CVaR and Credit Risk Measurement

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ABSTRACT

The link between credit risk and the current financial crisis accentuates the importance of measuring and predicting extreme credit risk. Conditional Value at Risk (CVaR) has become an increasingly popular method for measuring extreme market risk. We apply these CVaR techniques to the measurement of credit risk and compare the probability of default among Australian sectors prior to and during the financial crisis. An in depth understanding of sectoral risk is vital to Banks to ensure that there is not an overconcentration of credit risk in any sector. This paper demonstrates how CVaR methodology can be applied in different economic circumstances and provides Australian Banks with important insights into extreme sectoral credit risk leading up to and during the financial crisis.

Keywords: Conditional Value at Risk (CVaR); Banks; Structural modelling; Probability of default (PD)

1. Introduction

Value at Risk (VaR) has become an increasingly popular metric for measuring market risk. VaR measures potential losses over a specific time period within a given confidence level. The concept is well understood and widely used. Its popularity escalated when it was incorporated into the Basel Accord as a required measurement for determining capital adequacy for market risk. VaR has also been applied to credit risk through models such as CreditMetrics (Gupton, Finger, & Bhatia, 1997), CreditPortfolioView (Wilson, 1998), and *i*Transition (Allen & Powell, 2008).

Nevertheless, despite its popularity, VaR has certain undesirable mathematical properties; such as lack of sub-additivity and convexity; see the discussion in Arztner et al (1999; 1997). In the case of the standard normal distribution VaR is proportional to the standard deviation and is coherent when based on this distribution but not in other circumstances. The VaR resulting from the combination of two portfolios can be greater than the sum of the risks of the individual portfolios. A further complication is associated with the fact that VaR is difficult to optimize when calculated from scenarios. It can be difficult to resolve as a function of a portfolio position and can exhibit multiple local extrema, which makes it problematic to determine the optimal mix of positions and the VaR of a particular mix. See the discussion of this in Mckay and Keefer (1996) and Mauser and Rosen (1999).

Conditional Value at Risk (CVaR) measures extreme returns (those beyond VaR). Allen and Powell (2006; 2007) explored CVaR as an alternative method to VaR for measuring market and credit risk. They found that CVaR yields consistent results to VaR when applied to Australian industry risk rankings, but has the added advantage of measuring extreme returns (those beyond VaR). Pflug (2000) proved that CVaR is a coherent risk measure with a number of desirable properties such as convexity and monotonicity, amongst other desirable characteristics. Furthermore, VaR gives no indication on the extent of the losses that might be encountered beyond the threshold amount suggested by the measure. By contrast CVaR does quantify the losses that might be encountered in the tail of the distribution. A number of recent papers apply CVaR to portfolio optimization problems; see for example Rockafeller and Uryasev (2002; 2000), Andersson et.al (2000), Alexander et al (2003), Alexander and Baptista (2003) and Rockafellar et al (2006).

However, besides the studies by Allen & Powell there has been no use or application of CVaR in an Australian setting and its use, properties and applications are still in the early stages of their development.

This study compares credit risk prior to and subsequent to the onset of the financial crisis through the application of CVaR to the structural probability of default (PD) model of Merton. Examples of studies using structural methodology for varying aspects of credit risk include asset correlation (Cespedes, 2002; Kealhofer & Bohn, 1993; Lopez, 2004; Vasicek, 1987; Zeng & Zhang, 2001), predictive value and validation (Bharath & Shumway, 2004; Stein, 2002), and fixed income modelling (D'Vari, Yalamanchili, & Bai, 2003). The effect of default risk on equity returns has also been examined (Chan, Faff, & Koffman, 2008; Gharghori, Chan, & Faff, 2007; Vassalou & Xing, 2002). These papers also examine PD as an extension to the Fama and French (1992; Fama & French, 1993) three factor view of asset pricing which includes the market, size and book-to market. Ghargori et al. find that default risk is not priced in equity returns and that the Fama-French factors are not proxying for default risk. Vassalou and Xing find support for size and book to market as influences on default risk, but do not find strong linkage between default risk and return. Chan et al., using an extensive 30 year data sample of micro stocks, find significant linkage between default risk and returns. When conditioning for business cycles they find that default risk premium is twice as high during expansions than during contractions.

