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Optimal Abatement in Dynamic Multipollutant Problems when Pollutants can be Complements or Substitutes

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OPTIMAL ABATEMENT IN DYNAMIC MULTI-POLLUTANT PROBLEMS WHEN POLLUTANTS CAN BE COMPLEMENTS OR SUBSTITUTES *

ULF MOSLENER† AND TILL REQUATE‡

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Abstract. We analyze a dynamic multi-pollutant problem where abatement costs of several pollutants are not separable. The pollutants can be either technological substitutes or complements. Environmental damage is induced by the stock of accumulated pollution. We find that optimal emission paths are qualitatively different for substitutes and complements. We derive general properties governing optimal emission paths and present numerical examples to illustrate our main results. In particular we find that optimal emission paths need not be monotonic, even for highly symmetric pollutants. Finally, we describe a comparatively simple method to implement the optimal path without explicitly knowing its shape.

JEL Classification: Q2, L5

Keywords: Multi-pollution, abatement technology, accumulating pollutants

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1. Introduction

The environmental policy literature usually concentrates on regulation of a single pollutant or assumes that different pollutants can be treated separately. Many production processes, however, are not only accompanied by the emissions of more than one pollutant, but modern abatement technologies also allow for joint abatement of several pollutants. An example is the catalytic converter for cars which jointly reduces SO_2, CO, and NO_X. In this case we refer to pollutants which can be abated jointly as *complements.*

Other technologies reduce certain pollutants at the cost of increasing the output of at least one other pollutant. As an example particulate matter and SO_2 emitted from power plants are abated by using considerable amounts of energy which in turn leads to an increase of CO_2-emissions. Thus SO_2- and CO_2-emissions can be considered as *substitutes.* A further example of substitutable pollutants is waste treatment where combustion, on the one hand, and deposition of waste, on the other, cause either carbon dioxide or methane emissions, respectively.

Emissions can also be considered as complements or substitutes if the inputs that cause these emissions in the production process are complements or substitutes in the usual sense. Since the choice of the input mix has to be done simultaneously, it is clear that also abatement decisions can in general not be made separately for each pollutant.

This paper studies the *dynamic* properties of optimal joint abatement paths and the corresponding policy rules when pollutants accumulate and when the social damage is caused by the several stocks of pollution. We show that, even if pollutants are symmetric with respect to both, their abatement costs and the damage they cause, optimal abatement paths will look different for substitutes and complements. Besides presenting some instructive numerical examples of optimal abatement paths for complements and substitutes, we derive general properties of the optimal emission/abatement paths. In particular we show for the case of two pollutants that optimal steady state emissions of the less harmful pollutant rise with the degree of substitutability while this effect is ambiguous for the more harmful pollutant. We also show that the optimal paths need not be monotonic, a property which does not emerge in the case of a single accumulating pollutant.

Finally, we discuss policy rules which determine how to deal with several interacting pollutants. We suggest to regulate a multi-pollutant problem by issuing tradeable permits since those can be implemented in a relatively simple way. For n pollutants, the set of all optimal abatement paths – each path being generated by an $n-$vector of initial pollution stocks – can be considered as an n-dimensional manifold in a $2n$-dimensional space. Instead of calculating this rather complex optimal path, it suffices for the regulator to have at hand a table of numbers representing a hyperplane which contains the optimal emission path. In the case of two pollutants, the table tells the regulator how to pick the optimal pair of permit quantities given any pair of pollution stocks. If the permit markets are competitive, market prices for permits reveal the optimal marginal social cost of pollution. In principle, a corresponding table can also be created if the regulator prefers to charge emission taxes.

To date, only few authors have treated multi-pollution problems. In a static framework Cansier and Cansier (1999), Endres (1985) and Repetto (1987) examine the case of several pollutants by employing the concept of iso-cost and iso-damage curves. In a dynamic context, multiple pollutant aspects have been studied above all for the greenhouse problem. In his world model, Nordhaus (2000) accounts for different greenhouse gases (GHGs), however, he limits the *optimal choice* of emissions to carbon dioxide (CO_2). Michaelis (1996) minimizes additively separable abatement cost of several GHGs under a given CO_2 budget. Manne and Richels (2000) combine different model components, among those both economic and climate submodels, in order to form an intertemporal general equilibrium model to assess multi-gas- and CO_2-only scenarios. They find that a multi-gas strategy benefits all developed regions except the economies in transition. Reilly et al. (1999) study multi-gas abatement strategies in a framework of an integrated global systems model. Furthermore, Böhringer et al. (forthcoming) investigate the efficiency gains from "what-flexibility" with respect to the emissions of carbon dioxide and methane within the frame of an integrated computable general equilibrium assessment. Burniaux (2000) focuses on the empirical determination of abatement cost by estimating the costs to reach the Kyoto targets under different flexibility regimes. He finds that restricting attention to CO_2 only introduces an upward bias of cost estimations. Moslener and Requate (2001) characterize first best optimal emission paths if multiple accumulating pollutants interact with respect to social damage. By contrast to this paper they assume abatement costs to be additively separable. Different dynamic and physical properties of the pollutants are considered and found to influence both the behavior of optimal emissions and their shadow prices over time.

The paper is organized as follows. In the next section we outline the model. Section 3 characterizes optimal emission/abatement paths and possible policy rules followed by numerical simulations of optimal paths. Section 4 briefly discusses decentralized policy. In Section 5 we present our conclusions.

2. The Model

We consider a model where economic activity causes several pollutants. We denote by E_i the flow of emissions and by S_i the accumulated stock of pollutant i, respectively.

We assume that the damage depends on the stocks of the pollutants, whereas the abatement costs are a function of the flow of emissions. The pollutants accumulate according to the following equations of motion

$$\dot{S}_i = E_i - \beta_i S_i, \tag{2.1}$$

where $0 < \beta_i < 1$ is the (constant) decay rate of pollutant $i = 1, \dots, n$.

We rule out interaction on the damage side by assuming that the environmental damage function is additively separable, i.e. $D(S_1, ..., S_n) = \sum D_i(S_i)$. In other words, the damage caused by an additional ton of pollutant i is not influenced by any other pollutant.

