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CORPORATE TAX ASYMMETRIES UNDER INVESTMENT IRREVERSIBILITY

Paolo M. Panteghini*

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CESifo
Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224-1410 - Fax: +49 (89) 9224-1409
e-mail: office@CESifo.de
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Abstract

This article studies the effects of corporate tax asymmetries on irreversible investment. We discuss an asymmetric tax scheme where the tax base is given by the firm's return, net of an imputation rate. When the firm's return is less than this rate, however, no tax refunds are allowed. Contrary to common winsdom, this asymmetric scheme may be neutral even when assuming a long-lasting income uncertainty. Neutrality holds even if we add both capital and political uncertainty.

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Paolo M. Panteghini
Università di Scienze Economiche
Via San Faustino 74B
25122 Brescia
Italy
Panteghi@eco.unibs.it

1. Introduction

Most of the tax systems analysed in the existing literature were originally designed for fully reversible investment, and required the symmetric treatment of profits and losses. Instead, investment is at least partially irreversible[1], and the firm can decide when to undertake it.

Under irreversibility, Bernanke (1983) shows that investment decisions are affected only by bad news. Following this result, Panteghini (2001) proves that it is possible to design a tax system, where tax asymmetries offset the asymmetric effects of uncertainty. However, that paper suffers of some limitations, in that uncertainty is assumed to vanish after one period, capital depreciation is disregarded, and the firm's shareholders are risk-neutral.

In this paper we want to enrich the model in these directions. In particular, we assume that income uncertainty lasts to infinity. Moreover, we assume that capital depreciates randomly and that the representative firm may be owned by risk-averse shareholders. Finally, following some influential articles (see e.g. Cummins et. al. (1996) and Hassett et al. (1994)), we design policy uncertainty as a Poisson process, so that future tax rates are neither known nor certain. Despite the above generalisations, the asymmetric design remains neutral.

This article is structured as follows. Section 2 introduces a simple continuous-time model with income uncertainty. Section 3 introduces the asymmetric tax design and derives the basic neutrality result. In sections 4 both policy and capital uncertainty are introduced. Neutrality is shown to hold even with these extensions. Section 5 presents a numerical example and section 6 summarises the results.

2. The model

In this section we introduce a continuous-time model describing the behaviour of a representative firm. The following hypotheses hold[2]:

i) risk is fully diversifiable;

ii) the risk-free interest rate r is fixed;

iii) there exists an irreversible investment I, undertaken by the firm when it is willing to start production;

iv) current gross profits follow a geometric Brownian motion

$$d\Pi(t) = \alpha\Pi(t)dt + \sigma\Pi(t)dz$$

where α and σ are the growth rate and variance parameter, respectively;

[1] Irreversibility may be caused either by 'lemon effects', or by capital specificity. Even when brand-new capital can be employed in different productions, it may become specific once installed. Irreversibility may be caused by industry comovement as well: when a firm can resell its capital, but the potential buyers operating in the same industry are subject to the same market conditions, this comovement obliges the firm to resort to outsiders. Due to reconversion costs, however, the firm can sell the capital at a considerably low price than an insider would be willing to pay if it did not face the same bad conditions as the seller (see Abel et al., 1996, and Guiso and Parigi, 1999).

[2] The reader may find further details of the model in Dixit and Pindyck (1994, Ch. 2).

v) the firm is risk-neutral, but its owners may be risk-averse.

Assumption v), introduced by McDonald and Siegel (1985, 1986), deserves some comments. Following the contingent claims approach, these authors assume that the firm's option to delay irreversible investment is owned by well-diversified investors. To explain this point, let us define μ as the total expected rate of return. According to the Intertemporal Capital Asset Pricing Model, the rate μ must satisfy the equality $\mu = r + \lambda\sigma\rho_M$, where $\lambda \equiv (\mu_M - r)/\sigma_M$ is the market price of risk, with μ_M, σ_M^2 and ρ_M representing the expected return, the variance of the market portfolio and the correlation coefficient between the rate of return on the asset and that on the portfolio, respectively. The term $\lambda\sigma\rho_M$ is therefore an ad hoc compensation required by risk-averse shareholders.

Next compute the difference $\delta = \mu - \alpha$. As explained by Dixit and Pindyck (1994, Ch. 4 and 5) δ can be considered as an explicit or implicit dividend rate[3]. Using the above equalities, we can now write $\mu = \delta + \alpha = r + \lambda\sigma\rho_M$. Rearranging this relation the equality $r - \delta = \alpha - \lambda\sigma\rho_M$ is obtained. If the shareholders are risk-neutral, the term $\lambda\sigma\rho_M$ is omitted so that $r - \delta = \alpha$ holds. Therefore, the firm's problem under risk aversion is equivalent to the one under risk neutrality, except that $r - \delta = \alpha - \lambda\sigma\rho_M$ instead of $r - \delta = \alpha$. In other words, the risk-adjusted drift $\alpha - \lambda\sigma\rho_M$ allows the valuation of the firm as if it were risk neutral[4].

