A Dynamic Analysis of Water Savings from Advanced Irrigation Technology

Robert H. Hornbaker and Harry P. Mapp

A computerized grain sorghum plant growth model is combined with recursive programming to analyze the potential irrigation water savings from adopting irrigation scheduling and low pressure center pivot irrigation technology. Results indicate that irrigation pumping can be reduced with increased yields and net returns by adopting low energy precision application (LEPA) irrigation systems. Variations in input and output prices affect optimal irrigation quantities for low pressure irrigataion systems less than for high pressure systems.

Key words: irrigation scheduling, irrigation technology, plant growth model, recursive programming.

Throughout irrigated regions of the United States, farmers have responded to declining water supplies and increasing pumping costs by adopting improved irrigation technology. Many irrigators have converted older high pressure center pivot irrigation systems to more efficient low pressure systems. Additional savings may be gained by adopting the low energy precision application (LEPA) sprinkler system (Lyle and Bordovsky). Low pressure irrigation systems reduce variable pumping costs by reducing fuel consumed to create pressure in the irrigation nozzle. In addition, by moving the nozzle or deflector closer to the plants, water application efficiency is increased.

In a static model, it is assumed that production occurs in the area where the marginal product and average product are positive and diminishing. Figure 1 shows the impacts that changes in crop prices and irrigation water costs have on demand for water under profit-maximizing conditions. Because the marginal value product is the marginal physical product of water times the price of the commodity, a decrease in commodity price shifts the MVP curve downward to MVP_1. This shift results in a decrease from w to w_1 in the optimal quantity of irrigation water applied. As per acre-inch irrigation costs fall as a result of decreased fuel prices and/or increased application efficiency, the marginal factor cost falls from MFC to MFC', leading to an increased optimal water application of w'. The increase from w to w' represents an increase in effective water or water actually reaching the plant. With improvements in application efficiency associated with low pressure systems, increases in effective water may be possible while actual withdrawals from underground sources are reduced because less water is lost to evaporation and runoff. Of course, if low pressure systems are also more profitable, producers could expand irrigated production and increase overall water use. This study focuses on water savings which might accrue if irrigated acres remain constant.

Static marginal analysis is often used in water studies where the unit of time is the growing season. The dynamic and risky nature of irrigated crop production calls for analysis of a complex set of relationships on a weekly or daily basis. The yield response for irrigated crops varies with the timing and amount of irrigation water applied during the growing season (Hexem and Heady). Also, the production function for irrigated crops is dependent not only on soil moisture but on a host of other variables, including temperature, humidity, solar radiation, wind movement, soil fertility, and competitive insects and weeds.

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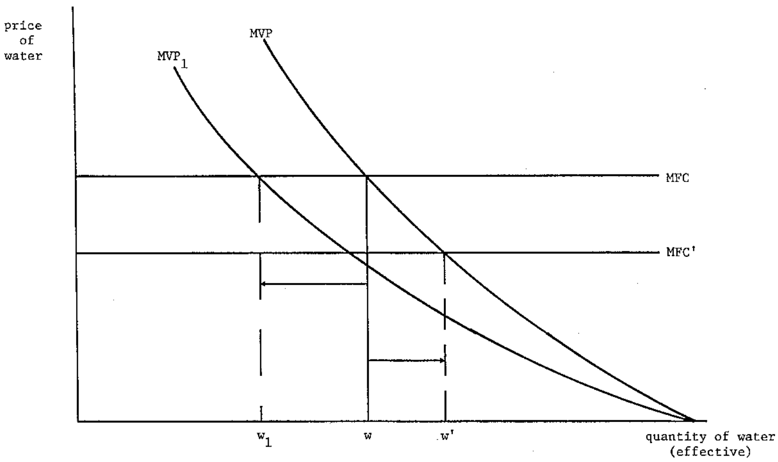

Figure 1. Static model marginal analysis of optimal water applications

Irrigation decisions are dynamic and recursive; a decision to initiate an irrigation application is considered daily during much of the growing season. If the soil water level is sufficient or if the plant is not susceptible to soil water stress during the current stage of development, the decision is made not to initiate an application. The same decision must be faced the next day and on subsequent days until conditions are such that an application is initiated. Once pumping begins, several days are required to complete the application. Once the application is completed, the decision maker again must consider the irrigation decision on a daily basis. Procedures designed to develop optimal irrigation decisions must consider the dynamic and recursive nature of the decision environment.