By contrast, the abatement costs $C(E_1, ..., E_n)$ depend on the whole vector of emissions. Restricting to the case of two pollutants we assume $C_i := \frac{\partial C}{\partial E_i} < 0$, where $E_j \leq E_j^{\max}$ for $j = 1, 2$, and $C_{ii} := \frac{\partial^2 C}{(\partial E_i)^2} > 0$. Moreover, C is twice continuously differentiable and strictly convex, implying $C_{11}C_{22} - [C_{12}]^2 > 0$. The sign of $C_{12}(= C_{21})$ may go either way in general. The two pollutants are called substitutes if $C_{12} > 0$ and complements if $C_{12} < 0$, respectively.[1]

The social planner minimizes the discounted sum (the integral) of social cost arising over time:

$$\min_{E_1(t), E_2(t)} \int_0^\infty [C(E_1, E_2) + D_1(S_1) + D_2(E_2)] e^{-rt} dt \tag{2.2}$$

subject to the equation of motion of the accumulating stocks (2.1) for $i = 1, 2$, where r as usual is the social discount rate. The current value Hamiltonian is then given by

$$H = C(E_1, E_2) + D_1(S_1) + D_2(S_2) + \lambda_1(E_1 - \beta_1 S_1) + \lambda_2(E_2 - \beta_2 S_2) \tag{2.3}$$

where λ_i are the costate variables. From the first-order conditions

$$\frac{\partial H}{\partial E_i} = C_i'(E_i, E_j) + \lambda_i = 0 \tag{2.4}$$

$$-\frac{\partial H}{\partial S_i} = \dot{\lambda}_i - \lambda_i r. \tag{2.5}$$

[1]This notion can be motivated as follows. Let us denote by λ_i is the shadow price of abatement of pollutant i under any kind of regulation. Then, given E_j, the firm sets $-C_i(E_i, E_j) = \lambda_i$ (*). If E_j is relaxed, we see by implicitly differentiating (*) that $\partial E_i / \partial E_j = -C_{ij}/C_{ii}$ is positive for $C_{ij} < 0$ (complements) and negative for $C_{ij} > 0$ (substitutes) as intuition would suggest.

and by eliminating the time derivative of the costate variable ($\dot{\lambda}$) the maximum principle leads to the corresponding Ramsey conditions. They read

$$\dot{E}_1 = \frac{C_{22}(D_1' + C_1(\beta_1 + r)) - C_{12}(D_2' + C_2(\beta_2 + r))}{C_{11}C_{22} - [C_{12}]^2}, \qquad (2.6)$$
$$\dot{E}_2 = \frac{C_{11}(D_2' + C_2(\beta_2 + r)) - C_{21}(D_1' + C_1(\beta_1 + r))}{C_{11}C_{22} - [C_{12}]^2}.$$

Together with the dynamic constraints (2.1) these Ramsey conditions (2.6) form a system of four differential equations. To calculate the optimal paths explicitly, we additionally need the initial conditions (i.e., initial stocks) and the transversality conditions (i.e., $\lim_{T\to\infty} \lambda_i(T) = 0$) to rule out diverging paths. Note, that for additively separable costs, i.e. $C_{ij} = 0$ for $i \neq j$, we obtain the case of two separate single pollutants, and (2.6) turns into the standard textbook case.

The goal of this analysis is to highlight the different behavior of the optimal emission paths and of the shadow prices of pollution (which correspond to the optimal emission taxes in a first-best scenario) for the cases where emissions are either complements or substitutes. For our purposes it suffices to take an easy specification for both the damage and the cost functions which allows us to characterize pollutants as complements or substitutes but which keeps the system linear. Hence, we assume a quadratic damage function, such that

$$D_i(S_i) = (\alpha_i S_i)^2 \qquad (2.7)$$

for $i = 1, 2$ where α_i indicates the harmfulness of pollutant $= i$. Abatement costs are taken bi-quadratic:

$$C(E_1, E_2) = \frac{\eta_1}{2}(\bar{E}_1 - E_1)^2 + \frac{\eta_2}{2}(\bar{E}_2 - E_2)^2 + \omega(\bar{E}_1 - E_1)(\bar{E}_2 - E_2) \qquad (2.8)$$

for $E_i < E_i^{\max}$. Here $\bar{E}_i$ present the *laisser-faire* emission levels of the two pollutants.[2] The parameters η_i describe the marginal abatement cost (disregarding the other pollutant). The parameter ω is crucial and determines whether the pollutants are substitutes ($\omega > 0$) or complements ($\omega < 0$). To assure $\frac{\partial C}{\partial E_i} < 0$ we assume $|\omega| < \eta_i$ for $i = 1, 2$.

Note, that w.l.o.g. the marginal abatement costs for pollutant 1 are linear in E_1 but shifted upwards or downwards by a higher level of E_2. The direction is determined by the sign of ω (upwards for substitutes, downwards for complements). The size of the shift is determined by the absolute value of ω and by the current emission level E_2 of the other pollutant.

For our purposes it would be ideal to vary the substitutability in the cost function without changing the marginal abatement costs. But since substitutability relates to

[2]For the usual assumptions ($C' < 0, C'' > 0$) it is sufficient to set $E_1^{max} = \bar{E}_1 - |\frac{\omega}{\eta_1}|\bar{E}_2$ and $E_2^{max} = \bar{E}_2 - |\frac{\omega}{\eta_2}|\bar{E}_1$.

a curvature property it cannot be altered without changing the first derivative as well. A variation of the substitutability in our model will therefore always induce two effects on the emissions E_i:

(1) A level-effect, changing the overall emissions level due to the overall change in marginal abatement costs.
(2) A substitution-effect,changing the allocation of emissions between the two pollutants.

Accordingly, changing ω will shift the steady state in a twofold way.

