# A Strategic R\&D Investment with Flexible Development Time in Real Option Game Analysis 

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# A Strategic R\&D Investment with Flexible Development Time in Real Option Game Analysis 


#### Abstract

The real option theory provides a useful tool to evaluate an R\&D investment under uncertainty because, unlike the NPV (Net Present Value), it considers the managerial flexibility that may be expand the investment opportunity value. However, most R\&D investment projects are open to competing firms in the same industry or line of business, and so the strategic considerations become extremely important. In this paper we analyze a real option game between two firms that invest in R\&D. The firm that invests first, defined as the Leader, acquires a first mover advantage that we assume as a higher market share than other one, namely the Follower, that postpones its R\&D investment decision. But, several R\&D investments present positive externalities and so, the option exercise by the Leader generates an "Information Revelation" that benefits the Follower. Moreover, to value the flexibility time to realize the development phase, we consider the American-Exchange type options.


JEL Code: G13, C72, C15, O32, D80.
Keywords: American Exchange options, game theory, Montecarlo simulation, R\&D, information revelation.

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## 1 Introduction

The innovation is one of the important key strategies for firms to survive. Therefore, Research and Development ( $\mathrm{R} \& \mathrm{D}$ ) investment plays an important role in the successful performance for a firm. During the last two decades, the application of option pricing formula to $\mathrm{R} \& D$ has become of interest and numerous studies have attempted to address how the real options analysis can help draw the proper line between knowledge building and strategic positioning. In fact it is widely recognised that the conventional NPV rule could in principle underestimate the value of an R\&D project because this method fails to take the managerial flexibility into account. From a modelling perspective, real R\&D options valuation methods have tended to follow financial option pricing techniques. Analogous to financial options on stocks, real options are options on real or physical assets such as technologies, production facilities and so on. When a firm "invests" means that it exercises its option by involving an initial cost to exchange for a real asset. According to Copeland \& Antikarov (2003), a real option is "the right, but not the obligation, to take an action (e.g. deferring, expanding, contracting, or abandoning) at a predetermined cost called the exercise price, for a predetermined period of time - the life of option".
Several models, such as is assumed to be in Majd \& Pindyck (1987), Trigeorgis (1991), Lee (1997), are based on this definition, in which the exercise price is fixed. But, for the evaluation of real R\&D investment opportunity, it is appropriate to consider that also the investment cost is uncertain since the manager cannot make an accurate estimate of the future costs. So the $\mathrm{R} \& D$ investment opportunity corresponds to an exchange option: it's the exchange of an uncertain investment cost for an uncertain gross project value. The most relevant models that value investment opportunities with two stochastic variables are given in Margrabe (1978), McDonald \& Siegel (1985), Carr (1988), Carr (1995), Armada et al. (2007).
Margrabe (1978) developed a model to price the simple European exchange option (SEEO) to exchange one risky asset for another one at maturity date $T$ and McDonald \& Siegel (1985) considered that the assets distribute dividends. In a real options context, "dividends" are the opportunity costs inherent in the decision to defer an investment project. Furthermore, in a real options context, deferment implies the loss of the project's cash flows. Carr (1988) model, building on Margrabe (1978) and Geske (1979), provided the valuation of compound European exchange options (CEEO). This model may be interpreted as a combination of a time-to-build option (growth option) and an option to exchange (operating option). In addition, Carr (1988), Carr (1995) Armada et al. (2007) provided an approximation to value a simple American exchange option (SAEO). When the asset to be received in the exchange pays large dividend yields, there is always a probability that the American exchange option will be exercised prior to expiration. This means that managers have the timing choice for the development phase realization that gives the opportunity to capture the project's cash flows.
Moreover, competitive interaction becomes fundamentally important in the valuation and exercise of real options, while it may not be such a significant concern for financial options. Such competitive interactions may have profound effects on option exercise decisions and the resulting equilibrium. Real options and game-theory thinking have been embraced by strategic decision-makers who recognise the importance of making an early investment commitment (game theory) while maintaining managerial flexibility (real options) to adapt their choices to a changing market environment.
The aim of this paper is to analyse a real option game model between two firms that
invest in R\&D. The first firm that invests, defined as the Leader, acquires a first mover advantage that we assume as a highest market share. But, several R\&D investments, present positive externalities and so, the option exercise by the Leader, generates an "Information Revelation" that benefits the Follower. Moreover, to consider the managerial flexibility to realize the development investment $D$, we assume that the opportunity to entry in the market is like an American exchange option.
This paper follows the Dias \& Teixeira (2004), Villani (2008) and Cortelezzi \& Villani (2008) models that analyze the equilibrium strategies of two firms that invest in R\&D assuming the uncertainty about the R\&D implementation and also considering the information revelation process. We differentiate from them because we use American exchange options to value the stochastic processes for $R \& D \operatorname{costs}(D$ and $R)$ and for overall market value $V$ deriving by R\&D innovations and also to consider the managerial flexibility to realize the development investment $D$.
The paper is organized as follows. Section 2 reviews some of the relevant American option pricing literature while Section 3 derives the final payoffs of two firms. In Section 4, we present a real model implementation with computation of critical market values that delimit the several Nash equilibriums and, in Section 5, we analyze the effects that the most important parameters have on the game ranges. Finally, Section 6 concludes.

## 2 Exchange Options Methodology

In this section we present the final results to value American exchange options.

### 2.1 Simple American exchange option (SAEO)

Carr (1988) and Carr (1995) models give us the value of a Pseudo American exchange option (PSAEO). In particular way, let $t_{0}=0$ the evaluation date and $T$ the maturity date of the exchange option, we assume that $V$ and $D$ follow a geometric Brownian motion process given by:

$$
\begin{gather*}
\frac{d V}{V}=\left(\mu_{v}-\delta_{v}\right) d t+\sigma_{v} d Z_{v}  \tag{1}\\
\frac{d D}{D}=\left(\mu_{d}-\delta_{d}\right) d t+\sigma_{d} d Z_{d}  \tag{2}\\
\operatorname{cov}\left(\frac{d V}{V}, \frac{d D}{D}\right)=\rho_{v d} \sigma_{v} \sigma_{d} d t \tag{3}
\end{gather*}
$$

where $V$ and $D$ are the Gross Project Value and the Investment Cost, respectively, $\mu_{v}$ and $\mu_{d}$ are the equilibrium expected rate of return on asset $V$, and the expected growth rate of the investment cost, $\delta_{v}$ and $\delta_{d}$ are the "dividend-yields" of $V$ and $D$, $Z_{v}$ and $Z_{d}$ are the Brownian standard motions of asset $V$ and $D, \sigma_{v}$ and $\sigma_{d}$ are the volatility of $V$ and $D$ respectively, $\rho_{v d}$ is the correlation between changes in $V$ and $D$. Carr (1988) shows that the value of a PSAEO $\left(S_{2}\right)$ exercisable at time $\frac{T}{2}$ or $T$ is:

$$
\begin{align*}
S_{2}(V, D, T)= & V e^{-\delta_{v} T} N_{2}\left(-d_{1}^{*}, d_{1} ;-\rho_{1}\right)-D e^{-\delta_{d} T} N_{2}\left(-d_{2}^{*}, d_{2} ;-\rho_{1}\right) \\
& +V e^{-\delta_{v} \frac{T}{2}} N\left(d_{1}^{*}\right)-D e^{-\delta_{d} \frac{T}{2}} N\left(d_{2}^{*}\right) \tag{4}
\end{align*}
$$

where:

$$
\text { - } P=\frac{V}{D} ; \quad \sigma=\sqrt{\sigma_{v}^{2}-2 \rho_{v, d} \sigma_{v} \sigma_{d}+\sigma_{d}^{2}} ; \quad \delta=\delta_{v}-\delta_{d}
$$

