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Abstract: In the 1896-97 *Cours d'Économie Politique* and the 1906 *Manuale di Economia Politica*, Vilfredo Pareto made no use of the 'Fisherian' type quantity theory equations of exchange that Walras developed in the 1874 edition of the *Éléments d'Économie Politique Pure* and completely ignored Walras' more mature 'Cambridge' type *encaisse desirée* (demand for real cash balances) approach that Walras integrated within general equilibrium in the 1900 edition of his *Éléments*. This paper critically examines a fragmented manuscript that Pareto wrote in 1920-21, and which was first published in 2005, for the purpose of clarifying the reasons why he did not follow Walras in integrating monetary theory within general equilibrium. In many respects, the manuscript follows Walras more closely than Pareto's major published works, but the substantive point is that Pareto was unable to introduce money within general equilibrium theory along the lines envisaged by Walras because he explicitly recognized the interdependence between money and the real economy and abandoned the quantity theory of money.

1   Introduction

Arthur Marget (1935, p 152.) pointed to the 'extravagant praise' bestowed on Pareto for his work on monetary theory by members of the Lausanne school.[2] This greatly irritated Marget as there were no such words of praise for Léon Walras, whose innovative and original work on monetary theory was left to languish by Vilfredo Pareto and his followers. In regard to Pareto specifically, Marget (1935, pp. 152-3) points to two deficiencies: first, he appears to have made no use of the Fisherian type quantity theory equations of exchange that Walras developed in the first edition of the *Éléments d'Économie Politique Pure* (1874); and second, he completely ignored Walras' Cambridge type *encaisse desirée* (demand for real cash balances) approach to monetary theory, which was developed over subsequent editions of the *Éléments* and culminated in

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[1] I would like to thank Darrell Turkington for his correction of Pareto's equation (23) in the manuscript. Any errors remain the responsibility of the author.

[2] For example, in 'Vilfredo Pareto e la teoria della moneta', Guido Sensini concludes:

> "all that we now know to be scientifically correct with respect to the economic theory of money is found, as is known, in the *Cours* of Pareto himself … in addition to various passages of his *Manuel*, while the sociological theory of money, which Pareto founded … , remains entrusted to several points in the *Trattato di Sociologia generale*, which would be well known to students of the social sciences"
>
> (Sensini 1929[1932], p. 215)

an attempt to fully integrate money within general equilibrium theory in the 1900 edition of Walras' great work.

In the subsequent secondary literature, the context of Pareto's position on monetary economics has been clarified. In particular, Pascal Bridel (1997, 2000) has shown that Pareto's approach to monetary theory was considered rather than arbitrary: monetary economics was maintained as applied economics and not pure economic theory largely because of Pareto's adoption of a 'successive approximations' methodology and his intention of accounting for the non-logical aspects of behaviour related to monetary phenomena. Nevertheless, from the published works of Pareto that Marget and Bridel were able to consider, it is still correct to conclude that Pareto did not use Walras' exchange equations for the purposes of monetary theory and that he did not utilize, or comment on, Walras' *encaisse désirée* approach to monetary theory. Nevertheless, it is also relevant to note, as Bridel (1997, p 157) has, that Pareto supported the quantity theory of money in his *Cours d'Économie Politique* (Pareto [1896-97] 1971), albeit as an approximation.[3]

More recently, though, thanks to the investigations and editorial work of Fiorenzo Mornati, a fragmented and previously unpublished manuscript written by Pareto on monetary theory has been published (Pareto 2005). This manuscript, which was located in the University of Lausanne's archives, was written in 1920-21 when Pareto was 72 years of age. In the decade prior to writing it, he was almost exclusively engaged in his long inquiry into general sociological theory. Even though economics is a branch within Pareto's sociological theory, and sociological discussions were often directed towards economic and monetary phenomena, this manuscript is unusual in that it marks one of the few occasions that the elderly Pareto set about formally investigating a fundamental issue in economic theory.

Publication of this manuscript provides an opportunity to update and clarify some of the conclusions mentioned earlier on Pareto's approach to monetary theory. Most importantly, it reveals that Pareto systematically: (i) reflected on quantity theory of money in terms of equations of exchange that were similar to, though not identical to, those developed by Walras in the first edition of the *Éléments*; (ii) interrogated these equations, while remaining within the context of the general equilibrium theories of exchange and transformations in production, to critically assess the quantity theory of money; and (iii) concluded that the quantity theory must be adopted because it does not account for interdependencies between money and the real economy.

This study is in two main parts. The manuscript is critically overviewed in the first part of the study. As Pareto alters the meaning of the coefficient representing the 'velocity of circulation' without warning part way through his analysis, and provides only incomplete explanations of aspects of his analysis, the main contribution of this part is to clarify the detail and character of Pareto's critical assessment of the quantity theory of money. In the second part of the study, the Walrasian and non-Walrasian elements of the manuscript

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[3] Pareto's most detailed published discussion on the economics of monetary theory is included in the *Cours*.

are identified and investigated. Identification of the Walrasian antecedent in the manuscript is significant because it is missing from, or obscured in, Pareto's previously published economic studies. To clarify why Pareto did not follow Walras more fully on matters of money, notable non-Walrasian elements are identified, in Pareto's manuscript in the first instance, and then in discussion of his work on net returns and rents from his two major books on economic. It is concluded that, while Pareto went further in accommodating Walras ideas on monetary theory in general equilibrium than was previous thought, his explicit investigation of the fundamental interdependence between real and monetary variables and the economic significance of divergent net rates of return on items of capital prohibited him from integrating money within general equilibrium theory along the lines that the mature Walras envisaged.