3. Optimal Policy for Compliments and Substitutes

Since the system of differential equations to b analised is linear, the general solution can be written in terms of the steady state $(E_1^*, E_2^*, S_1^*, S_2^*)$ and a linear combination of the four eigenvectors $(\vec{v}_a, \ldots, \vec{v}_d)$:

$$\begin{pmatrix} E_1^*(t) \\ E_2^*(t) \\ S_1^*(t) \\ S_2^*(t) \end{pmatrix} = \begin{pmatrix} E_1^* \\ E_2^* \\ S_1^* \\ S_2^* \end{pmatrix} + c_a \vec{v}_a e^{\lambda_a t} + c_b \vec{v}_b e^{\lambda_b t} + c_c \vec{v}_c e^{\lambda_c t} + c_d \vec{v}_d e^{\lambda_d t}. \tag{3.1}$$

Here $\lambda_a, \ldots, \lambda_d$ are the eingenvalues corresponding to the the eigenvectors $\vec{v}_a, \ldots, \vec{v}_d$ with the same index. The coefficients $c_a, \ldots, c_d$ have to be determined by using the transversality condition, i.e. restricting to solutions which satisfy the initial conditions and converge towards the steady state. As it is usual for these differential equations (and as we will see later from equation (5.15) in Appendix A), two of the eigenvalues have a negative sign and two have a positive sign. W.l.o.g., let $\lambda_c > 0$ and $\lambda_d > 0$. Hence, the corresponding coefficients (c_c, c_d) have to be zero, otherwise these terms in equation (3.1) would diverge. Optimal motion *towards the steady state* can therefore be represented as[3]:

$$\begin{pmatrix} E_1^*(t) \\ E_2^*(t) \\ S_1^*(t) \\ S_2^*(t) \end{pmatrix} = \begin{pmatrix} E_1^* \\ E_2^* \\ S_1^* \\ S_2^* \end{pmatrix} + c_a \vec{v}_a e^{\lambda_a t} + c_b \vec{v}_b e^{\lambda_b t}, \tag{3.2}$$

where $\lambda_{a,b} < 0$. The two coefficients $c_{a,b}$ have to be chosen such that the equation matches the initial pollutant stocks at $t = 0$.

One can furthermore see from (3.2) that all optimal paths are located in a plane which is spanned by the two eigenvectors $\vec{v_a}$ and $\vec{v_b}$. This form also shows that it is possible to

[3]For further details on the role of eigenvalues and eigenvectors in optimal control see Chiang (1992) or Feichtinger and Hartl (1986).

represent the optimal paths as phase-space trajectories moving simultaneously along $\vec{v}_a$ and $\vec{v}_b$ with speeds λ_a and λ_b, respectively.

3.1. **Simulation Results for Complements and Substitutes.** Before deriving some general properties of the optimal emission paths, we present some numerical simulations. Since the system is autonomous, we can display the properties of the optimal paths by a phase space diagram. For each numerical simulation in our analysis we will have to choose specific parameter values. Therefore, it is helpful to first consider the following special case:

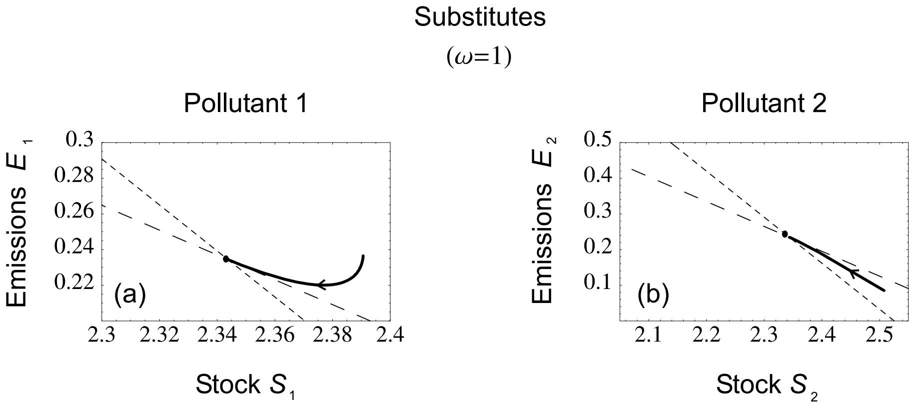

FIGURE 1. *Substitutes ($\omega = 1$): Optimal paths for a numerical example ($\eta = 2; \beta = 0.1; \alpha = 1; r = 0.06$) in $E_i - S_i$-phase diagrams. Initial stocks are chosen to be 2% (S_1) and 7% (S_2), respectively, above the steady state.*

Example 3.1. (1) The cost function is symmetric, i.e.

$$\eta_1 = \eta_2 = \eta \text{ and } \bar{E}_1 = \bar{E}_2 = \bar{E}. \tag{3.3}$$

(2) The two pollutants have identical decay rates, i.e.

$$\beta_1 = \beta_2 = \beta. \tag{3.4}$$

(3) The two pollutants are equally harmful, i.e. the damage functions are identical

$$\alpha_1 = \alpha_2 = \alpha. \tag{3.5}$$

Note, that these three conditions define a two-pollutant-problem where the pollutants are identical, but distinguishable. We are free yet to choose ω, which defines whether the pollutants are complements or substitutes.

For all our numerical simulations we will assume that (3.3) - (3.5) hold and we will set $\eta = 2, \beta = 0.1, \alpha = 1$, and $r = 0.06$. Figure (1) illustrates the optimal paths of the example for the case of substitutes ($\omega = 1$) whereas figure (2) displays those for complements ($\omega = -1$). In both scenarios initial stocks are chosen approximately two percent above the steady state level S_1^* for pollutant 1, and seven percent above the steady state level S_2^* for pollutant 2. The bold lines with an arrow show the phase-space trajectories towards the steady state. The dashed lines mark the (projections of the) eigenvectors where the long-dashed line corresponds to the small (slow) stable eigenvector while the short-dashed line corresponds to the larger (fast) stable eigenvector. Correspondingly, the trajectories show that in the neighborhood of the steady state the fast movement is basically completed and the trajectory is governed by the direction of the slow eigenvector. Obviously there are striking qualitative differences between optimal emissions over time in the two cases displayed in the Figures 1 and 2. These differences must be driven by the complementarity and substitutability properties, respectively, since the two pollutants are identical in all other aspects.

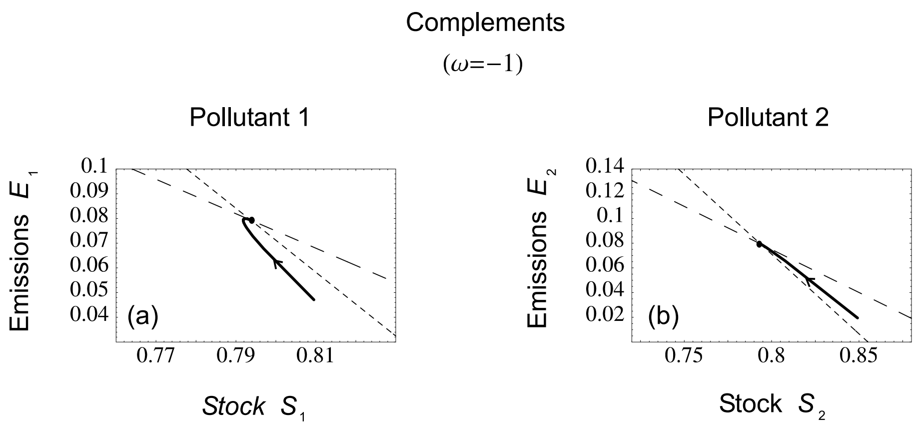

FIGURE 2. *Complements ($\omega = -1$): Optimal paths for a numerical example ($\eta = 2; \beta = 0.1; \alpha = 1; r = 0.06$) in $E_i - S_i$-phase diagrams. Initial stocks are chosen to be 2% (S_1) and 7% (S_2), respectively, above the steady state.*

