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Modelling the Fisher hypothesis: World wide evidence

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Abstract

In this paper we follow an empirical approach to examine the implications of the Fisher hypothesis, namely cointegration linking interest rates and inflation, and stationarity of the real interest rate implying in turn homogeneity of the potential equilibrium relation. The considered sample is an unbalanced panel and comprises monthly time series data from more than 100 economies covering at most a period of about 45 years. In total more than 31000 observations enter our empirical analysis. From cross sectional error correction and dynamic OLS regressions we find that the presumed dynamic relation is hardly homogeneous over the cross section. Therefore, building on cross sectional parameter homogeneity nonstationary panel data models are provided merely as a complement to cross section specific analyses. Apart from standard between regressions we exploit the cross section dimension to infer on parameter homogeneity over particular economic states. For this purpose we rely on semiparametric implementations of so-called functional coefficient models. The latter are suitable to relate key model parameters on economic states, as e.g. periods of higher vs. lower inflation or inflation risk. From the latter approach we find that time or state invariance of key model parameters is not supported empirically. Moreover the evidence in favor of cointegration is weak over periods of high inflation. The Fisher coefficient turns out to be remarkably stable and is, over most considered states, significantly less than unity.

Keywords: Fisher hypothesis; Panel cointegration analysis; Functional coefficient models;

JEL Classification: C32,C33,E40

1 Introduction

The relationship between interest rates and inflation has been frequently investigated in both theoretical and empirical economics. Fisher (1930) formalized a model according to which nominal interest rates respond one-to-one to expected changes of the price level. The latter relationship is typically referred to as the Fisher hypothesis. The Fisher coefficient may differ from unity when interest income is subjected to taxation (Crowder and Hoffman 1996). Moreover, in case of accelerating inflation the basic link between interest rates and inflation might be disturbed by agents shifting out of nominal assets and acquiring real assets (Tobin 1965, 1969). Lucas (1980) points out, however, that under the postulate of long run money neutrality the so-called Tobin effect is a short term phenomenon. In this scenario the interest rate elasticity of money holdings is zero and the quantity theory of money holds. Recent contributions to model monetary policy via policy rules (Taylor 1993, Woodford 2003) could also imply that the Fisher coefficient differs from unity.

Empirical studies for the US by Fama (1975) and Fama and Schwert (1977) find evidence in favor of approximately constant real interest rates as implied by the Fisher hypothesis. In contrast, Summers (1983) rejects the Fisher hypothesis for the period before WWII. Beginning in the 1990s the Fisher hypothesis has undergone empirical tests that take the potential nonstationarity and cointegration of the involved time series explicitly into account (Mishkin 1992). Finding evidence in favor of the postulated cointegration relationship for the US Evans and Lewis (1995) document a long-run coefficient less than unity whereas a coefficient estimate exceeding unity is found by Crowder and Hoffman (1996).

When analyzing a set of macroeconomies pooling is a promising device to improve the efficiency of statistical procedures, and, thus, panel cointegration techniques are a natural means to uncover links between interest rates and inflation over a cross section. Panel cointegration methods have been attracting a huge interest in the recent econometric literature (Banerjee 1999). Phillips and Moon (1999) underscore that for panel data models typical challenges posed by the statistical analysis of nonstationary processes, as e.g. spurious regression, may be addressed by exploiting the variation of parameter estimates over the cross section. In fact, under sufficient parameter homogeneity holding over the cross section, country specific estimates of model parameters can be assumed to stem from the same distribution. Having such an intermediate result, then, asymptotic normality of average parameter estimates is easily established.

From the perspective outlined in Phillips and Moon (1999) the adoption of panel data econometrics might be regarded as particularly fruitful for empirical work if an underlying economic theory implies specific and unique parametric restrictions. For instance, under purchasing power parity (PPP) the postulated stationarity of real exchange rates formalizes a particular cointegrating relationship linking a nominal exchange rate and the domestic and foreign price level. If, however, economic theory is in line with a range of admissible parameters the a-priori merits of a panel approach are less clear. As outlined above for the relation between nominal interest rates and inflation the current state of economic theory is not conclusive with respect to the size of the Fisher coefficient. Thus, the intrinsic homogeneity assumption made for panel data methods may be violated. In addition to the case of cross sectional heterogeneity it is worthwhile to point out that also in the time dimension the homogeneity assumption is rather strong if an empirical model is thought to approximate economic behavior over a long sample period. In the sequel of changing macroeconomic policies institutional changes may involve time variation of the parameters of an econometric model. In the light of the theory on the relation between interest rates and inflation, for instance, one may expect that the parameters of the Fisher equation vary according to intertemporal changes of tax regulations.

