should be a choice variable of a utility maximizing breadwinner who has the beneficiaries' best interest in mind.

¹The emphasis in Fischer is comparative statics. Other theoretical works in the area include, for example, Richard (1975), Campbell (1980), Pissarides (1980), and Karni and Zilcha (1985). Richard extended Merton's (1971) consumption and portfolio rules to include life insurance. Campbell used the technique of stochastic calculus to derive an explicit demand for life insurance function. Pissarides studied the age-bequest (with life insurance) relationship. Karni and Zilcha studied life insurance and the measures of risk aversion within a state-dependent framework. Also see Borch (1991) for the institutional development of life insurance.

Recall that in Fischer’s model, if the breadwinner lives, he chooses his own consumption and *nothing* for his beneficiaries. If, however, the breadwinner dies, the beneficiary receives the contingent bequest (savings plus the net value of life insurance). The departure of our model from Fischer’s is that we assume the breadwinner is altruistic towards his dependents while he is alive, not just after his death. To accomplish this, we include the recipient’s utility function up to the “age of independence” in the breadwinner’s optimization problem. Then, the age profiles of income transfer, contingent bequest, and the breadwinner’s own consumption are jointly determined in the model. In contrast, Lewis approached the problem by treating the breadwinner’s consumption and income transfer as given. Our model enables us to address the problem of life insurance purchase by itself or as a component of intergenerational transfer.

The literature of intergenerational transfer,² which emphasizes the interaction between generations, typically include the beneficiary’s income and other strategic actions in the model. Because these models apply mainly to adult offspring, they often have to exclude young children in the analysis. For example, in testing the altruistic theory, Wilhelm (1996) excluded children under age 25. Laitner and Juster (1996, p.895) recognized this point and presented a three-period model in which the middle period is a time of giving (to young offspring). Our model goes even further to study the issues arising from the ages of the young dependents. In this sense the proposed model emphasizes those “missing” years and hence complements the literature of intergenerational transfer.

²There is a debate on whether intergenerational transfer is derived from altruism or self-interest. See, for example, Becker (1974, 1981), Bernheim, Shleifer and Summers(1985), Cox (1987), Gale and Scholz (1994), Wilhelm (1996), Laitner and Juster (1996), and Altonji, Hayashi and Kotlikoff (1997). Most of these models are cast in an overlapping generation framework.

The proposed model adds new results to the life cycle theory. We show that there is precautionary saving arising from the breadwinner's lifetime uncertainty, *when there are dependent children in the family*. This precautionary saving rises with the breadwinner's hazard rate and the loading factor of the insurance market. The key to this result is that the Euler equation governing the breadwinner's consumption is discounted by the *actuarial rate of interest*, not the market interest rate. If the actuarial rate is greater than the subjective discount rate, then consumption rises with age. The presence of the loading factor in the Euler equation implies an additional rate of growth of consumption compared to the standard life cycle model. We work out a numerical example, using a CDC's Life Table, to illustrate the significance of this delayed consumption effect. The theory also predicts that an across-the-board increase in the loading factor produces an upward tilt to the consumption profile, i.e., as the insurance market becomes less fair, more precautionary saving is encouraged.

The inclusion of the beneficiary's utility function in the objective function enables us to draw implications on the income transfer to young children. We show that the income transfer rises as the child ages and that the magnitude of the aggregate income transfer is significantly influenced by the number of children. In particular, the age profile of the aggregate income transfer tends to peak just before the oldest child reaches independence. These results support Espen-shade's (1984) findings that older children are more expensive because, among other things, they are physically larger and have more activities. Our results are attributed to the breadwinner's altruism and lifetime uncertainty, and thus provide *additional* explanations for these observations.

The implication on the age profile of bequests is of particular interest. Unlike other profiles, the age profile of bequest is not necessarily rising over time even if the marginal utility of bequest is declining with age. This is because, in our model, life insurance purchase is to maintain the beneficiary's standard of living until his age of independence in the event of the breadwinner's untimely death. Such a need for protection declines as the child ages. If this age factor is the dominant factor, then the size of the bequest shrinks as the child grows up. We show that this is indeed the case if the beneficiary's utility function is isoelastic.

