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OPTIMAL PRIVATE AND PUBLIC HARVESTING UNDER SPATIAL AND TEMPORAL INTERDEPENDENCE*

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Abstract

This paper extends the Hartman model to include the case where two adjacent stands may be interdependent in the provision of amenity services. We show first that the relationship between the focal and exogenous rotation age depends on the nature of their temporal interdependence, i.e., on what happens to the degree of substitutability or complementarity between the stands when the rotation age of the private focal stand changes. We then apply this analysis to the determination of public rotation age in a two-stage game where the government first decides upon its harvesting and private harvesting is chosen in the second stage. Several new rules are derived for the socially optimal design of public harvesting depending on the nature of interdependence between private and public stands as well as on whether citizens have access to private forests for recreation or not.

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1. Introduction

Conventionally, the rotation period of a forest stand has been analyzed independently of that of other adjacent stands. Current focus on ecosystem management in forestry has put the possibility of stand interdependence on the research agenda. A typical example of spatial interdependence between stands is a forest area, comprising many stands, which sustains a given ecosystem, so that harvesting a stand would have a considerable impact on the whole ecosystem. To give another example, consider a private landowner producing timber and amenity services. Amenity services, however, can be produced either jointly by his own stand and the adjacent stand, or in either of them, while the adjacent stands may be owned by the forest landowner himself or by another landowner (private or public).

Both examples open a number of questions, the most crucial one being how the rotation age of a stand should be adjusted to those of adjacent stands when the stands are interdependent in the production of amenity services? If the landowner owns all spatially relevant stands the optimization problem is different from the case in which the adjacent stands are owned by other agents. The former case is often plausible in the management of public forests, while the latter is more typical of dispersed, private, nonindustrial land ownership.

The first analysts to point out the problem of potential interdependency between adjacent forest stands and its implications for forest management were Bowes and Krutilla (1985, 1989), who extended the standard single stand analysis to account for the age class distribution of the forest. Swallow and Wear (1993, 1997) reformulated the Hartman model for spatial interactions by defining the cases of substitutability and complementarity both for a forest landowner who does not own the adjacent stand and for a forest landowner who owns all stands. They concentrated, however, mostly on numerical simulations and did not fully develop the analytics of stand interdependence.[1] Koskela and Ollikainen (1999) offers an analysis of the interdependence in a two-period framework which suits to the case of uneven-aged

[1] Interestingly enough, their numerical simulations show that the optimal harvest schedule in the multiple stand ecosystem management problem does not necessarily converge to a single

forest management. In this paper we show, however, that this analysis does not hold for the more complex case of even-aged forests with an infinite series of rotations.

We use the Hartman model to re-examine the existing literature on stand interdependence and extend it in several directions. We draw on Swallow and Wear (1993), but formalize spatial interdependence so as to fit a characterization of substitutes and complements suggested in the economics literature. In the same vein as Swallow and Wear (1993) and Vincent and Binkley (1993) we start by considering a forest landowner who owns one focal stand and faces an exogenous adjacent stand which affects the amenity production of his stand. In the case of a single rotation the effect of the exogenous stand on the private rotation age depends on the spatial interdependence between the stands, i.e., on whether they are substitutes, independents or complements in the valuation of amenity services. For ongoing rotations we show that the rotation age of the focal private stand does not react to changes in exogenous stand if the amenity services are either independent or if the temporal interdependence does not change with the private rotation age, even though the stands were substitutes or complements. Private rotation age reacts negatively (positively) to the harvesting change of the exogenous stand under temporal dependence, when rotation age substitutability or rotation age complementarity increases (decreases) with private rotation age.

After having analyzed the private rotation age, we apply this generalized Hartman model to the determination of public rotation age. The interaction between private and public agents is described as a two-stage game.

In the first stage the Forest Service acts as a Stackelberg leader and decides about its harvesting, and private harvesting is chosen in the second stage. In our social welfare analysis we take into account the fact that the economy also consists of non-forest owners (recreators), who value amenities from private and public forests. Amenity services of public forest stands are a public good, while those of private forest stands may or may not be, depending on whether recreators have access to private forests or not. For socially optimal public harvesting, several new rules are derived depending

stand solution of the traditional Hartman-type forest management models.

on the nature of the interdependence between public and private stands, as well as on whether the recreators have access to private forests to enjoy amenities or not.

The rest of the paper is organized as follows. Section 2 conceptualizes the interdependence between private and exogenous forest stands in the "static," spatial sense and in the "dynamic," intertemporal sense. Section 3 is devoted to the study of optimal private harvesting under ongoing rotations, which requires the specification of temporal interdependence between the two stands. Optimal public harvesting, when the Forest Service is assumed to be a Stackelberg leader, is studied in section 4. Finally, there is a brief concluding section.

2. Spatial and Temporal Interdependence between Forest Stands

This section provides characterization of the spatial and temporal interdependence between the focal private stand and an exogenous adjacent stand in the valuation of amenity services. As the benchmark case we also describe the relationship between private harvesting and the exogenous stand for a single rotation.

2.1. Spatial Interdependence between the Private Stand and An Exogenous Stand

For a single rotation the representative private forest owner is assumed to choose the optimal harvesting time so as to maximize the utility from net harvest revenue and amenity services according to the following quasi-linear objective function

[1] $$\Omega = V^J + v(T,\tau),$$

in which $V^J = pf(T)e^{-rT} - c$, p is the timber price, $f(T)$ describes the growth of timber as a function of its age with the conventional convex-concave properties ($f'(T) > 0$ and $f''(T) > 0$ for $t < \bar{t}$ and $f''(T) < 0$, $t > \bar{t}$, where $\bar{t}$ is the inflexion point of the growth function) and c denotes regeneration cost.[2] The present value of

[2] In what follows the derivatives are noted by primes for functions with one argument and the partial derivative by subscripts for functions with many arguments. Hence, e.g.

amenity services from a private stand under a single harvest cycle of length T can be expressed as

[2] $$v(T,\tau) = \int_0^T F(s,\tau)e^{-rs}ds,$$

where $F(s,\tau)$ is the flow of amenities from the focal stand of age s when it is potentially affected by an adjacent exogenous stand of age τ.[3]

From equation [2] we get the discounted marginal valuation from amenity services as a function of the age of the private stand by differentiation

[3] $$v_T(T,\tau) = F(T,\tau)e^{-rT}.$$

It is often assumed that amenity valuation increases with the stand age, i.e. $F_T(T,\tau) > 0$ (see e.g. Hartman 1976). Depending on the specific amenities the valuation function F can have other properties as well. We may have $F_T(T,\tau) < 0$, indicating that a young forest is valued more than an old one; or if only the site-specific features of the forests count, then $F_T(T,\tau) = 0$.[4]

The sign of $v_{T\tau}(T,\tau)$ indicates how the discounted marginal valuation of amenity services from a private forest stand depends on the age of an exogenous stand. To explore this interdependence more precisely, we define the "static" concept of *Auspitz-Lieben-Edgeworth-Pareto (ALEP) complementarity or substitutability between forest stands* in our framework as follows.