Another difficulty of macroeconomic panel data models is that it is often not clear how results from specification tests have to be interpreted. Consider, for instance, the IPS-statistic (Im, Pesaran and Shin 2002) to test the null hypothesis of a unit root. Here the conclusions from the test are only clear under the null hypothesis since the procedure allows heterogeneity under the alternative. In the extreme case a rejection of the unit root hypothesis might go back to one stationary series entering the cross section. At the other end, statistics building upon homogeneity under both the null and the alternative hypothesis, as e.g. the LLC test (Levin, Lin and Chu 2002), face the risk of power loss in case of heterogeneity under the alternative hypothesis. Similar arguments apply to cointegration testing via a panel approach. Note that the latter arguments might become quite important when analyzing a large cross section such that cross sectional homogeneity is likely exceptional.

In this paper we will investigate the relation between nominal interest and inflation rates for an unbalanced cross section including the majority of the world's economies. Proceeding from a conjecture that cross sectional homogeneity of the Fisher relation hardly holds for the considered sample our empirical analysis will mainly rely on country specific regressions. Panel data methods are employed complementary. Moreover, adopting functional coefficient models (Cai, Fan and Yao 2000) we will allow for time variation by relating the parameters of interest to country and time specific factors describing the state of the economy.

Our empirical analysis obtains that interest rates and inflation are likely cointegrated over most considered conditioning scenarios. The key parameters of the econometric model seem to depend on economic conditions as, for instance, the level of inflation. Particular states, as periods of extreme inflation or interest rate levels, however, may disconnect interest rates and price changes. Moreover, our results show that from a world wide perspective the (average) Fisher coefficient is less than unity.

The remainder of this paper is organized as follows: In the next Section we will provide the economic model along with its common econometric implementation. In Section 3 the data used for the empirical analysis are described and empirical results from standard single equation and panel data econometrics will be given. Functional coefficient models are in the focus of Section 4 which also gives a detailed motivation for our approach. In addition, Section 4 provides a discussion of specification issues and briefly sketches the implementation of the semiparametric model. Empirical results obtained from the latter approach are discussed in Section 5. Section 6 summarizes our main findings and concludes.

2 The economic framework and parametric methods

This section will first briefly sketch the basic model linking nominal interest rates and inflation in a one-to-one manner. In the second place more general frameworks are mentioned that allow for deviations of the Fisher coefficient from unity. Thirdly, the econometric model is given and the applied methods to estimate the parameters of interest are noted. Finally, the section will provide a condensed review of recent empirical literature on the Fisher hypothesis.

2.1 A basic derivation of the Fisher hypothesis

According to the Fisher equation the nominal interest rate in time t (R_t) is composed of the ex-ante real interest rate ($E_{t-1}[r_t]$) and the expected inflation rate ($E_{t-1}[\pi_t]$) (Mishkin 2003), i.e.

$$R_t = E_{t-1}[r_t] + E_{t-1}[\pi_t] + u_t, \tag{1}$$

where $E_t[\bullet]$ denotes the conditional expectations operator. Following Rose (1988) it is assumed that under rational expectations the expected and the actual inflation rate differ by a stationary, zero mean forecast error v_{1t} obtaining

$$\pi_t = E_{t-1}[\pi_t] + v_{1t} \,. \tag{2}$$

and, similarly, the ex-post real interest rate is the sum of the ex-ante real rate and a forecast error v_{2t} such that

$$r_t = E_{t-1}[r_t] + v_{2t} \,. \tag{3}$$

The inflation rate and nominal interest rate are observable. Thus, the ex-post real interest rate is

$$r_t = R_t - \pi_t + v_t^{(1)}, \tag{4}$$

where $v_t^{(1)} = u_t - v_{1t} - v_{2t}$. The last equation provides a basis for the econometric strategy to test the Fisher hypothesis as outlined e.g. in Rapach and Weber (2004). Assuming $v_t^{(1)}$ to be stationary the integration properties of r_t are determined by the integration properties of R_t and π_t. If the latter variables are both stationary (R_t, $\pi_t \sim I(0)$), then $r_t \sim I(0)$. In case one variable is nonstationary and the other variable is stationary, the real rate is nonstationary ($r_t \sim I(1)$). If both variables are nonstationary (R_t, $\pi_t \sim I(1)$) and the linear combination $R_t - \pi_t$ is stationary the Fisher hypothesis implies a $(1, -1)'$ cointegrating vector linking interest rates and inflation.

