A Comparison of Parametric and Sampling Approaches to Portfolio Investment Selection using FTSE100 stocks

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ABSTRACT

In this paper we assess the effectiveness of two approaches to portfolio selection: the more customary parametric approach and a sampling approach using a sample of two years of daily data for the top 100 UK stocks for a period from the beginning of 2006 to the end of 2007. The portfolios are selected on a variance, VaR and CVaR basis: with the latter approach dominating. The sampling approach; involving repeated random one-month return sampling from the data set, un-encumbered by distributional assumptions, applying CVaR is optimal; possibly because it considers only one tail of a potentially non-symmetric PDF.

Keywords: Portfolio selection; Variance, VaR; CVaR; Random sampling

1. Introduction

The Selection of the optimal or efficient set of portfolios with a minimum level of risk is a customary investment strategy which is done by minimizing various risk measures from the universe of feasible securities/portfolios at each level of investors' expected return. The technology and methods available has come along way since Markowitz (1952) first suggested this algorithm using mean/variance analysis and founded modern portfolio theory (MPT).

An enduring problem has been the prediction of security and portfolio characteristics. It is clear that the selected portfolios will certainly not be optimal for ever, if at all. This is a particular problem if the selection is done on the basis of historical information. This leads to the problem of estimation risk. (See the discussions in Bawa et al (1979) and Alexander and Resnick (1985).

The durability and degree of ex-post optimality of selected portfolios directly depends on the length of the period for which the probability density function (PDF) of returns/risks/losses used to select the optimal portfolios is forecasted or simulated and the degree of accuracy of these forecasts or simulations.

It is clear that the first and most important step in optimal portfolio selection, if it is to be achieved by the calculation and minimization of risk levels; is the correct specification of an appropriate probability density function (PDF) with a high level of accuracy in terms of explaining the probable loss conditions that are likely to be experienced during the future investment period.

In this paper we compare and evaluate the efficacy of both the parametric and sampling approaches two portfolio selection. These are two customary alternative methods for constructing the loss PDFs for the considered investment horizon. The parametric approach is based on the construction of loss PDFs according to a specific distribution with specific parameters such as the normal distribution. By contrast in the sampling approach, the loss PDFs are consistent with different simulated samples of losses for the investment period without the prior assumption of any pre-determined distribution.

The main purpose of this study is to investigate the effects of choosing these two different alternative approaches to the problem of optimal portfolio selection whilst using alternative portfolio selection criteria. We adopt these two approaches and select optimal portfolios with respect to the respective levels of Variance, Value at Risk (VaR) and Conditional VaR (CVaR) of potential losses in a risk-return framework.

Many prior studies have focused on application of these risk measures to optimal portfolio selection problems. For example see Allen (2005), Alexander et al. (2006), Campbell et al. (2001), Consigly (2002), Duffie and Pan (2001), Fusai and Luciano (2001), Gaivoronski and Pflug (2005), Kluppelberg and Korn (1998), Rockafeller and Uryasev (2000), Rockafeller and Uryasev (2002), Szego (2002), Yiu (2004) and more recently, Fabian and Veszpremi (2008) for a dynamic stochastic programming algorithm.

2. Methodology

Consider an investor who wants to select an optimal portfolio between m ($i = 1,2,...,m$) stocks for investing in time horizon $\overline{T}$. This investor could select different positions for a decision vector $X \in R^s$ such as:

$$X = (X_1, X_2, ..., X_m) \qquad (1)$$

where $X_i \geq 0$ shows that all the positions taken are long. If the initial prices of these stocks are $p = (p_1, p_2, ..., p_m)$ the initial value of selected portfolio is determined by the investor's budget limit:

$$X^T p = \upsilon \qquad (2)$$

The subsequent prices of the selected stocks over the next few days are an unknown quantity for investors. These prices at the end of investment time horizon $\overline{T}$ might be:

$$Y = (Y_1, Y_2, ..., Y_m)$$

Then the investor is confronted by a random price vector of $Y \in R^n$ in his/her optimization. Assuming the rationality of this investor, he/she looks for a portfolio with a low probability level of loss. The amount of loss is a function of both the decision vector and the market price vector:

$$\Lambda = f(X, Y)$$

This function could be shown to be something like:

$$\Lambda = \upsilon^{-1}(\upsilon - X^T Y) = \frac{\upsilon - \sum_{i=1}^{m} X_i Y_i}{\upsilon} \qquad (3)$$

Then, for each specific portfolio $X \in R^s$ the random vector of loss has a distribution function of $F(u)$:

$$F(u) = P\{\Lambda \in R : \Lambda \leq u\} = P\{Y \in R^n : X^T Y \geq \upsilon - u\upsilon\} \qquad (4)$$

The expected level of prices at the end of time horizon $\overline{T}$ determines the expected loss of the selected portfolio:

$$E(\Lambda) = E\left[\upsilon^{-1}(\upsilon - X^T Y)\right] = \upsilon^{-1}(\upsilon - X^T E(Y)) \qquad (5)$$

Selecting the portfolio with the minimum level of expected loss is inefficient in terms of returns foregone. The investor could choose a level of expected loss such as ρ, higher than that minimal level, but one that reduces his/her risk in terms of replacement value, or terminal value of the selected portfolio. If the level of risk in the selected portfolio is $\Re(\Lambda)$ the optimization problem of this investor could be shown to be equivalent to:

$$\begin{aligned} &\underset{X}{Min} \; \Re(\Lambda) \\ &E(\Lambda) \le \rho \\ &X^T p = \upsilon \\ &X \ge 0 \end{aligned} \tag{6}$$

In this paper, we want to choose optimal portfolios drawn from the top 100 stocks in the UK market for an investor who enters the market with a view to achieving portfolio gains over a one month investment horizon. The end of day data for these top 100 stocks was taken from the Datastream database. As at the time of the data query, the last data available from this database was up to the end of 2007, the end of day price data for all these top 100 stocks was downloaded for all the working days of 2006 and 2007. We decided to use the prices data of the 100 stocks in the month of December 2007 for an evaluation of the results in terms of a hold out sample and consequently do not use them in the optimization procedure.

The Variance and CVaR are two risk measures that are used in this paper for selecting the optimal portfolio in a risk-return framework. Also the corresponding VaR values of portfolios are calculated for the selected portfolios.

The Variance of losses ($\nu(\Lambda)$) according to definition is:

$$\Re(\Lambda) = Variance(\Lambda) = \nu(\Lambda) = E(\Lambda - E(\Lambda))^T (\Lambda - E(\Lambda)) \tag{7}$$

The VaR of losses ($\zeta_\beta(\Lambda)$) is defined variously in literature, but this does not affect the outcomes.

$$\Re(\Lambda) = VaR(\Lambda) = \zeta_\beta(\Lambda) \tag{8}$$

VaR could be defined as "*a loss that will not be exceeded at some specified confidence level*"[Hull (2000)]. In the other word, "*the 100 a% h-day VaR is that number x such that the probability of losing x, or more, over the next h days equals 100 a%*"[Alexander (2001)]. But formally $\zeta_\beta(\Lambda)$ is defined as the β percentile of the loss distribution function [Gaivoronski and Pflug (2005)], then $\zeta_\beta(\Lambda)$ is the smallest value such that the probability that loss does not exceed this value is bigger or equal to β [Rockafeller and Uryasev (2000)].

$$\zeta_\beta(\Lambda) = Min\{\zeta \in R : P\{\Lambda \in R : \Lambda \leq \zeta\} \geq \beta\} \qquad (9)$$

CVaR of losses $\omega_\beta(\Lambda)$ is the expectation of losses conditioned on exceeding or being equal to the level $\zeta_\beta(\Lambda)$ [Gaivoronski and Pflug (2005)].

$$\Re(\Lambda) = CVaR(\Lambda) = \omega_\beta(\Lambda)$$

$$\omega_\beta(\Lambda) = \frac{P\{\Lambda \in R : \Lambda \geq \xi_\beta(\Lambda)\}}{1-\beta} E(\Lambda | \Lambda \geq \xi_\beta(\Lambda)) + \left(1 - \frac{P\{\Lambda \in R : \Lambda \geq \xi_\beta(\Lambda)\}}{1-\beta}\right)\xi_\beta(\Lambda)$$

(10)

where, $\omega_\beta(\Lambda) = E(\Lambda | \Lambda \geq \xi_\beta(\Lambda))$ if $P\{\Lambda \in R : \Lambda \geq \xi_\beta(\Lambda)\} = 1-\beta$.

In order to facilitate the calculation and the optimization of these risk measures, in this paper the loss PDFs are constructed using both the parametric and sampling approaches. Once we have obtained the realized average and standard deviation of the prices data, the loss PDF of the selected portfolio with the assumption of a normal distribution is created simply by using the relation below:

$$f(\Lambda) = \frac{\exp\left(-\frac{\left(\Lambda - \overline{\Lambda}_R\right)^2}{2\sigma^2(\Lambda_R)}\right)}{\sigma(\Lambda_R)\sqrt{2\pi}} \qquad (11)$$

where $\overline{\Lambda_R}$ and $\sigma(\Lambda_R)$ are respectively the realized average and standard deviation values of losses of the selected portfolios. It is clear that we should put the corresponding monthly standard deviation value of losses in this relation as in this study the investment period is the next one month investment period.

By contrast, in the sampling approach the simulated samples of losses are calculated using the actual historical end of day prices data without assuming any specific distribution of loss function. As the target for our supposed investor are the gains obtained during the next Δt days (1 month) of investment days, a necessary requirement is the specification of a sample PDF of losses up to that date via simulations of the probable loss conditions. If the historical end of day price of m stocks in time t is $h^t = (h_1^t, h_2^t, ..., h_m^t)$, $r_j = (r_{1j}, r_{2j}, ..., r_{mj})$ is the j^{th} scenario of probable rates of price changes over the next Δt days.

$$r_j = \frac{h^t}{h^{t-\Delta t}}, \qquad (12)$$

$$t = 1 + \Delta t, 2 + \Delta t, ..., T$$

Then, there are $N = T - \Delta t$ scenarios when T end of day historical data are used for simulating the sample PDF of losses.

