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ASYMMETRIC TAXATION UNDER INCREMENTAL AND SEQUENTIAL INVESTMENT

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Abstract

This article discusses the effects of an asymmetric tax scheme on incremental and sequential investment strategies. The tax base is equal to the firm's return, net of an imputation rate. When the firm's return is less than this rate, however, no tax refunds are allowed. This scheme is neutral under both income and capital uncertainty.

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1 Introduction

Most articles on corporate taxation assume fully reversible investment (see e.g. Boadway and Bruce, 1984). However, evidence shows that investment is far from being fully reversible and that firms' strategies are much more complex. Due to reconversion costs, in fact, the firm can sell the capital at a considerably low price (see Abel et al., 1996). This makes investment at least partially irreversible.

Other articles discuss tax policy implications by assuming a single investment decision, undertaken at an exogenously given time (see e.g. Bond and Devereux, 1995). Though irreversibility reduces the flexibility of business strategies, managers are well aware that investment opportunities are not obligations, but rather option-rights[1]. Thus, they can decide when to undertake investment.

In our previous works (Panteghini, 2001 and 2002), we proposed an asymmetric tax system ensuring neutrality under irreversibility and endogenous timing. Neutrality is a direct implication of Bernanke's (1983) Bad News Principle, according to which irreversible decisions are affected only by unfavourable events. Under the tax system proposed, the corporate tax is levied in the good states. Thus, tax asymmetries exploit the asymmetric effects of uncertainty to guarantee neutrality.

Our previous articles rely on the hypothesis that investment is a single project. In reality, not only managers can decide when to invest but also they have some limited ability to expand the firm's capacity. In this paper, therefore, we assume a two-stage investment framework, which account for the option to expand. This is a crucial generalisation because incremental strategies typically cause a twofold path dependence. On the one hand, the firm's current tax position will depend on the past accumulation of capital. On the other hand, the firm's tax position might depend on the project's size as well. Due to this path dependence, therefore, the neutrality result might fail. Using an option pricing approach, instead, this paper shows that neutrality still holds under both income and capital uncertainty.

Moreover, we will show that neutrality holds even when assuming sequential investment. In many cases, in fact, firms collect no revenues until more than one or, even, all the investment stages have been undertaken. Though this is an important case, it is usually disregarded by the current literature.

[1] See Amram and Kulatilaka (1999).

This article is structured as follows. Section 2 discusses the related literature. Section 3 introduces the two-stage model and discusses its main assumptions. Section 4 presents the asymmetric tax system and shows that it is neutral. Finally, section 5 summarises the results and discusses their implications.

2 The related literature

In the traditional literature on corporate taxation there are two basic neutral tax schemes. The first one, called 'imputed income method' (see Samuelson, 1964), defines true economic profits as its tax base. The second one, proposed by Brown (1948), is the cash-flow method. However, both methods are not easy to implement. The former is informationally very demanding. The computation of true economic profits, in fact, requires the knowledge of the firm-specific rate of return (see Sandmo, 1979). The latter, re-proposed by the Meade Committee (1978) in the UK, has at least three practical disadvantages. First, it is quite far from the standard notion of economic profits. Second, it may cause double taxation when other countries do not apply it[2]. Third, the cash-flow tax may fail to contain both tax evasion and tax avoidance, which may be caused by transfer pricing practices and intracompany arrangements (see e.g. Shome and Schutte, 1993).

Given the above difficulties, Boadway and Bruce (1984) proposed 'a simple and general result on the design of a neutral and inflation-proof business tax' [p. 232]. According to their proposal, the business tax base was given by the firm's current earnings, net of the accounting depreciation rate and of the nominal cost of finance. As shown by Fane (1987), neutrality can be achieved by using the risk-free nominal interest rate as the deductible cost of finance. This sharply reduces the information required.

Bond and Devereux (1995, 1999) have proven that a business tax scheme, based on the Boadway-Bruce Principle, is neutral even when income, capital and bankruptcy uncertainty are introduced. They have also proven that the imputation rate ensuring neutrality remains the nominal interest rate on default-free bonds[3].

[2] As argued by McLure and Zodrow (1998), who worked as tax advisors of the Bolivian government, the US International Revenue Service was not convinced to make the cash-flow tax eligible for the foreign tax credit.

[3] Bond and Devereux' (1995, 1999) theoretical papers are closely linked to the tax device

Despite the above generalisations, the existing neutrality results are based on at least two restrictive assumptions. The first regards the symmetric treatment of profits and losses. The second concerns the unrealistic assumptions on investment.

Tax symmetry is seldom implemented in the existing systems. In fact, this device entails that if the tax charge is negative in any period, then a subsidy should be granted to the firm. As the firm's earnings are not accurately observable, however, this rebate might lead to tax evasion or avoidance. Moreover, future positive revenues might be non sufficient to offset previous losses (see Ball and Bowers, 1983).

Bond and Devereux (1995) respond to the above objections by proposing alternative treatments of tax losses. In particular, the firm might be allowed to sell its tax losses to other firms with taxable profits. Alternatively, the losses might be carried forward marked up at the default-free nominal interest rate to set against future taxable profits. Moreover, in the event that the firm winds up with accumulated tax losses, Bond and Devereux (1995) propose the payment of a rebate.

The above devices are not fully satisfactory. As we know, a firm might compensate for tax asymmetries in several ways. First, it might change its accounting policies to shift income over time. Second, it might acquire another firm that has taxable income. Third, it might sell its tax shields by means of a leasing arrangement. However, these transactions may be fairly expensive. Using a sample of US nonfinancial firms, in fact, Auerbach and Poterba (1987) find that if selling tax losses were feasible, the transaction price would not be attractive for 90% of the firms.

Moreover, Isaac (1997) argues that "...there is both survey and anecdotal evidence that both governments and companies commonly place considerably more value on cash flow than is measured by conventional NPV arithmetic" [pp. 308-9]. For this reason, companies are stimulated to make tax-motivated (rather than business-motivated) take-overs. As we know, the tax motivation for merger is twofold. On the one hand, *after* a loss, the firm might decide to merge in order to exploit its losses (see Brealey and Myers, 2001). On the other hand, even *before* the realisation of losses, mergers can be used as an insurance against future redundant losses (see Green and Talmor, 1985). By

proposed by the IFS Capital Taxes Group (1991) for the UK system, called Allowance for Corporate Equity (ACE). According to this proposal, the ACE tax base should be set equal to the firm's current earnings net of: i) an arbitrary tax allowance for capital depreciation (not necessarily the cost of economic depreciation) and ii) the opportunity cost of finance.

eliminating any carry device, therefore, any urge to tax-motivated mergers vanishes.