Previous Studies

Boggess et al. discuss the objectives of irrigation managers and relate them to recent studies of irrigation water use. Most of the studies cited imply that the irrigation manager has a single-dimensional objective, such as maximization of unconstrained yield or unconstrained profit. Much of the research on risk management has relied on the expected utility model and a general assumption of risk aversion. Antle, however, argues that dynamic risk-neutral models may prove more useful than conventional static risk-averse models for understanding the role of production risk in farm management analysis. Dynamic analysis of ground water management and irrigation scheduling have typically been conducted using dynamic programming or simulation techniques.

Burt is among the first to apply dynamic programming to groundwater management problems. One of the earlier irrigation scheduling models, presented by Burt and Stauber, is designed to analyze the feasibility of irrigation investment in a subhumid climate. By concentrating on a limited time horizon, the number of stages and states is reduced to manageable proportions. However, their use of sixty-day stages seriously inhibits the accuracy of the response model because the sensitivity of a unit of water stress can differ significantly within a stage.

In another irrigation scheduling study (Dudley, Howell, and Musgrave) utilizing stochastic dynamic programming, plant growth is determined by chronological dates. This study also incorporates a simulation routine to estimate transition probabilities and the transition function from one state of soil moisture and crop yield system to another. Their specification does not account for differing magnitudes of daily growth and implies that growth in one period is independent of past growth patterns of the crop.

A number of other dynamic programming models have been developed to optimize irrigation schedules (Hall and Butcher; Howell, Hiler, and Reddell; Yaron et al.; Morgan, Biere, and Kanemasu; Raju et al.). Of the dynamic programming studies, McGuckin et al. provides perhaps the most thorough treatment of the soil moisture and plant growth complexities. Still, it does not consider the implications of timing irrigations relative to the key stages of plant development for each crop or of alternative irrigation technologies.

There are a number of studies which use other methodologies to evaluate irrigation schedules based on plant growth relationships. The primary contributions of these studies are improved specifications of the agronomic relationships describing the irrigation-plant growth environment and incorporation of multiple crops in the decision framework. However, the decision environment is typically nonoptimizing, and irrigation applications are based on specified priority criteria.

An interesting study which considers both the stochastic environment and dynamics of the irrigation scheduling optimization problem is that of Zavaleta, Lacewell, and Taylor. They use the grain sorghum growth model by Maas and Arkin to consider stochastic weather and allow irrigation timing and quantity decisions to be based on an expected profit maximization criterion. Numeric search procedures, referred to as open-loop stochastic control, are used to derive irrigation strategies which maximize expected profits over the eight discrete irrigation periods of the crop year.

Harris and Mapp (1980, 1986) use the same grain sorghum plant growth model to analyze intensive and water-conserving irrigation strategies. A number of irrigation strategies were simulated with their modifications to the plant growth model. The quantity of each irrigation application was held constant, and the timing of the irrigations was based on the stage of grain sorghum development and the extractable soil water level. Stochastic dominance procedures were used to identify risk efficient irrigation strategies.

None of these studies looks at irrigation timing and quantity decisions on a daily basis for alternative irrigation technologies and input and output price combinations. The study presented in this paper evaluates the potential water savings which might be generated by low pressure delivery systems using irrigation scheduling to time water applications in accordance with plant needs. The approach taken utilizes a daily plant growth model for grain sorghum and twenty-three years of daily historical weather data to generate yields and net returns. Recursive programming is used to optimize water use within the growing season. Results focus on both quantity and timing of water applications. In addition, the model is used to analyze the potential impact of variations in input and output prices on the optimal irrigation schedule for high pressure, low pressure and LEPA center pivot irrigation systems.