As equity forms a key component of structural modelling, we commence by applying CVaR to equity prices and then incorporate CVaR into structural credit modelling to obtain Conditional Probability of Default (CPD). The study is important in that it uses the CVaR credit methodology developed by the authors to understand extreme risk among sectors both prior to and during the financial crisis. This provides investors and lenders with a greater understanding of extreme sectoral equity and credit risk across different economic circumstances.

2. Data and Methodology

2.1 Data

We divide our data sample into 3 periods. Our first period relates to pre-financial crisis for which we use the 7 years prior to 2007. Seven years aligns with Basel Accord advanced model requirements for measuring credit risk. Periods 2 (2007) and 3 (2008) are our financial crisis years. The study includes entities listed on the Australian Stock Exchange (ASX) All Ordinaries Index (All Ords) for which equity prices and Worldscope balance sheet data are available in Datastream. Entities with less than 12 months data in any of the 3 periods were excluded. Industries with less than 5 companies were also excluded. Our sample is considered a fair representation of Australian listed entities given that the All Ords includes more than 90% of listed Australian Companies by market capitalisation, and our data sample includes approximately 90% of All Ords Entities.

2.2 VaR and CVaR

Prior to calculating CVaR of equity prices, we calculate VaR. We follow the method used by RiskMetrics (J.P. Morgan & Reuters, 1996), who introduced and popularised VaR. This is the most commonly used VaR method. Daily equity returns are calculated for each of the years in our data sample by using the logarithm of daily price relatives:

$$\ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

i.e. the logarithm of the ratio between today's price and the previous price. VaR is calculated at a 95% confidence level. Based on standard tables $\text{VaR}_x = 1.645\sigma_x$. CVaR uses the same methodology as VaR, except we use the average of the returns beyond VaR (i.e. the worst 5% of returns).

2.3 Credit Risk PD Methodology

We use the Merton approach to estimating default, and then in section 2.4 modify this calculation to incorporate CVaR. The Merton model measures distance to default (DD) and probability of default (PD) as

$$DD = \frac{\ln(V/F) + (\mu - 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}} \tag{2}$$

$$PD = N(-DD) \tag{3}$$

where

V = market value of firm's debt

F = face value of firm's debt

μ = an estimate of the annual return (drift) of the firm's assets

N = cumulative standard normal distribution function.

To estimate market value of assets, we follow approaches outlined by KMV (Crosbie & Bohn, 2003) and Bharath & Shumway (2004). Equity returns and their standard deviation are calculated exactly the same as for our market approach. Initial asset returns are estimated from our historical equity data using the following formula:

$$\sigma_V = \sigma_E \left(\frac{E}{E+F} \right) \tag{4}$$

These asset returns derived are applied to equation 4 to estimate the market value of assets every day. The daily log return is calculated and new asset values estimated. Following KMV, this process is repeated until asset returns converge (repeated until difference in adjacent σ's is less than 10^{-3}). These figures are then applied to the DD and PD calculations in equation 2 and 3. We measure μ as the mean of the change in lnV as per Vassalou & Xing (2002). We measure historical asset volatility using a combination of current balance sheet data, and historical equity values which are then used to estimate historical asset values as described in earlier in this section. This allows us to examine how the current distance to default would change if asset volatilities reverted to historical levels. Anchoring the default variable allows the loss distribution to shift with changes in another variable, as is noted by Pesaran et al. (2003) whose credit risk model anchors default and determines loss distribution changes brought about by changes in macroeconomic factors. The authors note that "the problem is not properly identified if we allow both to be time varying".