The second objection to traditional neutrality results concerns the assumptions on investment strategies. In fact, these results are obtained by assuming that investment is either fully reversible (Boadway and Bruce, 1984) or that it cannot be delayed (Fane, 1987; Bond and Devereux, 1995, 1999). However, evidence shows that investment is, at least, partially irreversible and that it does not entail a now-or-never decision.

Over the last decade, tax economists departed from the standard neoclassical model and adopted option pricing techniques to study the effects of taxation on investment. Apart from one exception[4], investment was treated either as a discrete one one-off variable[5] or as a continuous variable[6].

Once again, empirical evidence shows that, in most cases, investment is neither a one-off nor a continuous variable. Rather, firms usually face fixed adjustment costs and undertake a limited number of investment stages. Furthermore, firms have a limited ability to expand[7]. To make our analysis as realistic as possible, therefore, we will use a two-stage model, which allows us to deal with both investment lumpiness and limited expandability.

3 The model

In this section we present a continuous-time model describing a two-stage investment strategy undertaken by a representative firm. The following hypotheses hold:

[4]See Böhm and Funke (2000).

[5]In a pioneering work, MacKie-Mason (1990) showed that, under irreversibility, an asymmetric corporation tax always reduces the value of the investment project. However, it may happen that the decrease in the value of the investment project is more than offset by a decrease in the option value. Thus investment is stimulated by taxation. Alvarez and Kanniainen (1997, 1998) showed that, under irreversibility, the Johansson-Samuelson Theorem (see Sinn, 1987) fails to hold if taxation leaves the project's value unchanged but raises the option value of the project. In this case investment is discouraged. The reader will find further delatils on the relationship between a one-off investment and taxation in Pennings (2000) and Zhang (1997).

[6]See McKenzie (1994) and Faig and Shum (1999), who show that imperfect loss-offset provisions lead to underinvestment.

[7]This may be due to limited land or natural resource reserves, or to the need for permits and licenses (Dixit and Pindyck, 1998).

1. risk is fully diversifiable;

2. the risk-free interest rate r is fixed;

3. the firm is risk-neutral, but its owners may be risk-averse;

4. there exists an irreversible investment, which can be split into two parts, I_1 and I_2. The firm has the opportunity but is not obliged to undertake the total amount I_1+I_2 immediately. Rather, it can decide to undertake I_1 when the current payoff is sufficiently high. By investing I_1, it acquires an option[8] to undertake investment I_2.

5. The firm's payoff follows a geometric Brownian motion. In particular we assume that

$$\Psi\left(K\left(t\right)\right)\left[d\Pi(t)\right]=\Psi\left(K\left(t\right)\right)\left[\alpha\Pi(t)dt+\sigma\Pi(t)\right]dz.$$

   where α and σ are the growth rate and variance parameter, respectively and z is a Wiener process, and where $\Psi\left(K\left(t\right)\right)$ is a function of capital $K\left(t\right)$, i.e.

$$\Psi\left(K\left(t\right)\right)=\begin{cases}1 & \text{if } K\left(t\right)=I_1,\\ \Psi>1 & \text{if } K\left(t\right)=I_1+I_2.\end{cases}$$

   Namely, after investing I_1, the firm receives a payoff $\Pi(t)$. When the firm undertakes I_2, total payoff raises to $\Psi\Pi(t)$.

6. The lifetime of investment follows a Poisson process. At any time t there is a probability λdt that the existing project dies during the short internal dt[9].

Assumptions 1 and 2 are standard ones and do not deserve any comment. Assumption 3 was previously introduced by McDonald and Siegel (1985, 1986). Using the option pricing approach, they assume that the firm's option to delay irreversible investment is owned by well-diversified investors. Therefore, the firm's problem under risk aversion is equivalent to the one

[8]In technical terms, this opportunity to expand is an American call option. For further details see Trigeorgis (1996).

[9]The cumulative probability distribution function at any instant T is $(1-e^{-\lambda T})$.

under risk neutrality, if an appropriately risk-adjusted discount rate is used for valuation[10].

Assumptions 4 and 5 are the most important novelty of this work as they introduce path dependence. With assumption 4 incremental investment strategies are introduced[11]. Assumption 5 introduces the scale effect. As argued by Dixit (1995), the investment decision depends on whether the returns to scale are increasing or decreasing. In the former case, investment is lumpy, in order to cross the region of increasing returns. Thus, the thresholds that trigger these jumps are such that 'the expected total return exceeds the Marshallian normal return by the same factor that captures the option value of waiting' [p. 328]. In the latter case (namely, when the region with increasing returns has been crossed), investment undertakes infinitesimal increments (see e.g. Bertola, 1998, and Caballero, 1991).

Assumption 6 introduces capital uncertainty. As argued by Bulow and Summers (1984) capital risk is the most important source of risk involved in holding an asset. To deal with capital risk these authors assume a deterministic depreciation rate with an uncertain future price of capital. Here we propose a slightly different way of treating capital uncertainty. Namely, the price of capital is normalised to 1 and the investment's lifetime is assumed to be random. More importantly, we assume that when the investment project expires, the firm gets an option to restart. In this case, immediate restarting may not be profitable. Rather, the firm may find it profitable to wait until current profits will rise. With such an option to restart, therefore, the firm regains a limited degree of reversibility.

Given the above assumptions, the firm owns the option to delay investment I_1, the option to expand production (by investing I_2), and, the option to restart production after the investment's death. The firm's investment strategy is thus analogous to the exercise of compound options. The firm's problem will then be one of finding the optimal trigger points above which

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[10] Majd and Myers (1987) explain this certainty equivalent valuation by arguing that options are not valued in absolute terms, but rather relative to the underlying asset. If, therefore, the value of the asset is market down for risk, then the call option must be marked down as well. For further details see Panteghini (2002).

[11] It is worth noting that the firm's payoff $\Psi(K(t))\,\Pi(t)$ might be thought of as the reduced form of a more general function which incorporates both a market structure (i.e. imperfect competition) and variable inputs. This generalisation, however, would not change the quality of the results. For further details, see Dixit and Pindyck (1994, Ch. 10).

investing I_1 and I_2, respectively, and restarting production is profitable.