Model Development

The model developed for this study combines simulation and a recursive optimization model. The simulation component consists primarily of a daily plant growth model for grain sorghum, an important irrigated crop in the study area. The grain sorghum growth model, which has been reported in the literature (Arkin, Vanderlip, and Ritchie) and is available for use by researchers (Mass and Arkin), was modified to fit local growing conditions. Modifications of the grain sorghum model have been used in other economic analyses and reported elsewhere (Zavaleta, Lacewell, and Taylor; Harris and Mapp 1980, 1986). The growth model utilizes daily climatic data, including minimum and maximum temperature, precipitation, and solar radiation, to determine the phenological growth stage for the plant and to calculate leaf development and daily plant growth. To simulate the impact of weather variability on grain sorghum development, twenty-three years of actual weather data are used to generate yields and net returns. Beginning soil moisture levels for each year of simulation are calculated from an equation estimated by Mapp et al. An irrigation subroutine was added to the plant growth model to calculate the number of hours required for various center pivot irrigation systems to irrigate a specified number of acres.

A recursive optimization model is formulated to permit solution on a microcomputer. The recursive procedure works forward through the season to solve for quantities and timing of irrigation applications while maximizing expected net returns. Each day during the growing season is the beginning of a decision period or stage. The state of the system at any point in time is described by the accumulated growth of the grain sorghum plant, soil moisture level, and climatic conditions. Initially, perfect knowledge of future weather conditions is assumed, although alternative assumptions may be made.[1]

The recursive formulation is specified as

$$(1)\quad G_n(S_n, M_n, C_n) = \text{Max}[R_n(S_n, M_n, C_n \mid W_{ni})]$$

for $n = 1 + e, 1 + e + 1, \ldots 1 + e + m$, where G_n is the maximum value function in stage n; R_n, the expected return function in stage n; S_n, the set of state variables describing the condition of the grain sorghum plant; M_n, the set of soil moisture state variables; C_n, the set of climatic state variables; W_{ni}, the amount of irrigation water applied for irrigations; $i = 1, 2, \ldots, 6$; e, the number of days from planting to emergence and m, the number of days from emergence to plant maturity.

Transition equations for the state variables consist of the calculations performed by the daily plant growth model. The model calculates plant emergence, leaf appearance, daily total leaf area, daily light interception, daily potential photosynthesis, daily total evapotranspiration, daily water stress, daily temperature stress, daily net photosynthesis, daily dark respiration, daily stage of plant development, and daily dry matter gain. The grain sorghum plant does not produce grain until the fourth stage of its five stages of plant growth. Preceding the fifth stage of plant growth, expected grain weight is estimated as a function of the accumulated plant dry matter and the stage of plant growth.

To include varying irrigation application quantities, a number of irrigation alternatives are included in the formulation: a no-irrigation option and three different levels of irrigation (1.4 acre-inches, 2.1 acre-inches, and 2.8 acre-inches). The timing and quantities of irrigation are determined by the model. All irrigation levels are specified in terms of the gross quantity of water applied in acre-inches.

Because approximately eight days are required to apply the 2.8-acre-inch application, six scenarios are considered within the eight-day period: (*a*) no irrigation; (*b*) irrigate 1.4 acre-inches in the first four days; (*c*) irrigate 1.4 acre-inches in the last four days; (*d*) irrigate 1.4 acre-inches in the first four days and 1.4 acre-inches in the last four days; (*e*) irrigate 2.1 acre-inches in the first six days; and (*f*) irrigate 2.8 acre-inches over the eight days.

The return function R_n is computed based on information calculated previous to day $n - 8$. In a dynamic fashion the model computes information for different irrigation alternatives for day $n - 8$ to n. Once the choice of irrigation timing and quantity is made, the recursive program updates the state conditions using the selected irrigation alternative and considers the next eight-day decision period. The growth year is not divided into a unique set of eight-day stages. The decision stages are based upon the previous irrigation decision such that the transition equations are updated by the plant growth model for only the numbers of days required for the chosen alternative. If the expected return function is maximum for no irrigation during the eight-day decision period, the next decision period will begin after only one day of nonirrigated plant growth.

