serie ad

WP-AD 2010-25

Limited memory can be beneficial for the evolution of cooperation

Gergely Horváth, Jaromír Kovářík and Friederike Mengel

Ivie

Working papers

Working papers

Working papers

Los documentos de trabajo del Ivie ofrecen un avance de los resultados de las investigaciones económicas en curso, con objeto de generar un proceso de discusión previo a su remisión a las revistas científicas. Al publicar este documento de trabajo, el Ivie no asume responsabilidad sobre su contenido.

Ivie working papers offer in advance the results of economic research under way in order to encourage a discussion process before sending them to scientific journals for their final publication. Ivie's decision to publish this working paper does not imply any responsibility for its content.

La Serie AD es continuadora de la labor iniciada por el Departamento de Fundamentos de Análisis Económico de la Universidad de Alicante en su colección "A DISCUSIÓN" y difunde trabajos de marcado contenido teórico. Esta serie es coordinada por Carmen Herrero.

The AD series, coordinated by Carmen Herrero, is a continuation of the work initiated by the Department of Economic Analysis of the Universidad de Alicante in its collection "A DISCUSIÓN", providing and distributing papers marked by their theoretical content.

Todos los documentos de trabajo están disponibles de forma gratuita en la web del Ivie http://www.ivie.es, así como las instrucciones para los autores que desean publicar en nuestras series.

Working papers can be downloaded free of charge from the Ivie website http://www.ivie.es, as well as the instructions for authors who are interested in publishing in our series.

Edita / Published by: Instituto Valenciano de Investigaciones Económicas, S.A.

Depósito Legal / Legal Deposit no.: V-2924-2010

Impreso en España (julio 2010) / Printed in Spain (July 2010)

WP-AD 2010-25

Limited memory can be beneficial for the evolution of cooperation*

Gergely Horváth, Jaromír Kovářík and Friederike Mengel**

Abstract

We study a dynamic process where agents in a network interact in a Prisoner's Dilemma. The network not only mediates interactions, but also information: agents learn from their own experience and that of their neighbors in the network about the past behavior of others. Each agent can only memorize the last h periods. Evolution selects among three preference types: altruists, defectors and conditional cooperators. We show - relying on simulation techniques - that the probability of reaching a cooperative state does *not* relate monotonically to the size of memory h. In fact it turns out to be optimal from a population viewpoint that there is a finite bound on agents' memory capacities. We also show that it is the interplay of local interactions, direct and indirect reputation and memory constraints that is crucial for the emergence of cooperation. Taken by itself, none of these mechanisms is sufficient to yield cooperation.

Keywords: evolution, reputation, bounded memory, cooperation.

JEL Classification: C70, C72, C73

---

* We are grateful to Jonas Hedlund, Hubert János Kiss, Marco van der Leij, Peter Vida, Nick Vriend and seminar participants at IMEBE 2009 (Granada), ESEM 2009 (Barcelona), ASSET 2009 (Istanbul), Summer Workshop EI-HAS (Budapest), Alicante, EUI Micro Working Group (Florence) for helpful comments and suggestions during the development of this project. J. Kovářík acknowledges Spanish Ministry of Science and Innovation (SEJ2006-06309/ECON) and the Government of the Basque Country (IT-223-07) for financial support. F. Mengel thanks the European Union for financial support (PIEF-2009- 235973-LAG).

** G. Horváth: University of Alicante. Contact author: gergely@merlin.fae.ua.es. J. Kovářík: University of the Basque Country. F. Mengel: Maastricht University.

1 Introduction

1.1 Motivation

The evolution of human culture has presented some puzzle to evolutionists. Complex, cumulatively evolving culture is rare in nature and in most species traditions are fairly simple. Human capacities to store and transmit information about others, to communicate, share experiences, form beliefs or to teach others are not common in other species. People living in human populations, though, are heirs to a pool of socially transmitted information, for which there is little room in classical models of evolution. Studies of cultural evolution have partially filled these gaps by acknowledging the social context in which human evolution takes place. But still there are few studies modeling the coevolution of cognitive capacities and behavior.

In this study we are concerned with one important and well studied type of behavior namely cooperation in social dilemma situations. Human cooperation differs from cooperation observed in other species through the use of mechanisms such as direct or indirect reputation building. Direct reputation is built through own experience, while indirect reputation is learned through communication with others. In fact human abilities, such as (large) memories and language, which are needed for such mechanisms to develop, might be crucial for the evolution of cooperation (Trivers 1971, Nowak and Sigmund, 1998). All reputation-based theories implicitly assume that people remember past interactions. Indirect reputation-building is clearly facilitated through language, but also has a larger demand for memory capacity than first-hand experience. Memory capacity seems a key requirement for these mechanisms to work.

We model memory constraints explicitly and show that, maybe somewhat surprisingly, having a larger memory is not always better for the evolution of cooperation. We abstract from the (energetic and reasoning) costs of having larger memories and show that even if having larger memories were costless, such larger memories need not favor cooperative outcomes. In fact we will show that some forgetting will typically help cooperation in non-trivial ways.

To analyze the relevance of (limited) memory for prosocial behavior, our model combines two natural features of real-life interactions. First, we assume that people's beliefs about each other's behavior are determined through reputation. Second, both interactions of agents as well as the spread of information are mediated through a social network. Reputation is created both directly, through repeated interactions of the same individuals, and indirectly, through information from neighbors in the social network. Agents also interact with their neighbors in the network, but which neighbor they meet is randomly determined in each interaction.[1] This allows us to capture situations where people are simultaneously engaged in various relationships, which are not clearly separated in time. This certainly realistic feature of the model interacts with memory constraints in obvious and important ways. Both longer time intervals between potential cooperative encounters and having to recall the past play of multiple opponents are challenges for reputation-based mechanisms.[2] Our agents have limited memory and remember only their last h interactions.

Our model is set in the tradition of the indirect evolutionary approach.[3] In these models, evolution selects among preference types, rather than strategies. There are three types of agents in our model: altruists, defectors and conditional cooperators. We are interested in which of the three types survive evolutionary selection, when cooperation will survive and how this depends on memory constraints.

Our main findings are the following. Firstly we find that the optimal memory level is often interior, i.e. it is optimal for a population if its members have strong bounds on their memory constraints. In fact in (almost all) our simulations the optimal memory level is around 6-11 periods. Quite amazingly, this is consistent with Miller's (1956) "magical number seven +/- two" from psychology, where it is widely accepted that human long term memory is limited to storing seven +/- two items simultaneously. Our model provides one possible story for why such memory constraints might have emerged.

Secondly we show that the twofold role of network is crucial for the emergence of cooperation. If meetings

[1] In the appendix we we show that our results extend to a situation, where there is a positive probability (decreasing in distance) to interact with *anyone* in the same connected component of the network.

[2] Milinski and Wedekind (1998) provide experimental evidence on the effects of memory constraints on cooperative behavior, reporting that these constraints challenge cooperative behavior. See also Stevens and Hauser (2004).

[3] See, for instance, Bester and Gűth (1998).

are random, then cooperation cannot survive. If information is not transmitted through the network and the reputation-building is exclusively based on ones own experience, then cooperation is only residual. Hence, the spatial structure is essential for both matching and information transmission.

We also observe an interesting interplay between direct and indirect reputation-building. The long-run level of cooperation is maximal when agents rely equally on the their own and their neighbors' experience. However, the level of cooperation in the population is asymmetric with respect to the weights put on each type of information. If people rely on both direct and indirect experience, cooperation is more likely to stabilize when direct experience is weighted more heavily (than second-hand information). This may provide some indication for why people tend to exhibit a bias toward their own experience, while counterweighting such first- and second-hand information (Weizsaecker, 2009)

In order to check whether the limited levels of memory are optimal from the individual point of view, we inject into the model agents with different memory spans. Defectors have hard time surviving in these environments, as long as the network exhibit small-world property (Watts and Strogatz, 1998). On the contrary, conditional cooperators tend to survive, irrespective of their memory level. Hence in terms of individual selection there are no strong evolutionary pressures favoring any of the memory sizes, while from a population viewpoint bounded memory emerges as clearly being optimal.

These results add few new interesting viewpoints to the discussion of why evolution did not endow us with unlimited brain capacity or, more loosely speaking, did not make us "infinitely" smart. Not only is it the case that larger brain or memory capacity involves a direct cost (from which we abstract), our results also show that it can be optimal at least from a population viewpoint to be endowed with finite memory in strategic situations. In line with the group selection literature (see e.g. Boyd and Richerson, 1990), limited memory in our model can emerge from the conflict of two populations endowed with different memory capacity.[4] This is enhanced by the fact that within-groups there are no strong evolutionary pressures on larger memories to develop as our analysis will make precise.

The paper is organized as follows. Next we will discuss related literature. In Section 2 we will present the model, in Section 3 explain our simulation strategy and Sections 4 and 5 present our main results. Section 6 concludes. In Appendix, we provide some extensions of the model and some additional figures.

1.2 Related Literature

Cooperative behavior based on reputation has been studied for decades especially in repeated bilateral interactions. So called tit-for-tat players cooperate if and only if their partner has cooperated in all past interactions. Studies include Trivers (1971), Rubinstein (1979), Axelrod and Hamilton (1981), Kreps et al. (1982) or Fudenberg and Maskin (1986). Kreps et al. (1982) for example show that if for any agent there is a small probability to interact with such a tit-for-tat agent cooperation can be sustained even in the finitely repeated Prisoner's Dilemma.

However, cooperative behavior is also observed in one-shot interactions with unknown individuals. As a result, literature, which explores the role of reputation in non-regular encounters, has emerged. This led researchers to include indirect reputation building in their models (Nowak and Sigmund, 1998; Leimer and Hammerstein, 2001; Wedekind and Milinski, 2000; Seinen and Schram, 2004; Milinski et al., 2001). The drawback of these studies is that the history of each player's actions is *public* knowledge. Hence, there needs to be an institution, which will disseminate information. An exception are Nowak and Sigmund (1998) who simulate a situation, where just a subset of the population learns the play in past bilateral encounters and show that public knowledge is not necessary for cooperation to establish.

In our paper there is a natural mechanism to disseminate information. Agents are located on a network and obtain information either through their own experience or through their direct neighbors in the network.