2.2 Extensions of the basic model

Generalizing the basic formulation in (4) Crowder (2003) departs from a standard asset pricing model where both real and nominal default free bonds are traded. Then, intertemporal optimization yields the following Euler equations:

$$U'(C_t) = (1+\rho)^{-1}(1+r_t)E_t[U'(C_{t+1})], \tag{5}$$

$$\frac{U'(C_t)}{P_t} = (1+\rho)^{-1}(1+r_t)E_t\left[\frac{U'(C_{t+1})}{P_{t+1}}\right], \tag{6}$$

where $U'(C_t)$ is the marginal utility of consumption and P_t is the price of the consumption bundle. Noting that $E_t[XY] = E_t[X]E_t[Y] + \text{Cov}_t[X, Y]$, where Cov_t is the conditional covariance operator, some algebra obtains

$$\frac{1}{1+R_t} = \frac{1}{1+r_t}E_t\left[\frac{P_t}{P_{t+1}}\right] + (1+\rho)^{-1}\text{Cov}_t\left[\frac{U'(C_{t+1})}{U'(C_t)}, \frac{P_t}{P_{t+1}}\right]. \tag{7}$$

Equation (7) implies a nonlinear relation between inflation and interest rates. The conditional covariance of consumption growth and inflation may be interpreted as a risk premium. Moreover,

(7) formalizes time varying real interest rates. If preferences are characterized by hyperbolic absolute risk aversion and, in addition, consumption growth and inflation are jointly log-normally distributed, then equation (7) simplifies to

$$R_t = r_t + E_t[\pi_{t+1}] + 0.5\text{Var}_t[\pi_{t+1}] - \psi\text{Cov}_t[\Delta c_{t+1}, \pi_{t+1}], \tag{8}$$

where c_t is log consumption, $\pi_{t+1} = \ln P_{t+1} - \ln P_t$ and ψ is the coefficient of relative risk aversion. Subjecting nominal interest to income taxation equation (8) changes to:

$$R_t(1-\tau) = r_t + E_t[\pi]_{t+1} + 0.5\text{Var}_t[\pi_{t+1}] - \psi\text{Cov}_t[\Delta c_{t+1}, \pi_{t+1}], \tag{9}$$

with τ denoting the marginal tax rate. The after tax nominal interest rate is positively related to the real rate and the expected inflation rate. According to Crowder and Hoffman (1996) the tax adjusted observable Fisher equation is

$$R_t = \frac{\pi_{t+1} + \rho - \zeta}{(1-\tau)} + v_t^{(2)}. \tag{10}$$

In (10) ρ denotes the mean of the real short term interest rate plus one-half the conditional variance of inflation, $\rho = \bar{r} + 0.5\text{Var}_t[\pi_{t+1}]$, ζ is the mean of the risk premium, and $v_t^{(2)}$ is composed of stationary stochastic terms including rational expectations forecast errors as well as approximation errors involved when evaluating the conditional variance $\text{Var}_t[\pi_{t+1}]$. If $\rho \neq \zeta$ the empirical implementation of equation (10) will require an intercept term, i.e.

$$R_t = \theta\pi_{t+1} + \frac{\rho - \zeta}{(1-\tau)} + v_t^{(2)},\ \theta = \frac{1}{1-\tau}. \tag{11}$$

Also arguing against an one-to-one linkage between inflation and interest rates Mishkin (1992) relates the Fisher coefficient θ to second order moments of ex-ante inflation and real interest rates:

$$\theta = \frac{\tilde{\sigma}^2 + \vartheta\tilde{\sigma}}{1 + \tilde{\sigma}^2 + 2\vartheta\tilde{\sigma}}. \tag{12}$$

In (12) $\tilde{\sigma}$ is the ratio of the unconditional standard deviation of expected inflation to the unconditional standard deviation of the real interest rate. Similarly, ϑ is short for the unconditional correlation coefficient between the expected inflation and the real interest rate. Equation (12) indicates that θ is determined by the uncertainty associated with future inflation relative to the variation in the real interest rate. Mishkin (1992) illustrates if the variability of expected inflation exceeds real interest rate variation θ is larger than 0.5 and, generally, increases with $\tilde{\sigma}$. In case inflation is nonstationary its standard deviation and, thus, θ will grow with the sample size under the presumption of an existing Fisher effect, i.e. stationarity of the real rate. In contrast, if inflation and interest rates do not have trends, we might expect $\tilde{\sigma}$ to be less than unity and θ closer to zero.

