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THE HARTWICK RULE: MYTHS AND FACTS

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Abstract

We consider the Hartwick rule for capital accumulation and resource depletion, provide semantic clarifications and investigate whether this rule indicates sustainability and requires substitutability between manmade and natural capital. In addition to shedding light on the meaning of the Hartwick rule by reviewing established results, we establish the following novel finding: The value of net investments being negative does not imply that utility is unsustainable. Throughout we make the assumption of a constant technology, without which the Hartwick rule does not apply.

Keywords: Hartwick rule, natural resources, sustainability

JEL Classification: D9, Q3

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1. Introduction

In resource economics two intertemporal allocation rules have attracted particular attention: the *Hotelling rule* and the *Hartwick rule*. The Hotelling rule provides the fundamental no-arbitrage condition that every efficient resource utilisation path has to meet. In its basic form it indicates that along such a path the price of an exhaustible resource has to grow with a rate that equals the interest rate. Although the Hotelling rule is in principle relevant for all models of non-renewable resource use, its simplest application is that of a cake-eating economy where consumption results from depleting a given stock of natural capital. The Hartwick rule, in contrast, was formulated for a production economy where consumption at any point of time t depends not only on the extraction of natural capital but also on the stock of manmade capital available at t. In such a *Dasgupta-Heal-Solow model* Hartwick (1977) showed that, given the Hotelling rule as condition for local efficiency, a zero value of aggregate net investment will entail constant consumption over time. This result was the heart of what later on was called the Hartwick rule.

Hartwick's result became so attractive because it gave an extension to a basic message of neoclassical resource economics (cf. Solow (1974)): Exhaustible natural resource inputs can be substituted by manmade capital in a way that depleting these natural resources does not harm future generations. Substitutability between natural and manmade capital thus, in spite of the exhaustibility of natural resources, may allow for equitable consumption for all generations, and Hartwick (1977) seemed to have found the investment policy that would bring about sustainability in this way.

In the meantime, however, doubts have been raised concerning the true status of Hartwick's results and thus of the Hartwick rule. So following Asheim (1994) and Pezzey (1994) it has been claimed that the Hartwick rule is, contrary to the first impression, not a prescriptive but rather a descriptive rule (cf. Toman, Pezzey & Krautkraemer (1995, p. 147)). But the wording of the investment policy underlying the Hartwick rule undoubtedly gives a prescription. And even if one tends to see the Hartwick rule as a description, it is not exactly clear what is described by it. So more than 20 years after Hartwick's pioneering work everyone in resource economics will have *some* understanding of the Hartwick rule, but astonishingly there is no real consensus on what the Hartwick rule in fact is. This is partly a semantic problem, which can be solved by more precise formulations, including all specific assumptions. Beyond that, however, the ambiguous status of the Hartwick rule has also led to false beliefs concerning the material content of the rule. In order to give a correct interpretation of the Hartwick rule, we will confront two myths on this rule that are pertinent in the literature.

Myth 1: *The Hartwick rule indicates sustainability.*

This myth was already suggested by Hartwick (1977, pp. 973–974) himself when he stated that “investing all net returns from exhaustible resources in reproducible capital … implies intergenerational equity”.

Myth 2: *The Hartwick rule requires substitutability between manmade and natural capital.*

This myth is implicit in many contributions on the Hartwick rule. An explicit formulation can, e.g., be found in Spash & Clayton (1997, p. 146): “... the... Hartwick rule depends upon man-made capital ... being a substitute for, rather than a complement to, natural capital.”

We will demonstrate that neither of these two assertions is true, showing that an adequate understanding of the Hartwick rule is still pending. The structure of our argument will be as follows: After introducing the general technological framework in section 2, we give some semantic clarifications in section 3 where we, e.g., distinguish between the Hartwick investment rule, the Hartwick result and its converse. In sections 4 and 5 we will separately deal with the two myths described above. In section 4 we use the Dasgupta-Heal-Solow model to illustrate that consumption may exceed or fall short of the maximum sustainable level even if capital management is guided by the Hartwick investment rule in the short run. In section 5 we show how the Hartwick rule applies in models with no possibility for substitution between manmade and natural capital. On this basis we then try in section 6 to give an interpretation of the Hartwick rule that indicates in which sense an adequately conceived Hartwick rule can be used as a prescription or whether it should be seen as a description. We leave some technical derivations for an appendix, where we also refer to the interesting, but somewhat inaccurate, analysis by Hamilton (1995).

2. The technological setting

To concentrate on issues that are central to this paper (and to the analysis of the Hartwick rule), we will make the following simplifying assumptions:

- *Constant population.* We will assume that each generation lives for one instance; i.e., generations are not overlapping nor infinitely lived, implying that any intertemporal issue is of an intergenerational nature. Distributional issues within each generation will not be discussed.
- *Constant technology*. This means that any technological progress is *endogenous*, being captured by accumulated stocks of knowledge. Hence, the technology is time-*in*dependent, meaning that there is no *exogenous* technological progress in the sense of a time-dependent technology.

The analysis will allow for *multiple capital goods* since it is evident that the central question motivating the Hartwick rule — "is our accumulation of man-made capital sufficient to make up for the decreased availability of natural capital?" — is less interesting in a setting with one aggregate capital good.

In the real world environmental externalities are not always internalised. This is one of many causes which prevent market economies from being fully efficient. Furthermore, for many capital stocks (e.g. stocks of natural and environmental resources or stocks of accumulated knowledge) it is hard to find market prices (or to calculate shadow prices) that can be used to estimate the value of such stocks. In the present setting, we will abstract from these problems by assuming the

- *existence of an intertemporal competitive equilibrium* that leads to efficiency and that provides market prices for all capital goods.

Such an assumption is needed for a discussion of the Hartwick rule, which compares the market value of the net investments in different capital goods.

Following Dixit, Hammond & Hoel (1980) (henceforth referred to as DHH), we assume that the vector of consumption goods at time t, $\mathbf{c}(t)$, the vector of capital stocks at time t, $\mathbf{k}(t)$, and the vector of investments at time t, $\dot{\mathbf{k}}(t)$, is feasible if $(\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t))$ is in the set of feasible triples $\mathfrak{F}$. Here, $\mathbf{c}(t)$ includes both ordinary material consumption goods (measured as positive quantities) and labour inputs (measured as negative quantities), as well as environmental amenities, while $\mathbf{k}(t)$ comprises not only different kinds of manmade capital, but also stocks of natural capital and stocks of accumulated knowledge (thereby capturing *endogenous technological progress*). Since $\mathcal{F}$ is time-independent, the analysis does not allow for *exogenous technological progress*. We will assume that $\mathcal{F}$ is a closed and convex set that satisfies: (a) Capital stocks are non-negative ($(\mathbf{c},\mathbf{k},\dot{\mathbf{k}}) \in \mathcal{F}$ implies $\mathbf{k} \geq 0$) and (b) free disposal of investment flows ($(\mathbf{c},\mathbf{k},\dot{\mathbf{k}}) \in \mathcal{F}$ implies $(\mathbf{c},\mathbf{k},\dot{\mathbf{k}}') \in \mathcal{F}$ if $\dot{\mathbf{k}}' \leq \dot{\mathbf{k}}$). The latter assumption means e.g. that stocks of environmental resources are considered instead of stocks of pollutants. Lastly, the vector of consumption goods generates utility, $u(t) = u(\mathbf{c}(t))$, where u is a time-invariant, strictly increasing, concave, and differentiable function.

Given the assumption of an intertemporal competitive equilibrium, there are, at each t, prices for consumption and capital goods as well as utility. Let $\mathbf{p}(t)$ denote the present value prices of the consumption goods at time t, let $\mathbf{q}(t)$ denote the vector of present value prices of the capital stocks at time t, and let $\lambda(t)$ denote the present value price of utility (i.e. the utility discount factor) at time t. The term 'present value' reflects that discounting is taken care of by the prices. If $\lambda(t)$ is an exponentially decreasing function — i.e. $\lambda(t) = \lambda(0)e^{-\delta t}$ — then there is one constant (utility) discount rate: $\delta = -\dot{\lambda}(t)/\lambda(t) = \lambda(t)/\left(\int_t^\infty \lambda(s)ds\right)$. If not, there is a term structure of discount rates. The *instantaneous discount rate* is $\delta_0(t) = -\dot{\lambda}(t)/\lambda(t)$, while the *infinitely long-term discount rate* is $\delta_\infty(t) = \lambda(t)/\left(\int_t^\infty \lambda(s)ds\right)$.

The notion of a competitive path can now be defined.

DEFINITION 1. The path $(\mathbf{c}^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^\infty$ is *competitive* at present value prices $(\mathbf{p}(t), \mathbf{q}(t))_{t=0}^\infty$ and positive utility discount factors $(\lambda(t))_{t=0}^\infty$ if, at each t,

C1 instantaneous *utility* is maximized (i.e. $\mathbf{c}^*(t)$ maximizes $\lambda(t)u(\mathbf{c}) - \mathbf{p}(t)\mathbf{c}$),

C2 instantaneous *profit* is maximized (i.e. $(\mathbf{c}^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))$ maximizes $\mathbf{p}(t)c + \mathbf{q}(t)\dot{\mathbf{k}} + \dot{\mathbf{q}}(t)\mathbf{k}$ subject to $(\mathbf{c},\mathbf{k},\dot{\mathbf{k}}) \in \mathcal{F}$).

Refer to C1 and C2 as the *competitive conditions*.