Given the above assumptions, the firm's problem is one of choosing the optimal timing of irreversible investment. To solve this problem both the Value Matching Condition (VMC) and the Smooth Pasting Condition (SPC) are necessary. The former requires the equality between the present value of the project (net of the investment cost), $[V(\Pi) - I]$, and the value of the option to delay investment, $O(\Pi)$:

$$V(\Pi) - I = O(\Pi). \qquad \text{(VMC)}$$

The latter condition requires the equality between the slopes of $[V(\Pi) - I]$ and $O(\Pi)$

$$\frac{\partial}{\partial \Pi}[V(\Pi) - I] = \frac{\partial O(\Pi)}{\partial \Pi}. \qquad \text{(SPC)}$$

Conditions (VMC) and (SPC) thus yield the trigger point Π^* above which investment is immediately profitable.

3. The asymmetric tax system

Let us now introduce the asymmetric tax system discussed in Panteghini (2001). This tax design is based on an imputation method[5]. As in Garnaut and Ross (1975), the tax base is given by the firm's return, net of an imputation rate, r_E. Contrary to Garnaut and Ross' proposal, however, when the firm's return is less than the imputation rate,

[3]Note that δ must be positive in order for the net value of the firm to be bounded.

[4]For further details see also Merton (1990, Ch. 15).

[5]It is worth noting that, in the Nineties, tax systems based on the imputation method were introduced in the Nordic countries (see Sϕrensen, 1998), in Croatia (see Rose and Wiswesser, 1998), and in Italy (see Bordignon et al., 1999 and 2001). Recently, an imputation tax design was also proposed for Germany (see Fehr and Wiegard, 2001).

no tax refunds are allowed. Given the current tax burden, i.e. $\tau \max\left[\Pi(t) - r_E I, 0\right]$, net instantaneous profits (or losses) are equal to

$$\Pi^N(t) = \Pi(t) - \tau \max\left[\Pi(t) - r_E I, 0\right]$$

Let us then write the firm's value as a Bellman function

$$V(\Pi(t)) = \Pi^N(t)dt + e^{-rdt} E\left[V(\Pi(t) + d\Pi(t))\right]. \tag{3.1}$$

Hereafter, we will omit the time variable t. As shown in the Appendix, the value of the investment project is

$$V(\Pi) = \begin{cases} \frac{\Pi}{\delta} + A_1 \Pi^{\beta_1}, & \text{if } \Pi < r_E I \\ (1-\tau)\frac{\Pi}{\delta} + \tau \frac{r_E}{r} I + B_2 \Pi^{\beta_2}, & \text{if } \Pi > r_E I \end{cases} \tag{3.2}$$

where $\beta_1 > 0$ and $\beta_2 < 0$. It is worth noting that both terms $A_1\Pi^{\beta_1}$ and $B_2\Pi^{\beta_2}$ depend on the tax rate and are negative (see the Appendix). The former represents the present discounted value of future tax payments if current profits are less than $r_E I$. The latter measures the present discounted value of the loss due to the lack of refundability, if current profits are greater than $r_E I$.

As shown by Dixit and Pindyck (1994), the option function has the following form

$$O(\Pi) = H\Pi^{\beta_1} \tag{3.3}$$

where H is an unknown parameter (see the Appendix). The following proposition can easily be proven.

Proposition 1-*Under the assumption that current gross profits follow a geometric Brownian motion, if the imputation rate is high enough, i.e.* $r_E \geq r_E^* \equiv \frac{\beta_1}{\beta_1 - 1} \cdot \delta$, *the asymmetric tax regime is neutral, namely the trigger point*

$$\Pi^* \equiv \frac{\beta_1}{\beta_1 - 1} \cdot \delta I \tag{3.4}$$

is unaffected by the taxation.
Proof - See the Appendix.
Proposition 1 shows that investment timing is not distorted by taxation. This neutrality result can be explained as follows. The asymmetric tax device entails the elimination of a tax benefit (i.e. the loss-offset arrangement). In order for neutrality to hold, therefore, the loss of this tax benefit must be compensated with a new relief, namely by an increase in the imputation rate. If this rate is high enough (i.e. $r_E \geq r_E^*$), the firm enjoys a sufficiently long tax holiday, and is able to offset the lack of symmetry.

Notice that an increase in the tax rate reduces the present value of future discounted profits and induces the firm to delay investment. However, the decrease in $V(\Pi)$ is offset by a decrease in the option value, which, instead, stimulates investment. Proposition 1 thus shows that these effects neutralise each other. Namely, the difference $V(\Pi) - O(\Pi)$ is unaffected by changes in τ, and neutrality holds[6].

[6]For a survey on the effects of corporate taxation on irreversible investment see Niemann (1999).

Following Domar and Musgrave (1944) and van Wijnbergen and Estache (1999), we can also argue that the corporate tax is equivalent to *equity participation.* Under tax asymmetries, the government is also endowed with a *put option with zero strike price* written on the firm's profits. Namely, if the firm's return is less than $r_E I$, the government acts as if it sold its equity participation at zero price, thereby sharing no losses. The government's participation will then be rebought (at zero price) when the firm's profits exceed $r_E I$.