- $d_{1} \equiv d_{1}(P, T)=\frac{\log P+\left(\frac{\sigma^{2}}{2}-\delta\right) T}{\sigma \sqrt{T}} ; \quad d_{2}(P, T)=d_{1}(P, T)-\sigma \sqrt{T} ;$
- $d_{1}^{*} \equiv d_{1}\left(\frac{P}{P^{*}}, \frac{T}{2}\right)=\frac{\log \left(\frac{P}{P^{*}}\right)+\left(\frac{\sigma^{2}}{2}-\delta\right) \frac{T}{2}}{\sigma \sqrt{\frac{T}{2}}} ;$
- $d_{2}^{*} \equiv d_{2}\left(\frac{P}{P^{*}}, \frac{T}{2}\right)=d_{1}^{*}-\sigma \sqrt{\frac{T}{2}} ; \quad \rho_{1}=\sqrt{\frac{T}{2 \cdot T}}=\sqrt{0.5} ;$
- $N(d)$ is the cumulative standard normal distribution;
- $N_{2}\left(x_{1}, x_{2} ; \rho\right)$ is the standard bivariate normal distribution function evaluated at $x_{1}$ and $x_{2}$ with correlation $\rho$;
- $P^{*}$ is the unique value which makes indifferent the option exercise or not at time $\frac{T}{2}$ and it solves the following equation:

$$
\begin{equation*}
P^{*} e^{-\delta_{v} \frac{T}{2}} N\left(d_{1}\left(P^{*}, \frac{T}{2}\right)\right)-e^{-\delta_{d} \frac{T}{2}} N\left(d_{2}\left(P^{*}, \frac{T}{2}\right)\right)=P^{*}-1 \tag{5}
\end{equation*}
$$

Moreover, Armada et al. (2007) correct the two-moments extrapolation given in Carr (1988) and Carr (1995) to approximate the value of a simple American exchange option $S(V, D, T)$. So, using the Armada et al. (2007) formula, we have that:

$$
\begin{equation*}
S(V, D, T) \simeq S_{2}(V, D, T)+\frac{S_{2}(V, D, T)-s(V, D, T)}{3} \tag{6}
\end{equation*}
$$

where $s(V, D, T)$ is the value of a simple European exchange option (SEEO) given by McDonald \& Siegel (1985):

$$
\begin{equation*}
s(V, D, T)=V e^{-\delta_{v} T} N\left(d_{1}(P, T)\right)-D e^{-\delta_{d} T} N\left(d_{2}(P, T)\right) \tag{7}
\end{equation*}
$$

### 2.2 Compound American exchange option (CAEO)

Exchange option are simple or compound. If the underlying asset is another option, then the option is called compound. The underlying asset of a CAEO is the SAEO $S(V, D, \tau)$, the expiration date is $t_{1}$ and, following Carr (1988), the exercise price of a CAEO is a proportion $\varphi$ of asset $D$. Using Armada et al. (2007) extrapolation, we can approximate the value of a CAEO as:

$$
\begin{equation*}
C\left(S(V, D, \tau), \varphi D, t_{1}\right) \simeq \frac{4 c_{2}\left(S_{2}(V, D, \tau), \varphi D, t_{1}\right)-c\left(s(V, D, \tau), \varphi D, t_{1}\right)}{3} \tag{8}
\end{equation*}
$$

where:

- $\tau=T-t_{1}$ is the time to maturity of the SAEO with $t_{1}<T$;
- $c_{2}\left(S_{2}(V, D, \tau), \varphi D, t_{1}\right)$ is the Pseudo compound American exchange option (PCAEO) whose underlying asset is the PAEO $S_{2}(V, D, \tau)$ that can be exercised at middle $\frac{\tau}{2}$ and final time $T$, the maturity date is time $t_{1}$ and the exercise price is a proportion $\varphi$ of asset $D$;
- $c\left(s(V, D, \tau), \varphi D, t_{1}\right)$ is the value of a compound European exchange option (CEEO) whose underlying asset is the simple European exchange option (SEEO) $s(V, D, \tau)$.
The value of PCAEO can be determined using Montecarlo simulation as illustrated in Cortelezzi \& Villani (2009).


## 3 The Basic Model Game

In our model we consider a competitive interaction between two firms $(A$ and $B)$ face an R\&D investment opportunity. Both firms can decide to invest at time $t_{0}$ or to wait to invest and so to postpone their decision at time $t_{1}$. As it is know, the $\mathrm{R} \& \mathrm{D}$ investments are uncertain and so, assuming by $q$ and $p$ the $\mathrm{R} \& \mathrm{D}$ success probability of firms A and B respectively, we can represent this situation by two Bernoulli distributions $Y$ and $X$ :

$$
Y:\left\{\begin{array}{cc}
1 & q \\
0 & 1-q
\end{array} \quad X:\left\{\begin{array}{cc}
1 & p \\
0 & 1-p
\end{array}\right.\right.
$$

The value of $q$ and $p$ depend by the Know-How that each player holds on. Moreover, as it shown in Dias (2004), the R\&D success or failure of one firm generates an information revelation that influences the investment decision of the other firm. So, if firm A's R\&D is successful, the firm B's probability $p$ changes in positive information revelation $p^{+}$, while $p$ changes in negative information revelation $p^{-}$in case of A's failure. Symmetrically, the firm A's R\&D success changes in $q^{+}$or in $q^{-}$in case of firm B success or failure at time $t_{0}$. Using Dias (2004) model, it results that:

$$
\begin{aligned}
& p^{+}=\operatorname{Prob}[X=1 / Y=1]=p+\sqrt{\frac{1-q}{q}} \cdot \sqrt{p(1-p)} \cdot \rho(X, Y) \\
& p^{-}=\operatorname{Prob}[X=1 / Y=0]=p-\sqrt{\frac{q}{1-q}} \cdot \sqrt{p(1-p)} \cdot \rho(X, Y) \\
& q^{+}=\operatorname{Prob}[Y=1 / X=1]=q+\sqrt{\frac{1-p}{p}} \cdot \sqrt{q(1-q)} \cdot \rho(Y, X) \\
& q^{-}=\operatorname{Prob}[Y=1 / X=0]=q-\sqrt{\frac{p}{1-p}} \cdot \sqrt{q(1-q)} \cdot \rho(Y, X)
\end{aligned}
$$