2.4 CPD Calculation

For the purposes of this study we define conditional probability of default (CPD) as being PD on the condition that standard deviation of asset returns exceeds standard deviation at the 95% confidence level, i.e. the worst 5% of asset returns. We calculate the standard deviation of the worst 5% of daily asset returns for each period to obtain a conditional standard deviation (CStdev). We then substitute CStdev into the formula used to calculate DD, to obtain a conditional DD (CDD). CPD is calculated by substituting DD with CDD into the CPD formula.

$$CDD = \frac{\ln(V / F) + (\mu - 0.5\sigma_V^2)T}{CStdev_V \sqrt{T}} \tag{5}$$

and

$$CPD = N(-CDD) \tag{6}$$

3. RESULTS

Table 1 compares equity CVaR values prior to the financial crisis period with values during 2007 and 2008. All industries showed an increase in CVaR, but there have been major changes in rankings. The most significant negative shifts (industries most badly affected) are seen in Diversified Financials, Real Estate, Banks, Mining and Capital Goods. Industries least affected were Insurance, Healthcare and Technology which showed a significant improvement in their CVaR ranking status.

Table 2 shows DD and CD values, with rankings shown in table 3. Diversified Financials, Real Estate, Banks and Mining have fared the worst in terms of movement in rankings, which matches closely with movements in CVaR per Table 1. In terms of actual default probabilities Banks and Diversified Financials come precariously close to default. This is due to a combination of the high volatility and high leverage as shown by the equity ratios. Banks are operating on capital ratios of approximately 16%, which is much higher than other sectors.

Table 1. Equity CVaR Results

CVaR represents the average of the worst 5% of asset returns. Figures for 2007 and 2008 are each based on daily returns for 12 months. Figures for Prior 2007 incorporate 7 years of data. Rankings are from 1 (lowest risk) to 20 (highest risk). A negative movement in rankings shows deterioration in risk ranking.

	CVaR Values			CVaR Rankings			
	Prior 2007	2007	2008	Prior 2007	2007	2008	movement
Automobiles & Components	0.0536	0.0671	0.1387	16	13	17	-1
Banks	0.0268	0.0301	0.0868	1	2	7	-6
Capital Goods	0.0428	0.0676	0.1208	9	14	15	-6
Commercial Services & Supplies	0.0530	0.0704	0.1085	15	15	13	2
Consumer Durables & Apparel	0.0506	0.0438	0.0865	14	7	6	8
Diversified Financials	0.0392	0.0942	0.1822	7	20	20	-13
Energy	0.0538	0.0705	0.1412	17	16	19	-2
Food & Staples Retailing	0.0343	0.0368	0.0787	2	4	5	-3
Food Beverage & Tobacco	0.0369	0.0418	0.0664	5	6	2	3
Healthcare Equipment & Services	0.0499	0.0511	0.0746	13	11	4	9
Insurance	0.0586	0.0461	0.0897	18	8	8	10
Media	0.0417	0.0392	0.1041	8	5	11	-3
Metals & Mining	0.0498	0.0720	0.1405	12	18	18	-6
Pharmaceuticals & Biotechnology	0.0656	0.0601	0.1059	19	12	12	7
Real Estate	0.0381	0.0716	0.1321	6	17	16	-10
Retailing	0.0469	0.0486	0.0897	11	9	9	2
Technology	0.0862	0.0770	0.1167	20	19	14	6
Telecommunication Services	0.0343	0.0296	0.0497	3	1	1	2
Transportation	0.0451	0.0498	0.1020	10	10	10	0
Utilities	0.0351	0.0366	0.0710	4	3	3	1
All	0.0421	0.0601	0.1059				

Table 2. DD and CDD Results

DD (measured by number of standard deviations) is calculated using equation 2 and PD using equation 3. CDD is based on the worst 5% of asset returns and is calculated using equation 5 and CPD using equation 6. Figures for 2007 and 2008 are each based on daily returns for 12 months. Figures for Prior 2007 incorporate 7 years of data. PD and CPD are shown in percentages (e.g. Banks have a PD in 2008 of 27%). The equity ratio in the final column is based on the book value of assets and capital.