[4] At first, there may seem to be a link between our result and the evolution of "forgiveness" (Axelrod, 1984, Nowak and Sigmund, 1992), which argue that forgiveness can help cooperation: to punish early defection for too long time easily traps the individuals in everlasting mutual retaliation. Our argument is fundamentally different though. In our case, people do not forgive. Limited memory affects both defectors and cooperators in the same way and is independent of changes in actions or types.

Direct reputation-building within simple network architectures, such as circles and lattices has been studied by Boyd and Richerson (1989), Eshel et al. (1998), or Nakamaru et al. (1997,1998). More recently, Ohtsuki et al. (2006) and Ohtsuki and Nowak (2007) have proceeded analytical results for regular graphs and simulation results for random and scale-free networks. Mohtashemi and Mui (2003) explicitly focus on the effect of social information that travels through network on cooperation. More precisely, direct links mutually share information in their model, spreading reputation of past encounters across neighbors. They rule out repeated interactions and show that non-direct assessment of reputation itself can stabilize cooperative behavior. However, they model a growing network, in which everybody ends up knowing everybody, which is crucial for the survival of cooperation in their setting.[5] Recently, Roberts (2008) introduced both direct and indirect reputation into one model. In contrast to our model though, he makes them compete in the evolutionary process to see, which one will survive in which conditions. The focus of our study is different. We allow agents to rely on both direct and indirect reputation building and are interested in optimal memory bounds.

There is very little research on the role of bounded memory for cooperation. A famous study from psychology is Miller (1956) mentioned above. He found that humans typically can remember seven objects of the same kind. Since then numerous studies have repeatedly reconfirmed this finding in a great variety of contexts, for a great variety of recalling objects, and across different age categories. Cowan (2001) presents a recent exhaustive review of this topic.

In the framework of the repeated prisoner's dilemma, the role of memory has already drawn some attention. Hauert and Schuster (1997) for example directly test the effect of memory in a simulated environment and conclude that larger memories promote cooperation. In a different setting, the role of memory for the emergence of cooperation has been analyzed by Kirchkamp (2000) and Sheng et al. (2008). In their approach, people play the same action against all their network neighbors. Agents imitate their neighbors and the memory length determines on how many past rounds the imitation is based. Kirchkamp (2000) observes that infinite memory in learning promotes cooperation. Sheng et al. (2008) provide a more complex relation between memory and cooperation. Lastly Cox et al. (1999) explore whether memory about past interactions can compensate the fact that people do not have information about others. As in our model, memory serves for discriminating among opponents, but neither information flows nor interactions are mediated through a network. Overall their findings suggest that there is no strong selection pressure on memory span in such an environment.

2 The Model

Our population consists of a finite number of agents interacting in a bilateral Prisoner's Dilemma through a given network. Agents are of three different types, having different preferences over the outcomes of the game (Subsection 2.1). Agents interact exclusively with their first-order neighbors in the network (Subsection 2.2). Given their type and their beliefs about the opponent agents choose their preferred action. Beliefs are determined explicitly through the reputation this opponent enjoys with the agent in question (Subsection 2.3). Ultimately, we assume that evolution selects among preference types (Subsection 2.4).

2.1 Game and Payoff Types

There is a population of individuals, $i \in \{1, 2, ..., N\}$, who are repeatedly matched to play a prisoner's dilemma game. The payoff of an agent of type τ when choosing action a_i and his opponent chooses a_j is given by

$a_i \backslash a_j$	C	D
C	a	0
D	$1 - w_\tau$	$d - w_\tau$

(1)

[5] They themselves remark that without the network growth cooperators are easily defeated by defectors (Mohtashemi and Mui, 2003, p. 527).

with $1 > a > d > 0$. We also assume that $d > 1 - a$ or equivalently $a + d > 1$. This implies that joint cooperation (yielding a payoff of a to both players) is efficient and exploitation undesirable for the society.

Payoffs are obtained from material payoffs as well as from psychological payoffs w_τ (resulting from shared views or moral norms of behavior). For example, if $w_\tau = 0$ this matrix represents a (standard) Prisoner's Dilemma for agents of type τ. Different values of w_τ can induce different types of behavior. We assume that there are three types in the population.

- D: Defectors ($w_D < 1 - a$) whose optimal strategy is always defect.
- CC: Conditional cooperators ($w_{CC} \in [1 - a, d]$). Conditional Cooperators cooperate whenever their belief σ that their interaction partner cooperates is larger than the critical value $\widehat{\sigma}$,[6]

$$\widehat{\sigma} = \frac{d - w_{CC}}{a + d - 1}.$$

- A: Altruists ($w_A > d$) who will always cooperate.

2.2 Interaction Structure

Agents are organized on a fixed undirected network, which mediates who meets whom. In each period, one agent is chosen randomly (with uniform probability) to play the Prisoner's Dilemma game (1). This player is matched with one of her direct neighbors in the network. To distinguish between the player that has been chosen uniformly from the population and her neighbour with whom she has been matched, we call the former *player* and the latter *match*.

For each player, there is an equal probability to meet each of her neighbors. Denote the degree of node i, i.e. the number of neighbours i has by d_i. Then the probability for i to interact with j given that i has been chosen as a player is the following:

$$P_{ij} = \begin{cases} \frac{1}{d_i} & \text{if } j \text{ is a neighbour of } i \\ 0 & \text{otherwise} \end{cases}$$

Note that with this function generally $P_{ij} \neq P_{ji}$, that is players with more neighbors might be matched more frequently. Hence to avoid confounding effects of the degree (i.e. number of neighbours) of a player on evolutionary fitness, our definition of fitness only considers the payoffs from interactions, in which the agent plays as player.

When playing the game, altruists (A) always choose cooperate and defectors (D) always choose defect. For conditional cooperators (CC), though, action choice depends on their beliefs on what their match will choose, as we have seen above. These beliefs in turn depend on the reputation their match enjoys with them.

2.3 Reputation and beliefs

Agents have bounded memory and can remember only the last h periods. Hence they form beliefs about the behavior of their match using their experience from these last h periods. In addition, they can use information they get from their direct neighbors in the network. Hence the network also channels information flow.

Denote by $\gamma_{ij}(h)$ the proportion of times that j cooperated in an interaction with i in the last h periods, where i was player and j her match.[7] If there was no such interaction between i and j in the last h periods, then we set $\gamma_{ij}(h) < 0$ as a matter of convention.

[6] This can be understood as follows. The expected payoff of cooperating is σa and that of defecting is $\sigma(1 - w_{CC}) + (1 - \sigma)(d - w_{CC})$. Now the former is higher than the latter whenever $\sigma > \widehat{\sigma}$.

[7] Hence if i was a match of j then she does not store j 's behavior in her memory. This is certainly not realistic. We make this assumption because we are not interested in the effect of the degree of a player (i.e. her number of neighbors) on fitness. Hence to control for such effects we want to avoid that better connected players have more precise information about others. (Again let us emphasize that we do believe that this is realistic and worthwhile studying, but in this study we are interested in the undistorted effect of memory on cooperation).

There is no global "reputation" in the model. Some players will think badly about a given player and others good. Denote by $\overline{\beta}_{ij}(h)$ the proportion of times j cooperated in an interaction with any of i's neighbours in the last h periods. Hence β refers to information from neighbours (where all neighbours are weighted equally) and γ refers to own experience. Again we set $\overline{\beta}_{ij}(h) < 0$ if none of i's neighbours has interacted with j in the last h periods. The reputation that player j has for player i (what i thinks about j) at time t is then given by

$$r_{ij}^t = \begin{cases} \lambda\gamma_{ij}(h) + (1-\lambda)\overline{\beta}_{ij}(h) & \text{if } \gamma_{ij}(h) \geq 0 \wedge \overline{\beta}_{ij}(h) \geq 0 \\ \gamma_{ij}(h) & \text{if } \gamma_{ij}(h) \geq 0 \wedge \overline{\beta}_{ij}(h) < 0 \\ \overline{\beta}_{ij}(h) & \text{if } \gamma_{ij}(h) < 0 \wedge \overline{\beta}_{ij}(h) \geq 0 \end{cases}$$

In words, the reputation that j enjoys with i is a weighted average of her direct reputation with i ($\gamma_{ij}(h)$) and her indirect reputation communicated from $i's$ neighbours ($\overline{\beta}_{ij}(h)$). A high value of λ means that she relies mostly on her own experience and low λ that she forms judgements, based on the information from others. Naturally, if i has not met j in last h periods but at least one of her neighbors has she relies on neighbors' experience, and vice versa.[8]

If nothing is known about j (i.e. if neither i nor any of her neighbors have information about j) there is "no reputation". In this case the agents use the average rate of cooperation in period $t-1$ (the state of the population), which is assumed to be always known to all players ($\overline{\sigma}^{t-1}$). In other words the default belief about others (when nothing is known about them) is the population average.

In sum, the belief that i has about the likelihood that j cooperates at t is given by

$$\sigma_{ij}^t = \begin{cases} r_{ij}^t & \text{if } \gamma_{ij}(h) \geq 0 \vee \overline{\beta}_{ij}(h) \geq 0 \\ \overline{\sigma}^{t-1} & \text{otherwise} \end{cases}$$

Our set up relates the network structure to the information accumulation in a natural way. Given the locally biased matching process, the more tightly knit communities are, i.e. the more likely it is that any two neighbors of an agent know each other themselves, the more effective should reputation building be.

Two remarks are at order. First, one may ask why we interpret bounded memory as being able to remember the last h periods rather than remembering any h periods. The reason is simply that in a changing environment this is the best thing an agent can do. Secondly one may wonder whether it may be in a neighbours interest to withhold information about other players. This is an interesting question and it might sometimes be the case. Remember, though, that at least in the short run agents have no incentives to lie since they don't receive payoffs from others' interactions. Hence incentives for lying only arise if agents are very forward looking or are anticipating the consequences of cultural evolution. This seems to speak against the basic idea of evolution and we rule out such effects for the moment. Furthermore our model as it stands applies also to situations where information is not explicitly communicated but simply observed.