Why is $\mathbf{p}(t)\mathbf{c} + \mathbf{q}(t)\dot{\mathbf{k}} + \dot{\mathbf{q}}(t)\mathbf{k}$ instantaneous *profit*? By writing $\mathbf{P}(t) = \mathbf{p}(t)/\lambda(t)$ and $\mathbf{Q}(t) = \mathbf{q}(t)/\lambda(t)$ for the consumption and capital prices in terms of current utility, we have that $\dot{\mathbf{Q}}(t) = d\left(\mathbf{q}(t)/\lambda(t)\right)/dt = \dot{\mathbf{q}}(t)/\lambda(t) - \left(\dot{\lambda}(t)/\lambda(t)\right)\left(\mathbf{q}(t)/\lambda(t)\right) = \left(\dot{\mathbf{q}}(t)/\lambda(t)\right) + \delta_0(t)\mathbf{Q}(t)$, which amounts to a no-arbitrage condition. In particular, it implies that the Hotelling rule will be satisfied in resource applications. It follows that $\left(\mathbf{p}(t)/\lambda(t)\right)\mathbf{c} + \left(\mathbf{q}(t)/\lambda(t)\right)\dot{\mathbf{k}} + \left(\dot{\mathbf{q}}(t)/\lambda(t)\right)\mathbf{k} = \mathbf{P}(t)\mathbf{c} + \mathbf{Q}(t)\dot{\mathbf{k}} - \left(r_0(t)\mathbf{Q}(t) - \dot{\mathbf{Q}}(t)\right)\mathbf{k}$, where $\mathbf{P}(t)\mathbf{c} + \mathbf{Q}(t)\dot{\mathbf{k}}$ is the current value of production and $\left(\delta_0(t)\mathbf{Q}(t) - \dot{\mathbf{Q}}(t)\right)\mathbf{k}$ is the current cost of holding capital.

It turns out that every competitive path is efficient given that the sum of discounted utilities is finite and a *capital value* transversality condition holds.

DEFINITION 2. The competitive path $(\mathbf{c}^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ is *regular* at present value prices $(\mathbf{p}(t), \mathbf{q}(t))_{t=0}^{\infty}$ and positive utility discount factors $(\lambda(t))_{t=0}^{\infty}$ if,

R1 $\int_0^{\infty} \lambda(t) u(\mathbf{c}^*(t)) dt$ exists (and is finite),

R2 $\mathbf{q}(t)\mathbf{k}^*(t) \to 0$ as $t \to \infty$.

PROPOSITION 1. *If* $(\mathbf{c}^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *is regular at present value prices* $(\mathbf{p}(t), \mathbf{q}(t))_{t=0}^{\infty}$ *and positive utility discount factors* $(\lambda(t))_{t=0}^{\infty}$, *then* $(\mathbf{c}^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *maximizes* $\int_0^{\infty} \lambda(t) u(\mathbf{c}^*(t)) dt$ *subject to* $(\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)) \in \mathfrak{F}$ *for all t and* $\mathbf{k}(0) = \mathbf{k}^0$.

Proof. Assume $(\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)) \in \mathfrak{F}$ for all t and $\mathbf{k}(0) = \mathbf{k}^0$. Then

$$\int_0^T \lambda(t)\big(u(\mathbf{c}(t)) - u(\mathbf{c}^*(t))\big)dt \le \int_0^T \mathbf{p}(t)\big(\mathbf{c}(t) - \mathbf{c}^*(t)\big)dt \text{ (by C1)}$$
$$\le \int_0^T \Big[\mathbf{q}(t)\big(\dot{\mathbf{k}}^*(t) - \dot{\mathbf{k}}(t)\big) + \dot{\mathbf{q}}(t)\big(\mathbf{k}^*(t) - \mathbf{k}(t)\big)\Big]dt \text{ (by C2)}$$
$$= \int_0^T \Big[d\Big(\mathbf{q}(t)\big(\mathbf{k}^*(t) - \mathbf{k}(t)\big)\Big)\Big/dt\Big]dt = \mathbf{q}(T)(\mathbf{k}^*(T) - \mathbf{k}(T)) - \mathbf{q}(0)(\mathbf{k}^*(0) - \mathbf{k}(0))$$
$$\le \mathbf{q}(T)\mathbf{k}^*(T)$$

since $\mathbf{k}^*(0) = \mathbf{k}(0) = \mathbf{k}^0$, $\mathbf{q}(T) \ge 0$ (by free disposal of investment flows) and $\mathbf{k}(T) \ge 0$. By R1 and R2 the result follows. □

Given that the utility discount factors are positive, this means that any competitive path satisfying the regularity conditions R1 and R2 is efficient.

For the analysis of the Hartwick rule, the following lemma turns out to be useful.

LEMMA 1. (*i*) *If* $(\mathbf{c}^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *is a competitive path with* $\mathbf{c}^*(t)$ *interior, then, for each consumption good i*, $\lambda(t)\partial u(\mathbf{c}^*(t))/\partial c_i = p_i(t)$. (*ii*) *(DHH) If* $\mathfrak{F}$ *is smooth and* $(\mathbf{c}^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *is a competitive path, then* $\mathbf{p}(t)\dot{\mathbf{c}}^*(t) + d\big(\mathbf{q}(t)\dot{\mathbf{k}}^*(t)\big)/dt = 0$.

Proof. (i) follows directly from C1. (ii) Since $\mathfrak{F}$ is time-invariant, C2 implies that

$$\mathbf{p}(t)\mathbf{c}^*(t + \Delta t) + \mathbf{q}(t)\dot{\mathbf{k}}^*(t + \Delta t) + \dot{\mathbf{q}}(t)\mathbf{k}^*(t + \Delta t) \le \mathbf{p}(t)\mathbf{c}^*(t) + \mathbf{q}(t)\dot{\mathbf{k}}^*(t) + \dot{\mathbf{q}}(t)\mathbf{k}^*(t).$$

Divide by Δt, and let Δt go to zero both from the right and from the left. This yields

$$0 = \mathbf{p}(t)\dot{\mathbf{c}}^*(t) + \mathbf{q}(t)\ddot{\mathbf{k}}^*(t) + \dot{\mathbf{q}}(t)\dot{\mathbf{k}}^*(t) = \mathbf{p}(t)\dot{\mathbf{c}}^*(t) + d\big(\mathbf{q}(t)\dot{\mathbf{k}}^*(t)\big)/dt,$$

where differentiability follows since $\mathfrak{F}$ is smooth. □

Hence, as pointed out by Aronsson et al. (1997, p. 105), if there is no exogenous technological progress and $(\mathbf{c}^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ is a competitive path satisfying that

$\mathbf{q}(T)\dot{\mathbf{k}}^*(T) \to 0$ as $T\to\infty$, then $\mathbf{q}(t)\dot{\mathbf{k}}^*(t) = \int_t^\infty \lambda(s)\dot{u}^*(s)ds$. Thus, the value of net investments at time t measures the present value of future changes in utility. The *investment value* transversality condition, $\mathbf{q}(T)\dot{\mathbf{k}}^*(T) \to 0$ as $T\to\infty$, needed for this result, follows from the optimality (and hence, from the regularity) of the path if there is a constant discount rate; i.e. if $\lambda(t) = \lambda(0)e^{-\delta t}$ (cf. Dasgupta & Mitra, 1999). However, if $\lambda(t)$ is not an exponentially decreasing function, then regularity will not imply this condition.

It will be instructive for the discussion that follows to introduce three different technologies that fit into the framework above. Each of the three models has only one consumption good, which thereby becomes an indicator of the quality of life. This means that the competitive condition C1 becomes less important. The first has also only one capital good, while the two others are two capital good models.

1. *The Ramsey model*. Let the set of feasible triples be given by $c(t) + \dot{k}(t) \leq f(k(t))$: The stock of the aggregate capital good ($k(t)$) leads to production $f(k(t))$ that can either contribute to the quality of life of generation t or be used to accumulate capital. We will assume that the production function f is twice continuously differentiable, with $f' > 0$ and $f'' < 0$. Furthermore, $f(0) = 0$, $\lim_{k\to 0} f'(k) = \infty$, and $\lim_{k\to\infty} f'(k) = 0$. It will turn out to be interesting to discuss issues relating to the Hartwick rule even in the setting of the Ramsey model.

In the remaining two models with two capital goods, the one capital good will be interpreted as manmade capital ($k_m(t)$) and the other as natural capital ($k_n(t)$). The production $F(k_m(t), e(t))$, that can either contribute to the quality of life of generation t or be used to accumulate manmade capital, depends both on the stock of manmade capital and the extraction ($e(t)$) of natural capital: $c(t) + \dot{k}_m(t) \leq F(k_m(t), e(t))$. The extraction of natural capital is counteracted by natural renewal $g(k_n(t))$ that depends on the stock of natural capital: $e(t) + \dot{k}_n(t) \leq g(k_n(t))$. If there is no natural renewal (i.e. k_n is a non-renewable exhaustible resource) and the production function F is of an ordinary neoclassical type, then we obtain a model investigated by Dasgupta & Heal (1974, 1979) and Solow (1974):

2. *The Dasgupta-Heal-Solow (DHS) model.* We will assume that F is linearly homogenous and twice continuously differentiable w.r.t. both arguments, with $F_m > 0$, $F_e > 0$, $F_{mm} < 0$, $F_{ee} < 0$, and $F_{me} = F_{em} > 0$. Furthermore, $\lim_{e\to 0} F_e(k_m, e) = \infty$, and $\lim_{e\to\infty} F_e(k_m, e) = 0$ hold for any $k_m > 0$, and $\lim_{k_m\to 0} F_m(k_m, e) = \infty$, and $\lim_{k_m\to\infty} F_m(k_m, e) = 0$ hold for any $e > 0$. Finally, we assume that the resource share of total production, $F_e(k_m, e)e / F(k_m, e)$, is bounded away from zero by some b. A Cobb-

Douglas function, $F(k_m, e) = k_m^a e^b$, with $0 < b < a + b = 1$, satisfies all these properties. If, in addition, $b < a$, then it follows from an analysis by Solow (1974) that a regular (hence efficient) path with constant and positive consumption exists, as long as the initial stocks, $k_m(0)$ and $k_n(0)$, are both positive. Such a path is feasible by letting the increasing stock of manmade capital substitute for the dwindling extraction of natural capital. The DHS model is of course the setting in which Hartwick (1977) first formulated the rule bearing his name.

