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Self-Insurance and Self-Protection as Public Goods

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Abstract

Many public goods like lighthouses and fire departments do not provide direct utility but act as insurance devices against shipwreck and destruction. They either diminish the size and/or the probability of the loss. We extend the public good model with this insurance aspect and generalize Samuelson's efficient allocation rule when self-insurance and self-protection expenditures are pure public goods. Some comparative static results with respect to changes in income and risk behavior are derived. We analyze the interaction of private market insurance with the public good level, both for efficient provision and for private provision equilibria. The privately provided levels of self-insurance and self-protection decrease when market insurance is available, which suggests that the state should invest more in preventing not insurable risks like wars. Additionally, the state should focus on self-protection expenditures if those are better observable than private self-protection effort.

Keywords: Self-insurance, self-protection, efficient provision of public goods, private provision of public goods, market insurance

JEL Classification: G22, H41

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1 Introduction

Many standard textbook examples for public goods like lighthouses and fire departments do not provide intrinsic direct utility but act as insurance devices against shipwreck and destruction. In their seminal contribution, Ehrlich and Becker (1972) coined the terms "self-insurance" for effort that reduces the size of the loss and "self-protection" for effort that reduces the probability of the loss. Thus, a fire department nearby does not prevent a fire, but it reduces the size of the loss.[1] Similarly, a lighthouse or a national army do not lower the size of the loss but they lower the probability of shipwreck or war.[2] Public fire stations can be seen as self-protection devices and lighthouses and national armies act as self-insurance devices.

The standard literature usually assumes that the level of the public good is a direct functional argument of the individual's utility function, irrespective of whether the public good is provided publicly or privately (for a survey, see e.g. Cornes and Sandler, 1996). However, this assumption is not always realistic because the public goods mentioned above do not provide utility by their sheer existence, but they act as self-protection and self-insurance devices. With few exceptions, the literature on public goods has not analyzed these insurance aspects. The public goods literature has dealt with uncertainty focusing on the private provision. The standard result is that uncertainty reduces the free-riding incentive depending on the properties of the third derivatives of the utility function.[3] So far, the literature has always concentrated on uncertainty about the contributions of the other individuals or about their contribution behavior.

One recent contribution by Ihori and McGuire (2007) considers the collective provision towards a self-protection device and shows how the contributions to risk collective reduction depend on the risk aversion of the individuals. Our paper complements and extends this approach by establishing the theoretical similarities between the standard model of private contributions to a public good and the insurance model of private contributions to a collective self-protection or self-insurance device. We show that the role of income

[1] Orszag and Stiglitz (2002) have analyzed the efficient provision level of fire departments as public goods.

[2] Here we follow textbook economics in modeling lighthouses as public goods, despite Coase's (1974) analysis of lighthouses as private goods. For an analysis of collective efforts of armies and terrorism, see Sandler (2005).

[3] See the contributions of Austen-Smith (1980), Sandler et al. (1987), Gradstein et al. (1993), among others.

normality in the standard model is analogous to the role of risk aversion in the insurance model.

Thus, we also extend the seminal contribution by Ehrlich and Becker (1972) to the situation where self-insurance and self-protection are public goods with non-rival consumption. We develop modified Samuelson conditions (Samuelson 1954, 1955) characterizing the efficient provision level of those public goods and analyze the provision level when self-insurance and self-protection as public goods are privately provided. Moreover, we investigate for both cases the impact of the presence of market insurance on the provision level of the public good. Our results show that the efficient level of the public good decreases if fair market insurance is available. It is well-known that individuals will buy full insurance if insurance premiums are fair (Mossin, 1968). In this case, the efficient level of the public good will maximize expected wealth; i. e., it equals the efficient level for risk neutral individuals which is lower than the efficient level for risk averse subjects. Consequently, the state should invest more in public self-insurance and self-protection in case of events which are not insurable, e.g. wars or nuclear incidents.

In the case of public self-protection in the presence of market insurance, we assume realistically that the level of the public good can be observed by insurers and hence reduces premiums in an actuarial fair way. This means that a moral hazard problem does not occur in the case of public self-protection, which may be an advantage compared to private self-protection efforts which are often not observable.

The paper proceeds as follows. The next section presents the model and the modified Samuelson conditions for the efficient provision level of self-insurance and self-protection as public goods. The comparative statics results for changes in income and risk behavior are presented in section 3. In section 4 the additional possibility of a market insurance is introduced and the relating efficiency conditions are derived. Section 5 presents the Nash equilibria when the self-insurance and self-protection are privately provided public goods. Section 6 introduces the individual choice that maximizes expected utility when public self-insurance and self-protection can be complemented or substituted with market insurance. Section 7 summarizes the results and concludes.

2 Efficient provision of the public good

Consider an economy with $i = 1, \ldots, n$ individuals facing two possible states of the world, 1 and 2. All individuals have the same probability p of suffering a loss L, while with residual probability $1 - p$ there is no loss. Each individual i is endowed with income m_i which she may spend on increasing the level of the public good G with a non-negative contribution $g_i \geq 0$. The public good G diminishes the size of the loss or the probability of the loss in a way to be described in the following sections. For convenience and without loss of generality, we set the marginal cost of contributing to the public good to 1. This leads to the following state contingent income levels:

$$y_{i1} = m_i - g_i \quad (1)$$

$$y_{i2} = m_i - g_i - L, \quad (2)$$

where y_{ij} denotes the income of individual i in state j. All n individuals have the same von Neumann utility function u with increasing and diminishing returns to state-contingent income, $u'(y_{ij}) > 0, u''(y_{ij}) < 0$. We further assume for our comparative static results that all individuals are prudent, $u'''(y_{ij}) > 0$. A positive third derivative concerns the optimal choice under uncertainty. Intuitively, a prudent individual reacts to uncertainty by increasing the choice variable to avoid extreme situations (see Kimball (1990) on precautionary saving).