We also show that the birth order matters. When there are multiple children, the younger child would inherit a larger bequest. In contrast, the theory of primogeniture predicts the opposite allocation of resources. These results are not contradictory to each other because primogeniture applies mainly to adult heirs. Our birth-order result is derived from the fact that the breadwinner provides equal protection for all children up to their respective ages of independence. Since the model assumes away child mortality, the younger one has a longer way to go before reaching independence and, therefore, needs more protection. Along this line of reasoning, the theory predicts that the breadwinner would provide handicapped children a longer period of income transfer if he lives, and leave a larger bequest if he dies, since handicapped children need more protection.

Finally, we derive a *closed-form* demand for life insurance if the utility function is isoelastic. We show that the demand for life insurance is negatively related to the loading factor and savings, but is positively related to the beneficiary's risk aversion – all of these are quite intuitive. More importantly, we show that the demand for life insurance is positively related to the number, the age differen-

tials, and the living standards of the beneficiaries. Shorrocks's criticisms are thus addressed.

2. The Model

We first present Fischer's (1973) celebrated model with a minor change in timing. Let \bar{T} be the maximal period that the breadwinner can live. Throughout the paper, the only source of uncertainty is the breadwinner's lifetime uncertainty. Let p_t be the conditional probability that the breadwinner dies at the beginning of period $t \leq \bar{T}$, given survival to period $t - 1$. By definition, $p_{\bar{T}+1} = 1$. The decision to purchase life insurance is made at the end of period $t - 1$, or the beginning of period t *before* the true state of nature is revealed. We make this change in timing so that when we present our model the insurance decision would resemble that of a static insurance problem.

The revelation of the true state of nature over time is described by an event tree. If the breadwinner lives through period $t - 1$, then he accumulates wealth w_t for period t from which he spends a portion of his wealth, $k_t w_t$, the insurance premium, to purchase term life insurance with face value $q_t k_t w_t$, where $1/q_t$ is the price of insurance. In short, the breadwinner enters period t with wealth $(1 - k_t) w_t$. If the breadwinner did not die at the beginning of period t , then he chooses consumption c_t for himself and accumulates $w_{t+1} = (1 + r) [(1 - k_t) w_t - c_t]$ for period $t + 1$. If he died, he left behind $(1 - k_t) w_t + q_t k_t w_t$ to his heirs. Then, the breadwinner solves the following recursive problem

$$J_t = \max_{c_t, k_t} \{ (1 - p_t) [u_t(c_t) + \beta J_{t+1}] + p_t B_t [(1 - k_t) w_t + q_t k_t w_t] \},$$

where u_t is the utility function, β is the discount factor, J_t is the current value of the discounted expected utility function at the beginning of period t , and B_t is the bequest function. Since $k_{\bar{T}} = 0$ and $J_{\bar{T}+1} = 0$, the problem is solved by backward induction.

The departure from Fischer's model is that we shall include the recipient's utility in the breadwinner's decision problem. For the purpose of exposition, we begin with only one dependent. In this context, the initial period is regarded as the beneficiary's year of birth. Then, the beneficiary's consumption becomes one of the breadwinner's choice variables. This inclusion is made for two reasons. First, many dependents live primarily on the breadwinner's support until they reach independence. We shall call T the beneficiary's *age of independence*,³ if he reaches independence at the beginning of period $T + 1$, with $T < \bar{T}$. Second, since life insurance purchase depends on how long the needs of the beneficiary are, we include the beneficiary's utility function in the breadwinner's optimization problem only up to period T , the age of independence.

Again, we consider only term life insurance that has no cash value and therefore no savings component. Let the degree of unfairness of the insurance market be summarized by ℓ_t ($\ell_t \geq 1$), the loading factor of period t . Since $\ell_t p_t$ is the price of life insurance (i.e., the price paid for one dollar coverage of life insurance), the insurance premium for face value $f_t \geq 0$ is $\ell_t p_t f_t$.

If the breadwinner died in the beginning of period t , the beneficiary would receive an asset, $w_t + (1 - \ell_t p_t) f_t$, that is composed of savings, $w_t - \ell_t p_t f_t$, and

³In the United States, child support was emphasized in the Family Support Act of 1988, which stipulates that the wages of an absent parent shall be subject to withholding. Altruistic or not, raising one's dependent children to a certain age seems to be the social norm. I owe this legal reference to Bob Michael.

the face value of life insurance, f_t . This is Kotlikoff's (1989) "contingent bequest." As we shall see later, another departure from Fischer's model is the specification of the bequest function $B_t[w_t + (1 - \ell_t p_t) f_t]$: Since the purchase of life insurance is based on the needs of helping the child reach independence, we assume that the beneficiary (or the guardian) will use this bequest *optimally* to achieve that goal.