2   The Manuscript: A Critical Overview

The manuscript commences by pointing to a set of equations that take the same general form:

$$a_0 p_0 + a_1 p_1 + ... - a_n p_n - ... = 0$$

where:

$a_0, a_1, ...a_n$ are quantities of goods and services

$p_0, p_1, ...p_n$ are the prices of the goods and services with corresponding subscripts.

This form of equation is used for the cases of exchange of goods and the transformation of goods in production. For simplicity, Pareto truncates the above relationship by using the letter S to indicate a sum of values within the above equation for an individual, reserving the Greek letter Σ to indicate the summation of the equations for all individuals when deriving economic aggregates.

(1) $a_0 p_0 + S(ap) = 0$

Equation (1)[4] provides the basis for developing a formal definition of money. However, to investigate the relationship between monetary and real phenomena, Pareto explicitly acknowledged that the range of entities engaged in monetary transactions must be so broad that the scope of equation (1) has to be extended beyond the usual limits of pure general equilibrium theory. Specifically, the group of individuals and enterprises must

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[4] When equations are number sequenced in this paper with Arabic numerals, they correspond to the equation numbers that Pareto utilised in his manuscript. In the cases where a group of equations are labelled in the manuscript with a single equation number, then a decimal point has been introduced and followed by a decimal number sequence so that specific equations from within the group can be identified. For example, equation (9.3) is used in this article to represent the third equation in the group of equations that Pareto labelled as equation (9). When roman numerals indicate the equation number sequence in this article, the corresponding equations are not from Pareto's original manuscript.

include entities like banks, local government, provincial governments and the State, so the full set of equations (1) can account for expenses and incomes, gifts given and gifts received, taxes paid and government spending, deposits made into banks and payments made by banks for all individuals and enterprises that act economically. In doing so, Pareto saw the strictly experimental basis of the pure theory as diminishing, mainly because redistributive process associated with public sector actions obscure relative economic valuations that may be revealed through voluntary choice. Nevertheless, this matter is raised as a caveat only – he did not raise it as a reason for excluding monetary analysis from general equilibrium theory.

Defining the Quantity of Money

When equation (1) is limited to individuals and entities other than banks, and money is commodity-money, it may be formally expressed as:

(i) $$a_0 p_0 + \sum_{c=1}^{\tau}(a_c p_c) - \sum_{s=1}^{\varepsilon}(a_s p_s) = 0$$

Where:

a_0 quantity of the numeraire good A_0 acquired by economic entities.

p_0, $= 1$ and equates to the physical unit of measurement in which a_0 is expressed,

Therefore, $a_0 p_0 = A_0$.

a_c quantity of goods and services acquired c where $c \in C = (1, 2, 3, \ldots \iota)$.

p_c, is the price of goods and services c acquired, relative to p_0.

a_s is the quantity of goods and services supplied, where $s \in S = (i, ii, iii, \ldots \varepsilon)$.

p_s, is the price of goods and services s supplied, relative to p_0.

Once the above equation is summed across all economic entities in the economy, say β for the illustrative purposes of this paper, the value of aggregate expenditure on acquisition of goods and services is evidently equal to the aggregate receipt of income received from goods and services provided:

(ii) $$\sum_{i=1}^{\beta}\left(a_0 p_0 + \sum_{c=1}^{\tau}(a_c p_c)\right) = \sum_{i=1}^{\beta}\left(\sum_{s=1}^{\varepsilon}(a_s p_s)\right)$$

That is, the value of goods and services demanded (the left hand side) and the value of goods and services supplied (the right hand side) are equal. The aggregate value of transactions may therefore be represented by either the left hand side of equation (ii) or the right hand side of equation (ii). On the face of it then, it is perhaps surprising that Pareto uses equation (1) to derive his second equation in the form:

(2) $$\frac{1}{2}\sum(a_0 p_0 + a_1 p_1 + \ldots) = \frac{1}{2}\sum(a_n p_n + \ldots)$$

Of course, in terms of the mathematical equality of the left and right hand sides of the equations, the multiplication of each side by a constant, in this case ½, is trivial.

However, as the left hand side of the equation is considered in isolation, as the measure for the value of transactions, the matter is not trivial. Pareto's explanation for this is that values figure twice for each and every exchange operation - once as a collection and again as a payment. Of course, equation (ii) also shows two acquisitions and payments, but this is developed from equation (i) which does not extend to banks. Equation (1), in contrast, explicitly extends to banks and records the 'debits and credits' associated with exchange and transformations in production as well as the receipt and payment of money - either money-as-a-good (numeraire) or paper-money (fiat money). In view of this, it appears that Pareto intended equation (2) to account for both the exchange of money (tendering/receiving currency or the debiting/crediting of accounts) and the exchange of goods and services (acquired and supplied).[5]

The *total sum of money in use*, σ, is defined in the manuscript as the aggregate value of economic transactions multiplied by two coefficients: α, representing the proportion of the aggregate value of economic transactions that is the subject of monetary exchanges; and, v, 'to obtain the sum (of money) really operating ... which many economists call the *velocity of circulation* of money' (2005, p. 261).

$$(3) \qquad \sigma = \left(\frac{\alpha}{v} \cdot \frac{1}{2} \right) \Sigma \left(a_0 p_0 + a_1 p_1 + ... \right)$$

The coefficient α is not only introduced to account for transactions undertaken through the banking system, as discussed previously, but also for gifts and other exchanges received without payment. The coefficient v is empirical, it is simply the coefficient that is empirically necessary to derive the quantity of money in the economy once the aggregate value of economic transactions has been determined and the α coefficient has been established. The conceptual meaning of coefficient v is consistent with the velocity of circulation in equation (3). However, this is not the case in subsequent equations when the same coefficient symbol is used with a modified conceptual meaning, as discussed in the forthcoming sub-section on *Criticisms of the Quantity Theory*.

