Evidence of Temporal Variation in Site-Specific Crop Response to Fertilizer Inputs

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ABSTRACT: Precision agriculture (PA) in crop production refers to the management approach in which inputs are applied according to the site-specific demand and timing in order to improve efficiency of inputs and farm profitability. Among the variety of techniques under the concept of PA, variable rate application (VRA) of both phosphorus and nitrogen fertilizers, and temporal as well as spatial variability of corn crop response to these fertilizers were studied for the potential contribution of VRA of fertilizers to farm profitability.

The crop response data were generated by an on-farm experiment in Windom (Cottonwood County, Minnesota) in 1997, 1999, and 2001, using a commercially cultivated field of 10 ha area, where corn and soybean were rotated. Using a fractional factorial design, three phosphorus fertilizer rates (0.0, 56.2, and 112.3 kg ha^{-1}) and five N-fertilizer rates (0.0, 67.4, 112.3, 157.3 and 202.2 kg ha^{-1}) were applied to the field before corn planting. The observed yields were regressed on these treatment rates and site-specific crop response functions (SS-CRFs) were estimated using maximum likelihood estimation. The SS-CRFs were employed for the optimization problem and the site-specific economically optimal rates (SS-EORs) of phosphorus and nitrogen were obtained as the solution of the profit maximization problem.

The temporal as well as spatial variations in these SS-EORs were observed; the estimated SS-EORs of phosphorus and nitrogen fertilizers varied among 33 sites from 0.0 to 112.3 kg ha^{-1} and from 0.0 to 202.2 kg ha^{-1}, respectively in each year. Moreover, the results indicate that applying the SS-EORs could have been more profitable than uniform rate. The total gains from changing to the SS-EORs from the uniform rates (P = 56.2 kg ha^{-1} and N = 157.3 kg ha^{-1}) would have been \$63.58 ha^{-1}, \$64.34 ha^{-1}, and \$117.42 ha^{-1} in 1997, 1999, and 2001, respectively. This study indicates that VRA of fertilizers potentially improve the profitability of corn production. In order for this technology to become practical, some obstacles need to be

removed, such as a less expensive method to obtain SS-EORs than on-farm experiences, less expensive machinery, more accurate functional form for SS-CRFs than quadratic functions. Once VRA technologies advance and become inexpensive, the "potential" gain from VRA can be fully realized.

INTRODUCTION: Precision agriculture (PA) for crop production is an approach to agriculture input management in which the levels of inputs are adjusted to the demand of specific parts of a large field. The introduction of PA technologies is to improve farm efficiency, which contributes both farm productivity and environmental soundness. The US agriculture has improved productivity constantly during the last century. For example, the labor productivity (per worker production) growth rate was 3.1% per year between 1961 and 1990 (Craig, Pardey, and Roseboom, 1997).[1] The persisting demand for productivity growth drives crop producer interest in better understanding the potential of PA technologies. In addition to the constant pressure to improve farm profitability, environmental and health concerns are playing a significant role in the promotion of PA technologies (NSTC, 2000). For example, excess fertilizers and runoff of accumulated animal waste are suspected to be the major sources of contamination of the river systems in the Midwest. As for reduction of excess fertilizers, the VRA of fertilizers among PA technologies seems to be a preferable management approach.

In terms of both profitability and environmental concern, the premise of VRA of fertilizers to corn production is promising. However, one of the drawbacks is that PA technologies can be expensive because they are highly information and capital intensive. In order to obtain the information of site-specific optimal input rates, it is common to conduct field experiments or extensive soil surveys. Due to the high technicality of experimentation, the cost

[1] A part of growth in efficiency is achieved through the scale of economy; the average US farm size increased by 38 percent points between 1964 and 1997, while the total number of farms decreased by 40 percent points during that period (USDA, 2002).

of a field experiment is usually considerably high. In addition, it is common that advanced and expensive equipment is used for both collecting information and implementing PA technologies. Examples of these expensive tools include remote sensing images, aerial photographs, global positioning systems (GPS), and satellite-guided navigation systems. The initial cost of real-time application of GPS for mobile equipment ranges from $10,000 to $40,000 or more (Wiedemann, 2005).

In spite of the likely higher cost of agriculture input management, PA technologies are likely rewarding. Intuitively speaking, they prevent over-application and under-application of inputs. Under conventional uniform rate application, there is loss of inputs from two types of sites: the sites that demand less than the applied amount and the sites that incur yield losses due to insufficient fertilizer inputs. In addition, due to likely social benefit from PA technologies, the USDA and the EPA provide farmers subsidies for adoption of PA technologies.[2] However, in order to realize this benefit, more study of PA technologies is necessary since how to do so is not yet found. This means that the effort to advance PA practices and the amount of resources allocated largely depend on political decisions because public research institutes are the main players in leading this effort (NRCS, 1997). This project is a joint effort between farmers and public organizations to understand the potential of PA technologies in order to identify profitable and environmentally friendly applications. In particular, the variable rate application (VRA) of fertilizers is the PA technology of interest, and in order to test the potential of this PA technique, there are two objectives. (1) Show there is significant temporal as well as spatial variation in crop response to fertilizer using longitudinal corn yield observations. This variation is

[2] As for promotion programs, the Natural Resources Conservation Service (NRCS) of USDA implements Conservation Security Program that provides financial and technical assistance to farmers who comply the farm management practices described by the agency (2005). This program is not directly promoting PA practices, but it reflects the public concern about the impact of farm operation on environment.

demonstrated by showing there is spatial and temporal variation in the coefficients of site-specific crop response functions (SS-CRFs), for which it is assumed that crop response, or yield, is a function of fertilizer inputs. (2) Estimate the potential gain from the VRA of fertilizers based on the estimated SS-CRFs. For this estimation, perfect information of cropping conditions is assumed. In other words, the total gain calculated for the period of the longitudinal experiment is presented as the sum of the year specific gains.

Achieving the first objective – to examine whether or not the temporal and spatial variations in crop response exist in the data –is prerequisite for VRA fertilizer strategies to be profitable. If there is no variation temporal or spatial variation is crop response to fertilizers, VRA fertilizer strategies can not increase farm profitability. The sufficient condition is that there is a significant gain from VRA fertilizer strategies. Thus, the second objective of this project intends to examine whether or not the gain is large enough to cover the potential cost of moving from uniform rate application to VRA of fertilizers.

LITERATURE REVIEW: A systematic application of PA takes three steps: "capture of data at an appropriate scale and frequency, interpretation and analysis of that data, [and] implementation of a management response at an appropriate scale and time" (NRC, 1997). Each step requires a high level of accuracy, which recently became possible due to the development of information technologies and advanced machinery, such as microcomputers, geographic information systems, global positioning systems (GPS), automatic control systems on farm machinery, and other electronic technologies (Robert et al., 1994; Ess and Morgan, 2003). In particular, the development history of GPS alone reflects the fact that PA technologies recently became available. The first GPS satellite of the United States, Navister 1, was launched in 1978, and the full 24-satellite constellation was completed in 1994 (NASA, 2005). Considering that GPS is an indispensable technology to locate the exact points of input application in large fields, its development has made modern PA technologies practical.

While a complete analysis of the social benefits of PA technologies would include an estimate of the environmental benefits, this project focuses on the private incentives for farmer adoption of PA technologies in terms of potential profitability. The majority of the previous studies present evidence that PA technologies improve the profitability of agricultural production. For instance, Lambert and Lowenberg-DeBoer (2000) reviewed 113 scientific articles and documents and found 108 of them included the estimation of the economic gains from PA technologies. Sixty-three percent supported the profitability of PA practices, 11 percent presented negative results, and 26 percent were inconclusive. Their review indicated that the profitability of PA technologies depended on the type of crop, the type of technology, and the method of analysis. In addition, the items included in the estimation of the implementation costs influenced profitability results significantly. Some examples of such items were the cost of the experiment used to acquire information on site-specific crop response and the cost to train workers to use PA technologies.

Arguments against the profitability of PA tend to focus on the implementation costs of the technology. For example, Doerge (2004) argued that the cost of switching to PA technologies from conventional uniform management strategies would exceed the potential benefits due to the cost of acquiring advanced information technologies, such as GPS and satellite guided and automated farm machinery. Therefore, he concludes that applying uniform rates of inputs is a better than applying variable rates.

Precision agriculture technologies are essentially applicable to any type of agricultural production. Nonetheless, studies of PA technologies are commonly done using land intensive crops, such as barley, corn, cotton, oats, potato, rice, soybean, sugar beet, and wheat. In their literature review, Lambert and Lowenberg-DeBoer (2000) reported that, of 108 studies with economic analysis, 50 percent looked at corn, 13 percent wheat, nine percent corn-soybean rotations, four percent potato, three percent sugar beet, and three percent soybean. The most studied PA technology is the VRA of fertilizers.