On the other hand, if the breadwinner lived through period t , then he would earn income y_t (exogenously given) and choose consumption c_t for himself and transfer $g_t - \ell_t p_t f_t \geq 0$ to the beneficiary, where g_t represents gifts *inter vivos*. Notice that we make insurance premium $\ell_t p_t f_t$ a component of gifts *inter vivos* so that the insurance purchase *viewed from the beneficiary's perspective* would resemble that in the static model. More precisely, the demand for insurance can be derived from comparing the income of the good state, $g_t - \ell_t p_t f_t$, to that of the bad state, $w_t + (1 - \ell_t p_t) f_t$. Such an advantage would be lost if we put the insurance premium in the budget equation.

For simplicity, we assume that there is no human capital investment in the model, even though part of the income transfer may be used for dependent's education. Then the budget equation is

$$w_{t+1} = (1 + r)(w_t + y_t - c_t - g_t), \quad (2.1)$$

given the initial wealth, w_0 . Let the beneficiary's utility function be $v_t(\cdot)$. As usual, $u_t(\cdot)$, $B_t(\cdot)$ and $v_t(\cdot)$ are assumed strictly increasing and strictly concave. Then, for $t \leq T$, the breadwinner solves the recursive problem

$$J_t = \max_{c_t, f_t, g_t} \left\{ \begin{array}{l} (1 - p_t) [u_t(c_t) + v_t(g_t - \ell_t p_t f_t) + \beta J_{t+1}] \\ + p_t B_t(w_t + (1 - \ell_t p_t) f_t) \end{array} \right\}, \quad (2.2)$$

subject to (2.1). For $t \geq T + 1$, Fischer's formulation applies.

Assuming *interior* solutions, the first order conditions for (2.2) are, for $t \leq T$,

$$\frac{p_t B'_t [w_t + (1 - \ell_t p_t) f_t]}{(1 - p_t) v'_t (g_t - \ell_t p_t f_t)} = \frac{\ell_t p_t}{1 - \ell_t p_t}, \quad (2.3)$$

and

$$u'_t (c_t) = v'_t (g_t - \ell_t p_t f_t) = \beta (1 + r) \frac{\partial J_{t+1}}{\partial w_{t+1}}. \quad (2.4)$$

(It should be noted that $u'_t (c_t) = \beta (1 + r) \partial J_{t+1} / \partial w_{t+1}$ is valid for *all* $t \leq \bar{T}$). Condition (2.3) says that, in any period, the marginal rate of substitution between the two states is equal to the relative price of insurance. Equation (2.4), on the other hand, says that the marginal utility of current consumption (or that of income transfer) is equal to the discounted expected marginal utility of future income. In short, equation (2.3) is the optimal condition across states, while equation (2.4) is the optimal condition across time.

An immediate corollary is that

$$B'_t [w_t + (1 - \ell_t p_t) f_t] = \left[\frac{\ell_t (1 - p_t)}{1 - \ell_t p_t} \right] v'_t (g_t - \ell_t p_t f_t). \quad (2.5)$$

It follows that the contingent bequest, $w_t + (1 - \ell_t p_t) f_t$, is positively related to the beneficiary's consumption level when the breadwinner is alive, $g_t - \ell_t p_t f_t$. As such, the purchase of life insurance is to guarantee the beneficiary's standard of living as claimed.

A remark on the model is in order. While it bears some resemblance to Lewis's (1989) model, there is a fundamental difference in modeling. To see this, we set $\beta = 1$ and convert the breadwinner's problem into a two-stage maximization problem. In the first stage, the breadwinner maximizes (2.2) for a given $\{f_t\}$. Let $\{c_t^*\}$ and $\{g_t^*\}$ be the corresponding optimal solutions. The second stage

optimization problem is to maximize

$$\sum_{t=1}^T \left\{ \left[\prod_{i=1}^t (1 - p_i) \right] v_t (g_t - \ell_t p_t f_t) + \left[p_t \prod_{i=1}^{t-1} (1 - p_i) \right] B_t (w_t + (1 - \ell_t p_t) f_t) \right\}.$$

subject to $w_{t+1}^* = (1 + r) (w_t^* + y_t - c_t^* - g_t^*)$ by choosing $\{f_t\}$. Then the purchase of life insurance is indeed to maximize the beneficiary's utility from birth to age T , as contended by Lewis. The difference, however, is that the breadwinner's consumption, $\{c_t^*\}$, and gifts to the dependents, $\{g_t^*\}$, are *not exogenously given* as assumed in Lewis, but functions of f_t .

