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The Dixit-Pindyck and the Arrow-Fisher-Hanemann-Henry option values are not equivalent

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The Dixit-Pindyck and the Arrow-Fisher-Hanemann-Henry Option Values are not Equivalent*

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Abstract

Fisher [2000, this journal] offers a unifying framework for two concepts of (quasi-) option value suggested by Arrow, Fisher, Hanemann, and Henry (AFHH), on the one hand, and by Dixit and Pindyck (DP), on the other, and claims these two concepts to be equivalent. We show that this claim is not correct and point out the flaws in Fisher's proof. We further suggest a decomposition of the DP option value into two components, one of which corresponds exactly to the AFHH option value which captures the value of obtaining new information, and a second one which captures the postponement value irrespective of uncertainty.

JEL classification: D81, Q20

Key-words: option value, quasi-option value, decision under uncertainty, irreversible investment

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1 Introduction

In the context of irreversible decision making under uncertainty concepts of option values have been developed independently in different strands of literature and in very different contexts. Most prominent are the approaches by Arrow and Fisher [1974], Henry [1974], and Fisher and Hanemann [1986], on the one hand, and by Dixit [1992], Pindyck [1991] and Dixit and Pindyck [1994], on the other. Arrow, Fisher, Hanemann, and Henry developed the concept of quasi-option value mainly in the context of the economics of the environment, in particular when irreversible economic decisions, to be made under uncertainty, may cause irreversible environmental damage. Dixit and Pindyck were mainly interested in business investment decisions when the future value of an irreversible investment is uncertain. Hanemann [1989] captured the concepts of Arrow and Fisher [1974] and of Henry [1974] in a unifying framework. More recently, Fisher [2000] tried to unify the two concepts by Arrow, Fisher, Henry, and Hanemann (AFHH), on the one hand, and by Dixit and Pindyck (DP), on the other, and claimed the two concepts to be equivalent.[1]

In this note we show that the concept of the AFHH quasi-option value and the concept of DP option value are not equivalent and that the argument by Fisher is flawed. We confirm that AFHH quasi-option value, as precisely defined by Hanemann [1989], captures only the informational aspect of postponing an irreversible investment decision. It does not capture

[1]Other authors have developed similar concepts, many of them drawing on the seminal paper by Arrow and Fisher [1974]. Since in this note we want to point out that Fisher's claim is not correct, we will refrain from giving a further survey on the wide literature on option and quasi-option values.

another important issue of postponing a decision, namely the trade-off between foregoing the benefit of investing today against either gaining better information about the value of investment tomorrow or taking advantage of the fact that investing tomorrow may yield a higher payoff even in the absence of uncertainty.

In order to link the concepts of (quasi-)option value as suggested by AFHH and DP, we start from the definition of the DP option value and propose a decomposition of that value into two parts. The first part corresponds exactly to the AFHH concept of quasi-option value and captures the pure value of obtaining new information about the value of the investment tomorrow given that no investment takes place today. The second part accounts for both, better conditions of investment in the second period and for the benefit foregone by postponing the decision, independently of information.

Our note is organized as follows: in the next section we recall the basic AFHH model as set out by Hanemann and adopted by Fisher [2000]. We then define the DP concept of option value in terms of the AFHH-model and show that it is different from Hanemann's definition of quasi-option value and we point out two flaws in Fisher's proof. In section 2.3 we suggest a decomposition of the Dixit-Pindyck option value. We then illustrate the difference between the two concepts at the example of the Dixit-Pindyck widget-model.

2 The Model

There are two periods, where the second period can physically be interpreted as a whole sequence of further periods. An irreversible (investment) decision can be made in the first period or it can be postponed to the second period. We denote the decision variable by $d_i \in \{0,1\}$, where i denotes the two periods 1 and 2, and $d_i = 1$ means that the investment takes place, whereas $d_i = 0$ means that no investment takes place in period i. The (certain) benefit of the decision in period 1 is written as $B_1(d_i)$.