Utilizing r_j for $j = 1,2,..., N$ in:

$$p^T r_j = y_j$$

We have a sample of $Y \in R^n$ with N members that the j^{th} member is $y_j = (y_{1j}, y_{2j}, ..., y_{mj})$. Also $\overline{y} = (\overline{y_1}, \overline{y_2}, ..., \overline{y_m})$ is the vector of mean values of each stock price in all scenarios:

$$\overline{y_i} = \frac{1}{N}\sum_{j=1}^{N} y_{ij} \qquad (13)$$

Now, there are N members in the sample of $\Lambda \in R$ corresponding y_j, with the j^{th} member of:

$$\Lambda_j = \frac{\upsilon - \sum_{i=1}^{m} X_i y_{ij}}{\upsilon} \qquad (14)$$

Having the obtained sample PDF of losses in both the parametric and sampling approach, it is possible to optimize the portfolio selected using the previously mentioned risk measures and thereby select the optimal portfolio for the supposed investor by calculating the efficient risk-return frontier.

According to the previous definition given (relation (10)), CVaR in a normal distribution of losses can be calculated as shown below [Huang (2000)]:

$$\omega_\beta(\Lambda) = \frac{\exp\left(-\frac{q_{1-\beta}^2}{2}\right)}{(1-\beta)\sqrt{2\pi}}\sigma(\Lambda_R) \qquad (15)$$

where $q_{1-\beta}$ is the tail $100(1-\beta)$ percentile of a standard normal distribution. As the only variable part of relation (14) is $\sigma(\Lambda_R)$, the optimal portfolio with the minimum level of CVaR in parametric approach could be achieved by solving the nonlinear model below:

$$\begin{aligned} \underset{X}{Min} \quad & \nu(\Lambda) = \upsilon^{-2} X^T \left[\frac{1}{T}\sum_{t=1}^{T}(h^t - \bar{h})(h^t - \bar{h})\right] X \\ & \upsilon^{-1}(\upsilon - X^T \bar{h}) \le \rho \qquad (16) \\ & X^T p = \upsilon \\ & X \ge 0 \end{aligned}$$

Having the minimum level of Variance in each optimal portfolio, the corresponding minimal CVaR of losses could be calculated by relation (15). Also the VaR value of each portfolio given a normal assumption of loss distribution could be simply calculated as suggested by [Hull (2000)]:

$$\zeta_{\beta}(\Lambda) = q_{1-\beta}\sigma(\Lambda_R) \qquad (17)$$

In the sampling approach in order to choose the optimal portfolio with a minimal level of Variance, the nonlinear model below is used in the paper:

$$\begin{aligned} &\underset{X}{Min} \quad \nu(\Lambda) = \upsilon^{-2} X^T \left[\frac{1}{N} \sum_{j=1}^{N} (y_j - \overline{y})(y_j - \overline{y}) \right] X \\ &\upsilon^{-1}(\upsilon - X^T \overline{y}) \le \rho \\ &X^T p = \upsilon \\ &X \ge 0 \end{aligned} \qquad (18)$$

It was illustrated in the literature [Rockafeller and Uryasev (2000)] that the minimum level of CVaR is achieved by minimizing the following:

$$Min\,\omega_{\beta}(\Lambda) = Min\left\{\zeta_{\beta}(\Lambda) + (1-\beta)^{-1} E \max\left[\Lambda - \zeta_{\beta}(\Lambda), 0\right]\right\} \qquad (19)$$

Then this linear model can be utilized in order to select the optimal portfolio with a minimum level of CVaR [Gaivoronski and Pflug (2005), Rockafeller and Uryasev (2000) and Rockafeller and Uryasev (2002)]:

$$\begin{aligned} &\underset{X,\zeta,Z}{Min} \quad \omega_{\beta}(\Lambda) = \zeta + \frac{1}{(1-\beta)N} \sum_{j=1}^{N} Z_j \\ &\upsilon^{-1}(\upsilon - X^T y_j) - \zeta - Z_j \le 0 \\ &\upsilon^{-1}(\upsilon - X^T \overline{y}) \le \rho \\ &X^T p = \upsilon \\ &Z_j \ge 0 \\ &X \ge 0 \end{aligned} \qquad (20)$$

Z_j is a auxiliary variable that is used to selecting the $Max\left[\Lambda - \zeta_{\beta}(\Lambda), 0\right]$ in the above model. This is because when we proceed according to definition, $\omega_{\beta}(\Lambda)$ is the expectation of losses conditioned on exceeding or being equal to the level $\zeta_{\beta}(\Lambda)$.

3. Results and Discussions

In order to make a comparison of the optimal portfolios achieved using both the parametric and sampling approaches, outlined in the first part of this study; the optimal portfolios were selected by running the model (16). The minimum amounts of CVaRs of losses and the corresponding VaR values and the actual losses of each portfolio in different scenarios of expected returns are shown in Figure 1. These values up to a level of 100% of the maximum value of expected return (the last point on the right side of horizontal axis) have been calculated by maximizing the expected return of portfolio (minimizing the expected loss) regardless of the risk level of portfolio. As it is clear in this figure the risk level of portfolio (which is calculated by the CVaR and VaR of losses) is decreased by reducing the target expected return for the portfolio. (The customary risk/return trade-off). The model using declining target expected returns was run for about 700 scenarios. So the corresponding model was run about 700 times[1]. As the differences between the CVaR and VaR values in this parametric approach are just related to the coefficients of $\sigma(\Lambda_R)$ term in the relations (15) and (17) the size of the gap between these two risk measures can be seen to decrease exponentially in figure 1. The actual loss line in this figure is the locus of loss amounts corresponding to the selected optimal portfolio at each level of expected return. These are calculated at the end of our supposed one month investment period. Our supposed investment period (using out of sample data) in this study is from the beginning to the end of December 2007.

The optimal selected portfolio according to the amounts of actual loss incurred in the hold out sample is the portfolio which is chosen at a level of 92% of the maximum expected return. The actual loss (or actual return) of this portfolio at the

[1] Using the GAMS software

end of a 1 month investment horizon is -3.3% loss (or a 3.3% gain) of the initial investment value. However, decreasing the expected return by more than 8% of its maximum level leads to an increase in the amount of actual losses. In so much as the selection of portfolios with levels of expected returns between 73% down to 22.5% of the maximum return actually result in positive losses for the investor. (Depicted by the portion of the graph of losses above the horizontal line in Figure 1: the positive losses). The worst portfolio is related to an expected return level of 50.5% of the maximum return with an actual loss rate of 1.7% of the initial investment. (The peak of the actual loss line in Figure 1).

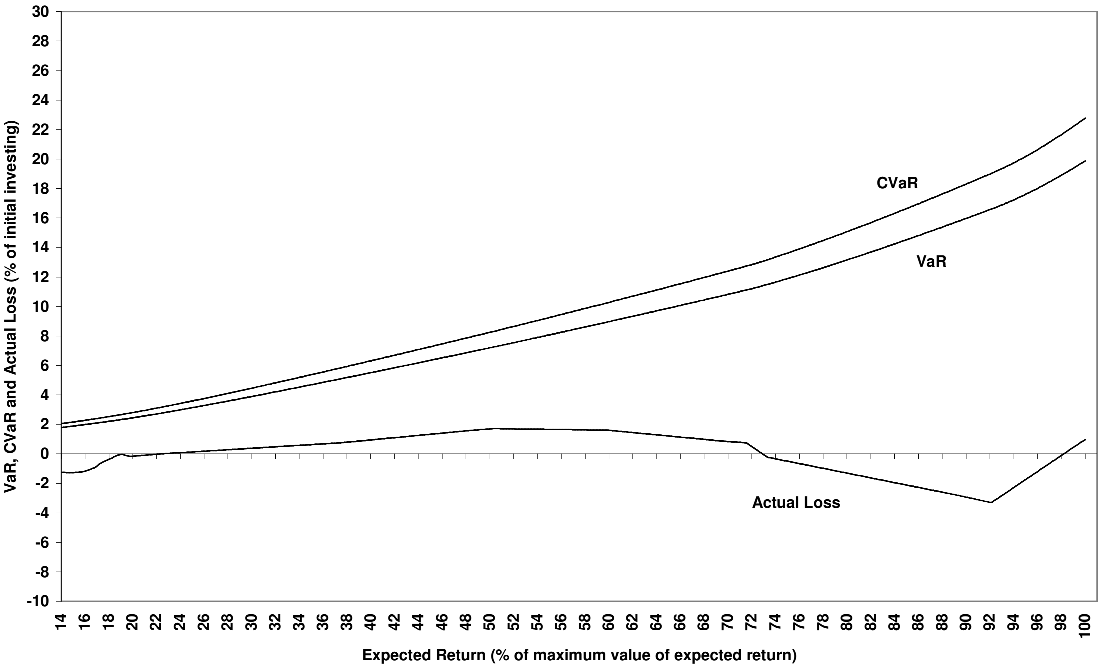

Figure 1. The Risk-Return frontier and actual loss values obtained by minimizing the CVaR of losses on the assumption of Normal distributions, using information and portfolio weights obtained from historical data but applied in the hold-out sample period: December 07. (Note: gains are shown as negative losses).

All of the selected portfolios are shown again in figure 2 which depicts how the optimal portfolio composition changes at different levels of required returns.

According to this figure the riskiest portfolio just contains one stock which is TW. Also the best and the worst portfolios (according to the results of realised actual losses) contain respectively 3 stocks (TW and WOS) and 5 stocks (BGY, ITV, KGF, PSN and SBRY).