	DD				CDD				
	Prior 2007	2007	2008	PD 2008	Prior 2007	2007	2008	CPD 2008	Equity ratio
Automobiles & Components	5.8001	1.3631	0.8042	0.2106	3.2563	0.3570	0.1728	0.4314	0.5222
Banks	8.2566	1.8069	0.5993	0.2745	5.1948	0.5653	0.1962	0.4222	0.1568
Capital Goods	8.4873	4.0467	2.1466	0.0159	4.9938	1.0531	0.5895	0.2778	0.7548
Commercial Services & Supplies	7.7998	6.7021	4.0492	0.0000	4.5327	1.8274	1.1854	0.1179	0.7183
Consumer Durables & Apparel	9.2748	7.2292	3.8112	0.0001	5.1630	2.1630	1.0959	0.1366	0.8346
Diversified Financials	11.6528	0.8197	0.3978	0.3454	5.1679	0.2111	0.1092	0.4565	0.3329
Energy	9.5162	8.5776	4.3734	0.0000	5.3553	2.4001	1.1983	0.1154	0.8063
Food & Staples Retailing	10.0591	8.1612	3.9090	0.0000	5.3267	2.4259	1.1334	0.1285	0.7414
Food Beverage & Tobacco	9.4412	10.4043	6.7108	0.0000	5.0638	3.3381	2.0991	0.0179	0.6218
Healthcare Equipment & Services	8.8620	13.3022	8.2336	0.0000	5.8645	3.6940	2.5280	0.0057	0.7227
Insurance	3.7028	2.8945	1.3450	0.0893	3.3801	0.7907	0.4061	0.3424	0.2864
Media	9.9655	7.7556	3.3284	0.0004	5.0000	2.5181	0.9484	0.1715	0.6884
Metals & Mining	8.5029	5.5021	2.6429	0.0041	5.8637	1.4598	0.7484	0.2271	0.7684
Pharmaceuticals & Biotechnology	7.6369	8.0968	5.1111	0.0000	4.5370	2.7075	1.5365	0.0622	0.8454
Real Estate	11.5424	4.5887	2.1613	0.0153	6.2634	1.2130	0.6576	0.2554	0.6523
Retailing	7.1157	6.0159	3.0494	0.0011	4.3520	1.8774	1.0165	0.1547	0.7134
Technology	5.6425	5.7531	4.7943	0.0000	3.8445	1.8135	1.3302	0.0917	0.8487
Telecommunication Services	9.4000	8.9649	5.6834	0.0000	6.5891	2.6524	1.5827	0.0568	0.6732
Transportation	8.3119	6.5817	3.1007	0.0010	4.3088	1.9103	0.9334	0.1753	0.5897
Utilities	13.9258	11.5900	5.7225	0.0000	6.1668	3.4240	1.7619	0.0390	0.5337
All	8.5442	6.6091	2.2626	0.0479	4.9486	0.5127	0.5707	0.1843	0.3820

Table 3. DD and CDD Rankings

The table provides sector rankings for the outputs in Table 2. Sectors are ranked from 1 (lowest risk) to 20 (highest risk). Movement is the difference between 2008 rankings and Prior 2007 rankings. Negative movement indicates a deterioration in ranking and positive movement shows an improvement.

	DD				CDD			
	Prior 2007	2007	2008	movement	Prior 2007	2007	2008	movement
Automobiles & Components	18	19	18	0	20	19	19	1
Banks	14	18	19	-5	8	18	18	-10
Capital Goods	12	16	16	-4	13	16	16	-3
Commercial Services & Supplies	15	10	8	7	15	12	8	7
Consumer Durables & Apparel	9	9	10	-1	10	9	10	0
Diversified Financials	2	20	20	-18	9	20	20	-11
Energy	6	5	7	-1	6	8	7	-1
Food & Staples Retailing	4	6	9	-5	7	7	9	-2
Food Beverage & Tobacco	7	3	2	5	11	3	2	9
Healthcare Equipment & Services	10	1	1	9	4	1	1	3
Insurance	20	17	17	3	19	17	17	2
Media	5	8	11	-6	12	6	12	0
Metals & Mining	11	14	14	-3	5	14	14	-9
Pharmaceuticals & Biotechnology	16	7	5	11	14	4	5	9
Real Estate	3	15	15	-12	2	15	15	-13
Retailing	17	12	13	4	16	11	11	5
Technology	19	13	6	13	18	13	6	12
Telecommunication Services	8	4	4	4	1	5	4	-3
Transportation	13	11	12	1	17	10	13	4
Utilities	1	2	3	-2	3	2	3	0

Figure 1 shows CPD (measured in number of standard deviations), with Diversified Financials being the highest risk and Healthcare the lowest. Figure 2 shows the changes in CPD risk rankings (2008 compared to the pre financial crisis period), with Real Estate having the largest negative shift in rankings and Technology the largest positive shift.