Malzer et al. (1996) was the first to apply ordinary least square (OLS) estimation to estimate site-specific crop response functions (SS-CRFs) and use these SS-CRFs to calculate site-specific economically optimal nitrogen rates (SS-EONRs). They then used these results to calculate the gain in nitrogen returns from using the SS-EONRs instead of the uniform rate recommended by the University Extension Service. This gain ranged from \$11 to \$72 ha^{-1} for different field experiments. Mamo et al. (2003) conducted a similar study and reported gains ranging from \$8 to \$23 ha^{-1}. Some researchers have criticized these results because the OLS method used to estimate the SS-CRFs did not account for the spatial autocorrelation observed in the data (Bongiovanni and Lowenberg-DeBoer, 2001). With spatial autocorrelation, OLS estimates are inefficient, which reduces the reliability of the results.

In order to improve estimation methods for the gains from a VRA fertilizer strategy, Bongiovanni and Lowenberg-DeBoer (2001) used the spatial autoregressive econometrics approach (SARE) to analyze data from on-farm experiments in Argentina. A randomized complete block design experiment was employed and replicated in two different fields to examine crop yield response to nitrogen fertilizer. They found that applying VRA of fertilizers could have been profitable and the net gain estimated was \$4.15 for one field and \$29.09 for the other. Using the same data, Lambert et al. (2002) compared a SARE model with a group-wise (site-specific) heteroskedasticity correction (SARE-GHET) and geostatistical models (GEO) with semivariograms that had one of three different distance functions: the spherical (GEO-S), Gaussian (GEO-G), and exponential (GEO-E) (McBratney and Webster, 1986). The net gain presented in Lambert et al. (2002) ranged from \$3.28 ha^{-1} for OLS to \$7.66 ha^{-1} for both SARE-GHET and GEO-S models. In addition, they tested the fit of these different models and concluded that SARE-GHET fitted better than any of the three GEO models. However, while the SARE-GHET model corrected for spatial autocorrelation and site-specific heteroskedasticity, it did not treat heteroskedasticity and correlation in the errors due to experimental design effects identified by Hurley, Malzer, and Kilian (2004, hereafter referred to as HMK).

Not only spatial autocorrelation and site-specific heteroskedasticity, HMK (2004) found heteroskedasticity and correlation in the regression errors that could be attributable to a systematic lack of randomization in the experimental designs commonly used to generate data for estimating SS-CRFs. HMK (2004) modified the GEO-S and GEO-G models to account for this lack of randomization. Comparing the fit of the two models, they found the GEO-G model fitted better. Using the GEO-G model, they estimated site-specific economically optimal nitrogen rates (SS-EONRs) for two corn fields and showed that applying the SS-EONRs could have resulted in a net gain of \$14.5 ha^{-1}.

As statistical tests have shown in previous work, the application of more sophisticated statistical models, such as spatial econometrics and geostatistics, to spatially distributed data yields more reliable estimates for SS-CRFs. Since varying fertilizer rates spatially has been the concern, the majority of the previous work focused only on spatial variation in crop response. There is now increasing concern among researchers about the effect of temporal variation on crop response. For instance, Whelan and MacBratney (2000) reported that the temporal variation was twice to three times as large as the spatial variation so that applying uniform rates was the best fertilizer management because the site-specific economically optimal rates (SS-EORs) obtained in the past years were not the SS-EORs for the years that followed. Thus, in order to improve site-specific fertilizer management, it is necessary to expand our understanding to the effect of temporal variation in crop response.

Liu and Swinton (2005) and Lambert, Lowenberg-DeBoer, and Malzer (2006; hereafter referred to as LLM) specified the SS-CRFs to incorporate temporally variable factors of crop yield response to fertilizer input(s). A random coefficient model was the choice of Liu and Swinton (2005), who assumed that the coefficients are determined by temporally variable weather conditions, in order to analyze corn's year-specific yield response to nitrogen fertilizer and other uncontrolled input variables. In addition, their specification incorporated the stochastic part of random coefficients into the error terms of the SS-CRFs, which means that the coefficients of the SS-CRFs were assumed to be determined by the average effect of weather conditions. Spatial autocorrelation was assumed to be partially attributed to the stochasticity of random coefficients and was corrected using both a spatial lag model and a spatial error model. By appending site-specific field characteristic variables to the quadratic response functions, and estimating year-specific SS-CRFs, Liu and Swinton (2005) found that there were field characteristics that determined site-specific crop response, such as nitrogen credit, organic matter, cation exchange capacity, electric conductivity, and wetness. In addition, they estimated the net gain from changing to VRA from uniform rates, which was, at most, approximately \$2 ha^{-1}.

An important weakness of Liu and Swinton (2005) that this project addresses is the justification of their empirical methods, which confounds the interpretation of their results. Their OLS approach fails to account for the types of spatial autocorrelation and heteroskedasticity identified by Bongiovanni and Lowenberg-DeBoer (2001). It is also subject to missing variable bias if the variables in their model other than nitrogen do not reasonably describe the primary sources of variation in the SS-CRFs. Their spatial error model fails to account for heteroskedasticity and is also subject to missing variable bias. Their spatial lag approach fails to account for heteroskedasticity. In addition, their specification is inconsistent with their description of the production technology, which makes it difficult to determine how well it controls for any potential missing variable bias.

LLM (2006) analyzed the data generated by a six-year long field experiment (which is the same data used for this project) using the corn-soybean crop rotation (three years for corn and two years for soybean) in southwestern Minnesota. They estimated separate site-specific crop response functions for each year using a General Method of Moments (GMM) estimator when the OLS errors test positively for heteroskedasticity. When the OLS errors also tested positively for spatial correlation, a geostatistically weighted GMM model was estimated. From their analysis, they conclude that there is significant spatial variation in site-specific crop response. They also conclude that phosphorus response was temporally stable for some regions of the field, while nitrogen response was not temporally stable. Finally, using the estimated SS-CRFs, they obtained the site-specific economically optimal rates of nitrogen and phosphorus (SS-EORs) and the potential gain from switching to the SS-EORs from the university recommended uniform rates for three years of corn production. The gains in the three corn years were \$14.19 ha^{-1}, \$14.93 ha^{-1}, and \$21.38 ha^{-1}, respectively.

In order to expand and improve our understanding of the effect of temporal variation as well as spatial variation in crop response to fertilizer, this project aims to add the following contributions to the literature: (1) it will demonstrate the temporal and spatial variation in crop response to nitrogen and phosphorus even when accounting for type of experimental design effects identified by HMK in addition to heteroskedasticity and (2) it will determine the degree to which unpredictable temporal variation in crop response diminishes the potential value of VRA.

METHOD AND MATERIAL

Experiment: The goal of this project is to examine the profitability of the VRA of both phosphorus and nitrogen fertilizers. In order to generate data of corn crop response to these fertilizers, a controlled experiment in a commercially cultivated field was conducted. The experiment field was located near Windom, Cottonwood County, Minnesota (43.63N, 92.87W) – henceforth it is referred to as the "Windom field." This part of the state is located in the northern prairie plain, where the corn-soybean rotation cropping system is very common among large commercial farms, and the Windom field is one such large scale crop field. Using the Windom field, the experiment was conducted between fall 1996 and fall 2001. The corn-soybean rotation was continued during this experiment period, and the yields of corn (*Zea may*, L) were recorded as crop response to different levels of nitrogen and phosphorus fertilizer inputs. In order to identify the other determinants of yield level, field characteristics were enumerated and soil tests were conducted. However, weather conditions such as precipitation and sunlight were not recorded, and it was assumed that the effect of weather conditions was homogenous because of their geographically homogeneous distributions within the Windom field.