For substitutes (figure 1) we see that initially there will be a higher level of abatement for the pollutant for which the original stock is higher (S_2) than for the other pollutant. As can be seen from the diagram for pollutant 1, this is done at the expense of initially higher emissions E_1 of pollutant 1. This reflects the substitutability. As the larger stock S_2 shrinks, abatement of this pollutant will slightly be relaxed while abatement of the other pollutant will be increased. Eventually emissions of both pollutants converge from below toward the steady state on very similar paths.

For complements (figure 2) we observe a different behavior: The emission paths of both pollutants start with relatively sharp abatement which will then be relaxed fairly quickly. In the long run the emissions E_1 converge towards the steady state from above while E_2 converges from below.

Note, that although in both cases the initial stocks of pollutant 1 and pollutant 2 are chosen by 2% and 7% *above* the steady state levels, respectively, emissions E_1 of pollutant 1 undershoot the steady state level for substitutes and overshoot that level for complements.

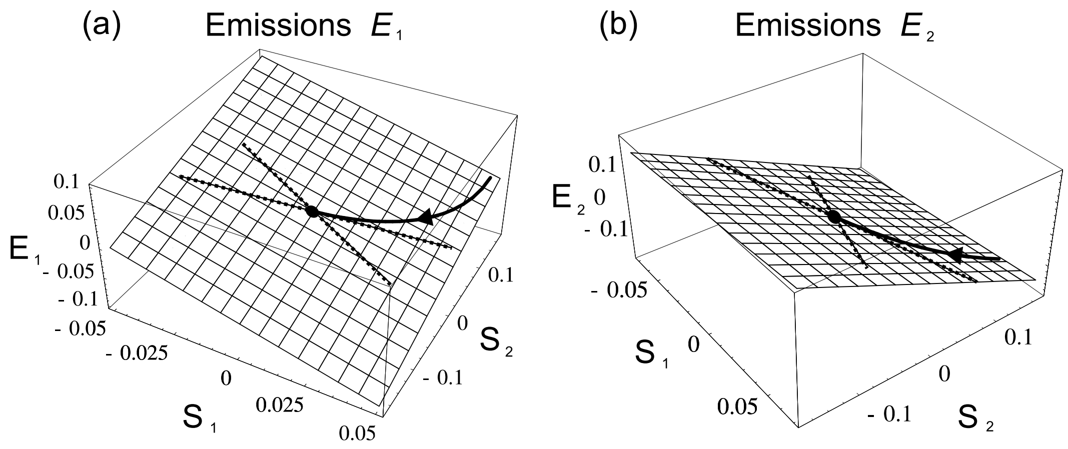

FIGURE 3. *Optimal path, eigenvectors and optimal policy plane for substitutes ($\omega = 1$). The scenarios are identical to those shown in the previous figures. The axes E_i, S_1, S_2 indicate the deviation from the steady state.*

Since the two-dimensional phase diagrams hide the impact of the second pollutant, it is useful to draw three-dimensional diagrams, plotting emissions against both stocks. We know from equation (3.1) that all optimal paths are located in a plane which is spanned by the eigenvectors $\vec{v}_a$ and $\vec{v}_b$. This plane - showing optimal emissions E_i - will be called the *optimal policy plane* for pollutant i. For each pair of stocks S_1 and S_2 the planes in (a) and (b) show the optimal emissions E_1 and E_2, respectively. It is important to notice that this does not only hold for the initial stocks but for the whole optimal path: For a given pair of initial stocks the optimal path is fully described by *(i)* the planes (in (a), (b) of figures 3 and 4) and *(ii)* by the dynamic constraints (for the motion on the planes). Note, that not every path one can draw on a policy plane is physically feasible, i.e. compatible with the dynamic constraints, but every physically feasible path that lies on the plane is optimal. This will be helpful later on for the implementation of the optimal path.

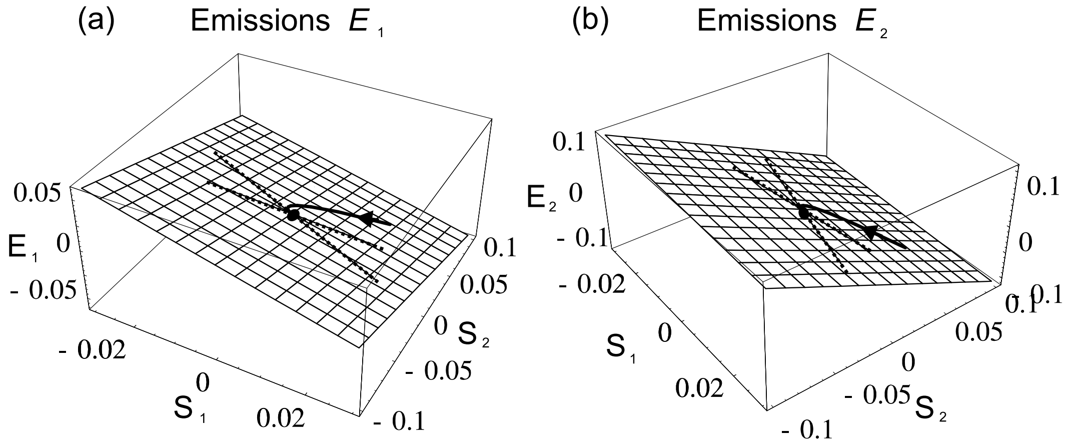

FIGURE 4. *Optimal path, eigenvectors and optimal policy plane for complements ($\omega = -1$). The scenarios are identical to those shown in the previous figures. The axes E_i, S_1, S_2 indicate the deviation from the steady state.*