$f'(T) = \frac{\partial f(T)}{\partial T}$ for $f(T)$, while $A_x(x,y) = \frac{\partial A(x,y)}{\partial x}$ for $A(x,y)$, etc.

3 Swallow and Wear (1993) originally suggested this formulation. Snyder and Bhattacharyya (1990) have analyzed the situation where consideration is given to the maintenance costs associated with a flow of non-timber values by assuming that otherwise they would vanish via a process of decay. Abstracting from the maintenance costs and from their assumption of the decay of non-timber values leads to the same formulation, which is used in this paper.

4 Calish, Fight and Teeguarden (1978) studied several alternative forest non-timber benefits for Douglas fir and found that they included a variety of increasing and decreasing time paths. Swallow, Parks and Wear (1990) extended their analysis by providing functional forms for various types of non-timber benefits and by presenting numerical simulations.

Definition 1. **Spatial interdependence.**

Two forest stands are substitutes, independents or complements in the ALEP sense when an increase in the age of the exogenous forest stand decreases, leaves unchanged, or increases the marginal valuation of the focal private forest stand, respectively.[5]

As one can see from equation [3], this definition is equivalent to the following mathematical formulation:

[4] $$v_{T\tau}(T,\tau)\begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \quad \Leftrightarrow \quad F_{\tau}(T,\tau) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \, .$$

What happens to the private rotation age T^J when the age τ of the adjacent stand changes? Given that the second-order condition $\Omega_{TT} < 0$ holds, the first-order condition $\Omega_T = 0$ defines implicitly the privately optimal rotation age in terms of the age τ of the adjacent stand, i.e. $T^J = T^J(\tau,..)$. Substituting this for T in $\Omega_T = 0$ gives an identity. Its partial differentiation with respect to τ gives $T_\tau^J = (-\Omega_{TT})^{-1}\Omega_{T\tau}$, so that the sign T_τ^J = sign $\Omega_{T\tau}$, where $\Omega_{T\tau} = F_\tau(T,\tau)$. Using Definition 1 yields the following

Proposition 1 *For a single rotation a change in the age of the exogenous forest stand will decrease, have no effect or increase, private rotation age when the exogenous forest stand is an ALEP substitute, independent or complement to the private forest stand, respectively.*

This result is qualitatively similar to that obtained from the two-period models with uneven-aged forest management (see Koskela and Ollikainen 1999) and has a natural interpretation. If forest stands are substitutes, the higher age of the adjacent forest allows the private forest owner to make his own harvest sooner and to enjoy the amenities from the adjacent stand for a longer period. For complements the opposite

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[5] See Samuelson (1974) and further discussions in Chipman (1977), Kannai (1980) and Weber (2000).

holds. Finally, when the forest stands are independents in terms of amenity valuation, changes in exogenous harvesting will have no effect on private harvesting.[6]

2.2. Temporal Interdependence between Private and Exogenous Forest Stands

Next we characterize how the spatial interdependence between the private and exogenous adjacent stands might evolve over time. The following definition turns out to be the key element in interpreting the results for the response of the focal private rotation age to changes in the age of the exogenous adjacent stand in the Hartman model for ongoing rotations.

Definition 2. **Temporal interdependence**

Temporal interdependence between two stands is constant, increases or decreases when substitutability or complementarity between the stands remains unchanged, increases or decreases with a higher private rotation age, i.e., when $F_{\tau T} = 0, F_{\tau T} > 0$ *or* $F_{\tau T} < 0$.

Constant temporal dependence holds when substitutability or complementarity is merely associated with site-specific properties, which remain the same regardless of the age of the endogenous private forest stand.[7] Increasing temporal dependence between the stands means that for ALEP complements the complementarity between stands increases with private rotation age, while for ALEP substitutes the substitutability decreases. Decreasing temporal dependence implies just the opposite: complementarity weakens, while substitutability becomes stronger. In Appendix 1 we

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[6] As for examples of substitutes and complements, we can offer the case on which Swallow and Wear (1993) base their simulations. They assume that a landowner values forage production consistently with big game production, where big game requires both forage and cover. The focal stand and the exogenous adjacent stand function as substitutes in their production by providing simultaneously both. The stands become complements if the focal stand provides forage, and the adjacent stand provides cover.

[7] See the discussion about this and several other cases in Calish, Fight and Teeguarden (1978) and in Swallow, Parks and Wear (1990). One should also note that if the amenity valuation is site-specific in the sense of $F_{\tau} = 0$, it is also temporally independent in the sense of $F_{\tau T} = 0$, but not necessarily the other way round.

present the parametric specifications of the amenity valuation function $F(T,\tau)$, which produce all the possibilities given in Definition 2.[8]

3. Optimal Private Rotation Age under Ongoing Rotations

In the Hartman model, extended to an exogenous adjacent stand, the private forest owner begins with bare land, plants trees and clear-cuts so as to maximize the present value from future harvest revenue and the utility of amenity services over an infinite cycle of rotations. This is given by

[5] $$\max_{\{T\}} W = V + E,$$

where $V = (1-e^{-rT})^{-1}V^J$ and $E = (1-e^{-rT})^{-1}\int_0^T F(s,\tau)e^{-rs}ds$.

The first-order condition $W_T = V_T + E_T = 0$ for the maximization of [5] can be expressed as follows:

[6] $$W_T = pf'(T) - rpf(T) - rV + F(T,\tau) - rE = 0.$$

The second-order condition is

[7] $$W_{TT} = pf''(T) - rpf'(T) + F_T(T,\tau) < 0,$$

which we assume to hold. According to [6] the private forest owner chooses the rotation age so as to equate the marginal benefit to delaying the harvest to age *T*, defined by $pf'(T) + F(T,\tau)$, to the marginal opportunity cost of delaying the harvest, defined by $rpf(T) + r(V+E)$.

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8 Swallow and Wear (1993) say that the stands are substitutes or complements when $F_{T\tau}$ is negative or positive, respectively (see 1993, p. 108). This corresponds to our definition 2 of decreasing or increasing temporal dependence, when the valuation function of amenities is continuously differentiable. Later on, Swallow and Wear (1993) say that the substitutability exists when F_τ is negative (p. 109), which corresponds to our definition 1 of ALEP substitutes. We know from the previous Result that Definition 1 matters only for the case of a single rotation, but – as we will show -- not for the case of ongoing rotations.