Asymmetric tax devices have been considered as sources of distortion, e.g. by Ball and Bowers (1983) and Auerbach (1986). However, these authors implicitly assumed fully reversibility. Under this unrealistic assumption, neutrality could be achieved only for firm-specific values of r_E, depending on the riskiness of each firm. For this reason, any asymmetric tax device was considered as informationally too demanding. Under irreversibility, instead, an entire region of neutral imputation rates, i.e. $r_E \in [r_E^*, \infty)$, can be derived[7]. In the $r_E \in [r_E^*, \infty)$ region, the effects of an increase in the imputation rate are twofold. On the one hand, the government's equity participation (i.e. the expected tax burden) decreases. On the other hand, the value of the government's put option increases, namely, the non-refundability arrangement is more valuable. These two effects neutralise each other. If the imputation rate is high enough, therefore, the government does not need to compute ad hoc imputation rates.

4. Policy and capital uncertainty

In this section we extend the neutrality result by introducing both policy[8] and capital uncertainty. As we know, policy uncertainty may result in a time inconsistency[9]. The government may either announce a tax rate change, which is not implemented after, or undertake an unexpected tax change. In either case, firms would learn that the government may undertake actions different from those initially planned and try to anticipate its choices. As will be shown, the inconsistency problem vanishes if the asymmetric tax device is employed.

To make the model more realistic we also introduce capital uncertainty. In particular, we assume that investment's lifetime is random. Moreover, we assume that when the investment project expires, the firm gets a non-depreciable option to restart. In this case, immediate restarting may not be profitable. Rather, the firm may find it profitable to wait until Π will rise. With such an option, therefore, firm regains a

[7] Note that $r_E^* > r$ always holds. Under the non-refundability system, therefore, the differential $r_E^* - r$ is sufficient to neutralise the effects of the asymmetric treatment of profits and losses.

[8] As argued by Sandmo (1979, p.176), "academic discussions of tax reform in a world of unchanging tax rates is something of a contradiction in terms". As shown in Panteghini (2001), under this asymmetric regime, the firm's investment choice is affected neither by current nor by uncertain future taxation. In line with Sandmo's assessment, therefore, neutrality holds from a dynamic point of view. Forfurther details on policy uncertainty, see Böhm and Funke (2000).

[9] Mintz (1995, p. 61) argues that "When capital is sunk, governments may have the irresistible urge to tax such a capital at a high rate in the future. This endogeneity of government decisions results in a problem of *time consistency* in tax policy whereby governments may wish to take actions in the future that would be different from what would be originally planned...".

limited degree of reversibility.

Both policy and capital uncertainty are modelled as Poisson processes.

Proposition 2 - *Assume that:*

i) the lifetime of investment follows a Poisson process, namely at any time t there is a probability $\lambda_1 dt$ that the existing project dies during the short internal dt;

ii) if the project dies, the firm gets the original opportunity to invest back again (see Dixit and Pindyck (1994), p.210);

iii) the tax rate τ follows a Poisson process

$$d\tau = \begin{cases} 0 & \text{w.p.} \quad 1 - \lambda_2 dt, \\ \Delta\tau & \text{w.p.} \quad \lambda_2 dt, \end{cases}$$

where $\Delta\tau = \tau_{new} - \tau_{old}$ (irrespective of the sign of the differential $\tau_{new} - \tau_{old}$). If the imputation rate is high enough, i.e. $r_E > r_E^{*'} \equiv \frac{\beta_1(\lambda_1)}{\beta_1(\lambda_1)-1}(\delta + \lambda_1)$, the trigger point

$$\Pi^{*'} = \frac{\beta_1(\lambda_1)}{\beta_1(\lambda_1) - 1}(\delta + \lambda_1)I \tag{4.1}$$

is unaffected by taxation.

Proof- See the Appendix.

Proposition 2 represents an extension of Proposition 1, and is interesting in at least three respects. First, a comparison between (3.4) and (4.1) shows that the trigger point is affected only by capital uncertainty, whereas policy uncertainty does not matter[10]. If the imputation rate is high enough, the firm investing immediately will not pay any tax (because of the tax holiday); nor will it benefit from any tax refund (because of the elimination of tax refundability). Like the firm postponing investment, it will take into account only future taxes. Irrespective of whether the firm invests immediately or waits, therefore, it will face the same expected tax burden. This implies that uncertain taxation does not affect the firm's propensity to invest.

The second interesting aspect of Proposition 2 regards the imputation rate. It is shown that policy uncertainty does not affect $r_E^{*'}$. This implies that the amount of information required to compute $r_E^{*'}$does not change.

The third aspect regards time consistency. Since policy uncertainty affects neither the trigger point nor the minimum imputation rate, the asymmetric design is equivalent to pre-committing by the government. Hence, time inconsistency cannot take place.

5. A numerical example

Let us next propose a numerical example based on long-term data. As will be shown, the decision to employ an asymmetric tax device could be implemented on the basis of

[10] The net effect of depreciation on the trigger point is ambiguous. To show this, let us compare (3.4) and (4.1). On the one hand, depreciation entails partial reversibility, which, in turn requires a lower multiple, so that $\frac{\beta_1}{\beta_1 - 1} > \frac{\beta_1(\lambda_1)}{\beta_1(\lambda_1)-1}$. On the other hand, depreciation requires a higher per-period rate of return, namely $\delta + \lambda_1$ instead of δ. For further details see Dixit and Pindyck (1996, p. 200-207).

realistic values of r_E^*. By realistic values we mean values able to gather tax revenues and reflecting the long-term (and, thus, statistically significant) performances of stock markets.