Figures 3 (substitutes) and 4 (complements) display our scenarios from figures 1 and 2 in such a way. The axes do not explicitly show the values of E_i and S_i but rather their deviation from the steady state ($E_i - E_i^*$, $S_i - S_i^*$). In addition to the steady state (black dot), the optimal path (black line with arrow) and the eigenvectors (black dotted straight lines), which we know already from the previous figures 1 and 2, figures 3 and 4 also display the planes spanned by the eigenvectors $\vec{v}_a$ and $\vec{v}_b$ of equation (3.2). As we know, this is the plane that contains all the optimal paths (including the one that is drawn). Each optimal path is generated by a particular pair of initial conditions $(S_1(0), S_2(0))$. The three-dimensional plots also display the the optimal paths from our previous example.[4]

3.2. **Qualitative differences of optimal paths for complements and substitutes.** To derive some qualitative differences between optimal paths for complements and substitutes we now abstract from the specific paths and directly examine the policy planes and eigenvectors instead. In figure 5 we display the planes for both complements (grey) and substitutes (white) in one diagram for the emissions of pollutant 1 (figure 5 (a)) and pollutant 2 (figure 5 (b)).[5] These diagrams illustrate the difference between

[4]Note, that, e.g., figure 1 (a) is a projection of figure 3 (a) into the $E_1 - S_1$ plane.

[5]The stead state shifts with ω, but in this representation (deviation from steady state) the steady states for substitutes and complements coincide.

optimal policy for complements and substitutes. The planes are tilted in different directions intersecting along one steady state stock.

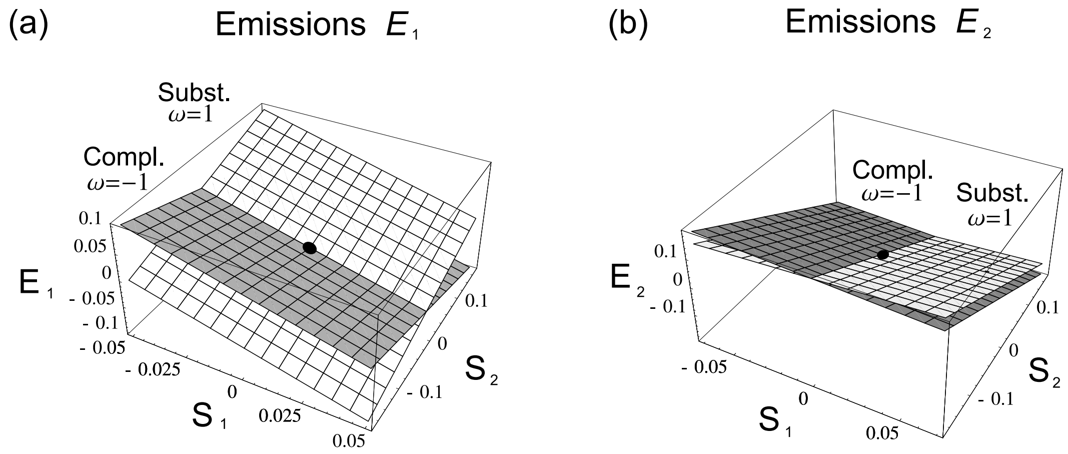

FIGURE 5. *Three-dimensional policy rules for pollutants which are substitutes ($\omega = 1$) or complements ($\omega = -1$) shown in one diagram for each pollutant.*

We are now ready to derive some general properties of the actual optimal paths by analyzing the planes and eigenvectors for complements and substitutes. In figure 5 we notice that the intersection line of the optimal emission planes for complements and substitutes in the emissions E_1-diagram is located at $S_2 = 0$. The two planes are mirror-images of each other. Correspondingly, the intersection line in the emissions E_2-diagram is located at $S_1 = 0$. In the appendix we show that this property also holds without the restrictions (3.3) to (3.5). This observation already characterizes an important difference between optimal emissions in case of substitutes and complements:

(1) If S_2 lies above the steady state, then E_1 should always be chosen *higher* for substitutes than it would be chosen if those pollutants were complements, regardless of S_1:
$$S_2(t) > S_2^* \rightarrow E_1(S_1, S_2, \omega = 1) > E_1(S_1, S_2, \omega = -1) \tag{3.6}$$
(2) If S_2 lies below the steady state, then E_1 should always be chosen *lower* for substitutes than it would be chosen if those pollutants were complements, regardless of S_1:
$$S_2(t) < S_2^* \rightarrow E_1(S_1, S_2, \omega = 1) < E_1(S_1, S_2, \omega = -1) \tag{3.7}$$

The corresponding rule also holds for E_2 and S_1. The larger the difference between the stocks and their steady state values, the larger the difference between optimal emissions if those are complements or substitutes, respectively, with respect to the abatement cost function.

To focus on the issue of complementarity and substitutability we will restrict the analytical studies to the case of symmetric abatement costs and decay rates. We will allow, however, for different harmfulness of the pollutants, by assuming (3.3) and (3.4) but not (3.5). Thus the values for α_i may be different. W.l.o.g. the second pollutant is more harmful than the first ($\alpha_1 = 1$, $\alpha_2 \geq 1$).

The steady state obviously depends on ω. However, enhancing ω (which also increases the overall abatement cost) does not necessarily increase the optimal steady state levels of both pollutants:

Proposition 3.2. Let (3.3) and (3.4) hold and assume (w.l.o.g.) $\alpha_2 \geq \alpha_1 = 1$. Then the following holds:

(1) If ω rises, both the optimal steady state emissions and the steady state pollution stock rise for the less harmful pollutant, i.e., $\frac{dS_1^*}{d\omega} > 0$ and $\frac{dE_1^*}{d\omega} > 0$.
(2) For the more harmful pollutant the corresponding effects are ambiguous, i.e., $\frac{dS_2^*}{d\omega} \gtrless 0$ and $\frac{dE_2^*}{d\omega} \gtrless 0$.

The proofs of all propositions can be found in the appendix. Referring to the terminology introduced in section 2 (overlap of a level-effect and a substitution-effect) proposition 3.2 can be interpreted as follows: For the less harmful pollutant the level- and substitution-effect go into the same direction. For the more harmful pollutant these effects are counteracting. Here, it is in general not possible to determine which effect is dominating.[6] Furthermore, one can show that the optimal paths need not to approach the steady state monotonically:

Proposition 3.3. Under assumptions (3.3) and (3.4) there exist initial stocks of pollution such that optimal emission paths do not behave monotonically.