3.1 Comparative statics: regeneration costs, interest rate and timber price

We derive here the comparative statics of the Hartman model. Substituting $T^H = T^H(p,r,c,\tau)$, defined implicitly by the first-order condition (6), for T in $W_T = 0$ and differentiating the resulting identity partially with respect to exogenous parameters gives the comparative statics. The effects of parameters c, r and p on private rotation age are derived in Appendix 2. It turns out that $T_c^H > 0$, which is qualitatively the same as in the Faustmann model. Also in the Hartman model the interest rate affects the private rotation age negatively, i.e., $T_r^H < 0$. As for the relationship between the private rotation age and the timber price p, it turns out to be useful to characterize first how the relative size of the amenity benefits at the harvest time and its opportunity cost depends on the precise type of amenity valuation. This is given in the following

Lemma 1. $F(T,\tau) - rE \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \;\; as \;\; F_T(T,\tau) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$.[9]

Proof. *See Appendix* 3.

According to Lemma 1 the site-specific amenity valuation of the private stand, $F_T = 0$, implies that $F(T,\tau) = rE$, so that the Faustmann and the Hartman rotation age are the same (see equation [6]). If the marginal valuation increases (decreases) with the age of the private forest stand, then the valuation at the time of harvest dominates (is dominated by) its opportunity cost over an infinite series of rotations. Therefore, the Hartman rotation age is longer (shorter) than the Faustmann rotation age.

As for timber price p we get $T_p^H = (-W_{TT})^{-1} W_{Tp}$, and it can be shown that

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[9] The content of Lemma 1 can be found also in Bowes and Krutilla 1985, p. 539, and in Johansson and Löfgren (1988).

[8] $$T_p^H \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \quad as \quad rc(1-e^{-rT})^{-1} + F(T,\tau) - rE \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 .$$

To summarize, we have

Proposition 2.

In the Hartman model, regeneration cost affects positively and interest rate negatively optimal rotation age, while the effect of the timber price is ambiguous a priori.

Proposition **2** is new and shows that the timber price effect may differ from that given in the Faustmann model, in which $T_p^F < 0$.[10] Under positive regeneration costs the effect of the timber price on rotation age depends both on the discounted regeneration costs and on the sign of $F_T(T,\tau)$. Since private rotation age and timber price can now be also positively related, the effects of forest taxes on private rotation age may change qualitatively when we allow for non-timber services.[11]

3.2 The Response of Private Rotation Age to a Change in the Age of the Exogenous Stand

What happens to the focal private rotation age when the age of the exogenous adjacent stand changes? Using the similar procedure as above we get $T_\tau^H = -(W_{TT})^{-1} W_{T\tau}$, where

[9] $$W_{T\tau} \quad = \quad F_\tau(T,\tau) - r(1-e^{-rT})^{-1} \int_0^T F_\tau(s,\tau) e^{-rs} ds .$$

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[10] Bowes and Krutilla (1985, p. 540-541) present a part of this result when they say that under zero regeneration costs, $T_p^H < 0$ if the Hartman solution is above the Faustmann solution, i.e., if $F_T(T,\tau) > 0$.

[11] For the effects of forest taxes on the optimal private rotation age in the Faustmann and Hartman models, see Johansson and Löfgren (1985, Chapter 5), and Koskela and Ollikainen

Applying integration by parts yields

$$\int_0^T F_\tau(s,\tau)e^{-rs}ds \quad = \quad \frac{1}{r}\left[F_\tau(0,\tau) - F_\tau(T,\tau)e^{-rT} + \int_0^T F_{\tau T}(s,\tau)e^{-rs}ds\right],$$

so that equation $[9]$ can be re-expressed as

[10] $$W_{T\tau} \quad = \quad F_\tau(T,\tau) - (1-e^{-rT})^{-1}\left[F_\tau(0,\tau) - F_\tau(T,\tau)e^{-rT} + \int_0^T F_{\tau T}(s,\tau)e^{-rs}ds\right].$$

The terms in equation [10] have a natural interpretation. The first term describes the effect of the adjacent stand on the amenity valuation of the private stand at the time of the first harvest. The first and the second RHS bracket terms give the present value effect over all rotations of τ on the marginal amenity valuation of private bare land and of the stand during the harvesting period. Finally, the third RHS (integral) term captures the present value effect of the temporal interdependence of private and adjacent forests. It describes whether the complementarity or substitutability of the stands becomes stronger, weaker or remains unchanged when the private rotation period changes.

The response in the focal private rotation age to a change in τ is given by the following theorem:

Theorem. $T_\tau^H \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \text{ as } F_{\tau T} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0.$

Proof. *See Appendix* 4.

According to the Theorem the response of the private rotation age will depend only on how the temporal dependence between the stands in the amenity valuation will be affected by the change in the private rotation age. Therefore, unlike for the single

(2000), respectively.

rotation, the sign of the term $F_{\tau T}$ alone, not the notion of ALEP independence, substitutability and complementarity per se, is crucial. In the following three corollaries we apply the notion of temporal interdependence developed in Definition 2 to provide interpretation of the theorem.

Note first that if $F_{\tau} = 0$, we have the case of ALEP independence, so that a change in the rotation of the exogenous stand has no effect on private rotation age, as ALEP independence implies that $F_{\tau T} = 0$. Consider next the case of $F_{\tau} \neq 0$, for which there are three possibilities. The first one is presented in

Corollary 1. *If the ALEP substitutability or complementarity is temporally independent, i.e., if* $F_{\tau T} = 0$*, then a change in the adjacent stand will have no effect on the focal private rotation age.*

Temporal independence between stands means that the complementarity or substitutability relationship is merely due to their site-specific characteristics. Since a change in the exogenous harvesting will affect neither the marginal valuation nor opportunity costs, the private forest owner has no reason to change the privately optimal rotation age.

If the relationship between private and exogenous stands is temporally dependent, we have for increasing dependence

Corollary 2. *If the temporal interdependence increases, i.e., if* $F_{\tau T} > 0$, *implying that either ALEP complementarity becomes stronger or that ALEP substitutability becomes weaker, then a rise in the exogenous adjacent stand lengthens the focal private rotation age*.

Increasing temporal interdependence means that for ALEP complements a rise in the rotation age of the exogenous stand increases both the marginal valuation at harvest time and the opportunity cost effect of the future amenity valuation, the former effect being stronger. For ALEP substitutes, a rise in the rotation age of the exogenous stand decreases both the marginal valuation and the opportunity cost of amenity services, the latter effect being stronger. Therefore, in both cases the forest owner lengthens the private rotation age.