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[5] On that basis, the variables in Pareto's equation (2) would correspond to:

$a_0 p_0$ the quantity of money received, either directly as currency or indirectly through the crediting of an account, in exchange for the supply of goods and services;

$+a_1 p_1 + ...$ the value of economic goods and services acquired through exchange or transformation; and

$a_n p_n$ the quantity of money paid, either directly as money or indirectly through the debiting of an account, from the acquisition of goods and services, with sum of the *ap* terms (with alphabetic and not numeric subscripts) subsequent to $a_n p_n$ representing the value of goods and services supplied for exchange or transformation.

Money and the Arbitrary Determination of General Equilibrium Values

When the unit of price, (p_0) in equation (1), is a defined unit of an economic good A_0, such that a_0 is the quantity of good A_0 measured in the same units that price is set,[6] then Pareto concluded that 'all is determined and nothing in the equations of [general] equilibrium is left arbitrary' (Pareto 2005, p.261). However, when paper (fiat) money is introduced into the numerous equations (1), with the unit of price p_0 redefined as a unit of paper money (either money received directly, as currency, or indirectly, as credits to bank accounts), prices alter across the entire economic system with, among other things, the quantity of money. Pareto classes this equilibrium as 'entirely arbitrary' (Pareto 2005, p.261).

Importantly, Pareto's distinction between arbitrary and non-arbitrary determination of equilibrium is not introduced to suggest that money is a veil - the basic purpose of the manuscript is to demonstrate that money is not a veil. Rather, consistent with what Bridel (1997, p. 153) has identified in Pareto's earlier published works, it would appear that comments on the 'determination' of equilibrium are related to the fundamental relationship between value and utility (ophelimity). When prices are determined non-arbitrarily, valuations are considered with respect to a numeraire good and each of the exchanged quantities within equations (1) are included within utility functions, including good a_0 which plays both a monetary and a non-monetary role in the economy. When equilibrium is 'entirely arbitrary' because valuations are expressed in terms of fiat money, Pareto appears to acknowledge that the price p_0 of paper money a_0 is unrelated to utility and the absolute price of economic goods in monetary terms is not directly related to utility.

The Quantity Theory of Money

Pareto defines the *quantity theory of money* in terms of equations (8.2) and (9.1):

(8.2) $$a_0\left(\frac{p_0}{\mu}\right) + S\left[a\left(\frac{p}{\mu}\right)\right] = 0$$

(9.1) $$\sigma_1 = \left(\frac{1}{\mu}\right)\sigma$$

For an individual (or entity), equation (8.2) is algebraically equivalent to equation (1), but it represents economic relationships in a new period: p_0 in the first left hand term (the unit of currency) and p in the second left hand term (the prices of economic goods and services) are both discounted with respect to the initial period by the same coefficient, μ. For the economy in aggregate, equation (9.1) is equation (3) after adjustment for the

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[6] For example, if p_0 was an ounce of gold, then a_0 would be the quantity of gold expressed in ounces.

difference between equation (1), in the initial period, and equation (8.2) in the new period. In effect, the parameter μ is implicitly assumed to be 1 in equations (1) and (3), which define the initial period, whereas, in the new period, a change in the quantity of money and prices is indicated by a movement in the parameter value μ away from 1. When μ is less that one, the quantity of money and prices rise between the initial and the new period. After reflecting on this relationship Pareto notes that:

> "one has a theory, complete, simple, beautiful. A shame it does not accord too much with the facts."
>
> (Pareto 2005, p. 264).

Criticisms of the Quantity Theory

The manuscript reveals that Pareto had two main concerns with the quantity theory of money. First, and most fundamentally, the theory excludes interdependent influences between money and real economic phenomena. Second, a single homogenous change in the price of all economic goods and services is correlated with changes in the quantity of money, which is at odds with the observation of multiple and diverse price variations across a range of goods.

To interrogate the relationship between the quantity of money and prices, Pareto undertakes a number of exploratory algebraic investigations to consider whether equations (8.2) and (9.1) can be modified to provide a more realistic representation of the relationships between the monetary and real phenomena. His primary purpose was to modify these equations to account for the interdependence between these phenomena:

> "The reality is that there is only a relationship of interdependence and to know the particulars [of this relationship] considerations other than monetary circulation and prices are needed."
>
> (Pareto 2005, p. 264)

He finally settled on dividing the monetary and real terms in equations (8.2) and (9.1) by another coefficient, v:

(10)
$$\frac{a_0}{v\mu} + S\left(\frac{a}{v}\left(\frac{p}{\mu}\right)\right) = 0$$

(9.3)
$$\sigma_1 = \left(\frac{1}{v\mu}\right)\sigma$$

It is necessary to recognise a change in the character of the coefficient v in the new period compared to its use in the initial period. In equation (3), which refers to an initial period, the co-efficient v is readily linked to the conventional concept of velocity of circulation of money. In equation (10), for an individual or entity, and in equation (9.3), for the aggregate, reference is to the 'new' period rand the coefficient v changes in two ways.