where the correlations $\rho(X, Y)$ and $\rho(Y, X)$ are a measure of information revelation from $Y$ to $X$ and from $X$ to $Y$, respectively. Obviously, the information revelation is considerable when the investment is not realized in the same time. So, if both players invest simultaneously in $\mathrm{R} \& \mathrm{D}$ or they wait to invest, there is not information revelation and consequently it results that $p=p^{+}=p^{-}$and $q=q^{+}=q^{-}$.
Under the threat of competition, the exercise of options strategically depends on the trade-off between the benefits and costs of going ahead with an investment against waiting for more information. So we state that the Leader is the pioneer firm (A or B) that invests in $\mathrm{R} \& D$ at time $t_{0}$ earlier than other one, namely the Follower, that defers exercising its option at time $t_{1}$ to receive better information. Leader can take an advantage of being first in the market and, in particular way, we suppose that it achieves the market share opportunity $\alpha \in\left(\frac{1}{2}, 1\right]$ of V higher than Follower's one, that is $1-\alpha$. But, if the investment is realized in the same time, both players share the market equally and so $\alpha=\frac{1}{2}$.
We denote by $R$ the R\&D investment for the development of a new product, $V$ the overall market value deriving by $\mathrm{R} \& \mathrm{D}$ innovations and $D$ is the total investment cost to realize new goods. We consider that the production investment of each firm is proportional to its market share and it can be realized at anytime before $T$ so we consider the managerial flexibility to realize the investment $D$. Therefore, the option to enter in the market is like an American exchange option. In particular, we assume that $V$ and $D$ follow the geometric Brownian motion defined in the Eqs.(1) and (2) respectively, and $R=\varphi D$ is a proportion $\varphi$ of asset $D$, so $R$ assumes the identical
stochastic process of $D$ except that it can be spent only at initial time $t_{0}$ or at time $t_{1}$.

### 3.1 The Follower's payoff.

First of all, we analyze the game in which the firm A (Leader) invests in R\&D at time $t_{0}$ and the firm B (Follower) decides to delay its $\mathrm{R} \& \mathrm{D}$ investment decision at time $t_{1}$. So, assuming the Leader's R\&D success, the Follower's R\&D success probability changes in $p^{+}$and, after the investment $R$, the Follower holds the development option $S((1-\alpha) V,(1-\alpha) D, \tau)$ to invest $(1-\alpha) D$ at anytime from $t_{1}$ and $T$ and claims a share $1-\alpha$ of the overall market $V$. Of course, the investment $R$ will be realized at time $t_{1}$ if the development option $p^{+} S((1-\alpha) V,(1-\alpha) D, \tau)$ is bigger than $R$. So, the Follower's payoff at time $t_{0}$ is a CAEO with maturity $t_{1}$, exercise price equal to $R$ and the underlying asset is the development option $S((1-\alpha) V,(1-\alpha) D, \tau)$, as shown in Fig.1(a).


Figure 1: Follower's payoffs
The CAEO payoff at expiration date $t_{1}$ with positive information revelation is:

$$
C\left(p^{+} S((1-\alpha) V,(1-\alpha) D, \tau), R, 0\right)=\max \left[p^{+} S((1-\alpha) V,(1-\alpha) D, \tau)-R, 0\right]
$$

Considering that $R=\varphi D$ is a proportion $\varphi$ of asset $D$ and denoting with $C\left(p^{+}\right)$the CAEO at time $t_{0}$, i.e.:

$$
C\left(p^{+}\right) \equiv C\left(p^{+} s((1-\alpha) V,(1-\alpha) D, \tau), \varphi D, t_{1}\right)
$$

we can write, using the Eq.(8), the value of CAEO with positive information:

$$
\begin{equation*}
C\left(p^{+}\right) \simeq \frac{4 c_{2}\left(p^{+} S_{2}((1-\alpha) V,(1-\alpha) D, \tau), \varphi D, t_{1}\right)-c\left(p^{+} s((1-\alpha) V,(1-\alpha) D, \tau), \varphi D, t_{1}\right)}{3} \tag{9}
\end{equation*}
$$

Alternatively, in case of Leader's R\&D failure, the Follower success probability changes in $p^{-}$and the Follower holds, after the investment $R$ at time $t_{1}$, the development option $S((1-\alpha) V,(1-\alpha) D, \tau)$ to invest $(1-\alpha) D$ at anytime between $t_{1}$ and $T$ and claims the market value $(1-\alpha) V$. So the Follower's payoff at time $t_{0}$ is a CAEO with maturity $t_{1}$, exercise price equal to $R$ and the underlying asset is the development option $S((1-\alpha) V,(1-\alpha) D, \tau)$ as shown in Fig. 1(b). Hence, the CAEO payoff with negative information revelation at expiration date $t_{1}$ is:

$$
C\left(p^{-} S((1-\alpha) V,(1-\alpha) D, \tau), R, 0\right)=\max \left[p^{-} S((1-\alpha) V,(1-\alpha) D, \tau)-R, 0\right]
$$

So, denoting with $C\left(p^{-}\right)$the CAEO at time $t_{0}$, i.e.:

$$
C\left(p^{-}\right) \equiv C\left(p^{-} S((1-\alpha) V,(1-\alpha) D, \tau), \varphi D, t_{1}\right)
$$

we can write, using the Eq.(8), the value of CAEO with negative information:
$C\left(p^{-}\right) \simeq \frac{4 c_{2}\left(p^{-} S_{2}((1-\alpha) V,(1-\alpha) D, \tau), \varphi D, t_{1}\right)-c\left(p^{-} s((1-\alpha) V,(1-\alpha) D, \tau), \varphi D, t_{1}\right)}{3}$
The Follower obtains the CAEO $C\left(p^{+}\right)$in case of Leader's success with a probability $q$ or the CAEO $C\left(p^{-}\right)$in case of Leader's failure with a probability $(1-q)$. Hence, the Follower's payoff at time $t_{0}$ is the expectation value:

$$
\begin{equation*}
F_{B}(V, D)=q C\left(p^{+}\right)+(1-q) C\left(p^{-}\right) \tag{11}
\end{equation*}
$$

Similarly, if we consider that firm B (Leader) invests in R\&D at time $t_{0}$ and firm A (Follower) decides to wait to invest it results:

$$
\begin{equation*}
F_{A}(V, D)=p C\left(q^{+}\right)+(1-p) C\left(q^{-}\right) \tag{12}
\end{equation*}
$$

Using Cortelezzi \& Villani (2009) model, we are able to determine the Follower's payoff through Montecarlo simulation. In particular way, the appendix (A) shows the Matlab algorithm to obtain the values given by Eqs. (11) and (12).