Given the above assumptions, we can thus say that the incremental strategy is preferred to the simultaneous strategy if inequality $\Psi < \frac{I_1+I_2}{I_1}$ holds. This condition can be easily interpreted. As we know, if the firm invests I_1 it earns Π. If, instead, it invests $(I_1 + I_2)$, its payoff grows to $\Psi\Pi$. Compare now the returns on the undertaken investment. In the first case we have $\frac{\Pi}{I_1}$ and in the second case we obtain $\frac{\Psi\Pi}{I_1+I_2}$. If we have $\Psi\frac{\Pi}{I_1+I_2} < \frac{\Pi}{I_1}$, then $\Psi < \frac{I_1+I_2}{I_1}$, namely decreasing returns to scale are obtained and an incremental strategy may be optimal. If the converse is true, the returns to scale are increasing and the firm's strategy is one-off.

4 The asymmetric tax system

In this section we will discuss the effects of an asymmetric tax system on investment strategies. This tax design, presented in Panteghini (2001 and 2002), is based on an imputation method[12]. As in Garnaut and Ross (1975), in fact, the tax base is given by the firm's return, net of an imputation rate, r_E, times the amount of investment. Contrary to Garnaut and Ross' proposal, when the firm's return is less than the imputation rate, no tax refunds are allowed[13]. As argued by Green and Talmor (1985), in fact, loss carry devices are a tax claim for the government. Namely, the government owns a portfolio of call options on the firm's earnings with a variable exercise price. Current tax revenues, in fact, depend on either the past or the future firm's tax position. This makes the effective tax rate path dependent and causes a distortion. The elimination of any carry device thus eliminates a source of path dependence.

By eliminating any carry device only incremental and scale effects cause path dependence. As explained by Auerbach (1986), in fact, a multistage project introduces additional state variables in the form of previous realiza-

[12]It is worth noting that, in the Nineties, tax systems based on the imputation method were introduced in the Nordic countries (see Sørensen, 1998), in Croatia (see Rose and Wiswesser, 1998), and in Italy (see Bordignon et al., 1999 and 2001). Recently, an imputation tax design has also been proposed for Germany (see Fehr and Wiegard, 2001).

[13]This asymmetric system is also related to the well-known R-based cash-flow tax analysed by Brown (1948) and Meade (1978). As we know, the main difference between the R-based and the S-based tax is that, under the latter, interest payments are deductible. As shown by Bond and Devereux (1999), however, the S-based tax requires a greater amount of information and may lead to tax avoidance more easily.

tions of the stochastic process. Moreover, Majd and Myers (1985) argue that for small projects, the firm's tax position may be exogenous. Large projects, instead, may affect the overall status of the firm. Thus, interactions between the firm's tax status and its capacity might be distortive. As will be shown, instead, the asymmetric system under study remains neutral.

Hereafter, for simplicity, we will omit the time variable. Given the amount of capital accumulated so far, K, tax payments are

$$\tau \max\left[\Psi(K)\Pi - r_E K, 0\right]. \tag{1}$$

Given (1), net instantaneous profits (or losses) are therefore equal to

$$\Pi^N = \Psi(K)\Pi - \tau \max\left[\Psi(K)\Pi - r_E K, 0\right].$$

The firm's solution to the optimal stopping time is one of choosing the after-tax trigger points, ${\Pi_1^*}^T$ and ${\Pi_2^*}^T$, at which it is optimal to invest. Given the above tax system we can show that the following Proposition holds:

Proposition 1 When incremental investment is a feasible strategy, the asymmetric tax device is neutral, i.e. the post-tax trigger points are equal to the pre-tax ones (${\Pi_i^*}^T = \Pi_i^*$, $i = 1, 2$) provided that the imputation rate is high enough, i.e.

$$r_E \geqslant r_E^* \equiv \max\left(\frac{\Pi_1^*}{I_1}, \frac{\Psi\Pi_2^*}{I_1 + I_2}\right). \tag{2}$$

Proof- *See the Appendix.*

In the Appendix, the reader will find the formal proof of the above Proposition. Here we will give the intuition behind the result. As we know, corporate taxation is equivalent to equity participation (see Domar and Musgrave, 1944). Under asymmetric taxation, however, the government's tax claim is equivalent to a portfolio of European call options, one on each year's cash flow[14], and $r_E K$ is the exercise price. If, therefore, the firm's payoff reaches $r_E K$, the government exercises the call option and shares profits.

Let us next define $V_i(\Pi)$ and $V_i^T(\Pi)$ as the pre- and post-tax present discounted value of the firm's stage $i = 1, 2$, respectively. Moreover, define

[14] For further details on this interpretation see Majd and Myers (1987) and van Wijnbergen and Estache (1999).

$C_i(\Pi)$ and $C_i^T(\Pi)$ as the pre- and post-tax value of the firm's compound options, at stage i =1,2. In the Appendix we will show that the neutrality result derives from the following conditions

$$V_i^T(\Pi) - I_i - C_i^T(\Pi) = [V_i(\Pi) - I_i - C_i(\Pi)] = 0, \tag{3}$$

$$\frac{\partial\left[V_i^T(\Pi) - I_i - C_i^T(\Pi)\right]}{\partial\Pi} = \frac{\partial\left[V_i(\Pi) - I_i - C_i(\Pi)\right]}{\partial\Pi} = 0. \tag{4}$$

The former condition arises from the Value Matching Condition, which requires the equality between the net present value of the project[15], $\left[V_i^T(\Pi) - I_i\right]$, and the value of the compound option $C_i^T(\Pi)$. The latter is derived from the Smooth Pasting Condition and requires the equality between the slopes of $\left[V_i^T(\Pi) - I_i\right]$, and the value of the compound option $C_i^T(\Pi)$. Given the above equalities, the pre-tax and post-tax case are equivalent.

Equations (3) and (4) yield a sufficient neutrality condition that accounts for the firm's ability to modify its strategies by exercising options[16].

It is worth noting that an increase in the tax rate reduces not only the present value $V_i^T(\Pi)$ but also the option value $C_i^T(\Pi)$. However, Proposition 1 shows that these decreases neutralise each other. Namely, the equalities $V_i^T(\Pi) - C_i^T(\Pi) - I_i = [V_i(\Pi) - C_i(\Pi) - I_i]$ and $\frac{\partial\left[V_i^T(\Pi) - C_i^T(\Pi)\right]}{\partial\Pi} = \frac{\partial[V_i(\Pi) - C_i(\Pi)]}{\partial\Pi}$ hold irrespective of the tax rate applied. This neutrality result is an application of Bernanke's (1983) Bad News Principle (BNP): under investment irreversibility, bad events affect the firm's propensity to invest[17]. If the corporate tax is levied only in the good state, therefore, investment decisions are not affected.