2.1 Self-insurance as a public good

In the self-insurance case, for all individuals the size of the loss L depends on the level of the public good G, $L(G)$, where G is the sum of all private contributions to the public good; i. e., $G = \sum_{i=1}^{n} g^i$. One can think, for instance, of the loss due to a fire. The size of the loss depends on the number of fire stations and on the distance to the next fire station. Thus, the existence of fire stations is a public good. It is reasonable to assume that the public good reduces the size of the loss with diminishing productivity: $L'(G) < 0$ and $L''(G) > 0$. We further assume that it is worthwhile to invest in loss reduction, i. e. $\lim_{G \to 0} L'(G) \to -\infty$, and that it does not pay to spend all income on self-insurance effort, i. e. $\lim_{y \to 0} u'(y) \to \infty$. The state contingent income levels are given by

$$y_{i1} = m_i - g_i \quad (3)$$

$$y_{i2} = m_i - g_i - L(G), \quad (4)$$

where G acts as an self-insurance device: it involves redistributing income from the good state of the world to the bad state.

The individual i maximizes her expected utility given by

$$EU_i(g_i, G) = (1-p)U(m_i - g_i) + pU(m_i - g_i - L(G)) = (1-p)U_{i1} + pU_{i2}, \tag{5}$$

where $g_i = G - G_{-i}$ and G_{-i} is the level provided by all other contributors but i; i. e., $G_{-i} = \sum_{j=1, j\neq i}^{n} g_j$.

The first-best, Pareto efficient outcome for $n > 1$ is found when the expected utility level of individual 1 EU_1 is maximized, given the restrictions that individuals 2 to n obtain given expected utility levels $\overline{EU}_j, j = 2, \dots, n$ and that $G = \sum_{i=1}^{n} g_i$. The resulting Lagrangian for this problem is

$$\mathcal{L} = EU_1 + \sum_{j=2}^{n} \mu_j (EU_j - \overline{EU}_j) + \lambda(G - \sum_{i=1}^{n} g_i) \tag{6}$$

The first-order conditions with respect to G and g_i are

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_{i=1}^{n} \mu_i p U'_{i2}(-L'(G^*_{SI})) + \lambda = 0 \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial g_i} = \mu_i((1-p)U'_{i1} + pU'_{i2}) + \lambda = 0, \qquad \text{for } i = 1, \dots, n, \tag{8}$$

where $\mu_1 = 1$. Let the superscript $*$ denote the efficient level of the public good, and the subscript SI refer to self-insurance. Marginal expected utility $(1-p)U'_{i1} + pU'_{i2}$ is abbreviated as EU'_i. Solving each of the n equations (8) for μ_i, substituting into (7) and canceling out λ, we obtain

Proposition 1 (Efficient level of public self-insurance)
The Pareto efficient level of a public good G which acts as a self-insurance device is given by the modified Samuelson condition

$$\sum_{i=1}^{n} \frac{-L'(G^*_{SI})pU'_{i2}}{EU'_i} = 1, \tag{9}$$

where the Inada assumptions imply that $p \cdot (-L'(G^*_{SI})) > 1$, i. e., that the expected marginal value of the efficient self-insurance effort level is larger than its marginal cost. The left hand side reflects the willingness to pay for the public good G: the marginal positive effect of an additional unit of G, measured in units of forgone income in both states of the world (marginal expected utility EU'_i). Since an additional unit of G benefits all individuals, it is the sum of the marginal willingness to pay for public self-insurance of *all* individuals which should equal the marginal cost of an additional unit of G. The second-order conditions are fulfilled by the assumptions on u and L.

2.2 Self-protection as a public good

In the self-protection case, the size of the loss L is fixed and uniform for all individuals. Now the collective effort G reduces the probability of the loss for all individuals which will be denoted by $p(G)$. The probability of a bad state can be reduced by contributing to the public good. For the relationship between the public good level and the probability of the bad state, we again assume realistically that increasing G reduces its probability with diminishing returns: $p'(G) < 0$ and $p''(G) > 0$. We further assume that it pays to invest in the reduction of the loss probability, i. e. $\lim_{G \to 0} p'(G) \to -\infty$, and that it does not pay to spend all income on self-protection effort, i. e. $\lim_{y \to 0} u'(y) \to \infty$. Additionally, in the self-protection case we need to assume that the probability $p(G)$ of the bad state of the world is sufficiently small. The loss is relatively seldom in the following sense:

Assumption 1

The slope of the line connecting the utility levels in the good and in the bad states of the world is larger than the average of the slopes at those utility levels, i. e., than the expected marginal utility, for all income levels:

$$\frac{U_{i1} - U_{i2}}{L} > EU_i' > 0. \tag{10}$$

A similar condition applies to the slope of the line connecting the marginal utility levels in the good and in the bad states of the world, which is smaller than the average of the slopes at those marginal utility levels, for all income levels:

$$-\frac{U_{i2}' - U_{i1}'}{L} < EU_i'' < 0. \tag{11}$$

The first part of Assumption 1 concerns the slopes of the utility function, while the second part concerns analogously the case of marginal utility function. Notice that for a concave utility function, both equations (10) and (11) are always fulfilled if $p \to 0$ and are never fulfilled if $p \to 1$.

The state contingent income levels are given by

$$y_{i1} = m_i - g_i \tag{12}$$

$$y_{i2} = m_i - g_i - L, \tag{13}$$

where G acts as an self-protection device by affecting the probabilities of the good and the bad state of the world. Note that self-protection does not involve the redistribution of income. Since the absolute size of the loss does not change, self-protection expenditures even increase the relative size of the loss.