This result contrasts with earlier findings on additively separable abatement costs and interaction of the pollutants in the damage function, where in case of identical decay rates all optimal emission paths behave monotonically over time (Moslener and Requate (2001)). Further analysis of both the eigenvalues and the eigenvectors leads to the following result:

Proposition 3.4. Let (3.3) and (3.4) hold. Further, let $\lambda_s = \min[|\lambda_a|, |\lambda_b|]$ denote the "slow" eigenvalue. Then for the corresponding eigenvector ($\vec{v}_s = (E_1^s, E_2^s, S_1^s, S_2^s)$), satisfying equation (3.2) we obtain the following property concerning its components:

[6]In fact, for most parameter values optimal stationary emissions will rise with ω. However, they might fall, if the pollutants are very close substitutes ($\omega \approx \eta$) and if the pollutant is comparatively more harmful than the other one (i.e. $\alpha_2 >> \alpha_1$).

(1) For $\omega > 0$, $\text{sign}(E_1^s) = \text{sign}(E_2^s)$, i.e., for substitutes either both emissions converge from above or both emissions converge from below towards the steady state.
(2) For $\omega < 0$, $\text{sign}(E_1^s) \neq \text{sign}(E_2^s)$, i.e., for complements optimal emissions of *one* pollutant converge from above, while optimal emissions of the *other* pollutant converge from below towards the steady state.

The idea of the proof of propositions 3.3 and 3.4, given in the appendix, is to show that in the representation of the optimal paths, given by equation 3.2, the E_1- and E_2-components of the "*fast*" eigenvector have opposite signs for substitutes and identical signs for complements. By contrast, the eigenvectors of the "*slow*" eigenvector have identical (opposite) signs.

Put differently, for substitutes where an increase of E_1 along the iso-cost line tends to lower E_2 the allocation of emissions between E_1 and E_2 (say, $\frac{E_2}{E_1}$) is comparatively more important and happens on a faster time scale (motion along an eigenvector where the E_1 and E_2 components have opposite signs) than the aggregate pollution. The aggregate level of emissions (along the eigenvector where the E_1 and E_2-components have identical signs) is comparatively less important and converges towards the steady state on a slower time scale. Since in the neighborhood of the steady state the motion of the slow time scale always dominates, E_1 and E_2 will eventually converge towards the steady state either both from below or both from above. (In figure 1 (substitutes) both converge from below.)

For complements, by contrast, where along the iso-cost line a decrease of E_1 tends to lower E_2 as well, we would intuitively expect E_1 and E_2 to behave very similar. This means that the aggregate level of emissions is more important and should be adjusted on a faster time scale than the allocation between E_1 and E_2. As a consequence, the emissions of one pollutant will eventually converge towards the steady state from below, while the other will come from above. (See, e.g., figure 2.) The following result tells us how ω influences this motion:

Proposition 3.5. Let assumptions (3.3) and (3.4) hold. Let $\lambda_{\text{f}} = max[|\lambda_a|, |\lambda_b|]$ denote the "fast" eigenvalue of (3.2). Then $\frac{\partial \lambda_{\text{f}}}{\partial |\omega|} > 0$. Therefore, a rising absolute value of ω induces the different optimal emissions to move faster in opposite (identical) directions if the pollutants are substitutes (complements).

A rising absolute value of ω means that the substitutability (or complementarity) is more pronounced. This implies a stronger (positive or negative) correlation between the two pollutants. Consequently, this increases the difference in the speed of the convergence processes described after proposition 3.4.

3.3. **Remarks on the general case.** Finally, some heuristic remarks on the general case without the symmetry restrictions (3.3) to (3.5) are in order: A higher harmfulness of pollutant i (larger α_i) leads to optimal emissions rules for E_1 and E_2 which are both steeper in the S_i-direction. (The planes in figure (5) would become steeper into the S_i-direction.) This is what one would expect since the stock S_i of a more harmful pollutant should be more important for the optimal emissions. The angle between the optimal-emission-planes of pollutant j for complements ($\omega = -1$) and substitutes ($\omega = 1$) is increased if the harmfulness of the *other* pollutant i rises (i.e., through an increase of α_i). That means a rising α_i inflates the difference between the optimal policy for pollutants being complements or substitutes. An increased ratio $\frac{\beta_i}{\beta_j}$ will have a similar effect on the policy rule. It means that pollutant i decays comparatively slower than pollutant j, therefore being more harmful for the environment. This will again cause an increased difference between $\omega = 1$ and $\omega = -1$ for the optimal emission level E_j.

We also refrain from pursuing a detailed analysis of the impact of the cost parameters. Intuitively one would expect that if abatement of, say pollutant 1 is comparatively costly (i.e. a higher η_1, other things equal), the optimal level of E_1 will vary less with S_1. In other words, for a pollutant with more costly abatement the influence of the pollutant stock is less strong (other things equal). This again feeds back into the optimal emissions rule for E_2 by making S_1 less important and therefore declining the difference between the optimal value of E_2 for complements and substitutes, respectively.

4. Implementation of the optimal policy

The optimal path, which can be non-monotonic, as formally described by (3.2) and as exemplified in figures 1 through 4 looks comparatively complex. Even if (as in our model) abatement cost and damage functions are perfectly known and both the steady state and the optimal emission path could be calculated ex ante, it is not obvious how such a path could be implemented.

As we have noted above, for each pair of initial stocks of the two pollutants, there is an optimal path, as displayed in figures 3 and 4. All these optimal paths are located in a 4-dimensional hyperplane. For each single pollutant $i = 1, 2$ we can draw a hyperplane in the $S_1/S_2/E_i-$space as we have done in figures 3 and 4. These two hyperplanes can also be considered as functions $F_i : S_1 \times S_2 \to E_i$ which assign the optimal emission level (flow) of pollutant i to each pair of pollution stocks. Note that once we are on that plane, the Ramsey conditions keep the emission path *on* the plane whereas the equations of motion for the pollution stocks (2.1) let the optimal path move *along* the plane.

Thus it is sufficient for the regulator to have at hand a pair of tables which represent the two hyperplanes F_i and which exactly assign the optimal emission levels to each pair of pollution stocks observed by the regulator. The regulator can then issue two quantities of permits which exactly correspond to those optimal emission levels. The permits can only be used in the same period, i.e. banking and borrowing should not be allowed in this framework of perfect information.[7] If the permit markets are competitive, the permit price will exactly reveal the optimal shadow price of pollution. If the regulator prefers a system of emission taxes instead of tradeable permits, similar tables can be set up for the optimal tax rates. In this way the regulator forces the emissions on the optimal path while the specific shape is determined by the dynamic nature of the pollutants.