It is well-known that the elimination of a tax benefit (e.g. the loss-offset arrangement) must be offset by a new benefit (e.g. a higher imputation rate) in order for neutrality to hold. However, many authors (see e.g. Ball and Bowers (1983) and Auerbach (1986)) argue that it is hard to compute the

[15] This is the relevant measure of profitability in the absence of any option to change investment strategies.

[16] For details on the sufficient conditions under tax symmetry, see Niemann (1999).

[17] As stated by Bernanke (1983), "The investor who declines to invest in project i today (but retains the right to do so tomorrow) gives up short-run returns. In exchange for this sacrifice, he enters period *t+1* with an "option" (...). In deciding whether to "buy" this option (...), the investor therefore considers only "bad news" states in *t+1 (...)*" [p. 92-3].

neutral rate r_E . Since each firm has its own risk profile, the neutral imputation rate should be risk-specific. This result holds under the unrealistic assumption of full reversibility of investment.

In our model, instead, an *entire* region of neutral imputation rates can be derived. Thus, it is sufficient to find the minimum imputation rate r_E^* [18]. If, therefore, $r_E \in [r_E^*, \infty)$, neutrality holds. To explain this result, let us assume an increase in r_E. The effects of this increase are twofold. On the one hand, the increase in r_E increases the government's exercise price $r_E K$, thereby reducing the value of its options and increasing the post-tax firm's value project. On the other hand, the increase in r_E raises both the option to wait (related to investment I_1) and the option to expand (related to investment I_2). Proposition 1 shows that, if $r_E \in [r_E^*, \infty)$, these two effects offset each other thereby ensuring neutrality.

Let us next focus on the scale effect. As shown in the proof of Proposition 1, the relationship $\Pi_1^* - \Pi_2^* \propto \left(\frac{I_1+I_2}{I_1} - \Psi\right)$ is obtained. If, therefore, $\Pi_1^* < \Pi_2^*$ $\left(\text{i.e. } \Psi < \frac{I_1+I_2}{I_1}\right)$, then the returns to scale are decreasing and the incremental strategy may be optimal. In this case, inequality $\frac{\Pi_1^*}{I_1} < \frac{\Psi\Pi_2^*}{I_1+I_2}$ holds. Thus, the minimum imputation rate is $r_E^* = \frac{\Psi\Pi_2^*}{I_1+I_2}$. If, instead, $\Pi_1^* < \Pi_2^*$ the converse is true and minimum imputation rate is $r_E^* = \frac{\Pi_1^*}{I_1}$. Therefore, the tax rule (2) ensures neutrality irrespective of the returns to scale.

Sequential investment is a special but important case. In many circumstances, firms earn no revenues until more than one or, even, all the investment stages have been undertaken[19]. Oil production is a good example of sequential investment. In the first stage exploration takes place. When oil has been found, a pipeline can be built and, subsequently, oil can be sold. Exploitation of natural resources and R&D are other interesting examples. Despite their importance, they are disregarded by the current literature.

To discuss the effects of taxation on sequential investment, let us modify assumption 5 as follows:

5'. After investing I_1 the firm earns no revenues, but it acquires an option to undertake the second stage I_2. When it invests I_2, it starts to earn $\Psi\Pi$.

[18] Note that $r_E^* > r$ always holds. Under the non-refundability system, therefore, the differential $r_E^* - r$ is sufficient to neutralise the effects of the asymmetric treatment of profits and losses.

[19] Dixit and Pindyck (1994, Ch. 10) report other examples of sequential investment, such as pharmaceutical and aircraft companies.

Thus we have

$$\Psi(K) = \begin{cases} 0 & \text{if } K(t) = I_1, \\ \Psi > 1 & \text{if } K(t) = I_1 + I_2. \end{cases}$$

The following Proposition can be proven:

Proposition 2 *Under sequential investment neutrality holds if*

$$r_E \geqslant r_E^* = \frac{\Psi \Pi_1^*}{I_1 + I_2}. \tag{5}$$

Proof - See the Appendix.

Under sequential investment, inequality $\Psi\Pi_1^* > \Psi\Pi_2^*$ holds. If, therefore, the firm's payoff reaches $\Psi\Pi_1^*$ the project can be immediately completed. Thus, this case would collapse to the one-off strategy. However, Proposition 2 is not banal. Dixit and Pindyck (1994) argue that the study of sequential strategies case is important for, at least, two reasons. First, undertaking investment takes time. Thus firms often complete the early stages and then wait before undertaking the following stages. Second, different investment stages may require different skills or they may be located in different places. In all these cases, therefore, a firm might find it profitable to sell a partially completed project.

Given Propositions 1 and 2 one can finally derive a comprehensive neutrality rule. Let us compare the sufficient neutrality conditions obtained under both incremental and sequential, i.e. (2) and (5). Since an entire region of neutral imputation rates can be obtained, it is sufficient to set

$$r_E \geq \max\left(\max\left(\frac{\Pi_1^*}{I_1}, \frac{\Psi\Pi_2^*}{I_1 + I_2}\right), \frac{\Psi\Pi_1^*}{I_1 + I_2}\right) \tag{6}$$

to ensure neutrality. The following Corollary thus holds.

Corollary 3 *If the imputation rate is high enough (i.e. inequality (6) holds), the asymmetric tax device is neutral irrespective of whether investment is an one-off, an incremental or a sequential decision.*

5 Conclusion

In this article, we have discussed the effects of an asymmetric tax system on a two-stage investment project. Although incremental investment strategies and scale effects lead to path dependence, it is shown that taxation is neutral. Namely, neither the minimum imputation rate nor the trigger point are affected by the tax rate. The result has an important policy implication. In order to obtain neutrality, in fact, the corporate tax should be levied only when the investment project is completed or, equivalently, the firm is mature.

Not only is the system proposed neutral but it is also easy to manage. In fact, the computation of the firm's tax liabilities is based on accounting data. Moreover, the absence of any carry device eliminates a source of path dependence, thereby reducing tax compliance costs.

The generalisation to N stages is left to future research.

6 Appendix

In this Appendix the proofs of Propositions 1 and 2 are discussed.