3. Precautionary Saving and Income Transfer

To study the age profiles of the breadwinner's consumption and income transfer, consider the case that the breadwinner live through period T . For $t \leq T$, the Euler equations are

$$u'_{t-1}(c_{t-1}) = \beta (1 + r) \left(\frac{1 - p_t}{1 - \ell_t p_t} \right) u'_t(c_t), \quad (3.1)$$

and

$$v'_{t-1}(g_{t-1} - \ell_{t-1} p_{t-1} f_{t-1}) = \beta (1 + r) \left(\frac{1 - p_t}{1 - \ell_t p_t} \right) v'_t(g_t - \ell_t p_t f_t). \quad (3.2)$$

If the insurance market is actuarially fair, then (3.1) is reduced to $u'_{t-1}(c_{t-1}) = \beta (1 + r) u'_t(c_t)$, which is standard in the life cycle theory. See, for example, Tobin (1967). Similarly, (3.2) becomes $v'_{t-1}(g_{t-1} - p_{t-1} f_{t-1}) = \beta (1 + r) v'_t(g_t - p_t f_t)$.

Recall that β stands for the time preference derived from factors other than lifetime uncertainty. Then, *the* subjective discount factor is $\beta (1 - p_t)$. By ignoring $r \ell_t p$ (a small number) and the higher order terms of p_t , we have

$$\frac{1 + r}{1 - \ell_t p_t} \approx 1 + r + \ell_t p_t.$$

Since $\ell_t p_t$ is the price of life insurance, $r + \ell_t p_t$ is the actuarial rate of interest. In other words, with the purchase of life insurance, the proper discount rate is not the market interest rate, but the actuarial rate of interest. This is standard in uncertain lifetime literature. See, for example, Chang (1991).

The presence of the loading factor in the Euler equations is a new result. It has an interesting implication on the breadwinner's consumption profile. Under the assumptions of $u_t(\cdot) = u(\cdot)$ and $\beta(1+r)\left(\frac{1-p_t}{1-\ell_t p_t}\right) \geq 1$, for all $t \leq T$, the age profile of consumption of the breadwinner is upward sloping. In other words, if the preference is stable over time and the actuarial rate of interest is greater than or equal to the subjective discount rate (i.e., $r + \ell_t p_t \geq \rho + p_t$, where $1 + \rho = 1/\beta$), then there is delayed consumption. More importantly, an increase in the loading factor, and hence an increase in the actuarial rate of interest, produces an upward tilt to the breadwinner's consumption profile. Simply put, an increase in market unfairness increases precautionary saving.

To see the significance of the loading factor effect on precautionary saving, we assume $u_t(x) = u(x) = x^{1-\alpha}/(1-\alpha)$, $\alpha > 0$. Then the consumption profile satisfies $c_t = R_t c_{t-1}$, where

$$R_t = \left(\frac{1}{\alpha}\right) [r - \rho + (\ell_t - 1)p_t] > 0,$$

is the growth rate of consumption. In contrast, $R_t = (r - \rho)/\alpha$ in Tobin's model ($\ell_t = 1$). The additional growth rate, $(\ell_t - 1)p_t/\alpha$, which rises with the loading factor ℓ_t and the mortality rate p_t , is thus attributed to the unfairness of the insurance market and the breadwinner's lifetime uncertainty. From the table 6-1 of CDC's Life Tables (of Vital Statistics of the United States, 1992, Vol. II, section 6), we obtain

Table 1. Conditional probability of death

age	30-35	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75
p_t	.8%	1.0%	1.3%	1.8%	2.8%	4.4%	6.8%	10.1%	14.7%

An immediate inference to be drawn from Table 1 is this: Since the loading factor and the mortality rate rises with the breadwinner's age, so does the growth rate of consumption. Notice that this result is true *when the dependents are young*.