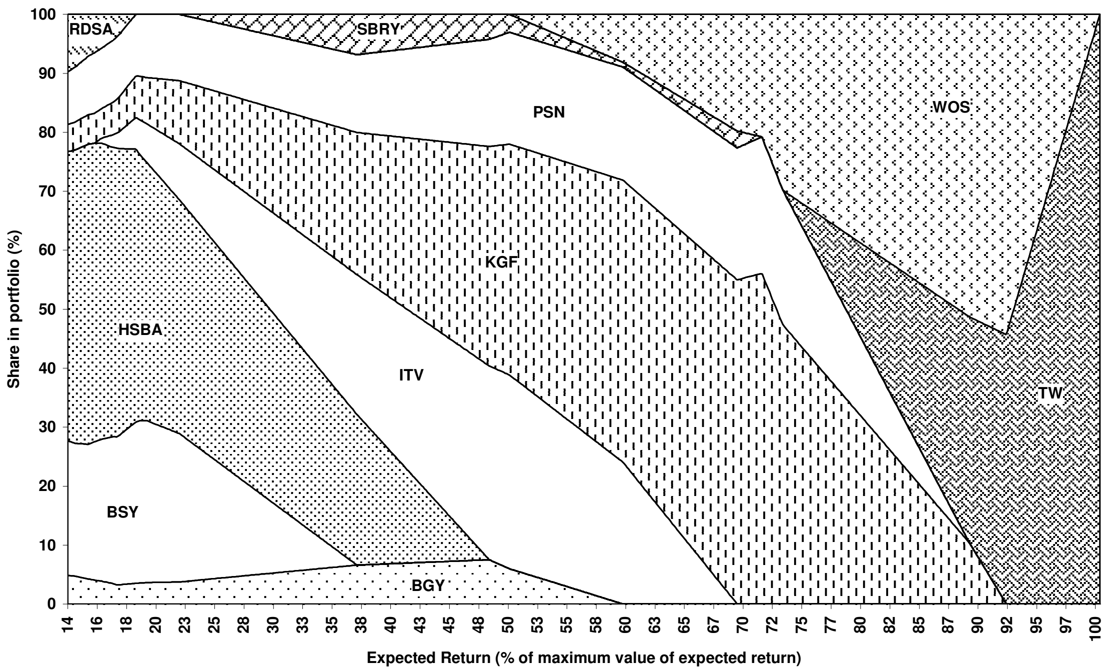

Figure 2. The composition of optimal portfolios at different levels of expected returns achieved by minimizing the CVaR of losses on the assumption of Normal Distributions. using information and portfolio weights obtained from the historical data but applied in the hold-out sample period: December 07.

It is clearly shown in Figure 2 that, as should be expected, diversification decreases the risk of the portfolio. There is just one stock in the portfolio with the highest level of risk whilst there are portfolios with 7 stocks in the lower risk levels.

Given that diversification using the stocks with smaller correlation coefficients has an effect on the reduction of risk in portfolio we have presented information about the stocks which were present in the selected portfolios from correlation point of view in figure 3. We saw in figures 1 and 2 that replacing the company WOS and using it instead of TW not only decreases the risk of the portfolio but also decreases the actual risk of selecting the portfolio. Yet the positively correlation between these two stocks (according to figure 3) shows that this is most likely a result of better subsequent returns on WOS rather than from any risk diversification in the portfolio. If we follow the replacement process in the portfolios that result from decreasing the risk of the portfolio it is clear that in the initial stages of risk reduction what is happening is the replacement by stocks with lower levels of volatility instead of stocks with higher levels of expected return. But as we continue we see the replacement of stocks with both lower and mostly negatively correlated stock returns replacing the old stocks.

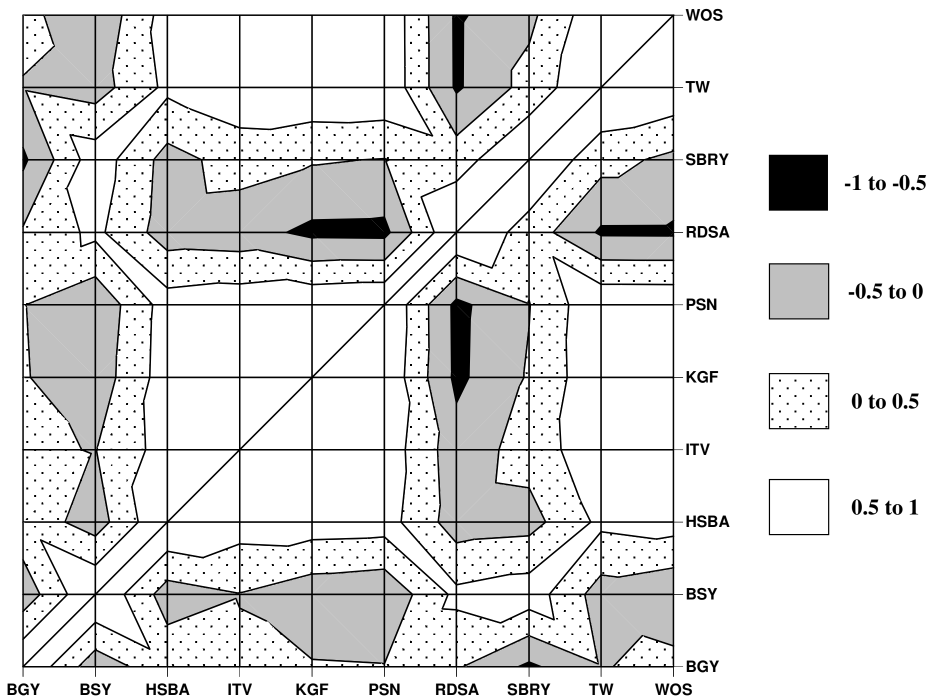

Figure 3. Schematic view of correlation coefficients of the stocks contained in the optimal portfolios achieved by minimizing the CVaR of losses assuming Normal distributions

For example in the left hand side portfolios in figure 2 there are stocks like as BGY, BSY, HSBA and RDSA. According to figure 3, BGY and BSY, BSY and HSBA and also HSBA and RDSA are negatively correlated stocks which are selected as ingredients of the optimal portfolio. Then, it is clear that the selected portfolios are really optimal according to the supposed loss distribution in the parametric approach. However, why are these portfolios not optimal according to the amounts of actual losses obtained which are calculated when we use the out of sample price data? The answer could be the weakness of the assumption of a normally distributed loss function used in forecasting what will happen in future.

To evaluate this issue we compare the above results with the results of optimal portfolio selection using a sampling approach without the prior assumption of any specific distribution of loss function. It is not surprising that the results of the selection of portfolios from the minimization of Variance and CVaR are not same when we use the sampling approach as opposed to the parametric method. We present these results of the sampling approach in separate Figures constructed in an identical fashion to the Figures showing the results of the parametric approach.

Figures 4 and 5 display the amounts of CVaR, VaR, standard deviation and actual losses of the selected portfolios in varfious scenarios of expected return which are achieved in the sampling approach framework by minimizing the CVaR and Variance of expected losses respectively. Although the absolute amounts of the CVaR and VaR of expected losses decrease when we decrease the expected returns of the portfolios the size of gap between them does not decrease in the same proportionate manner as was evident in the parametric approach shown in Figure 4. Indeed the size of the difference between them displays no specific trend over the CVaR and VaR lines as the target return is reduced and remains relatively constant.

When we compare the actual losses shown in Figures 4 and 5 with those shown in Figure 1 it is clear that the selected portfolios obtained using the sampling approach are more efficient than those which are chosen by parametric approach. It is also apparent in the sampling approach that the selection of optimal portfolios obtained by minimizing the CVaR of potential losses provides better results comparing with those obtained by minimizing the Variance. The actual losses of portfolios which are achieved by minimizing the CVaR never have positive values. In effect this means they are all positive gains because the loss line plots below the horizontal axis in Figure 4. Also in the case of the portfolios selected by variance minimization with the exception of a few portfolios with expected returns between 42.5% and 36% -which have smaller losses of less than 0.1%- there are no portfolios with positive amounts of actual loss. Whilst it will be recalled that it was shown in Figure 1 that there were positive amounts of actual losses, especially for those risk averse investors choosing portfolios with expected returns of less than 73% of the maximum return.

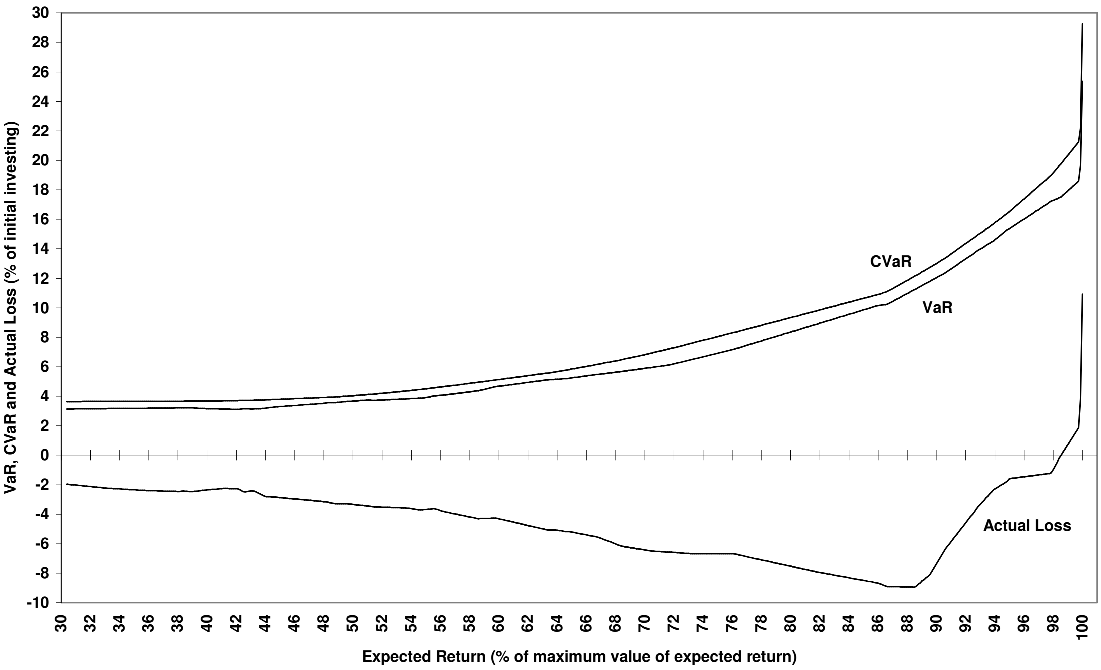

Figure 4. The Risk-return frontier and the actual loss values obtained by minimizing the CVaR of losses using the sampling approach in the hold-out sample period of the month of December 07.