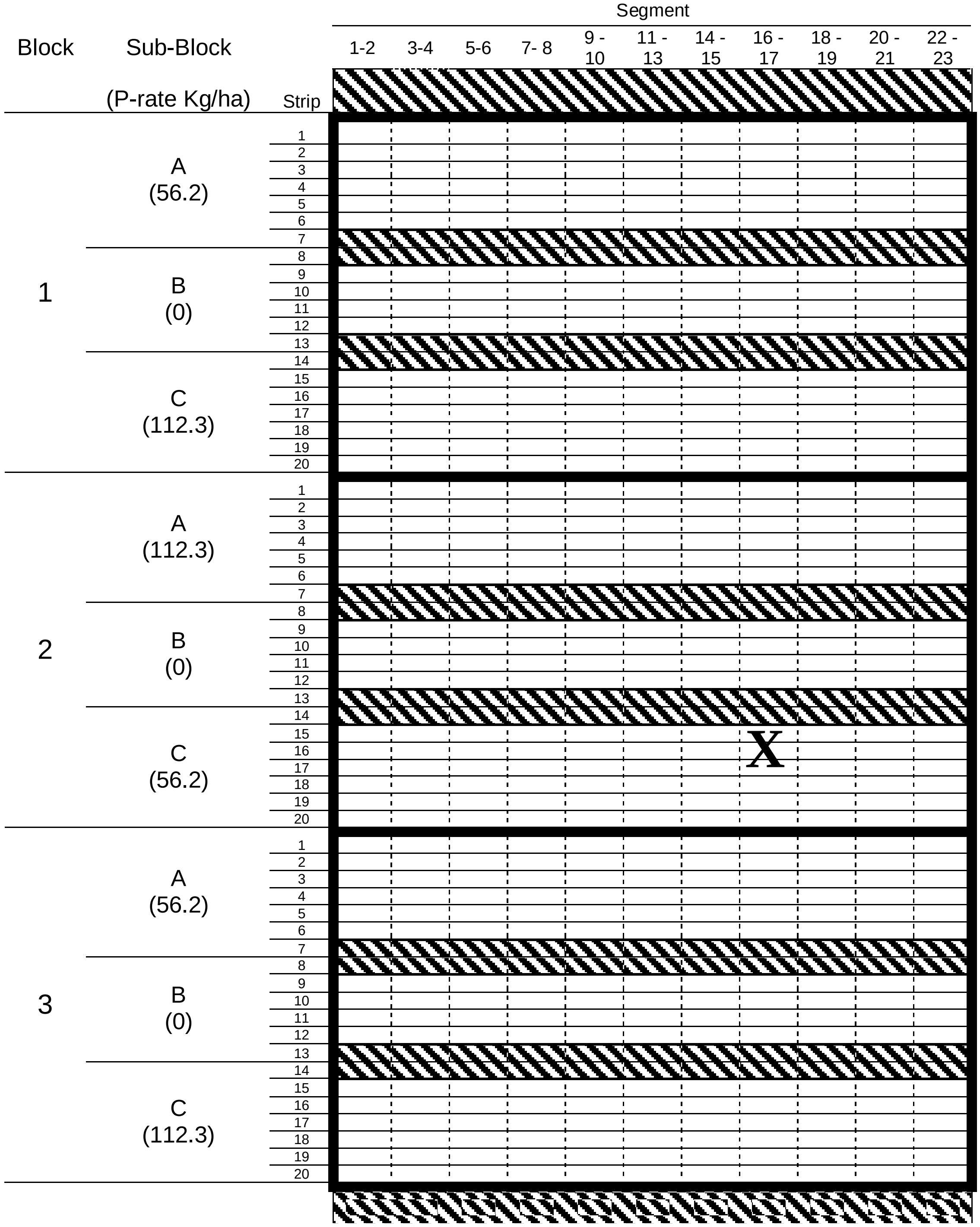
Since weather conditions influence corn yields, it is necessary to understand the general climate condition of the experiment field in Cottonwood County. The climate of the county is interior continental, which is characterized by cold winters (the average temperature for January is approximately -11˚C) and moderately hot summers (the average temperature for July is approximately 22˚C). While this area is relatively dry, crop production does not rely on irrigation, and fields are rain fed. The monthly average precipitation is less than 50 mm between November and February, and in the wettest month, June, it is about 110 mm. The precipitation per annum is about 740 mm. The precipitation in the growing season (May to September) was 566, 533, 391, 353, 457, and 382 mm in years from 1996 to 2001, respectively. This temporal fluctuation in the precipitation, combined with variation in soil and landscape characteristics within the experimental field, likely caused variation in crop yield response.

The ranges of variation in field characteristics and soil conditions are partly determined by the size of the field, topographical characteristics, soil type, and water availability. The size of the experiment field was 381.0 m × 295.7 m (1,250 ft × 970 ft) with the larger dimension in the north-south direction (Figure 1, which is not to scale). The field has its high point in the southeast corner, and from this corner it slopes down toward the northwest corner, which is the lowest point of the Windom field. The difference in the relative elevation between the highest and lowest points was 8.3 meters (27 feet). The average slope was 1.3 percent. The parent material of the soil in the field is glacial deposits, called till, and outwash sediments of till. Identified soils in the Windom field are Clarion and Webster loam in the higher part and Jeffers clay loam in the lower part (USDA, 1977). Due to this soil composition, the drainage of the Windom field is generally good. However, lower points of this field tend to be moister than higher points. In order to improve drainage, a pipe-drain system runs diagonally through the northeast corner of the field, and its inlet is located in the lowest point of that cone-shaped area (marked with a bold X in Figure 1).

Figure 1: Experiment Layout; Three Levels of Phosphorus and Five Levels of Nitrogen in Three Replications; there are 11 segments and 60 treatment strips

Legend: Replication; Buffer zone and Headland;

South → North



In order to implement a fractional factorial design, the Windom field was divided into blocks, sub-blocks, and treatment strips. First, to avoid edge effects, 10.7 m (35 feet)-wide headlands were established on the east and west ends, and likewise 15.2 m (50 feet) on the north and south ends of the field, and yield from the head lands was not recorded. The yield observations used for this study were obtained from the remaining area of 273.6 m (900 feet) width by 345m (1,150 feet) length. This rectangle was divided into three 91.2 m (300 feet)-wide blocks and used to replicate the treatments. Each block was divided into 20 4.6 m (15 feet)-wide treatment strips. A treatment strip contained six 0.76m-wide rows of corn. As shown in Figure 1, these strips were bundled up to form three sub-blocks; the number of strips per sub-block varied because of the size of sub-block B, which was smaller than those of sub-blocks A and C. As Figure 1 shows, the three sub-blocks consisted of strips 1 to 6 (sub-block A), 9 to 12 (sub-block B), and 15 to 20 (sub-block C), respectively. Strips between sub-blocks A and B and B and C (7^{th}, 8^{th}, 13^{th} and 14^{th} strips) were also reserved as buffer zones to separate treatments between replications.

A "treatment" refers to a combination of two fertilizer rates: phosphorus and nitrogen (hereafter referred to as P- and N). These fertilizers were applied in order to supplement phosphorus and nitrogen, but the shares of these nutrients in fertilizers vary depending on fertilizer forms. The P and N rates were calibrated to supplement the nutrients at 0, 56.2, and 112.3 kg ha^{-1} (0, 50, and 100 lb $acre^{-1}$, respectively) of phosphorus and 0, 67.4, 112.3, 157.3 and 202.2 kg ha^{-1} (0, 60, 100, 140, 180 lb $acre^{-1}$, respectively) of nitrogen. Hence, there were 15 possible different treatments. These treatments were assigned to the selected sub-blocks and strips. However, there are treatments omitted from sub-block B of every replication, which made this experimental design fractional factorial. Sub-block B was chosen not to receive some of the treatments because the zero P rate was assigned to sub-block B, and non-zero P rates were assigned to sub-blocks A and C (Figure 1).

Thus, sub-block B was expected to provide the least amount of information regarding the effect of interaction between P and N on corn yield. The buffer zones between sub-blocks A and B and B and C were established to separate the effect of different P rates. No buffer zone was necessary between blocks (replications) because adjacent sub-blocks in different replications were treated with the same P rate. N rates were more diverse. Two strips of every sub-block were reserved for the zero N rate. Then, in sub-blocks A and C, four N rates among five non-zero rates were applied to the rest of the strips. In sub-block B, two non-zero N rates, 112.3 and 202.2 kg ha^{-1}, were assigned to the rest of the strips.

The method of fertilizer application and their forms and sources varied, and therefore they deserve extra attention. The sources of phosphorus and nitrogen were phosphate (P_2O_5), anhydrous ammonia (NH_3), or urea ($CO(NH_2)_2$).[3] Phosphate was applied to the field in the fall of years 1996, 1998, and 2000 using a broadcast applicator. In the fall of year 1996, anhydrous ammonia was injected into the field using an applicator for liquefied anhydrous ammonia, and in the fall of year 1998 and in spring of year 2001, urea was spread by a broadcast applicator. Again, these fertilizers were applied only before corn years (1997, 1999, and 2001) and before seeds were sown. No fertilizers were applied in 1998 and 2000.