Firstly, it is now used in a 'relative' sense: it has become the ratio of 'velocity' in the new period to 'velocity' in an initial period. This is not stated explicitly in the manuscript, but it is nevertheless clear from the discussion later in the manuscript concerning a sequence of monetary periods (as summarised in Appendix 1). Secondly, and more fundamentally, in equations (10) and (9.3), the meaning of 'velocity' changes: v is no longer a coefficient for the velocity of circulation of money, rather, it becomes the coefficient that represents the interdependent link between real and monetary phenomena.

When the relative form of the coefficient v is 1 in equation (10) and (9.3), the 'quantities' of economic transactions in the new period are said by Pareto to be the same as they were in the original period: the real economy is unchanged between periods. However, when v is greater than one, the quantity of economic goods and services in the new period diminishes relative to the original period (i.e. real economic contraction). When v is less than one, the quantity of economic goods and services in the new period rises relative to the original period (i.e. real economic expansion). When a state is characterised by $v > 1$ and $\mu < 1$, Pareto surmises that:

> a [from equation (1)] becomes a/v which indicates that a falls, or, to be precise for instance, part of the a is destroyed; p becomes p/μ, which indicates that p rises; therefore equation (10) corresponds to the case in which some of goods a are destroyed while prices rise.
>
> (Pareto 2005, p 263)

But the case where $v > 1$ and $\mu < 1$ not only marks a decline in the real economy ('a falls') and a rise in prices ('p rises'), the coefficient v also denotes a decline in the quantity of money when expressed in values from the initial period (a_0 is divided by, among other things, v). Of course, the decline in the real economy would be of the same relative magnitude as the rise in prices when the product of v and μ is 1. In Pareto's illustrative example, $v = 2$ and $\mu = 1/2$, when the nominal quantity of money is left unchanged between periods, as the quantity of real economic transactions is halved but prices double (Pareto 2005, p 263).

The character of coefficient v in equation (10) and (9.3) may be illustrated by the simplified case in which there are no price changes, $\mu = 1$. In this case, it is readily evident that coefficient v is the inverse of what would be required for it to be interpreted conceptually as a change in the velocity of circulation. For example, from equation (10), a 10% increase in real transactions would be associated with a v coefficient of 0.90909, but if this coefficient were to also represent the relative change in the velocity of circulation as the quantity of money remains fixed, it would imply a 9.0909% decrease in circulation (and not the 10% increase in the velocity of circulation required for the quantity of money, which would imply that the relative coefficient for the velocity of circulation coefficient should be 1.1). In short then, there does not appear to be any direct link between the coefficient v in equations (10) and (9.3) and the change in the velocity of money between periods (at least not as it is usually understood). Assuming again that there is no inflation, $\mu = 1$, the first term ratios a_0/v and σ/v from these two

respective equations indicate that the quantity of 'real' money, i.e. money in the new period expressed in terms of the purchasing power of the initial period, which is determined on the implicit assumption that the velocity of circulation remains constant between periods. In contrast, when the quantity of money is defined for the initial period, as it is in equation (3), the coefficient v is expressed in an absolute form that is consistent with the conventional meaning of the velocity of circulation.

Needless to say, this multi dimensional conceptual role for coefficient v is far from satisfactory and will no doubt confuse readers. Perhaps Pareto's comment in the manuscript that v is the 'coefficient to obtain the sum (of money) really operating' (Pareto 2005, p. 261) is his way of suggesting that v is only intended to have an empirical character to calculate the quantity of money without any conceptual foundation. Alternatively, as the manuscript is only fragmentary and incomplete, it may be that a fuller discussion of the coefficient v may be contained in the parts of the manuscript that have been lost.

Clarification of the role of Pareto's v in the new period

The role of v in Pareto's analysis may be clarified when it is recast in the context of the Fisher equation $MV = PT$. The elements of Pareto's equation (3) broadly equate with the elements of the Fisher equation, with σ corresponding to M and the value of transactions, $\frac{1}{2}\sum\left(a_0 p_0 + a_1 p_1 + ...\right)$, corresponding to PT.[7] As equation (3) is limited to the initial period, v here broadly corresponds to Fisher's V – it is the velocity of money plus any residual empirical factors that influence the resulting quantity of money.[8] If Fisher's upper case notation is adopted for the initial period variables and Pareto's lower case relative coefficients v and μ are used to represent between period ratios for the new period equations, then Pareto's equations for the quantity of money in the initial period (equation 3) and in the new period (equation 9.3) may be represented respectively as:

(iii) 'Initial' Period $\qquad M = \dfrac{PT}{V} \qquad$ or $\qquad MV = PT$

(iv) 'New' Period $\qquad M_1 = \dfrac{M}{\mu v} = \left(\dfrac{P}{\mu} \cdot \dfrac{T}{v}\right)\dfrac{1}{V}$

When the coefficients μ and v vary from 1, the quantity of money alters between periods. In this general circumstance, the change in prices μ and the change in quantities v both contribute to changes in the quantity of money. As noted earlier, when there is no change in prices ($\mu = 1$) and there is growth in the real economy ($v < 1$), the impact of the coefficient v flows directly through to the quantity of money: Pareto's 'relative'

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[7] For Fisher, the sum of transactions is not halved, but Pareto was forced to do so for the reasons discussed earlier. Consequently, Pareto's $\frac{1}{2}\sum\left(a_0 p_0 + a_1 p_1 + ...\right)$ is substantively equivalent to Fisher's PT.