Finally, for decreasing temporal dependence, the private rotation age response is shown in

Corollary 3. *If the temporal interdependence decreases, i.e., if* $F_{\tau T} < 0$*, implying that either ALEP complementarity becomes weaker or that ALEP substitutability becomes stronger, then a rise in the exogenous adjacent stand shortens the focal private rotation age.*

If temporal dependence becomes weaker, then under decreasing ALEP complementarity a rise in the rotation age of the exogenous stand increases both the marginal valuation and the opportunity cost effects, but the latter is stronger. If ALEP substitutability increases, a rise in the rotation age of the exogenous stand decreases both the marginal valuation and the opportunity cost effects, the former effect being stronger. In both cases the forest owner shortens the private rotation age.

4. Optimal Public Harvesting with Amenity Externalities

We next apply our analysis to the determination the optimal public rotation age.[12] Forest Service adopts a harvesting policy will maximize the social welfare from public harvesting by accounting also for citizens' recreation as a component of the social welfare and for the presence of private harvesting response. We assume that public forest is a public good, which means that citizens have full access to enjoy the amenity services from public forests. As for private forests we assume that they may or may not be **a** public good, reflecting different practices of various countries as regards to possible access restrictions to private forests. Moreover, in both cases we assume that there are no congestion effects associated with enjoying amenity services of forests.

[12] The determination of public harvesting is analyzed also in Amacher (1999) and in Koskela and Ollikainen (1999), but in the context of a two-period model.

In the spirit of traditional public finance literature we assume that the policy maker credibly commits to a future policy (see e.g. Atkinson and Stiglitz 1980). Technically this means that before any private harvest decisions are made the Forest Service announces its harvest policy and thereby acts as a Stackelberg dominant player by taking into account the response of private forest owners when choosing the public rotation age.[13] We start by analyzing the case when there is free access not only to public but also to private forests.

4.1. Free access to private forests

The government chooses the optimal rotation age τ* so as to maximize the social welfare function when citizens have free access to both public and private forests. The quasi-linear social welfare function consists now of the present value of the indirect utility function of the representative forest owner ($W^*(\tau, p, r, c)$),[14] the present value of the utility of recreators from private forests ($(n-1)E$), the present value of the utility of all citizens (n) from public forests (nE^g), and of the net present value of the public harvest revenue over an infinite series of rotations (V^g).

In formulating the social welfare maximization problem we have to take into account that the private rotation age T^H may depend on the public rotation age, such that $T^H = T^H(\tau, ...)$. Hence public harvesting will affect social welfare both directly and indirectly by changing private harvesting behavior. While only the direct effect matters to the representative forest owner because he has optimized with respect to T, both the direct and indirect effects are relevant for the other agents. Therefore, the social welfare function can be written as

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[13] If government cannot enter into binding commitments regarding its future harvesting policy, then it re-optimizes at the beginning of each period. The representative landowner decides about harvesting given his (or her) expectations concerning government policy. Government in turn sets its harvesting policy, taking the behavior of the landowner as given. Equilibrium in this non-commitment environment is the Nash equilibrium (see. e.g. Persson and Tabellini (1990) for an introduction).

[14] The indirect utility function $W^*(\tau, p, r, c)$ can be obtained by substituting the optimal private rotation age $T^H(\tau, p, r, c)$, implicitly defined by equation [6], for T in equation [5]. Here we are interested in the effect of τ on the components of social welfare defined in equation [11].

[11] $$SW = \underbrace{W^*(\tau,..) + (n-1)E(\tau,T^H(\tau...),...)}_{private\ \ forests} + \underbrace{V^g + nE^g(\tau,T^H(\tau,...),...)}_{public\ \ forests},$$

where

[12a] $$E(\tau,T^H(\tau,...)...) = (1-e^{-rT^H(\tau...)})^{-1}\int_0^{T^H(\tau,...)} F(s,\tau)e^{-rs}ds,$$

[12b] $V^g = (1-e^{-r\tau})^{-1}\left[pg(\tau)e^{-r\tau} - c\right]$, and

[12c] $$E^g(\tau,T^H(\tau,...)...) = (1-e^{-r\tau})^{-1}\int_0^{\tau} F^g(T^H(\tau,...),x)e^{-rx}dx,$$

in which $g(\tau)$ is the growth of the public stand as a function of its age with the usual properties (see page 6) and F^g describes the valuation of amenities from the public stand.

The first-order condition for the socially optimal public rotation age can be expressed in a general form as

[13] $$SW_\tau = W_\tau^* + (n-1)E_\tau + V_\tau^g + nE_\tau^g + \left\{ (n-1)E_T + nE_T^g \right\} T_\tau^H = 0.$$

The first-order condition describes various channels through which changes in public rotation will affect social welfare. The first four terms characterize the direct effects of the change in public stand τ on the welfare of the forest owner and recreationalists, on public timber revenues, and on the welfare of citizens, respectively. The last two terms characterize the indirect social welfare effects via the amenity valuation of private and public stands, which result from the response of the private rotation age to public harvesting. The detailed expressions for the partial derivatives in equation [13] are developed in Appendix 5.

We assume that the second-order condition holds and start the analysis of optimal public harvesting by considering cases 1-3 outlined in the previous section, beginning with the simplest case 1 and then progressing to the more complex ones, 2 and 3.

If there is no interdependence between private and public forest stands in the provision of amenity services (i.e. since the amenity valuation of the private landowner holds that $F_\tau = 0$), then public harvesting does not affect private rotation (Case 1). As one can see from the Appendix 5, the optimal public harvesting is given by

$$\textbf{[14]} \qquad SW_{\tau|F_\tau=0,n>1} = 0 \quad \Leftrightarrow \quad \left[pg'(\tau) - rpg(\tau) - rV^g\right] + n\left[F^g(T,\tau) - rE^g\right] = 0 .$$

The terms in the first RHS brackets give the familiar Faustmann rule for the rotation of the public stand in the absence of amenity valuation. Recognizing, however, the social benefits of amenity services from the public stand introduces the second RHS bracket term into [14] to characterize the socially optimal public harvesting. According to the first-order condition [14], the Forest Service equates the marginal benefit of delaying public harvest until age τ ($pg'(\tau) + nF^g(T,\tau)$) with the marginal opportunity cost of delaying public harvest ($rpg(\tau) + r(V^g + nE^g)$).

How does equation [14] relate to the Faustmann rule? The answer depends on whether the marginal valuation of the public stand at harvest time dominates the opportunity cost of the public stand or not. The following Lemma provides the answer.

Lemma 2. $F^g(T,\tau) - rE^g \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ as $F^g_\tau(T,\tau) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$.

Proof. *See Appendix* 6.