As an illustration, if we employ Lewis's point estimate, $\ell = 2$, then the additional growth rate of consumption is p_t/α . If we employ Szpiro's (1986) estimates that α is between 1.2 and 1.8, then p_t/α lies between .7% and 1.1% for the breadwinner who is 40-45, 1% and 1.5% for the breadwinner who is 45-50, and 1.6% and 2.3% for the breadwinner who is 50-55. In practice, the additional rate of growth of consumption $(\ell_t - 1)p_t/\alpha$ should exceed the range of .7% to 2.3%, since the loading factor rises with the breadwinner's age.

Similarly, the theory predicts that income transfer would rise with age if $v_t(\cdot) = v(\cdot)$ and $\beta(1+r)\left(\frac{1-p_t}{1-\ell_t p_t}\right) \geq 1$ for all $t \leq T$. In his well-known study, particularly his Table 3, Espenshade (1984) shows that as children age (up to age 18) they tend to become more expensive. Older children cost more because, among others, they are physically larger and have more activities. Our model does not take into account any of these factors. Rather, the result is implied by the breadwinner's lifetime uncertainty and the unfairness of the insurance market. In other words, our result provides an *additional* reason for Espenshade's findings. The inclusion of college expenditures would only strengthen, not weaken, the result.⁴

⁴In empirical studies, high school students are generally treated as dependents, but not necessarily college students. Whether or not college expenses should be considered as transfers, the reader is referred to Gale and Scholz (1994) for an interesting discussion.

The theory also predicts that income transfer would be further delayed if the loading factor increases.

4. Contingent Bequest

The implication on the bequest-age relation, however, is different. As mentioned earlier, a departure of the paper from Fischer's model is that we specifically assume that the breadwinner leaves the bequest with a clear purpose of helping the child reach independence and that the beneficiary will use the bequest optimally. To make it tractable, we ignore the fact that the beneficiary may want to save some of this bequest for the use beyond age T . We also assume the beneficiary's subjective discount rate equals the market interest rate, i.e., assume the discount factor is $(1 + r)^{-1}$. Then

$$\begin{aligned} & B_t [w_t + (1 - \ell_t p_t) f_t] \\ &= \max_{\{x_j\}} \sum_{j=t}^T (1 + r)^{t-j} v(x_j), \text{ s.t. } \sum_{j=t}^T (1 + r)^{t-j} x_j = w_t + (1 - \ell_t p_t) f_t. \end{aligned} \quad (4.1)$$

It is easy to verify that, with $r = 0$,

$$B_t [w_t + (1 - \ell_t p_t) f_t] = (T - t + 1) v \left(\frac{w_t + (1 - \ell_t p_t) f_t}{T - t + 1} \right). \quad (4.2)$$

See, e.g., Philipson and Becker (1998). Note that this bequest function is age dependent. Then

$$B'_t(x_t) = v' \left(\frac{x_t}{T - t + 1} \right).$$

If the marginal utility of bequest rises with age, then

$$\forall s < t, \frac{x_s}{T - s + 1} > \frac{x_t}{T - t + 1} \Rightarrow x_s > \left(\frac{T - s + 1}{T - t + 1} \right) x_t > x_t,$$

i.e., contingent bequest falls with time. Next, assume the age profile of the marginal utility of bequest is downward sloping. Since $T - s + 1 > T - t + 1$ and $\frac{x_s}{T-s+1} < \frac{x_t}{T-t+1}, \forall s < t$, we cannot conclude that $x_s < x_t$. In fact, the opposite could happen, i.e., contingent bequest falls with time, if the rate of decline in the marginal utilities of bequest is fairly small. This is quite possible because ℓ_t , p_t , and hence $\frac{\ell_t(1-p_t)}{1-\ell_t p_t}$, rise with age, which implies that the slope of $\{B'_t(x_t)\}$ is smaller than that of $\{v'_t(g_t - \ell_t p_t f_t)\}$. The intuition behind this result is that the beneficiary's need for protection declines as the child grows up, which makes a declining age profile of bequest possible. This bequest-age relation is germane to Pissarides's (1980) finding (his b_3 curve).