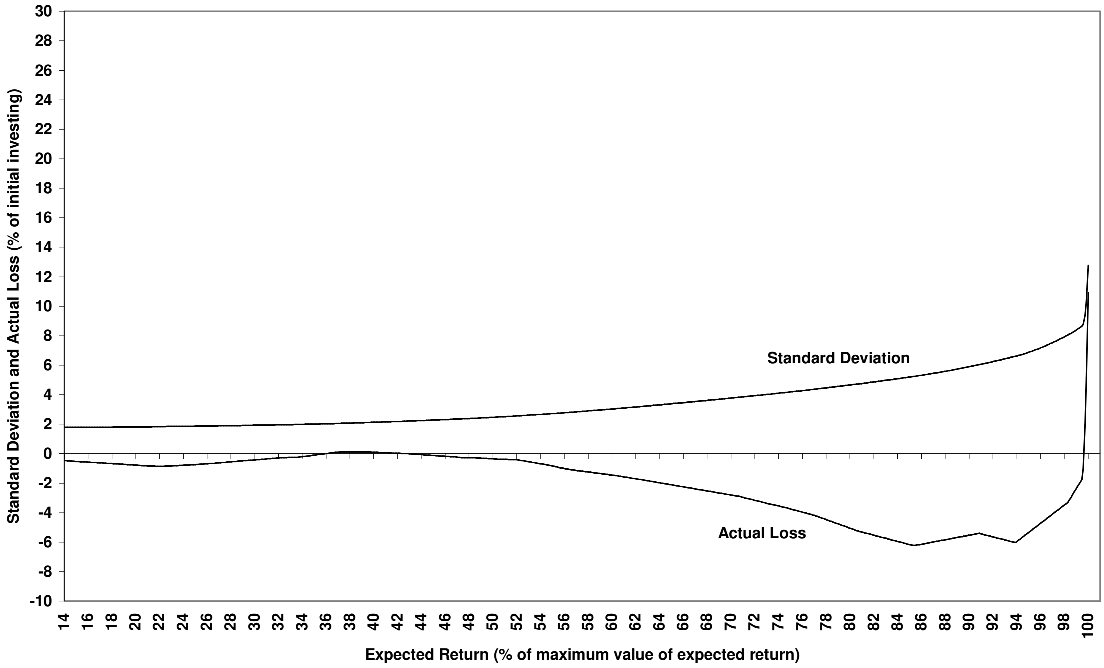

Figure 5. The risk-return frontier and the actual loss values obtained by minimizing the Variance of losses in the sampling approach in the hold-out sample period of the month of December 07

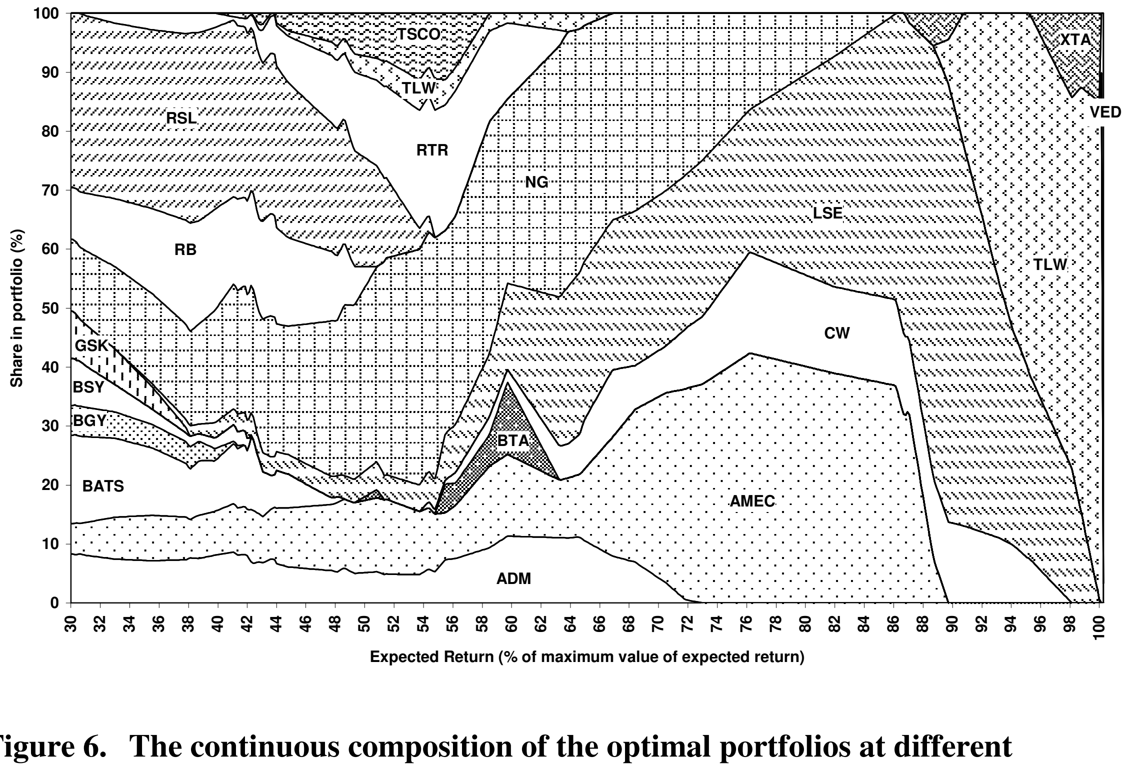

Figure 6. The continuous composition of the optimal portfolios at different levels of target expected return achieved by minimizing the CVaR of losses in the sampling approach

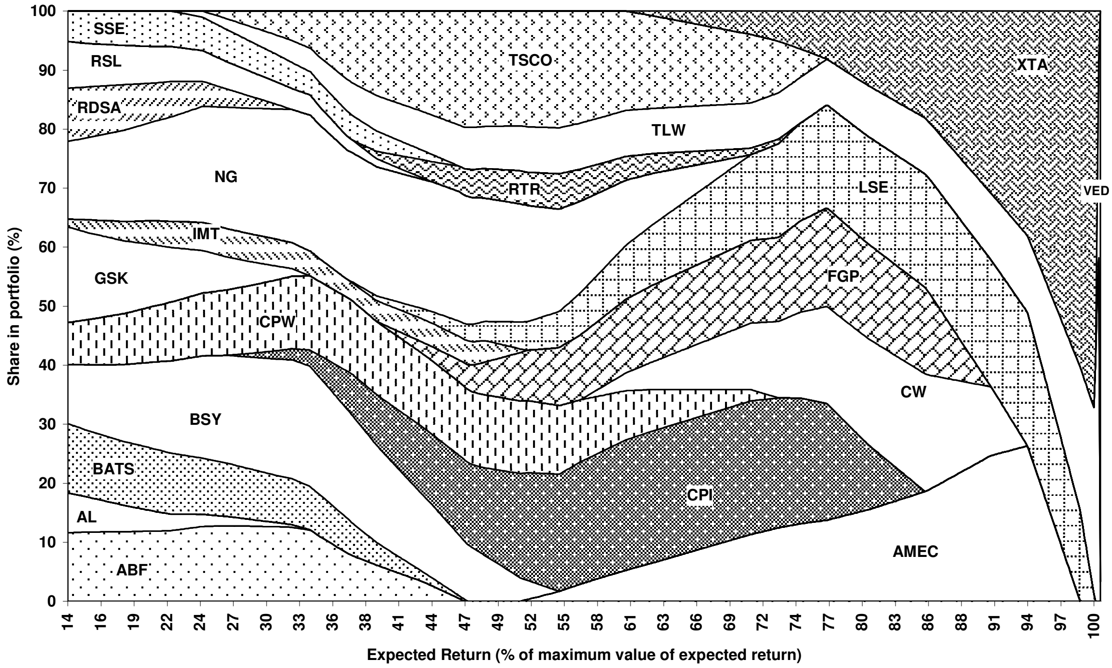

Figure 7. The continues composition of the optimal portfolios at different target levels of expected return achieved by minimizing the Variance of losses in the sampling approach

The compositions of the related portfolios at each level of expected return obtained in the sampling approach using the minimization of CVaR and the Variance of losses are shown respectively in Figures 6 and 7.

If we compare these Figures with Figure 2 it is clear that the diversification effects in the optimal portfolios chosen in the sampling approach are stronger than in the parametric approach. Also in the sampling method, the selected portfolios with a minimum level of variance are more diversified than those chosen with a minimum level of CVaR. There are optimal portfolios containing 14 stocks in the case of the Variance minimization portfolios selected. However, diversification does not necessarily improve the efficiency of the selected portfolios. Decreasing the risk of portfolio should be in proportion with decreasing the expected return of portfolio. In the other word if two or more stocks with high correlation coefficients and low expected returns replace one stock with a higher expected rate of return just because they have less volatility, the risk of portfolio may not decrease meaningfully but the expected return may decreases more than proportionately. For example, the optimal portfolios with a minimum level of variance and a target expected return between 42.5% and 36% contain the greatest numbers of stocks but have positive losses at the end of the investment period. An examination of the correlation coefficients of stocks contained in these portfolios illustrates the above points.

Figures 8 and 9 show the orders of the correlation coefficients between the stocks contained in the portfolios which are selected by minimizing the CVaR and Variance of losses respectively.

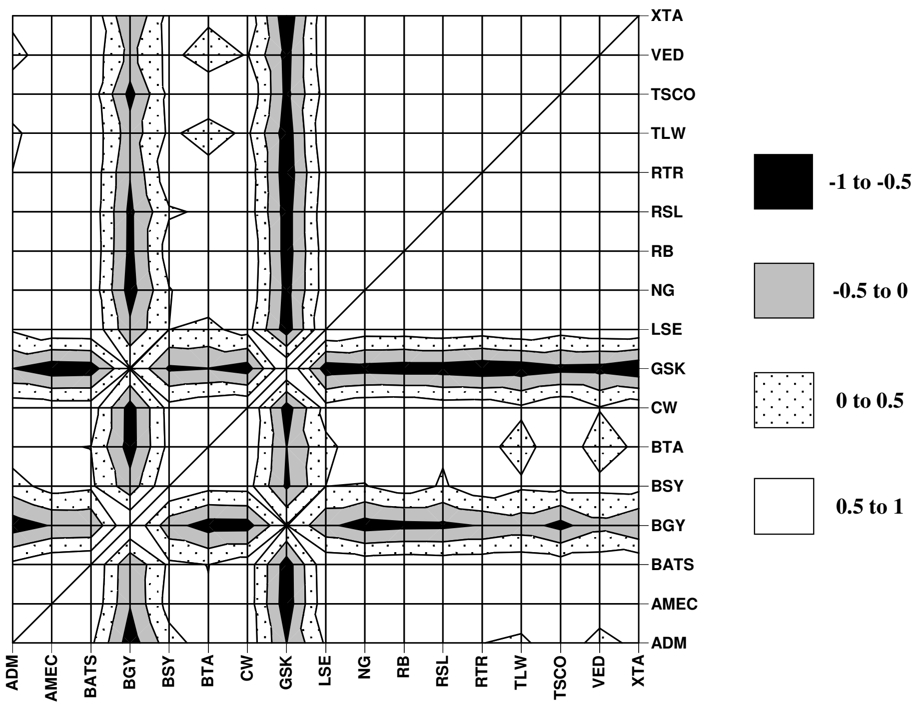

Figure 8. A schematic view of the correlation coefficients of the stocks contained in the optimal portfolios achieved by minimizing the CVaR of losses in the sampling approach