Also within the recursive algorithm is a modification to allow a preplant or post plant preemergence irrigation if insufficient moisture is available to germinate the plant. If emergence has not occurred under the no irrigation alternative four days after planting, then the objective is to maximize the value of only the five irrigation alternatives. This modification insures that the first plant leaf appears within four days of planting.

This recursive model is formulated as a "real time" dynamic risk-neutral model. The model simulates the dynamic and recursive irrigation conditions which a decision maker faces.[2] The model uses all available climatic and crop growth information up to the current point in time and expectations for the next eight days. The formulation is flexible enough to allow longer expectation periods (20 days, 30 days, etc.). However, the computer computation time increases dramatically as the expectation period is lengthened. Moreover, the accuracy of weather forecasts diminishes as the period is lengthened.

Data and Analysis

In this study, three different center pivot irrigation systems are analyzed. The high pressure system is assumed to have a water dis-

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[1] The model is run deterministically over the twenty-three years of weather data. Results are means and standard deviations of twenty-three years of simulation.

[2] The model was designed on a microcomputer to examine the impact of decision which could be made by producers or irrigation consultants if provided with the appropriate climatic, agronomic, and economic relationships.

charge pressure of 60 pounds per square inch (psi) and a relatively low application efficiency of 60%. Water application efficiency is the ratio of the quantity of water effectively put into the crop root zone and utilized by the growing crop to the quantity delivered to the field, expressed as a decimal. An irrigation system properly designed for a particular field and soil condition should eliminate runoff and possibly any loss to deep percolation. The discharge pressure assumed for the low pressure system is 30 psi with an application efficiency of 75%. Recent studies on LEPA sprinkler systems have shown application efficiencies of 95% to 98% (Stoecker; Ellis, Lacewell, and Reneau). Thus, the discharge pressure and application efficiency for the LEPA center pivot irrigation system are 10 psi and 95%, respectively. For the LEPA system, furrow dike tillage technology is assumed to achieve an application efficiency of 95%.

Irrigation fixed and variable costs are calculated using the Oklahoma State University Irrigation Cost Generator (Kletke, Harris, and Mapp). Irrigation costs are derived on a per acre-inch and per acre basis under various assumptions regarding the irrigation well, fuel source, distribution system, and water requirements. For this analysis, variable costs are computed for a typical quarter-mile center pivot system capable of irrigating 130 acres. The pump is assumed to provide 900 gallons of water per minute to the irrigation system. A light, industrial, natural gas engine is used to draw the water from 360 feet below the land surface.

For the LEPA system an additional $6,000 investment in the irrigation system is included in the calculation of the irrigation fixed and variable costs. However, investment analysis for choosing between irrigation system technology is not provided in this paper. The added cost of the LEPA technology is included for computing the variable costs for the irrigation system, well, pump, and motor. Variable costs for the LEPA irrigation system are higher than for the high and low pressure irrigation systems. However, increased efficiency of the LEPA system leads to lower variable cost for the well, pump, and motor. Therefore, total variable costs for the LEPA system are less than for the others.

The range of natural gas (the irrigation system fuel source) prices is based on prices being paid by irrigators withdrawing water from the Ogallala Formation. Three discrete prices of $2.60, $3.80, and $5.00 per thousand cubic foot (mcf) are used in the analysis. The plant growth model uses climatic data from the first of May to the end of October for simulating soil and plant conditions. Precipitation for this area of the country is highly variable. Over twenty-three years, rainfall for the six-month growing season averaged 15.39 inches, with a standard deviation of 5.17. Average monthly rainfall during the growing season ranged from 3.26 to 1.37 inches, with less precipitation during the later, more critical stages of plant growth. Irrigations are most beneficial during these latter months of grain sorghum production (Harris and Mapp).