[8] There is no clear place for the coefficient α (the proportion of transactions that are monetarised) in the Fisher equation. However, as Pareto does not discuss variations in α between periods, it is implicitly treated as constant between periods and can be set aside for the purposes of this illustrative comparison.

coefficient v is not a general indicator of changes in the velocity of money as it is understood in any conceptual sense. For it to represent the velocity of money, manipulation of equation (iv) would be required:

$$\text{(v) 'New' Period} \qquad M_1 V v = \frac{P}{\mu} \cdot T \qquad \text{or} \qquad M_1 = \frac{P}{\mu} \cdot T \cdot \frac{1}{Vv}$$

The transformation from equation (iv) to (v) is obviously algebraically sound. But Pareto's equations for the quantity theory of money, equations (8.2) and (9.1), are also the algebraic equivalents of equations (9.3) and (10), which he employed to reject the quantity theory. It is the economics, and not the algebra, that is at issue. Equation (v) would not have been acceptable to Pareto because the empirical character of the coefficient v would change: instead of being empirical estimated with respect to the real economy T, as Pareto does, it would be estimated with direct reference to velocity V. In Pareto's manuscript, money is not presented as a veil, money and real transactions are interdependent, and the coefficient v represented in his new period equations provides that link.

Heterogeneous Price Variations

Pareto's second objection to the quantity theory of money concerned the literal proposition that a single price adjustment, $(1/\mu)$ in equations (8.2) and (9.1), is revealed as equal for all economic goods. He does not interpret the quantity theory of money as a revealing a relationship between money and the general price level, rather, he interprets the theory as literally suggesting that money is a veil that falls over the entire system of prices. When money is a veil, an implication of the 'price level' interpretation of the quantity theory is that observed variations in the relative price of goods and services are due to changes in market conditions, which are deliberately set aside in the quantity theory comparative statics (when equations (1) and (3) are contrasted with equations (8.2) and (9.1)). Pareto, in contrast, considers that at least some of this observed variation in relative prices is due to different degrees of real-monetary interdependence across the range of economic goods.

To 'take a step closer to reality', he distinguishes between two broad classes of goods: goods A which vary in price with monetary phenomena; and goods B which do not vary in prices with respect to monetary phenomena. On that basis, equation (1) is revised to:

$$a_0 p_0 + \sum (ap) + \sum (bp) = 0$$

The expression for the quantity of money changes accordingly. Money in the 'original' period is now given by equation (22.1) and money in the 'new' period is given by equation (22.2), or its equivalent, equation (22.3).

$$(22.1) \qquad \sigma = \left(\frac{\alpha}{2v}\right)\left(\sum ap + \sum bp\right)$$

$$(22.2) \qquad \sigma_1 = \left(\frac{\alpha}{2v}\right)\left(\sum a'p' + \sum bp\right)$$

$$(22.3) \qquad \sigma_1 = \sigma\frac{\left(\sum a'p' + \sum bp\right)}{\left(\sum ap + \sum bp\right)}$$

The quantity of money is then defined, in equation (23.3), with reference to ratios of the value of commodities unrelated to, and related to, monetary phenomena for each period

$$(23) \qquad \sigma_1 = \left(\frac{\sigma}{n}\right)\frac{(1+mn)}{(1+m)} \qquad \text{when: } m = \frac{\sum bp}{\sum ap}, \ n = \frac{\sum bp}{\sum a'p'}$$

The general purpose of equation (23), which is partly the result of an algebraic error,[9] was to demonstrate that when, prices rise by more than real economic growth, $vu<1$, the equality revealed in equation (9.3) will not hold when prices rise are confined to a subset of goods. Rather, the quantity of money in the new period will be less that what equation (9.3) indicates, as only values that figure in the ratio n are influenced by the coefficients v and μ - values that figure in ratio m are not influenced by these coefficients. By way of illustrative (and rough) approximation, the variable n in equation (23) is replaced with the product of the coefficients v and μ:

$$\sigma_1 = \left(\frac{\sigma}{\mu v}\right)\frac{(1+m\mu v)}{(1+m)}$$

$$\text{when } \mu v < 1, \frac{(1+m\mu v)}{(1+m)} < 1 \text{ and:}$$

$$(24) \qquad \sigma_1 < \frac{\sigma}{\mu v}$$

Of course, the error in equation (23) flows through, but equation (24) still remains valid in general terms.[10] Nevertheless, it should be noted that this analysis of Pareto is very restrictive as it is limited to two classes of relationships. Instead of limiting his

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[9] After correction (by my colleague Darrell Turkington) equation (23) becomes:

$$(\text{xxiii}) \qquad \sigma_1 = \left(\frac{\sigma}{n}\right)\frac{(m+mn)}{(1+m)}$$

Consequently, Pareto's specification of equation (23) only holds strictly when the value of transactions unaffected by price changes is equal to the initial value of transaction that will be affected by price changes ($m = 1$).

[10] After correcting equation (23), the approximate relationship between the quantity of money in the new period and the initial period reflected in equation (24) holds subject to $(m + m\mu v) < (1+ m)$.

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considerations to the case of interdependence between monetary phenomena and some commodities (goods A) and the case of independence between monetary phenomena and some commodities (goods B), commodity specific subscripted coefficients *v* and *u* could be introduced which cover the full spectrum of interdependencies between real and monetary phenomena. However, the restriction is deliberate and appears to be due to Pareto's concerns over the experimental difficulty of identifying coefficients. In this regard, Pareto even saw empirical problems with the general situation represented by his equation (9.3) - direct observation of monetary data over time will only reveal the product $v\mu$ and not the separate coefficients for each parameter (2005, p. 263). Any further use of coefficients would further compound the experimental problem.