Another model, which is a variant of a model appearing in Asheim (1978) and Hannesson (1986), is obtained by assuming a positive regenerative capacity for natural capital, and by assuming that the extraction of natural capital is limited by the extractive capacity.

3. *The complementarity model.* Let the regenerative capacity for natural capital be given by a logistic growth model, $g(k_n(t)) = k_n(t)(\bar{k}_n - k_n(t))$, and let the extractive capacity be given by $f(k_m(t))$, where f is twice continuously differentiable, with $f' > 0$ and $f'' < 0$, and satisfies $f(0) = 0$, $\lim_{k\to 0} f'(k) = \infty$, and $\lim_{k\to\infty} f'(k) = 0$. Since the extraction of natural capital is limited by the extractive capacity, it follows that $F(k_m(t), e(t)) = \min\{f(k_m(t)), e(t)\}$. As long as production is smaller than the maximal level of natural renewal, this model behaves as the Ramsey model. However, when one tries to sustain production above such a level, this model has interesting features to which we will return in Section 5.

The two technologies with heterogeneous capital — models 2 and 3 — have the following feature in common: The stock of manmade capital is to a certain degree complementary to the extraction of natural capital. In the first of these technologies (the DHS model) the marginal productivity of manmade capital is positively related to the extraction of natural capital. In model 3, the complementarity is, however, more extreme: Manmade capital can only be used for extracting natural capital. With such extreme complementarity, the accumulation of manmade capital is a mixed blessing. Following Richard Norgaard's (1991) analogy: if the livelihood of a society depends on the harvesting of a forest, future generations can gain more if the current generation invests by letting trees grow rather than accumulating saws.

It can be shown that these models essentially satisfy the general technological assumptions we made above when introducing the setting of DHH.

3. What is the Hartwick rule?

The term 'the Hartwick rule' has been used in different meanings. E.g. DHH in their first paragraph (p. 551) associated this term with both the *investment rule* of keeping "the total value of net investment under competitive pricing equal to zero" and the *result* that following such a investment rule "yields a path of constant consumption". In particular, it will be clarifying to differentiate between

- *the Hartwick investment rule* – which we will associate with the prescription of holding the value of net investments constant and equal to zero – and
- *the Hartwick result* – which we will associate with the finding that following such a prescription leads to constant utility.

Both 'the Hartwick investment rule' and 'the Hartwick result' require that the economy satisfies the competitive conditions C1 (when there are multiple consumption goods) and C2 along the interval of time in question. This means that there will, at any time, be a vector of present value prices of capital, $\mathbf{q}(t)$. Furthermore, the vector of capital stocks, $\mathbf{k}^*(t)$, will be superscripted by a star, to indicate that the competitive conditions apply. The term '(present) value of net investments' as used above corresponds to $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$. We can now state the definitions that we will suggest, present the results that follow from the analysis of Section 2, and provide a partial review of the relevant literature.

DEFINITION 3. Say that the *Hartwick investment rule* is followed if $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ is constant and equal to zero.

PROPOSITION 2. (The *Hartwick result*; Hartwick (1977) and later contributions.) If the Hartwick investment rule is followed in an economy with constant population and constant technology, then utility is constant (provided that the assumptions of Lemma 1 are satisfied).

Proof. Assume that C1, C2, and $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)=0$ is satisfied for all $t\in(t_1,t_2)$. Then

$$\lambda(t)\dot{u}(t) = \mathbf{p}(t)\dot{\mathbf{c}}^*(t) \quad \text{(by Lemma 1(i))}$$
$$= -d\big(\mathbf{q}(t)\dot{\mathbf{k}}^*(t)\big)/dt \quad \text{(by Lemma 1(ii))}$$
$$= 0 \quad \text{(since } \mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)=0\text{)}$$

for all $t\in(t_1,t_2)$. □

DHH made the observation that the Hartwick result can be generalised. For the statement of this more general result we first need to define 'the generalised Hartwick investment rule'.

DEFINITION 4. Say that the *generalised Hartwick investment rule* is followed if $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^{*}(t)$ is constant.

PROPOSITION 3. (The *generalised Hartwick result*, DHH.) If the generalised Hartwick investment rule is followed in an economy with constant population and constant technology, then utility is constant (provided that the assumptions of Lemma 1 are satisfied).

Proof. The proof of Proposition 2 applies even if $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^{*}(t)=\nu$ for all $t\in(t_1,t_2)$. □

DHH posed the question of whether the converse of the Hartwick result can be established. It is instructive to observe that the converse of the (ordinary) Hartwick result is not correct.

INCORRECT CLAIM. (The *converse of the Hartwick result*.) If a path satisfying the competitive conditions and yielding constant utility is followed in an economy with constant population and constant technology, then the Hartwick investment rule is followed (provided that the assumptions of Lemma 1 are satisfied).

Counter-example. Consider the Ramsey model. Here the competitive condition C2 implies that $c^{*}(t)+\dot{k}^{*}(t)=f(k^{*}(t))$, $p(t)=q(t)$, and $q(t)f'(k^{*}(t))=-\dot{q}(t)$. Hence,

$$p\dot{c}^{*}=q\dot{c}^{*}=-q\ddot{k}^{*}+qf'(k^{*})\dot{k}^{*}=-q\ddot{k}^{*}-\dot{q}\dot{k}^{*}=-d\left(q\dot{k}^{*}\right)/dt\,,$$

where the time-dependency has been surpressed. Hence, $\dot{c}^{*}(t)=0$ for all $t\in(t_1,t_2)$ is compatible with $q(t)\dot{k}^{*}(t)=\nu\neq0$ for all $t\in(t_1,t_2)$. In particular, if $\nu<0$, then $c^{*}=c^{*}(t)>f(k(t))$, which is feasible in the short run.

However, the converse of the generalised Hartwick result can be established:

PROPOSITION 4. (The *converse of the generalised Hartwick result*, DHH.) If a path satisfying the competitive conditions and yielding constant utility is followed in an economy with constant population and constant technology, then the generalised Hartwick investment rule is followed (provided that the assumptions of Lemma 1 are satisfied).

Proof. Since C1 and C2 imply that $\lambda(t)\dot{u}(t)=\mathbf{p}(t)\dot{\mathbf{c}}^{*}(t)=-d\left(\mathbf{q}(t)\dot{\mathbf{k}}^{*}(t)\right)/dt$, as shown in the proof of Proposition 2, it follows from the constancy of $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^{*}(t)$ that utility is constant. □

Applying these results at all times along infinite horizon paths yields some observations concerning the relationship between the (generalised) Hartwick result and the concept of sustainable development, as a precursor to the discussions of sections 4 and 5. For the statement of these results, we introduce the notion of

- *the Hartwick rule for sustainability*,

and say that a utility path $\{u(t)\}_{t=0}^{\infty}$ is *egalitarian* if utility is constant for all *t*.

PROPOSITION 5. (The *Hartwick rule for sustainability*.) If the Hartwick investment rule is followed for all *t* in an economy with constant population and constant technology, then the utility path is egalitarian (provided that the assumptions of Lemma 1 are satisfied).

Proof. This is an immediate consequence of the Hartwick result. □

PROPOSITION 6. (The *generalised Hartwick rule for sustainability*.) If the generalised Hartwick investment rule is followed for all *t* in an economy with constant population and constant technology, then the utility path is egalitarian (provided that the assumptions of Lemma 1 are satisfied).

Proof. This is an immediate consequence of the generalised Hartwick result. □

One may wonder whether Proposition 6 is an empty generalisation of Proposition 5, in the sense that any feasible competitive path with constant utility does in fact satisfy the (ordinary) Hartwick investment rule. This is not the case since in the Ramsey model there exist feasible competitive paths with constant utility for which $q(t)\dot{k}^*(t) = v > 0$ for all $t \in (0,\infty)$, provided that $v < q(0) f(k(0))$. Then $c^* = c^*(t) < f(k(t))$ for all *t*, so that the path is inefficient since capital is over-accumulated. It is, however, true that the (ordinary) Hartwick investment rule must be satisfied for all *t* if the egalitarian utility path is efficient.

PROPOSITION 7. (The *converse of the Hartwick rule for sustainability*, DHH, Withagen & Asheim (1998).) If the utility path is egalitarian along a regular path in an economy with constant population and constant technology, then the Hartwick investment rule is followed for all *t* (provided that the assumptions of Lemma 1 are satisfied).

Proof. The proof of Withagen & Asheim (1998) is too extensive to be reproduced here. The result means that a regular path with constant utility satisfies $\mathbf{q}(T)\dot{\mathbf{k}}^*(T) \to 0$ as $T \to \infty$. Combining this transversality condition with the results of Lemma 1 means that $\mathbf{q}(t)\dot{\mathbf{k}}^*(t) = \int_t^{\infty} \lambda(s)\dot{u}^*(s)ds$, as already noted in the discussion following the Lemma. From this it can be easily seen that the Hartwick investment rule is satisfied for all *t* if the utility path is egalitarian. □

The fact – shown above – that there exist egalitarian, but inefficient, utility paths in the Ramsey model, means that Proposition 7 does not hold if regularity is not assumed. If only the competitive conditions C1 and C2 are assumed to hold at any *t*, then a weaker result obtains:

PROPOSITION 8. (The *converse of the generalised Hartwick rule for sustainability*, DHH) If the utility path is egalitarian along a competitive path in an economy with constant population and constant technology, then the generalised Hartwick investment rule is followed for all *t* (provided that the assumptions of Lemma 1 are satisfied).