The individual i maximizes her expected utility given by

$$EU_i(g_i, G) = (1-p(G))U(m_1-g_1)+p(G)U(m_1-g_1-L) = (1-p(G))U_{i1}+p(G)U_{i2}. \quad (14)$$

Notice that, strictly speaking, we use the same notation U_{i1} and U_{i2} for the different settings self-insurance and self-protection. Since it is always clear how the utility argument looks like, we will use this notation for the sake of a clear exposition with parsimonious notation. The first-best, Pareto efficient outcome is found when the expected utility level of individual 1 EU_1 is maximized given the restrictions that individuals 2 to n obtain given expected utility levels $\overline{EU}_j, j = 2, \ldots, n$ and that $G = \sum_{i=1}^n g_i$. The resulting Lagrangian for the self-protection problem is

$$\mathcal{L} = EU_1 + \sum_{j=2}^{n} \mu_j(EU_{j1} - \overline{EU}_j) + \lambda(G - \sum_{i=1}^{n} g_i) \quad (15)$$

and leads to the following first-order conditions with respect to G and g_i:

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_{i=1}^{n} \mu_i p'(G^*_{SP})(U_{i2} - U_{i1}) + \lambda = 0 \quad (16)$$

$$\frac{\partial \mathcal{L}}{\partial g_i} = \mu_i(p(G^*_{SP})U'_{i2} + (1 - p(G^*_{SP}))U'_{i1}) + \lambda = 0, \qquad \text{for } i = 1, \ldots, n, \quad (17)$$

where again $\mu_1 = 1$, the superscript $*$ stands for efficiency and the subscript SP for self-protection. We obtain analogously to the self-insurance case

Proposition 2 (Efficient level of public self-protection)
The Pareto efficient level of a public good G which acts as a self-protection device is given by the modified Samuelson condition

$$\sum_{i=1}^{n} \frac{-p'(G^*_{SP})(U_{i1} - U_{i2})}{EU'_i} = 1, \quad (18)$$

where the Inada assumptions imply that $(-p'(G^*_{SP})) \cdot L > 1$, i. e., that the expected marginal value of the efficient self-protection effort level is larger than its marginal cost. This condition resembles again the Samuelson condition. Since the reduction in the probability of the loss accrues to all individuals, the left hand side is the sum of the marginal

willingness to pay of *all* individuals for this reduction. The marginal willingness to pay is the difference in utility between both states of the world, weighted with the marginal change in the probability of the loss and measured in units of forgone income as given by the marginal expected utility EU'_i in the denominator. This sum of marginal benefits must equal the right hand side, which is the marginal cost of the public good.
As usual in the self-protection (and moral hazard) literature, under the assumptions made so far the second-order condition does not always hold.[4] In the following, we assume the Hessian matrix $H(\mathcal{L})$ of the Lagrangian function to be negative definite, therefore conditions (18) describe the Pareto efficient outcome.

3 Comparative statics of risk behavior and income

In the following we will analyze the comparative static effects of increasing income and increasing risk aversion. It turns out that the effect of increased risk influences the interaction of public self-insurance and self-protection with market insurance, while the income comparative statics results affect the interaction of public self-insurance and self-protection with private provision efforts.

3.1 The effect of risk behavior

Another important effect is the role of the attitude towards risk. How does the efficient level of provision of the public good change when society becomes more risk-averse? For answering this question, we adapt an approach of Dionne and Eeckhoudt (1985). Suppose the utility function V represents more risk-averse preferences than the utility function U. Then, according to Pratt (1964), there exists a function f with $f'(\cdot) > 0$ and $f''(\cdot) < 0$ such that $V = f(U)$.

(i) The case of self-insurance

Under the same endowed incomes and size of loss as in section 2.1, the appropriate first-order condition for the more risk averse society is given by

$$\sum_{i=1}^{n} \frac{-L'(\tilde{G}^*_{SI})pf'(U_{i2})U'_{i2}}{pf'(U_{i2})U'_{i2} + (1-p)f'(U_{i1})U'_{i1}} = 1 \tag{19}$$

[4]See, e. g., Ehrlich and Becker (1972) and Shavell (1979).

and characterizes the efficient level $\tilde{G}^*_{SI}$, where the tilde denotes the increased risk aversion. Now, we substitute the original G^*_{SI} in (19). Clearly, if

$$\sum_{i=1}^{n} \frac{-L'(G^*_{SI})pf'(U_{i2})U'_{i2}}{pf'(U_{i2})U'_{i2} + (1-p)f'(U_{i1})U'_{i1}} > 1 \tag{20}$$

holds, then $\tilde{G}^*_{SI} > G^*_{SI}$ follows. The intuition of (20) is straightforward. The current level of the public self-insurance is G^*_{SI}, and the cost of an additional unit of G is 1. But as society has become more risk-averse, the sum of the marginal willingness to pay for public self-insurance of all individuals exceeds the additional cost. Hence, the efficient level of the provision of the public good must be higher than G^*_{SI}.

(ii) The case of self-protection

For an increase in risk-aversion, consider again a concave transformation as described above. The resulting first-order condition is

$$\sum_{i=1}^{n} \frac{-p'(\tilde{G}^*_{SP})(f(U_{i1}) - f(U_{i2}))}{pf'(U_{i2})U'_{i2} + (1-p)f'(U_{i1})U'_{i1}} = 1 \tag{21}$$

and gives $\tilde{G}^*_{SP}$. Now, substitute G^*_{SP} in (21). Then, if

$$\sum_{i=1}^{n} \frac{-p'(G^*_{SP})(f(U_{i1}) - f(U_{i2}))}{pf'(U_{i2})U'_{i2} + (1-p)f'(U_{i1})U'_{i1}} > 1 \tag{22}$$

is fulfilled, we must have $\tilde{G}^*_{SP} > G^*_{SP}$. As before, the sum of the marginal willingness to pay in this more risk-averse society exceeds the additional cost for unit of G at the level G^*_{SP}.

Lemma 1 (Effect of risk behavior on self-insurance and self-protection)
Increasing risk aversion as reflected by a concave transformation of the original utility function leads to higher efficient levels of public self-insurance and public self-protection.

For both situations, an increase in risk aversion leads to a higher efficient level of the public good. Naturally, this result also means that when the individuals become less risk-averse, the efficient provision level of public self-insurance decreases. This will be an important case in the following sections.