3   Walrasian and non-Walrasian Ideas

For historians of the Lausanne tradition in economics, the significance of this manuscript is that it defines the limited extent to which Pareto was prepared to go when integrating monetary theory with general equilibrium theory. Notwithstanding the total lack of references in the manuscript to monetary studies by Walras or any other scholar, it is possible to deduce the aspects of Walras' monetary theory that Pareto accepted, from his discussion of money within the context of the exchange equation, and to be more specific about the reasons why he did not follow Walras in attempting to fully money within general equilibrium.

Walrasian elements of the Manuscript

As Pareto considered monetary issues in the context of general equilibrium, there was some basis for him to follow Walras along the lines of the exchange equation introduced in the first edition of his *Éléments* (Walras 1874, p. 200):[11]

$$\alpha'' Q_a'' = \alpha' Q_a' p_a + \beta Q_b p_b + \gamma Q_c p_c + \delta Q_d p_d + ...$$

Where Q_a'' = stock of money-metal A that is used as money

Q_a' = part of the money-metal stock A used doe commodity use

α_a'', α_a' = respective coefficient (velocities) of circulation for Q_a'' and Q_a'

Q_b, Q_c, Q_b = quantities of economci goods B, C and D

p_a = the price of A relative to itself (i.e. $p_a = 1$)

p_b, p_c, p_b = the relative prices (in terms of A) of goods B, C and D

β, γ, δ = respective coefficient (velocities) of circulation of B, C and D

Monetary equation (3) in Pareto's manuscript has many similarities with Walras' exchange equation above and both can be substantively reduced to the Fisher exchange equation. The differences between the two specifications are largely contextual (in the particular case above Walras was dealings with money-as-a-good and isolated the

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[11] This source for this reference is Bridel (1997, p. 43).

monetary component of that good from its other uses), technical (Walras explicitly included the velocities of circulation for goods to account for the influence of repeat sales of goods on money, whereas, from the time of his 1906 *Manuale di Economia Politica* onwards, Pareto recognised that the same physical good at different points in time become different economic goods) [12] or trivial (Pareto's adjustment to the value of exchange for his double counting of monetary and real transfers).

The only substantial difference between Pareto's equation (3) and Walras' original monetary exchange equation is associated with Pareto's introduction of an exogenous variable (α) for the proportion of transactions that are monetarised. However, the significance of this should be seen in a broader context because Pareto's exogenous adjustment is largely irrelevant when his analysis is considered with respect to the more mature *encaisse desirée* approach that Walras finalised in his 1900 edition of the *Éléments*, in which case the demand for real cash balances is treated as an endogenous element within the general equilibrium system.

Non-Walrasian Elements of the Manuscript

The *Cours*, which contains Pareto's most extensive discussion of monetary issues, includes discussion of real and monetary interdependencies in the equilibrium adjustment process following a monetary shock. While this did not prevent him from accepting the quantity theory as a guiding rule, [13] he nevertheless refrained from formally integrating money within general equilibrium theory.

In his 1920-21 manuscript, however, the fundamental reason for Pareto not following Walras's in *encaisse desirée* approach and attempt to fully integrate money within general equilibrium theory is evidently grounded in the recognition of interactions between the real and monetary sectors, which lead him to abandon the quantity theory. As such, the primary difference in the mature position of the two masters of Lausanne on monetary economics is evident from Pareto's introduction, in equation (10), of the 'relative' form of coefficient *v* to the equations of exchange to link real and monetary phenomena. Full Walrasian integration of money within general equilibrium theory would, in Pareto's eyes at least, require empirical support for the view that money does not disturb the relationships of general equilibrium. However, observation suggested to Pareto that money does indeed disturb the relationships of general equilibrium: it is not a veil and the propositions of the quantity theory for money are not empirical uniformities

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[12]   '… transformation in time consists of substituting a good that is available at a certain time for a good available at another time. …[e.g.] two economic combinations for production — (A) 100 kilograms of wheat to be consumed at seed time; (B) 100 kilograms of wheat to be consumed at the time of the next crop — are not identical; they are different *commodities*; consequently, (A) may have a price that is different from that of (B) … . The difference between the price of (A) and that of (B) is the price of transformation in time…' (Pareto 1906 [2006], p. 220).

[13] As Pascal Bridel has noted, Pareto continued to consider money as 'approximately' neutral, arguing that:

'if left to its own devices, an economic system submitted to a monetary shock is 'naturally' (though approximately) self adjusting. In a Friedmanite way, Pareto's careful theorizing seems to have been slightly overtaken by the ultra liberal 'vision' displayed in the *Cours*' (Bridel 1997, p. 157).

that belong within general theory. Moreover, as Pareto relied on the coefficients ν and μ to link the real economic quantities, nominal quantities and nominal prices without establishing empirical regularities for modelling a theoretical relationship between ν and μ, there was no scope for Pareto to consider integrating money and pure general equilibrium in a fundamentally non-Walrasian way. Monetary theory simply remained outside the scope of pure economic theory as part of a necessary second order approximation of the economic phenomenon.

Returns on Capital and Rent: on Re-reading the 'Cours' and 'Manuale'

It should also be recognised that, even if the fundamental concerns with the quantity theory of money evident from the manuscript in question could be set aside, Pareto's approach to capital alone would have prevented him from fully integrating money within the pure theory of general equilibrium. Bridal (1997) has identified the main general reasons by Pareto did not follow Walras, but the relevance of capital and rents to this issue have not been discussed. For that reason, some comment on the relevance of Pareto's work on capital and rents to monetary economics is discussed below.