Proof. This follows from the converse of the generalised Hartwick result. □

In the following two sections we will discuss the implications of these results along two dimensions. Firstly, we note that these results are weak since they are based on strong premises involving the properties of the entire paths. In section 4 we therefore pose the question: can stronger results be obtained by weakening the premises – i.e. by relating sustainability of a path to only the current value of net investment – thereby addressing Myth 1. Secondly, in section 5 we discuss whether the Hartwick rule for sustainability requires substitutability between manmade and natural capital, thereby addressing Myth 2.

4. Myth 1: The Hartwick investment rule indicates sustainability

What makes Hartwick's investment rule so appealing in the framework of resource economics is its alleged relation with intergenerational fairness. Hartwick himself purported to have found a prescription how "to solve the ethical problem of the current generation shortchanging future generations by 'overconsuming' the current product, partly ascribable to current use of exhaustible resources" (Hartwick (1997, p. 972)). By invoking Hartwick's result the Hartwick investment rule then seemed to provide a sufficient condition for intergenerational justice. Although Hartwick's result is undoubtedly correct, this interpretation is not quite precise because the assumptions underlying it are not completely worked out. What in fact is not correct is to draw a close link between Hartwick's result and intergenerational equity without taking notice of additional conditions. There are more or less sophisticated versions of this precipitate interpretation.

INCORRECT CLAIM. *(trivial version)*: If the competitive conditions C1 and C2 hold and $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ is non-negative for all $t\in(t_1,t_2)$ in an economy with constant population and constant technology, then the constant utility level realised during the interval (t_1,t_2) is sustainable forever.

Whether this claim is correct or incorrect crucially depends on the underlying technology. Even in this simplistic version, which combines short-term considerations with long-term results, the claim is correct for, e.g. the Ramsey technology. To see this, note that in the Ramsey technology the utility level $\bar{c}$ that can be sustained forever from time *t* on is equal to $u(\bar{c})=u(f(k(t)))$, where $k(t)$ is the stock of capital at t. Having non-negative value of net investment at t, i.e. $q(t)\dot{k}^*(t)\geq 0$, it follows from the technological constraint that $f(k^*(t))-c^*(t)\geq\dot{k}^*(t)\geq 0$ or $u(c^*(t))\leq u(f(k^*(t)))=u(\bar{c})$, which proves the claim.

The claim is, however, not true in the DHS model. To give a counter-example, choose any consumption level c^* exceeding the maximum consumption level $\bar{c}$, which can be sustained for the underlying production function *F*, and the given stocks of manmade and natural capital $k_m(0)$ and $k_n(0)$. Then consider the path where consumption is held constant at c^* for some interval during which the competitive condition C2 is fulfilled and the Hartwick investment rule is followed, i.e. at any t in this interval $F(k_m^*(t),e^*(t))-c^* = F_e(k_m^*(t),e^*(t))e^*(t)$ has to hold. Such a path is uniquely determined as demonstrated in the Appendix (cf. Lemma A3). But as $c^*>\bar{c}$ the consumption level c^* can be maintained only for an interval of finite length. At some $T<\infty$ the stock of the natural capital is exhausted, and the sum of future consumption is limited to $k_m^*(T)$, as continued production is not feasible without resource extraction. Hence, during the interval $(0,T)$ the competitive condition C2 is satisfied (while C1 does not apply) and the value of net investments is non-negative; still, the constant consumption during this interval is not sustainable forever.

Hartwick (1977) does not say much about efficiency requirements going beyond competitiveness conditions, i.e. the Hotelling rule. In this context he only remarks that the entire stock of the exhaustible resource has to be used up in the long run in order to achieve an optimal solution. But it does not seem appropriate to neglect efficiency requirements going beyond competitiveness in looking for counter-examples. The path described above for the DHS model is in fact not efficient. Even the Hotelling rule is not fulfilled everywhere along that path, as there exist arbitrage possibilities by which the total length of the period when consumption c^* is possible can be prolonged. At time T a certain stock of manmade capital $k_m^*(T)$ has been accumulated, which can be used to maintain consumption c^* even for some interval following T. If – as in the Cobb-Douglas case – the marginal productivity of extraction tends to infinity when extraction goes to zero, then there are profits to be made by shifting resource extraction from right *before T* to right *after T*. Hence, there are profitable opportunities for arbitrage at *T*, implying that the Hotelling rule is not satisfied at that time. As the path in this counter-example thus is not efficient, the possibility arises that the missing link between the Hartwick investment rule and sustainability might be attributed to lack of efficiency. However, this is not true either. The claim above does not become valid even if we refer to regular – and thus efficient – paths for which not only competitiveness but also transversality conditions hold.

INCORRECT CLAIM. *(sophisticated version)*: If along a regular path $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ is non-negative for all $t \in (t_1, t_2)$ in an economy with constant population and constant technology, then the constant utility level realised during the interval (t_1, t_2) is sustainable forever.

Again counter-examples can be provided in the framework of the DHS model. Asheim (1994) and Pezzey (1994) gave a counter-example to this statement by considering paths in the DHS model, where the sum of utilities discounted at a constant discount rate is maximised. If, for some discount rate, the initial consumption level along such a discounted utilitarian optimum exactly equals the maximum sustainable consumption level given $k_m(0)$ and $k_n(0)$, then there exists an initial interval during which the value of net investments is strictly positive while consumption is unsustainable given the current capital stocks $k_m^*(t)$ and $k_n^*(t)$. It is, however, not obvious that the premise of this statement can be fulfilled; i.e. that there exists some discount rate such that initial consumption along the optimal path is barely sustainable. This has subsequently been established for the Cobb-Douglas case by Pezzey and Withagen (1998). The fact that their proof is quite intricate indicates, however, that this is not a trivial exercise.

Consequently, we wish to provide another type of counter-example here, which resembles our first counter-example given above. Moreover it can be used to show that even if in a DHS model in which the maximum sustainable consumption level is zero there exist regular paths that have a non-negative value of net investments in an initial period.

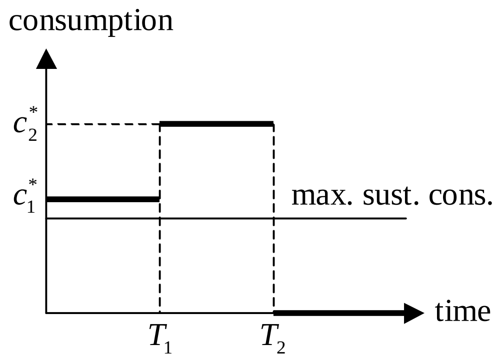

Figure 1.

This example, which is illustrated in Figure 1, consists of three separate phases with constant consumption, spliced together so that the Hotelling rule is satisfied at any time, even at the two points in time, T_1 and T_2, when consumption is not continuous. The initial stock of natural capital, $k_n(0)$, is determined so that both capital stocks are exhausted at T_2, implying that consumption equals zero for (T_2,∞).

Let $k_m(0)$ be given, fix some consumption level $c_1^*>0$ and some terminal time T_1 of the first phase of the path. Then construct, as described in the proof of Lemma A3 of the appendix, the unique path that has constant consumption c_1^* and obeys the Hartwick investment rule in the interval $(0,T_1)$. Let $k_m^*(T_1)$ be the stock of manmade capital at time T_1, and let $e^*(T_1)=\lim_{t->T_1^-} e^*(t)$. To satisfy the Hotelling rule at time T_1, extraction must be continuous; i.e. the continuation of the path must be constructed so that $e^*(T_1)=\lim_{t->T_1^+} e^*(t)$. Hence, the constant consumption during the interval (T_1,T_2), c_2^*, and the constant (present) value of net investment during this phase, v, must satisfy $F_e(k_m^*(T_1),e^*(T_1))(e^*(T_1)+v) = F(k_m^*(T_1),e^*(T_1))-c_2^*$. Since $F_e(k_m^*(T_1),e^*(T_1))e^*(T_1) = F(k_m^*(T_1),e^*(T_1))-c_1^*$, these two equalities are fulfilled if

$$v=\frac{c_1^*-c_2^*}{F_e(k_m^*(T_1),e^*(T_1))}.$$

By choosing an arbitrary $c_2^*>F(k_m^*(T_1),e^*(T_1))\,(>c_1^*)$ and by choosing v according to the equation above, a path can be determined along which investment in manmade capital is strictly negative at each point in time (cf. Lemma A7 of the appendix). This path is terminated at some finite point of time T_2 when the stock of manmade capital is completely depleted (cf. Lemma A8). For the two open intervals $(0,T_1)$ and (T_1,T_2) the Hotelling rule is fulfilled (cf. Lemma A1). By the construction of v given c_2^* a jump of the marginal productivity of extraction at T_1 is avoided so that the Hotelling rule obtains everywhere along this path. As the second part of this path is regular for an appropriate choice of $k_n(0)$ (cf. Lemma A9), regularity then holds for the whole path.

First note that the construction given above is completely independent of whether the underlying production function F allows for sustaining a strictly positive consumption level forever given finite initial stocks of manmade and natural capital. If F does not allow for a positive level of sustainable consumption, we have thus shown that having non-negative value of net investments during an initial phase of a regular path is well compatible with consumption exceeding the sustainable level.