The experiment maintained the corn-soybean rotation system, and both corn and soybeans were seeded during the first half of May each year. The corn cultivars planted were Asgrow® XP–4967 or RX495BT, at a rate of approximately 68,000 seeds ha^{-1} in 1997, 1999, and 2001 following P and N applications. The soybean cultivars planted were Asgrow® AG–

[3] Following the agronomic convention, we assumed that there is no ecological and physiological difference between these sources of nitrogen so that crop response to these fertilizers is identical.

2301RR or 2201RR at the rate of approximately 452,000 seeds ha^{-1} in 1998 and 508,000 seeds ha^{-1} in 2000.

Crops were harvested in the fall of each year using a Massey Ferguson® plot combine equipped with a ground distance monitor and a computerized Harvestmaster® weigh cell. The weight of crop yield was recorded every 15.2 m (50 feet) of the two center rows of a treatment strip. For a corn year, there were 23 yield observations per treatment strip. There were three blocks of 13 strips each. Thus, there were a total of 879 corn yield observations. Corn was dried and yield records were adjusted to reflect 15.5 % moisture.

In order to study site-specific crop response to the VRA of fertilizers, the data were divided into smaller units (or sites), which were assigned after the experiment was completed. Then, the year and site-specific crop response functions (SS-CRFs) were estimated.

Figure 2: Post-Expriment Site Division

Site borders

South → North

Block	Sub-block	Segment 1-2	3-4	5-6	7-8	9-10	11-13	14-15	16-17	18-19	20-21	22-23
	A											
1	B	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
	C											
	A											
2	B	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
	C											
	A											
3	B	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
	C											

We chose 33 sites because it was the fewest number of sites we could estimate while still being able to identify the models spatial, site, treatment, and treatment strip effects. There are two different sizes for our sites due to the odd number of harvest segments (Figure 2). Three sites (6, 17, and 28) were 91.2m wide by 45.7m long, which combined three segments for each replication resulting in 39 observations each. The remaining 30 sites were 91.2m wide by 30m long, which combine two harvest segments for each replication resulting in 26 observations each.

Conceptual Framework

Farmers' Problem: Before discussing the estimation method for the SS-CRFs, we need to understand the economic framework of the farmer's decision making problem for fertilizer inputs. In short, a farmer wants to choose site-specific fertilizer rates in order to maximize the return to fertilizer inputs. A mathematical representation assists with this problem.

Let $t \in \{1,3,5\}$ represent the years 1997, 1999, and 2001 and $r \in \{1,\ldots,R\}$ represent a site within a field that is divided into R contiguous sites. Let P_r^t and N_r^t be the site and time specific application rates for phosphorus and nitrogen. Let y_r^t be the site and time specific yield where $y_r^t = f(P_r^t, N_r^t; z_r^t)$ such that z_r^t represents site and time specific field characteristics that influence yield other than phosphorus and nitrogen. Let a_r represent the number of acres represented by site r. Finally, let p_y^t, w_P^t, and w_N^t be the price of corn, phosphorus, and nitrogen at time t. The farmer's optimization problem for the net return to site-specific fertilizer applications in time t can be written as

$$
\begin{aligned}
& \underset{\{P_r^t, N_r^t\}_{r\in\{1,\ldots,R\}}^{R}}{Max} \sum_{r=1}^{R} a_r \left\{ p_y^t y_r^t - w_P^t P_r^t - w_N^t N_r^t \right\} \\
(1) \qquad & subject\ to \qquad y_r^t = f\left(P_r^t, N_r^t; z_r^t\right) \qquad . \\
& \qquad\qquad\qquad\quad P_r^t \geq 0 \\
& \qquad\qquad\qquad\quad N_r^t \geq 0
\end{aligned}
$$

If $f(P, N; z)$ is globally concave in P and N for all z, the solution to this problem will solve the first order conditions

$$(2) \qquad p_y^t f_P\left(P_r^t*, N_r^t*; z_r^t\right) - w_P^t \leq 0 \text{ with equality if } P_r^{t\,*} > 0 \text{ and}$$

$$p_y^t f_N\left(P_r^t*, N_r^t*; z_r^t\right) - w_N^t \leq 0 \text{ with equality if } N_r^{t\,*} > 0,$$

for $t \in \{1,2,3\}$ and $r \in \{1,\ldots,R\}$ where $f_X(P, N; z)$ is the partial derivative of $f(P, N; z)$ with respect to $X \in \{P, N\}$. The optimal net return to VRA in year t is then

$$p_{SS\text{-}EOR}{}^t = \sum_{r=1}^{R} a_r \left\{p_y^t f\left(P_r^t*, N_r^t*; z_r^t\right) - w_P^t P_r^t * - w_N^t N_r^t *\right\}.$$

Estimating Site-Specific Crop response Functions

The solution to this problem depends of the how phosphorus, nitrogen, and other field characteristics affect yield, $y = f(P, N; z)$, which can be determined by using experimental field data to estimate SS-CRFs. To understand how, consider the Taylor approximation

$$(3) \qquad \begin{aligned} f(P,N,z) = f(\mathbf{0}) &+ \left[\sum_{k_z=0}^{\infty} \frac{\partial^{k_z+1} f(\mathbf{0})}{\partial z^{k_z} \partial P} \frac{z^{k_z}}{(k_z+1)!}\right] P + \left[\sum_{k_z=0}^{\infty} \frac{\partial^{k_z+1} f(\mathbf{0})}{\partial z^{k_z} \partial P^2} \frac{z^{k_z}}{(k_z+1)!}\right] P^2 \\ &+ \left[\sum_{k_z=0}^{\infty} \frac{\partial^{k_z+1} f(\mathbf{0})}{\partial z^{k_z} \partial P} \frac{z^{k_z}}{(k_z+1)!}\right] N + \left[\sum_{k_z=0}^{\infty} \frac{\partial^{k_z+1} f(\mathbf{0})}{\partial z^{k_z} \partial N^2} \frac{z^{k_z}}{(k_z+1)!}\right] N^2 \\ &+ \left[\sum_{k_z=0}^{\infty} \frac{\partial^{k_z+2} f(\mathbf{0})}{\partial z^{k_z} \partial P \partial N} \frac{z^{k_z}}{(k_z+2)!}\right] PN \end{aligned}$$

where $\mathbf{0} = (0,0,0)$. Assuming the approximation error is negligible, equation 3 implies the effect of phosphorus and nitrogen on yield can be written as

$$(4) \qquad y = a_0(z) + a_1(z)P + a_2(z)P^2 + a_3(z)N + a_4(z)N^2 + a_5(z)\,PN,$$

where $a_k(z)$ for $k \in \{0,\ldots,5\}$, are parameters that only vary with the site-specific field characteristics.

Now consider the experimental field data collected for this project: $\{y_i,\ P_i,\ N_i\}$ for i = 1,…M where M is the number of observations; y_i is the observed yield for observation i, P_i is the phosphorus application rate associated with y_i, and N_i is the nitrogen application rate associate with y_i. Equation 4 implies we can use this data and regression analysis to estimate the SS-CRFs:

$$(5)\quad y_i = a_0\left(z_{r_i}^{t_i}\right) + a_1\left(z_{r_i}^{t_i}\right)P_i + a_2\left(z_{r_i}^{t_i}\right)P_i^2 + a_3\left(z_{r_i}^{t_i}\right)N_i + a_4\left(z_{r_i}^{t_i}\right)N_i^{\,2} + a_5\left(z_{r_i}^{t_i}\right)P_iN_i + e_i$$

where e_i is a random error such that $E(e_i) = 0$.

Bongiovanni and Lowenberg-DeBoer (2001) showed that estimating equation 5 using ordinary least squares (OLS) can be problematic due to spatial correlation in the errors. Lambert et al. (2002) showed that OLS can also be problematic due to site-specific heteroskedasticity in addition to spatial correlation. Finally, Hurley, Malzer, and Kilian (2004) showed that OLS can be problematic due to a lack of randomization in the experimental design used to generate the site-specific yield response data for this project. These problems imply that OLS estimates for equation 5 will be inefficient.