At the general level, Pareto is at one with Walras on presenting general equilibrium as a successive integration of the theories of exchange, equilibrium and capital formulation. This is readily evident in the *Cours* and even the abstraction from the concrete notion of capital in the *Manuale*, to focus on the transformation of goods and services in production as obstacles to the satisfaction of tastes, alters the form of pure theory but not its substantive character. Walras' attempt to integrate circulating capital and money with the related theories of capital formation, production and exchange, by introducing real monetary balances within the utility function and determining the quantity of cash balances demanded through a sequence of equations that relate back to real variables, as reviewed in Bridel (1997). But this is predicated on the equilibration of the interest rate with the net rate of return on all capital items. This alone would represent a major sticking point for Pareto, who repeatedly pointed to issues of importance that relate to the failure of interest rates and net rates of return on heterogeneous capital items to equalise.

In the *Cours,* rents are presented as the obstacle to the equalisation of the net rates of interest (Pareto 1896-97 [1971], p.769, § 780), which may be due to differences between old and new capital goods following a change in economic circumstances, such as a variation in the interest rate on the transformation of savings into capital (1896-97 [1971, p.747, § 747). More generally, rents on specific types of heterogeneous capital and land are associated with the different periods of time required to transform economic goods into other goods.[14] In the language of the *Cours*, when an economic system generates rents, free competition is associated with 'incomplete equilibrium'. The point that needs to be underlined here is that Pareto regarded incomplete equilibrium generally and the theory of rent specifically as:

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[14] See Bird and Tarascio (1992) for clarification of the difference between Pareto's theory of rent and what is more commonly referred to as 'Paretian' rent theory.

pertinent to pure economics, but for didactic reasons, it is convenient to delay its study until after studying fixed capital (Pareto 1896-97 p.1090).

In the *Manuale*, when discussing the abstraction from the concrete notion of capital in favour of transformations in production, Pareto also emphasised that his analysis of transformations does not presume that there is a single net yield on all capital goods (Pareto 1906 [2006], p. 226). Of course, he recognised that complete general equilibrium depends on the equalization of rates of return across the diverse types of capital and, by temporarily positing that capital items are assumed to be produced at the same time (1906 [2006], p. 240), he investigated the properties of complete equilibrium. But this was a convenient device for closing a theoretic system in a manner that created a platform, or point of departure, from which the issue of rents could be investigated when net returns are not equalised. As an end point of pure theory, Pareto regarded the equalisation of returns is an acceptable approximation because it would serve as the point of departure for the development of his theory of rent based on variations in net rates of return across capital goods. Pareto would not have even contemplated further developments in pure theory that depended on the re-introduction of an equalisation assumption pertaining to net rates of return on capital items and interest.[15]

4   Conclusion

There are many references in Pareto's work to the weakness of monetary theory. By the time that he wrote 'Economia sperimentale' (Pareto 1918), there is absolutely no doubt that Pareto regarded the development of monetary theory by economists as totally lacking in scientific foundation. The discovery of a manuscript drafted in 1920-21 that formally investigates the quantity theory of money, instead of extending his sociological analysis of money, is therefore rather surprising.[16] More importantly, the manuscript is significant, as it represents the first 'formal' economic analysis to explain the shift in Pareto's position from the quantity theory of money.

From the manuscript it is now clear that Pareto went further in accommodating Walras' ideas on monetary theory in general equilibrium than was previous thought. Specifically, he introduced a set of equations of exchange for the purpose of monetary theory that was very similar to, though not identical to, that developed by Walras in the first edition of the *Éléments*. However, his use of this equation as the basis for a critical assessment of the quantity theory reveals why Pareto would have been unable to follow along the lines that Walras in integrating money within general equilibrium theory. His recognition of the interdependence between real and monetary variables, and the consequent abandonment of the quantity theory of money, would prohibit such a development. This is also

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[15] One may object that Alfred Marshall was comfortable including his theory of rent and the Cambridge equation for the quantity theory within his broad economic framework, but Marshall dealt in partial equilibrium, and the issues that Pareto would have confronted when reflecting on the roles of rent and money in a system of general equilibrium would not have been issues of any consequence for Marshall. Pareto's theory of rent is also quite distinct from Marshall's theory of rent (see Bird and Tarascio 1992).

[16] See McLure (2007, pp 91-93) for discussion if Pareto's observations on monetary theory and the 'sociological part' of economic phenomena.

consistent with the argument, advanced in this paper, that the approach that Pareto took in the *Cours* and *Manuale* on rent theory and diverse rates of return on heterogeneous capital would have further dissuaded Pareto from following Walras' mature approach to monetary theory.

Of course, Pareto did not publish his manuscript. Nor did he liaise widely with his interlocutors on the manuscript. It was referred to in his letters to Stanislao Scalfati and Guido Sensini written between November 1920 and February 1921 (Mornati in Pareto 2005, p. 268). However, on all occasions the comments were very general and oblique. From this, it must be conceded that Pareto may have had reservations about the paper or that he did not consider it significant. It is also plausible that the failure to publish the manuscript was due to the inability of an elderly man in poor health to marshal the energy necessary to re-engage in formalist economic discourse. There are certainly significant ambiguities in the manuscript, which have hopefully been clarified in this study, and a number of errors. Redrafting of the manuscript would have had to be extensive. But, irrespective of the reason why he did not redraft and publish the manuscript, historians are indeed fortunate now to have access to a document that provides considerable insight into Pareto's monetary thought.

References

Bird, Ronald and Vincent J. Tarascio. 1992. 'Paretian Rent Theory Versus Pareto's Rent Theory: A Clarification and Correction', 24(4), pp. 909-23.