### 3.2 The A and B payoffs when both firms invest simultaneously in R\&D.

In this situation, both players decide to realize the $R \& D$ investment simultaneously at time $t_{0}$. Hence, we can setting that there is not information revelation and consequently it results that $\rho(Y, X)=\rho(X, Y)=0$. Since the investment $R$ is equal for both firms, we assume that A and B can capture the same fraction $\alpha=\frac{1}{2}$ of the overall market value. So, after the investment $R$ in $t_{0}$, A and B hold with a probability $q$ and $p$ respectively, the development option $S\left(\frac{1}{2} V, \frac{1}{2} D, T\right)$ to invest $\frac{1}{2} D$ at anytime before $T$, as illustrated in the Figs. 2(a) and 2(b).


Figure 2: A and B payoffs in case of simultaneous investment
According to Eq.(6), we can write the A and B payoffs in case of simultaneous R\&D investment at time $t_{0}$ as:

$$
\begin{align*}
S_{A}(V, D) & =-R+q \cdot S\left(\frac{1}{2} V, \frac{1}{2} D, T\right) \\
& \simeq-R+q\left(\frac{4 S_{2}\left(\frac{1}{2} V, \frac{1}{2} D, T\right)-s\left(\frac{1}{2} V, \frac{1}{2} D, T\right)}{3}\right)  \tag{13}\\
S_{B}(V, D) & =-R+p \cdot S\left(\frac{1}{2} V, \frac{1}{2} D, T\right) \\
& \simeq-R+p\left(\frac{4 S_{2}\left(\frac{1}{2} V, \frac{1}{2} D, T\right)-s\left(\frac{1}{2} V, \frac{1}{2} D, T\right)}{3}\right) \tag{14}
\end{align*}
$$

### 3.3 The Leader's payoff

Now we analyse the game in which firm A (Leader) invests in R\&D at time $t_{0}$, assuming that firm B (Follower) decides to postpones its decision waiting better information. In this case, the Leader spends the investment $R$ at time $t_{0}$ and obtains, in case of success with a probability $q$, the development option $S(\alpha V, \alpha D, T)$ that gives the opportunity to invest $\alpha D$ at anytime before $T$ and to claim a market share $\alpha>\frac{1}{2}$, as illustrated in the Fig. 3. Thus the Leader's payoff (firm A) will be:

$$
\begin{align*}
L_{A}(V, D) & =-R+q \cdot S(\alpha V, \alpha D, T) \\
& \simeq-R+q\left(\frac{4 S_{2}(\alpha V, \alpha D, T)-s(\alpha V, \alpha D, T)}{3}\right) \tag{15}
\end{align*}
$$

Symmetrically, if we consider that firm B (Leader) realizes the $R \& D$ investment at time $t_{0}$ and player A postpones its decision, the firm B payoff will be:

$$
\begin{aligned}
L_{A}(V, D) & =-R+p \cdot S(\alpha V, \alpha D, T) \\
& \simeq-R+p\left(\frac{4 S_{2}(\alpha V, \alpha D, T)-s(\alpha V, \alpha D, T)}{3}\right) \\
& \begin{array}{ccc}
q S(\alpha \mathrm{~V}, \alpha \mathrm{D}, \mathrm{~T}) & \alpha \mathrm{V} \\
\hline \mathrm{t}_{0} \uparrow & \uparrow \\
\mathrm{R} & \uparrow \mathrm{D} & \mathrm{~T}
\end{array}
\end{aligned}
$$

Figure 3: Leader's payoff

### 3.4 The A and B payoffs when both firms wait to invest.

Finally, we suppose that both players decide to delay their R\&D investment decision at time $t_{1}$ and, specifically, we can assume that there is not information revelation and consequently $\rho(Y, X)=\rho(X, Y)=0$. As we have seen in simultaneous case, we can setting that A and B share the market equally and so $\alpha=\frac{1}{2}$. Then, after the investment $R$ in $t_{1}$, each player holds in case of $\mathrm{R} \& \mathrm{D}$ success the development option $S\left(\frac{1}{2} V, \frac{1}{2} D, \tau\right)$ to invest $\frac{1}{2} D$ at anytime before $T$ and claims a market share $\frac{1}{2} V$. So, at time $t_{0}$, the A and B payoffs are CAEO with maturity $t_{1}$, exercise price equal to $R=\varphi D$ and the underlying asset is the development option $S\left(\frac{1}{2} V, \frac{1}{2} D, T\right)$ with probability $q$ and $p$ respectively, as illustrated in the Figs 4(a) and 4(b). Thus, A and B payoffs at time $t_{0}$ are given by:

$$
\begin{align*}
& W_{A}(V, D)=C\left(q \cdot S\left(\frac{1}{2} V, \frac{1}{2} D, \tau\right), \varphi D, t_{1}\right)  \tag{17}\\
& W_{B}(V, D)=C\left(p \cdot S\left(\frac{1}{2} V, \frac{1}{2} D, \tau\right), \varphi D, t_{1}\right) \tag{18}
\end{align*}
$$

Using the Eq.(8) we can determine the firms A and B waiting payoffs as:

$$
\begin{equation*}
W_{A}(V, D) \simeq \frac{4 c_{2}\left(q S_{2}\left(\frac{1}{2} V, \frac{1}{2} D, \tau\right), \varphi D, t_{1}\right)-c\left(q S\left(\frac{1}{2} V, \frac{1}{2} D, \tau\right), \varphi D, t_{1}\right)}{3} \tag{19}
\end{equation*}
$$



Figure 4: A and B payoffs in case waiting to invest

$$
\begin{equation*}
W_{B}(V, D) \simeq \frac{4 c_{2}\left(p S_{2}\left(\frac{1}{2} V, \frac{1}{2} D, \tau\right), \varphi D, t_{1}\right)-c\left(p S\left(\frac{1}{2} V, \frac{1}{2} D, \tau\right), \varphi D, t_{1}\right)}{3} \tag{20}
\end{equation*}
$$

So, the appendix (A) shows the Matlab algorithm to determine the firms A and B waiting payoffs through Montecarlo simulation. It's sufficient to consider that information revelation $\rho(X, Y)=0$ and $\alpha=\frac{1}{2}$.

### 3.5 Final payoffs at time $t_{0}$

The two-by-two matrix represented in the Fig. 5 summarizes the final payoffs. The first value in each cell indicates the strategic investment opportunity for A at time $t_{0}$, while the second represents the firm B's value. We can distinguish four basic cases: (i) when both firms decide to postpone the R\&D investment at time $t_{1}$; (ii) and (iii) when one firm invests first (as a Leader) and the other decides to invest later (as a Follower); (iv) when both firms decide to invest simultaneously in $\mathrm{R} \& \mathrm{D}$ at time $t_{0}$.