Obviously establishing such tables in practice will not be as easy if the functional forms are not second order polynomials. However, since all differentiable functions can be approximated by a second order Taylor series around the steady state, the curvature properties are the same and the results remain valid. If one wants to deal with non-linear costs and damages explicitly (or with stock-dependent decay rates) it is possible to generalize the notion of the policy plane to the notion of a policy 2-manifold. In this case it would not be possible to represent the optimal paths in terms of the eigenvectors but explicit simulations would have to be performed and the hyperplane would turn into a bent 2-dimensional manifold. However, in the tables this would merely change the numbers but not the mechanism.

The problem becomes more difficult, of either the cost or the damage function are explicitly time dependent. This would generate a non-autonomous system of coupled differential equations which cannot be represented in the phase space, meaning that the policy planes would no more exist in such a simple form.

5. Conclusions and further research

We have set up a model for several accumulating pollutants where abatement of different pollutants (causing different externalities) does not happen independently. For illustrative purposes we have focused on the case of two pollutants for the most part of the paper. For a given (abatement) technology the pollutants can be either complements or substitutes. Examples for complements may inter alia be efficiency improvements, examples for substitutes may be end-of-pipe technologies which cause other emissions, such as CO_2 from energy use.

[7]Banking and borrowing could be useful in a framework of aggregate uncertainty. See Yates and Cronshaw (2001), or Phaneuf and Requate (2001).

For pollutants with symmetric abatement costs and identical decay rates optimal steady state emissions turned out to rise with the degree of substitutability for the less harmful pollutant while this effect was ambiguous for the more harmful one. Moreover, optimal paths for complements and substitutes show different behavior. This remains valid even when the pollutants and other model variables are highly symmetric.

In general our analysis has shown that environmental policy based on the isolated analysis of only one pollutant may be qualitatively misleading if other pollutants are involved.

Appendix: Proofs of the Propositions

Policy planes of complements and substitutes are mirror images. We will now show that the policy planes of complements and substitutes through the steady state are mirror images of each other with respect to the corresponding S_i-plane. The eigenvalue problem from equations (2.1) and (2.6) which yields the eigenvalues and eigenvectors spanning the policy planes reads

$$\Sigma(\omega) \cdot \begin{pmatrix} E_1 \\ E_2 \\ S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} \lambda E_1 \\ \lambda E_2 \\ \lambda S_1 \\ \lambda S_2 \end{pmatrix} \tag{5.1}$$

with the Jacobian $\Sigma(\omega)$ depending on ω:

$$\Sigma(\omega) =$$

$$\begin{pmatrix} [(\beta_1+r)\eta_1\eta_2 - (\beta_2+r)\omega^2] & \omega\eta_2((\beta_1+r)-(\beta_2+r)) & \eta_2 & -\alpha_2^2\omega \\ \omega\eta_1((\beta_2+r)-(\beta_1+r)) & [(\beta_2+r)\eta_1\eta_2 - (\beta_1+r)\omega^2] & -\omega & \alpha_2^2\eta_1 \\ 1 & 0 & -\beta_1 & 0 \\ 0 & 1 & 0 & -\beta_2 \end{pmatrix}. \tag{5.2}$$

To establish that the eigenvectors are transformed into their mirror images when moving from ω to $-\omega$ we need to show: (E_1, E_2, S_1, S_2) solves the eigenvalue equation for ω if $(E_1, -E_2, S_1, -S_2)$ solves it for $-\omega$ lst[8].

$$\Sigma(\omega) \cdot \begin{pmatrix} E_1 \\ E_2 \\ S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} \lambda E_1 \\ \lambda E_2 \\ \lambda S_1 \\ \lambda S_2 \end{pmatrix} \Leftrightarrow \Sigma(-\omega) \cdot \begin{pmatrix} E_1 \\ -E_2 \\ S_1 \\ -S_2 \end{pmatrix} = \begin{pmatrix} \lambda E_1 \\ -\lambda E_2 \\ \lambda S_1 \\ -\lambda S_2 \end{pmatrix} \tag{5.3}$$

[8]Eigenvectors are invariant with respect to their sign and multiplying by a scalar. It is therefore equivalent to have the minus-sign at E_2 and S_2 or at E_1 and S_1.

We now investigate what will happen to the Jacobian if ω changes sign: The parameter ω only occurs in the two upper rows. If it occurs only quadratic, its sign does not matter. Therefore, the sign only changes (i) in row 1 (column 2 and 4); (ii) in row 2 (columns 1 and 3). The equivalence (5.3) can be confirmed by simple calculation.

Proposition 3.2. Setting $E_i^* = S_i^* = 0$ the steady-state values turn out to be:

$$S_i^* = \frac{1}{det} \cdot e^{max}(r+\beta)\Big[\alpha_i^2 + \beta(r+\beta)(\eta-\omega)\Big](\eta-\omega) \tag{5.4}$$

where

$$det = \alpha_2^2 + \eta\beta(1+\alpha_2^2)(r+\beta) + \beta^2(r+\beta)^2(\eta^2+\omega^2). \tag{5.5}$$

By assumption $a_2 \geq a_1 = 1$ we show $\frac{\partial S_1^*}{\partial \omega} > 0$ and $\frac{\partial S_2^*}{\partial \omega}$ to be ambiguous in sign.

$$\frac{dS_1^*}{d\omega} = \frac{1}{det^2} \cdot e^{max}(r+\beta) \tag{5.6}$$

$$\cdot \big[\alpha_2^4 + \alpha_2^2\beta(r+\beta)(\eta+\alpha_2^2\eta-2\omega) + \beta^2(r+\beta)^2(\alpha_2^2(\eta^2+\omega^2)-2\eta\omega)\big] \tag{5.7}$$

The denominator is clearly positive. Recalling $|\eta| > |\omega|$ we see that the numerator is positive as well, since $\eta + \alpha_2^2\eta - 2\omega \geq 2\eta - 2\omega > 0$, and $\alpha_2^2(\eta^2+\omega^2) - 2\eta\omega > \eta^2 - 2\eta\omega + \omega^2 = (\eta-\omega)^2 > 0$.

$$\frac{dS_2^*}{d\omega} = \frac{1}{det^2} \cdot e^{max}(r+\beta) \tag{5.8}$$

$$\cdot \big[\alpha_2^2 + \beta(r+\beta)(\eta+\alpha_2^2\eta-2\alpha_2^2\omega) + \beta^2(r+\beta)^2(\eta^2+\omega^2-2\alpha_2^2\eta\omega)\big] \tag{5.9}$$

It is easy to confirm that $\frac{\partial S_2^*}{\partial \omega}$ may be positive or negative: Setting $\omega = 0$, we see that the enumerator is positive. Setting $\omega \approx \eta$, we see that α_2 can be chosen large enough to lead to a negative enumerator.