In this example we use Jorion and Goetzman's (1999) estimates of five stock markets: Denmark (1923-95), Sweden (1926-95), Switzerland (1921-95), the UK (1921-95) and the USA (1921-95). In particular, we use country-specific estimates regarding the total expected rate of return μ, the dividend rate δ, and the standard deviation σ. Moreover, we need Homer and Sylla's (1991) data on the long-term government bond rates of interest, as a proxy of r. These data cover the 1920-89 period, with the exception of Denmark (1930-89).

For simplicity, we assume that capital does not depreciate and that shareholders are not risk-averse. Following Proposition 1, we compute the country-specific imputation rate r_E^*. Then, we compare it with μ and r.

Country	μ	δ	r_E^*	$r_E^* - \mu$	$r_E^* - r$	standard error
Denmark	4.88	4.24	9.41	4.53	1.51	1.51
Sweden	7.13	3.83	8.34	1.21	2.56	1.99
Switzerland	5.57	3.45	6.49	0.92	2.34	1.71
UK	8.16	5.17	8.98	0.82	2.70	1.75
USA	8.22	4.84	8.27	0.05	3.42	1.95

Table 1- A comparison of Stock Market performances and the minimum opportunity cost (in %). Sources: Jorion and Goetzman (1999) (for Stock Markets data) and Homer and Sylla (1991) (for r).

As shown in Table 1, at least three interesting results can be found. First, despite the relatively high stock volatility over the past century (the standard deviation ranges from 12.88% in Denmark to 16.85% in the USA), the range of r_E^* is fairly narrow (from 6.49% for Switzerland to 9.41% for Denmark).

Second, the difference $r_E^* - \mu$ is low, except for Denmark (453 basis points). In this country, however, the stock market registered quite poor performances. In Sweden, Switzerland and the UK, the same difference is about one hundred basis points (121, 92 and 82 respectively). Finally, the USA show a difference of just 5 basis points: in terms of statistical significance this difference is null, and the neutral imputation rate is equal to the average rate of return[11].

The third result regards the difference $r_E^* - r$, which measures the ad hoc additional benefit able to neutralise the non-refundability asymmetry. Unlike the second result, the Danish parameter is not an outlier, as it would require only an additional relief of 151 basis points. In the USA, the differential $r_E^* - r$ is relatively higher. As we have seen, however, rate r_E^* is almost equal to the average rate of stock return and, thus, its value looks realistic.

To sum up, the gap between r_E^* and, respectively, μ and r is small. Therefore, the asymmetric system looks implementable.

[11] It is easy to ascertain that the lower the difference $r_E^* - \mu$ the greater is the present discounted value of tax revenues, for a given τ.

6. Conclusion

In this article, an asymmetric tax system has been discussed. Since neither the minimum imputation rate nor the trigger point are affected by the tax rate, neutrality holds.

The system discussed is neutral from a dynamic point of view as well. We have shown that the effects of future uncertain taxation on both the project and the option value neutralise each other. Under this regime, therefore, time consistency problems are not present and the government may benefit from a higher degree of freedom.

The model presented describes an once-and-for-all investment decision. The study of incremental investment is left to future research.

7. Appendix

7.1. Computation of the project's function

Expanding the right-hand side of (3.1) and using Itô's lemma yields

$$rV(\Pi) = \Pi^N + (r-\delta)\Pi V_\Pi + \frac{\sigma^2}{2}\Pi^2 V_{\Pi\Pi}.$$

If no financial bubbles exist and the condition $V(0) = 0$ holds, it is straightforward to obtain

$$V(\Pi) = \begin{cases} \frac{\Pi}{\delta} + A_1\Pi^{\beta_1}, & \text{if } \Pi < r_E I \\ (1-\tau)\frac{\Pi}{\delta} + \tau\frac{r_E}{r}I + B_2\Pi^{\beta_2}, & \text{if } \Pi > r_E I \end{cases} \tag{7.1}$$

where β_1 and β_2 are, respectively, the positive and negative roots of the characteristic equation $\frac{\sigma^2}{2}\beta(\beta-1)+(r-\delta)\beta-r=0$. Substituting (7.1) into the (VMC) and (SPC) we make the two branches of this function meet tangentially at point $\Pi = r_E I$, and find

$$\begin{aligned} A_1 &= -\tau \cdot \frac{r-\beta_2(r-\delta)}{(\beta_1-\beta_2)r\delta} \cdot (r_E I)^{1-\beta_1} < 0, \\ B_2 &= -\tau \cdot \frac{r-\beta_1(r-\delta)}{(\beta_1-\beta_2)r\delta} \cdot (r_E I)^{1-\beta_2} < 0. \end{aligned}$$

7.2. Proof of Proposition 1

Let us set the imputation rate high enough, in order for inequality $\Pi < r_E I$ to hold. Given this inequality a tax holiday is assured to the investing firm. Next, focus on the first branch of function (7.1). Substituting (7.1) and (3.3) into (VMC) and (SPC) we obtain a two-equation system

$$\begin{aligned} \tfrac{\Pi}{\delta} + A_1\Pi^{\beta_1} - I &= H\Pi^{\beta_1}, \\ \tfrac{1}{\delta} + \beta_1 A_1 \Pi^{\beta_1-1} &= \beta_1 H \Pi^{\beta_1-1}. \end{aligned}$$

Solving the above system yields the trigger point (3.4) and the coefficient

$$H = A_1 + (\frac{\Pi^*}{\delta} - I)\cdot \Pi^{*-\beta_1}. \tag{7.2}$$

As can be seen, Π^* is unaffected by taxation. Moreover, using equations (3.3), (7.1), and (7.2) it is straightforward to show that the difference $[V(\Pi) - O(\Pi)]$ is unaffected by the tax rate.