FIRM B


Figure 5: Final payoffs at time $t_{0}$

## 4 Real Applications

### 4.1 Assumptions and Inputs

This model can be applied to analyse industries such as high-tech, pharmaceutical, telecommunication, oil, in which competitors can substantially influence a firms investment opportunity. In fact, a firm may pre-empt competition and capture a significant share of the market $\alpha>\frac{1}{2}$ by setting the $\mathrm{R} \& \mathrm{D}$ investment early on. This is
an important source of advantage that may establish a sustainable strategic position. But, the firm that delays investment, can derive information about its R\&D success from observing the R\&D performance of the other player.
So, to illustrate the concepts and equations presented, we develop a numerical example for the competitive $\mathrm{R} \& \mathrm{D}$ game between firms A and B with the following parameters:

- R\&D Investment: $R=150000 \$$;
- Development Investment: $D=400000 \$$;
- Market and Costs Volatility: $\sigma_{v}=0.90 ; \quad \sigma_{d}=0.23$;
- Proportion of $D$ required for $R: \varphi=\frac{R}{D}=0.375$
- Correlation between $V$ and $D: \rho_{v d}=0.15$;
- Dividend-Yields of $V$ and $D: \quad \delta_{v}=0.15 ; \quad \delta_{d}=0 ;$
- Expiration Time of Compound Option: $t_{1}=0.5$ years;
- Expiration Time of Simple Option: $T=3$ years;
- A and B success probability: $q=0.60 ; p=0.55$;
- Information Revelation: $\quad \rho(X, Y)=\rho(Y, X)=0.70$;
- Leader's Market Share: $\alpha=0.60$;
- Critical Price $K=1.6722$ :

We consider five expected total market values $V: 800000 \$$ (low expected return), $1000000 \$, 1200000 \$$ (medium expected return) and $1400000 \$$ and $1600000 \$$ (high expected return). $V$ corresponds to present value of the expected cash flows deriving by R\&D innovations. We assume that $V$ follows the Brownian motion presented in Eq.(1).
The total investment cost $D$ is the exercise price for the development option. We consider that the investment cost is proportional to market share, namely if the firm's market share is $\alpha$ then its investment cost will be $\alpha D$. We assume that $D$ follows the Brownian motion process defined in Eq.(2). The total current value of $D$ is $400000 \$$ and it can be spent at anytime before $T$.
The $\mathrm{R} \& \mathrm{D}$ investment $R$ can be realized at time $t_{0}$ or $t_{1}$. If it is made in $t_{0}$, then $R=150000 \$$ otherwise the investment $R$ assumes the identical stochastic process of $D$, except that it occurs at time $t_{1}$ and it is proportional to $\varphi=0.375$ of $D$.
Appropriately, we assume that the volatility of quoted shares and traded options is an adequate proxy for the volatility of asset $V$ and investment cost $D$. As the R\&D investments present a high uncertainty about their results, we assume that $\sigma_{v}=0.90$ and the cost volatility is $\sigma_{d}=0.23$.
According to financial options, $\delta$ denotes the opportunity cost in holding the option instead of the stock. So, in real option world, $\delta_{v}$ is the opportunity cost of deferring the project and $\delta_{d}$ is the "dividend yield" on asset $D$. As at the beginning the cash flows are very low, so we assume that $\delta_{v}=0.15$ and $\delta_{d}=0$.
The time to maturity $T$ denotes project's deferment option after that each opportunity disappears and we adopt $T=3$ years. Moreover, we state that Follower needs about six months to know the Leader's outcome and consequently to receive the information revelation. So we assume that $t_{1}=0.5$ years.
$K$ denotes the critical price value that makes indifferent the exercise or not at middle time $\frac{\tau}{2}$ of a PSAEO $S_{2}\left(V, D, \frac{\tau}{2}\right)$. So to determine $K$ it is sufficient to use Eq. (5) with $T=\tau$. For our adapted numbers it results $K=1.6722$.
Finally, we consider that firm A has an higher and more efficient Know-How than firm B and so, the firm A's success probability is $q=0.60$ while the firm B's one is $p=0.55$.

### 4.2 Empirical Results

The Table 1 shows the Montecarlo simulation assuming the several overall market values. In particular way we compute, for each player, four Montecarlo simulations and, to determine the final Follower and the Waiting strategic payoffs, we compute the average value. We assume that the number of simulations $n$ is equal to 100000 . As it is shown in Cortelezzi \& Villani (2009), this simulations number allows us to obtain a very low variance and to improve the efficiency of computations.

| Strategy | $1^{s t} \mathrm{MC}$ | $2^{\text {nd }} \mathrm{MC}$ | $3^{\text {rd }} \mathrm{MC}$ | $4^{\text {th }} \mathrm{MC}$ | Average Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $F_{A}(800000)$ | 26620 | 26525 | 26573 | 26663 | 26595 |
| $F_{B}(800000)$ | 23936 | 23862 | 23916 | 23999 | 23928 |
| $W_{A}(800000)$ | 30760 | 30675 | 30777 | 30875 | 30772 |
| $W_{B}(800000)$ | 25191 | 25133 | 25227 | 25323 | 25219 |
| $F_{A}(1000000)$ | 47146 | 47147 | 47103 | 47087 | 47120 |
| $F_{B}(1000000)$ | 43232 | 43060 | 43024 | 42988 | 43076 |
| $W_{A}(1000000)$ | 56355 | 56123 | 56089 | 56004 | 56143 |
| $W_{B}(1000000)$ | 47146 | 46925 | 46900 | 46780 | 46938 |
| $F_{A}(1200000)$ | 72288 | 72286 | 71908 | 72176 | 72164 |
| $F_{B}(1200000)$ | 66707 | 66711 | 66359 | 66608 | 66596 |
| $W_{A}(1200000)$ | 87566 | 87618 | 87150 | 87484 | 87455 |
| $W_{B}(1200000)$ | 74349 | 74369 | 73977 | 74261 | 74239 |
| $F_{A}(1400000)$ | 100510 | 100750 | 100510 | 100420 | 100548 |
| $F_{B}(1400000)$ | 93460 | 93687 | 93460 | 93356 | 93491 |
| $W_{A}(1400000)$ | 123240 | 123530 | 123240 | 123030 | 123260 |
| $W_{B}(1400000)$ | 105810 | 106060 | 105810 | 105650 | 105833 |
| $F_{A}(1600000)$ | 130940 | 131290 | 131430 | 131440 | 131275 |
| $F_{B}(1600000)$ | 122380 | 122720 | 122830 | 122870 | 122700 |
| $W_{A}(1600000)$ | 161490 | 162000 | 162020 | 162130 | 161910 |
| $W_{B}(1600000)$ | 139850 | 140290 | 140330 | 140460 | 140233 |