To address the problems of spatial correlation, site-specific heteroskedasticity, and lack of randomization in the experimental design, Hurley, Malzer, and Kilian (2004) proposed defining the covariance of errors as

$$(6)\qquad \mathrm{E}[e_ie_j] = \begin{cases} s_{r_is_i}{}^{2} & \text{for } i = j \\ s_{r_is_i}\, s_{r_js_j}\left\{C_1[1 - h(d_{ij})] +\ C_TT_{ij}\ +\ C_SS_{\mathrm{ij}}\right\} & \text{for } i\ \neq\ j \end{cases}$$

where s_i denotes the treatment strip for the ith observation; $s_r > 0$ and $s_s > 0$ are site-specific and treatment strip-specific standard deviations; d_{ij} is the spatial distance between yield observations i and j; $h(d_{ij})$ is a distance function that relates Euclidean distance to spatial correlation; T_{ij} is a dummy variable equal to one if observations i and j had the same phosphorus and nitrogen treatments; S_{ij} is a dummy variable equal to one if observations ***i*** and ***j*** were in the same treatment strip; and C_1, C_T, and C_S are parameters that capture the degree of spatial, treatment, and treatment strip correlation in the errors. Furthermore, they use the common Gaussian distance function to describe the relationship between spatial correlation and distance:

$$(7) \qquad h(d_{ij}) = \begin{cases} 0 & if \;\; i = j \\ 1 - \exp\left[-\left(\dfrac{d_{ij}}{a}\right)^2\right] & if \;\; i \neq j \text{'} \end{cases}$$

where the ***a*** parameter referred to as the range.

Equation 6 accounts for heteroskedasticity by defining the variance of error ($s_{r_i s_i}{}^2$) as dependent on the site and treatment strip associated with the yield observations. If we divide equation by $s_{r_i s_i} s_{r_j s_j}$, we get the correlation coefficient $r_{ij} = C_1(1 - h(d_{ij})) + C_T T_{ij} + C_S S_{ij}$. Notice that this correlation coefficient depends on the distance between yield observations, which accounts for spatial correlation. It also depends on whether two yield observations received the same fertilizer treatment or were within the same treatment strip, which takes into account the lack of randomization in the experimental design.

Equation 6 can also be interpreted in terms of the geostatistical concept of the semivariance: $\dfrac{E\left((e_i - e_j)^2\right)}{2}$. Equation 6 implies the semivariance is

$$(8) \;\; \gamma[d_{ij}] = \begin{cases} 0 & \text{for } i = j \\ s_{r_i s_i} s_{r_j s_j} \left\{ C_0 \; + \; C_1\, h(d_{ij}) + \; C_T(1 - T_{ij}) + \; C_S(1 - S_{ij}) \right\} & \text{for } i = j \end{cases}$$

where $C_0 = 1 - C_1 - C_T - C_S$. Two important features of the semivariance are the nugget and sill. The nugget describes background variation unrelated to distance. It is the lower limit of the semivariance as the distance between random observations approaches zero. In the context of equation 6, this occurs when $h(d_{ij})$, $1 - T_{ij}$, and $1 - S_{ij}$ are 0, which implies the nugget is $s_{r_i s_i} s_{r_j s_j} C_0$. The sill describes the upper bound of the semivariance as the distance between random observations becomes large. In the context of equation 6 this occurs when $h(d_{ij})$, $1 - T_{ij}$, and $1 - S_{ij}$ approach 1, which implies the sill is $s_{r_i s_i} s_{r_j s_j}$. In standard geostatistical models, the nugget and sill are constants. In equation 6, the nugget and sill depend on the site and treatment strip. We used equations 5, 6, and 7 to estimate SS-CRFs for 33 sites with the Windom field data (Figure 2).

With 33 sites, we must estimate 198 parameters (33 sites multiplied by six parameters for each site) for equation 5. For equations 6 and 7, we need to estimate 33 region specific variance parameters, 36 treatment strip variance parameters (39 strips minus 3 for identification), two spatial correlation parameters (C_1 and a), one treatment correlation parameter (C_T), and one treatment strip correlation parameter (C_S). Therefore, for each year we must estimate at most a total of 271 parameters. To estimate these parameters, we have 897 total observations for each year.

Maximum likelihood estimation (MLE) was used estimate our parameters. The likelihood function is

$$(9) \qquad \ln L = -\frac{M}{2}\log(2p) - \frac{1}{2}\log\left(\left|\boldsymbol{\Sigma}(\boldsymbol{\Theta})\right|\right) - \frac{1}{2}(\mathbf{Y} - \mathbf{XA})'\boldsymbol{\Sigma}(\boldsymbol{\Theta})^{-1}(\mathbf{Y} - \mathbf{XA}),$$

where S(Q) is the covariance matrix defined by equations and 6, which depends on the vector of covariance parameters Q = {$s_{r_i s_i}$, a, C_0, C_1, C_T, C_S}; **Y** is a column vector of observed yields; **X** is the matrix of explanatory variables based on equation 6, and **A** is a vector of crop response parameters based on equation 5. Solving 9 for **A** given Q, we obtain $\hat{\mathbf{A}}(\boldsymbol{\Theta})$ = (**X**'S(Q)$^{-1}$**X**)$^{-1}$**X**'S(Q)$^{-1}$**Y**. If we substitute $\hat{\mathbf{A}}(\boldsymbol{\Theta})$ for **A** in equation 9, we get the concentrated log-likelihood

(Greene, 2000, p 426), which depends only on the parameters Q. Optimizing the concentrated log-likelihood with respect to Q yields $\hat{\mathbf{\Theta}}$, such that $\hat{\mathbf{A}} = (\mathbf{X}'\mathrm{S}(\hat{\mathbf{\Theta}})^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathrm{S}(\hat{\mathbf{\Theta}})^{-1}\mathbf{Y}$. The optimization of the concentrated log-likelihood was accomplished using Matlab®.

Hypotheses Test

There are two null hypotheses to be tested regarding both spatial and temporal variation in the site-specific crop response functions (SS-CRFs) as represented by equation 5. To test the hypothesis that the coefficients of the SS-CRFs do not vary in space, two estimated models were compared using likelihood ratio (LR) test for each time period. The first model, referred to as the unrestricted model, is the model defined by equations 5 to 7. The second model, referred to as the restricted model, restricts the first by assuming $a_k(z) = a_k$ for all $k \in \{0,..5\}$ and homoskedasticity (i.e., $s_{r_i s_i} s_{r_j s_j} = \sigma^2$ for all r_i, r_j, s_i, and $s_j \in \{1,..,33\}$ and in the specification for the variance-covariance, $C_1 = 1$, $h(d_{ij}) = 1$, $C_T = 0$, and $C_S = 0$). The LR value is obtained from the formula,

$$LR = 2(\ln L_U - \ln L_R), \tag{10}$$

where L_U and L_R are likelihood values of unrestricted and restricted models, respectively. This value is asymptotically distributed chi-squared with degrees of freedom equal to the number of coefficient restrictions.

We also explore the degree of spatial variation in the crop response parameters by comparing individual coefficient estimates with the average estimate for the field using 90 percent confidence intervals (CI) for the individual estimates. If the field wide average of a coefficient falls outside the 90 percent CI for an individual coefficient estimate, we conclude the coefficient varied significantly in space.

Two approaches were used to explore temporal variation in crop response functions. First, we compared individual coefficients for a region to the temporal average for that region using the 90 percent CIs for the individual estimates. If the temporal average fell outside the CI for a specific year, we conclude the coefficient varied significantly in time.

The second approach normalized the SS-CRF coefficient estimates for each year using the estimated covariance matrix. Then for each type of coefficient a paired *t*-test was used to compare the normalized estimates across two different years. If two sets of the normalized coefficients are different, the t-statistics will be significantly large.

Site-specific economically optimal rates and the returns

Site-specific economic optimal rates (SS-EORs) of fertilizers are obtained by solving the first order conditions after plugging the estimated SS-CRFs into the objective function in equations 1.

Let P_{UMN} and N_{UMN} be the P- and N- fertilizer rates recommended by the Extension Service of the University of Minnesota. The returns to these rate can be written as

$p_{UMN}{}^{t} = \sum_{r=1}^{R} a_r \left\{ p_y^t f\left(P_{UMN}, N_{UMN}; z_r^t\right) - w_P^t P_{UMN} - w_N^t N_{UMN} \right\}$. The difference in this return and the return to the SS-EORs, $p_{SS\text{-}EOR}{}^{t}$ - $p_{UMN}{}^{t}$, provides an estimate of the potential benefit of variable rate fertilizer applications. In addition to the estimation of the year specific returns on the SS-EORs, the returns for SS-EORs from previous years are estimated to examine the profitability of the using optimal variable rate from a previous years (i.e., $p_{SS\text{-}EOR}{}^{t}$ - $p_{UMN}{}^{t'}$ for $t' > t$).

Our estimate of the gain to VRA compared to the Extension recommendation is a measure of the potential profitability of VRA, not the actual profitability. It is only the potential profitability because it does not consider increases in costs due to switching from a uniform fertilizer application to VRA strategy not related to change in the cost of fertilizer. For example, it does not consider the cost of any special equipment or services that might be needed to conduct a VRA strategy, such as a variable rate applicator, GPS unit, and soil sampling. Still, this estimate of potential profitability is useful because it provides an estimate of how big these other costs can get before VRA is no longer profitable.