Figure 1. CDD in 2008

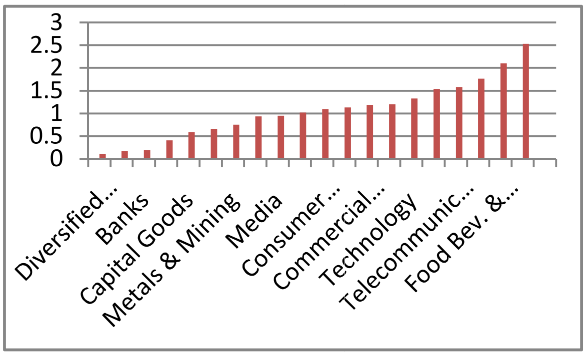


Figure 2. Change in CDD rankings

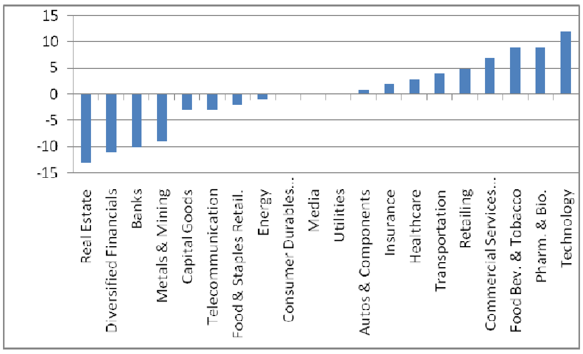
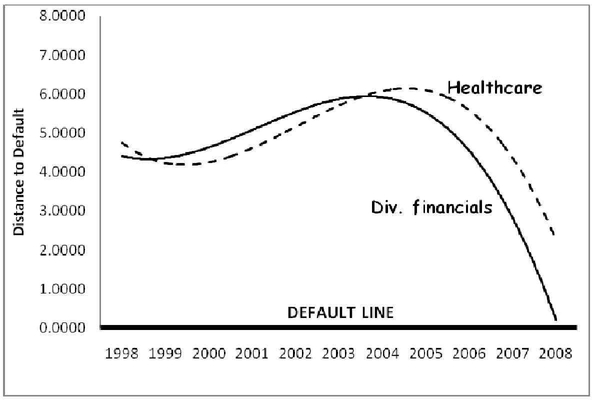


Figure 3. CDD Trend



To illustrate CDD movements, Figure 3 compares the industry with the highest CPD in 2008 (Diversified Financials) to the industry with the lowest CPD (Healthcare). Both industries move further away from default during the mid-2000's and closer to default in 2007 and 2008. Healthcare fares better in 2008 due to a lower volatility and higher equity (72% as compared to 33%). This translates into a much lower CPD for Healthcare (0.57%) as compared to Diversified financials (45%). This CPD calculates the probability of default based on the worst 5% of asset value movements.

Prior to the financial crisis, Allen and Powell (2007) found that there is significant correlation between those industries that are risky from a market perspective (share price volatility) and those industries that are risky from a credit perspective (PD). In the current study, we apply a Spearman Rank Correlation test to 2008 equity CVaR rankings and credit CPD rankings figures to see if this relationship continues to hold. We find that there continues to be a strong relationship (99% confidence) between market and credit risk. There is however, no correlation between CPD rankings prior to the financial crisis and CPD rankings during the financial crisis. This shows that relative risk between sectors changes over different economic conditions.

4. Conclusions

CVaR techniques have been applied to credit risk measurement, which provides lenders with an insight into changes in extreme risk across industries since the onset of the financial crisis. We find significant deterioration in default probabilities across all industries since the onset of the financial crisis. There has also been significant movement in sector risk rankings, meaning that those industries that were risky prior to the financial crisis are not the same of industries that were most risky during the financial crisis. The Basel Accord advanced model requires Banks to measure credit risk over a 7 year period. However, long periods of data tend to smooth or ’average’ credit risk across periods. Our findings show that it is also important for Banks to divide their data tranches into shorter time frames to compare risk across different economic circumstances.

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