Table 1: Simulated Values of Follower and Waiting Strategies
The Tables 2 and 3 summarize the strategic A and B payoffs considering the several expected total market values. The Figs. 6 and 7 show the A and B strategic values. We can observe that, when the expected market value $V=0$, the simple and the compound American exchange option values are zero and so it results that $L_{i}(0)=S_{i}(0)=-R$ and $F_{i}(0)=W_{i}(0)=0$, for $i=A, B$. Now, to determine the several Nash equilibriums, we introduce the critical market values that realize the equality among the four strategic values. We define by $V_{W A}^{*}$ and $V_{W B}^{*}$ the critical market values that make $L_{i}\left(V_{W i}^{*}\right)=W_{i}\left(V_{W i}^{*}\right)$, for $i=A, B$ and by $V_{S A}^{*}$ and $V_{S B}^{*}$ the critical market values such that $F_{i}\left(V_{S i}^{*}\right)=F_{i}\left(V_{S i}^{*}\right)$, for $i=A, B$. Through Figs. 6 and

| Market | Leader's Value | Follower's Value | Simultaneous Value | Waiting Value |
| :--- | :---: | :---: | :---: | :---: |
| Value $V$ | $L_{A}$ | $F_{A}$ | $S_{A}$ | $W_{A}$ |
| 800000 | -4474 | 26595 | -28728 | 30772 |
| 1000000 | 48152 | 47120 | 15126 | 56143 |
| 1200000 | 102894 | 72164 | 60745 | 87455 |
| 1400000 | 159113 | 100548 | 107594 | 123260 |
| 1600000 | 216402 | 131275 | 155335 | 161910 |

Table 2: Firm A's final payoffs assuming $\alpha=0.60$ and $\rho(X, Y)=0.70$

| Market | Leader's Value | Follower's Value | Simultaneous Value | Waiting Value |
| :---: | :---: | :---: | :---: | :---: |
| Value $V$ | $L_{B}$ | $F_{B}$ | $S_{B}$ | $W_{B}$ |
| 800000 | -16601 | 23928 | -38834 | 25219 |
| 1000000 | 31639 | 43076 | 1366 | 46938 |
| 1200000 | 81819 | 66596 | 43183 | 74239 |
| 1400000 | 133354 | 93491 | 86128 | 105833 |
| 1600000 | 185869 | 122700 | 129891 | 140233 |

Table 3: Firm B's final payoffs assuming $\alpha=0.60$ and $\rho(X, Y)=0.70$

7, we obtain:

$$
V_{W A}^{*} \simeq 1070000 ; \quad V_{W B}^{*} \simeq 1130000 ; \quad V_{S A}^{*} \simeq 1320000 ; \quad V_{S B}^{*} \simeq 1490000
$$

When the expected market value $V<V_{W A}^{*}$, we have the following inequality among the strategic values:

$$
L_{A}(V)<W_{A}(V) ; \quad L_{B}(V)<W_{B}(V) ; \quad F_{A}(V)>S_{A}(V) ; \quad F_{B}(V)>S_{B}(V) ;
$$

So, using this inequality, we have one Nash equilibrium $\left(W_{A}, W_{B}\right)$. For instance, assuming that the expected market value is equal to $V=800000$ (low return), the two by two matrix represented in Fig. 8(a) shows the ( $W_{A}, W_{B}$ ) Nash equilibrium in which firms A and B prefer to wait for best market evolutions and so they decide to delay their $\mathrm{R} \& \mathrm{D}$ investment decision at time $t_{1}$.
Instead, if the expected market value $V>V_{S B}^{*}$, it results the following inequality among the stratigic values:

$$
L_{A}(V)>W_{A}(V) ; \quad L_{B}(V)>W_{B}(V) ; \quad F_{A}(V)<S_{A}(V) ; \quad F_{B}(V)<S_{B}(V)
$$

So, assuming that the expected market value $V=1600000$ (high return), there is one Nash equilibrium $\left(S_{A}, S_{B}\right)$ as shown in the Fig. 8(d). Both firms decide to invest simultaneously in $\mathrm{R} \& \mathrm{D}$ at time $t_{0}$ to take advantage of high market value.
If we consider that the overall expected market value $V \in] V_{W A}^{*}, V_{W B}^{*}[$, the relation among the strategic payoffs is:

$$
L_{A}(V)>W_{A}(V) ; \quad L_{B}(V)<W_{B}(V) ; \quad F_{A}(V)>S_{A}(V) ; \quad F_{B}(V)>S_{B}(V)
$$

In this case we have one Nash equilibriums $\left(L_{A}, F_{B}\right)$. Specifically, the firm with the highest success probability (firm A) realizes the R\&D investment at time $t_{0}$ earlier


Figure 6: Firm's A Strategic Values
that other one (firm B) that postpones its $\mathrm{R} \& \mathrm{D}$ investment decision at time $t_{1}$ waiting better information. Moreover, if we assume that $V \in] V_{S A}^{*}, V_{S B}^{*}$ [, we have the following relation among the strategic payoffs:

$$
L_{A}(V)>W_{A}(V) ; \quad L_{B}(V)>W_{B}(V) ; \quad F_{A}(V)<S_{A}(V) ; \quad F_{B}(V)>S_{B}(V) ;
$$

Also in this case, using the above relations, we have one Nash equilibrium $\left(L_{A}, F_{B}\right)$. For instance, if $V=1400000$, the Fig. 8(c) shows that there exists one Nash equilibrium $\left(L_{A}, F_{B}\right)$.
Finally, if we assume that $V \in] V_{W B}^{*}, V_{S A}^{*}[$, we have the following inequality among the strategic values:

$$
L_{A}(V)>W_{A}(V) ; \quad L_{B}(V)>W_{B}(V) ; \quad F_{A}(V)>S_{A}(V) ; \quad F_{B}(V)>S_{B}(V) ;
$$

In this case we have two Nash equilibriums: $\left(L_{A}, F_{B}\right)$ and $\left(F_{A}, L_{B}\right)$. In the first equilibrium firm A invests immediately at time $t_{0}$ while B postpones its $\mathrm{R} \& \mathrm{D}$ decision at time $t_{1}$ waiting better information, vice versa in the second equilibrium. If we consider that $V=1200000$, we have two Nash equilibriums as it is represented in the Fig. 8(b).