The plant growth/recursive programming model is solved for the three types of irrigation systems, each with a unique pressure and application efficiency, under three different prices for grain sorghum and for three levels of price for natural gas. Since interest lies in analyzing the potential water savings and increase in net returns from new technology, the actual weather (perfect knowledge) is used during the eight-day decision periods.[3]

Results

Results from this study indicate the impact that new irrigation technology and irrigation timing can have on water use and net returns for grain sorghum producers. Figure 2 depicts the pattern and quantity of irrigations during the growing season for the three irrigation systems with the price of grain sorghum at $4.40 per cwt. and a natural gas price of $3.80 per mcf. From this figure it is apparent that early in the growing season the three types of systems will pump approximately the same amount of water. However, in the last two stages of plant growth the LEPA system irrigates much less water than the low pressure and high pressure systems. The reason for this is shown in figure 3, where the amount of effective water reaching the plant is depicted. The LEPA system delivers the largest quantity of effective water to the plant in every stage of

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[3] The analysis was also performed using expected climatic conditions. Yields and net returns, using expected weather, were slightly lower but not significantly different from those shown in the following section. More important, the relative variations between types of irrigation systems did not differ from the analysis using perfect knowledge of weather events.

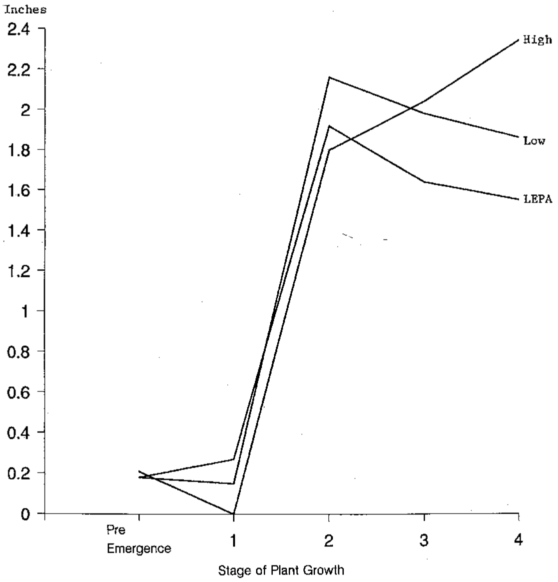

Figure 2. Average gross quantity of irrigation by stage of plant growth for high pressure, low pressure, and LEPA irrigation systems

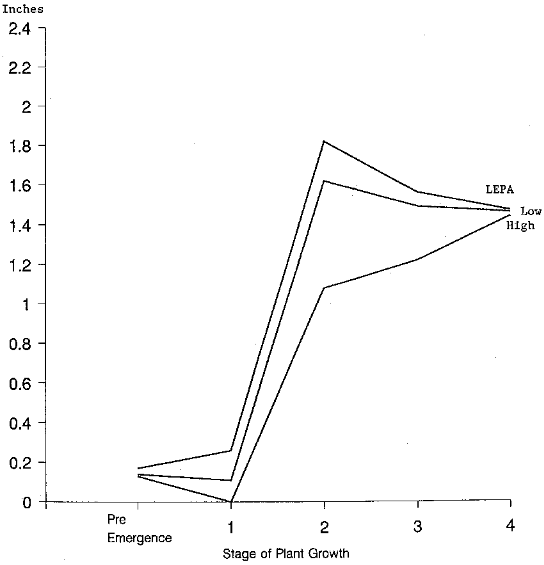

Figure 3. Average effective quantity of irrigation by stage of plant growth for high pressure, low pressure, and LEPA irrigation systems

growth. Quantities for the low pressure system follow the pattern for the LEPA system very closely. However, the high pressure system provides much less water to the plant in the late stages of growth when soil and atmospheric conditions are more severe.

Figure 4 indicates the average number of 1.4-, 2.1-, and 2.8-inch irrigations applied by each of the irrigation systems over the 23 years used to replicate the analysis. The more efficient the irrigation system, the less water is lost to evaporation and runoff. The LEPA system has a higher number of 1.4-inch applications, whereas the less efficient high pressure system applies more 2.1-inch and 2.8-inch irrigations than the other two types of center pivot systems.