6.1 Proof of Proposition 1

The proof of Proposition 1 consists of three steps. First, we will compute the functional form of the option to delay. Then we will turn to the value function. Finally, we will compute the trigger points ${\Pi_1^*}^T$ and ${\Pi_2^*}^T$. As will be shown they are unaffected by taxation, namely ${\Pi_i^*}^T = \Pi_i^*$ $i = 1, 2$.

6.1.1 The option to delay

Let us start with the option function. Using dynamic programming we have

$$O(\Pi) = e^{-rdt} \left\{ \xi \left[O(\Pi + d\Pi) \right] \right\}. \tag{7}$$

Expand the RHS of (7). Using Itô's Lemma, eliminating all the terms multiplied by $(dt)^2$ and dividing by dt, one obtains

$$rO(\Pi) = (r - \alpha)\Pi O_\Pi + \frac{\sigma^2}{2}\Pi^2 O_{\Pi\Pi}. \tag{8}$$

The solution of $O(\Pi)$ has the standard general form

$$O(\Pi) = \sum_{i=1}^{2} A_i \Pi^{\beta_i}, \tag{9}$$

where

$$\beta_1 = \tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2} + \sqrt{\left(\tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}} > 1,$$
$$\beta_2 = \tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2} - \sqrt{\left(\tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}} < 0.$$

Since the boundary condition $O(0) = 0$ holds[20], the solution (9) reduces to

$$O(\Pi) = A_1 \Pi^{\beta_1}. \tag{10}$$

6.1.2 The value function

Let us now turn to the value function. We first study the value function when investing I_1. Then we will turn to the increment I_2. Using dynamic programming we can write the value function as

$$\begin{aligned} P(\Pi) &= \Pi dt + e^{-rdt}(1 - \lambda dt)\xi \left[P(\Pi + d\Pi)\right] + \\ &+\lambda dt \begin{cases} \xi \left[O(\Pi + d\Pi)\right] & if \ \Pi \in [0, \Pi_1^{*^T}), \\ \xi \left[P(\Pi + d\Pi) - I_1\right] & if \ \Pi \in (\Pi_1^{*^T}, \Pi_2^{*^T}), \end{cases} \end{aligned} \tag{11}$$

where the trigger points $\Pi_1^{*^T}$ and $\Pi_2^{*^T}$ are to be determined. As can be seen, $P(\Pi)$ consists of two branches. Depending on whether Π is in the $[0, \Pi_1^{*^T})$ or in the $(\Pi_1^{*^T}, \Pi_2^{*^T})$ region the form of the value function changes. If, in fact, the project dies when $\Pi \in [0, \Pi_1^{*^T})$, then restarting production immediately is not profitable. If, instead, the project's death takes place when $\Pi \in (\Pi_1^{*^T}, \Pi_2^{*^T})$, the value function is given by the second branch.

Let us start with the first branch of $P(\Pi)$. Using Itô's Lemma and simplifying, one obtains

$$(r + \lambda)P(\Pi) = \Pi + (r - \delta)\Pi P_\Pi(\Pi) + \frac{\sigma^2}{2}\Pi^2 P_{\Pi\Pi}(\Pi) + \lambda O(\Pi). \tag{12}$$

[20] This boundary condition is an implication of the stochastic process followed by Π. It implies that if Π goes to zero, it will stay at zero. For further details see Dixit and Pindyck (1994, Ch. 5).

Next subtract equation (8) from (12) and define $X(\Pi) \equiv P(\Pi) - O(\Pi)$, so as to obtain

$$(r+\lambda)X(\Pi) = \Pi + (r-\delta)\Pi X_{\Pi}(\Pi) + \frac{\sigma^2}{2}\Pi^2 X_{\Pi\Pi}(\Pi).$$

The function $X(\Pi)$ has the standard general form

$$X(\Pi) = \frac{\Pi}{\delta+\lambda} + \sum_{i=1}^{2} X_i \Pi^{\beta_i(\lambda)}, \tag{13}$$

where

$$\beta_1(\lambda) = \left(\tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2}\right) + \sqrt{\left(\tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2}\right)^2 + 2\tfrac{r+\lambda}{\sigma^2}} > 1,$$
$$\beta_2(\lambda) = \left(\tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2}\right) - \sqrt{\left(\tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2}\right)^2 + 2\tfrac{r+\lambda}{\sigma^2}} < 0.$$

Setting the boundary condition $X(0) = 0$, function (13) reduces to

$$X(\Pi) = \frac{\Pi}{\delta+\lambda} + X_1 \Pi^{\beta_1(\lambda)} \tag{14}$$

Next, using (10) and (14) we obtain the solution of the first branch

$$P(\Pi) = \frac{\Pi}{\delta+\lambda} + A_1 \Pi^{\beta_1} + X_1 \Pi^{\beta_1(\lambda)}. \tag{15}$$

Let us now turn to the second branch of $P(\Pi)$. Using Itô's Lemma and simplifying yields

$$rP(\Pi) = \Pi - \lambda I_1 + (r-\delta)\Pi P_{\Pi}(\Pi) + \frac{\sigma^2}{2}\Pi^2 P_{\Pi\Pi}(\Pi). \tag{16}$$

Given equation (16) it is straightforward to obtain the second branch

$$P(\Pi) = \frac{\Pi}{\delta} - \lambda\frac{I_1}{r} + \sum_{i=1}^{2} P_i \Pi^{\beta_i}. \tag{17}$$

Let us next study the value function after investment I_2. It consists of three branches

$$Q(\Pi) = \{\Psi(K)\Pi - \tau \max[\Psi(K)\Pi - r_E K, 0]\} dt + e^{-rdt}(1-\lambda dt)\xi[Q(\Pi + d\Pi)] + \tag{18}$$

$$+e^{-rdt}\lambda dt \begin{cases} \xi[O(\Pi + d\Pi)] & if\ \Pi \in (0, \Pi_1^{*^T}), \\ \xi[P(\Pi + d\Pi) - I_1] & if\ \ \Pi \in (\Pi_1^{*^T}, \Pi_2^{*^T}), \\ \xi[Q(\Pi + d\Pi) - (I_1 + I_2)] & if\ \Pi \in (\Pi_2^{*^T}, \infty). \end{cases}$$

If the project expires when $\Pi \in (0, \Pi_1^{*^T})$, immediate investment is not optimal. If expiration takes place when $\Pi \in (\Pi_1^{*^T}, \Pi_2^{*^T})$, investment I_1 is immediately profitable. If, finally, $\Pi \in (\Pi_2^{*^T}, \infty)$, it is optimal to invest both I_1 and I_2 immediately.