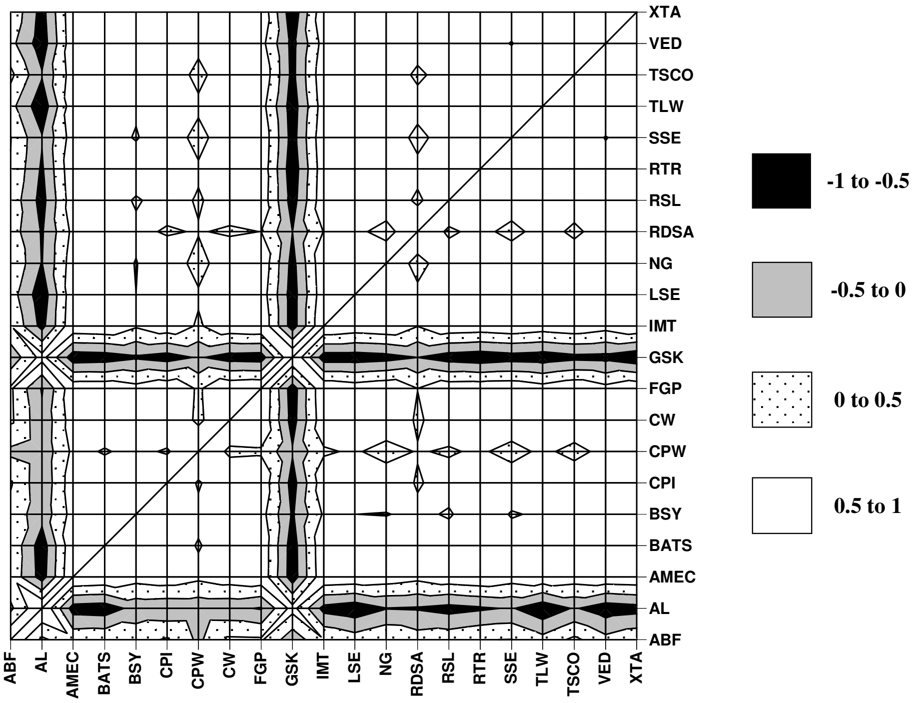

Figure 9. A schematic view of the correlation coefficients of the stocks contained in the optimal portfolios achieved by minimizing the Variance of losses in the sampling approach

We have tried to investigate the affects of utilizing different loss PDFs in the construction and the use of portfolio optimization methods on the quality of selected portfolios in terms of their hold-out sample performance. In order to shed more light on this subject, the distribution of the loss functions and the selected optimal portfolios obtained from each previously described approach are now considered at various specific levels of target expected returns (100, 95, 90, 85, 80, 75 and 70 percent of the maximum return). These are shown in Figures 10 to 23. In all of these figures *Normal* shows the results of the parametric approach assuming a normal distribution and *Historical-CVaR* and *Historical-Variance* illustrate the results of the sampling approach using historical simulations to obtain the pdfs which are then utilised minimizing CVaR and Variance respectively.

It was seen previously in Figures 2, 6 and 7 that the portfolios which are selected using the parametric and the sampling approaches are completely different. These portfolios are different in composition at the maximum level of target expected return which also has the maximum level of risk. This is because of differences between the results of simulations of the future obtained by these two methods. These differences in the maximum level of expected return are shown in figure 10. The loss distributions of the portfolios in the sampling approach are exactly the same in both the Variance and CVaR minimization cases because we actually have not minimized the risk at the maximum level of expected return. It is clear in Figure 10 that the historically simulated samples of losses do have not a specific distribution like a normal distribution. Non real assumptions about the distribution of losses may be a most important reason causing inefficiency in the parametric approach.

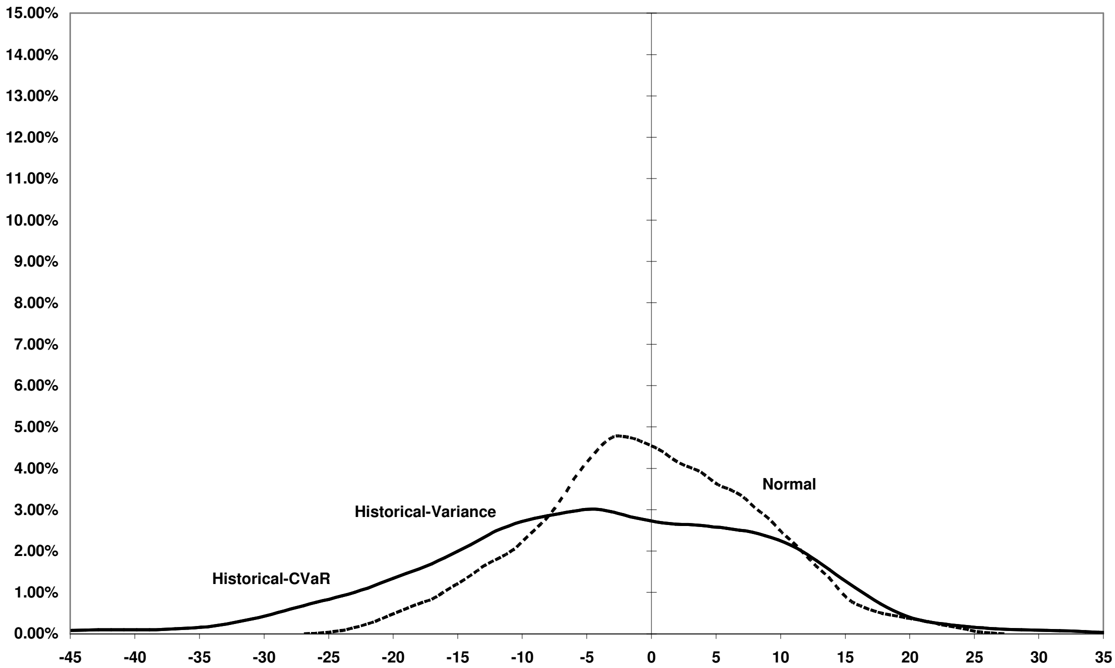

Figure 10. The loss distribution of the optimal portfolios selected by the different approaches obtained at the 100% level of maximum target return

As is shown in Figure 11 at the highest level of target expected return, the portfolios obtained by all methods contain just one stock in their combinations. These portfolios would be too much risky in practice and their potential efficiency is mostly dependent on chance. In the case of our out of sample data none of these portfolios were lucky and all of them actually had a positive loss. The intuition is straightforward though, if you want to maximize returns irrespective of risk, you invest in the one stock with the highest expected return.

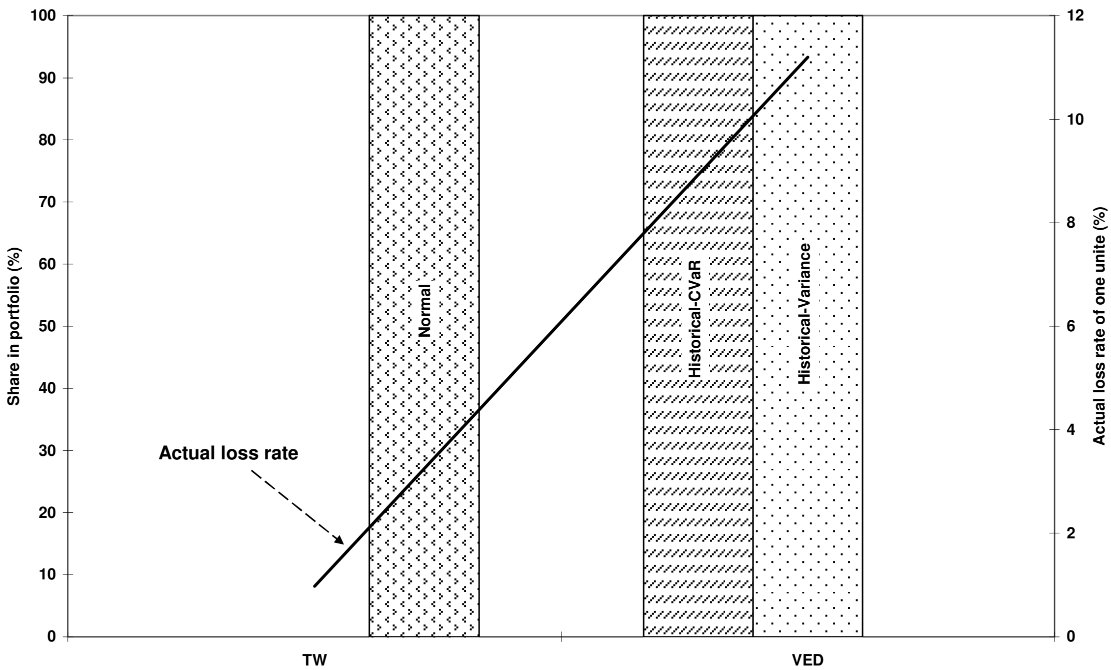

Figure 11. Optimal portfolios and the actual loss rate of one unit of each stock in the portfolios chosen by the different approaches at the 100% level of maximum target return

Once we start the minimization of the risk level of the portfolios the shapes of the loss distributions change in the cases of the Variance and CVaR minimization. If we follow these changes across the Figures 12, 14, 16, 18, 20 and 22 will see that although in the variance minimization models the volatility of losses of the portfolios chosen decreases by decreasing the expected return but the amount of the downside risk (the right tail of distribution) more than decreases in the case of CVaR minimizations. In other words, the risk-return trade off in the case of the CVaR minimizations is mostly to the benefit of the returns of the portfolios compared with the Variance minimizations. This is the most important reason for the advantage of the CVaR minimization of losses instead of the variance as a decision criterion.

This advantage becomes clearer if we follow the changes in the compositions of the portfolios achieved by decreasing the expected returns amongst the Figures 13, 15, 17, 19, 21 and 23.