2.4 Selection process

Ultimately, we are interested in which of the three types survive in an evolutionary model. Evolutionary success (fitness) is measured by per interaction material payoffs of the agents (disregarding w_τ), as we want to know which preferences will be selected.[9] As mentioned above, we use the last interaction payoffs agents get as players, such that more connected agents and agents who were chosen to play more than once do not *per se* enjoy higher fitness.[10]

The selection process is modeled as follows. Every N periods, i.e. once each agent had the chance to play the Prisoner's Dilemma exactly once in expectation, k agents are selected randomly for selection. For each of these agents another agent is randomly chosen from the whole population and their fitness is

[8] One may think that an optimal λ would be to weigh the information proportional to how much experience i and her neighbours have with j. We will see below, though, that this is not true.

[9] This is the objective measure for fitness. If we did not disregard the w_τ, then agents with a value of w_τ tending to minus infinity would be the most fit in our population, which is clearly an absurd conclusion.

[10] In terms of preference selection our results are robust to considering more periods for fitness.

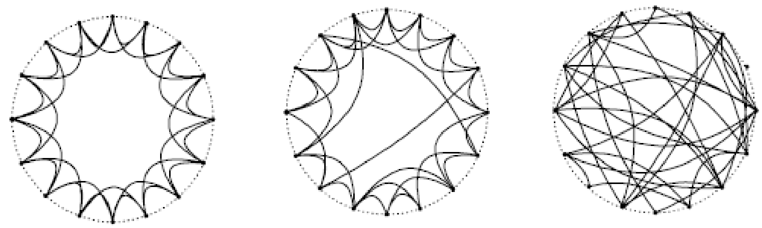

Figure 1: Network structures for $\rho = 2$ and different values of rewiring probability. Left: $\theta = 0$, middle: $\theta = \epsilon$, right: $\theta = 1$. Taken from Vega-Redondo (2007) p. 59.

compared.[11] If the randomly chosen agent has higher fitness, the agent adopts her type. This process could be interpreted as cultural evolution, which could take place e.g. via imitation learning. In this case the randomly chosen agent could be thought of as a cultural role model for our agent. (See e.g. Mengel, 2008). Note also that since types are in general not observable (other than through reputation), type changes can also not be observed, but only be learned over time.[12]

3 Computational Analysis

In this section, we present the conditions used for the numerical analysis. Whenever possible, conditions are chosen such that they match interaction patterns typically observed in reality.

3.1 The Networks

The network structure used in the simulations is a well-known small world network (Watts and Strogatz, 1998). Small-world networks are known to display very similar characteristics to most networks observed in real life. In our network there are $N = 100$ nodes/agents.[13] The network is generated from a one-dimensional lattice where the agents have links to their neighbors of up to distance ρ. Let us call ρ the connection radius. Starting from this network, each link is rewired with probability θ in the following way: the link between agents i and j is destroyed and then agent i is connected to a randomly chosen other agent who is not already connected to i. Figure 1 shows an example of such networks for rewiring probabilities $\theta = 0$, small ϵ, and 1. The resulting networks are regular lattice, small-world and random networks, respectively.

The rewiring procedure has an important impact on network measures, such as the average clustering coefficient and the average distance. The clustering coefficient of a network refers to the average of individual clustering coefficients of nodes, which is the fraction of pairs of neighbors of a given node that are neighbors themselves.[14] In connected networks, the average distance is the average of the length of the shortest path between any two nodes. When the network is disconnected, the average distance is infinite according to the conventions.

With $\theta = 0$, we have a local network which is characterized by high clustering and high average distances, and each node has exactly 2ρ neighbors. Local neighborhoods are relatively isolated in such a case. With small positive θ, the average distance decreases dramatically: few shortcuts interconnect local neighborhoods, which makes it easier to reach all nodes in the network from a particular one. On the other hand, this does

[11] An alternative approach would be to compare the payoff to a payoff of a randomly chosen neighbor (Ohtsuki and Nowak, 2007). In such a case, cooperation can stabilize thanks to local clusters of cooperators. Our model of local interaction, but global selection makes it harder for cooperation to survive. In Appendix 7.1 we parametrize the degree of local vs global interaction (maintaining global selection) and show that our results are robust.

[12] In the Appendix we also consider an alternative model where this is not the case.

[13] Simulations are programmed and carried out using the program RePast.

[14] See, for instance, Vega-Redondo (2007, Chapter 2) for a formal definition.

$\rho = 2$		
θ	Cluster.	Av. Dist.
0	0.500	13.009
0.01	0.475	8.808
0.05	0.384	5.219
1.00	0.037	3.488

$\rho = 4$		
θ	Cluster.	Av. Dist.
0	0.643	6.765
0.01	0.611	4.391
0.05	0.498	3.144
1.00	0.080	2.450

Table 1: Measures of the networks (averaged over 10.000 observations of the network for each value of the network parameters). Average distance computed for connected graphs.

not affect the clustering of the network. This "small-world" phenomenon has been observed in many real social networks (Watts and Strogatz, 1998; Goyal, 2007). When the rewiring probability is high, the local neighborhoods get dissolved. As a result, both the average clustering coefficient and distances are small, and the degree distribution becomes heterogeneous.

3.2 Parameters for the Numerical Analysis

In the simulations, we use two values for the connection radius, $\rho = 2$ and 4; that is, each node has on average 4 and 8 neighbors, respectively. We focus on four values of the rewiring probability: $\theta = 0, 0.01, 0.05$, and 1.[15] In order to illustrate the impact of θ and ρ on the network architecture, Table 1 reports the average clustering coefficients and average distances for various simulated networks.

In each simulation, there is one third of agents of each type in the beginning and the types are distributed randomly over the whole network. The starting default belief is set to 2/3 (the sum of the shares of altruists and conditional cooperators in the population).[16]

In the numerical analysis, the game parameters are $a = 0.8$, $d = 0.5$, $w_{CC} = 0.35$ and $\lambda = 0.5$ for the benchmark simulations. After $N = 100$ periods of interaction, each agent had the chance to move once on expectation. Then, $k = 20$ agents are selected to update their type, according to the described selection procedure. The default belief is also updated to the actual average number of cooperations in the last N periods. Agents "forget" the information about the h^{th} oldest interaction in their memory. Afterwards, the process repeats. All the parameter values are summarized in Table 2.

We simulated the system for up to 3000000 periods (implying that each agent has moved 30000 times, on average) and we run each parameter constellation 100 times.[17] In the following section, we look at the number of cooperating agents and at the type distribution, after the system has converged. Particular interest is put on the effect of (i) memory (h), (ii) the weights put on own and neighbors' experience (λ), and (iii) the network structure (θ) on cooperation and the resulting type distribution. Later, in the sensitivity analysis, we analyze the effect of the other parameters of the model (k, w_{CC} and ρ).

[15] We also performed all the analysis for $\theta = 0.1$ and 0.5. Since all the effects of the rewiring probability occur with the small-world transition (that is, when θ changes from 0 to 0.05), we do report these results here. These simulation results are available upon request.

[16] It is worth mentioning that these initial conditions creates a great initial advantage for defectors, because initially others believe that they will cooperate with probability 2/3. This makes it easy for them to exploit cooperators and makes the survival of cooperative traits very difficult in early rounds.

[17] All the simulations have converged to a steady state before the 3000000$^{\text{th}}$ period.

Parameter	Value
N	100
a	0.80
d	0.50
w_{CC}	0.35
λ	0.50
k	20
ρ	$2, 4$
h	$\{1, 2, ..., 20\}$
θ	$0, 0.01, 0.05, 1$

Table 2: Parameters in basic runs.

4 Results

In this section, we study the long-run frequency of cooperation in the population and how it relates to the memory constraint. Before we describe our main result on bounded memory we will first show that all three, direct and indirect reputation as well as local interaction (the network) are needed to ensure cooperation.

4.1 Reputation and Local Interaction

We are aware that the incorporation of three mechanisms, direct reputation, indirect reputation and spatial matching, creates a rather complex model. Still all three mechanisms are observed in reality. In this subsection, we show that if we remove any of the three mechanisms the cooperation either decreases drastically or disappears completely. The definition of the model allows us to test these issues in a very straightforward way.

To this aim, we compare four following scenarios:

1. *Benchmark*. In this case, network matching, direct and indirect ($\lambda = 0.5$) reputation are all present in the model.

2. *Direct and indirect reputation* ($\lambda = 0.5$). In this scenario, people are matched to play randomly with anybody in the population, rather than meeting exclusively their neighbors, while both reputation mechanisms are maintained.

3. *Network matching and direct reputation* ($\lambda = 1$). People do not base their reputation estimates on the experience of their neighbors. Only own experience is considered.

4. *Network matching and indirect reputation* ($\lambda = 0$). In this case, only the experience of neighbors matter for the reputation agents consider for their matches.

Figures 2 and 3 shows the long-run levels of cooperative behavior under the above four scenarios for different lengths of memory, separated for connection radius $\rho = 2$ and 4, respectively. The x- and y-axes respectively reflects the length of memory and the level of cooperation in the steady state. The green (highest) plot represent the benchmark case, where all the mechanisms are present.

We do not include Scenario 2 into the graph, since cooperation never survives under random matching. Network matching is clearly crucial for cooperation to stabilize. The red (lowest) graph plots Scenario 3, where reputation is built exclusively on basis of own experience. We see in the graphs that the levels of cooperation decline dramatically. For $\rho = 2$, cooperation is only residual and it almost never survives if $\rho = 4$, despite network matching.

There is slightly larger cooperation under the conditions of Scenario 4 (network matching and indirect reputation). The fact that removing indirect reputation impacts cooperation more than removing direct

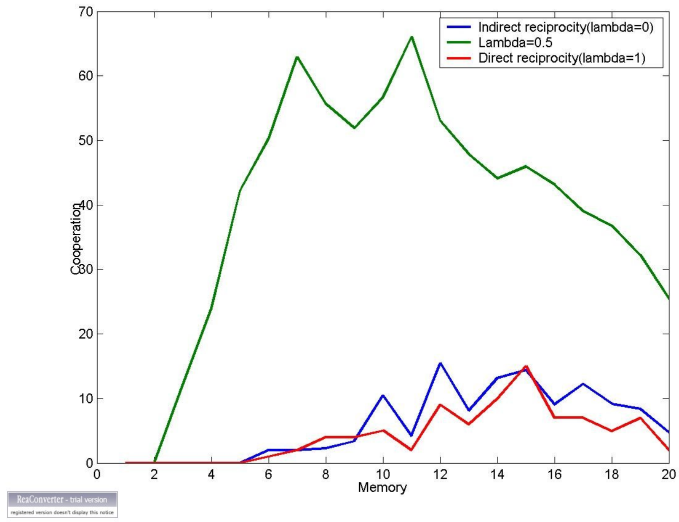

Figure 2: Effects of removing mechanisms for $\theta = .05$ and $\rho = 2$.