Now, let us compute the minimum imputation rate ensuring neutrality. Rewrite condition $\Pi < r_E I$ as:

$$r_E > \frac{\Pi}{I} \tag{7.3}$$

Substituting (3.4) into condition (7.3) yields the minimum imputation rate ensuring neutrality $r_E^* \equiv \frac{\beta_1}{\beta_1 - 1} \cdot \delta$. Inequality $r_E \geq r_E^*$ represents a sufficient neutrality condition. Proposition 1 is thus proven.■

7.3. Proof of Proposition 2

Let us define $O_0(\Pi)$ and $O_1(\Pi)$ as the option functions before and after the reform, respectively. Similarly, $V_0(\Pi)$ and $V_1(\Pi)$ are the pre- and post-reform value functions. Finally, $\Pi^{*'}$ and $\Pi^{*''}$ are the trigger points under the pre- and post-regime. These trigger points will be computed later.

Let us start with the option function. Using dynamic programming we have[12]

$$O_1(\Pi) = e^{-rdt} \left\{ \xi \left[O_1(\Pi + d\Pi) \right] \right\}, \tag{7.4}$$

and

$$O_0(\Pi) = e^{-rdt} \left\{ \lambda_2 dt \xi \left[O_1(\Pi + d\Pi) \right] + (1 - \lambda_2 dt) \xi \left[O_0(\Pi + d\Pi) \right] \right\}, \tag{7.5}$$

respectively. Expand the RHS of (7.4). Using Itô's Lemma, eliminating all the terms multiplied by $(dt)^2$ and dividing by dt, one obtains

$$rO_1(\Pi) = (r - \delta)\Pi O_{1_\Pi} + \frac{\sigma^2}{2}\Pi^2 O_{1_{\Pi\Pi}} \tag{7.6}$$

The solution of $O_1(\Pi)$ has the standard form

$$O_1(\Pi) = G\Pi^{\beta_1}, \tag{7.7}$$

where parameter G is unknown. Expand now the RHS of (7.5). Using Itô's Lemma we have

$$(r + \lambda_2)O_0(\Pi) = (r - \delta)\Pi O_{0_\Pi} + \frac{\sigma^2}{2}\Pi^2 O_{0_{\Pi\Pi}} + \lambda_2 O_1(\Pi). \tag{7.8}$$

Next, define the difference $O_s(\Pi) \equiv O_0(\Pi) - O_1(\Pi)$, which measures the effect of policy uncertainty on the firm's option value. Subtracting equation (7.6) from (7.8) one obtains

$$(r + \lambda_2)O_s(\Pi) = (r - \delta)\Pi O_{s_\Pi} + \frac{\sigma^2}{2}\Pi^2 O_{s_{\Pi\Pi}}$$

which has the standard solution

$$O_s(\Pi) = G_s \Pi^{\beta_1(\lambda_2)}, \tag{7.9}$$

[12] For further details on the mathematical steps, see Dixit and Pindyck (1994, pp. 200-207).

and where G_s is unknown[13]. Using (7.7) and (7.9) we can thus compute

$$O_0(\Pi) = O_s(\Pi) + O_1(\Pi) = G_s\Pi^{\beta_1(\lambda_2)} + G\Pi^{\beta_1}. \tag{7.10}$$

Let us now turn to the value function. Start with the post-reform function. Following Proposition 1, set a sufficiently high imputation rate, namely $r_E > \frac{\Pi^{*'}}{I}$, where $\Pi^{*'}$ is the trigger point to be determined. Thus the value function consists of three branches. If the project dies over the range $\Pi < \Pi^{*'}$, the firm regains the right to re-undertake investment, but it waits. Over the range $r_E I > \Pi > \Pi^{*'}$, immediate investment is undertaken, and the firm enjoys a tax holiday. Finally, when $\Pi > r_E I$ immediate investment is profitable and taxes are paid since the beginning. Thus, we have:

$$V_1(\Pi) = \begin{cases} \Pi dt + e^{-rdt}\left\{(1-\lambda_1 dt)\xi\left[V_1(\Pi + d\Pi)\right] + \lambda_1 dt \xi\left[O_1(\Pi + d\Pi)\right]\right\} & if\ \Pi \in [0, \Pi^{*'}) \\ \Pi dt + e^{-rdt}\left\{(1-\lambda_1 dt)\xi\left[V_1(\Pi + d\Pi)\right] + \lambda_1 dt \xi\left[V_1(\Pi + d\Pi) - I\right]\right\} & if\ \Pi \in (\Pi^{*'}, r_E I) \\ \Pi^N dt + e^{-rdt}\left\{(1-\lambda_1 dt)\xi\left[V_1(\Pi + d\Pi)\right] + \lambda_1 dt \xi\left[V_1(\Pi + d\Pi) - I\right]\right\} & if\ \Pi \in (r_E I, \infty) \end{cases} \tag{7.11}$$