To complete the argument for $r > 0$, we assume that the utility function is of the form $v(x) = x^{1-\lambda}/(1-\lambda), \lambda > 0$. Then the bequest function can be written as

$$B_t(x) = b(t, T)^\lambda v(x), \text{ where } b(t, T) = \sum_{j=t}^T (1+r)^{t-j}. \quad (4.3)$$

Then

$$B'_s(x_s) > B'_t(x_t) \Leftrightarrow \frac{x_s}{b(s, T)} < \frac{x_t}{b(t, T)}, \forall s < t.$$

By definition, $b(t, T)$ is *increasing* in $T - t + 1$. Clearly, the aforementioned argument applies and the bequest could fall as the beneficiary grows up.

In reality, the beneficiaries are minors and therefore do not receive direct bequests. Often the funds are placed in the hands of the surviving parents, legal guardian or placed in trust for children until they reach adulthood. It seems reasonable to assume that whoever is in charge would have the beneficiary's best interest in mind and that the beneficiary's utility up to the age of independence is maximized, as the decedent would have wished.

5. Birth Order Effect

When there are two or more children, the model becomes quite complicated within the framework of the expected utility maximization, because there are issues arising from infertility, child mortality and time inconsistency. To make the model tractable and to make the breadwinner's lifetime uncertainty the only source of uncertainty, we shall make some simplifying assumptions. Specifically, we assume away child mortality so that all dependents will reach independence with certainty, and assume perfect family planning such that the number of children and their age differentials are nonstochastic and exogenously given.

For the purpose of exposition, the model is set for two dependents. The first period represents the year the first child is born. This child reaches independence at $T_1 = T$. The second child will be born in period $t_2 \leq T_1$ and reaches independence at age $T_2 = T + t_2 - 1$. The division of assets at the breadwinner's death is denoted by ξ_t^i , the percentage of assets given to beneficiary i , with $\xi_t^1 + \xi_t^2 = 1$. The choice variables for the breadwinner are $\{c_t\}$, $\{g_t^i\}$, $\{f_t^i\}$, and $\{\xi_t^i\}$, $i = 1, 2$. Then the breadwinner's problem, for $t \leq T_2$, is

$$J_t = \max_{c_t, g_t^i, f_t^i, \xi_t^i} \left\{ \begin{array}{l} (1 - p_t) \left[u_t(c_t) + \sum_{i=1}^2 v_t^i (g_t^i - \ell_t p_t f_t^i) + \beta J_{t+1} \right] \\ + p_t \sum_{i=1}^2 B_t^i (\xi_t^i w_t + (1 - \ell_t p_t) f_t^i) \end{array} \right\}, \quad (5.1)$$

with the understanding that, for $t < t_2$, $g_t^2 = f_t^2 = \xi_t^2 = v_t^2 = B_t^2 = 0$, and for $T_1 < t \leq T_2$, $g_t^1 = f_t^1 = v_t^1 = 0$. Naturally, for $t > T_2$, the problem is simplified to

$$J_t = \max_{c_t, \xi_t^i} \left\{ (1 - p_t) [u_t(c_t) + \beta J_{t+1}] + p_t \sum_{i=1}^2 B_t^i (\xi_t^i w_t) \right\}.$$

The total transfer is $g_t = g_t^1 + g_t^2$, while the total purchase of life insurance is $f_t = f_t^1 + f_t^2$. The law of motion is still (2.1).

For $t_2 \leq t \leq T_1$, assume $v_t^i(x) = v^i(x) = x^{1-\lambda_i}/(1-\lambda_i)$, $\lambda_i > 0$, $i = 1, 2$. By (4.3), $B_t^i(x) = b(t, T_i)^{\lambda_i} v^i(x)$. The first order conditions are

$$u'(c_t) = (g_t^i - \ell_t p_t f_t^i)^{-\lambda_i} = \beta J'_{t+1}, i = 1, 2, \quad (5.2)$$

$$\xi_t^i w_t + (1 - \ell_t p_t) f_t^i = \left[\frac{\ell_t (1 - p_t)}{1 - \ell_t p_t} \right]^{-1/\lambda_i} b(t, T_i) (g_t^i - \ell_t p_t f_t^i), i = 1, 2, \quad (5.3)$$

and

$$\left(\frac{b(t, T_1)}{\xi_t^1 w_t + (1 - \ell_t p_t) f_t^1} \right)^{\lambda_1} = \left(\frac{b(t, T_2)}{\xi_t^2 w_t + (1 - \ell_t p_t) f_t^2} \right)^{\lambda_2}. \quad (5.4)$$