Before solving the above function let us apply the tax-holiday rule ensuring an exemption for at least $\Pi \geq \Pi_2^{*^T}$. This implies that the first and second branches do not account for current taxation, namely the current net payoff is simply $\Psi\Pi$. Start with the first branch of $Q(\Pi)$. Using Itô's Lemma and simplifying yields

$$(r+\lambda)Q(\Pi) = \Pi + (r-\delta)\Pi Q_{\Pi}(\Pi) + \frac{\sigma^2}{2}\Pi^2 Q_{\Pi\Pi}(\Pi) + \lambda O(\Pi). \tag{19}$$

Next, define $Y(\Pi) \equiv Q(\Pi) - O(\Pi)$ and subtract equation (8) from (19) thereby obtaining

$$(r+\lambda)Y(\Pi) = \Pi + (r-\delta)\Pi Y_{\Pi}(\Pi) + \frac{\sigma^2}{2}\Pi^2 Y_{\Pi\Pi}(\Pi).$$

The general solution of $Y(\Pi)$ is

$$Y(\Pi) = \frac{\Psi\Pi}{\delta+\lambda} + \sum_{i=1}^{2} Y_i \Pi^{\beta_i(\lambda)}. \tag{20}$$

Setting the boundary condition $Y(0) = 0$, yields

$$Y(\Pi) = \frac{\Psi\Pi}{\delta+\lambda} + Y_1 \Pi^{\beta_1(\lambda)}. \tag{21}$$

Next using (10) and (21) yields the first branch

$$Q(\Pi) = \frac{\Psi\Pi}{\delta+\lambda} + A_1\Pi^{\beta_1} + Y_1\Pi^{\beta_1(\lambda)}. \tag{22}$$

Let us now turn to the second branch. To compute it we follow the same procedure. Namely, we start with the Bellman equation in the $(\Pi_1^{*^T}, \Pi_2^{*^T})$ region, i.e.

$$(r+\lambda)Q(\Pi) = \Psi\Pi + (r-\delta)\Pi Q_\Pi(\Pi) + \frac{\sigma^2}{2}\Pi^2 Q_{\Pi\Pi}(\Pi) + \lambda P(\Pi). \tag{23}$$

Then, we define $Z(\Pi) \equiv Q(\Pi) - P(\Pi)$ and subtract (16) from (23), thereby obtaining

$$(r+\lambda)Z(\Pi) = (\Psi-1)\,\Pi + (r-\delta)\Pi Z_\Pi(\Pi) + \frac{\sigma^2}{2}\Pi^2 Z_{\Pi\Pi}(\Pi). \tag{24}$$

Solving (24) yields

$$Z(\Pi) = \frac{(\Psi-1)\Pi}{\delta+\lambda} + \sum_{i=1}^{2} Z_i\Pi^{\beta_i(\lambda)} \tag{25}$$

Using (17) and (25) one obtains

$$Q(\Pi) = \left[\frac{\Pi}{\delta} + \frac{(\Psi-1)\Pi}{\delta+\lambda}\right] - \lambda\frac{I_1}{r} + \sum_{i=1}^{2} P_i\Pi^{\beta_i} + \sum_{i=1}^{2} Z_i\Pi^{\beta_i(\lambda)}. \tag{26}$$

Let us finally turn to the third branch. If we apply the tax-holiday rule we have to split the branch into two parts, which measure the value function in the $\left(\Pi_2^{*^T}, \frac{r_E(I_1+I_2)}{\Psi}\right)$ and $\left(\frac{r_E(I_1+I_2)}{\Psi}, \infty\right)$region, respectively. Using equation (18) and applying Itô's Lemma yields

$$\begin{aligned} rQ(\Pi) = \Psi\Pi - \tau\max\left[\Psi\Pi - r_E\left(I_1+I_2\right), 0\right] - \lambda\left(I_1+I_2\right) + \\ +(r-\delta)\Pi Q_\Pi(\Pi) + \tfrac{\sigma^2}{2}\Pi^2 Q_{\Pi\Pi}(\Pi). \end{aligned} \tag{27}$$

Solving (**??**) yields

$$Q(\Pi) = \begin{cases} \frac{\Psi}{\delta}\Pi - \lambda\frac{(I_1+I_2)}{r} + \sum_{i=1}^{2} Q_i\Pi^{\beta_i} & if\ \Pi \in \left(\Pi_2^{*^T}, \frac{r_E(I_1+I_2)}{\Psi}\right) \\ \\ \frac{(1-\tau)\Psi}{\delta}\Pi + \left(\tau\frac{r_E}{r} - \frac{\lambda}{r}\right)(I_1+I_2) + \sum_{i=1}^{2} R_i\Pi^{\beta_i} & if\ \Pi \in \left(\frac{r_E(I_1+I_2)}{\Psi}, \infty\right) \end{cases} \tag{28}$$

The term $R_1\Pi^{\beta_1}$ measures a speculative bubble as Π goes to ∞. If we assume that no bubbles exist, namely $R_1 = 0$, the second part of (28) is simplified[21].

6.1.3 The solution

Let us next find the trigger points Π_1^{*T} and Π_2^{*T}. Given the above functions, we have ten unknown parameters (A_1, X_1, P_1, P_2, Z_1, Z_2, Q_1, Q_2, Y_1 and R_2), plus Π_1^{*T} and Π_2^{*T}. To compute their solutions we thus need twelve equations. Eight equations can be found by stitching together the branches of the value functions. The remaining four equations will be found using the VMC and SPC.