The computation of the trigger point $\Pi^{*'}$ is less complex than one would think. Following Dixit and Pindyck (1994, pp. 203-204), we know that the first and second branch of function (7.11) meet tangentially at point $\Pi^{*'}$. Analogously, the second and third branch will meet tangentially at point $\Pi = r_E I > \Pi^{*'}$. However, this latter part of the problem is not relevant for our purposes, since in the $(r_E I, \infty)$ region investment has already been made. Thus, let us focus on the first two branches. To compute $\Pi^{*'}$ we could use either the first or the second branch of function (7.11). Since the first one yields an easier solution we use it. Let us expand its RHS. Using Itô's Lemma yields

$$(r + \lambda_1)V_1(\Pi) = \Pi + (r - \delta)\Pi V_{1_\Pi} + \frac{\sigma^2}{2}\Pi^2 V_{1_{\Pi\Pi}} + \lambda_1 O_1(\Pi). \tag{7.12}$$

Next, define $X_z(\Pi) \equiv V_1(\Pi) - O_1(\Pi)$. Using equations (7.11) and (7.6) one obtains

$$(r + \lambda_1)X_z(\Pi) = \Pi + (r - \delta)\Pi_{z_\Pi} + \frac{\sigma^2}{2}\Pi^2 X_{z_{\Pi\Pi}}. \tag{7.13}$$

Since the boundary condition $X_z(0) = 0$ must hold (see Dixit and Pindyck, 1994, p.141), eq. (7.13) has the following solution

$$X_z(\Pi) = \frac{\Pi}{\lambda_1 + \delta} + G_z\Pi^{\beta_1(\lambda_1)}, \tag{7.14}$$

[13] $\beta_1(\lambda_2)$ is the positive root of the characteristic equation $\frac{\sigma^2}{2}\beta(\beta - 1) + (r - \delta)\beta - (r + \lambda_2) = 0$. It is easy to ascertain that $\beta_1(\lambda_2) > \beta_1 > 1$.

where G_z is unknown[14]. Using equations (7.7) and (7.14), we have

$$V_1(\Pi) = \frac{\Pi}{\lambda_1 + \delta} + G_z \Pi^{\beta_1(\lambda_1)} + G\Pi^{\beta_1}. \tag{7.15}$$

Let us turn to the pre-reform value function. Similarly to function (7.11), it consists of three branches which meet tangentially at the common points $\Pi^{**'}$ and $r_E I$:

$$V_0(\Pi) = \begin{cases} \begin{array}{c} \Pi dt + e^{-rdt}(1-\lambda_1 dt)\left\{\lambda_2 dt \xi\left[V_1(\Pi + d\Pi)\right] + \right. \\ \left. +(1-\lambda_2 dt)\xi\left[V_0(\Pi + d\Pi)\right]\right\} + \\ +e^{-rdt}\lambda_1 dt\left\{\lambda_2 dt \xi\left[O_1(\Pi + d\Pi)\right] + (1-\lambda_2 dt)\xi\left[O_0(\Pi + d\Pi)\right]\right\} \end{array} & if\ \Pi \in [0, \Pi^{**'}) \\ \\ \begin{array}{c} \Pi dt + e^{-rdt}(1-\lambda_1 dt)\left\{\lambda_2 dt \xi\left[V_1(\Pi + d\Pi)\right] + \right. \\ \left. +(1-\lambda_2 dt)\xi\left[V_0(\Pi + d\Pi)\right]\right\} + \\ +e^{-rdt}\lambda_1 dt\left\{\lambda_2 dt \xi\left[V_1(\Pi + d\Pi)\right] + (1-\lambda_2 dt)\xi\left[V_0(\Pi + d\Pi)\right] - I\right\} \end{array} & if\ \Pi \in (\Pi^{**'}, r_E I) \\ \\ \begin{array}{c} \Pi^N dt + e^{-rdt}(1-\lambda_1 dt)\lambda\left\{_2 dt \xi\left[V_1(\Pi + d\Pi)\right] + \right. \\ \left. +\ (1-\lambda_2 dt)\xi\left[V_0(\Pi + d\Pi)\right]\right\} + \\ +e^{-rdt}\lambda_1 dt\left\{\lambda_2 dt \xi\left[V_1(\Pi + d\Pi)\right] + (1-\lambda_2 dt)\xi\left[V_0(\Pi + d\Pi)\right] - I\right\} \end{array} & if\ \Pi \in (r_E I, \infty) \end{cases} \tag{7.16}$$

To compute $\Pi^{**'}$, let us expand the RHS of the first branch of (7.16) and use Itô's Lemma. We thus obtain

$$(r + \lambda_1 + \lambda_2)V_0(\Pi) = \Pi + (r-\delta)\Pi V_{0_\Pi} + \frac{\sigma^2}{2}\Pi^2 V_{0_{\Pi\Pi}} + \lambda_2 V_1(\Pi) + \lambda_1 O_0(\Pi). \tag{7.17}$$