In table 1, irrigation scheduling results are presented for the three types of irrigation systems, with the price of grain sorghum varied between $3.80 and $5.00 per cwt. The results show that the LEPA system produces higher yields and net returns with lower levels of irrigation water applied. The per acre yield from the LEPA system is significantly higher (at the 5% level) than the yield from the conventional low pressure irrigation system and significantly higher (at the 1% level) than the high pressure system. Lower variable irrigation costs result in significantly higher (at the 1% level) net returns for theh LEPA irrigation system.

The lower price of grain sorghum has much less effect on yield and irrigation quantity under the LEPA system than under the low pressure and high pressure systems. A change in output price is expected to have less impact on production decisions under the LEPA system because its marginal cost of water is lower than for the low and high pressure systems. For the LEPA system the average optimal irrigation application declines by only .18 inches with a 24% reduction in the price of grain sorghum from $5.00 to $3.80 per cwt. The slight decline in water applied leads to a reduction in average yield of 1% from 56.82 to 56.20 cwt. per acre. Changes in grain sorghum price have a more pronounced impact on the other irrigation systems. A lower grain sorghum price reduces the optimal irrigation application for the high pressure system from 6.54 acre-inches to 5.75 acre-inches. Grain sorghum yield declines by 2.8% or 1.50 cwt. per acre. Results for the low pressure system fall between those for the LEPA and high pressure systems with a reduced optimal water application averaging 0.58 acre-inches and a reduced yield of .81 cwt. per acre.

Table 2 shows the impacts of variations in variable irrigation costs resulting from two different prices of natural gas. With variable irrigation costs ranging from $3.28 to $4.58 per acre-inch, the optimal irrigation applications from the LEPA system result in significantly higher yields than those from the low and high pressure systems. Net revenue is likewise significantly higher, at the 1% level, for the LEPA system. The reductions in yield associated with the higher variable costs are 1.63, 1.51, and 1.14 cwt. per acre (3.0%, 2.7%, and 2.0%) for the high pressure, low pressure, and LEPA systems, respectively. The impact of a change in natural gas prices ($2.60–$5.00) is not as great, among the three irrigation systems, as the impact caused by the change in the price of grain sorghum ($3.80–$5.00).

Summary and Conclusions

This study analyzes the potential of low pressure irrigation systems for improving the timing of applications, reducing water use, and increasing net returns from irrigated grain sorghum production. A grain sorghum plant growth model is combined with a recursive programming algorithm to optimize net returns from irrigated production. The analysis is conducted for the high pressure center pivot irrigation system, a low pressure center pivot and a low energy precision application (LEPA) center pivot irrigation system. Irrigation schedules, including timing and quantity, are

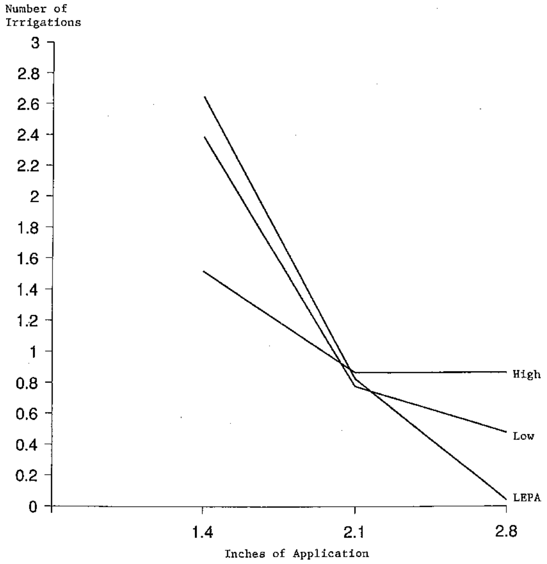

Figure 4. Average number of 1.4-, 2.1-, and 2.8-inch applications for high pressure, low pressure, and LEPA irrigation systems

Table 1. Statistics for Twenty-three Years of Simulated Grain Sorghum Yield, Revenue, Irrigation Quantities, Net Revenue Under Various System Types for the Recursive Programming Irrigation Scheduling Model