Apart from accounting for tax effects or second order properties, the Fisher hypothesis of an one-to-one relation between nominal interest and inflation rates may be criticized in the light of recent approaches to model the monetary policy of central banks by means of policy rules (Clarida, Galí and Gertler 1999). Here, the macroeconomic framework is characterized by an aggregate demand and supply equation (IS curve and Phillips curve, respectively) and policy rules determining the short term interest rate. For instance, the latter is formalized to depend on the long-run equilibrium interest rate, deviations of expected inflation from some target level and the output gap (Taylor 1993). In an alternative setting the interest rate depends mainly on deviations of inflation from some target level π_t^*,

$$R_t = \phi(\pi_t/\pi_t^*; \mu_t),$$

with μ_t denoting exogenous shifts (Woodford 2003). In a steady state situation, where both π_t and R_t fluctuate within a neighborhood of their steady state a log-linear approximation yields

$$\hat{R}_t = \phi_\pi(\ln(\pi_t) - \ln(\pi_t^*)) + \mu_t, \tag{13}$$

where ϕ_π is the elasticity of the interest rate with respect to inflation evaluated at the steady state. Measuring the total exogenous shift in the central bank's reaction function as $\tilde{R}_t = \mu_t - \phi_\pi \ln(\pi_t^*)$ the model in (13) may be reorganized to obtain

$$\hat{R}_t = \tilde{R}_t + \phi_\pi \ln(\pi_t).$$

For this setting Woodford (2003) shows that to obtain a determinate solution it is necessary that the coefficient of the inflation gap in the policy rule exceeds unity. Woodford (2003) refers to this result as the Taylor principle, since Taylor (1993) states an inflation gap coefficient of 1.5.

2.3 The econometric model

To investigate the Fisher relation over a cross section indexed with $i = 1, \ldots, N$ it is appropriate to start from a bivariate vector error correction model (VECM)

$$\Delta \mathbf{y}_{it} = \nu_i + F_i \mathbf{y}_{it-1} + \Gamma_{i1} \Delta \mathbf{y}_{it-1} + \ldots + \Gamma_{ip} \Delta \mathbf{y}_{it-p} + e_{it},\ t = 1, \ldots, T, \tag{14}$$

where $\mathbf{y}_{it} = (R_{it}, \pi_{it})'$ collects observations of short term nominal interest rates and inflation for country i in time t. Note that the vector of intercept parameters ν_i, the (2×2) dimensional parameter matrices governing short run dynamics $\Gamma_{ik}, k = 1, \ldots, p$, and the matrix F_i are allowed to vary over the cross section. For convenience of model representation and implementation we assume that presample values required to implement the VECM in (14) are available and that the autoregressive order p is unique over the cross section. In our empirical analysis of monthly data we will use $p = 2$ throughout. According to diagnostic statistics for some (randomly) considered members of the cross section we regard this particular model order as suitable and refrain from a cross section specific selection of the order parameter. For the particular issue of unit root testing it turned out that doubling the lag order leaves almost all conclusions on the features of the data unchanged. By assumption, e_{it} is a serially uncorrelated residual vector with mean zero and covariance matrix Ω_i. In case the variables in $\mathbf{y}_{it}$ are integrated of order 1 and cointegrated with cointegration rank $r = 1$ the matrix F_i allows a factorization as $F_i = a_i b_i'$, where both a_i and b_i are 2×1 vectors.

Instead of a bivariate analysis we will consider two univariate empirical models that can be derived from the VECM in (14). For this purpose we denote the typical elements in $\mathbf{y}_{it}$ as y_{it} and z_{it} and normalize b_i with respect to its first element. In the first place we regard the interest rate or, respectively, its changes as the left hand side variable (i.e. $y_{it} = R_{it}$, $z_{it} = \pi_{it}$) thereby providing a common analysis of the Fisher relation by means of the following single equation ECM:

$$\Delta y_{it} = c_i^{(1)} + \alpha_i^{(1)}(y_{it-1} - \theta_i^{(1)} z_{it-1}) + \sum_{k=1}^{2} \gamma_{ik}^{(1)} \Delta z_{it-k} + \sum_{k=1}^{2} \delta_{ik}^{(1)} \Delta y_{it-k} + e_{it}^{(1)}. \tag{15}$$

Moreover, as a second set of empirical models we consider specifications where changes of inflation serve as the left hand side variable. To model the so-called 'inverted Fisher relation' we denote in the general VECM (14) $y_{it} = \pi_{it}$, $z_{it} = R_{it}$. According to Ng and Perron (1997) the latter regression is preferable in terms of a reduced small sample bias for those members of the cross section where the long run variation of interest rates exceeds the corresponding feature of inflation. In analogy to (15) the typical parameters describing the latter ECM are $c_i^{(2)}$, $\alpha_i^{(2)}$, $\theta_i^{(2)}$, $\gamma_{ik}^{(2)}$ and $\delta_{ik}^{(2)}$.