Now, define $X_T(\Pi) \equiv [V_0(\Pi) - V_1(\Pi)] - [O_0(\Pi) - O_1(\Pi)]$. Using function $X_T(\Pi)$ and equations (7.6), (7.8), (7.12), and (7.17) it is straightforward to obtain

$$(r + \lambda_1 + \lambda_2)X_T(\Pi) = \Pi + (r-\delta)\Pi X_{T_\Pi}(\Pi) + \frac{\sigma^2}{2}\Pi^2 X_{T_{\Pi\Pi}}(\Pi) \tag{7.18}$$

which has the following solution

$$X_T(\Pi) = G_T \Pi^{\beta_1(\lambda_1 + \lambda_2)}, \tag{7.19}$$

where G_T is unknown[15]. Substituting (7.9), (7.15), and (7.19) into (7.17) we obtain

$$V_0(\Pi) = G_T \Pi^{\beta_1(\lambda_1+\lambda_2)} + \left[\frac{\Pi}{\delta + \lambda_1} + G_z \Pi^{\beta_1(\lambda_1)} + G\Pi^{\beta_1}\right] + G_s \Pi^{\beta_1(\lambda_2)}. \tag{7.20}$$

So far, we have computed the solutions of the option and value functions. Using the (VMC) and (SPC), we can now compute the trigger points. Start with the post-reform regime. Substitute equations (7.7) and (7.15) into the (VMC) and (SPC). A

[14] Note that $\beta_1(\lambda_1)$ is the positive root of the characteristic equation $\frac{\sigma^2}{2}\beta(\beta-1)+(r-\delta)\beta-(r+\lambda_1) = 0$. It is easy to ascertain that $\beta_1(\lambda_1) > \beta_1 > 1$ and that $[\beta_1(\lambda_1) - \beta_1(\lambda_2)] \propto (\lambda_1 - \lambda_2)$.

[15] $\beta_1(\lambda_1 + \lambda_2) > 1$ is the positive root of the characteristic equation $\frac{\sigma^2}{2}\beta(\beta-1) + (r-\delta)\beta - (r + \lambda_1 + \lambda_2) = 0$. Note also that $\beta_1(\lambda_1 + \lambda_2) > \beta(\lambda_i) \quad i = 1, 2$.

two-equation system is obtained

$$\begin{aligned}\frac{\Pi}{\delta+\lambda_1}+G\Pi^{\beta_1}+G_z\Pi^{\beta_1(\lambda_1)}-G\Pi^{\beta_1}-I &= 0,\\ \frac{1}{\delta+\lambda_1}+G\beta_1\Pi^{\beta_1-1}+G_z\beta_1(\lambda_1)\Pi^{\beta_1(\lambda_1)-1}-G\beta_1\Pi^{\beta_1-1} &= 0.\end{aligned}$$

As can be seen, the option value $G\Pi^{\beta_1}$ is embodied in the value function. Thus the value of the unknown parameter G does not affect the difference $[V_1(\Pi)-O_1(\Pi)]$. Solving the above two-equation system yields

$$\begin{aligned}\Pi^{*'} &= \tfrac{\beta_1(\lambda_1)}{\beta_1(\lambda_1)-1}\cdot(\delta+\lambda_1)I,\\ G_z &= -\tfrac{1}{\beta_1(\lambda_1)}\cdot\tfrac{1}{\delta+\lambda_1}\cdot\Pi^{*'\,1-\beta_1(\lambda_1)}<0.\end{aligned}$$

The computation of the trigger point under policy uncertainty, $\Pi^{**'}$, is trickier. Substitute equations (7.10) and (7.20) into the (VMC) and (SPC). After some manipulation, we obtain

$$\frac{\Pi}{\delta+\lambda_1}+G_z\Pi^{\beta_1(\lambda_1)}+G_T\Pi^{\beta_1(\lambda_1+\lambda_2)}-I=0 \tag{7.21}$$

$$\frac{1}{\delta+\lambda_1}+G_z\beta_1(\lambda)\Pi^{\beta_1(\lambda_1)-1}+G_T\beta_1(\lambda_1+\lambda_2)\Pi^{\beta_1(\lambda_1+\lambda_2)-1}=0 \tag{7.22}$$

Multiply equation (7.22) by Π, and divide it by $\beta_1(\lambda_1+\lambda_2)$. Then, substitute it into (7.21). Finally, substituting the solutions of G_z and $\Pi^{*'}$ into (7.21) yields the following non-linear equation

$$\left[1-\frac{1}{\beta_1(\lambda_1+\lambda_2)}\right]\frac{\Pi}{\delta+\lambda_1}-\frac{\beta_1(\lambda_1+\lambda_2)-\beta_1(\lambda_1)}{\beta_1(\lambda_1+\lambda_2)\left[\beta_1(\lambda_1)-1\right]}\cdot I\cdot\left(\frac{\Pi}{\Pi^{*'}}\right)^{\beta_1(\lambda_1)}-I=0.$$