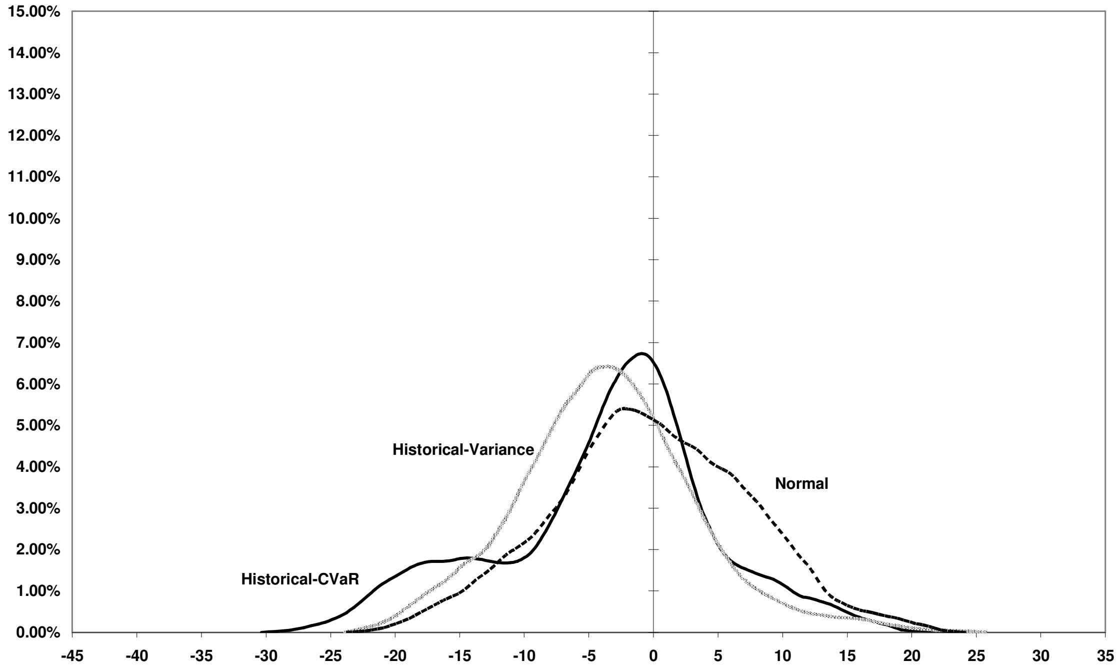

Figure 12. The loss distributions of the optimal portfolios selected by the different approaches at a 95% level of the target maximum return

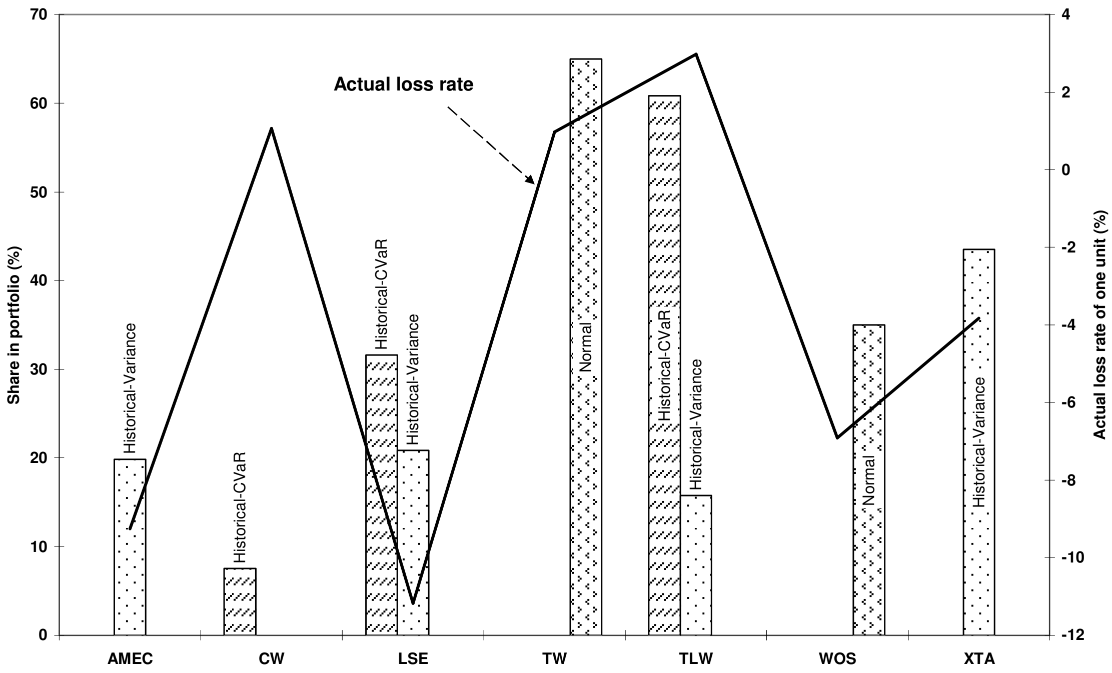

Figure 13. The optimal portfolios and the actual loss rates of one unit of each stock in the portfolios selected by the different approaches at the 95% level of the maximum target return

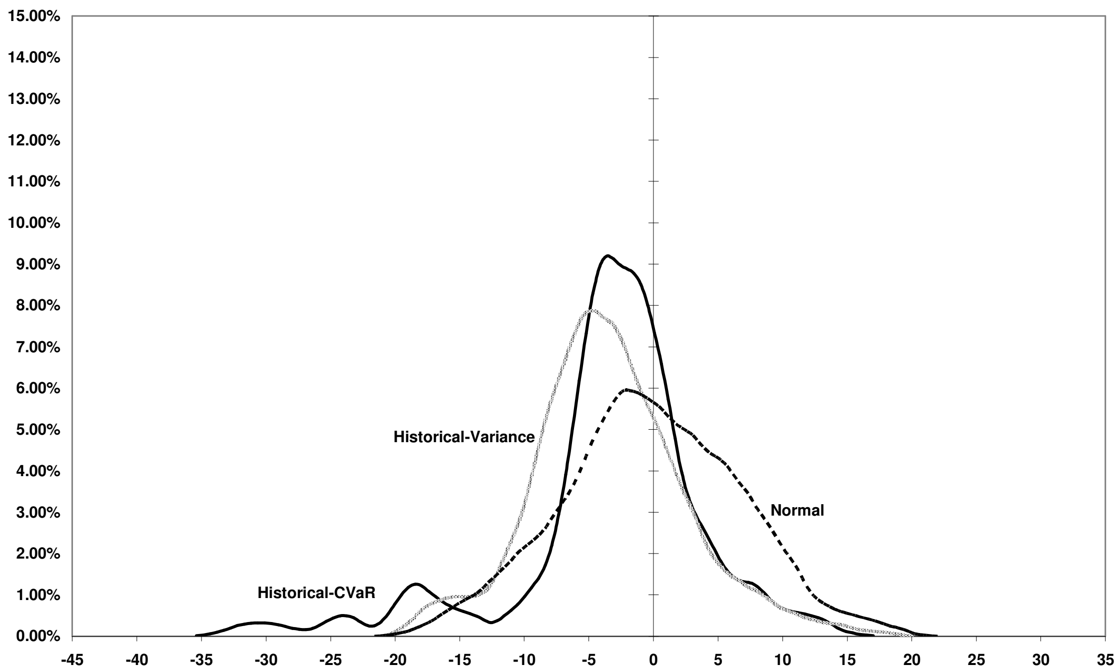

Figure 14. The loss distribution of the optimal portfolios selected by the different approaches at the 90% level of the maximum target return

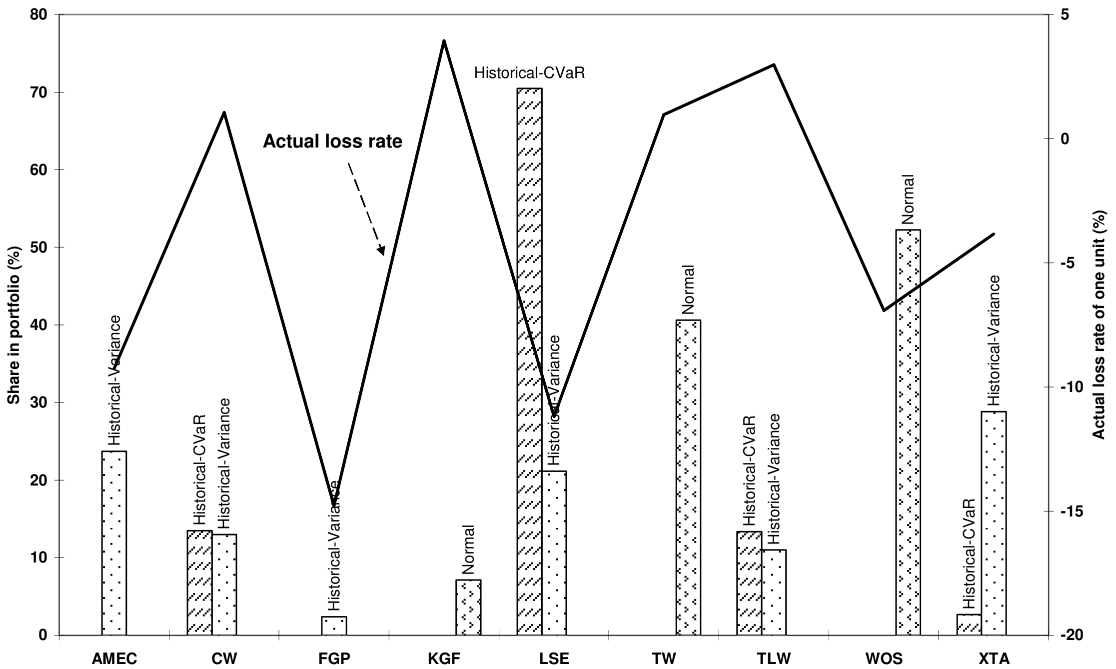

Figure 15. The optimal portfolios and the actual loss rates of one unit of each stock in portfolios selected by the different approaches at the 90% level of the maximum target return