## 5 The effects of $\rho(X, Y), \alpha$ and $\delta_{v}$ on the equilibriums

As we have seen above, in the range game $\left[V_{W A}^{*}, V_{S B}^{*}\right]$ we have one Nash equilibrium $\left(L_{A}, F_{B}\right)$ or two Nash equilibriums $\left(L_{A}, F_{B}\right) ;\left(F_{A}, L_{B}\right)$ that we can solve by mixed strategies. Now we are interested to analyse the effects that the information revelation $\rho(X, Y)$, the first mover's advantage $\alpha$ and the dividend yield $\delta_{v}$ have on Nash equilibriums of both players.
First of all, it is obvious that the strategic payoffs using American exchange options are bigger then European one since American options give the managerial flexibility value to realize the investment $D$ prior to maturity $T$. In particular way, comparing the results given in Villani (2008), we can remark that the critical market values using


Figure 7: Firm's B Strategic Values

American exchange options $V_{W A}^{*}$ and $V_{S B}^{*}$ go down with respect to European options and the length of range game $\left[V_{W A}^{*}, V_{S B}^{*}\right] \simeq[1070000,1490000]=420000$ is smaller then $[1349400,1898700]=549300$ using European options. So we can state that, using the managerial flexibility, both firms reduce the critical market values that bound both the opportunity to delay the R\&D investment decision (wait and see policy) and the simultaneous investment implementation. So, with American options, the R\&D investment can be realized at time $t_{0}$ when $V=1070000 \$$ instead of $V=1349400 \$$. Moreover, when the dividend yields $\delta_{d}$ and $\delta_{v}$ go to zero, then the CAEO and SAEO prices are equal to CEEO (see Carr (1988)) and SEEO (see McDonald \& Siegel (1985)) respectively, since there is not the incentive to exercise the American option prior to maturity date $T$. So for our adapted number, assuming that $\delta_{v}=0$, we have that $V_{W A}^{*} \simeq 860000$ and $V_{S B}^{*} \simeq 1305000$.
The Table 4 shows the effects that the information revelation has got on the game ranges. To simplify, we assume that $\rho(X, Y)=\rho(Y, X)$. The conditions to respect to have $0 \leq p^{+} \leq 1$ and $0 \leq p^{-} \leq 1$ is that:

$$
\begin{equation*}
0 \leq \rho(X, Y) \leq \min \left\{\sqrt{\frac{p(1-q)}{q(1-p)}}, \sqrt{\frac{q(1-p)}{p(1-q)}}\right\} \tag{21}
\end{equation*}
$$

In our applications it results that $0 \leq \rho(X, Y) \leq 0.9026$. We can observe that the Leader and Waiting payoffs are independent by $\rho(X, Y)$ and so the critical market values $V_{W A}^{*}$ and $V_{W B}^{*}$ do not change and therefore the length of range [ $V_{W A}^{*}, V_{W B}^{*}$ ] is always about $60000 \$$. But, if the information revelation increases, then the game ranges $] V_{W B}^{*}, V_{S A}^{*}[$ (in which we have two Nash equilibriums) and $] V_{S A}^{*}, V_{S B}^{*}[$ (in which we have one Nash equilibrium) enlarge.
The Table 5 shows the effects that the first mover's advantage has on the critical market values and in particular way we can note that, if the Leader's market share $\alpha$ increases, then all the critical market values go down. When $\alpha=1$ then the Follower's strategy values zero since its market share is $1-\alpha=0$.


Figure 8: Final payoffs

| $\rho(X, Y)$ | $V_{S A}^{*}$ | $V_{S B}^{*}$ | $V_{W B}^{*}-V_{W A}^{*}$ | $V_{S A}^{*}-V_{W B}^{*}$ | $V_{S B}^{*}-V_{S A}^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1155000 | 1228000 | 60000 | 25000 | 73000 |
| 0.10 | 1165000 | 1262000 | 60000 | 35000 | 97000 |
| 0.30 | 1203000 | 1307000 | 60000 | 73000 | 104000 |
| 0.50 | 1235000 | 1380000 | 60000 | 105000 | 145000 |
| 0.70 | 1320000 | 1490000 | 60000 | 190000 | 170000 |
| 0.90 | 1439000 | 1690000 | 60000 | 309000 | 251000 |

Table 4: Variation of Information Revelation with $\alpha=0,60$ and $\delta_{v}=0.15$

| $\alpha$ | $V_{W A}^{*}$ | $V_{W B}^{*}$ | $V_{S A}^{*}$ | $V_{S B}^{*}$ |
| :--- | ---: | ---: | ---: | ---: |
| 0.60 | 1070000 | 1130000 | 1320000 | 1490000 |
| 0.70 | 858000 | 906000 | 1070000 | 1161000 |
| 0.80 | 742000 | 791000 | 975000 | 1042000 |
| 0.90 | 662000 | 703000 | 935000 | 998000 |
| 1 | 609000 | 645000 | 932000 | 993000 |

Table 5: Variation of Leader's Market Share with $\rho(X, Y)=0.70$ and $\delta_{v}=0.15$

## 6 Concluding Remarks.

The R\&D investment is an important successful key for the firm performance. An R\&D investment opportunity is not held by one firm in isolation and so the competitive considerations become extremely important. The theory of option games combines two successful theories, namely real options and game theory. By real options we value an R\&D investment opportunity using financial techniques and, in particular way, we use Montecarlo simulations to value an American exchange options that take into account the managerial flexibility to realize the investment $D$ at anytime before the maturity $T$. By the game theory, we consider strategic interactions between two firms. The first firm that invests, defined as the Leader, acquires a first mover advantage that we assume as a higher market share then Follower's one, that postpones the R\&D investment. But, in our model, we assume that Follower receives an information revelation from Leader's $\mathrm{R} \& \mathrm{D}$ investment. Through the critical market values $V_{W A}^{*}, V_{W B}^{*}, V_{S A}^{*}$ and $V_{S B}^{*}$, we are able to determine the range game in which is optimal each strategy policy in Nash meaning and we have showed the effects that most important parameters have on the game. So, when $V<V_{W A}^{*}$ we have one Nash equilibrium ( $W_{A}, W_{B}$ ) and if $V>V_{S B}^{*}$ the optimal Nash policy is the simultaneous investment $\left(S_{A}, S_{B}\right)$ at time $t_{0}$. Moreover, if $V$ is in the ranges $] V_{W A}^{*}, V_{W B}^{*}[$ and $] V_{S A}^{*}, V_{S B}^{*}[$ we have one Nash equilibrium $\left(L_{A}, F_{B}\right)$ in which the firm with the highest success probability realizes the $\mathrm{R} \& \mathrm{D}$ investment earlier then other one, while in the interval $] V_{W B}^{*}, V_{S A}^{*}$ [ we have two Nash equilibriums: $\left(L_{A}, F_{B}\right)$ and $\left(F_{A}, L_{B}\right)$. In this case we need to use the mixed strategies to solve the game.