However, even if the production function F allows for a positive level of sustainable consumption, we obtain a counter-example as desired. For this purpose, increase c_2^* beyond all bounds so that $-v$ increases (i.e. v becomes more negative). Then T_2 decreases and converges to T_1, and the aggregate input of extracted natural capital in the interval (T_1,T_2)

converges to zero. This in turn means that c_1^* cannot be sustained forever for large enough c_2^* given the choice of $k_n(0)$ needed to achieve exhaustion of natural capital at time T_2.

This example shows that a non-negative value of net investments during a time interval need not entail that consumption is sustainable. Although this result is not new, it is here established through a counter-example that it is simpler than those that have previously been available. However, it has up to now been an open question whether negative value of net investments during a time interval implies that consumption exceeds the sustainable level. We are able to show that not even this conjecture is true.

INCORRECT CLAIM: If along a regular path $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ is negative for all $t\in(t_1,t_2)$ in an economy with constant population and constant technology, then the constant utility level realised during the interval (t_1,t_2) is not sustainable forever.

Also in this case we will provide a counter-example in the framework of the DHS model. Let the production function F allow for a positive level of sustainable consumption. Again, the example (cf. Figure 2) consists of three separate phases with constant consumption, spliced together so that the Hotelling rule is satisfied at any time, even at the two points in time, T_1 and T_2, when consumption is not continuous.

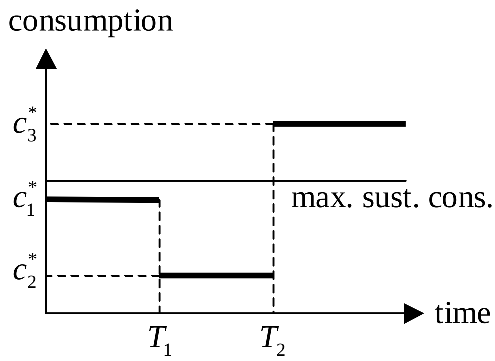

Figure 2.

Let $k_m(0)$ be given, fix some consumption level $c_1^*>0$ and some terminal time T_1 of the first phase of the path. Construct, as described in the proof of Lemma A7 of the appendix, a path that has constant consumption c_1^* and obeys the generalised Hartwick investment rule with $\nu_1<0$ in the interval $(0,T_1)$, where T_1 is small enough to ensure that $k_m^*(T_1)>0$. Let, as the second phase, the path have a constant consumption c_2^* and a constant (present) value of net investments $\nu_2>0$ in the interval (T_1,T_2), To satisfy the Hotelling rule at time T_1, c_2^* and ν_2, must fulfil $c_2^*+F_e(k_m^*(T_1),e^*(T_1))\nu_2 = c_1^*+F_e(k_m^*(T_1),e^*(T_1))\nu_1$. E.g. we can set

$$\nu_2=\frac{1}{2}\left(\frac{F(k_m^*(T_1),e^*(T_1))}{F_m(k_m^*(T_1),e^*(T_1))}-e^*(T_1)\right)>0,$$

so that $c_2^*=\frac{1}{2}\left(F(k_m^*(T_1),e^*(T_1))-F(k_m^*(T_1),e^*(T_1))e^*(T_1)\right)>0$. Construct, as described in the proof of Lemma A3 of the appendix, the unique path that has constant consumption c_2^* and obeys the generalised Hartwick investment rule in the interval (T_1,T_2). Let $k_m^*(T_2)$ and $e^*(T_2)$ be the stock of manmade capital and the flow of extraction at time T_2. Then, at T_2, the path turns over to the third phase where the (ordinary) Hartwick path is followed with c_3^* = $c_2^* + F_e(k_m^*(T_2), e^*(T_2))v_2$.

Since the production function F allows for a positive level of sustainable consumption, there exists an appropriate choice of $k_n(0)$ that makes the third phase of the path – and hence the whole path – regular. This stock of natural capital depends on T_1 and T_2, but it is finite in any case. Keep T_1 fixed and increase T_2. If T_2 goes to infinity, then the stock $k_n(0)$ will also tend to infinity (by Lemma A4). The same holds true for the maximum sustainable consumption level $\bar{c}$ that can be attained given $k_m(0)$ and $k_n(0)$. Hence, by shifting T_2 far enough into the future, a regular path can be constructed which has a first phase where the value of net investments is negative and a consumption level c_1^* which is sustainable given $k_m(0)$ and $k_n(0)$.

In these counter-examples (and in the analysis of the appendix) we have not invoked the competitive condition C1, which is somewhat superfluous in the one-consumption case. However, for any time-invariant strictly increasing, concave, and differentiable function u one can find a path of utility discount factors so that C1 is satisfied at any point in time. If u is strictly concave, the examples above will not lead to continuous paths of discount factors.

Both our counter-examples are consistent with the result for regular paths noted subsequently to Lemma 1 of section 2, namely that the value of net investments at time t measures the present value of all future changes in utility. It follows directly from that result that if along an efficient path utility is monotonely decreasing/increasing indefinitely, then the value of net investments will be negative/positive, while utility will exceed/fall short of the sustainable level. The value of net investments will thus indicate sustainability correctly along such monotone utility paths. Hence, the counter-examples above are minimal by having consumption (and thus, utility) be constant except at two points in time.

Moreover, such paths with piecewise constant consumption would not yield counter-examples if constant consumption would lead to a constant consumption interest rate (as it does in the Ramsey model). In the DHS model, however, it follows from the competitive conditions (cf. (A1)–(A5) of the appendix) that the consumption interest rate, $-\dot{p}(t)/p(t)$, measures the marginal productivity of manmade capital and is decreasing whenever consumption is constant. It is therefore the non-monotonicity of the paths – combined with the property that the consumption interest rate is decreasing along a constant consumption path in the DHS model – that leads to the negative results established above concerning the connection between the value of net investments and the sustainability of utility.

It is also worth to emphasise the point made in Asheim (1994) and elsewhere that the relative value of different capital stocks in an intertemporal competitive equilibrium depends on the property of the whole path. The counter-examples above show how the relative value of natural capital depends positively on the consumption level of the generations in the distant future. Thus, the future development – in particular, the distribution of consumption between the intermediate and the distant future – affects the value of net investments today and, thereby, the usefulness of this measure as an indicator today of sustainability.

Hence, in order to link the (generalised) Hartwick investment rule to sustainability we cannot avoid letting this rule apply to investment behavior at *all points in time*. We can present a correct claim concerning the value of net investments and the sustainability of utility by restating the generalised Hartwick rule for sustainability (Proposition 6) as follows.

CORRECT CLAIM: If along a competitive path $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ is constant for all $t\in(0,\infty)$ in an economy with constant population and constant technology, then the constant utility level at time *t* is sustainable forever.

Proof. From the generalised Hartwick rule for sustainability, it follows that the utility path is egalitarian. Hence, utility at any time is sustainable. □

If the path is regular, it follows from Proposition 7 that an egalitarian utility path is consistent only with $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ being equal to zero for all $t\in(0,\infty)$. In the Ramsey model, it is feasible, but not efficient to have $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ being constant and positive for all $t\in(0,\infty)$. As established in Lemma A5 in the appendix, this case is not even feasible in the DHS model. In both the Ramsey model and the DHS model, feasibility rules out $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ being constant and negative for all $t\in(0,\infty)$.

It is an open question whether the claim can be strengthened to: "if along a competitive path $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ is *non-negative* for all $t\in(0,\infty)$ in an economy with constant population and constant technology, then the constant utility level at time *t* is sustainable forever." We cannot prove this under general assumptions, but does not have a counter-example either.

5. Myth 2: The Hartwick rule for sustainability requires substitutability between manmade and natural capital

Hartwick (1977) concentrated his attention on economics where substitution of manmade capital and resource extraction is feasible. In the wake of his contribution an impression appears to have been formed to the effect that the Hartwick rule for sustainability requires that manmade capital can substitute for natural capital; i.e. that the production possibilities are consistent with the beliefs held by the proponents of 'weak sustainability' (cf. the citation from Spash and Clayton (1997) reproduced in the introduction). If, on the other hand, natural capital has to be conserved in order for utility to be sustained (i.e. the world is as envisioned by the proponents of 'strong sustainability'), then – it is claimed – the Hartwick rule for sustainability does not apply.

The *relevance* of the Hartwick rule for sustainability is related to the question of whether a constant utility path exists. Since a false premise does not falsify an implication, the Hartwick rule for sustainability as an implication is true even if, in some specific model, there does not exist any path with $\mathbf{q}(t) \cdot \dot{\mathbf{k}}^*(t)$ being constant and equal to zero for all *t*. What the Hartwick rule for sustainability entails is that if no constant utility path exists, then there cannot exist any path with $\mathbf{q}(t) \cdot \dot{\mathbf{k}}^*(t)$ being constant and equal to zero for all *t*. Still, even though the non-existence of an egalitarian path does not falsify the Hartwick rule for sustainability, it is interesting to discuss in what kind of technologies there exists an egalitarian utility path, implying that the result is relevant (i.e. not empty).

It turns out, however, that such substitutability is not necessary even for the relevance of the Hartwick rule for sustainability.

INCORRECT CLAIM: The Hartwick rule for sustainability is relevant only if manmade capital can substitute for natural capital.