Applying Lemma 2 to [14] shows that if the valuation of the public forest is merely site-specific so that $F^g_\tau = 0$, the optimal public rotation age is equal to the Faustman rotation age. If the marginal valuation of public stand increases (decreases) with its age ($F^g_\tau > 0(<)$), then the optimal public rotation age is longer (shorter) than the Faustmann rotation age. Therefore, one gets from equation [14]

Proposition 3.*If private and public stands are independents in the valuation of amenity services* $(F_\tau = 0)$, *allowing for free access to a public forest lengthens (shortens or leaves unchanged), the public rotation age if the valuation of the amenity services increases (decreases, or remains unchanged) with the age of the public forest.*

What happens if private and public stands are interdependent in the marginal valuation of amenity services, but this interdependence does not change with the age of the private stand? Under these circumstances, public harvesting has no effect on private harvesting age ($T_\tau^H = 0$), and the first-order condition for the socially optimal public rotation age can be written as

[15] $$SW_{\tau|F_\tau \neq o, F_{\tau T}=0, n>1} = (1-e^{-rT^H(\tau,\ldots)})^{-1} n \int_0^{T^H(\tau,\ldots)} F_\tau(s,\tau)e^{-rs}ds$$
$$+(e^{r\tau}-1)^{-1}\left\{ \left[pg'(\tau) - rpg(\tau) - rV^g\right] + n\left[F^g(T,\tau) - rE^g\right] \right\} = 0 .$$

Utilizing equation [14] yields the following connection

[16] $$SW_{\tau|F_\tau \neq o, F_{\tau T}=0, n>1} = 0 \quad \Leftrightarrow$$
$$SW_{\tau|F_\tau = 0, F_{\tau T}=0, n>1} + \frac{(e^{r\tau}-1)}{(1-e^{-rT^H(\tau,\ldots)})} n \int_0^{T^H(\tau,\ldots)} F_\tau(s,\tau)e^{-rs}ds = 0 .$$

Equation [16] gives a generalized Hartman rule under temporal independence between public and private forest stands.

Proposition 4.*Compared with independent stands, temporal independence between stands* $(F_{\tau T} = 0)$ *implies a longer public rotation age when public and private forests are complements* ($F_\tau > 0$), *and a shorter public rotation age when they are substitutes* ($F_\tau < 0$).

Proposition 4 has a natural interpretation. For complements, compared to independence, it is optimal for the Forest Service to provide a longer public rotation age, because increased public harvesting induces higher private harvesting, which would reinforce the decrease in the amenity benefits of harvesting for citizens. Analogously, for substitutes the public rotation age will be shortened, because private forest owners will lengthen their rotation age, which yields amenity benefits to citizens.

Finally, we consider the general case [13], which allows either substitutability or complementarity to evolve over time, so that $F_\tau \neq 0$ and $F_{\tau T} \neq 0$. Rearranging equation [13] (see Appendix 5) yields

[17] $$SW_{\tau|F_\tau \neq 0, F_{\tau T} \neq 0, n>1} = 0 \quad \Leftrightarrow \quad SW_{\tau|F_\tau \neq o, F_{\tau T} = 0, n>1}$$

$$+T_\tau^H \frac{(e^{r\tau}-1)}{(e^{rT^H(\tau,...)}-1)}\left[(n-1)\left[F(T,\tau)-rE\right]+\frac{e^{rT^H(\tau,...)}-1}{1-e^{-r\tau}}n\int_0^\tau F_T^g(T,x)e^{-rx}dx\right]=0$$

Comparing equation**s** [16] and [17] allows us to infer how temporal interdependence in the valuation of amenity services affects optimal public harvesting. Recall first from the theorem presented in Section 3 that $T_\tau^H > 0$ for increasing and $T_\tau^H < 0$ for decreasing temporal dependence and that according to Lemma 1 it holds that $F(T,\tau)-rE \geq (<)\ 0$ as $F_T \geq (<)\ 0$. Finally, the sign of the last term depends on the sign of F_T^g, i.e., on whether the private stand is an independent, a substitute or a complement to the public stand in the marginal valuation of amenities from public stand. On the basis of these considerations we can see that equation [17] gives rise to several cases. In the following we characterize two alternatives by providing sufficient conditions for them.

Proposition 5.*Compared with temporally independent stands, increasing temporal interdependence* $(F_{\tau T} > 0)$ *implies that the public rotation age is a) longer if* $F_T > 0$ *and* $F_T^g \geq 0$*, and b) shorter if* $F_T < 0$ *and* $F_T^g \leq 0$.

Proposition **5** provides an interesting qualification to Proposition 4. Consider part (a) of Proposition **5.** Increasing temporal interdependence between stands implies a longer private rotation period when the marginal amenity valuation of a private stand increases with its age. A longer private rotation age benefits recreators provided that the private stand enters into their marginal valuation of the public stand as an independent or complement. Hence public rotation age will increase. Likewise decreasing marginal valuation of a private stand with age implies a shorter private rotation period. This benefits recreators if they regard the private stand as a substitute or independent in the marginal valuation of the public stand. Then the Forest Service promotes a beneficial change in the private rotation age by decreasing the public rotation age compared with the temporal independence.[15]

4.2 No Access to Private Forest

Thus far we have assumed that citizens have full access to private forests for recreation, which is the case e.g. in Finland and Sweden. One may ask how the lack of access to private forests, as is the case in many parts of the United States, affects the socially optimal public harvesting? Given no access to private forests for recreation, the term $(n-1)E$ is no longer relevant and the social welfare function can now be written as

[18] $$SW = \underbrace{W^*(\tau,..)}_{private\ forests} + \underbrace{V^g + nE^g}_{public\ forests},$$

The first-order condition for the social optimum is given by

[19] $$SW_\tau = W_\tau^* + V_\tau^g + nE_\tau^g + nE_T^g T_\tau^H.$$

Assume first that the stands are independent. Then the last term in [19] is zero, and comparing the first-order conditions [13] and [19] allows one to conclude, in comparison with the case of free and no access, that

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[15] Proposition **5** does not exhaust the possibilities inherent in equation (17). One can provide an analogous characterization of sufficient conditions for the case of decreasing temporal

Proposition 3'. *If private and public forest stands are independent, denying access to private forests will have no effect on optimal public harvesting relative to open access to them.*

If there is no link between private and public stands in the valuation of amenities, or if a change in private rotation age does not affect the marginal valuation of amenities from public stands, then accessibility to private stands affects neither recreators' nor forest owner's utility. Therefore optimal public harvesting remains unaffected as well.

Assume next that forest stands depend on each other, but the degree of this dependence does not change in time, i.e., they are temporally independent ($F_\tau \neq 0$, but $F_{\tau T} = 0$). Noting that [19] will be identical to equation [16], when we set $n = 1$ due to access restriction, we have

Proposition 4'. *If the relationship between private and public forest stands is temporally independent, denying access to private forest will shift the optimal public rotation age up to that of independent stands from below (complements) or down from above (substitutes).*

The interpretation is straightforward. Denying recreators' access to private forests reduces the size of the externality caused by public harvesting through the marginal valuation of private forests from (n-1) to 1. Therefore, compared to the open access case, the optimal public rotation age will be closer to the age in the absence of externalities, i.e., the age of independent stands.