3.2 The effect of income

To derive the comparative statics of the first best results given in Propositions 1 and 2, i.e., how the efficient provision level of the public good G reacts to a change in income m_i,

$\frac{dG}{dm_i}$ we denote with FOC the first-order condition and get

$$\frac{dG}{dm_i} = -\frac{\frac{\partial FOC}{\partial m_i}}{\frac{\partial FOC}{\partial G}}. \tag{23}$$

If the first-order conditions fulfill the sufficient conditions for a maximum, the denominator is negative. Therefore, the sign of $\frac{dG}{dm_i}$ depends on the sign of $\frac{\partial FOC}{\partial m_i}$.

(i) The case of self-insurance

Consider the first-order condition FOC (9) and take the partial derivative with respect to income i:

$$\frac{\partial FOC(9)}{\partial m_i} = -L'(G^*_{SI})p \sum_{i=1}^{n} \left(\frac{EU'_i U''_{i2} - U'_{i2} EU''_i}{EU'^2_i} \right) \tag{24}$$

It suffices to consider only one addend. The sign of each addend depends on the sign of the numerator. After rearranging terms we obtain

$$EU'_i U''_{i2} - U'_{i2} EU''_i = \underbrace{(1-p)U'_{i1}U'_{i2}}_{(+)}(A_1 - A_2) \tag{25}$$

where A_1 and A_2 denote the Arrow-Pratt measures of absolute risk aversions calculated at the state contingent income levels $y_1 > y_2$: $A_j := -\frac{U''_{ij}}{U'_{ij}}$ (Pratt, 1964). Thus, the effect of an income change on the efficient public level of self-insurance depends on how the Arrow-Pratt measure of absolute risk aversion changes with income.

Lemma 2 (Effect of income on public self-insurance)

If income rises, the efficient provision of public self-insurance depends on how the Arrow-Pratt measure of absolute risk aversion $A_j, j = 1, 2$ changes with income:

1. *stays constant for constant absolute risk aversion (CARA): $A^1 = A^2$.*
2. *increases for increasing absolute risk aversion (IARA): $A^1 > A^2$.*
3. *decreases for decreasing absolute risk aversion (DARA): $A^1 < A^2$.*

(ii) The case of self-protection

For self-protection, we can proceed in an analogous way and take the partial derivative of the first-order condition (18) with respect to income i to determine its sign:

$$\frac{\partial FOC(18)}{\partial m_i} = \sum_{i=1}^{n} \left(\frac{-p'(G^*_{SP})EU'_i(U'_{i1} - U'_{i2}) + p'(G^*_{SP})EU''_i(U_{i1} - U_{i2})}{EU'^2_i} \right). \tag{26}$$

Consider the expression in the numerator of addend i:

$$\begin{aligned} & -p'(G^*_{SP})EU'_i(U'_{i1} - U'_{i2}) + p'(G^*_{SP})(U_{i1} - U_{i2})EU''_i \\ = & -p'(G^*_{SP})EU'_i(U'_{i1} - U'_{i2}) - EU'_i EU''_i = EU'_i[p'(G^*_{SP})(\underbrace{U'_{i2} - U'_{i1}}_{> -L \cdot EU''_i}) - EU''_i] \\ < & \; EU'_i[p'(G^*_{SP})(-L \cdot EU''_i) - EU''_i] = EU'_i EU''_i[(\underbrace{-p'(G^*_{SP})L - 1}_{>0})] < 0, \end{aligned}$$

where we have used Assumption 1 and $-p'(G^*_{SP})L > 1$ follows from the FOC (18) and the Inada assumptions. Thus, expression (26) is negative:

Lemma 3 (Effect of income on public self-protection)
Given Assumption 1, the efficient provision of public self-protection is decreasing in income.

4 Efficient provision with market insurance

Up to now, we have confined our analysis to a setting in which only a public insurance via the public good exists. However, it may also be possible to cover the loss, to some extent, by buying private market insurance. In the case of fire stations, one may buy fire insurance. In the case of shipwreck, one may privately insure the ship and the load. How does the availability of market insurance influence the efficient level of provision of the public good? In a first step, we will analyze the efficient provision level if both market insurance and self-insurance or self-protection are available, the latter two as public goods. Individual i can buy coverage $s_i \in [0, L]$ at a uniform price π, and can contribute to the public device G at the marginal cost of 1. For coverage s_i, a premium of πs_i has to be paid. Since we want to focus on the relationship between public insurance through the public good and private market insurance, we assume that market insurance is fair; i. e., the expected payoff of the insurance is zero. Hence, its price equals the probability of a loss. Our results carry over with only quantitative changes if we assume a positive loading factor when buying insurance.

4.1 Efficient self-insurance with market insurance

Since the individuals have the possibility to insure the loss at a fair premium, a risk averse subject will always choose to buy full insurance. In the case of self-insurance fair private insurance means $\pi = p$. The resulting utility level is given by

$$U_i(g_i) = U(m_i - g_i - pL(G)), \qquad \text{for } i = 1, \ldots, n. \tag{27}$$

Efficient public self-insurance can be derived by maximizing the following Lagrangian:

$$\mathcal{L} = U_1 + \sum_{j=2}^{n} \mu_j (U_j - \overline{U}_j) + \lambda (G - \sum_{i=1}^{n} g_i) \tag{28}$$

and the first-order conditions with respect to G and g_i are given by

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_{j=1}^{n} \mu_j p U_j'(-L'(\hat{G}_{SI}^*)) + \lambda = 0 \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial g_i} = \mu_i U_i' + \lambda = 0, \qquad \text{for } i = 1, \ldots, n, \tag{30}$$

where $\mu_1 = 1$ and a hat indicates the efficient public good level that is obtained in the presence of private market insurance. Rearranging yields

$$\sum_{j=1}^{n} \frac{-\lambda}{U_j'} p U_j'(-L'(\hat{G}_{SI}^*)) + \lambda = 0. \tag{31}$$

This leads to the following proposition:

Proposition 3 (Efficient public self-insurance with market insurance)
If beside public self-insurance fair market insurance is available, the Pareto efficient level of a public good G which acts as a self-insurance device is given by the modified Samuelson condition

$$n \cdot p(-L'(\hat{G}_{SI}^*)) = 1. \tag{32}$$

The left hand side of condition (32) is the expected marginal benefit of an additional unit of self-insurance, while the right hand side is its marginal cost. Since G is a public good, the expected marginal benefit $p(-L'(\hat{G}_{SI}^*))$ accrues to all n individuals and thus has to be multiplied by n.