Bridel, Pascal. 1997. *Money and General Equilibrium Theory*, Cheltenham: Edward Elgar

Bridel, Pascal. 2000 ''From Walras's to Pareto's Teachings: the case of monetary theory', in *Economia, Sociologia e Politica nell' Opera di Vilfredo Pareto*, edited by Corrado Malandrino and Roberto Marchionatti, Florence: Leo S Olschki editore.

Fisher, Irving 1911 [1913]. *The Purchasing Power of Money, its Determination and Relation to Credit, Interest and Crises,* New York, Macmillan.

Marget, Arthur 1931. 'Léon Walras and the "Cash-Balance Approach" to the problem of the value of money', *Journal of Political Economy*, 39(5), pp 571-600.

Marget, Arthur 1935. 'The monetary aspects of the Walrasian system', *Journal of Political Economy*, 43(2), pp 145-86.

McLure, M. 2007. *The Paretian School and Italian Fiscal Sociology*, Basingstike: Palgrave

Pareto, Vilfredo [1896-97] 1971. *Corso di Economia Politica [Cours d'Économie Politique]*, Torino: Unione Tipographico-Editrice Torinese *Cours* .

Pareto, Vilfredo 1906 [2006]. *Manuale di Economia Politica*, Edizione Critica, Aldo Montesano, Alberto Zanni and Luigino Bruni (eds), Milan: EGEA – Università Bocconi Editore.

Pareto, Vilfredo 1918 [2007]. 'Economia Sperimentale', *Giornale degli Economisti e Rivista di Statistica*, 1918, 28(57), pp. 1-18, translated by Vincenzo Savini and Rocco Loiacono as 'Experimental Economics' in Michael McLure 2007. *The Paretian School and Italian Fiscal Sociology*, Basingstike: Palgrave, pp 185-202.

Pareto, Vilfredo 2005. 'Note critiche di teoria monetaria', in Firenzo Mornati (ed) Vilfredo Pareto *Œuvres Complètes* Tome XXXIII, pp. 259-68.

Sensini, Guido 1929. Vilfredo Pareto e la teoria della moneta, *Il Giornale Economico*, reprinted in G. Sensini, 1932. *Studi di Scienza Sociali*, Roma: Maglione, pp. 213-15.

Walras, Léon 1874. *Éléments d'Économie Politique Pure - 1ˢᵗ edition*, Lausanne: L. Corbaz & Co.

Walras, Léon 1926 [1954]. *Éléments d'Économie Politique Pure ou Théorie de la Richesse Sociale - Édition Définitive,* Paris: R. Pichon et R. Durand-Auzias, Translated by William Jaffé, *The Elements of Pure Economics or the Theory of Social Wealth,* Fairfield: Augustus M. Kelly.

Appendix 1

Pareto's Three Period Analysis

Some illustration of the general relevance of equations (10) and (9.3) is provided by Pareto in his analysis of a multi-period sequence. For three periods, I, II and III:

v' parameter that scales for change in the real economy between periods I and II;

v'' parameter that scales for change in the real economy between periods I and III;

μ' parameter that scales for nominal price changes between periods I and II, and

μ'' parameter that scales for nominal price changes between periods I and III;

the respective equations may be represented as:

I   representative equation: $\dfrac{a_0}{v\mu} + S\left(\dfrac{ap}{v\mu}\right) = 0$

       aggregate equation $\sigma_1 = \dfrac{\sigma}{(v\mu)}$

II   representative equation: $\dfrac{a_0}{(v\mu v'\mu')} + S\left(\dfrac{ap}{(v\mu v'\mu')}\right) = 0$

       aggregate equation $\sigma_1' = \dfrac{\sigma}{(v\mu v'\mu')}$

III   representative equation: $\dfrac{a_0}{(v\mu v''\mu'')} + S\left(\dfrac{ap}{(v\mu v''\mu'')}\right) = 0$

       aggregate equation $\sigma_1'' = \dfrac{\sigma}{(v\mu v''\mu'')}$

The relevance of this to Pareto is that it illustrates, again, that the quantity theory of money is a very partial theory because the quantity of money in period II relative to period III is not $(\mu'')/(\mu')$, as the quantity theory implies, but:

$$\frac{\sigma_1'}{\sigma_1''} = \frac{\dfrac{\sigma}{(v\mu v'\mu')}}{\dfrac{\sigma}{(v\mu v''\mu'')}} = \frac{(v''\mu'')}{(v'\mu')}$$

Notation Errors in 'Note Critiche di Teoria Monetaria'

Section I

Page 261

Pareto's manuscript: Correction

$$\alpha = \frac{1}{2}\sum\left(a_0 p_0 + a_1 p_1 + ...\right) \qquad \sigma = \frac{1}{2}\sum\left(a_0 p_0 + a_1 p_1 + ...\right)$$

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Pareto's manuscript: Correction

…   …

$$(8) \qquad \left(\frac{a_0}{v}\right) + S\left(\frac{a}{v}\right) = 0 \qquad \left(\frac{a_0}{v}\right) + S\left[\left(\frac{a}{v}\right)p\right] = 0$$

…   …

Section II

Page 263

Pareto's manuscript: Correction

$$\sigma_1 = \left(\frac{1}{\sigma}\right)\mu \qquad \sigma_1 = \left(\frac{1}{\mu}\right)\sigma$$

Section IV

Page 267

Pareto's manuscript: Correction[17]

$$(23) \qquad \sigma_1 = \left(\frac{\sigma}{n}\right)\frac{(1+mn)}{(1+m)} \qquad \sigma_1 = \left(\frac{\sigma}{n}\right)\frac{(m+mn)}{(1+m)}$$

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[17] Derived courtesy of Darrell Turkington