The latter proves the proposition and shows that for good substitutes we tend to have falling stationary state emissions of the more harmul pollutant when ω rises.

q.e.d.

Proposition 3.3. Recall that the optimal path into the steady state can be described by motion along different directions (eigenvectors) with different characteristic speeds (eigenvalues). It is therefore sufficient to show that there exists no eigenvector which has vanishing components in the E_i-directions, i.e. $E_1 = E_2 = 0$. This can be done indirectly by making use of the eigenvalue-equations. (In principle it is also necessary to show that the eigenvalues are different, but this can directly be seen from their expressions which are explicitly displayed in the proof of proposition 3.) The eigenvalue problem is defined by

$$\mathbf{\Sigma} \cdot \begin{pmatrix} E_1 \\ E_2 \\ S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} \lambda E_1 \\ \lambda E_2 \\ \lambda S_1 \\ \lambda S =_2 \end{pmatrix} \tag{5.10}$$

where

$$\mathbf{\Sigma} = \begin{pmatrix} [(\beta_1 + r)\eta_1\eta_2 - (\beta_2 + r)\omega^2] & \omega\eta_2((\beta_1 + r) - (\beta_2 + r)) & \eta_2 & -\alpha_2^2\omega \\ \omega\eta_1((\beta_2 + r) - (\beta_1 + r)) & [(\beta_2 + r)\eta_1\eta_2 - (\beta_1 + r)\omega^2] & -\omega & \alpha_2^2\eta_1 \\ 1 & 0 & -\beta_1 & 0 \\ 0 & 1 & 0 & -\beta_2 \end{pmatrix} \tag{5.11}$$

Setting $E_1 = E_2 = 0$ would yield

$$\lambda = -\beta \tag{5.12}$$

$$\eta S_1 - \alpha_2^2\omega S_2 = 0 \tag{5.13}$$

$$-\omega S_1 + \alpha_2^2\eta S_2 = 0. \tag{5.14}$$

This implies $\omega = \eta$ which is in contradiction to the assumption of our cost function.

To construct a non-monotonic emission path one just has to choose the initial conditions in such a way that the optimal emission path first moves upwards along the eigenvector with the "fast" eigenvalue and downwards along the eigenvector with the "slow" eigenvalue.

q.e.d.

Proposition 3.4. We will show that for complements ($\omega > 0$) the negative eigenvalue of an eigenvector which has positive E_1 and E_2-components has a smaller absolute value than the one that corresponds to the eigenvector with different signs of E_1, E_2.

All four eigenvalues of the problem (5.1) can be represented as follows:

$$\lambda_{a,b,c,d} = \frac{1}{2}\left(r \pm \frac{\sqrt{(\eta^2-\omega^2)\left((r+2\beta)^2(\eta^2-\omega^2)+2\left(\eta(1+\alpha^2)\pm\sqrt{(\alpha^2-1)^2\eta^2+4\alpha^2\omega^2}\right)\right)}}{\eta^2-\omega^2} \right). \tag{5.15}$$

The eigenvectors of the stable branches are the negative ones, meaning that the first ($\pm$) (before the outer squareroot) has to be ($-$) for the stable branches. Instead of the indices a and b we will use s and f to indicate the eigenvalues belonging to the *slow* and *fast* motion.

The negative eigenvalues for the symmetric case are therefore:

$$\lambda_{f,s} = \frac{1}{2}\left(r - \frac{\sqrt{(\eta^2-\omega^2)\left((r+2\beta)^2(\eta^2-\omega^2)+2\left(\eta(1+\alpha^2)\pm\sqrt{(\alpha^2-1)^2\eta^2+4\alpha^2\omega^2}\right)\right)}}{\eta^2-\omega^2} \right). \tag{5.16}$$

with (+) for the fast and (−) for the slow motion. The corresponding eigenvectors have the components

$$E_1 = 1 \tag{5.17}$$

$$E_{2f,s} = \frac{\eta(1-\alpha^2) \mp \sqrt{\eta^2(\alpha^2-1)^2 + 4\alpha^2\omega^2}}{2\alpha^2\omega} \tag{5.18}$$

where we have chosen E_1 and calculated E_2 accordingly[9]. Here (−) represents the fast and (+) the slow motion. For $\omega > 0$ it follows that $E_2 < 0$ for the fast motion and $E_2 > 0$ for the slow motion. For $\omega < 0$ the signs are reversed. This proves the proposition.

q.e.d.

Proposition 3.5. Analyzing the comparative dynamics of $|\lambda_f|$ we obtain:

$$\frac{d\lambda_f}{d\omega} = \frac{1}{B(>0)} \cdot (-\omega)\left[(1+\alpha^4)\eta^2 + 2\alpha^2\omega^2 + (1+\alpha^2)\eta\sqrt{(\alpha^2-1)^2\eta^2 + 4\alpha^2\omega^2}\right]. \tag{5.19}$$

Where B is a lengthy but clearly positive expression. It is therefore irrelevant for the sign of $\frac{d\lambda_f}{d\omega}$ and we just show it for the sake of completeness:

$$\begin{aligned} B = \quad & (\eta^2-\omega^2)\sqrt{(\alpha^2-1)^2\eta^2+4\alpha^2\omega^2} \ \times \\ & \sqrt{(\eta^2-\omega^2)\left((r+2\beta)^2(\eta^2-\omega^2) + 2(\eta(1+\alpha^2) + \sqrt{(\alpha^2-1)^2\eta^2+4\alpha^2\omega^2}\right)}. \end{aligned}$$

For $\omega > 0$ we see $\frac{d\lambda_f}{d\omega} < 0$, and for $\omega < 0$ we can confirm $\frac{d\lambda_f}{d\omega} > 0$. Thus in both cases an increasing $|\omega|$ increases $|\lambda_f|$.

q.e.d.

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[9]This is always possible by renormalization of the eigenvector.

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