The prices used to estimate the returns on the VRA are summarized in Table 1. The producer prices of corn used were adopted from the National Agricultural Statistics Service (NASS): $92.32 t^{-1} ($2.35 bu^{-1}) in 1997, $60.50 t^{-1} ($1.54 bu^{-1}) in 1999, and $67.18t^{-1} ($1.71bu^{-1}) in 2001. The November retail prices of phosphorus fertilizer were $0.31 kg^{-1} ($0.141 lb^{-1}) in 1996, $0.31 kg^{-1} ($0.141 lb^{-1}) in 1998, and $0.26 kg^{-1} ($0.118 lb^{-1}) in 2000. Nitrogen fertilizer prices were $0.35 kg^{-1} ($0.159 lb^{-1}) in 1996, $0.32 kg^{-1} ($0.145 lb^{-1}) in 1998, and $0.27 kg^{-1} ($0.123 lb^{-1}) in 2000 (NASS).

Table 1: Prices for Return Calculation

Year	Commodity	Fertilizer	
	Corn	Phosphorus	Nitrogen
1996		$0.31 kg^{-1} ($0.141 lb^{-1})	$0.35 kg^{-1} ($0.159 lb^{-1})
1997	$92.32 t^{-1} ($2.35 bu^{-1})		
1998		$0.31 kg^{-1} ($0.141 lb^{-1})	$0.32 kg^{-1} ($0.145 lb^{-1})
1999	$60.50 t^{-1} ($1.54 bu^{-1})		
2000		$0.26 kg^{-1} ($0.118 lb^{-1})	$0.27 kg^{-1} ($0.123 lb^{-1})
2001	$67.18 t^{-1} ($1.71 bu^{-1})		

RESULTS

Agronomists have traditionally relied on the ordinary least squares estimation (OLS) to obtain crop response functions. However, OLS yields inefficient estimates although unbiased if the specification is correct. In order to improve estimation, the geostatistical approach (GEO) is adopted in this project. GEO uses a more complex variance-covariance matrix in order to improve the efficiency of the estimates.

Goodness-of-Fit

In order to determine the improvement in the goodness-of-fit between two different types of model, we use likelihood ratio test (equation 10). The likelihood test result implied that, the restricted GEO model significantly improved the goodness-of-fit for each year compared with the estimates given by the OLS equivalent model (simple maximum likelihood estimation)[4] (see Table 2).

Table 2: Log-likelihood statistics and Akaike's information criterion for the OLS equivalent and GEO models

	Year		1997	1999	2001
(1) OLS Equivalent	Log-likelihood		-4058.5	-4121.1	-4886.9
	Number of coefficients		7	7	7
(2) Restricted GEO Model	Log-likelihood		-3503.9	-3746.1	-4225.4
	Number of coefficients		45	45	45
	Likelihood Ratio (1) vs (2)		1109.2***	750.0***	1323.0***
(3) Unrestricted GEO Model	Log-likelihood		-3166.8	-3418.8	-3687.3
	Number of coefficients		271	271	271
	LR	(1) vs. (3)	1783.4***	1404.6***	2399.2***
		(2) vs. (3)	674.2***	654.6***	1076.2***

*** Significant at a 99% level

The unrestricted GEO model, which consists of 33 SS-CRFs as shown in equation 5, further improved the goodness-of-fit from the restricted model. Between the OLS equivalent model and the restricted GEO model, the LR values were 1109.2, 750.0, and 1323.0 for years 1997, 1999, and 2001, respectively (with degrees of freedom of 38). Between the OLS equivalent model and the unrestricted model, the LR values were 1783.4, 1404.6, and 2399.2 for year 1997, 1999, and

[4] The OLS equivalent model estimated six coefficients of the field-wide crop response function and a field-wide variance, where the simple GEO model estimated six coefficients of a field-wide crop response function, two coefficients of distance function in the Gaussian semivariogram, a field-wide variance, and 36 strip specific variances.

2001, respectively (with degrees of freedom of 264). Between the restricted and unrestricted models, the LR values were 674.2, 654.6, and 1076.2 for years 1997, 1999, and 2001, respectively (with degrees of freedom of 264). All of these LR values are significant at a 99 percent level, which indicate that there was significant spatial variation in crop response in each year.

Let us continue examining the result of the geostatistical model. The temporal variation was captured by the semivariogram parameters, the range, sill, strip effect, and treatment effect (Table 3). The treatment effect was negligible in all years, but nonetheless, the sizes of the sill and strip effect parameters were relatively large for some years, but not consistent over time. Interestingly, the size of the sill parameter was very close to one another (in fact, they are the same until third decimal place) between 1997 and 2001, and that for 1999 was very small. This was probably due to the fact that there was large spatial variation in crop response in years 1997 and 2001 due to unfavorable weather conditions, but there was not in 1999 since the weather conditions were more favorable. In addition, the smaller range parameter for 2001 than that for 1997 might be due to a larger variation in crop response; some sites experienced crop failure due to heavy rainfalls in 2001. Thus, the spatial variation appeared less spatially correlated in 2001 than other two years.

Table 3: Semivariogram Parameters of the Unrestricted GEO Model

Year	1997	1999	2001
Range (feet)	133.4	100.7	40.9
Sill	0.717	3.52×10^{-9}	0.717
Strip Effect	0.037	0.246	2.45×10^{-3}
Treatment Effect	1.18×10^{-23}	4.06×10^{-9}	2.66×10^{-3}

Spatial and temporal variations are also shown by comparing the estimated SS-CRFs with the temporal average SS-CRFs and the field-wide average crop response function. Table 4 shows the proportion of coefficients for which we cannot reject the null hypothesis when compared to the experiment, year-specific, or site-specific average. For 90 percent CIs, we would expect these proportions to be close to 0.9. What we see instead is that for all types of coefficients these proportion are less than 0.9, which leads us to conclude that there is significant temporal and spatial variation in the SS-CRFs.

Table 4: Proportion of coefficients that were not significantly different (90% confidence interval) for the whole field, year, and site average coefficient estimate

Coefficient	Proportion of Insignificant Coefficients: Experiment-wide	Spatial (Year-specific)	Temporal (Site-specific)
Constant	0.404	0.616	0.414
Phosphorus Linear	0.485	0.535	0.535
Phosphorus Squared	0.515	0.576	0.525
Nitrogen Linear	0.687	0.768	0.677
Nitrogen Squared	0.727	0.848	0.636
Phosphorus × Nitrogen	0.525	0.586	0.596

Table 5: P-values for Paired t-test

Coefficient	Year	1997	1999
ALL	1999	0.046	1.000
	2001	0.139	0.585
Constant	1999	0.000	1.000
	2001	0.004	0.383
Phosphorus Linear	1999	0.014	1.000
	2001	0.042	0.660
Phosphorus Squared	1999	0.010	1.000
	2001	0.007	0.739
Nitrogen Linear	1999	0.344	1.000
	2001	0.256	0.910
Nitrogen Squared	1999	0.055	1.000
	2001	0.758	0.091
Phosphorus × Nitrogen	1999	0.947	1.000
	2001	0.872	0.840

The results of paired t-test between year-specific SS-CRFs also indicate temporal variation in crop response. Table 5 presents the probabilities that the year specific SS-CRFs are not different between years. When all coefficients of the SS-CRFs were tested jointly, they were different between 1997 and 1999. Comparisons for individual coefficients indicated that the constant, linear, and quadratic terms for phosphorus, and the quadratic term for nitrogen were temporally variable for a 90 percent confidence level. Although the linear term of nitrogen and the interaction term were not significantly temporally variable, we point out that the optimal SS-EORs of nitrogen are temporally variable because of temporally variable quadratic coefficient for nitrogen.

Site-Specific Economically Optimal Rates and Crop Responsiveness

Plugging the best fitting model, the unrestricted GEO, into the farmer's problem (equations 1), we estimated the SS-EORs of P- and N- fertilizer rates, solving the first order conditions (equations 2) for each year. The temporal variation in the SS-CRFs is reflected in the estimated SS-EORs. Most of the sites showed temporally variable SS-EORs; there was only one site (site 14) for which the SS-EOR of phosphorus fertilizer was consistent over time, and it was zero kg ha^{-1} (Figure 3). Also there was only one site (site 4) for which the SS-EOR of nitrogen fertilizer was consistent over time, and it was 202.17 kg ha^{-1} (Figure 4).