Finally, in the general case, where $F_\tau \neq 0$ and $F_{\tau T} \neq 0$, we get from $[19]$

[20] $$SW_{\tau|F_\tau \neq 0, F_{\tau T} \neq 0, n>1} = 0 \quad \Leftrightarrow \quad SW_{\tau|F_\tau \neq o, F_{\tau T} = 0, n>1} + T_\tau^H e^{-r\tau} n \int_0^\tau F_T^g (T, x) e^{-rx} dx = 0 .$$

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dependence $(F_{\tau T} < 0)$.

Comparing this with equation [17] shows that, as opposed to Proposition 3' denying access to private forest affects the adjustment of public rotation age compared with Proposition **5**. More specifically, we have

Proposition 5'. *If the relationship between the private and public forest stands is temporally interdependent* ($F_{\tau T} > (<)\ 0$), *denying access to private forests will shift the optimal private rotation age towards that of temporally independent stands from above,* if $F_T^g > (<) 0$, *or below, if* $F_T^g < (>) 0$.

As Proposition 5' indicates, again the size of the externality is decreased such that the rotation age becomes closer to that of independent stands.

The analysis of public harvesting as a two-stage game with the Forest Service being a Stackelberg leader gives a set of harvesting rules that differ substantially from those usually given in forest economics. The socially optimal harvesting rules depend on i) whether recreators have free access private forests or not, ii) how amenity valuation changes with the age of forest stands, and iii) how the degree of complementarity or substitutability between public and private forests in the valuation of amenity services evolves over time. The first aspect is institutional and varies from country to country, while the other two are basically empirical questions, and are interesting areas for research.

4. Conclusions

We have re-examined the literature of interdependent stands in the Hartman model, and applied our analysis to the determination of public rotation age. Our analysis contributes the existing literature in several ways. First, we have clarified the potential interdependence between two adjacent stands and offered a new interpretation of the concepts of spatial and temporal interdependence. While spatial interdependence means complementarity/substitutability between the stands, temporal interdependence means that the degree of substitutability/complementarity between stands, may increase, decrease or remain constant with the age of the stands. Second, we have shown that the comparative statics of private rotation age is not always conventional.

In contrast to Faustmann model, a higher timber price may lengthen the rotation age. Moreover, private rotation age reacts positively or negatively to exogenous harvesting, depending on how the degree of substitutability or complementarity, i.e. temporal dependence, between the stands evolves in time. Third, based on this analysis and on assumptions concerning access restrictions we derive several new rules for public harvesting. By and large they show that in the presence of stand interdependence the Hartman rotation age, derived under the single, stand has to be modified.

There are several avenues for further research. First, a natural step is to extend the analysis to include interdependent stands under sole ownership, which internalizes the (positive and negative) externality effects caused by changes in the age of an exogenous adjacent stand. Second, modeling the interaction between private and public agent as a two-stage game with government being a Stackelberg leader is only one - and not always the most plausible - alternative. In the absence of the Forest Service's commitment we would end up with a Nash game between the Forest Service and private forest owners. Third, we used a simple form of the 'felicity' function to describe the value of amenities in terms of the age of the focal stand and of the relationship between the focal stand and the exogenous stands. A next step is to reformulate the felicity function to describe the age factor in a more realistic way, e.g. in terms of a share of decaying trees, and tree species diversity. Finally, as the optimal rules for public harvesting depend on the precise type of amenity valuation, it would be an important topic to study empirically the interdependence between private and public stands in the provision of amenity services.

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Appendix 1. Parametric specifications for amenity valuation function $F(T,\tau)$

It was shown in section 2 that the precise role of the amenity valuation function matters. This Appendix specifies various possibilities.

***** ALEP independence can be described e.g. by an amenity valuation function

A.2.1 $F(T,\tau)=\dfrac{T^{1-\gamma}}{1-\gamma}+K$, where K is constant, $F_T=T^{-\gamma}>0$, $F_\tau=0$ and $F_{\tau T}=0$.

***** Temporal independence for ALEP complements can be described e.g. by

A.2.2 $F(T,\tau)=\dfrac{T^{1-\gamma}}{1-\gamma}+\dfrac{\tau^{1-\rho}}{1-\rho}$, where $F_T=T^{-\gamma}>0$, $F_\tau=\tau^{-\rho}>0$ and $F_{\tau T}=0$,

and temporal independence for ALEP substitutes respectively as

A2.3 $F(T,\tau)=\dfrac{T^{1-\gamma}}{1-\gamma}-\dfrac{\tau^{1-\rho}}{1-\rho}$, where $F_T=T^{-\gamma}>0$, $F_\tau=-\tau^{-\rho}<0$ and $F_{\tau T}=0$.

*Decreasing ALEP complementarity can be described e.g. by a valuation function

A2.4 $F(T,\tau)=\dfrac{(T+\tau)^{1-\gamma}}{1-\gamma}$, where $F_T=(T+\tau)^{-\gamma}>0$, $F_\tau=(T+\tau)^{-\gamma}>0$ and

$$F_{\tau T}=-\gamma(T+\tau)^{-(\gamma+1)}<0$$

and decreasing ALEP substitutability as

A2.5 $F(T,\tau)=\dfrac{(T-\tau)^{1-\gamma}}{1-\gamma}$, where $F_T=(T+\tau)^{-\gamma}>0$, $F_\tau=-(T-\tau)^{-\gamma}>0$ and

$$F_{\tau T}=\gamma(T-\tau)^{-(\gamma+1)}>0.$$

* Increasing ALEP complementarity can be described e.g. as

A.2.6 $F(T,\tau)=T^{\alpha}\tau^{1-\alpha}$, where $F_T=\alpha T^{\alpha-1}\tau^{1-\alpha}>0$, $F_\tau=(1-\alpha)T^{\alpha}\tau^{-\alpha}>0$ and

$$F_{\tau T}=(1-\alpha)\alpha T^{\alpha-1}\tau^{-\alpha}>0$$

and increasing ALEP substitutability as

A.2.7 $F(T,\tau)=e^{T-\beta\tau}$, where $F_T=e^{T-\beta\tau}>0$, $F_\tau=-\beta e^{T-\beta\tau}<0$ and

$$F_{\tau T}=-\beta e^{T-\beta\tau}<0.$$

Appendix 2. Comparative statics of the Hartman model

The first-order condition for $\max_{\{T\}} W$ can be written as

A1.1 $W_T = pf'(T) - rpf(T) - rV + F(T,\tau) - rE = 0$,

where $V = (1-e^{-rT})^{-1} V^J$, $V^J = pf(T)e^{-rT} - c$. Assuming that the second-order condition $W_{TT} = pf''(T) - rpf'(T) + F_T(T,\tau) < 0$ holds, A1.1. defines implicitly the privately optimal rotation age as a function of exogenous parameters, i.e. $T^H = T^H(p,r,c,\tau)$. Substituting this for T in $W_T = 0$ gives an identity and its partial differentiation gives $T^H_\alpha = (-W_{TT})^{-1} W_{T\alpha}$, where $\alpha = p,r,c,\tau$. One gets for c and p