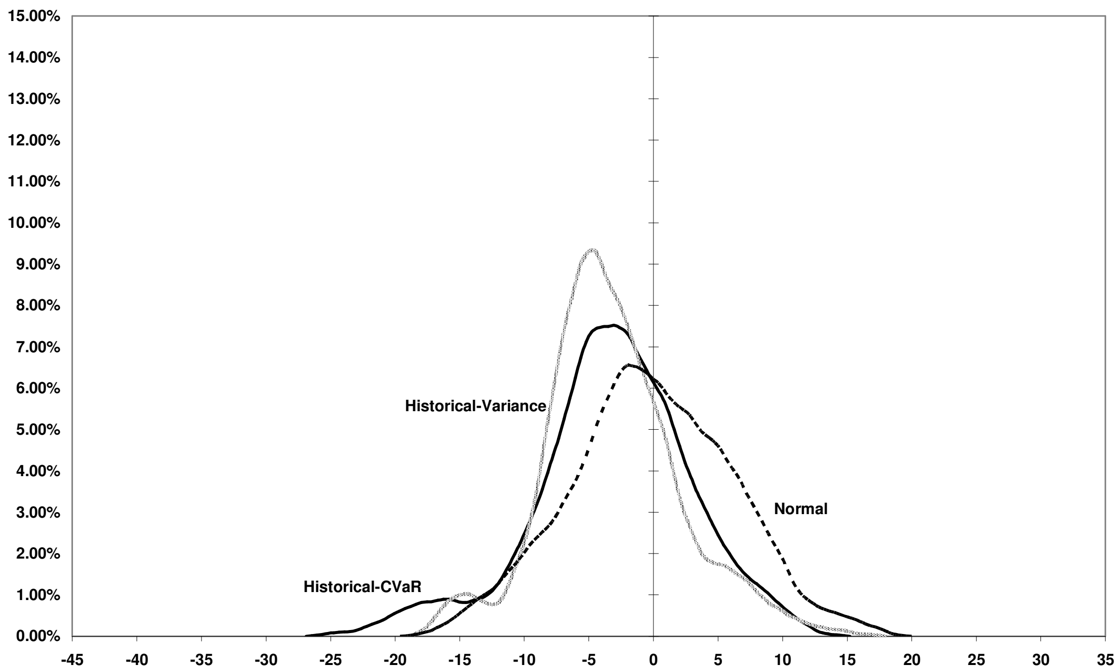

Figure 16. The loss distribution of the optimal portfolios selected by the different approaches at the 85% level of the maximum target return

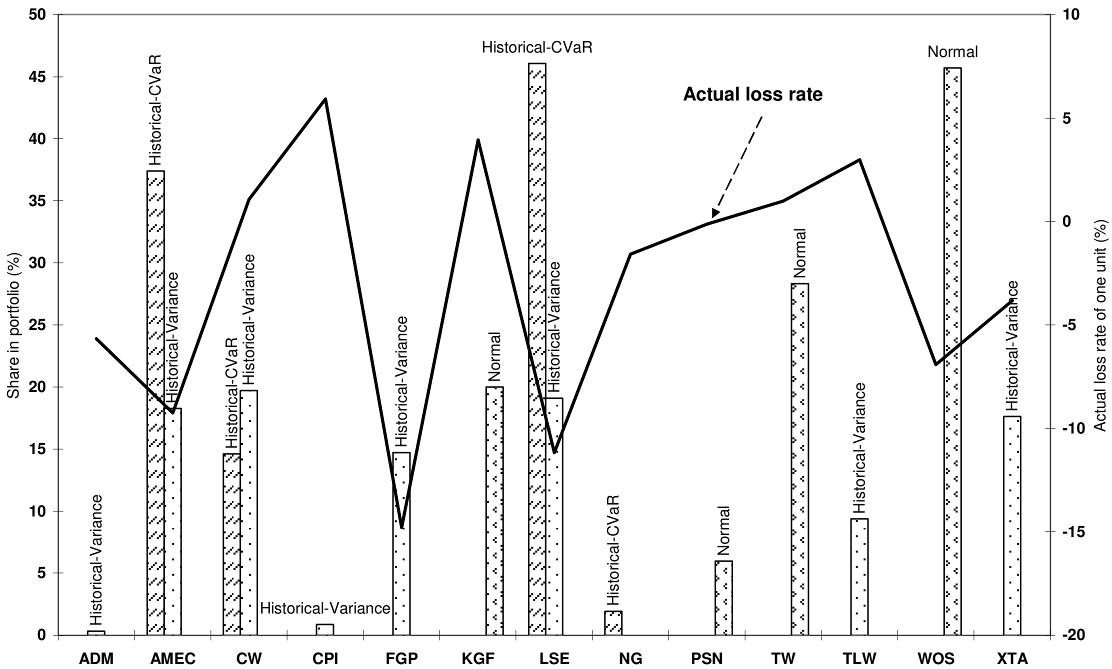

Figure 17. **The optimal portfolios and the actual loss rates of one unit of each stock in the portfolios selected by the different approaches at the 85% level of the maximum target return**

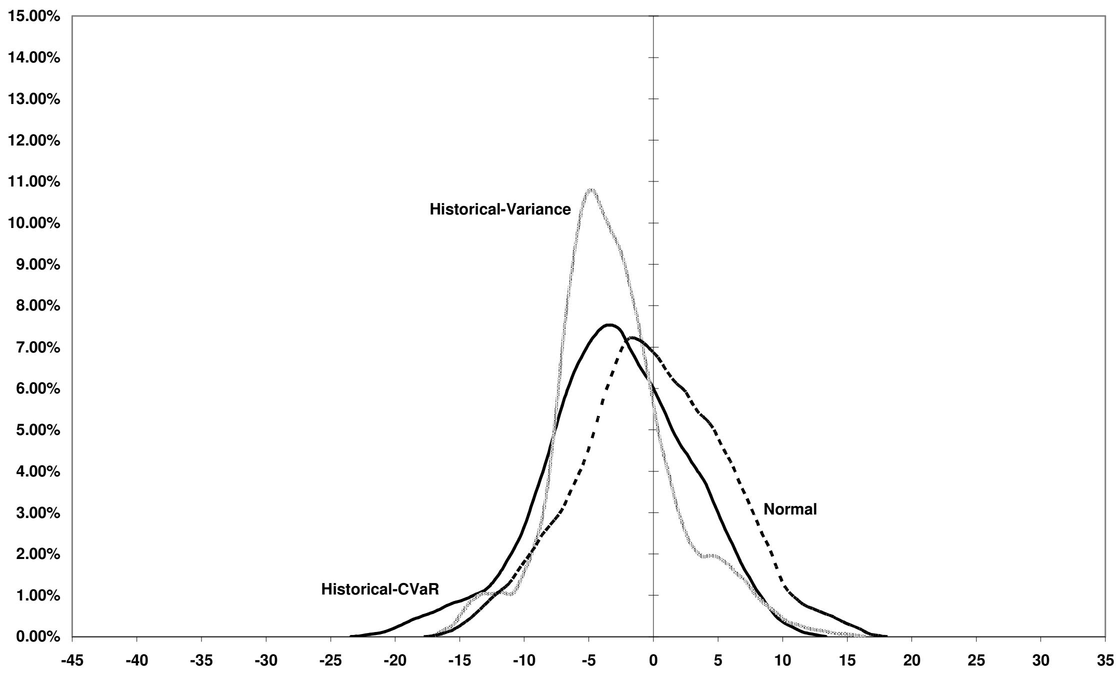

Figure 18. **The loss distributions of the optimal portfolios selected by the different approaches at the 80% level of the maximum target return**

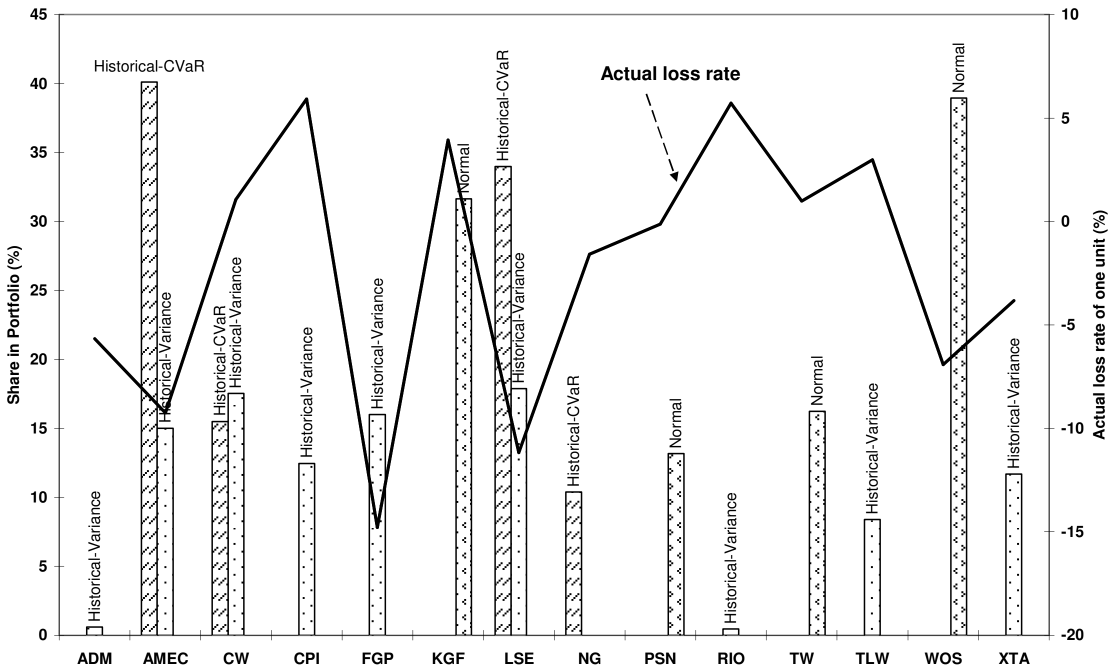

Figure 19. **The optimal portfolios and the actual loss rates of one unit of each stock in the portfolios selected by the different approaches at the 80% level of the maximum target return**