The expenditure on each child is governed by (5.2). Under the usual conditions, the expenditure rises with age. An interesting implication is this: Given the initial wealth and earning profile $\{y_t\}$, an increase in the number of children lowers each consumption profile, including the breadwinner's. Moreover, the aggregate expenditure on children tends to peak just before the oldest child reaches independence. These results reinforce Espenshade's (1984) findings that the number of children has a greater impact on parental expenditures than the parents' socioeconomic status and wife's employment status. Equation (5.3) says that the contingent bequest for child i is positively related to $g_t^i - \ell_t p_t f_t^i$ (the standard of living if the breadwinner survives), $b(t, T_i)$ (the time it takes child i to reach independence), and λ_i (the taste parameter), but negatively related to ℓ_t (the loading factor). Equation (5.4) reflects the optimal division of wealth (ξ_t^i) such that the marginal utilities of bequest are equal for all children.

To highlight the birth order effect, we assume the beneficiaries have identical tastes, i.e., $\lambda_1 = \lambda_2$. Then both beneficiaries would receive the same amount of income transfer while the breadwinner is alive. However, it is not so for contingent

bequests. More precisely, from (5.4) and $b(t, T_1) < b(t, T_2)$, the bequest to the older child is smaller than the bequest to the younger child⁵, i.e.,

$$\xi_t^1 w_t + (1 - \ell_t p_t) f_t^1 < \xi_t^2 w_t + (1 - \ell_t p_t) f_t^2. \quad (5.5)$$

The intuition is that each child would receive “equal protection” from the breadwinner up to the age of independence. The older child, who has been protected for some time, clearly needs less protection than the younger sibling. An immediate corollary is that the breadwinner will leave a larger bequest to the handicapped children than to those who are not handicapped, since it takes a handicapped child a longer time to reach independence.

The theory remains valid for $T_1 \leq t \leq T_2$ if $B_t^1(x) = v^1(x) = v(x)$. In this case, equation (5.4) becomes

$$\xi_t^1 w_t = \frac{\xi_t^2 w_t + (1 - \ell_t p_t) f_t^2}{b(t, T_2)}.$$

Since $b(t, T_2) > 1$, we have $\xi_t^2 w_t + (1 - \ell_t p_t) f_t^2 > \xi_t^1 w_t$. Once again, the younger child would receive a larger bequest. For $t > T_2$, equal division ($\xi_t^1 = \xi_t^2$) is the rule if $B_t^i(x) = v(x)$, $i = 1, 2$.

This birth order result may appear contradictory to the theory of primogeniture and many empirical findings. But, it is not. On the theoretic front, Chu (1991) shows that in the pursuit for lineal succession, when dependents’ lifetimes are uncertain, the older child would receive a larger bequest. Since we assume away the issue of child mortality, our result is not at odds with his theory. On empirical

⁵In practice, the designated beneficiary of the life insurance may be the spouse. Not only it blurs the distinction of differential bequest discussed above, but also it ushers in the principal-agent problem of resource allocation. No attempt is made to deal with these issues in this paper.

front, most findings of primogeniture are based on the bequests to adult beneficiaries, while our theory applies to the very young and financially constrained children.

6. Demand for Life Insurance

The demand for life insurance, f_t^i , is jointly determined with consumption c_t and income transfer g_t^i . Consequently, the comparative statics should be jointly determined as well. To facilitate the analysis, we follow Lewis's approximation method of setting $p_t \approx 0$. This can be justified from Table 1, since the conditional probability of death is generally below 2% for ages 50 and younger, the time to raise a family. Then $\frac{\ell_t(1-p_t)}{1-\ell_t p_t} \approx \ell_t$ and equation (5.3) is simplified to

$$\xi_t^i w_t + f_t^i = \ell_t^{-1/\lambda} b(t, T_i) g_t^i, i = 1, 2.$$

Consequently, the (aggregate) demand for life insurance is

$$f_t = f_t^1 + f_t^2 = \ell_t^{-1/\lambda} [b(t, T_1) g_t^1 + b(t, T_2) g_t^2] - w_t. \quad (6.1)$$

The aggregate demand for life insurance thus obtained is negatively related to loading factor ℓ_t and savings w_t , but positively related to risk aversion λ . These are fairly standard results.