After having determined the efficiency condition it is now of practical interest to analyze if the efficient provision level of the public good has changed due to the availability of market insurance. Considering again the case of conflagration, we are interested to see how buying fire insurance affects the efficient spending on collective fire fighting squads. Consequently, one has to compare the efficient public good levels G_{SI}^* and $\hat{G}_{SI}^*$ resulting from conditions (9) and (32).

On both right hand sides of the conditions (9) and (32) we have 1, the marginal cost of an additional unit of public self-insurance. The left hand side of (9) can be written as

$$p(-L'(G^*_{SI}))\sum_{i=1}^{n}\frac{U'_{i2}}{EU'_i}. \tag{33}$$

As income is lower in state 2, marginal utility U'_{i2} is greater than expected marginal utility, which is the probability average of both marginal utilities. Thus, all the fraction summands are greater than 1, the sum is greater than n. Compared to condition (32) above, we obtain

$$-L'(G^*_{SI}) \quad < \quad -L'(\hat{G}^*_{SI}) \tag{34}$$

$$G^*_{SI} \quad > \quad \hat{G}^*_{SI} \tag{35}$$

The availability of private insurance decreases the efficient provision level of the public self-insurance. Given that fair market insurance is available, the individuals behave as if they were risk neutral expected income maximizers. This changes the efficient equilibrium level G_{SI} in the direction established in section 3.1. A decrease in risk aversion decreases the efficient provision level of public self-insurance, market insurance and public self-insurance are strategic substitutes.

4.2 Efficient self-protection with market insurance

In the self-protection case, the individuals analogously choose to buy full fair insurance at a price of $\pi = p(G)$. This leads to utility

$$U_i(g_i) = U(m_i - g_i - p(G)L), \qquad \text{for } i = 1, \ldots, n, \tag{36}$$

The resulting Lagrangian for this problem is

$$\mathcal{L} = U_1 + \sum_{j=2}^{n}\mu_j(U_j - \overline{U}_j) + \lambda(G - \sum_{i=1}^{n} g_i) \tag{37}$$

and the first-order conditions with respect to G and g_i are

$$\frac{\partial\mathcal{L}}{\partial G} \quad = \quad \sum_{j=1}^{n}\mu_j U'_j(-p'(\hat{G}^*_{SP})L) + \lambda = 0 \tag{38}$$

$$\frac{\partial\mathcal{L}}{\partial g_i} \quad = \quad -\mu_i U'_i - \lambda = 0, \qquad \text{for } i = 1, \ldots, n, \tag{39}$$

where $\mu_1 = 1$. Rearranging terms yields

$$\sum_{j=1}^{n}\frac{-\lambda}{U'_j}pU'_j(-p'(\hat{G}^*_{SP})L) + \lambda = 0, \tag{40}$$

which leads to the following proposition:

Proposition 4 (Efficient public self-protection with market insurance)
If beside self-protection fair market insurance is available, then the Pareto efficient level of a public good G which acts as a self-protection device is given by the modified Samuelson condition

$$n \cdot (-p'(\hat{G}^*_{SP}))L = 1. \tag{41}$$

Condition (41) can be interpreted as follows. The left hand side is the expected marginal benefit of an additional unit of the self-protection public good to the n individuals, while the right hand side is its marginal cost.
To compare the efficient public good levels G^*_{SP} and $\hat{G}^*_{SP}$ without and with market insurance, we analyze conditions (18) and (41). Using Assumption 1 to rearrange condition (18) yields

$$1 = -p'(G^*_{SP}) \sum_{i=1}^{n} \underbrace{\frac{(U_{i1} - U_{i2})}{EU'_i}}_{>L} > -p'(G^*_{SP}) \sum_{i=1}^{n} L = n \cdot (-p'(G^*_{SP}))L \tag{42}$$

Combining (41) and (42) leads to

$$-p'(G^*_{SP})L \quad < \quad -p'(\hat{G}^*_{SP})L \tag{43}$$

$$\iff \qquad G^*_{SP} \quad > \quad \hat{G}^*_{SP}. \tag{44}$$

By Assumption 1, market insurance and public self-protection are also strategic substitutes. It is plausible to assume that the publicly provided level of self-protection can be observed better (because it is provided publicly by the state) than private self-protection effort. Thus it can be observed by private insurers who reduce risk premia in an actuarial fair way. This means that the moral hazard problem does not occur in the case of public self-protection, which may be an advantage compared to private self-protection expenditures.

5 Private provision of self-insurance and self-protection as public goods

Suppose now that there are $n > 1$ individuals, but there is no social planner or other coordinating institution who might provide the efficient provision level of the insurance public good G. Thus, the individuals contribute privately to the public good. In this setting, we make two assumptions. As usual in most private provision games, we assume

Nash behavior, i. e. the individuals take the contributions of the other players as given and react to the others' behavior with their best response. We will denote the resulting equilibrium levels with the superscript N for Nash. Additionally, we make the simplifying assumption that all individuals are rich enough to be contributors, or, alternatively, that income is so evenly distributed such that there are no pure free-riders in our game and all individuals are included in the set of contributors. This assumption means that, in equilibrium, all individuals are at an inner solution and allows us to disregard corner solutions. As a by-product, assuming that all individuals are included in the set of contributors excludes the anomaly of overprovision of a public good (see Buchholz and Peters, 2001). This assumption implies no loss of generality for our results below and, by greatly simplifying the analysis, allows us to focus on the interaction between the private contributions to the public good and the contributions to market insurance.