That this assertion is not correct can be seen even if one considers the Ramsey model. In that model there is only one capital good such that substitution between different kinds of capital stocks is a priori not possible. Surprisingly, the general treatment of the Hartwick result and its converse given by DHH carries over to the Ramsey model if one replaces vectors of capital goods $\mathbf{k}(t)$ and their prices $\mathbf{q}(t)$ by scalars describing the size of the stock of manmade capital $k(t)$ and its present value price $q(t)$. As we have seen, when analysing the Ramsey model in section 3, it is feasible to follow forever the generalised Hartwick rule

($q(t)\dot{k}^*(t) = \nu$ for all $t \in (0,\infty)$) as long as the constant (present) value of net investment, ν, satisfies $0 \le \nu < q(0)f(k(0))$. And the resulting path has constant consumption as

$$p\dot{c}^* = q\dot{c}^* = -q\ddot{k}^* + qf'(k^*)\dot{k}^* = -q\ddot{k}^* - \dot{q}\dot{k}^* = -d\left(q\dot{k}^*\right)/dt .$$

Seen in this way, one could even turn things around by deriving the Hartwick result and its converse first for the Ramsey model and then generalising it in a very straightforward way to the many capital goods case. This would not only serve didactical purposes but would, more importantly, highlight that the DHS model is by no means the only field of application for the Hartwick result and its converse. This trivial insight alone sheds light on the Hartwick rule.

As the most important subcase this general treatment of the Ramsey model includes the situation where the stock of manmade capital $k^*(t)$ is time invariant, which, by the feasibility constraint, immediately implies constant consumption. This is the only efficient sustainable constant consumption path given an initial capital stock $k(0)$. The generalised Hartwick investment rule with positive or negative net investment either leads to an efficient path with over-accumulation of capital, or to a non-sustainable path.

Even within a model with multiple capital goods it can be shown that an ability to substitute manmade capital for natural capital is not necessary for the relevance of the Hartwick rule for sustainability. For this purpose, consider the complementarity model introduced in section 2. Here, the regenerative capacity for natural capital depends on the stock of natural capital, $g(k_n(t)) = k_n(t)(\bar{k}_n - k_n(t))$, while the extractive capacity depends on manmade capital $f(k_m(t))$.

The competitive condition C2 implies that

$$\begin{aligned}
&c^*(t) + \dot{k}_m^*(t) = \min\{f(k_m(t)), e(t)\},\\
&e^*(t) + \dot{k}_n^*(t) = g(k_n^*(t)),\\
&p(t) = q_m(t),\\
&(q_m(t) - q_n(t))\, f'(k_m^*(t)) = -\dot{q}_m(t),\\
&q_n(t)g'(k_n^*(t)) = -\dot{q}_n(t).
\end{aligned}$$

If, in this model, one tries to sustain production above the maximal level of natural renewal, then natural capital will be exhausted in finite time, undermining the productive capabilities. Any competitive path with constant consumption forever will satisfy the (ordinary) Hartwick investment rule by having the stock of manmade capital remain constant and the value of investments in natural capital be equal to zero. Hence, constant consumption along a competitive path is characterised by $c^* = f(k_m^*)$, implying that $\dot{k}_m^* = 0$, while $q_n(t)\dot{k}_n^*(t) = 0$. If, along such a path, the stock of natural capital converges to a size larger than the one corresponding to the maximal level of natural renewal, then $q_n(t) \equiv 0$ and the productivity of manmade capital measures the consumption interest rate: $f'(k_m^*) = -\dot{q}_m(t)/q_m(t)$. If, on the other hand, the stock of natural capital is constant and smaller than the size corresponding to the maximal level of natural renewal, then $c^* = f(k_m^*) = e^* = g(k_n^*)$ and $q_n(t) > 0$. And the productivity of natural renewal measures the consumption interest rate: $f'(k_m^*) > -\dot{q}_m(t)/q_m(t) = -\dot{q}_n(t)/q_n(t) = g'(k_n^*)$. In this latter case, the application of the Hartwick investment rule leads to a feasible egalitarian path by keeping both capital stocks constant. Hence, the model is consistent with the world as envisioned by the proponents of 'strong sustainability'; still, the Hartwick rule for sustainability applies.

In order to state a correct claim concerning the relevance of the Hartwick rule for sustainability, we must define the concept of 'eventual productivity'.

DEFINITION 5. Say that a model with preferences *u* and technology *F* satisfies *eventual productivity* given the vector of initial stocks $\mathbf{k}(0)$ if starting from these initial stocks there exists a regular path with constant utility forever.

CORRECT CLAIM. The Hartwick rule for sustainability is relevant under the assumptions of Lemma 1, if eventual productivity is satisfied given the vector of initial stocks $\mathbf{k}(0)$.

Proof. From eventual productivity and the converse of the Hartwick rule for sustainability, it follows that there exists a path with $\mathbf{q}(t) \cdot \dot{\mathbf{k}}^*(t)$ being constant and equal to zero for all *t*. □

The question of whether manmade capital can substitute for natural capital is important for the relevance of the Hartwick rule for sustainability only to the extent that lack of such substitutability means that eventual productivity cannot be satisfied.

6. Prescription or description?

The preceding analysis naturally leads to a more profound discussion of the following questions that are raised in the literature: Can the Hartwick investment rule be used as a prescription? Or is the Hartwick rule for sustainability (and its converse) a description of an egalitarian utility path; i.e. a characterisation result?

In Section 4 we have shown that a generation may well obey the Hartwick investment rule but nevertheless consume more than the maximum sustainable consumption level. On the other hand, a generation with a negative value of net investments will not necessarily undermine the consumption possibilities of its successors. It is thus an important message of the analysis of Section 4 that the Hartwick investment rule as such cannot serve as a *prescription* for sustainability, as capital management that is guided by the Hartwick investment rule in the short run may be compatible with quite different consumption levels. Hence, it is not enough to know whether the current investment in manmade capital in value makes up for the current depletion of natural capital, since the Hartwick result (Proposition 2) only says that following the Hartwick investment rule will entail constant consumption for an interval of time. This is clearly not sufficient for development to be sustainable, thereby ensuring intergenerational justice. Rather, a judgement on whether short-run behaviour is compatible with sustainable development must be based on the long-run properties of the path and the technological environment. By Proposition 6 of Section 3 (the generalised Hartwick rule for sustainability) these long-run properties are:

1. *Feasiblity*. The generalised Hartwick rule for sustainability requires that constant consumption can be sustained indefinitely. How can we know *now* that a path with constant consumption for some interval of time can be sustained forever? The DHS model of capital accumulation and resource depletion shows that it can be problematic to determine whether it is feasible to sustain a given level of constant consumption. As illustrated by a counter-example to the trivial version of the incorrect claim of Section 4, one can construct paths where feasibility breaks down due to an underestimation of the availability of natural capital.

2. *Competitive conditions.* The generalised Hartwick rule for sustainability requires that competitive conditions hold indefinitely. How can we know *now* that competitive conditions will be followed at any future point in time? Within the context of the DHS model the remaining examples of Section 4 illustrate that it is quite demanding to assume that competitive conditions (in particular, the Hotelling rule) hold for all t so there is no possibility for arbitrage.

3. *Constant present value of net investments.* The generalised Hartwick rule for sustainability requires that $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ is constant indefinitely. It is not sufficient to have *current* price-based information about the path in order to prescribe sustainable behaviour; rather such information has to be available at all future points in time. How can we know *now* that $\mathbf{q}(t)\cdot\dot{\mathbf{k}}^*(t)$ will be constant for all t?

4. *No exogenous technological progress.* The generalised Hartwick rule for sustainability applies only if the technology remains constant. Constant utility requires that any

> technological progress is *endogenous*, being captured by accumulated stocks of knowledge. How can we know *now* that we will be able to attribute any future technological progress to accumulated stocks of knowledge?

Moreover, if this amount of information were available about the long-run properties of paths as well as the future technological environment, and a constant consumption path is desirable, then the price-based information entailed in Hartwick rule would hardly seem necessary nor convenient for the social planning of such a path. Therefore, it is our opinion that the Hartwick investment rule is of limited use as a prescription for decision-makers trying to ensure that development is sustainable.

The Hartwick investment rule is, however, of interest when it comes to *describing* an efficient path with constant utility. It follows from the converse of the Hartwick rule for sustainability (Proposition 7) that any such egalitarian path will be characterised by the Hartwick investment rule being satisfied at all points in time. Note that the importance of this result is not that it tells decision-makers anything concerning how to steer the economy along such a path; rather, it describes how the path would look like *if* it were followed. It is therefore our view that it seems more natural to consider $\mathbf{q}(t) \cdot \dot{\mathbf{k}}^*(t) = 0$ for all t as a descriptive result, characterising an efficient and egalitarian utility path. This characterisation result is of high generality so that, as was seen in Section 5, it does impose any particular requirements on the possibility of substitution between manmade and natural capital. The DHS model is only one application among many others.

Even the interpretation of the Hartwick rule as a descriptive device needs a couple of caveats. First, the existence of an efficient and egalitarian path requires the assumption of eventual productivity to be satisfied so that such a path is in fact feasible. Without eventual productivity, sustainable and price supported paths need not exist, so that the Hartwick rule loses its relevance. Secondly, the unrealistic assumption that future technological progress can be contributed to accumulated stocks of knowledge, the value of which can be measured in market prices, is needed for all results relating to the Hartwick rule. Without being able to attribute the evolving technology to the augmentation of identifiable stocks, it becomes, for obvious reasons, a deficient exercise to account for the value of net investments.

7. Concluding remark

As proposed by Hartwick (1977) and further refined by Dixit, Hammond & Hoel (1980), the Hartwick result – based on the Hartwick investment rule – is a most important finding within resource economics that focused attention on the close relationship between constant consumption and zero net investment. Still, it appears that the meaning and implications of this result are often misunderstood or misinterpreted in the literature. Here we have attempted to clarify the underlying assumptions for the result, and tried to show what its significance in fact is. Our theoretical analysis and interpretative discussion shed light on the converse of the Hartwick rule for sustainability as the important result, giving a useful characterization of regular paths with constant utility forever. The existence of such paths has, however, to be ensured by additional technological assumptions that are not in necessarily implied by having capital management be guided by the Hartwick investment rule at some interval of time.