Let us start with function $P(\Pi)$. The first and second branch of this function meet tangentially at point $\Pi = \Pi_1^{*T}$, namely both the value and the derivative of the two branches are equal (see Dixit and Pindyck, 1994). Substituting (15) and (17) into the (VMC) and (SPC) we have

$$\frac{\Pi_1^{*T}}{\delta+\lambda} + A_1 \Pi_1^{*T\,\beta_1} + X_1 \Pi_1^{*T\,\beta_1(\lambda)} = \frac{\Pi_1^{*T}}{\delta} - \lambda\frac{I_1}{r} + \sum_{i=1}^{2} P_i \Pi_1^{*T\,\beta_i}, \qquad (29)$$

$$\frac{1}{\delta+\lambda} + \beta_1 A_1 \Pi_1^{*T\,\beta_1 - 1} + \beta_1(\lambda) X_1 \Pi_1^{*T\,\beta_1(\lambda)-1} = \frac{1}{\delta} + \sum_{i=1}^{2} \beta_i P_i \Pi_1^{*T\,\beta_i - 1}. \qquad (30)$$

Let us turn to function $Q(\Pi)$. Its meeting points are Π_1^{*T} and Π_2^{*T}, plus, if the tax-holiday rule holds, $\frac{r_E(I_1+I_2)}{\Psi}$. Start with the first two branches (22) and (26). They meet tangentially at point Π_1^{*T}, namely

$$\begin{gathered}\frac{\Psi\Pi_1^{*T}}{\delta+\lambda} + A_1\left(\Pi_1^{*T}\right)^{\beta_1} + Y_1\left(\Pi_1^{*T}\right)^{\beta_1(\lambda)} = \\ = \left(\frac{\Psi-1}{\delta+\lambda}+\frac{1}{\delta}\right)\Pi_1^{*T} - \lambda\frac{I_1}{r} + \sum_{i=1}^{2} P_i\left(\Pi_1^{*T}\right)^{\beta_i} + \sum_{i=1}^{2} Z_i\left(\Pi_1^{*T}\right)^{\beta_i(\lambda)},\end{gathered} \qquad (31)$$

$$\begin{gathered}\frac{1}{\delta+\lambda} + \beta_1 A_1\left(\Pi_1^{*T}\right)^{\beta_1-1} + \beta_1(\lambda) Y_1\left(\Pi_1^{*T}\right)^{\beta_1(\lambda)-1} = \\ = \left(\frac{\Psi-1}{\delta+\lambda}+\frac{1}{\delta}\right) + \sum_{i=1}^{2} \beta_i P_i\left(\Pi_1^{*T}\right)^{\beta_i-1} + \sum_{i=1}^{2} \beta_i(\lambda) Z_i\left(\Pi_1^{*T}\right)^{\beta_i(\lambda)-1}.\end{gathered} \qquad (32)$$

[21] For further details on this absorbing barrier see Dixit and Pindyck (1994, Ch. 6).

Similarly, the second branch (26) and the first part of the third branch (28) meet tangentially at point $\Pi_2^{*^T}$

$$\begin{aligned}\left(\tfrac{\Psi-1}{\delta+\lambda}+\tfrac{1}{\delta}\right)\Pi_2^{*^T}-\lambda\tfrac{I_1}{r}+\textstyle\sum_{i=1}^2 P_i\left(\Pi_2^{*^T}\right)^{\beta_i}+\textstyle\sum_{i=1}^2 Z_i\left(\Pi_2^{*^T}\right)^{\beta_i(\lambda)}= \\ =\tfrac{\Psi}{\delta}\Pi_2^{*}-\lambda\tfrac{I_1+I_2}{r}+\textstyle\sum_{i=1}^2 Q_i\left(\Pi_2^{*^T}\right)^{\beta_i},\end{aligned} \tag{33}$$

$$\left(\frac{\Psi-1}{\delta+\lambda}+\frac{1}{\delta}\right)+\sum_{i=1}^{2}\beta_i P_i\left(\Pi_2^{*^T}\right)^{\beta_i-1}+\sum_{i=1}^{2}\beta_i(\lambda)Z_i\left(\Pi_2^{*^T}\right)^{\beta_i(\lambda)-1}=\frac{\Psi}{\delta}+\sum_{i=1}^{2}Q_i\left(\Pi_2^{*^T}\right)^{\beta_i}. \tag{34}$$

Finally, let the two parts of the branch (28) meet tangentially at point $\frac{r_E(I_1+I_2)}{\Psi}$

$$\begin{aligned}\tfrac{\Psi}{\delta}\left(\tfrac{r_E(I_1+I_2)}{\Psi}\right)-\lambda\tfrac{I}{r}+\textstyle\sum_{i=1}^2 Q_i\left(\tfrac{r_E(I_1+I_2)}{\Psi}\right)^{\beta_i}= \\ =\tfrac{(1-\tau)\Psi}{\delta}\left(\tfrac{r_E(I_1+I_2)}{\Psi}\right)+\left(\tau\tfrac{r_E}{r}-\tfrac{\lambda}{r}\right)(I_1+I_2)+R_2\left(\tfrac{r_E(I_1+I_2)}{\Psi}\right)^{\beta_2},\end{aligned} \tag{35}$$

$$\frac{\Psi}{\delta}+\sum_{i=1}^{2}\beta_i Q_i\left(\frac{r_E\left(I_1+I_2\right)}{\Psi}\right)^{\beta_i-1}=\frac{(1-\tau)\Psi}{\delta}+R_2\left(\frac{r_E\left(I_1+I_2\right)}{\Psi}\right)^{\beta_2-1}. \tag{36}$$

We have thus obtained eight equations. Now let us recall the option function (10) and concentrate on the investment choices. Investment I_1 is undertaken when the (VMC) and (SPC) conditions hold, i.e.

$$P(\Pi)-O(\Pi)=I_1, \tag{37}$$

$$P_\Pi(\Pi)-O_\Pi(\Pi)=0. \tag{38}$$

Substituting (10) and (15) into conditions (37) and (38), respectively, yields[22]

$$\frac{\Pi_1^{*^T}}{\delta+\lambda}+X_1\Pi_1^{*^T\,\beta_1(\lambda)}=I_1, \tag{39}$$

[22]Note that the two branches of $P(\Pi)$ meet tangentially at point $\Pi_1^{*^T}$. Thus we could use either of them. However, we will use its first branch (15) since computations are easier.

$$\frac{1}{\delta+\lambda}+\beta_1(\lambda)X_1{\Pi_1^*}^{T\,\beta_1(\lambda)-1}=0. \tag{40}$$

It is worth noting that equations (39) and (40) correspond to conditions (3) and (4), when the firm is deciding to invest I_1. In fact, $X_1\Pi^{\beta_1(\lambda)}$ measures the post-tax compound option which embodies the option to delay, the option to expand and the option to restart. Solving the sub-system (39)-(40) yields

$${\Pi_1^*}^T=\frac{\beta_1(\lambda_1)}{\beta_1(\lambda_1)-1}(\delta+\lambda)I_1, \tag{41}$$

$$X_1=-\frac{1}{\beta_1(\lambda)}\frac{1}{\delta+\lambda}{\Pi_1^*}^{1-\beta_1(\lambda)}.$$

Since the point ${\Pi_1^*}^T$ is not affected by taxation, we have equality ${\Pi_1^*}^T=\Pi_1^*$. According to conditions (3) and (4), therefore, both the post- and pre-tax value of the project, net of investment I_1 and of the compound options, are null at point $\Pi={\Pi_1^*}^T=\Pi_1^*$.