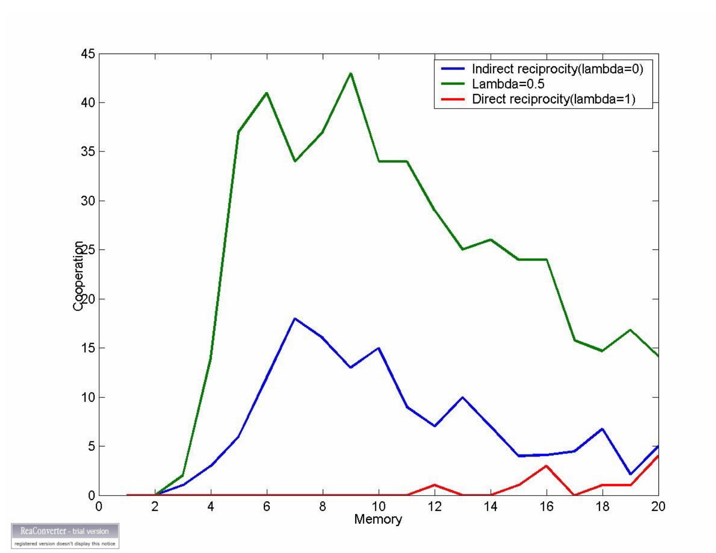

Figure 3: Effects of removing mechanisms for $\theta = .05$ and $\rho = 4$.

reputation is very intuitive, since neighbours as a group typically have more information than any single agent. This is confirmed by the difference between the red and blue plots in Figures 2, where each agent collects second-hand information from four neighbors, and 3, where there are eight neighbours.

In sum, we find that locally-biased matching is crucial for the stability of cooperative behavior in the population. The role of networks for matching has already been recognized in the literature. However, in the present model, the network also performs an important role as an information-transmission device. If we abstract from this role of the network cooperation breaks down under most parameter constellations. Nevertheless, the multiple role of social networks, being both channels of interaction and information, has only earned minor attention in the literature so far.[18]

4.2 Bounded Memory and Cooperation

In this section, we focus on how memory affects the proliferation of cooperation. Note from Figures 2 and 3 that there seems to be a non-monotonic relation between memory length and cooperation. In Result 1, we first describe the case, where agents have few connections.

Result 1 *For $\rho = 2$, there exist values of memory h_1 and $h_2 > h_1$, both non-decreasing in θ,*[19] *s.t.*

- *for $h < h_1$, a cooperative state is never reached;*
- *for $h \in [h_1, h_2)$, the frequency of cooperative states is increasing in h and*
- *for $\forall h \geq h_2$, the frequency is non-increasing in h.*

Figure 4 illustrates these findings. Each plot corresponds to the average frequencies of cooperative states (over 100 simulation runs) for a given value of rewiring probability θ in function of memory (x-axis). Note that, in order to have positive frequency of cooperation in the steady state, memory has to be at least two for $\theta = 0$, larger than two for for $\theta = .01$ and .05, but larger than four if $\theta = 1$, confirming the first part of Result 1.

Regarding the maximal level of cooperation, it is reached for $h = 9$, 11 and 15 when $\theta = 0$, .05 and 1, respectively. For $\theta = .01$, the maximal level is achieved with $h = 16$. Overall, though, the results suggest that the maximal average values are around $h = 9$ and that afterwards cooperation decreases with h. Still, in Result 1 we prefer to state that the share of cooperative outcomes is non-decreasing in h as $h \geq h_2$, because of the outliers seen in the graph. For $\rho = 4$, though, results are much clearer and there is a strictly decreasing trend after the optimal memory level is reached.

Figure 5 gives a more detailed account of Result 1. Each panel in the figure reports the final distribution of types in the population and each line plots one of the following final distributions: (1) *Zero cooperation* (blue), where conditional cooperators and defectors coexist, but always defect, (2) *Full cooperation* (green), where conditional cooperators and altruists survive, and always cooperate, (3) *"CC survive"* (red), in which only conditional cooperators survive and cooperate. The graphs show how the shares of these scenarios change with different values of memory, x-axes, and rewiring probability, panels. The results go in line with the analysis from Figure 4. There is no cooperation in the population for extremely low values of memory, independently of other parameters. Conditional cooperators can actually survive in these cases, but never cooperate. As memory increases, defectors extinct more frequently, but as memory gets very large, the scenario with only conditional cooperators present in the long run increases steadily.

Finally, Figure 6 plots the time to convergence as a function of memory and the rewiring probabilities for $\rho = 2$.[20] Interestingly, irrespective of whether cooperation survives or not the time to convergence increases monotonically in memory. Hence there is an additional sense in which larger memory can be detrimental to achieving cooperative outcomes.

[18] Mohtashemi and Mui (2003) stress the information-channeling capacity of social structures, but they completely abstract from its matching role.

[19] Obviously, h_1 and h_2 are also functions of the other parameters of the model. We analyze the remaining effects below.

[20] The convergence result are very similar for all the parameter constellations.

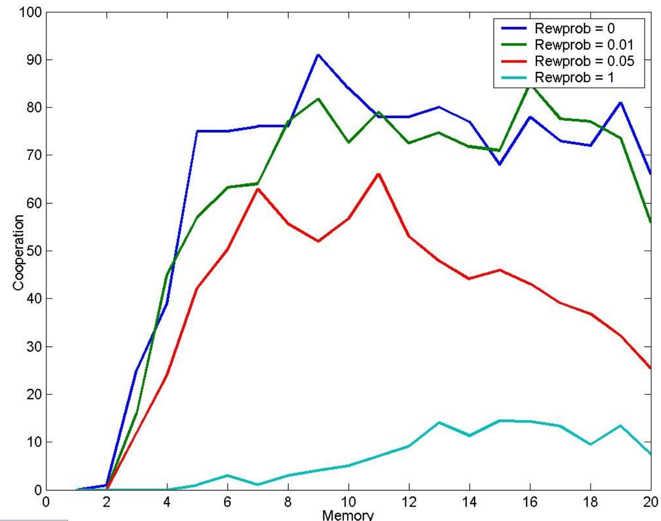

Figure 4: Frequency of cooperative outcomes as a function of memory for different rewiring probabilities and $\rho = 2$.

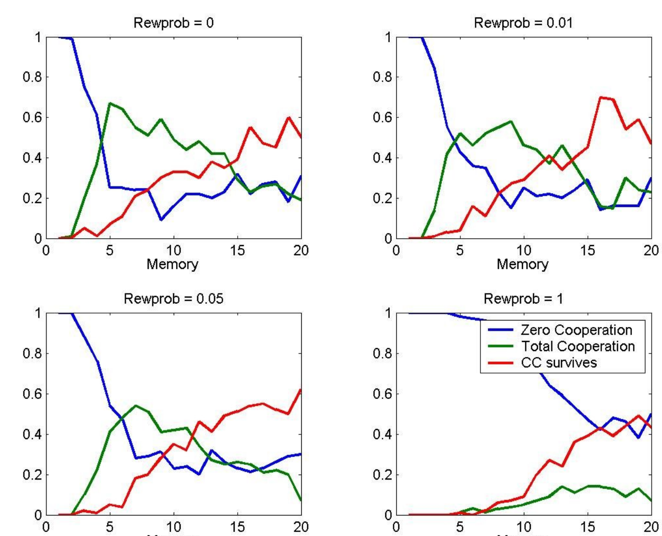

Figure 5: Shares of the different scenarios for $\rho = 2$. *Blue plot*: zero cooperation. *Green plot*: full cooperation. *Red plot*: CC survive.

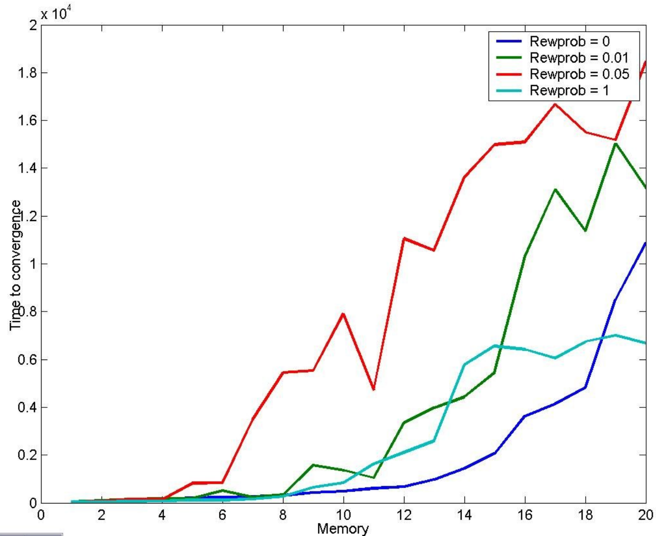

Figure 6: Average time to convergence for $\rho = 2$.

Result 2 describes how things change when agents have more connections. The simulation results are plotted in Figure 7.

Result 2 *When $\rho = 4$, there exist h_1 and $h^* > h_1$, both increasing in the rewiring probability, θ, and memory, h, such that*

- *if $h < h_1$, a cooperative state is never reached,*
- *if $h \in [h_1, h^*]$, the frequency of cooperative states is increasing in h, and*
- *if $h > h^*$, cooperation monotonically decreases with h.*

The main difference between the $\rho = 2$ and the $\rho = 4$ case is that, in the latter, the optimality of bounded memory is more evident, since the share of cooperative regimes drastically declines for $h > h^*$. As before clustering (or low θ) is essential for reaching high levels of cooperation.

Figure 8 focuses on the final type distribution. As in Figure 5, we classify the steady-state type distribution into the three categories: Zero cooperation, Full cooperation, and CC survive. Most importantly, scenarios with full defection start to decrease for a certain value of memory, but as h reaches the optimal memory level they start to gain ground over cooperative scenarios.