The uncertain benefit of the investment decision in the second period is written as $B_2(d_1, d_2, \vartheta)$, where $d_1 + d_2 \in \{0,1\}$, and ϑ is a random variable.[2] To define the value of different decision rules we best work backwards, starting in period 2.

2.1 The Decision in Period 2

First we define the *open-loop second period expected value,*

$$B_2^*(d_1) = \max_{d_2, d_1+d_2 \leq 1} E_\vartheta[B_2(d_1, d_2, \vartheta)] \tag{1}$$

which represents the *expected value of the ex ante optimal investment decision* at the beginning of period 2 when the decision is made *before* nature has drawn the state of the world ϑ, given any investment decision d_1 in period 1.

Secondly, we define the *closed-loop second period expected value,* given any investment decision d_1 in period 1 by

$$\widehat{B}_2(d_1) = E_\vartheta[\max_{d_2, d_1+d_2 \leq 1} \{B_2(d_1, d_2, \vartheta)\}] \tag{2}$$

[2] Hanemann assumes that B_2 does not depend directly on d_1, i.e. he writes $B_2(d_1, d_2, \vartheta) = B_2(d_2, d_1 + d_2, \vartheta)$. In many environmental applications the direct impact of d_1 on the second period benefit is, however, more important than the direct effect of d_2. E.g., in case of investment in abatement of an accumulating pollutant (a GHG), abatement in period 1 has a stronger impact on the benefit than abatement in period 2. We use the most general set-up $B_2(d_1, d_2, \vartheta)$. For our main argument, the special kind of relationship is not crucial.

This is the expected value of the benefit in period 2, when at the beginning of period 2 the decision maker can observe the state of the world ϑ and then make his optimal investment decision $d_2(d_1, \vartheta)$ contingent on ϑ. Since $d_2(1, \vartheta) = 0$ for each ϑ, we clearly obtain

$$B_2^*(1) = \widehat{B}_2(1) \tag{3}$$

For $d_1 = 0$, denote by $d_2^*(0) = \arg\max_{d_2} E_\vartheta[B_2(0, d_2, \vartheta)]$ the optimal open loop decision for period 2, given that no investment took place in period 1. Then

$$\begin{aligned} B_2^*(0) &= \max_{d_2} E_\vartheta[B_2(0, d_2, \vartheta)] = E_\vartheta[B_2(0, d_2^*(0), \vartheta)] \\ &\leq E_\vartheta[\max_{d_2} B_2(0, d_2, \vartheta)] = \widehat{B}_2(0) \end{aligned} \tag{4}$$

(4), says that if no investment took place in period 1 or if there is simply no first period, then one can never be worse off by making the decision *with* further information than making it *without*.[3] The difference $VI = \widehat{B}_2(0) - B_2^*(0)$ can be interpreted as the value of information, or the *option value for receiving information* about the true state of the world in period 2 for $d_1 = 0$. This value exists even if there is no preceding period 1. Thus this kind of quasi-option value is actually *not* a value which stems from intertemporally postponing a decision.

2.2 The Decision in Period 1

Now we study the possible decisions in period 1. If it is not possible to obtain further information at the beginning of period 2, the payoff to the decision

[3] Note that by contrast to Fisher [2000] and Hanemann [1989] we obtain this inequality without refering to Jensen's inequality. Thus we also do not need any restricitve assumptions on the benefit function $B_2(d_1, d_2, \vartheta)$ such as concavity.

maker in period 1 is defined as:

$$V^*(d_1) = B_1(d_1) + B_2^*(d_1) \tag{5}$$

The decision maker then chooses

$$\max_{d_1} V^*(d_1) = \max \{V^*(0), V^*(1)\} \tag{6}$$

If gathering further information at the beginning of period 2 *is* possible, the payoff to the decision maker in period 1 is defined as:

$$\widehat{V}(d_1) = B_1(d_1) + \widehat{B}_2(d_1) \tag{7}$$

The decision maker then chooses

$$\max_{d_1} \widehat{V}(d_1) = \max \left\{ \widehat{V}(0), \widehat{V}(1) \right\} \tag{8}$$