Appendix: The generalised Hartwick investment rule in the DHS model

Recall the assumptions that we make for the DHS model: F is linearly homogenous and twice continuously differentiable w.r.t. both arguments, with $F_m > 0$, $F_e > 0$, $F_{mm} < 0$, $F_{ee} < 0$, and $F_{me} = F_{em} > 0$. Furthermore, $\lim_{e \to 0} F_e(k_m, e) = \infty$, and $\lim_{e \to \infty} F_e(k_m, e) = 0$ hold for any $k_m > 0$, and $\lim_{k_m \to 0} F_m(k_m, e) = \infty$, and $\lim_{k_m \to \infty} F_m(k_m, e) = 0$ hold for any $e > 0$. Since, by linear homogeneity $F_e(k_m, e) = F_e(1, e/k_m)$ and $F_k(k_m, e) = F_k(k_m/e, 1)$, this implies that $\lim_{k_m \to \infty} F_e(k_m, e) = \infty$, and $\lim_{k_m \to 0} F_e(k_m, e) = 0$ hold for any $e > 0$, and $\lim_{e \to \infty} F_m(k_m, e) = \infty$, and $\lim_{e \to 0} F_m(k_m, e) = 0$ hold for any $k_m > 0$. Finally, the resource share of total production, $F_e(k_m, e)e / F(k_m, e)$, is bounded away from zero by some b. A Cobb-Douglas function, $F(k_m, e) = k_m^a e^b$, with $0 < b < a + b = 1$, satisfies all these properties.

The competitive condition C2 implies that

(A1) $$c^*(t) + \dot{k}_m^*(t) = F(k_m^*(t), e^*(t)),$$

(A2) $$e^*(t) + \dot{k}_n^*(t) = 0,$$

(A3) $$p(t) = q_m(t),$$

(A4) $$q_m(t) F_m(k_m^*(t), e^*(t)) = -\dot{q}_m(t),$$

(A5) $$q_m(t) F_e(k_m^*(t), e^*(t)) = 1,$$

where (A5) follows from $q_m(t)F_e(k_m^*(t),e^*(t)) = q_n(t)$ and $0 = \dot{q}_n(t)$ by choosing resource extraction as numéraire. Note that (A4) and (A5) entail that the Hotelling rule (HOR) is satisfied:

HOR $$F_m(k_m^*(t),e^*(t)) = \frac{dF_e(k_m^*(t),e^*(t))/dt}{F_e(k_m^*(t),e^*(t))}.$$

Since $p(t)$ and $q_m(t)$ are present value prices, (A2) and (A5) implies that the generalised Hartwick investement rule (GHIR) is satisfied if

GHIR $$\dot{k}_m^*(t) = F_e(k_m^*(t),e^*(t))(e^*(t)+\nu),$$

where ν is the constant present value of net investments, a result that has previously been observed by Hamilton (1995). It is of interest to note that constant consumption (CC),

CC $$c^*(t) = c^*,$$

and (GHIR) imply that (HOR) is satisfied; this is a generalisation of the main result of Buchholz (1980).

LEMMA A1. Every path that satisfies CC and GHIR on an open interval where $F(k_m^*(t),e^*(t)) \neq c^*$ fulfils HOR on this interval.

Proof: Taking derivatives w.r.t. time we obtain from (A1), CC and GHIR that

$$F_m\dot{k}_m^* + F_e\dot{e}^* = \ddot{k}_m^n = \dot{F}_e(e^* + \nu) + F_e\dot{e}^*,$$

which implies that

$$F_m\dot{k}_m^* = \dot{F}_e(e^* + \nu) = \frac{\dot{F}_e}{F_e}F_e(e^* + \nu) = \frac{\dot{F}_e}{F_e}\dot{k}_m^*.$$

Cancelling $\dot{k}_m^* = F(k_m^*(t),e^*(t)) - c^* \neq 0$ gives HOR: $F_m = \dot{F}_e / F_e$. □

When describing paths that fulfil CC and GHIR we take a certain consumption level $c^* > 0$ and an initial stock of manmade capital as exogenously given and then endogenously determine the stock of the natural resource that is used along such a path. Such a path will be called a GHIR path. Depending on the sign of the constant ν we distinguish two subcases: $\nu \geq 0$ and $\nu < 0$. We start with the former of these cases.

LEMMA A2. If $\nu \geq 0$ and $c^* > 0$, then, for any $k_m > 0$, there is exactly one $e^*(k_m)$ that fulfils $F_e(k_m,e^*(k_m))\,(e^*(k_m)+\nu) = F(k_m,e^*(k_m)) - c^*$.

Proof: Given k_m consider the function

$$h(e;k_m) = F_e(k_m,e)(e+\nu) - (F(k_m,e) - c^*) = c^* + F_e(k_m,e)\nu - F_k(k_m,e)k_m ,$$

where the second equality follows from linear homogeneity. As $\lim_{e\to 0} F_e(k_m,e) = \infty$, and $\lim_{e\to 0} F_m(k_m,e) = 0$ hold for any $k_m > 0$, we have that $h(e;k_m) > 0$ for small values of *e*, as $\lim_{e\to\infty} F_e(k_m,e) = 0$, and $\lim_{e\to\infty} F_m(k_m,e) = \infty$ hold for any $k_m > 0$, we have that $h(e;k_m) < 0$ for *e* high enough. By continuity of $h(\cdot;k_m)$, there is at least one $e^*(k_m)$ that fulfils $h(e^*(k_m);k_m) = 0$. As $dh(e;k_m)/de = F_{ee}(k_m,e)(e+\nu) < 0$, for $\nu \geq 0$ and $e > 0$, $e^*(k_m)$ is uniquely determined. □

LEMMA A3. Let a consumption level $c^* > 0$, an initial stock of manmade capital $k_m(0)$, and a constant $\nu \geq 0$ be given. Then a corresponding GHIR path is uniquely determined. Along such a path investment in manmade capital is strictly positive at each point in time.

Proof: This result follows from Lemma A2, since the development of the stock of manmade capital is determined by the differential equation

$$\dot{k}_m^*(t) = F(k_m^*(t), e^*(k_m^*(t))) - c^*$$

starting from the initial value $k_m(0)$, while the path of the resource extraction is given by $e^*(k_m^*(t))$. It follows from GHIR that $\dot{k}_m^*(t) > 0$ as $\nu \geq 0$ and $e^*(k_m^*(t)) > 0$. □

LEMMA A4. If $\nu > 0$ and $c^* > 0$ there exists a $\gamma > 0$ so that $e^*(k_m) \geq \gamma$ for any $k_m > 0$.

Proof: The GHIR condition can be transformed to

$$c^* = F(k_m, e^*(k_m))\left(1 - \frac{F_e(k_m, e^*(k_m))e^*(k_m)}{F(k_m, e^*(k_m))}\left(1 + \frac{\nu}{e^*(k_m)}\right)\right).$$

Since, by assumption $F_e(k_m,e)e / F(k_m,e)$ is bounded away from zero by some *b*, it follows that

$$0 < c^* \leq F(k_m, e^*(k_m))\left(1 - b\left(1 + \frac{\nu}{e^*(k_m)}\right)\right),$$

implying that $1 - b\left(1 + \left(\nu / e^*(k_m)\right)\right) > 0$. This gives $\gamma = b\nu/(1-b)$ as a lower bound for $e^*(k_m)$. □

On these grounds the next lemma will give, for the considered class of production functions, an impossibility result for GHIR paths having $\nu > 0$.

LEMMA A5. Let $\nu > 0$, $c^* > 0$, $k_m(0)$, and $k_n(0)$ be given. Then the GHIR path is not sustainable.

Proof: It follows from Lemma A3 that the aggregate extraction along such a GHIR path will approach infinity as time goes to infinity. Hence, any finite stock of natural capital will be exhausted in finite time, implying that the GHIR cannot be sustained indefinitely. □

In this respect the DHS model is different from the Ramsey model, where – as we have seen in section 3 – there exist sustainable GHIR paths with $\nu > 0$ and $c^* > 0$. In the DHS model, in contrast, any such path can be followed only for a finite period of time. Note also that following a GHIR path with $\nu > 0$ and $c^* > 0$ all along to exhaustion is not efficient since a positive stock of manmade capital will be left over at this point in time. This implies that profits are to be made by shifting resource extraction from right *before* exhaustion to right *after* exhaustion, meaning that there are profitable opportunities for arbitrage at that time.

Hamilton (1995) also analyses GHIR paths having $\nu > 0$ for different classes of technologies. For the class that overlaps with the one treated here ($\sigma \leq 1$), he incorrectly claims (1995, pp. 397–398 & Table 1) that – along a GHIR path with $\nu > 0$ – the level of consumption has to become negative at a finite point in time, which clearly contradicts Proposition 3. This as well as many other inaccuracies seem to be caused by his implicit and inappropriate assumption that variables are continuous functions of time throughout, even in the case when a GHIR path cannot be sustained indefinitely. For the case of a GHIR path with $\nu > 0$, the GHIR path (which yields constant consumption by Proposition 3) can be sustained up to the point when the stock of natural capital has been exhausted. The path from then on must be a completely different path, which cannot be governed by GHIR with $\nu > 0$. E.g., it is not correct, as claimed by Hamilton (1995, pp. 397–398), that resource extraction goes continuously to zero as the stock of natural capital approaches exhaustion.