Multiply by $\frac{\beta_1(\lambda_1)-1}{\beta_1(\lambda_1)}\frac{1}{I}$, thereby obtaining

$$\left[\frac{\beta_1(\lambda_1+\lambda_2)-1}{\beta_1(\lambda_1+\lambda_2)}\right]\left(\frac{\Pi}{\Pi^{*'}}\right)-\left[\frac{\beta_1(\lambda_1+\lambda_2)-\beta_1(\lambda_1)}{\beta_1(\lambda_1+\lambda_2)\beta_1(\lambda_1)}\right]\left(\frac{\Pi}{\Pi^{*'}}\right)^{\beta_1(\lambda_1)}-\frac{\beta_1(\lambda_1)-1}{\beta_1(\lambda_1)}=0.$$

Multiply by $\frac{\beta_1(\lambda_1+\lambda_2)}{\beta_1(\lambda_1+\lambda_2)-1}$ so as to obtain

$$\left(\frac{\Pi}{\Pi^{*'}}\right)-\frac{\left[\beta_1(\lambda_1+\lambda_2)-\beta_1(\lambda_1)\right]}{\left[\beta_1(\lambda_1+\lambda_2)-1\right]\beta_1(\lambda_1)}\left(\frac{\Pi}{\Pi^{*'}}\right)^{\beta_1(\lambda_1)}-\frac{\left[\beta_1(\lambda_1)-1\right]\beta_1(\lambda_1+\lambda_2)}{\left[\beta_1(\lambda_1+\lambda_2)-1\right]\beta_1(\lambda_1)}=0.$$

Adding and subtracting 1 from the LHS yields

$$\left[\left(\frac{\Pi}{\Pi^{*'}}\right)-1\right]-\frac{\left[\beta_1(\lambda_1+\lambda_2)-\beta_1(\lambda_1)\right]}{\left[\beta_1(\lambda_1+\lambda_2)-1\right]\beta_1(\lambda_1)}\cdot\left[\left(\frac{\Pi}{\Pi^{*'}}\right)^{\beta_1(\lambda_1)}-1\right]=0. \tag{7.23}$$

Define $x \equiv \left(\frac{\Pi}{\Pi^{*\prime}}\right)$ and $\phi \equiv \left[\frac{\beta_1(\lambda_1+\lambda_2)-\beta_1(\lambda_1)}{\beta_1(\lambda_1)[\beta_1(\lambda_1+\lambda_2)-1]}\right] < 1$. Thus, eq. (7.23) can be rewritten as

$$x - 1 = \phi\left(x^{\beta_1(\lambda_1)} - 1\right). \tag{7.24}$$

Equation (7.24) has more than one solution. Thus, we compute them and identify the optimal one. As can be noted, solution $x = 1$ holds in equation (7.24), namely $\Pi^{**'} = \Pi^{*'}$. Substituting $\Pi^{*'}$ into system (7.21)-(7.22) one thus obtains $G_T = 0$. This is the first couple of solutions of system (7.21)-(7.22).

Define $x\prime$ as any other solution. Given inequalities $\beta_1(\lambda_1) > 1$, $\phi < 1$ and $\beta_1(\lambda_1)\phi < 1$, it is easy to show that any other solution is $x\prime > 1$. This implies that the trigger point obtained would be $\Pi^{**'} > \Pi^{*'}$. Substituting this new solution into system (7.21)-(7.22) yields $G_T > 0$. Thus $\left(\Pi^{**'} > \Pi^{*'}, G_T > 0\right)$ is the second couple of solutions. However this couple is sub-optimal. To show this, assume ab absurdo that $\left(\Pi^{**'} > \Pi^{*'}, G_T > 0\right)$ is the optimal solution. Then, define the pre-reform project's payoff, net of both the opportunity and the effective cost, as

$$F(\Pi) \equiv [V_0(\Pi) - O_0(\Pi) - I] \tag{7.25}$$

Using (VMC) and eq. (7.25) we obtain $F(\Pi^{**'}) = 0$. Recall now the definition of $X_T(\Pi)$ and equation (7.19), and rewrite (7.25) as

$$F(\Pi) = [V_1(\Pi) - O_1(\Pi) - I] + G_T\Pi^{\beta_1(\lambda_1+\lambda_2)}.$$

Since in $\Pi = \Pi^{*'}$ the post-reform project's payoff $[V_1(\Pi) - O_1(\Pi) - I]$ is nil, we know that $F(\Pi^{*'}) = G_T\Pi^{*'\beta_1(\lambda_1+\lambda_2)} > 0$. Namely, in the interval $\Pi \in (0, \Pi^{**'})$, there exists at least one point $(\Pi = \Pi^{*'})$ such that the project's payoff is strictly positive. Thus, a rational firm, facing a positive payoff in $\Pi = \Pi^{*'}$, immediately invests instead of waiting until the trigger point $\Pi^{**'}$ is reached. This contradicts the assessment that $\left(\Pi^{**'} > \Pi^{*'}, G_T > 0\right)$ is the optimal solution. Therefore, the remaining solution $(\Pi^{**'} = \Pi^{*'}, G_T = 0)$ is the optimal one.

Finally, note that the minimum imputation rate ensuring neutrality is not affected by policy uncertainty. Substituting (4.1) into condition (7.3), yields $r_E^{*'} \equiv \frac{\beta_1(\lambda_1)}{\beta_1(\lambda_1)-1}(\delta + \lambda_1)$. Setting an imputation rate $r_E \geq r_E^{*'}$ is a sufficient neutrality condition. Proposition 2 is thus proven.■

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