Finally we look at the *net present value decision rule*. For this purpose we define the default value $\overline{B}_0 = B_1(0) + E_\vartheta[B_2(0, 0, \vartheta)]$ which reflects the present value of the stream of payoffs which emerges if no investment decision is made for all times, here neither in period 1, nor in period 2. Dixit and Pindyck, who are mainly interested in the investment decision of a firm, consider the special case where this default value is zero. If a firm invests in order to substitute its old technology, the default value would in general be positive. In environmental applications, too, the default value is often different from zero. Think of the value of public benefits arising from some piece of land in case that no investment to develop the land is made.

Dixit and Pindyck formulate the net present value decision rule follows

$$NPV = \max \left\{ V^*(1), \overline{B}_0 \right\}. \tag{9}$$

2.3 The Option Value of Postponing the Investment Decision

We are now ready to define the option value of postponing the investment decision. Following exactly the definition of Dixit and Pindyck (1994, p. 96/97) and Fisher (2000) we write:

$$\begin{aligned} OV^{DP} \quad &: = \max\left\{\widehat{V}(0), \widehat{V}(1)\right\} - NPV \qquad (10) \\ &= \max\left\{\widehat{V}(0), \widehat{V}(1)\right\} - \max\left\{V^*(1), \overline{B}_0\right\} \end{aligned}$$

Note that $\widehat{V}(1) = V^*(1)$. This is so because, if the decision maker has invested in period 1, she is "locked in" with her decision. Hence there is nothing to decide in period 2 and the two values must coincide.[4]

If $\max\left\{\widehat{V}(0), \widehat{V}(1)\right\} = \widehat{V}(1)$, then clearly $OV^{DP} = 0$. Hence consider the interesting case where

$$\widehat{V}(0) > \widehat{V}(1) \qquad (11)$$

Let us also assume

$$V^*(1) > \overline{B}_0 \qquad (12)$$

Only if these two conditions are satisfied, there will be a non-trivial option value of postponing the investment decision.

Adding the "appropriate zero" (10) becomes

$$\begin{aligned} OV^{DP} &= \max\left\{\widehat{V}(0), \widehat{V}(1)\right\} - NPV \qquad (13) \\ &= \max\left\{\widehat{V}(0), \widehat{V}(1)\right\} - \max\left\{V^*(1), \overline{B}_0\right\} \qquad (14) \\ &= \widehat{V}(0) - V^*(1) \\ &= \widehat{V}(0) - V^*(1) + \left[\widehat{V}(0) - V^*(0)\right] - \left[\widehat{V}(0) - V^*(0)\right] \end{aligned}$$

[4]This has also been pointed out by Hanemann and Fisher.

The term $OV^{AFHH} := \widehat{V}(0) - V^*(0)$ corresponds exactly to the Arrow-Fisher-Henry-Hanemann concept of quasi-option value. It reflects the difference between closed-loop and open-loop decisions, i.e. with and without making use of (or being able to make use of) additional information about the state of the world in period 2, given that no investment took place in period 1. It is always non-negative by inequality (4).

By contrast we define

$$OV^{PP} := V^*(0) - V^*(1)$$

as the pure *postponement value.* This value may be positive for example due to technological progress. If the decision maker knows that tomorrow there will be a better technology available than the current one, he will possibly postpone the investment decision even if the currently available technology yields a positive payoff, i.e. if investing today is better than investing never.

Thus we can decompose the DP option value into the pure informational quasi-option value OV^{AFHH} and an option value purely due to postponing the decision OV^{PP} due to better conditions irrespective of the chance to get new information:

$$OV^{DP} = OV^{AFHH} + OV^{PP} \tag{15}$$

In our view, only the Dixit-Pindyck concept reflects the *economic trade-off* between the benefit resulting from the investment today and forgoing this benefit. The benefit need not necessarily materialize in the first period. With the example of investing in a faster computer, the benefit from investment will already materialize in the first period. In case of investment into abatement of greenhouse gases today, this benefit will materialize in the future.