A1.2 sign T^H_c = sign $r(1-e^{-rT})^{-1} > 0$,

A1.3 sign T^H_p = sign $A = f'(T) - rf(T) - rf(T)e^{-rT}(1-e^{-rT})^{-1}$**.**

On the basis of Lemma 1 presented in the text we have

1. If $F_T = 0$, then $F(T,\tau) - rE = 0$, $\Rightarrow$ A = 0.
2. If $F_T > 0$, then $F(T,\tau) - rE > 0$, $\Rightarrow$ A < 0.
3. If $F_T < 0$, then $F(T,\tau) - rE < 0$, $\Rightarrow$ the sign of A depends whether $rc(1-e^{-rT})^{-1} + F(T,\tau) - rE \geq (<)\ 0$ i.e.

A1.4 $T^H_p \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \;\; as \;\; rc(1-e^{-rT})^{-1} + F(T,\tau) - rE \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$.

As for the effects of the real interest rate *r*, one has

A1.5 sign T^H_r = sign B,

where $B = B_0 + B_1$, with $B_0 = V + r\frac{d}{dr}V$ describing the "Faustmann part" and $B_1 = E + r\frac{d}{dr}E$ the "Hartman part" of the problem, respectively. The Faustmann part is $B_0 = -pf(T) - V - r(1-e^{-rT})Tpf(T)e^{-rT} - T(pf(T)e^{-rT} - c)e^{-rT}$ and it can be rewritten as $B_0 = -(pf(T)+V)(1 - \frac{rT}{e^{-rT}-1})$. Applying the L'Hopital's rule one can prove that $(1 - \frac{rT}{e^{-rT}-1}) > 0$, i.e., $B_0 < 0$.

As for the Hartman part B_1, note first that

$\frac{d}{dr}E = -Te^{-rT}(1-e^{-rT})^{-1}E - (1-e^{-rT})\int_0^T sF(s,\tau)e^{-rs}ds$**.** Integrating the last term in

$\frac{d}{dr}E$ yields $(1-e^{-rT})\int_0^T sF(s,\tau)e^{-rs}ds = TE - E$, so that

$\dfrac{d}{dr}E = -Te^{-rT}(1-e^{-rT})^{-1}E - TE + E = Ed$ **,** where $d = (1+r-rT-rT(e^{rT}-1)^{-1}) =$

$\dfrac{(1+r)(e^{rT}-1)-rTe^{rT}}{e^{rT}-1}$. To apply the L'Hopital's rule for *d*, we differentiate its numerator and the denominator with respect to *rT* and get $\hat{d} = \dfrac{(1+r)e^{rT}-e^{rT}-rTe^{rT}}{e^{rT}}$ which gives that $\lim_{rT\to 0}\hat{d} = \dfrac{(1+r)-1}{1} = r > 0$. Hence, we have $-(E+rE) == E\left[1+r-rT-rT(e^{rT}-1)^{-1}\right] < 0$. Now the overall term

$B_1 = -(1-\dfrac{rT}{e^{rT}-1})(pf(T)+V)-(1+r-rT-\dfrac{rT}{e^{rT}-1})E < 0$, so that $T_r^H < 0$. Q.E.D.

Appendix 3. Proof of Lemma 2:

Note first that we can write

A.3.1. $F(T,\tau) - rE = \int_0^T F(s,\tau)e^{-rs}ds\left[\dfrac{F(T,\tau)}{\int_0^T F(s,\tau)e^{-rs}ds} - \dfrac{r}{(1-e^{-rT})}\right]$.

If $F_T > (\le)\ 0$, then $\int_0^T F(T,\tau)e^{-rs}ds > (\le)\int_0^T F(s,\tau)e^{-rs}ds \iff$

$\dfrac{F(T,\tau)}{r}(1-e^{-rT}) > (\le)\int_0^T F(s,\tau)e^{-rs}ds \iff \dfrac{F(T,\tau)}{\int_0^T F(s,\tau)e^{-rs}ds} > (\le)\dfrac{r}{(1-e^{-rT})}$.

Hence, $F(T,\tau) - rE > (\le)\ 0$ as $F_T(T,\tau) > (\le)\ 0$. Q.E.D.

Appendix 4. Proof of the Theorem: the sign of T_τ^H = the sign of $F_{\tau T}$

Recall from the text that the cross-derivative of equation [9] can be written as

A4.1 $W_{T\tau} = \int_0^T F_\tau(s,\tau)e^{-rs}ds \left[\frac{F_\tau(T,\tau)}{\int_0^T F_\tau(s,\tau)e^{-rs}ds} - \frac{r}{1-e^{-rT}} \right]$

- Temporal Independence**:** $F_{\tau T} = 0 \;\Rightarrow\; T_\tau^H = 0$

Proof. If $F_{\tau T} = 0$, equation [10] reduces to

$W_{T\tau} = F_\tau(T,\tau) - (1-e^{-rT})^{-1}\left[F_\tau(0,\tau) - F_\tau(T,\tau)e^{-rT}\right]$. There are two possibilities. If $F_\tau = 0$, then trivially $W_{T\tau} = 0$. Under $F_\tau \neq 0$, $F_{\tau T} = 0$ implies that

$\left[F_\tau(0,\tau) - F_\tau(T,\tau)e^{-rT}\right] = F_\tau(1-e^{-rT}) \Rightarrow$
$W_{T\tau} = F_\tau(T,\tau) - (1-e^{-rT})^{-1}F_\tau(T,\tau)(1-e^{-rT}) = 0$. Hence, $T_\tau^H = 0$.

- Increasing Temporal Dependence: $F_{\tau T} > 0 \;\Rightarrow\; T_\tau^H > 0$

Proof. i) Assume that $F_\tau > 0 \;\Rightarrow\; W_{T\tau} > 0 \;\Leftrightarrow\; \frac{F_\tau(T,\tau)}{\int_0^T F_\tau(s,\tau)e^{-rs}ds} > \frac{r}{1-e^{-rT}}$.