Overall bounded memory clearly emerges as optimal from a population viewpoint. Loosely speaking one may say that 'Holding grudges forever' turns out to be non-optimal. But the underlying intuition is a bit more complicated. The first thing to note is that bounded memory leads agents to forget both 'cooperative' and 'non-cooperative' behavior. Hence in which direction the overall effect will go will depend on the population dynamics.

Let us discuss the intuition behind this result in some more detail. First if memory is very low ($h < h_1$), conditional cooperators cannot effectively learn the type of their opponents and hence are prone to exploitation by defectors as long as their share is not too large. Altruists can be exploited by defectors in any case and in addition they receive the same treatment from conditional cooperators as defectors since h is too small for CC types to effectively memorize cooperative behaviors. Hence for small h the share of altruists will decrease steadily. Defectors outperform conditional cooperators as long as they cooperate (i.e. as long as the share of defectors in the overall population is quite small). Once there are more defectors, conditional cooperators will also start defecting and hence the system will converge to a state where everyone defects.

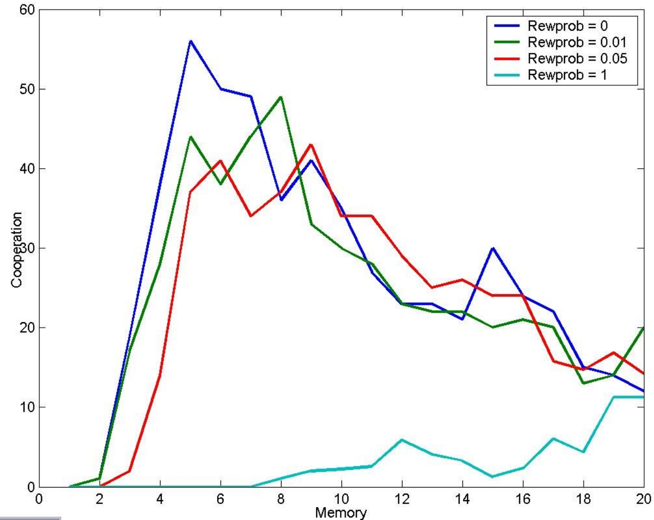

Figure 7: Frequency of cooperative outcomes as a function of memory for different values of rewiring probability when $\rho = 4$.

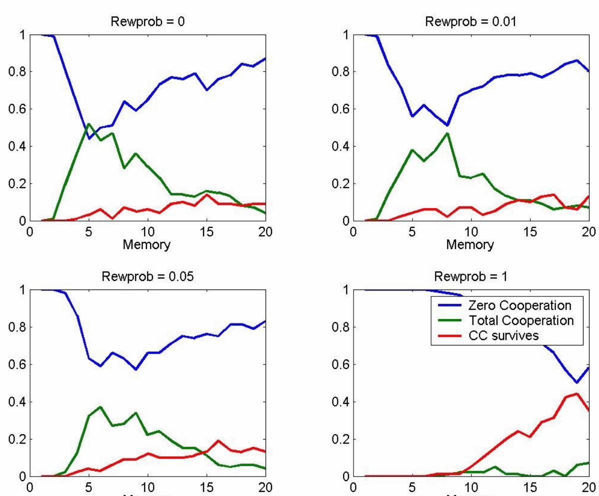

Figure 8: Shares of the different scenarios among the 100 runs per parameter setting as a function of memory for different rewiring probabilities for $\rho = 4$. Blue line: zero cooepration. Green line: total cooperation. Red line: CC survive.

For intermediate levels of memory ($h_1 < h < h_2$), conditional cooperators can prevent the extinction of altruists. After an initial decrease in the share of altruists, defectors are mostly matched with defectors and conditional cooperators. Defectors defect against each other and - due to larger memory - CCs are able to differentiate easier between D-types and types who display cooperative behavior. This effect can counterbalance the previous effect and even reverse it leading to the expansion of altruists in the society and to the extinction of defectors. The rate of cooperation in Figure 4 increases steeply with h.

Additionally, note that the length of memory necessary to establish cooperation depends on the network structure, represented by the rewiring probability θ. When the probability is low, clustering is high in the network and shorter memory is sufficient to stabilize cooperation. Clustering leads to convergence of opinions. Since neighbours interact more among each other, neighbors opinions become more similar to ones own experience. Due to this convergence of opinions shorter memory suffices to stabilize cooperation.

As memory gets larger ($h > h_2$), cooperation is sustained by conditionally cooperative (CC) agents who are more often the only survivors. Long memory enables conditional cooperators to distinguish between altruists and defectors, but now there is a drawback. Remember that initially defectors have a selective advantage over altruists because before reputations are established they are treated equally by conditional cooperators. This means that there is a point in which the system consists of quite many defectors. This temporary selective advantage of defectors has long-lasting consequences now. The reason is that if there are many defectors which are identified as such, conditional cooperators will quite often defect against such defectors. But since memory is very long this will never get forgotten and hence cooperation has a tough time emerging in this environment. But this means that many conditional cooperators will earn themselves a bad reputation among each other and will start defecting among themselves.

It is hence the interaction between reputation building and long memory which is detrimental to cooperation. Reputation building takes time and hence initially defection can spread. But in this 'hostile' environment conditional cooperators will see themselves forced to defect quite often which then (together with long memory) acts against them as a group, even if they now enjoy a selective advantage over defectors.

Another important observation we make here is that the lack of clustering cannot be offset by longer memory. When the rewiring probability is large ($\theta = 1$), for $h > h_2$ the average cooperation is still dramatically lower than with low θ, and cooperation monotonically decreases as clustering decreases (Figure 4).[21]

4.3 Direct and Indirect Reputation Building

We can analyze how the weight put on (in)direct reputation affects cooperation, by varying λ in our simulations. We have already shown above, that cooperation decreases significantly if we remove (in)direct reputation from the model entirely.

Here, we focus on the intermediate cases, where both matter at least a little. Figure 9 shows the levels of cooperation for different values of λ and $\theta = 0, .01, .05, 1$ (from the top-left-most to the bottom-right-most panel) for $\rho = 4$.

The largest levels of cooperative behavior are reached when equal weight is put on both direct and indirect reciprocity, irrespective of the values of other parameters. Hence, relying equally on both types of information promotes cooperative behavior in our model.

This is interesting because ex ante one might expect that the best λ should be smaller than 1/2 since neighbours in the aggregate have more experience with j than any player herself. On the other hand of course j's behavior towards me also depends on my reputation with j, making my own experience with j more informative. The latter effect would suggest $\lambda > 1/2$. On balance the two effects seem to cancel each other out as our evidence suggests.

We can observe in the real life though that people tend to rely more on their own experience than on the gossip or experiences of others. The question hence is whether relying more on own experience promotes

[21] Partial support for this can actually be found in an experimental study by Cassar (2007). She shows that people tend to cooperate more in a local network (when $\theta = 0$) than in a small world network. However, the effect of rewiring seems to be non-monotone in her experiments, as on random graphs (when $\theta = 1$) individuals cooperate more than on small-world networks.

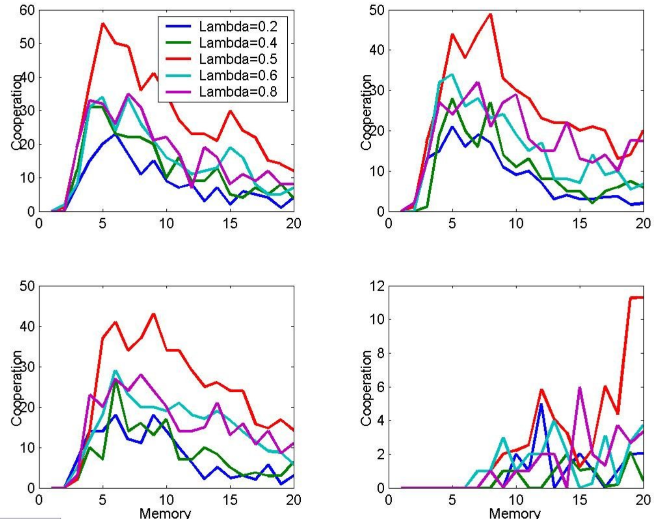

Figure 9: Frequency of cooperative outcomes as a function of memory for different values of θ and λ when $\rho = 2$.

cooperation from the evolutionary perspective. As Figure 9 shows there is an asymmetry. Giving more weight to own experience enhances cooperation more than weighting experience of others more heavily, as long as both direct and indirect reciprocity are considered. This is reflected by the violet and light blue plots ($\lambda = .8$ and $.6$, respectively), which lie above the green and dark blue lines basically for any parameter specification when θ is low.[22] Hence, there is a non-trivial role of the weights put on each type of information in our model.

On the other hand of course $\lambda = 1/2$ means already that an agent is weighting her own experience more than that of others, since the aggregate information obtained from neighbours is typically based on many more interactions. Overall, this allows us to conjecture that the fact that people generally rely more on their own experience may have emerged as natural adaptation and as a by-product toward the evolution of cooperative traits in the society.

4.4 Robustness

In this section, we analyze how robust the findings from previous sections are to changes of other conditions of the model. In particular, we perform two types of robustness checks. First, we introduce the possibility of mutations into the selection process. Second, we change some parameters held fixed until now such as the number of imitating agents (k), the preference bias of CC type for cooperation (w_{CC}) and the connection radius ρ. The corresponding graphs are reported in Appendix.

Regarding the possibility of mutations, we modify the selection process such that with a probability 0.02 the randomly chosen agent's fitness is not compared to the fitness of someone else, but she adopts a type randomly. In this way every behavioral type has the chance to be reintroduced to the population. Figure 13 shows the average frequency of cooperation in this case.[23] We may see that we obtain the same non-monotonic relationship between memory and the level of cooperation.

As for the sensitivity analysis, using the original model without mutations we hold most of the parameters

[22] For $\theta = 1$, the cooperation is very low in general and it is hard to make conclusions from Figure 9. However, we can still appreciate that equal share enhances cooperation well above cases and high values of λ still seem to lead to more cooperative societies than lower values.