Figure 3: The Site-Specific Economic Optimal Phosphorus Fertilizer Rates (kg ha^{-1})

	1997			1999			2001		
Segment	Block 1	Block 2	Block 3	Block 1	Block 2	Block 3	Block 1	Block 2	Block 3
22-23	112.32	112.32	112.32	76.23	73.10	72.22	112.32	70.36	77.32
20-21	112.32	76.90	99.90	85.97	78.30	71.29	112.32	78.59	99.54
18-19	112.32	77.19	100.17	99.63	66.99	69.64	82.60	49.31	38.77
16-17	112.32	65.22	101.44	112.32	0	74.20	70.10	47.50	67.61
14-15	0	55.50	69.45	112.32	112.32	72.10	0	67.19	61.80
11-13	0	112.32	57.65	112.32	65.91	69.88	112.32	0	0.
9-10	41.25	103.42	112.32	61.89	107.73	67.64	0	70.97	42.88
7-8	48.07	0	112.32	84.60	112.32	112.32	108.72	112.32	0
5-6	49.07	0	36.97	84.97	0	0	108.40	0	0
3-4	0	0	0	112.32	0	0	112.32	44.51	66.22
1-2	0	0	112.32	112.32	42.99	60.71	112.32	52.81	37.97

Figure 4: The Site-Specific Economic Optimal Nitrogen Fertilizer Rates (kg ha^{-1})

	1997			1999			2001		
Segment	Block 1	Block 2	Block 3	Block 1	Block 2	Block 3	Block 1	Block 2	Block 3
22-23	119.13	0	133.59	158.01	84.36	154.78	131.12	96.44	111.78
20-21	112.23	59.10	89.45	103.40	65.69	134.45	131.54	91.50	202.17
18-19	124.59	55.47	111.24	0	0	128.40	111.37	202.17	13.92
16-17	124.92	17.99	112.40	0	101.83	143.14	61.89	0	118.27
14-15	151.22	110.98	108.17	0	0	123.58	87.80	67.59	86.42
11-13	166.85	62.87	162.77	202.17	202.17	126.25	142.95	30.12	0
9-10	139.94	97.17	107.31	202.17	202.17	131.96	100.24	110.27	66.84
7-8	202.17	202.17	128.19	202.17	195.94	202.17	202.17	125.01	27.13
5-6	202.17	97.28	100.94	74.53	111.94	106.84	202.17	110.85	134.57
3-4	202.17	202.17	54.36	61.67	192.75	162.18	132.56	169.26	134.04
1-2	202.17	202.17	110.85	0	116.00	89.91	115.53	125.86	120.65

The best fertilizer application scheme

This report, so far, has presented evidence that the crop response varied both spatially and temporally. However, this is only a prerequisite to the VRA of fertilizers. For the VRA of fertilizers to be practical, it is necessary to present evidence that this technology potentially improves profitability. Changing to VRA from a uniform application management approach is justified if the return on the SS-EORs is greater than that on the UMN rates. For the Windom field, the UMN recommended rate for a uniform application of phosphorus and nitrogen were 56.2 kg ha^{-1} (50lb $acre^{-1}$) and N =157.3 kg ha^{-1} (140 lb $acre^{-1}$).

Figures 5 and 6 present the estimated site-specific returns on the SS-EORs and the UMN rates for each year. Then, Figure 7 shows the site-specific difference between these two sets of returns. Note that the sizes of the returns on the SS-EORs in 1997 were much larger than those in 1999 and 2001; these larger returns were caused by the higher corn price in year 1 than 1999 and 2001, with the prices of fertilizers almost constant for all years (Table 1). With the UMN rates, the returns were lower than those to SS-EORs, especially in 2001 (Figure 6), when there were sites that had a negative return. Thus, the returns on the SS-EORs were larger than those to the UMN rates in all sites in all three years. One can easily obtain the potential site-specific gains by subtracting the values in Figure 6 from those in Figure 5. The average site-specific gains were \$63.58 ha^{-1}, \$64.34 ha^{-1}, and \$117.42 ha^{-1} in 1997, 1999, and 2001 respectively. Those gains ranged from \$0.58 ha^{-1} in site 28 to \$170.05 ha^{-1} in site 10 in 1997, from \$16.43 ha^{-1} in site 33 to \$311.45 ha^{-1} in site 15 in 1999, and from \$2.97 ha^{-1} in site 13 to \$507.14 ha^{-1} in site 2 in 2001.

Figure 5: Estimated Return on the Site-Specific Economic Optimal Rate Application (\$$ha^{-1}$)

	1997 Block			1999 Block			2001 Block		
Segment	1	2	3	1	2	3	1	2	3
20-21	739.27	795.51	748.51	469.65	562.15	576.77	473.87	555.57	472.20
18-19	724.54	810.19	835.87	462.56	617.31	607.97	546.16	515.40	534.87
16-17	773.61	790.19	839.11	558.36	626.44	621.18	615.89	558.49	530.47
14-15	676.65	737.11	825.02	519.68	605.67	618.42	467.20	553.11	540.01
11-13	762.87	732.44	745.26	452.75	541.34	602.55	497.88	242.80	237.50
9-10	785.93	665.75	738.09	556.23	551.52	572.37	444.30	579.89	565.14
7-8	800.94	675.73	715.91	551.86	537.10	484.68	466.22	429.58	315.87
5-6	700.65	735.86	706.39	520.34	499.75	558.44	510.97	494.58	491.22
3-4	665.02	816.01	738.37	507.63	547.86	571.16	428.93	509.97	531.52
1-2	768.84	804.58	779.88	570.95	585.29	576.36	440.50	560.75	557.14

Figure 6: Site-Specific Return (\$$ha^{-1}$) on the UMN Rates (P = 56.2 kg ha^{-1} and N = 157.3 kg ha^{-1})

	1997 Block			1999 Block			2001 Block		
Segment	1	2	3	1	2	3	1	2	3
22-23	593.78	675.53	722.86	450.61	507.51	529.22	214.71	444.90	450.79
20-21	569.22	739.50	724.72	417.23	494.21	551.61	227.53	490.79	454.23
18-19	626.41	778.96	792.12	415.06	558.96	576.94	460.86	497.80	466.33
16-17	653.55	776.07	790.66	361.72	544.51	584.16	397.51	510.64	515.56
14-15	611.86	730.66	792.48	411.76	531.95	586.03	402.10	527.82	515.56
11-13	684.13	706.70	744.68	408.55	518.46	576.42	341.91	-148.86	-151.23
9-10	765.85	654.40	680.18	501.96	476.26	547.09	360.83	546.26	514.07
7-8	785.52	515.77	577.77	480.70	225.66	357.04	432.05	243.54	-148.22
5-6	696.18	678.09	690.51	490.36	445.33	514.81	455.54	452.12	469.42
3-4	605.50	674.64	690.91	442.50	501.80	536.40	-78.21	507.00	526.82
1-2	692.96	735.97	702.26	414.10	561.21	529.52	191.38	556.34	546.09

There was a tendency that the low-return sites in 1997 were low return sites in 1999 and the high return sites in 1997 were high return sites in 1999. For instance, both in 1997 and 1999, sites 2, 3, 6, 7, and 9 were among the ten lowest return sites, and sites 12, 19, 20, 29, 31, and 30 were among the 10 highest-return sites. In 2001, site 2 remained in the 10 lowest-return sites, and sites 12, 19, and 29 remained in the 10 highest return sites (Figure 5).

If the trend in site-specific demand persists over time, applying the SS-EORs of the previous years rather than the UMN rates might improve the return on fertilizer inputs. We found that applying the SS-EORs of immediately previous year is a better fertilizer input management approach than the UMN rates. Figure 7 shows the returns on the SS-EORs of 1997 if used in 1999 and 2001 (Figure 7, left and middle panels, respectively). All the potential returns are positive in year 1999, but some of them were negative in year 2001. The potential gains from applying the SS-EORs of previous years instead of the UMN rates are obtained by subtracting the values shown in Figure 8 from the values in Figure 7; the gains ranged from \$-82.21 ha^{-1} in site 27 to \$292.59ha^{-1} in site 15 in 1999 and ranged from \$-645.35 ha^{-1} in site 27 to \$249.31 ha^{-1} in site 11 in 2001. The potential gains from applying the SS-EORs of 1999 instead of the UMN rates in 2001 ranged from \$-105.70 ha^{-1} in site 16 to \$491.86 ha^{-1} in site 2.

The average site-specific gain from changing from the UMN rates to the SS-EORs of year 1997 if applied in year 1999 was \$21.38 ha^{-1}, whereas that in 2001 was \$-12.96 ha^{-1}.