5.1 Private provision of self-insurance

Each individual i maximizes her expected utility EU_i by her choice of g_i, taking the contributions of the other $n-1$ individuals, which already reduce the size of the loss, as given. $G_{-i}^{SI} = \sum_{j=1,j\neq i}^{n} g_j^{SI} = G_{SI}^N - g_i$ is the sum of the contributions of all other individuals but subject i. The first-order condition reads:

$$\frac{dEU_i}{dg_i} = pU'_{i2}(-1 - L'(G_{-i}^{SI} + g_i)) + (1-p)U'_{i1}(-1) \quad = \quad 0, \quad i = 1, \ldots, n \tag{45}$$

$$\Longleftrightarrow \qquad \frac{pU'_{i2}(-L'(G_{SI}^N))}{EU'_i} \quad = \quad 1, \quad i = 1, \ldots, n. \tag{46}$$

To express the marginal benefit and the marginal cost with respect to the public good the first-order condition can also be rearranged to

$$(-L'(G_{SI}^N))pU'_{i2} = (1-p)U'_{i1} + pU'_{i2}. \tag{47}$$

Each individual i contributes until the marginal benefit of an additional investment in the public good to reduce the size of the loss (left hand side) equals the marginal cost of this additional spending on the public good, which accrues in both states of the world (right hand side). From the FOC (45) we can calculate the slope of the reaction function:

$$\frac{dg_i}{dG_{-i}^{SI}} = -\frac{pU''_{i2}(-L'(G_{SI}^N))(-1 - L'(G_{SI}^N)) + pU''_{i2}(-L''(G_{SI}^N))}{pU''_{i2}(-1 - L'(G_{SI}^N))^2 + pU''_{i2}(-L''(G_{SI}^N)) + (1-p)U''_{i1}}. \tag{48}$$

The slope (48) of the reaction function is negative, which means that G_{-i}^{SI} and one's own contribution g_i are substitutes, a standard result of the theory of private provision of public goods. It obtains because both numerator and denominator in (48) are negative. Whether the slope is larger or smaller than -1 (i. e., whether one under or overcompensates the contributions of the other individuals) depends on the measure of absolute risk aversion. The difference between denominator and numerator is

$$-pU''_{i2}(-1 - L'(G^N_{SI})) + (1-p)U''_{i1}. \tag{49}$$

For the slope (48) to lie between -1 and 0, this difference must be negative, i. e., the denominator must be larger than the numerator in absolute terms, which using the FOC (45) means

$$\begin{aligned}
(1-p)U''_{i1} &< pU''_{i2}(-1 - L'(G^N_{SI})) \\
(1-p)U''_{i1} &< pU''_{i2}\frac{1-p}{p}\frac{U'_{i1}}{U'_{i2}} \\
\frac{U''_{i1}}{U'_{i1}} &< \frac{U''_{i2}}{U'_{i2}} \\
A(y_1) &> A(y_2),
\end{aligned}$$

which establishes the following

Lemma 4 (Privately provided self-insurance)
The slope of the reaction function in a setting of private provision of self-insurance depends on how the Arrow-Pratt measure of absolute risk aversion $A_j, j = 1, 2$ changes with income:

1. *is equal to -1 for constant absolute risk aversion (CARA): $A^1 = A^2$.*
2. *is smaller than -1 for increasing absolute risk aversion (IARA): $A^1 > A^2$.*
3. *lies between -1 and 0 for decreasing absolute risk aversion (DARA): $A^1 < A^2$.*

Proposition 5 (Equilibrium of privately provided self-insurance)
For individuals with decreasing absolute risk aversion, the private provision Nash equilibrium of self-insurance contributions exists and is unique. It leads to a privately provided level of a public good G^N_{SI} which is smaller than the Pareto-efficient level G^*_{SI}.

Proof. By Lemma 4, for decreasing absolute risk aversion the slope of the reaction function (48) lies between -1 and 0. Thus the reaction of individual i to a change in the sum of the contributions of the other individuals G_{-i} is normal in the sense of Cornes et al. (1999), who show that this normality ensures existence of a unique Nash equilibrium. If all individuals are included in the set of contributors (which means that all individuals are rich enough to contribute or, alternatively, that income is distributed evenly enough), there can be no underprovision anomaly, so the privately provided provision level is subefficient (Buchholz and Peters, 2001). QED.

This result confirms the usual intuition in private provision games. The contributions to G^{SI}_{-i} and G^{SP}_{-i} by the other players but i represent a de facto income transfer to i. While the efficiency conditions (9) require the sum of the willingness to pay of *all* individuals to equal the marginal cost of providing the public good, an individually rational contributor only takes into consideration the effect of his contribution on his individual utility, which decreases the resulting equilibrium cases for both self-insurance and self-protection.

5.2 Private provision of self-protection

In an analogous way, in the self-protection case each individual i maximizes her expected utility EU_i by her choice of g_i, taking the contributions of the other $n-1$ individuals, which already reduce the size of the loss, as given. $G^{SP}_{-i} = \sum_{j=1, j\neq i}^{n} g^{SP}_j = G^N_{SP} - g_i$ is the sum of the contributions of all other individuals but subject i. The first-order condition reads:

$$\frac{dEU_i}{dg_i} = (-p'(G^N_{SP}))(U_{i1} - U_{i2}) - EU'_i \;=\; 0, \quad i = 1, \ldots, n \tag{50}$$

$$\iff \qquad \frac{(-p'(G^N_{SP}))(U_{i1} - U_{i2})}{EU'_i} \;=\; 1, \quad i = 1, \ldots, n. \tag{51}$$

To express the marginal benefit and the marginal cost with respect to the public good the first-order condition can also be rearranged to

$$(-p'(G^N_{SP}))(U_{i1} - U_{i2}) = (1 - p(G^N_{SP}))U'_{i1} + p(G^N_{SP})U'_{i2}. \tag{52}$$