[23] Note that when mutations are possible, the system never settles down. Hence we take the average level of cooperation over 30000 periods of run. The reported values are the averages of these over 100 runs.

fixed as indicated in each plot and only change the variable of interest on a grid.

Figure 14 graphs the frequency of cooperative outcomes for different values of k. We can clearly observe that as k increases the cooperation monotonically decreases. In case of higher k, social overturn is higher, i.e. in a given round more agents update their type, which means that types change more frequently and hence reputation becomes less effective. On the other hand, the minimal memory necessary to reach positive cooperation seems to be unchanged for different values of k.

In the previous analysis the preference bias of CC type for cooperation has been set to $w_{CC} = .35$, which is the value for which CC agents cooperate with opponents, who have cooperated in 50% of last meetings, about which information is available. If we increase the value of w_{CC}, CC types cooperate more easily whereas for small w_{CC} they will defect most of the time. As it can be seen in Figure 15, below a certain level of w_{CC} cooperation does not arise. As expected, for larger values of w_{CC}, cooperation is monotonically rising with w_{CC}.

In Figure 16, we also illustrate the effect of the connection radius more extensively. For ρ larger than one, there is a monotonic effect of increasing the number of neighbors on cooperation as seen before. The more neighbors agents have the more complicated is to have information about a particular neighbor and more difficult is to establish cooperative regimes. The only exception is $\rho = 1$, where the cooperation is actually lower than with $\rho = 2$.

Generally we conclude that our results are qualitatively robust to changes in parameter values.

5 Evolution of Memory

The previous section has shown that limited memory can be optimal from a population viewpoint. Using group selection arguments, societies where optimal memory prevails will cooperate more and hence outperform less cooperative societies. The natural question that arises now is whether limited memory is also optimal at the individual level, i.e. within groups. Hence, in this section we explore this hypothesis. To this aim, we enrich our environment introducing heterogenous memory levels and let selection works on types and memory simultaneously.

The model has exactly the same features as the original specification, except for the number of behavioral types. In particular we introduce three types of conditional cooperators differing in their memory levels. We introduce three levels of memory: $h = 2$, 7 and 15. Hence, there are five types of agents initially, each representing 20% of the population.

These memory levels were carefully selected for the purpose of this section. We rarely observe cooperative states with two-period memory. Hence $h = 2$ types. The largest memory, $h = 15$, lies on the decreasing part of the frequencies of cooperative regimes, but still close to the optimal memory span. It therefore represents the case of too long memory. Since in the simulations, the optimal memory is around $h = 7$, we introduce this level as a representation of optimal memory. The main objective of this section is to verify whether $h = 7$ will outperform other types and memory levels. The selection process still works at the level of behavioral types, but each memory level is treated as a different type in this section. Table 3 summarizes the average steady state distributions over 100 runs of each parameter specification.

>From the macro perspective, we observe that the average survival probabilities of defectors are very low, as long as θ is low.[24] Conditional cooperators, on the other hand, prevail in the population. Interestingly, there seems to be no clear evolutionary advantage for a particular memory level. The exception is $\theta = 1$, where the large memory types survive systematically more often than other conditional cooperators, but do not cooperate and coexist with defectors.

These results may not be very surprising. Larger memory types are suitable for the protection of the society against defectors. However, once cooperating types reach a larger proportion of the population, the cooperating types in principle cooperate mutually and memory levels play a role neither for successive cooperation nor for evolutionary fitness. In such situations evolution does not select among memory types

[24] This again confirms that clustering is essential for the proliferation of cooperative traits.

			$\rho = 2$			
θ	Coop.	A	D	CC_2	CC_7	CC_{15}
0	92.00	.1237	.0308	.2910	.3149	.2396
.01	90.26	.1230	.0303	.2463	.2952	.3052
.05	79.40	.1154	.0355	.2291	.3603	.2597
1	6.13	.0202	.2490	.1382	.1960	.3966

			$\rho = 4$			
θ	Coop.	A	D	CC_2	CC_7	CC_{15}
0	90.00	.1338	.0533	.3004	.2474	.2651
.01	92.00	.1180	.0361	.2863	.2532	.3064
.05	89.00	.0991	.0690	.2824	.3046	.2770
1	2.39	.0040	.3314	.1702	.1897	.3043

Table 3: Simulation results with heterogenous memory levels.

and they coexist in the population. This also holds for altruists, who indeed have relatively large shares after convergence. The undiscriminating nature of their behavior actually confirms the above intuition.

Table 4 gives a more detailed account of the simulation results.[25] In analogy with the previous analysis, it lists the percentages of scenarios that appear in the long run. This time, the classification is much more complex since more combinations arise in steady states.[26]

		θ	
Scenarios	0	.01	.05
CC_2,CC_7,CC_{15}	7	5	4
CC_{15}	2	0	7
A,CC_2,CC_7,CC_{15}	**71**	**64**	**39**
A,CC_2,CC_{15}	7	5	6
A,CC_2,CC_7	1	6	3
A,CC_7,CC_{15}	**3**	**10**	**18**
A,CC_7	0	0	3
D,CC_2,CC_7,CC_{15}	**2**	**1**	**11**
D,CC_2,CC_{15}	1	1	4
D,CC_2,CC_7	2	4	1
Others	4	4	4

Table 4: Particular Scenarios with heterogenous memory levels. *rho=2*

In sum, within group selection does not lead to a clear evolutionary advantage of either memory level, at least as long as 'all' types of memory are present. Populations, where at least some agents have (optimal) bounded memory will do better than populations where all agents have very long memory. Hence, the optimal memory level provides an advantage across groups, but is not evolutionary costly within groups. Since there is no real trade-off between within- and between-group forces, optimal memory may be an outcome of evolutionary conflicts across groups.

[25] We do not report the $\theta = 1$ case in the table, because full defection mostly arise after convergence, characterized mainly by the coxistence of defectors and conditnional cooperators. It is easily visible in Table 3.

[26] In the table, *Others* refers to non-listed low-frequency final distributions. No type but CC_{15} ever survives on its own. Other scenarios that survive at least once and are not explicitly stated are: (CC_2,CC_{15}), (A,CC_7), and (A,CC_{15}).

6 Conclusion

We analyzed the evolution of cooperation in a setting that comes very close to actual human interactions. Agents with limited memory interact and share information through a given social network. We have seen that cooperation can but need not evolve in such a setting. Our results show that there exists an interior memory size maximizing the rate of cooperation in the long run. Moreover, we observe a non-trivial relation between the degree of reliance on direct and indirect reputation.

These results add an interesting new viewpoint to the discussion of why evolution did not endow us with unlimited brain capacity or, more loosely speaking, did not make us "infinitely" smart. Firstly, of course, larger memory capacities imply larger cognitive costs. However, our results also show that it can be optimal (at least from a population viewpoint) to be endowed with finite memory in strategic situations. In line with the group selection literature limited memory in our model can emerge from the conflict of two populations endowed with different memory capacity in the brain.

7 Appendix

7.1 Meeting strangers: Can cooperation still survive?

In the main text, people only interact with their first-order neighbors. This assumption can be too extreme, since in real life people meet more socially distant individuals or even complete strangers. In this section, we explore an alternative setup, concerning the matching mechanism. More precisely, we relax the extreme locality of matching and rather assume that people can meet *anybody* in their component. The probability to interact with a particular player is proportional to the shortest " geodesic distance" that separates the two players.[27]

Again, denote the shortest distance between i and j by d_{ij} ($d_{ij} = d_{ji}$). The probability for i to interact with j is the following:

$$P_{ij}(d_{ij}) = \begin{cases} \dfrac{e^{-\alpha d_{ij}}}{\sum\limits_{k \neq i} e^{-\alpha d_{ik}}} & \text{if } d_{ij} > 0 \\ 0 & \text{if } d_{ij} = 0 \text{ or } d_{ij} = \infty \end{cases}$$

Note that with this function $P_{ij}(d_{ij}) \neq P_{ji}(d_{ji})$, more central players in the network can still be matched more frequently, as in the original speficiation of the model in the main text, and we define the evolutionary fitness as before.[28]

Observe that the higher α the more likely it is to meet neighbors and the more unlikely to meet distant individuals. Hence, α parametrizes the effect of the network for matching. As $\alpha \to \infty$, everybody meets exclusively her neighbors. This is the original model. If $\alpha = 0$ matching is completely random and this would represent Scenario 2 in Section 4.1. Finally, note that if $d_{ij} = \infty$ (agents in disconnected components) then $p(d_{ij}) = 0$.

First, let us illustrate the effect of α on the matching probabilities for different networks. Imagine a regular lattice network of 20 individuals, where $\rho = 2$. For $\theta = 0$, each agent has four direct, second–order, third-order and fourth-order neighbors, and three neighbors of distance five. Table 5 lists the matching probabilities for an individual to meet each t^{th}- order neighbor (columns) for different values of α (rows).

α \ d_{ij}	1	2	3	4	5
0.5	.109	.066	.040	.0240	.0150
1	.160	.059	.022	.0080	.0030
1.5	.194	.043	.010	.0020	.0005
2	.216	.029	.004	.0005	.0007
2.5	.229	.019	.002	.0001	.0001

Table 5: Effect of alpha on matching probabilities.

Rewiring the links introduces heterogeneity, such that agents have different number of neighbors of any order. This obviously generates heterogeneity in the matching probabilities.

Intuitively, as α gets higher agents are matched most frequently with agents from the neighborhood which limits the number of possible opponents and facilitates the learning about their types. For a given values of h and θ, the cooperation increases monotonically in α. The value of memory necessary to have some cooperation also rises with α.

Generally, the overall pattern of cooperation as a function of memory is robust to changes in the locality of matching for both $\rho = 2$ and $\rho = 4$.

[27] Geodesic distance between any two nodes is the minimum number of links that connects the two nodes.

[28] Note that if i is chosen as row player, the probability that i is matched with somebody else still adds up to 1: $\sum_{j \neq i} P_{ij}(d_{ij}) = \sum_{j \neq i} \dfrac{e^{-\alpha d_{ij}}}{\sum\limits_{k \neq i} e^{-\alpha d_{ik}}} = 1$.