In principle, the ECM in (15) may be used to estimate both the error correction parameter $\alpha_i^{(\bullet)}$ and the long run coefficient $\theta_i^{(\bullet)}$. Since the latter estimate $(\hat{\theta}_i^{(\bullet)})$ is the ratio of two OLS estimators one may a-priori expect a few outlying estimates to be obtained over the cross section.

For the latter reason we will prefer the dynamic OLS (DOLS) estimator introduced by Saikkonen (1991) and Stock and Watson (1993), which is obtained from applying OLS to the typical regression

$$y_{it} = c_i^{(\bullet)} + \theta_i^{(\bullet)} z_{it} + \sum_{k=1}^{2} (\gamma_{ik_-}^{(\bullet)} \Delta z_{it-k} + \gamma_{ik_+}^{(\bullet)} \Delta z_{it+k}) + u_{it}^{(\bullet)}. \tag{16}$$

In (16) the error term $u_{it}^{(\bullet)}$ is by construction independent from Δz_{it-k} and Δz_{it+k}, $k = 1, 2$. Moreover, it is assumed that $u_{it}^{(\bullet)}$ is Gaussian and stationary (Stock and Watson 1993).

2.4 Empirical evidence

As already indicated above, the empirical results on the Fisher relation are at least ambiguous. Mishkin (1992) studies the relationship between the level of short-term interest rates and future inflation for the US over the period January 1953 until December 1990. He finds that a short-run relationship linking changes in expected inflation and interest rates does not exist. A long-run relation between interest rates and inflation is diagnosed in periods during which these variables exhibit trending behavior. Over such states cointegration is confirmed. In periods where interest rates and inflation do not show a trending behavior the long run linkage cannot be detected. The latter findings are supported for the case of Australia by Mishkin and Simon (1995). It is noteworthy that the results in Mishkin (1992) and Mishkin and Simon (1995) indicate that central conclusions concerning the long run behavior of interest rates and inflation might be state specific. As mentioned, Evans and Lewis (1995) provide evidence for a Fisher coefficient less than one for the US whereas Crowder and Hoffman (1996) estimate a coefficient exceeding unity.

Investigating a cross section of 18 industrialized countries Rose (1988) documents for quarterly post WWII nominal interest and inflation rates that $R_t \sim I(1)$ and $\pi_t \sim I(0)$ for each country and, hence, concludes that real interest rates are nonstationary. To explain the stochastic development of real interest rates Rose (1988) refers to the consumption based capital asset pricing model (CCAPM) which were in line with nonstationary real interest rates if also consumption growth would show a stochastic trend. Nonstationarity of consumption growth rates, however, lacks any empirical support. The empirical evidence in Rose (1988) is supported by Koustas and Serletis (1999) investigating 11 industrialized economies. Driffill and Snell (2003) examine important factors, which affect the development of real interest rates. Building on a 'cash in advance' assumption they formalize a real business cycle type of model where the effects of nominal and real shocks can be distinguished. Analyzing the time series properties of real short term interest rates for five major OECD members over the period 1957 to 1994 Driffill and Snell (2003) infer that four of five real interest rate series were nonstationary.

In contrast, Rapach and Weber (2004) reexamine the integration properties of postwar nominal long-run interest and inflation rates for 16 OECD countries by means of unit root tests recently introduced by Ng and Perron (2001). They support the view that both the nominal interest and the inflation rate are $I(1)$. With regard to cointegration, however, the documented evidence is not unique over the cross section. Crowder (2003) investigates the orders of integration of short term interest and inflation rates for nine industrial countries by means of panel unit root tests and concludes that both variables exhibit stochastic trends. Employing alternative deterministic specifications of the cointegration relation Crowder (2003) finds that for 80% of the empirical models the hypothesis of a unit Fisher coefficient cannot be rejected. For pooled regressions, however, the Fisher coefficient is found to differ significantly from unity in most cases. Using ECM based panel cointegration tests Westerlund (2005a) provides evidence in favor of the Fisher hypothesis for 14 OECD economies over a sample covering the period 1980:1 to 1999:12. On the pooled level the Fisher coefficient is not significantly different from unity (Westerlund 2005a Table 6).