Each individual i contributes until the marginal benefit of an additional investment in the public good to reduce the probability of the loss (left hand side) equals the marginal cost of this additional spending on the public good, which accrues in both states of the world (right hand side). From the FOC (50) we can calculate the slope of the reaction function:

$$\frac{dg_i}{dG^{SP}_{-i}} = -\frac{-p''(G^N_{SP})(U_{i1} - U_{i2}) - p'(G^N_{SP})(U'_{i2} - U'_{i1})}{-p''(G^N_{SP})(U_{i1} - U_{i2}) - 2p'(G^N_{SP})(U'_{i2} - U'_{i1}) + (1 - p(G^N_{SP}))U''_{i1} + p(G^N_{SP}))U''_{i2}}. \tag{53}$$

The denominator is negative by the second order condition. The sign of the difference between denominator and numerator can be determined in a similar way to Section 3.2

using Assumption 1:

$$
\begin{aligned}
& -p'(G^N_{SP})(\underbrace{U'_{i2} - U'_{i1}}_{> -L \cdot EU''_i}) + EU''_i > -p'(G^N_{SP})(-L \cdot EU''_i) + EU''_i \\
= \quad & EU''_i[(\underbrace{-p'(G^N_{SP})(-L) + 1}_{<0})] > 0,
\end{aligned}
$$

Thus, the numerator is also negative and larger than the denominator in absolute terms. If the second order condition is fulfilled, the slope (53) of the reaction function is negative, which again means that G^{SP}_{-i} and one's own contribution g_i are substitutes and, remarkably, the slope (53) is smaller than -1:

Lemma 5 (Privately provided self-protection)

The slope of the reaction function in a setting of private provision of self-protection is smaller than -1 if the second order condition and Assumption 1 are fulfilled.

Proposition 6 (Equilibrium of privately provided self-protection)

There exists a private provision Nash equilibrium of private self-protection contributions.

Proof. The existence proof follows Bergstrom et al. (1986). The conditions (50) define a best-response function which is a mapping of the compact and convex set $[0, m_i]$ to itself. By Brouwer's Fixed Point Theorem there must exist a fixed point, which is a Nash equilibrium of the contributions $g_i, i = 1, \ldots, n$. QED.

This result without uniqueness of equilibria is analogous to Ihori and McGuire (2007)'s multiple equilibria result. The missing normality and the multiplicity of equilibria does not allow to establish a general result regarding the underprovision of self-protection as a public good.

6 Interaction of private provision with market insurance

In the following, we analyze the interaction between a public good that is privately provided and private market insurance and specially whether it is individually optimal to contribute to a public good which acts as an insurance device when private insurance is available.

6.1 Market insurance and self-insurance

Individual i maximizes her expected utility

$$EU_i(g_i, G, s_i) = pU(m_i - g_i - L(G) + (1 - \pi)s_i) + (1 - p)U(m_i - g_i - \pi s_i) \tag{54}$$

by simultaneously choosing g_i and s_i. The first-order conditions are given by

$$FOC_g := \frac{\partial EU_i}{\partial g_i} = pU'_{i2}(-1 - L'(\hat{G}^N_{SI})) + (1-p)U'_{i1}(-1) = 0 \quad (55)$$

$$FOC_s := \frac{\partial EU_i}{\partial s_i} = pU'_{i2}(1-\pi) - (1-p)U'_{i1}\pi = 0, \quad (56)$$

the second-order conditions are fulfilled for $U''_i < 0$ and $L'' > 0$ as assumed. We write $\hat{G}^N_{SI}$ for the Nash equilibrium level of the public good in the self-insurance case with market insurance. Conditions (55) and (56) can be rearranged to

$$\frac{\pi}{1-\pi} = \frac{1}{-1 - L'(\hat{G}^N_{SI})}. \quad (57)$$

The optimum is reached when the shadow price of self-insurance, as given by the right hand side, is equal to the market price of insurance (left hand side). In other words, the individual is indifferent whether to spend an additional unit of income in self-insurance or market insurance. If the price for market insurance is fair, $\pi = p$, condition (57) leads to

Proposition 7 (Private provision of self-insurance with market insurance) *The privately provided efficient level of a public good G, which acts as a self-insurance device, in the presence of market insurance is implicitly defined by*

$$\frac{1}{-1 - L'(\hat{G}^N_{SI})} = \frac{p}{1-p} \qquad \Longleftrightarrow \qquad p \cdot (-L'(\hat{G}^N_{SI})) = 1. \quad (58)$$

Condition (58) is also the condition that maximizes expected income. However, in contrast to the efficient provision, expected income is maximized at the individual and not at the social level. We can calculate the comparative static effect of π on the first-order conditions (55) and (56). Let D be the determinant of the maximization problem (54). By the second-order condition and our assumptions, we have $D = FOC_{gg} \cdot FOC_{ss} - (FOC_{gs})^2 > 0$, where the index denotes the partial derivative(s) with respect to the corresponding variable(s). Then, we obtain by Cramer's rule

$$\frac{dg}{d\pi} = \frac{1}{D}\begin{vmatrix} FOC_{ss} & FOC_{s\pi} \\ FOC_{gs} & FOC_{g\pi} \end{vmatrix} > 0 \quad (59)$$

$$\frac{ds}{d\pi} = \frac{1}{D}\begin{vmatrix} FOC_{s\pi} & FOC_{sg} \\ FOC_{g\pi} & FOC_{gg} \end{vmatrix} < 0 \quad (60)$$

Thus, market insurance and self-insurance are strategic substitutes in the sense that a market price increase in market insurance decreases the demand for market insurance and increases the demand for self-insurance, which has become relatively cheaper.