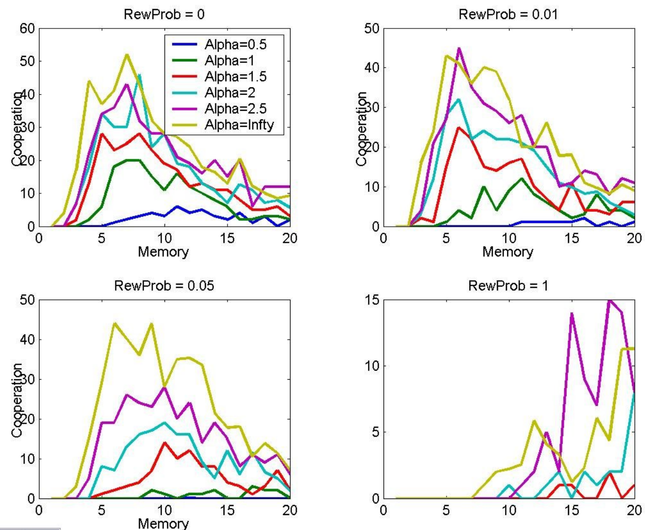

Figure 10: Effect of locality of encounters ($\rho = 4$). Larger α's corresonds to more local encounters.

Figure 10 presents the simulation results for differnt degrees of locality of matching. It generally confirms the theoretical results and the basic intuitions. Allowing for less frequent encounters of more distant individuals, about whom ones is less likely to have information, does not eradicate cooperation. The cooperation increases in α, but its levels are already very high for relatively low α.

These finding show that if we embed our model into a more realistic environment, where people engage in both regular contacts with their neighbors and more rare encounters with more distant individuals, cooperation still proliferates, despite limited memory capacity, which obviously interact negatively with the possibility to have information about all potentional encounters. Moreover, qualitatively the results from the main text still show up here.

7.2 Observability of Type Changes

What happens to our results if type changes are observable, i.e. if agents' reputations and memories are reset once they have changed their types. Note that since an agent's type is not observable it makes not much sense to assume that type changes are observable. Still let us be generous with modeling and explore this possibility as well.

Broadly speaking one could interpret the two cases as situations with and without "stigma". "Stigma" refers to situations, in which an agent after changing her type is still burdened by her "old" reputation and memory. This case can reflect cultural evolution among humans and is certainly the correct model if changes of type are not completely observable. The "No-Stigma" case refers to the case where an agent after changing her type enjoys a "clean" reputation and memory. The non-stigma case could also be interpreted as genetic evolution, but keep in mind that our model is essentially a model of human (cultural) evolution. This evolutionary setup should, in principle, help the proliferation of cooperation, since with large memory levels, agents are not burdened with the reputation of their ancestors.

Figure 11 shows that the average cooperation monotonically increases with memory, even for very large memory capacities for $\rho = 2$. However, even under such evolutionary specification, we observe that cooperation rises markedly in the begining, but after a certain memory level there seems to be a stagnation of the effect of memory lenght on cooperation. Cooperation still increases, but the advantages of larger memories are very low as h grows large. A careful comparison of Figures 4 and 11 also reveals that, for small values of memory there is no difference between the two cases. Figure 12 again provides the picture of the final type distribution for different parameter constellations.

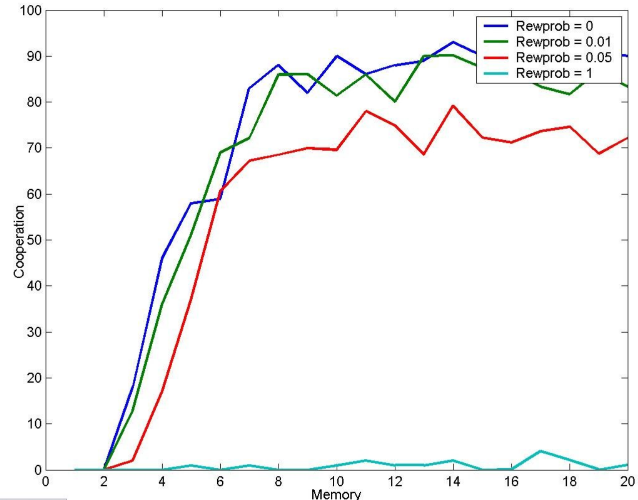

Figure 11: Average cooperation as a function of memory for different rewiring probability values when $\rho = 2$. No-Stigma.

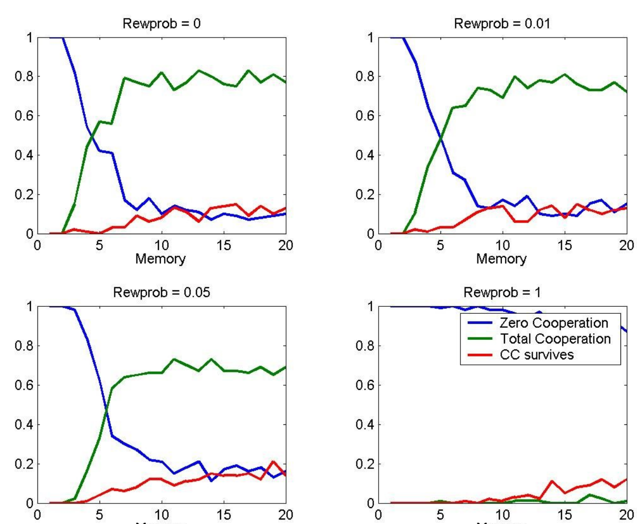

Figure 12: Shares of scenarios as a function of memory for different rewiring probabilities for $\rho = 2$. No-Stigma Case. Blue line: zero cooepration. Green line: full cooperation. Red line: CC survive.

7.3 Robustness checks: Figures

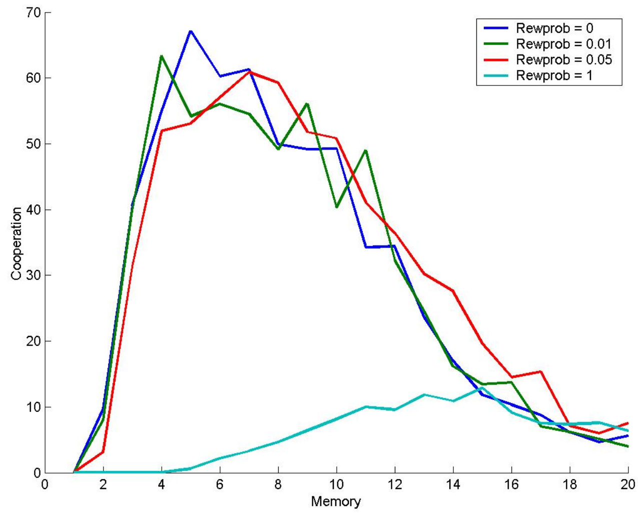

Figure 13: Average level of cooperation as a function of memory when mutations are introduced for $\rho = 4$.

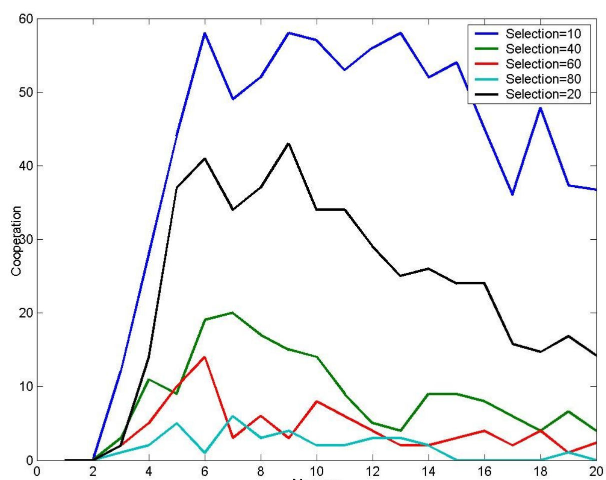

Figure 14: Frequency of cooperative outcomes as a function of memory for different values of k ($\theta = .05$ and $\rho = 4$).

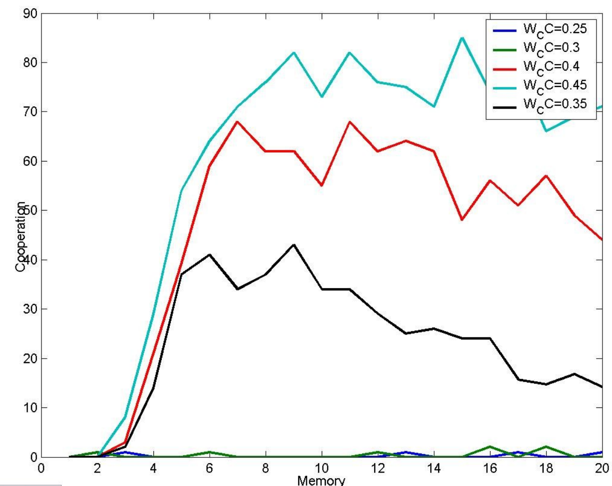

Figure 15: Frequency of cooperative outcomes as a function of memory for different values w_{CC} ($\theta = .05$ and $\rho = 4$).

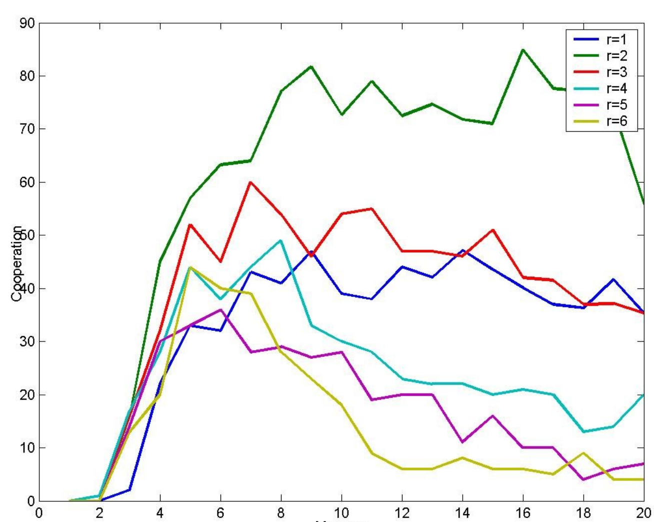

Figure 16: Frequency of cooperative outcomes as a function of memory for different values of ρ ($\theta = .05$).

References

[1] Axelrod R. (1984): The Evolution of Cooperation, New York: Basic Books.