Hanemann, who considers the investment problem in the context of irreversible land development, writes [p. 28]: "This [i.e. $OV^{AFHH} = \widehat{V}(0) - V^*(0) \geq 0$] does *not* mean that one should never develop in the first period; after all, it may happen that $\widehat{V}(0) < \widehat{V}(1)$. But it does mean that the case for preservation is *strengthened* when one recognizes the prospect of further information about the future consequences of development." So Hanemann is fully aware that his definition does not capture the trade off of postponing the investment decision. But this also means that AFHH quasi-option value as made precise by Hanemann and the Dixit Pindyck option value are different.

3 Two Mistakes in Fisher's Proof

In Fisher's equivalence proof we discovered two mistakes: Firstly, whereas in equation (13) on p. 201, he sets the $NPV = \max\left\{V^*(1), \overline{B}_0\right\}$, with the special case of $\overline{B}_0 = 0$, Fisher claims to derive $NPV = \max\left\{V^*(1), V^*(0)\right\}$, as set out in equation (13') on page 202. However, $\overline{B}_0 = B_1(0) + E_\vartheta[B_2(0, 0, \vartheta)]$, whereas $V^*(0) = B_1(0) + \max_{d_2}\{E_\vartheta[B_2(0, d_2, \vartheta)]\}$. Note that the value of $\overline{B}_0$ presupposes the open loop decision, to never invest. By contrast, $V^*(0)$ is the value of the optimal open loop decision in period 2 when there was no investment in period 1. Thus $V^*(0)$, allows for investment in period 2 albeit without taking advantage of new information, whereas $\overline{B}_0$ results from not investing at all.

The second reason why Fisher obtains equivalence of the two concepts is due to algebraic mistake. Fisher employs the following relationship:

$$\max\left\{A, B + C\right\} - \max\left\{A, B + D\right\} \quad "=" \quad C - D \tag{16}$$

where $A = B_1(1) + E_\vartheta[B_2(1,\vartheta)]$, $B = B_1(0)$, $C = E_\vartheta[\max\{B_2(0,\vartheta), B_2(1,\vartheta)\}]$, and $D = \max\{E_\vartheta[B_2(0,\vartheta)], E_\vartheta[B_2(1,\vartheta)]$.

A simple numerical counterexample shows that (16) does not hold in general: Let $A = 1$, $B = 0$, $C = 2$, and $D = 0$. Then $\max\{A, B + C\} - \max\{A, B + D\} = 2 - 1 = 1$, whereas $C - D = 2 - 0 = 2$.

4 The AFHH and DP Option Values in the Dixit-Pindyck Widget Model of Investment under Uncertainty

Mimicking the analysis of Fisher [2000], we consider the special problem, as set out in Dixit and Pindyck [1994], of investment under uncertainty. A firm faces a decision of whether or not to make an investment, with a sunk cost of I, in a factory that will produce one widget per period forever. The certain price of widgets in the first period is P_0 whereas in the second period (and thereafter), it will be θP_0, where $\theta = (1 + u)$ with probability q, and $\theta = (1 - v)$ with probability $(1 - q)$.

Expressing the respective benefits in terms of the widget model[5] yields $B_1(0) = 0$, $B_2(0, 0, \theta) = 0$, and $B_2(0, 1, \theta) = \frac{1}{1+r}[\frac{1+r}{r}\theta P_0 - I]$, where r is the interest rate.

To keep things simple we make a number of assumptions:

Assumption 1 $E_\vartheta[B_2(0, 1, \vartheta)] = \frac{1+r}{r}[q(1 + u) + (1 - q)(1 - v)] P_0 - I > 0$.

This assumption says that the expected value of investment is positive. Further we assume

[5]Including reframing the widget model into a two period model, a first period, and a second period that consists of the benefits in period 2 and there after.

Assumption 2 *$B_2(0,1,1+u) = \frac{1+r}{r}(1+u)P_0 - I > 0$, and $B_2(0,1,1+u) = \frac{1+r}{r}(1-v)P_1 - I < 0$.*

This assumption says that given no investment in period 1, investment in period 2 is profitable in the good state of the world and is not profitable in the bad state of the world. Both assumptions together imply that q, the probability of the good state, is bounded away from zero, i.e. q is greater than some $\overline{q} > 0$.