	Grain Sorghum Price = $3.80; Natural Gas Price = $3.80					
	High Pressure Irrigation Var. Cost = $4.78		Low Pressure Irrigation Var. Cost = $4.16		LEPA Irrigation Var. Cost = $3.93	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Yield (cwt./acre)	51.79	5.36	54.77	5.38	+56.20[a]	5.88
Total revenue ($/acre)	196.81	20.35	208.11	20.44	+213.56	22.34
Water pumped (inches)	5.75	3.44	5.81	3.39	5.39	2.97
Effective water (inches)	3.45	2.06	4.36	2.54	+5.12	2.82
Irrig. cost ($/acre)	27.49	16.42	24.18	14.12	21.17	11.66
Net revenue ($/acre)	*38.59	22.24	53.20	19.11	++61.66	19.87
	Grain Sorghum Price = $5.00; Natural Gas Price = $3.80					
Yield (cwt./acre)	53.29	4.45	55.58	5.11	+56.82	5.79
Total revenue ($/acre)	266.44	22.23	277.87	25.54	+284.13	28.93
Water pumped (inches)	6.54	3.97	6.39	3.63	5.57	3.20
Effective water (inches)	3.93	2.38	4.79	2.72	5.29	3.04
Irrig. cost ($/acre)	31.28	18.97	26.59	15.09	21.89	12.59
Net revenue ($/acre)	*104.43	23.04	120.56	22.23	++131.51	24.69

[a] Asterisk indicates a significant difference at 5% with the second scenario; single plus indicates a significant difference at 1% with the second scenario; double plus indicates a significant difference at 1% with the first scenario.

Table 2. Statistics for Twenty-three Years of Simulated Grain Sorghum Yield, Revenue, Irrigation Quantities, and Net Revenue Under Various System Types for the Recursive Programming Irrigation Scheduling Model

	Grain Sorghum Price = $4.40; Natural Gas Price = $2.60					
	High Pressure Irrigation Var. Cost = $3.86		Low Pressure Irrigation Var. Cost = $3.40		LEPA Irrigation Var. Cost = $3.28	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Yield (cwt./acre)	53.66	5.14	55.97	5.09	+57.12[a]	5.85
Total revenue ($/acre)	236.11	22.60	246.27	22.37	+251.33	25.74
Water pumped (inches)	6.91	3.81	6.67	3.57	5.81	3.22
Effective water (inches)	4.15	2.29	5.00	2.68	5.52	3.06
Irrig. cost ($/acre)	26.67	14.72	22.66	12.15	19.07	10.55
Net revenue ($/acre)	*78.71	22.62	92.88	19.69	++101.53	22.34
	Grain Sorghum Price = $4.40; Natural Gas Price = $5.00					
Yield (cwt./acre)	52.03	4.86	54.46	5.08	+55.98	6.07
Total revenue ($/acre)	228.92	21.40	239.61	22.33	+246.33	26.70
Water pumped (inches)	5.69	3.35	5.63	3.42	5.33	3.01
Effective water (inches)	3.41	2.01	4.22	2.56	+5.06	2.86
Irrig. cost ($/acre)	32.50	19.14	27.70	16.82	24.39	13.80
Net revenue ($/acre)	*65.70	23.75	81.18	20.27	++91.21	23.19

[a] Asterisk indicates a significant difference at 5% with the second scenario; single plus indicates a significant difference at 5% with the second scenario; double plus indicates a significant difference at 1% with the first scenario.

determined under varying prices for natural gas and grain sorghum.

The LEPA irrigation system permits the operator to apply less water per application and, with improvements in both timing and efficiency of application, generates yields and net returns which are higher and more stable than those generated by the low pressure and high pressure alternatives. In addition, the optimal irrigation schedules for the LEPA system are less affected by increases in the prices of natural gas and grain sorghum than are those of the high and low pressure irrigation systems. For a given 130-acre field under irrigation, less total water is applied using the LEPA system.

The potential benefits of applying reduced quantities of irrigation water but irrigating at the critical stages of plant development are often underestimated by producers and researchers. The type of daily plant growth model used in this study permits a more careful analysis of the value of timing of applications and improvements in irrigation efficiency than models which focus on annual water requirements and simplified yield response relationships.

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