[2] Axelrod R. and Hamilton, W.D. (1981). The evolution of cooperation, Science 211, 1390-1396.

[3] Bester H. and Güth, W. (1998): Is altruism evolutionary stable? Journal of Economic Behavior and Organization, 34(2), 193-209.

[4] Boyd, R. and Richerson, P.J. (1989): Evolution of Indirect Reciprocity, Social Networks 11, 213-236.

[5] Boyd, R. and Richerson, P.J. (1990): Group selection among alternative evolutionarily stable strategies, Journal of Theoretical Biology 145, 331-342.

[6] Cassar, A. (2007): Coordination and cooperation in local, random and small world networks: Experimental evidence, Games and Economic Behavior 58, 209-230.

[7] Cowan, N. (2001): The magical number 4 in short-term memory: A reconsideration of mental storage capacity, Behavioral and Brain Sciences, 24, 87-185.

[8] Cox, S.J., Sluckin, T.J. and Steele, J. (1999). Group Size, Memory, and Interaction Rate in the Evolution of Cooperation, Current Anthropology 40(3), 369-377.

[9] Eshel, I., Binmore, K., and Shaked, A. (1998): Altruists, egoists, and hooligans in a local interaction model, American Economic Review 88, 157-179.

[10] Fudenberg, D. and Maskin, E. (1986): The Folk theorem in repeated games with discounting or with incomplete information, Econometrica 50, 533-554.

[11] Goyal, S. (2007): Connections: an introduction to the economics of networks. Princeton University Press.

[12] Hauert, Ch. & Schuster, H. G. (1997). Effects of increasing the number of players and memory steps in the iterated Prisoner's Dilemma, a numerical approach, Proceedings of the Royal Society B 264, 513-519.

[13] Kirchkamp, O. (2000). Spatial Evolution of Automata in the Prisoner's Dilemma, Journal of Economic Behavior and Organization 43, 239-262.

[14] Kreps, D.M., Milgrom, P., Roberts, J., and Wilson, R. (1982): Rational cooperation in the finitely repeated Prisionner's Dillema, Journal of Economic Theory 27, 245-252.

[15] Leimar, O. and Hammerstein, P. (2001): Evolution of cooperation through indirect reciprocity, Proceeding of the Royal Society B, 268, 745-753.

[16] Mashuda, N. and Aihara, K. (2003): Spatial prisoner's dilemma optimally played in small-world networks, Phisics Letters A, 313, 55-61.

[17] Mengel, F. (2007): The Evolution of Function-Valued Traits for Conditional Cooperation, Journal of Theoretical Biology 245, 564-575.

[18] Mengel, F. (2008): Matching Structure and the Cultural Transmission of Social Norms", Journal of Economic Behavior and Organization 67, 608-623

[19] Milinski, M., Semmann, D., Bakker, T.C.M., and Krambeck, H. (2001): Cooperation through indirect reciprocity: image scoring or standing strategy?, Proceedings of the Royal Society B 268, 2495-2501.

[20] Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. Psychological Review, 63, 81-97.

[21] Milinski, M. and Wedekind, C. (1998): Working memory constrains human cooperation in the Prisioner's Dilemma, Proceedings of the National Academy of Sciences 95: 13755-58.

[22] Mohtashemi, M. and Mui, L. (2003): Evolution of indirect reciprocity by social information: the role of trust and reputation in evolution of altruism, Journal of Theoretical Biology 223, 523-531.

[23] Nakamaru, M., Matsuda, H., and Iwasa, Y. (1997): The evolution of cooperation in a lattice structured population, Journal of Theoretical Biology 184, 65-81.

[24] Nakamaru, M., Nogami, H. , and Iwasa, Y. (1998): Score-dependent fertility model for the evolution of cooperation in a lattice, Journal of Theoretical Biology 194, 101-124.

[25] Nowak, M.A., and Sigmund, K. (1992): Tit for tat in heterogeneous populations. Nature 355: 250-253.

[26] Nowak, M.A., and Sigmund, K. (1998): The Evolution of Indirect Reciprocity, Nature 437, 1291-1298.

[27] Ohtsuki, H., Hauert, C., Lieberman, E., and Nowak M.A. (2006): A simple rule for the evolution of cooperation on graphs and social networks. Nature 441, 502-505.

[28] Ohtsuki, H. and Nowak M.A. (2007): Direct reciprocity on graphs. Journal of Theoretical Biology 247, 462-470.

[29] Roberts, G. (2008): Evolution of diret and indirect reciprocity, Proceedings of the Royal Society B 275, 173-179.

[30] Rubinstein, A. (1979): Strong perfect equilibrium in supergames, International Journal of Game Theory 9, 1-12.

[31] Seinen, I, and Schram, A. (2004): Social status and group norms: Indirect reciprocity in a repeated helping experiment , European Economic Review 50, 581-602.

[32] Sheng Z.H., Hou Y.Z, Wang, X.L., and Du, J.G. (2008). The Evolution of Cooperation with Memory, Learning and Preferential Selection in Spatial Prisoner's Dilemma Game, Journal of Physics: Conference Series 96, 012107.

[33] Stevens, J.R. and Hauser, M.D. (2004): Why be nice? Psychological constraints on the evolution of cooperation, TRENDS in Cognitive Sciences 8(2): 60-65.

[34] Trivers, R. (1971): The evolution of reciprocal altruism. Quarterly Review of Biology 46, 35-57.

[35] Vega Redondo, F. (2007): Complex Social Networks, Cambridge University Press.

[36] Watts, D.J. and Strogatz, S.H. (1998). Collective dynamics of 'small-world' networks. Nature 393: 409–410.

[37] Wedekind, C. and Milinski, M. (2000): Cooperation through image scoring in humans, Science 288, 850-852.

[38] Weizsaecker, G. (2009), Do we follow others when we should? A simple test of rational expectations, American Economic Review, forthcoming.

PUBLISHED ISSUES*

WP-AD 2010-01 "Scaling methods for categorical self-assessed health measures"
P. Cubí-Mollá. January 2010.

WP-AD 2010-02 "Strong ties in a small world"
M.J. van der Leij, S. Goyal. January 2010.

WP-AD 2010-03 "Timing of protectionism"
A. Gómez-Galvarriato, C.L. Guerrero-Luchtenberg. January 2010.

WP-AD 2010-04 "Some game-theoretic grounds for meeting people half-way"
P.Gadea-Blanco, J.M. Jiménez-Gómez, M.C. Marco-Gil. February 2010.

WP-AD 2010-05 "Sequential city growth: empirical evidence"
A. Cuberes. February 2010.

WP-AD 2010-06 "Preferences, comparative advantage, and compensating wage differentials for job routinization".
C. Quintana-Domeque. February 2010.

WP-AD 2010-07 "The diffusion of Internet: a cross-country analysis"
L. Andrés, D. Cuberes, M.A. Diouf, T. Serebrisky. February 2010.

WP-AD 2010-08 "How endogenous is money? Evidence from a new microeconomic estimate"
D. Cuberes, W.R. Dougan. February 2010.

WP-AD 2010-09 "Trade liberalization in vertically related markets"
R. Moner-Colonques, J.J. Sempere-Monerris, A. Urbano. February 2010.

WP-AD 2010-10 "Tax evasion as a global game (TEGG) in the laboratory"
M. Sánchez-Villalba. February 2010.

WP-AD 2010-11 "The effects of the tax system on education decisions and welfare"
L.A. Viianto. March 2010.

WP-AD 2010-12 "The pecuniary and non-pecuniary costs of job displacement. The risky job of getting back to work"
R. Leombruni, T. Razzolini, F. Serti. March 2010.

WP-AD 2010-13 "Self-interest and justice principles"
I. Rodríguez-Lara, L. Moreno-Garrido. March 2010.

WP-AD 2010-14 "On spatial equilibria in a social interaction model"
P. Mossay, P.M. Picard. March 2010.

WP-AD 2010-15 "Noncooperative justifications for old bankruptcy rules"
J.M. Jiménez-Gómez. March 2010.

WP-AD 2010-16 "Anthropometry and socioeconomics in the couple: evidence from the PSID"
S. Oreffice, C. Quintana-Domeque. April 2010.

WP-AD 2010-17 "Differentiated social interactions in the US schooling race gap"
L.J. Hall. April 2010.

WP-AD 2010-18 "Things that make us different: analysis of variance in the use of time"
J. González Chapela. April 2010.

WP-AD 2010-19 "The role of program quality and publicly-owned platforms in the free to air broadcasting industry"
M. González-Maestre, F. Martínez-Sánchez. June 2010.

WP-AD 2010-20 "Direct pricing of retail payment methods: Norway vs. US"
F. Callado, J. Hromcová, N. Utrero. June 2010.

WP-AD 2010-21 "Sexual orientation and household savings.Do homosexual couples save more?
B. Negrusa, S. Oreffice. June 2010.

WP-AD 2010-22 "The interaction of minimum wage and severance payments in a frictional labor market: theory and estimation"
C. Silva. June 2010.

WP-AD 2010-23 "Fatter attraction: anthropometric and socioeconomic matching on the marriage market"
P.A. Chiappori, S. Oreffice, C. Quintana-Domeque. June 2010.

WP-AD 2010-24 "Consumption, liquidity and the cross-sectional variation of expected returns"
E. Márquez, B. Nieto, G. Rubio. July 2010.

WP-AD 2010-25 "Limited memory can be beneficial for the evolution of cooperation"
G. Horváth, J. Kovárík, F. Mengel. July 2010.

WP-AD 2010-26 "Competition, product and process innovation: an empirical analysis"
C.D. Santos. July 2010.

WP-AD 2010-27 "A new prospect of additivity in bankruptcy problems"
J. Alcalde, M.C. Marco-Gil, J.A. Silva. July 2010.

* Please contact Ivie's Publications Department to obtain a list of publications previous to 2010.

serie ad

Ivie

Guardia Civil, 22 - Esc. 2, 1º
46020 Valencia - Spain
Phone: +34 963 190 050
Fax: +34 963 190 055

Department of Economics
University of Alicante

Campus San Vicente del Raspeig
03071 Alicante - Spain
Phone: +34 965 903 563
Fax: +34 965 903 898

Website: www.ivie.es
E-mail: publicaciones@ivie.es