With these assumptions we obtain

$$\begin{aligned}
V^*(0) &= B_1(0) + \max\{E_\theta[B_2(0,\theta)], E_\theta[B_2(1,\theta)]\} \\
&= \max\{E_\theta[B_2(0,\theta)], E_\theta[B_2(1,\theta)]\} \\
&= \max\{0, \frac{1}{r}\left[q(1+u) + (1-q)(1-v)\right] P_0 - \frac{1}{1+r} I\} \\
&= \frac{1}{r}\left[q(1+u) + (1-q)(1-v)\right] P_0 - \frac{1}{1+r} I
\end{aligned}$$

and

$$\hat{V}(0) = B_1(0) + E_\theta[\max\{B_2(0,\theta), B_2(1,\theta)\}] = q[\frac{1}{r}(1+u)P_0 - \frac{1}{1+r}I] \quad (17)$$

The AFFH option value is then given by:

$$\begin{aligned}
& OV^{AFFH} = \hat{V}(0) - V^*(0) \\
&= -\frac{1-q}{r}(1-v)P_0 + \frac{1-q}{1+r} I \\
&= (1-q)[-\frac{1}{1+r}\underbrace{(\frac{1+r}{r}(1-v)P_0) - I)}_{<0}] \qquad (18)
\end{aligned}$$

which is positive for $\overline{q} < q < 1$. For $q = 1$ the AFHH quasi-option value of postponing investment in order to get more information is equal to zero as it should be. After all, it is an expression for the value of knowing the true value of the stochastic parameter conditional on non-investment in the first period.

Finally we assume

Assumption 3 *$\hat{V}(1) < \hat{V}(0)$, and $\hat{V}(1) > 0$.*

This assumption makes the option value non-trivial since it says that investing in period 1 is profitable, but waiting with investment after getting more information is better. Then we are ready to calculate the Dixit-Pindyck option value of postponing investment:

$$\begin{aligned} OV^{DP} &= \max\{\hat{V}(0), \hat{V}(1)\} - \max\{\hat{V}(1), 0\} \\ &= \hat{V}(0) - \hat{V}(1) \\ &= \left(q \left[\frac{1}{r}(1+u)P_0 - \frac{1}{1+r}I \right] \right) \\ &\quad - \left(P_0 + \frac{q(1+u) + (1-q)(1-v)}{r} P_0 - I \right) \\ &= - \left[1 + \frac{(1-q)(1-v)}{r} \right] P_0 + \frac{1+r-q}{1+r} I \end{aligned} \tag{19}$$

Finally we may decompose the DP option value into:

$$\begin{aligned} OV^{DP} &= - \left[1 + \frac{(1-q)(1-v)}{r} \right] P_0 + \frac{1+r-q}{1+r} I \\ &= \left(-\frac{1-q}{r}(1-v)P_0 + \frac{1-q}{1+r} I \right) + \left(P_0 - \frac{r}{1+r} I \right) \\ &= OV^{AFFH} + OV^{PP}. \end{aligned} \tag{20}$$

The difference between the two options values is highlighted if we consider the special case when there is no uncertainty, and thus no value of information. With $q = 1$ we have $OV^{AFHH} = 0$, and the DP option value is equal to $\max\{0, \frac{r}{1+r}I - P_0\}$ which equals the amount of money earned or lost by postponing the investment by one period.

5 Conclusions

In this note we demonstrated that in contrast to Fisher's claim the Arrow-Fisher-Hanemann-Henry quasi-option value and the Dixit-Pindyck option value are not equivalent. We have also stressed that the AFHH quasi-option value reflects only the value of postponing a decision due to the value of obtaining more information whereas the Dixit-Pindyck option value can be decomposed into the informational option value which corresponds to the AFHH option value, and a pure postponement value which even exists in the absence of uncertainty.

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