$F_{\tau T} > 0 \;\Rightarrow$

$$\int_0^T F_\tau(T,\tau)e^{-rs}ds > \int_0^T F_\tau(s,\tau)e^{-rs}ds \;\Leftrightarrow\; \frac{F_\tau(T,\tau)(1-e^{-rT})}{r} > \int_0^T F_\tau(s,\tau)e^{-rs}ds$$

$\Leftrightarrow \quad \frac{F_\tau(T,\tau)}{\int_0^T F_\tau(s,\tau)e^{-rs}ds} > \frac{r}{1-e^{-rT}}$. Hence, $W_{T\tau} > 0$ so that $T_\tau^H > 0$.

ii) Assume that $F_\tau < 0 \;\Rightarrow\; W_{T\tau} < 0 \;\Leftrightarrow\; \frac{F_\tau(T,\tau)}{\int_0^T F_\tau(s,\tau)e^{-rs}ds} < \frac{r}{1-e^{-rT}}$.

$F_{\tau T} > 0 \;\Rightarrow$

$$\int_0^T F_\tau(T,\tau)e^{-rs}ds > \int_0^T F_\tau(s,\tau)e^{-rs}ds \;\Leftrightarrow\; \frac{F_\tau(T,\tau)(1-e^{-rT})}{r} > \int_0^T F_\tau(s,\tau)e^{-rs}ds$$

$\Leftrightarrow \quad \frac{F_\tau(T,\tau)}{\int_0^T F_\tau(s,\tau)e^{-rs}ds} < \frac{r}{1-e^{-rT}}$. Hence, $W_{T\tau} > 0$ so that $T_\tau^H > 0$.

- Decreasing Temporal Dependence: $F_{\tau T} < 0 \;\Rightarrow\; T_{\tau}^{H} < 0$

Proof. i) Assume that $F_{\tau} > 0 \Rightarrow W_{T\tau} > 0 \;\Leftrightarrow\; \dfrac{F_{\tau}(T,\tau)}{\int_0^T F_{\tau}(s,\tau)e^{-rs}ds} < \dfrac{r}{1-e^{-rT}}$.

$F_{\tau T} < 0 \;\Rightarrow$

$$\int_0^T F_{\tau}(T,\tau)e^{-rs}ds < \int_0^T F_{\tau}(s,\tau)e^{-rs}ds \Leftrightarrow \frac{F_{\tau}(T,\tau)(1-e^{-rT})}{r} < \int_0^T F_{\tau}(s,\tau)e^{-rs}ds$$

$\Leftrightarrow \quad \dfrac{F_{\tau}(T,\tau)}{\int_0^T F_{\tau}(s,\tau)e^{-rs}ds} < \dfrac{r}{1-e^{-rT}}$. Hence, $W_{T\tau} < 0$ so that $T_{\tau}^{H} < 0$.

ii) Assume that $F_{\tau} < 0 \;\Rightarrow\; W_{T\tau} < 0 \;\Leftrightarrow\; \dfrac{F_{\tau}(T,\tau)}{\int_0^T F_{\tau}(s,\tau)e^{-rs}ds} > \dfrac{r}{1-e^{-rT}}$.

$F_{\tau T} < 0 \;\Rightarrow$

$$\int_0^T F_{\tau}(T,\tau)e^{-rs}ds < \int_0^T F_{\tau}(s,\tau)e^{-rs}ds \;\Leftrightarrow\; \frac{F_{\tau}(T,\tau)(1-e^{-rT})}{r} < \int_0^T F_{\tau}(s,\tau)e^{-rs}ds$$

$\Leftrightarrow \quad \dfrac{F_{\tau}(T,\tau)}{\int_0^T F_{\tau}(s,\tau)e^{-rs}ds} > \dfrac{r}{1-e^{-rT}}$. Hence, $W_{T\tau} < 0$ so that $T_{\tau}^{H} < 0$. Q.E.D.

Appendix 5. Optimal public harvesting

The optimal public rotation age τ* is implicitly defined by

A5.1 $\quad SW_{\tau} = W_{\tau}^{*} + (n-1)E_{\tau} + V_{\tau}^{g} + nE_{\tau}^{g} + \left\{ (n-1)E_{T} + nE_{T}^{g} \right\} T_{\tau}^{H} = 0$,

in which we have accounted for the fact that $W_T{*} = 0$ due to the envelope theorem. The individual terms in A5.1 are

$$W_{\tau}^{*} = (1-e^{-rT^{H}(\tau,...)})^{-1} \int_0^{T^{H}(\tau,...)} F_{\tau}(s,\tau)e^{-rs}ds$$

$$V_{\tau}^{g} = (1-e^{-r\tau})^{-1}\left[pg'(\tau) - rpg(\tau) - rV^{g}\right]$$

$$E^g_\tau = (e^{r\tau} - 1)^{-1}\left[F^g(T^H(\tau,...),\tau) - rE^g\right]$$

$$E_T = (e^{rT^H(\tau,...)} - 1)^{-1}\left[F(T^H(\tau,...),\tau) - rE\right], \text{ and}$$

$$E^g_T = (1 - e^{-r\tau})^{-1}\int_0^\tau F^g_T(T^H(\tau,...),\tau)e^{-rx}dx$$

Substituting these explicit formulas into equation A5.1 yields

A5.2 $$SW_\tau = (e^{-r\tau} - 1)^{-1}\left[pg'(\tau) - rpg(\tau) - rV^g\right] + (e^{r\tau} - 1)^{-1}n\left[F(T^H(\tau,...),\tau) - rE^g\right]$$

$$+ (1 - e^{-rT^H(\tau,...)})^{-1}n\int_0^{T^H(\tau,...)} F_\tau(s,\tau)e^{-rs}ds + (n-1)T^H_\tau(e^{rT} - 1)^{-1}\left[F(T^H(\tau,...),\tau) - rE\right]$$

$$(1 - e^{-r\tau})^{-1}nT_\tau{}^H\int_0^\tau F_T(T,x)e^{-rx}dx = 0,$$

This first-order condition A5.2 is a combination of direct and indirect effects on social welfare from private and public forest stands over infinite cycles of rotations.

Appendix 6. Proof of Lemma 2

Note first that we can write

A6.1 $$F^g(T,\tau) - rE^g = \int_0^\tau F^g(T,x)e^{-rx}dx\left[\frac{F^g(T,\tau)}{\int_0^\tau F^g(T,x)e^{-rx}dx} - \frac{r}{(1-e^{-r\tau})}\right].$$

If $F^g_\tau > (\leq)\ 0$, then $\int_0^\tau F^g(T,\tau)e^{-rx}dx > (\leq)\int_0^\tau F^g(T,x)e^{-rx}dx \Leftrightarrow$

$$\frac{F^g(T,\tau)}{r}(1-e^{-r\tau}) > (\leq)\int_0^\tau F^g(T,\tau)e^{-rx}dx \Leftrightarrow \frac{F^g(T,\tau)}{\int_0^\tau F^g(T,x)e^{-rx}dx} > (\leq)\frac{r}{(1-e^{-r\tau})}.$$

Hence, $F^g(T,\tau) - rE^g > (\leq)\ 0$ as $F^g_\tau(T,\tau) > (\leq)\ 0$. Q.E.D.