In the case with $\nu = 0$ – i.e. the (ordinary) Hartwick investment rule is followed – the answer to the question of whether some $c^* > 0$ is sustainable depends on the possibility for substitution between the stock of manmade capital and the flow of extraction. If F is in the CES class, constant and positive consumption is feasible if the coefficient of substitution, σ, is larger than 1, and infeasible if σ is smaller than 1. In this class, only the case of $\sigma = 1$ – i.e. F is a Cobb-Douglas function, $F(k_m, e) = k_m^a e^b$, with $0 < b < a + b = 1$ – is consistent with the general assumptions we made above. It then follows from an analysis by Solow (1974) that a regular (hence efficient) path with constant and positive consumption exists, as long as $b < a$ and the initial stocks, $k_m(0)$ and $k_n(0)$, are both positive. Such a path satisfies $\nu = 0$ and is feasible by letting the increasing stock of manmade capital substitute for the dwindling extraction of natural capital. For more general production functions, Cass & Mitra (1991) give a necessary and sufficient condition for the existence of a path with constant and positive consumption, while the analysis of Dasgupta & Mitra (1983) can be used to argue that this implies the existence of an *efficient* path. From Withagen & Asheim (1998) it follows that any such efficient path with constant and positive consumption must satisfy the (ordinary) Hartwick investment rule (i.e. $\nu = 0$).

Turn now to the case with $\nu < 0$. In this case it turns out that if ν is too negative, the GHIR path is not feasible even in the short run. For the statement of the following results, let $\bar{e}(k_m)$ be defined by $F(k_m, \bar{e}(k_m)) = c$ for any given k_m.

LEMMA A6. If $\nu < 0$ and $c^* > 0$, then, for any $k_m > 0$, there exists e^* that fulfils $F_e(k_m, e^*)\,(e^* + \nu) = F(k_m, e^*) - c^*$ if and only if $-\nu \le \bar{e}(k_m)$. There is a unique value, $e^*(k_m) = -\nu$, that fulfils this equation if $-\nu = \bar{e}(k_m)$, while there are two values, $e_1^*(k_m) \in (0, -\nu)$ and $e_2^*(k_m) > \bar{e}(k_m)$, that fulfil this equation if $-\nu \in (0, \bar{e}(k_m))$.

Proof. Given k_m consider again the function

$$h(e;k_m) = F_e(k_m, e)(e + \nu) - (F(k_m, e) - c^*) = c^* + F_e(k_m, e)\nu - F_m(k_m, e)k_m .$$

As $dh(e;k_m)/de = F_{ee}(k_m, e)(e + \nu)$, if it follows that $dh(e;k_m)/de > 0$ if $e < -\nu$ and $dh(e;k_m)/de < 0$ if $e > -\nu$. At $e = -\nu$, $h(e;k_m) = -(F(k_m, e) - c^*) \ge 0$ if and only if $e \le \bar{e}(k_m)$. As $\lim_{e \to 0} F_e(k_m, e) = \infty$, and $\lim_{e \to 0} F_m(k_m, e) = 0$ hold for any $k_m > 0$, we have that $h(e;k_m) < 0$ for small values of *e*. As $\lim_{e \to \infty} F_e(k_m, e) = 0$, and $\lim_{e \to \infty} F_m(k_m, e) = \infty$ hold for any $k_m > 0$, we have that $h(e;k_m) < 0$ for *e* high enough. By continuity of $h(\cdot;k_m)$ the results follow. □

LEMMA A7. Let a consumption level $c^* > 0$, an initial stock of manmade capital $k_m(0)$, and a constant $\nu \in (-\bar{e}(k_m(0)), 0)$ be given. Then a corresponding GHIR path is determined along which investment in manmade capital is strictly negative at each point in time.

Proof. Analogous to the proof of Lemma A3 except that the development of the stock of manmade capital is determined by the differential equation

$$\dot{k}_m^*(t) = F(k_m^*(t), e_1^*(k_m^*(t))) - c^* \;\; (= F_e(k_m^*(t), e_1^*(k_m^*(t)))(e_1^*(k_m^*(t)) + \nu)) ,$$

where it follows from Lemma A6 that $e_1^*(k_m^*(t)) < -\nu$; hence $\dot{k}_m^*(t) < 0$. □

An alternative GHIR path is determined by letting resource extraction be determined by $e_2^*(\cdot)$ during an initial phase. Such paths need not be considered in the present analysis.

LEMMA A8. Let $\nu \in (-\bar{e}(k_m(0)),0)$, $c^* > 0$, $k_m(0)$, and $k_n(0)$ be given. Then the GHIR path with strictly negative investment in manmade capital is not sustainable.

Proof: First observe that $e_1^*(k_m)$ falls if k_m decreases. This follows from taking the total differential in the GHIR condition, which gives

$$(F_{em}dk_m + F_{ee}de_1^*)(e_1^* + \nu) + F_e de_1^* = F_m dk_m + F_e de_1^*$$

or

$$\frac{de_1^*(k_m)}{dk_m} = \frac{\frac{F_e}{e_1^*(k_m)+\nu} - F_{em}}{F_{ee}} > 0$$

as $F_m > 0$, $F_{em} > 0$, $F_{ee} < 0$ and $e_1^*(k_m) - \nu < 0$. This in turn means that output decreases and that the negative investment in manmade capital accelerates. Hence, the stock of manmade capital is used up in finite time. □

As a consequence we get

LEMMA A9. Let $\nu \in (-\bar{e}(k_m(0)),0)$, $c^* > 0$, and $k_m(0)$ be given. Then there exists $k_n(0)$ such that the path consisting of the GHIR path with strictly negative investment in manmade capital up to the time when manmade capital is used up, and of a path with zero consumption, capital investment and resource extraction thereafter, is regular.

Proof: Let $k_n(0)$ equal the integral of $e_1^*(k_m^1(t))$ up the time when $k_m^*(t) = 0$. It follows from Lemma A1 that the competitive conditions are satisfied, while the regularity conditions R1 (by normalising $u(0) = 0$) and R2 (as both stocks are exhausted in finite time) are clearly fulfilled. □

Thus, since the path of Lemma A9 is efficient, it follows that c^* strictly exceeds the maximal consumption level that is sustainable given the initial stocks $k_m(0)$ and $k_n(0)$.

Hamilton's (1995) analysis of GHIR paths with $\nu < 0$ contains inaccuracies for reasons similar as those noted subsequent to Lemma A5.

References

Aronsson, T., P.-O. Johansson and K.-G. Löfgren (1997), *Welfare Measurement, Sustainability and Green National Accounting*, Cheltenham: Edward Elgar.

Asheim, G. B. (1978), *Renewable Resources and Paradoxical Consumption Behavior*, Ph.D. dissertation, University of California, Santa Barbara.

Asheim, G. B. (1994), 'Net national product as an indicator of sustainability', *Scandinavian Journal of Economics* **96**, 257–265.

Buchholz, W (1980), 'Intergenerational equity, a savings investment rule, and the efficient allocation of an exhaustible resource', *Jahrbücher für Nationalökonomie und Statistik* **195**, 271–274.

Cass, D. and T. Mitra (1991), 'Indefinitely Sustained Consumption Despite Exhaustible Natural Resources', Economic Theory **1**, 119-146.

Dasgupta, P. S. and G. M. Heal (1974), 'The optimal depletion of exhaustible resources', *Review of Economic Studies* **41** (Symposium issue), 3–28.

Dasgupta, P. S. and G. M. Heal (1979), *Economic Theory and Exhaustible Resources*, Cambridge, U.K: Cambridge University Press.

Dasgupta, S. and T. Mitra (1983), 'Intergenerational Equity and Efficient Allocation of Exhaustible Resources', International Economic Review **24**, 133-153.

Dasgupta, S. and T. Mitra (1999), 'On the welfare significance of national product for economic growth and sustainable develpment', *Japanese Economic Review* **50**, 422–442.

Dixit, A., P. Hammond and M. Hoel (1980), 'On Hartwick's rule for regular maximin paths of capital accumulation and resource depletion', *Review of Economic Studies* **47**, 551–556.

Hamilton, K. (1995), 'Sustainable development, the Hartwick rule and optimal growth', *Environmental and Resource Economics* **5**, 393–411.

Hannesson, R. (1986), 'The effect of the discount rate on the optimal exploitation of renewable resources', *Marine Resource Economics* **3**, 319–329.

Hartwick, J. (1977), 'Intergenerational equity and the investing of rents from exhaustible resources', *American Economic Review* **67**, 972–974.

Norgaard, R. (1991), 'Sustainability as intergenerational equity: The challenge to economic thought and practice', Report no. IDP 97, the World Bank.

Pezzey, J. (1994), The optimal sustainable depletion of nonrenewable resources, Discussion paper, University College of London, Department of Economics, March.

Pezzey, J. and C. Withagen (1998), 'The rise, fall and sustainability of capital-resource economies', *Scandinavian Journal of Economics* **100**, 513–527.

Solow, R. (1974), 'Intergenerational equity and exhaustible resources', *Review of Economic Studies* **41** (Symposium issue), 29–45.

Spash, C.L. and A. M. H. Clayton (1997), 'The maintenance of natural capital: Motivations and methods', in A. Light and J. M. Smith, eds., *Philosophy and Geography I: Space, Place and Environmental Ethics*, Lanham, Boulder, New York, London: Rowman & Littefield Publishers, Inc. I, 143–173.

Toman, M.A., J. Pezzey, and J. Krautkraemer (1995), 'Neoclassical economic growth theory and ‘sustainability’', in D. W. Bromley, ed., *Handbook of Environmental Economics*, Oxford UK and Cambridge USA: Blackwell, 139-165.

Withagen, C. and G. B. Asheim (1998), 'Characterizing sustainability: The converse of Hartwick's rule', *Journal of Economic Dynamics and Control* **23**, 159–165.