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## A Montecarlo Simulation to determine Follower's payoff

In this algorithm, we denote by ' f ' the proportion $\varphi$ of asset $D$ to determine the research investment $R$, by ' pL ' and ' pF ' the R\&D success probability of Leader and Follower respectively, and by 'rev' the information revelation. Moreover, ' $n$ ' is the number of simulations and ' K ' denotes the critical market value that makes indifferent the exercise or not at middle time $\frac{\tau}{2}$ a PSAEO $S_{2}(V, D, \tau)$.
function FOLLOWER=MCAmerComp(VO,D0,f,dV, dD,T1, T2, K, sigV, sigD,rhoVD, ...
pL, pF,rev, alpha,n);
\% R\&D success probability with positive and negative information revelation
$\mathrm{pp}=\mathrm{pF}+$ sqrt $((1-\mathrm{pL}) /(\mathrm{pL})) *$ sqrt $(\mathrm{pF} *(1-\mathrm{pF})) * \mathrm{rev}$;
$\mathrm{pm}=\mathrm{pF}-\mathrm{sqrt}((1-\mathrm{pL}) /(\mathrm{pL})) * \operatorname{sqrt}(\mathrm{pF} *(1-\mathrm{pF})) * r e v$;
sig=sqrt(sigV. $\left.{ }^{2} 2+s i g D . \wedge 2-2 * r h o V D . * s i g V . * s i g D\right) ; ~ \% V a r i a n c e ~ o f ~ a s s e t ~ P ;$
$\mathrm{u}=\mathrm{rand}(1, \mathrm{n})$; \%Random uniform values between 0 and 1 ;
P0=VO/D0;
$\mathrm{d}=(\mathrm{dV}-\mathrm{dD})$;
rho=sqrt (T1/T2);
$\%$ Value of asset $P$ at time t1 to compute the PAEO
PT1=P0*exp (norminv (u, -d*T1-sig ${ }^{\wedge} 2 * T 1 * 0.5$, sig*sqrt (T1))) ;
$\mathrm{ds} 1=\left(\left(\log ((\mathrm{PT} 1 * \exp (-\mathrm{d} *(0.5 *(\mathrm{~T} 2-\mathrm{T} 1)))) /(\mathrm{K}))+0.5 *\left(\right.\right.\right.$ sig. $\left.\left.{ }^{\wedge} 2\right) * 0.5 *(\mathrm{~T} 2-\mathrm{T} 1)\right) / \ldots$
(sig*sqrt(0.5*(T2-T1))));

```
d1=(log(PT1*exp(-d*(T2-T1)))+0.5*(sig^2)*(T2-T1))/(sig*sqrt(T2-T1));
ds2=((log((PT1*exp(-d*(0.5*(T2-T1))))/(K))-0.5*(sig. ^2)*0.5*(T2-T1))/...
(sig*sqrt(0.5*(T2-T1))));
d2=(log(PT1*exp (-d*(T2-T1))) -0.5*(sig^2)*(T2-T1)) /(sig*sqrt(T2-T1));
%Computation of simulations;
for i=1:n
R1(i)=bivnormcdf(-ds1(i),d1(i),-rho);
R2(i)=bivnormcdf(-ds2(i),d2(i),-rho);
end
%Payoff of PCAEO with positive and negative information revelation;
vpp=max(pp*(1-alpha)*(PT1*exp(-dV*(T2-T1)).*R1+PT1*...
exp(-dV*0.5*(T2-T1)).*normcdf(ds1)-exp(-dD*(T2-T1)).*R2...
-exp(-dD*0.5*(T2-T1)).*normcdf(ds2))-f,0);
vpm=max(pm*(1-alpha)*(PT1*exp(-dV*(T2-T1)).*R1+PT1*...
exp(-dV*0.5*(T2-T1)).*normcdf(ds1)-exp(-dD*(T2-T1)).*R2...
-exp(-dD*0.5*(T2-T1)).*normcdf(ds2))-f,0);
PCAEOpp=D0*exp(-dD*T1)*mean(vpp)
PCAEOpm=D0*exp (-dD*T1)*mean(vpm)
%Payoff of CEEO with positive and negative information revelation;
zpp=max(pp*(1-alpha)*(PT1*exp(-dV*(T2-T1)).*normcdf (d1)...
-exp(-dD*(T2-T1)).*normcdf(d2))-f,0);
zpm=max(pm*(1-alpha)*(PT1*exp(-dV*(T2-T1)).*normcdf(d1)...
-exp(-dD*(T2-T1)).*normcdf(d2))-f,0);
CEEOpp=D0*exp(-dD*T1)*mean(zpp)
CEEOpm=D0*exp (-dD*T1)*mean (zpm)
%Follower's payoff
FOLLpp=PCAEOpp+(PCAEOpp-CEEOpp)/3
FOLLpm=PCAEOpm+(PCAEOpm-CEEOpm)/3
FOLLOWER=pL*FOLLpp+(1-pL)*FOLLpm
```


## References

Armada, M.R., Kryzanowsky, L. \& Pereira, P.J., (2007). A Modified Finite-Lived American Exchange Option Methodology Applied to Real Options Valuation, Global Finance Journal, Vol. 17, Issue 3, 419-438.

Carr, P. (1988).The Valuation of Sequential Exchange Opportunities, The Journal of Finance, Vol. 43, Issue 5, 1235-1256.

Carr, P. (1995). The Valuation of American Exchange Options with Application to Real Options, in: Real Options in Capital Investment: Models, Strategies and Applications ed. by Lenos Trigeorgis, Westport Connecticut, London, Praeger.

Copeland, T. and V. Antikarov (2003). Real Options: a practitioner's guide, New York, Texere.

Cortelezzi, F. and G. Villani (2008). Strategic Technology Adoption and Market Dynamics as Option Games, The Icfai Journal of Industrail Economics, Vol. 5, Issue 4, 7-27.

Cortelezzi, F. and G. Villani (2009). Valuation of R\&D Sequential Exchange Options using Monte Carlo approach, Computational Economics, Vol. 33 Issue 3, 209-236.

Dias, M.A.G. (2004). Valuation of exploration and production assets: an overview of real options models, Journal of Petroleum Science and Engineering, Vol. 44, 93114.

Dias, M.A.G. and J.P. Teixeira (2004). Continuous-time option games part 2: oligopoly, war of attrition and bargaining under uncertainty. Working Paper, PUCRio, presented at 8th Annual International Conference on Real Options, Montreal, June 2004.

Geske, R. (1979). The Valuation of Compound Options, Journal of Financial Economics, Vol. 7, 63-81.

Lee, M.H. (1997). Valuing Finite-Maturity Investment-Timing Options, Financial Management, Vol. 26, Issue 2, 58-66.

Majd, S. and R.S. Pindyck (1987). Time to Build, Option Value and Investment Decisions, Journal of Financial Economics, Vol. 18, Issue 1, 7-27.

Margrabe, W. (1978). The Value of an Exchange Option to Exchange One Asset for Another, The Journal of Finance, Vol. 33, Issue 1, 177-186.

McDonald, R.L. and D.R. Siegel (1985). Investment and the Valuation of Firms When There is an Option to Shut Down, International Economic Review, Vol. 28, Issue 2, 331-349.

Trigeorgis, L. (1991), Anticipated competitive entry and early preemptive investment in deferrable projects, Journal of Economics and Business, Vol. 43, Issue 2, 143-156.

Villani, G. (2008). An RGD Investment Game under uncertainty in Real Option Analysis, Computational Economics, Vol. 32, Issue 2-3, 199-219.

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