3 Data and first empirical results

Data are taken from the International Financial Statistics of the International Monetary Fund (IMF). Time series are sampled at the monthly frequency and span at most a period from 1960:1 to 2004:6. The sample period of more than four decades covers different regimes and paradigms under which monetary policy took place. It starts in the Bretton Woods era during which the Federal Reserve targeted money market conditions (Mishkin 2003, Chapters 18 and 20). Relying on a presumably stable money demand function the Fed switched in the 1970s to targeting monetary aggregates. In the sequel of oil price shocks in the 1970s inflation rates accelerated world wide. Owing to factual instability of money demand functions the Fed weakened its focus on monetary aggregates in the 1980s and early 1990s. In the 1990s inflation targeting became world-wide an important and often successful strategy of monetary policy, resulting in a deceleration of inflation rates.

The annual inflation rate is determined from the national consumer price indices (P_{it}) as

$\pi_{it} = 100(\ln P_{it} - \ln P_{it-12})$. To measure the level of short term nominal interest rates we mostly use money market rates. In case the latter are not available for some member of the cross section we sample either the treasury bill or the discount rate. Missing values in a series are substituted by linearly interpolated data. The sample includes time series from 114 economies. A list encountering the countries under consideration is given the Appendix.

Mostly providing results for cross section specific regression models the remainder of this section will address the issue of cross sectional parameter heterogeneity vs. homogeneity with the latter often motivating a panel approach. We begin with a brief descriptive analysis contrasting average interest and inflation rates evaluated over the cross section. Then, we report on the outcomes of ADF regressions to infer on stochastic trends governing the variables of interest. Single equation error correction models as (15) will be informative for typical modeling steps involved in cointegration analysis as e.g. testing for cointegration or weak exogeneity. Results from DOLS regressions formalized in (16) are used to characterize the cross sectional pattern of the long run coefficient in both the Fisher and the inverted Fisher equation. After the discussion of country specific empirical models we will provide some complementary results from panel unit root and cointegration tests.

Table 1: Distribution of available observations

	abs	rel		abs	rel		abs	rel
$0 < T_i \leq 100$	6	.053	$200 < T_i \leq 300$	29	.254	$400 < T_i \leq 500$	15	.132
$100 < T_i \leq 200$	30	.263	$300 < T_i \leq 400$	27	.237	$500 < T_i$	7	.061

Distribution of available observations to estimate the dynamic OLS specification of the Fisher relation over the unbalanced cross section containing 114 countries. 'abs' and 'rel' give relative and absolute frequencies, respectively.

Since our investigation builds upon a cross section which is unbalanced with respect to the number of available observations it is first worthwhile to indicate the number of available observations used for the analysis. Table 1 provides a highly aggregated view at the distribution of available sample sizes (T_i) to implement DOLS estimation of the Fisher coefficients $\theta_i^{(1)}$. For the remaining empirical models (ECM regressions and DOLS estimation of the inverted Fisher specification) the results are almost identical. Regarding the tails of the empirical distribution of sample sizes it turns out that for 7 (6) members of the cross section more than 500 (less than 100) observations enter the regression. For one quarter of the cross section the relevant sample size is larger than $T_i = 200$ but less than or equal to $T_i = 300$. When aggregating all available observations over the cross section we arrive at a total of 31823 (31801, 31701, 31798) time points to estimate $\theta_i^{(1)}$ ($\theta_i^{(2)}$, $\alpha_i^{(1)}$, $\alpha_i^{(2)}$), $i = 1, \ldots, 114$.

3.1 Descriptive data analysis

The left hand side panel of Figure 1 shows a scatter plot of the average of short term interest rates ($\bar{R}_i = 1/T_i \sum_{t=1}^{T_i} R_{it}$) against the corresponding average inflation rate ($\bar{\pi}_i$) estimated for the entire cross section, $i = 1, \ldots, N$. Apparently, average interest rates increase with inflation. Estimates obtained from a between regression formalizing the latter relation are given in Table 2. As expected, the slope coefficient is significantly positive. The intercept term is negative owing to six observations characterized by some outlying average interest rates exceeding 100%. The corresponding economies are Argentina, Armenia, Brazil, Croatia, Democratic Republic of Kongo and Nicaragua. As documented in the right hand side panel of Table 2 removing these economies from the cross section yields a slope coefficient of 1.10 which does not differ from unity at a conventional significance level. Thus, from this highly aggregated perspective the Fisher hypothesis is supported in its original form.