Let us now concentrate on investment I_2. The second stage is undertaken when the following conditions hold

$$Q(\Pi)-P(\Pi)=I_2, \tag{42}$$

$$Q_\Pi(\Pi)-P_\Pi(\Pi)=0. \tag{43}$$

The above conditions are used for obtaining the last two equations, necessary for finding the solutions. Substitute the second branch of both $P(\Pi)$ and $Q(\Pi)$, i.e. (17) and (26), into the above conditions[23], so as to obtain

$$\left(\frac{\Psi-1}{\delta+\lambda}\right){\Pi_2^*}^T+\sum_{i=1}^{2}Z_i{\Pi_2^*}^{T\,\beta_i(\lambda)}=I_2, \tag{44}$$

$$\frac{\Psi-1}{\delta+\lambda}+\sum_{i=1}^{2}\beta_i(\lambda)Z_i{\Pi_2^*}^{T\,\beta_i(\lambda)-1}=0. \tag{45}$$

[23]As in the previous case we could use either the second or the third branch of $Q(\Pi)$, since they meet tangentially at point ${\Pi_2^*}^T$. However, the use of the second branch is easier.

It is worth noting that equations (44) and (45) represent conditions (3) and (4) when the firm decides to invest I_2. In other words, $\sum_{i=1}^{2} Z_i \Pi^{\beta_i(\lambda)}$ measures the post-tax compound option which embodies the option to expand and the option to restart.

Given the solutions for $\Pi_1^{*^T}$ and X_1, a ten-equation sub-system remains to be solved. Subtract (31) from (29) and (32) from (30), respectively. It is straightforward to obtain

$$(X_1 - Y_1 + Z_1)\Pi_1^{*^T\beta_1(\lambda)} + Z_2\Pi_1^{*^T\beta_2(\lambda)} = 0, \tag{46}$$

$$\beta_1(\lambda)(X_1 - Y_1 + Z_1)\Pi_1^{*^T\beta_1(\lambda)-1} + \beta_2(\lambda)Z_2\Pi_1^{*^T\beta_2(\lambda)-1} = 0. \tag{47}$$

which yields $X_1 = Y_1 - Z_1$ and $Z_2 = 0$. Given $Z_2 = 0$, solving the sub-set (46)-(47) yields

$$\Pi_2^{*^T} = \frac{\beta_1(\lambda_1)}{\beta_1(\lambda_1) - 1}\frac{\delta + \lambda}{\Psi - 1} I_2, \tag{48}$$

$$Z_1 = -\frac{1}{\beta_1(\lambda)}\frac{\Psi - 1}{\delta + \lambda}\Pi_2^{*^{1-\beta_1(\lambda)}},$$

and the post-tax compound option reduces to $Z_1\Pi^{\beta_1(\lambda)}$. Like the first trigger point, the second is unaffected by taxation, i.e. $\Pi_2^{*^T} = \Pi_2^*$.

We have thus obtained the relevant solutions $\Pi_1^{*^T}$ and $\Pi_2^{*^T}$, plus X_1, Z_1, Z_2. Then, using the equality $X_1 = Y_1 - Z_1$, we find Y_1.

The remaining six unknown parameters (A_1, P_1, P_2, Q_1, Q_2 and R_2) can be solved numerically by using the sub-system (29), (30), (32), (34), (35) and (36). However, their solution is not relevant for our purposes.

Finally, let us compute the minimum imputation rate ensuring neutrality. To do so, recall the solutions of trigger points (41) and (48). Easy computations yield the following relationship

$$\Pi_1^{*^T} - \Pi_2^{*^T} \propto \left(\Psi - \frac{I_1 + I_2}{I_1}\right), \tag{49}$$

which confirms Dixit's findings. If, therefore, for a given current payoff Π, inequality $\Psi < \frac{I_1+I_2}{I_1}$ holds (i.e. returns to scale are decreasing), then we have $\Pi_1^{*^T} < \Pi_2^{*^T}$. This implies that the incremental strategy is preferred to the simultaneous strategy. If, instead, we have $\Psi > \frac{I_1+I_2}{I_1}$, the converse is true and the two-stage project reduces to an one-off investment.

Recall now the tax-holiday condition. We know that when $\Psi(K)\Pi < r_E K$, no tax is paid. Thus in order to ensure a sufficiently long tax holiday after the firm's investment, both the following inequalities must hold

$$r_E > \max\left(\frac{\Pi}{I_1}, \frac{\Psi\Pi}{I_1+I_2}\right). \tag{50}$$

Namely, if the firm faces increasing returns to scale we have $r_E > \frac{\Pi}{I_1} > \frac{\Psi\Pi}{I_1+I_2}$. If instead it is characterised by decreasing returns to scale, then inequality $r_E > \frac{\Psi\Pi}{I_1+I_2} > \frac{\Pi}{I_1}$ must hold.

Now substitute the relevant trigger points (41) and (48) into condition (50). It is straightforward to obtain the minimum neutral imputation rate (2). Inequality $r_E \geq r_E^*$ represents a sufficient neutrality condition. Proposition 1 is thus proven.■

6.2 Proof of Proposition 2

The proof of Proposition 2 is straightforward. As shown by Dixit and Pindyck (1994, pp. 322-328), inequality $\Pi_1^* > \Pi_2^*$ always holds under sequential investment. Namely, once Π reaches Π_1^* the firm will be able to undertake both stages of the project. The investment decision thus might reduces to an one-off problem. Apply now Panteghini's (2002) Proposition 1, which has the same notation as that used in this article, except for the gross payoff which is $\Psi\Pi$ instead of Π. If the imputation rate is high enough, the trigger point is unaffected by taxation, i.e.

$$\Psi\Pi_1^* = \frac{\beta_1(\lambda)}{\beta_1(\lambda)-1}(\delta+\lambda)(I_1+I_2) \tag{51}$$

Recall in fact the tax-holiday condition. We know that when $\Psi\Pi < r_E(I_1 + I_2)$, no taxes are paid. Substituting (51) into condition (50) yields the minimum imputation rate ensuring neutrality (5). Proposition 2 is thus proven.■

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