Condition (58) defines implicitly a private provision level $\hat{G}^N_{SI}$ of public self-insurance in the presence of market insurance, which can be compared with the privately provided provision level G^N_{SI} without market insurance as given by equation (46):

$$-L'(\hat{G}^N_{SI}) = \frac{1}{p} > \frac{1}{p}\frac{EU'_1}{U'_{12}} = -L'(G^N_{SI}). \tag{61}$$

Since the marginal utility in the loss state 2 is larger than in non-loss state 1, $U'_{12} > U'_{11}$,

$$\frac{EU'_1}{U'_{12}} = \frac{(1-p)U'_{11} + pU'_{12}}{U'_{12}} < 1, \tag{62}$$

such that

$$-L'(\hat{G}^N_{SI}) \;>\; -L'(G^N_{SI}) \tag{63}$$

$$\hat{G}^N_{SI} \;<\; G^N_{SI}. \tag{64}$$

Thus, the possibility of buying market insurance and the strategic substitutability between self-insurance and market insurance decreases the privately provided level of the public good further.

To compare the efficient and the private provision level of self-insurance when market insurance is available, we use conditions (32) and (58). Since the efficiency condition (32) contains the size n of the population that benefits from public self-insurance and the private provision condition (58) does not reflect the positive external effect of the public good,

$$\hat{G}^N_{SI} < \hat{G}^*_{SI}. \tag{65}$$

Combining results (35), section 5.1, (64), and (65), we obtain the following rankings for the provision levels of the self-insurance public good:

$$\hat{G}^N_{SI} \;<\; G^N_{SI} < G^*_{SI} \tag{66}$$

$$\hat{G}^N_{SI} \;<\; \hat{G}^*_{SI} < G^*_{SI}. \tag{67}$$

6.2 Market insurance and self-protection

When the public good acts as a self-protection device the fair price for market insurance is given by $\pi = p(G)$. Hence, the public good does not only - to some extent - protect individuals, but decreases also the price of the insurance. However, as insurance is assumed to be fair, individuals always fully insure. In the case of a positive loading it depends on the intensity of competition whether a probability reduction leads to a reduction of the insurance price or not.

The individual maximizes her expected utility

$$EU_i(g_i, G, s_i) = p(G)U(m_i - g_i - L + (1-p(G))s_i) + (1-p(G))U(m_i - g_i - p(G)s_i) \quad (68)$$

by simultaneously choosing g_i and s_i. The first-order conditions are given by

$$\frac{\partial EU_i}{\partial s_i} = p(\hat{G}^N_{SP})U'_{i2}(1-p(\hat{G}^N_{SP})) - (1-p(\hat{G}^N_{SP}))p(\hat{G}^N_{SP})U'_{i1} = 0 \quad (69)$$

$$\frac{\partial EU_i}{\partial g_i} = p'(\hat{G}^N_{SP})(U_{i2} - U_{i1}) + p(\hat{G}^N_{SP})U'_{i2}(-1 - p'(\hat{G}^N_{SP})s_i) \quad (70)$$
$$+(1-p(\hat{G}^N_{SP}))U'_{i1}(-1 - p'(\hat{G}^N_{SP})s_i) = 0.$$

In the first condition, the probabilities cancel out and we obtain $U'_{i1} = U'_{i2}$, i. e., equal income in both states of the world: When insurance is fair, the individuals choose full cover $s_i = L$ independently of the additional self-protection effort. Since $U_{i1} = U_{i2}$, the second condition simplifies to

Proposition 8 (Private provision of self-protection with market insurance) *The privately provided efficient level of a public good G, which acts as a self-protection device, in the presence of market insurance is implicitly defined by*

$$-p'(\hat{G}^N_{SP})L = 1. \quad (71)$$

We can compare the Nash private provision equilibrium without market insurance as defined by (51) with the corresponding private provision equilibrium when market insurance is available as described by (71):

$$\frac{-p'(G^N_{SP})(U_{i1} - U_{i2})}{EU'_i} = -p'(\hat{G}^N_{SP})L \quad (72)$$

Using Assumption 1, $\frac{U_{i1}-U_{i2}}{EU'_i} > L$, we find

$$-p'(G^N_{SP}) < -p'(\hat{G}^N_{SP}) \quad (73)$$

$$G^N_{SP} > \hat{G}^N_{SP} \quad (74)$$

then market insurance reduces further the private provision level of the public good.

To compare the efficient and the private provision level of self-protection when market insurance is available, conditions (41) and (71) are relevant. As in the case of self-insurance, the efficiency condition (41) contains the size n of the population that benefits from public self-insurance and the private provision condition (71) does not reflect the positive external effect of the public good, the private provision level is inefficiently small:

$$\hat{G}^N_{SP} < \hat{G}^*_{SP}. \tag{75}$$

Combining results (43), (74), and (75), we obtain the following rankings for the public self-protection provision levels:

$$\hat{G}^N_{SP} < G^N_{SP} \tag{76}$$

$$\hat{G}^N_{SP} < \hat{G}^*_{SP} < G^*_{SP}. \tag{77}$$

7 Conclusion

Many public goods provide utility to the society only due to an insurance effect of reducing the size or probability of possible losses. This loss or probability reduction benefits all individuals and is a public good. Our paper extends and combines two strands of the literature: the public goods literature including the efficient and the private provision of public goods and the self-insurance and self-protection literature.

In a very intuitive way, more risk averse societies prefer higher levels of self-insurance and self-protection as public goods. In contrast to the standard framework, the comparative static effects of income are more elaborated. We show how the "normality" concept of the public goods literature can be interpreted in our risk model as decreasing absolute risk aversion (in the self-insurance case) and as a condition of the probability of the loss (in the self-protection case). These condition highlight the theoretical similarities and differences that our model brings out.

An interesting aspect of regarding public goods as insurance devices is the interaction with market insurance. The presence of market insurance decreases efficient provision of the public good since fully insured subjects behave as if they were risk neutral. The private provision of public goods is also reduced by the availability of market insurance. The publicly provided level of the public good will, in general, be observable by insurers. Consequently, in the case of self-protection, public goods may be superior to private self-protection activities if moral hazard problems are involved where private self-protection effort may be difficult to monitor.

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