Table 2: Results from selected between regressions

Full sample	Subsample ($\bar{R}_i < 100\%$ p.a.)
$\hat{\bar{R}}_i = \underset{(2.661)}{-13.03} + \underset{(12.948)}{2.385}\, \bar{\pi}_i, \quad R^2 = 0.599$	$\hat{\bar{R}}_i = \underset{(0.666)}{1.310} + \underset{(9.474)}{1.102}\, \bar{\pi}_i, \quad R^2 = 0.456$
$\hat{\bar{R}}_i = \underset{(1.004)}{-6.908} + \underset{(7.102)}{19.89}\, \overline{\lvert\Delta\pi\rvert}_i, \quad R^2 = 0.304$	$\hat{\bar{R}}_i = \underset{(2.025)}{5.528} + \underset{(4.340)}{6.926}\, \overline{\lvert\Delta\pi\rvert}_i, \quad R^2 = 0.144$

Linear relations summarizing the scatter diagrams in Figure 1. Cross sectional means of interest rates ($\bar{R}_i$) are related to corresponding measures of average inflation ($\bar{\pi}_i$) and average inflation risk ($\overline{\lvert\Delta\pi\rvert}_i$). Regression results are reported with $t-$ratios given in parentheses underneath the coefficient. R^2 is the degree of explanation adjusted for the number of explanatory variables.

In addition, the right hand side scatter diagram in Figure 1 visualizes a relation linking average interest rates and the unconditional level of absolute changes of inflation ($\overline{\lvert\Delta\pi\rvert}_i = 1/T_i \sum_{t=1}^{T_i} \lvert\Delta\pi_{it}\rvert$). Note that the latter quantity may be used to approximate the unconditional uncertainty in an economy concerning the development of inflation, or put differently, inflation risk. The estimated linear relation linking $\overline{R}_i$ and $\overline{\lvert\Delta\pi\rvert}_i$ is also given in Table 2. The significantly positive slope estimate underscores that, on average, interest rates are increasing in inflation risk.

The latter result also holds for the subsample excluding cross section members with outlying average interest rates. From the positive relation between average interest rates and inflation risk it appears that a stable inflation rate is an important factor to stabilize interest rates in an economy at a relatively low level. A similar result would be obtained when contrasting average interest rates with cross section specific empirical standard errors of inflation.

3.2 Some notes on integration and cointegration testing

After taking a highly aggregated view at the relation between interest rates and inflation or inflation risk, respectively, the following paragraphs will discuss standard inference on the involved time series' properties in some detail. Owing to the large cross sectional dimension we refrain from giving the entire batteries of cross section specific estimates or test statistics but rather provide a few summary measures like the empirical mean and median, or the interquartile range of particular statistics as e.g. ADF tests or the ECM based test of the null hypothesis of no cointegration. Moreover, by means of surface regressions we will try to relate cross section specific estimates or test statistics to some economic measures characterizing a particular member of the cross section. For the latter purpose we perform a multiple regression of common test statistics on the average level of inflation ($\overline{\pi}_i$) and interest rates ($\overline{R}_i$) their changes ($\overline{\Delta\pi}_i$, $\overline{\Delta R}_i$) and their absolute changes ($\overline{|\Delta\pi|}_i$, $\overline{|\Delta R|}_i$). To evaluate the outcome of the latter regression we will mainly discuss coefficient estimates, $t-$ratios and the partial degrees of explanation (R_p^2). In addition, the overall degree of explanation will be shown. Almost all empirical results obtained along this outline are displayed in Table 3. Since six members of the cross section show extreme average interest rates (more than 100%, see Figure 1) we decide to remove these economies from the cross sectional regressions summarized in Table 3. Thus, the overview given in Table 3 is informative for a cross section comprising $N = 108$ economies. When interpreting the results of the surface regressions one should take into account that all estimated models like (15) or (16) build upon time homogeneity. In case the true underlying relationship is time varying parameter estimates are likely biased which in turn might have adverse effects on the economic intuition of conclusions offered by the surface regression. Apart from cross section specific modeling the last paragraph of this section will briefly address the outcomes of various panel integration and cointegration tests.

3.3 Unit root tests

ADF tests for unit roots are implemented throughout with an intercept term and two or four lags of the respective left hand side variable. Since the obtained results are quite similar for both lag orders we only provide test results for ADF regressions implemented with two lags for space considerations. The relevant critical value at the 5% significance level is -2.892 (MacKinnon 1996). The first, second and third two columns of Table 3 provide an overview over ADF tests obtained for testing nonstationarity of interest rates (R_{it}), inflation rates (π_{it}), and real interest rates ($r_{it} = R_{it} - \pi_{it}$), respectively. Apart from descriptive features of ADF statistics the lower panel of this overview documents slope estimates obtained from a multiple regression of country specific ADF statistics on a constant and particular factors as the average level of interest or inflation rates, their average changes or their average absolute changes.