An interesting implication of equation (6.1) is that the aggregate demand for life insurance rises with $b(t, T_1) g_t^1 + b(t, T_2) g_t^2$. It shows that the demand for life insurance depends on the number, the age differentials, the time it takes for each child to reach independence, and the consumption need of the beneficiaries. In short, the demand for life insurance thus obtained reflects the "needs" and the

“circumstances” of the beneficiaries. It should be mentioned that equation (6.1) is a generalization of Lewis’ equation (11)

From (6.1), the elasticity of life insurance with respect to the loading factor is

$$-\frac{\ell_t}{f_t} \frac{\partial f_t}{\partial \ell_t} = \frac{1}{\lambda} \left(1 + \frac{w_t}{f_t} \right). \quad (6.2)$$

Recall that λ is between 1.2 and 1.8. If we employ Lewis’s finding that w_t/f_t averages 2.4, then the elasticity is between 1.8 and 2.8. This elastic demand for life insurance has an interesting economic interpretation. In the absence of market insurance, the breadwinner’s saving (w_t) has the effect of reducing the beneficiary’s losses in the event of the breadwinner’s death. Consequently, it is a form of *self-insurance* of Ehrlich and Becker (1972) even though it lags by one period and the buyer (the breadwinner) is different from the beneficiary. From this perspective, an elastic demand for life insurance suggests that there is a strong substitution effect between self-insurance and market insurance.

7. Conclusion and Comparisons to Lewis (1989)

In this paper we presented a theory of life insurance purchase of an altruistic breadwinner supporting his liquidity constrained children. The model takes the ages and the consumption need of the dependent children explicitly into account. It differs from Fischer’s (1975) model in that the choice between the breadwinner’s own consumption and that of the beneficiary is endogenous, there are gifts *inter vivos*, and that the purchase of life insurance depends on the number and the circumstances of the dependent children.

Lewis had the same objective. However, there are major differences. Lewis assumes that the breadwinner’s own consumption and the gifts to the heirs are

exogenously given. In contrast, our model allows the dependent's utility function to enter the breadwinner's optimization problem. In our dynamic setting, the breadwinner's consumption, gifts *inter vivos*, life insurance purchase, and contingent bequests are jointly determined in a single model. As a result, several interesting economic implications that go beyond Lewis' findings emerge.

First, our theory provides some new insights into the life cycle theory. We show that the breadwinner's own lifetime uncertainty and the altruism toward heirs are incentives for precautionary saving. Specifically, we show that the consumption profile is steeper than the one derived from the standard life-cycle model because the growth rate of consumption depends positively on the breadwinner's mortality rate and the unfairness of the insurance market. It may account for some of the observed delayed consumption than previously recognized.

Second, our theory sheds some lights on the role of the dependents' ages in the breadwinner's decision making. As the child ages, the need for protection is reduced, and hence, the time path of bequest to each child may decline with the child's age. When there are several children, the younger child have a larger bequest because the older child need less protection – a result opposite of primogeniture. We also show that the gifts *inter vivos* grow with each child's age as a result of the breadwinner's lifetime uncertainty, thereby provides an additional reason for this empirical fact. In addition, the demand for life insurance depends on the ages of children as shown in (6.1).

Third, we show that the number of children matters in a variety of ways. As the number of children increase, there is an additional saving to meet the future need. The age profile of consumption of the breadwinner is lowered as the number

of children increases, if the breadwinner's earning profile remains unchanged. The same is true for income transfer and bequest. Furthermore, the age profile of gifts *inter vivos* for each child rises with age, and the aggregate transfer peaks just right before the oldest child reaches the age of independence. Moreover, the demand for life insurance clearly reflects the number and the needs of the beneficiaries.

It should be mentioned that we have made some simplifying assumptions to make the model tractable. Specifically, we assume perfect family planning on the timing and age differentials of children, and assume away child mortality so that all dependents will live to their respective ages of independence. We also minimize the role played by the surviving spouse in the purchase of life insurance. Relaxing these assumptions would test the robustness of the theory developed in this paper. For example, it would be interesting to compare this theory of contingent bequest to Chu's theory of primogeniture in the presence of child mortality. These are for future research.

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