Figure 7: Return (\$ ha^{-1}) on Site-Specific Economic Optimal Rate of Previous Years

	If the SS-EORs of 1997 were used						If the SS-EORs of 1999 were used		
	1997 Block			1999 Block			2001 Block		
Segment	1	2	3	1	2	3	1	2	3
22-23	440.97	497.45	515.85	464.02	417.69	456.58	257.65	460.17	467.08
20-21	451.73	561.54	558.66	472.31	551.59	450.36	351.96	553.65	465.88
18-19	429.18	602.73	585.89	473.46	449.59	361.32	421.75	501.91	446.58
16-17	423.69	600.01	593.49	260.97	543.87	499.77	377.22	506.20	528.15
14-15	493.76	541.06	615.89	464.96	460.64	538.79	332.79	399.87	536.47
11-13	364.90	511.97	581.45	374.20	70.73	-194.53	471.39	-192.94	-164.06
9-10	481.28	507.52	464.88	406.59	462.82	-131.28	335.28	440.55	458.73
7-8	517.89	518.25	410.18	419.66	425.58	-154.82	459.29	408.91	-225.29
5-6	484.92	499.36	549.42	458.98	493.70	473.23	439.85	494.57	488.95
3-4	399.02	547.71	532.67	80.04	469.13	491.56	413.64	470.16	483.76
1-2	401.63	560.03	510.44	-147.65	445.56	456.95	369.42	558.41	533.72

The average site-specific gain from changing from the UMN rates to the SS-EORs of 1999 if applied in 2001 was \$19.93 ha^{-1}. This finding implied that if the SS- EORs from the immediately previous crop season were available, they would be more desirable than the UMN rate to improve the return on fertilizer application.

CONCLUSION: This chapter presented evidence of spatial and temporal variations in corn crop response to phosphorus and nitrogen fertilizer in a large scale field. The 10 ha field was divided into 33 sites, and quadratic site-specific crop response functions (SS-CRFs) were estimated using a geostatistical approach. According to the linear coefficient of phosphorus in the SS-CRFs, 46.5 percent of 33 SS-CRFs varied over time, and according to the quadratic coefficient of phosphorus, 47.5 percent of 33 SS-CRFs varied in space. These variable SS-CRFs resulted in temporally variable site-specific economically optimal rates (SS-EORs) of phosphorus and nitrogen fertilizers; there was only one site that demanded the same fertilizer rate every corn year for both phosphorus and nitrogen. As the chapter has shown, applying SS-EORs is potentially more profitable than the UMN rates; the average site-specific gains were \$63.58ha^{-1}, \$64.34 ha^{-1}, and \$117.42 ha^{-1} in years 1997, 1999, and 2001, respectively. Furthermore, applying the SS-EORs of the immediately previous year can be more profitable than the UMN rates; the gain from applying the SS-EORs of year 1997 instead of the UMN rates in year 1999 was \$21.38 ha^{-1} and the gain from applying the SS-EORs of year 1999 instead of the UMN rates in year 2001 was \$19.93 ha^{-1}. Thus, changing from the UMN rates to the SS-EORs improve returns in fertilizer inputs.

In order to calculate the returns on fertilizer inputs presented in this chapter, certainty in cropping conditions was assumed. This means that farmers knew all the necessary information for application of variable rates of fertilizer inputs in advance. Such an assumption is unrealistic. Some researchers argue that temporal variation diminishes the value of managing spatial variation in site-specific fertilizer demand. Therefore, estimation of the “expected” gains from the variable rate application of fertilizers under uncertainty in cropping conditions is left as a future study. In addition, in order to improve the PA practice regarding fertilizer application, it is necessary to predict future cropping conditions, but what can help us to do so? In the future, this question should be answered by identifying the determinants of crop response to fertilizer inputs. Furthermore, the results of the present and past econometric studies on crop response to fertilizer inputs can be capitalized once a method of predicting site-specific crop response functions relying on this secondary information is established.

REFERENCES

Bongiovanni and Lowenberg-DeBoer. 2001. "Economics of Nitrogen Management in Corn Using Site-specific Crop Response Estimates from a Spatial Regression Model." *Journal of Precision Agriculture* 5: 359-387.

Doerge, T. A. 2004. "Variable-rate nitrogen management for corn production – success proves elusive." Site-Specific Management Guidelines, SSMG-36, Potash and Phosphate Institute, Canada. Available online at http://www.ppi-far.org/ssmg, Accessed on July 23, 2004.

EPA (US Environmental Protection Agency). 2003. "Protecting America's Land Legacy, Stewardship Policies, Tools, and Incentives, to Protect and Restore America's Land Legacy, Environmental Financial Advisory Board (EFAB)." Available online at http://www.epa.gov/efinpage/efab/stewardship_2003.pdf, Accessed March 2006.

Graig, Barbara J., Phillp G. Pardey, Johannes Roseboom. 1997. "International productivity patterns: Accounting for input quality, infrastructure, and research." *American Journal of Agricultural Economics* 79 (4): 1064 – 1076.

Greene, William H. 2000. *Econometric Analysis. 4th Ed.* Prentice Hall International Inc., New Jersey.

Hurley T. M., G.L. Malzer, and B. Kilian. 2004. "Estimating Site-Specific Nitrogen Crop Response Functions: A Conceptual Framework and Geostatistical Model." *Agronomy Journal* 96 (1):1331-1343.

Hurley, T. M., K. Oishi, G.L. Malzer. 2005. "Estimating the Potential Value of Variable Rate Nitrogen Applications: A Comparison of Spatial Econometric and Geostatistical Models." *Journal of Agricultural and Resource Economic* 30 (2): 231-249.

Lambert, Dayton and J. Lowenberg-Debore. 2000. "Precision agriculture Profitability review." Site-specific Management Center, Purdue University. Available online at http://mollisol.agry.purdue.edu/SSMC/Frames/newsoilsX.pdf. Accessed March 2006.

Lambert, D. M., Lowenberg-DeBoer, J., and R. Bongiovanni. 2002. "Spatial Regression, An Alternative Statistical Analysis for Landscape Scale On-farm trials: Case Study of Variable Rate Nitrogen Application in Argentina." In: Proceedings of the 6th International Conference on Precision Agriculture. July 14-17, 2002, Bloomington, Minnesota. ASA, CSSA, and SSSA, Madison, WI.

Lambert, M. Dayton, Gary Malzer, and James Lowenberg-Deboer. 2004. "General Moment and Quazei-maximum likelihood estimation of a spatially autocorrelated system of equations: an empirical example using on-farm precision agriculture data." Staff Paper #04-02, Agricultural Economics, Purdue University.

Lambert, M. Dayton, James Lowenberg-Deboer, and Gary Malzer. 2006. "Economic Analysis of Spatial-Temporal Patterns in Corn and Soybean Response to Nitrogen and Phosphorus." *Agronomy Journal* 98:43-54.

Liu, Yanyan and Scott M. Swinton. 2006. "Is Site-Specific Response Consistent over Time? Does It Pay?" *American Journal of Agricultural Economics* 88: 471-483.

Malzer, G. L., P.J. Copeland, J.G. Davis, J.A. Lamb, P.C. Robert, T.W. Bruulsema. 1996. "Spatial variability of profitability in site-specific N management." In Proceedings of International conference, 3rd, Minneapolis, Minnesota. June 23-26, 1996: 967-976. ASA, CSSA, and SSSA, Madison, WI.

Mamo, M., G. L. Malzer, D.J. Mulla, D. R. Huggins, and J. Strock. 2003. "Spatial and temporal variation in economically optimum nitrogen rate for corn." *Agronomy Journal* 95: 958-964.

McBratney, A.B. and R. Webster. 1986. "Choosing functions for semi-variagrams of soil properties and fitting them to sampling estimates." *Journal of Soil Science* 37: 617-639.

NASA. 2005. Global Positioning System. Mission and Spacecraft Library http://samadhi.jpl.nasa.gov/msl/Programs/gps.html

NRCS (Natural Resource Conservation Service, USDA). 1997. Environmental Quality Incentives Program Final Rule, Federal Register: May 22, 1997 (Volume 62, Number 99) Rules and Regulations: pp 28257-28292. Available online at http://www.nrcs.usda.gov/programs/farmbill/2002/rules/eqipfinal970522.html. Accessed May 2006.

Robert, P.C., R.H. Rust & W.E. Larson. 1994. "Site-Specific Management for Agricultural Systems." In Proceedings of the third International Conference on Precision Agriculture: June 23-26, 1996, Minneapolis, Minnesota. ASA, CSSA, and SSSA, Madison, WI.

Whelan, B. M. and A.B. McBratney. 2000. "The 'null hypothesis' of precision agriculture management." *Precision Agriculture* 2: 265-279.