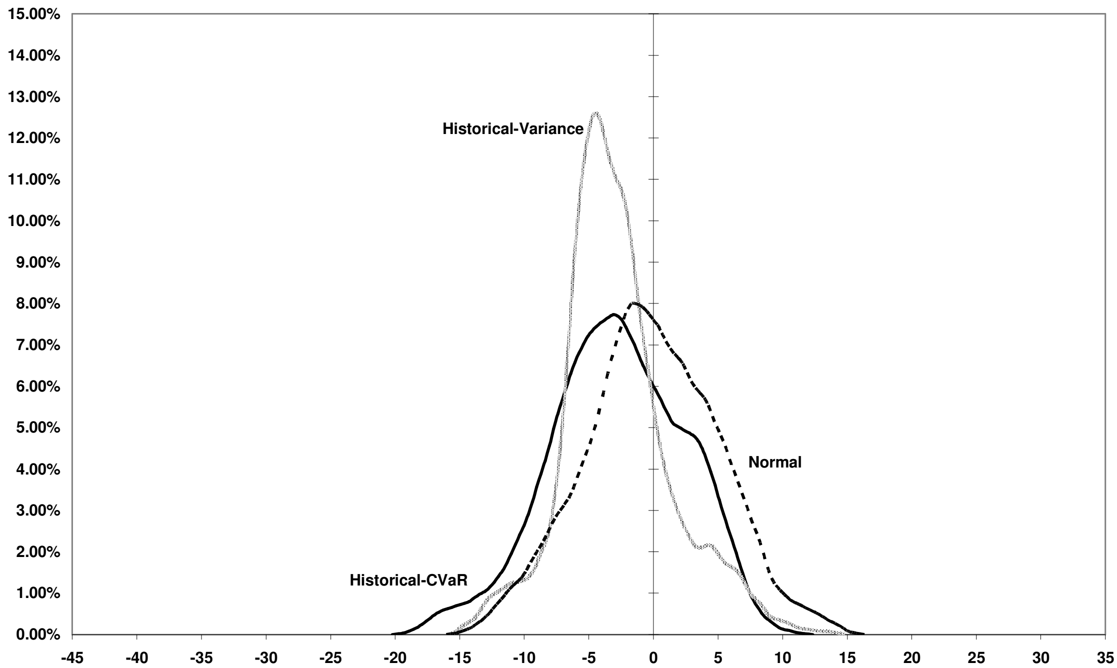

Figure 20. **The loss distributions of the optimal portfolios selected by the different approaches at the 75% level of the maximum target return**

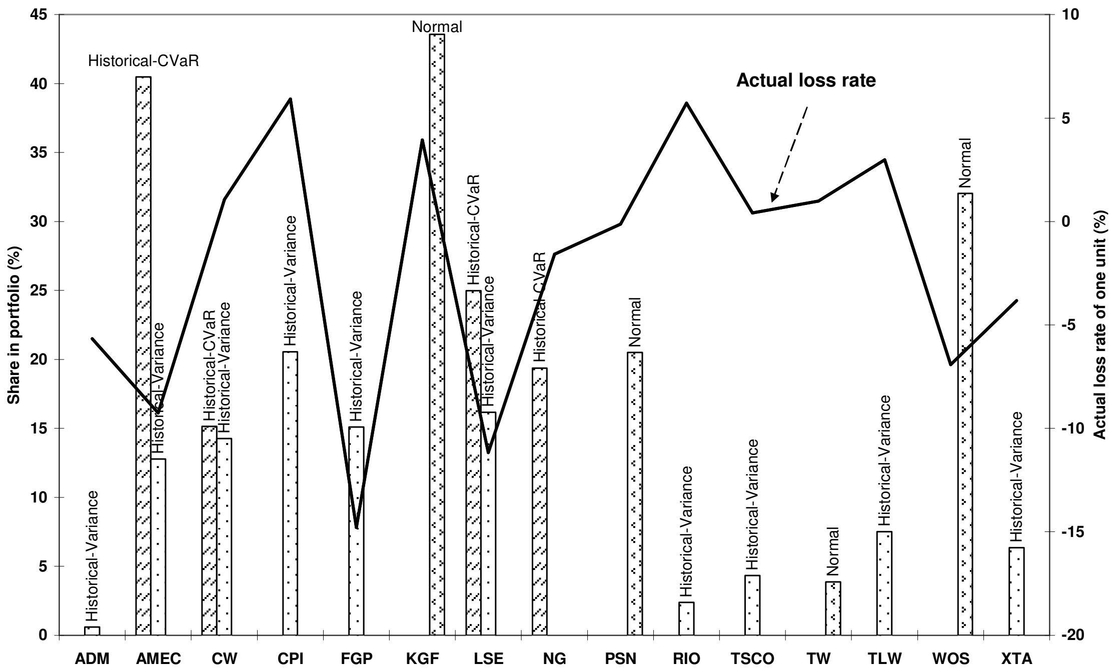

Figure 21. **The optimal portfolios and the actual loss rates of one unit of each stock in the portfolios selected by the different approaches at the 75% level of the maximum target return**

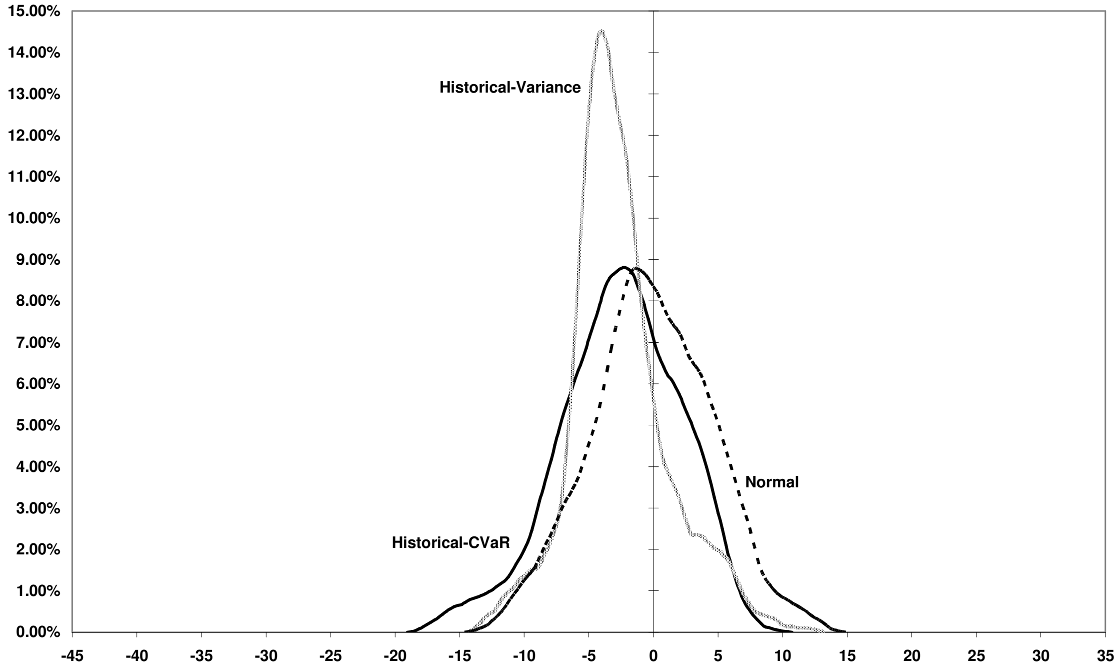

Figure 22. **The loss distribution of the optimal portfolios selected by the different approaches at the 70% level of the maximum target return**

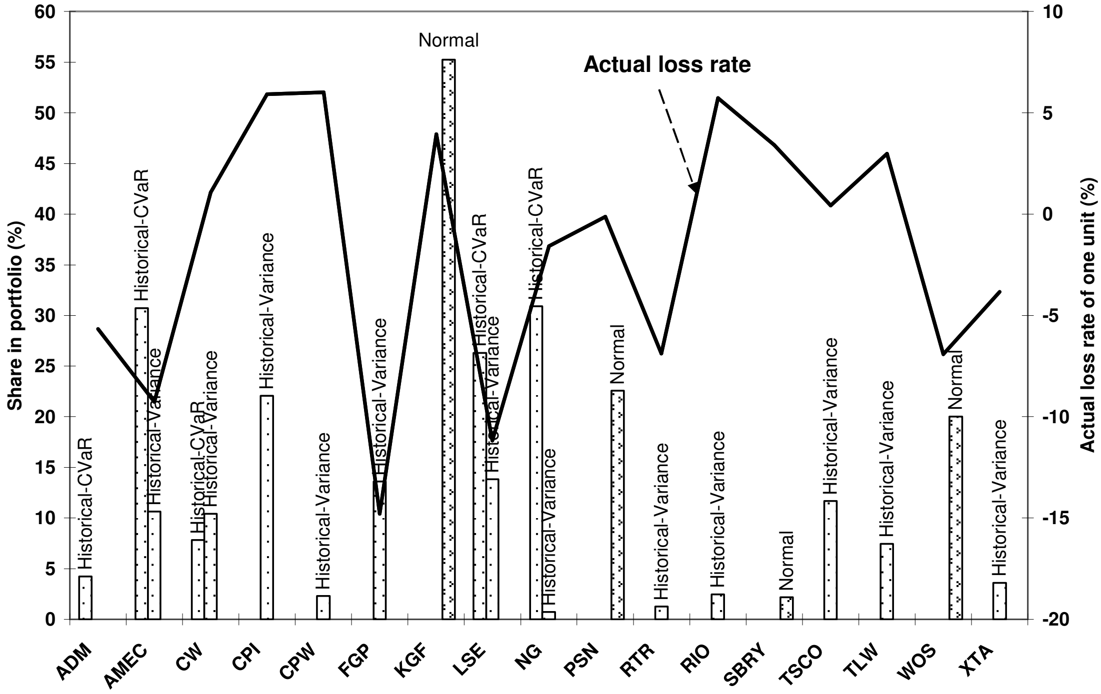

Figure 23. The optimal portfolios and the actual loss rates of one unit of each stock in the portfolios selected by the different approaches at the 70% level of the maximum target return

4. Conclusion

In this study the effects of choosing portfolios for a short-term investment horizon of one month, using two different approaches to construct the loss PDFs to make the projections: the Parametric and the Sampling approaches- have been investigated. This is a most important step in terms of the investigation of the selection of optimal portfolios in a risk-return framework. The results of this study showed that the wrong assumptions about the distribution of losses involved in the parametric approach appear to result in the selection of portfolios with a low probability of success in the subsequent hold-out investment period. The results confirm the advantage of the sampling approach which appears to result in more accurate calculations of the amounts of potential losses in the different subsequent scenarios. Our results suggest that the use of the parametric approach to the projection of the loss PDFs produces relatively poor results especially in the case of the more risk averse investors. The results also showed the advantage of using CVaR minimization rather than Variance minimization in the sampling approach. It seems to be the case that in CVaR minimization the downside risks are minimized, yet decreasing the risk has less of an effect on decreasing the returns. By contrast, when portfolios are selected using a Variance minimization criterion this effect appears to be stronger because of the impact of decreasing the total volatility of the portfolio.

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