Without almost any exception changes of interest or inflation rates are found to be stationary, i.e. the corresponding level series are at most integrated of order one. Implementing the ADF regressions with 2 lags the unit root hypothesis is rejected for 44.4% and 15.7% of all members of the cross section when testing the level of inflation and interest rates, respectively. The cross sectional means of ADF statistics obtained for inflation and interest rates are -2.87 and -2.69, respectively. A comparison of both means and selected quantiles given in Table 3 indicates that the evidence in favor of a nonstationary stochastic trend is stronger for interest rates in comparison with inflation. The latter result is in analogy to other empirical studies as, for instance, Rose (1988). Since with regard to interest rates our conclusion is clearly that these series are driven by stochastic trends it will be interesting to judge the outcome of ADF tests, ECMs and DOLS regressions jointly. In case inflation is stationary it will not enter a cointegration relationship with nonstationary interest rates. Taking this argument into account it is likely that estimates of the error correction coefficient in (15) or of the long run parameter in (16) will be close to zero since a single nonstationary variable (here the level of interest rates) can neither explain stationary changes of interest rates or inflation nor it can be explained by a stationary level of inflation.

Table 3: Cross section specific test statistics and regression results

	ADF tests						Fisher regression				Inverted Fisher			
	R_{it}	ΔR_{it}	π_{it}	$\Delta\pi_{it}$	r_{it}	Δr_{it}	$c_i^{(1)}$	$\theta_i^{(1)}$	$\alpha_i^{(1)}$	$t_{\alpha^{(1)}}$	$c_i^{(2)}$	$\theta_i^{(2)}$	$\alpha_i^{(2)}$	$t_{\alpha^{(2)}}$
mean	-2.629	-8.536	-2.872	-9.120	-3.106	-8.995	7.354	0.453	-0.051	-2.264	0.687	0.982	-0.051	-2.823
$q_{.25}$	-2.490	-9.993	-3.412	-11.32	-3.546	-10.64	4.368	0.050	-0.050	-3.090	-1.401	0.216	-0.069	-3.606
$q_{.50}$	-1.646	-7.531	-2.759	-9.380	-2.893	-8.928	6.517	0.307	-0.022	-2.094	1.425	0.497	-0.039	-2.823
$q_{.75}$	-1.152	-5.998	-2.214	-7.549	-2.252	-7.239	8.930	0.682	-0.006	-1.258	7.511	0.823	-0.022	-1.979
< -2.892	0.157	1.00	0.444	1.00	0.519	1.00								
	Multiple regression with partial R^2													
R^2	0.723	0.423	0.421	0.306	0.348	0.179	0.558	0.421	0.476	0.520	0.042	0.005	0.180	0.231
const	-1.585*	-8.717*	-2.501*	-10.10*	-2.880*	-10.01*	2.412*	0.477*	-0.059*	-2.126*	-5.432	1.029	-0.054*	-2.629*
	(-2.30)	(-10.7)	(-17.3)	(-25.3)	(-10.6)	(-23.0)	(2.50)	(6.44)	(-4.94)	(-7.55)	(-1.00)	(1.26)	(-7.25)	(-11.3)
$\overline{\pi}_i$	0.165*	0.152	0.096*	0.116*	0.041	0.034	0.162	0.015	-0.003*	-0.006	0.164	0.022	0.002*	0.076*
	(2.25)	(1.76)	(6.26)	(2.73)	(1.43)	(0.74)	(1.57)	(1.92)	(-2.11)	(-0.20)	(0.28)	(0.26)	(2.27)	(3.05)
R_p^2	0.048	0.030	0.279	0.069	0.020	0.005	0.024	0.035	0.042	0.000	0.001	0.001	0.048	0.085
$\overline{\Delta\pi}_i$	-2.168*	-1.351	0.720*	-0.327	-0.638*	-0.487	-3.162*	0.079	-0.034*	-0.811*	4.564	0.308	0.010	0.233
	(-2.73)	(-1.45)	(4.33)	(-0.71)	(-2.04)	(-0.97)	(-2.84)	(0.93)	(-2.47)	(-2.50)	(0.73)	(0.33)	(1.22)	(0.87)
R_p^2	0.116	0.044	0.037	0.028	0.066	0.017	0.112	0.001	0.032	0.062	0.004	0.001	0.002	0.000
$\overline{\lvert\Delta\pi\rvert}_i$	-1.354*	-0.829	-0.625*	-0.494	-0.234	0.235	1.626*	-0.255*	0.007	0.064	0.749	0.155	-0.015*	-0.483*
	(-2.45)	(-1.27)	(-5.40)	(-1.54)	(-1.08)	(0.67)	(2.10)	(-4.29)	(0.69)	(0.29)	(0.17)	(0.24)	(-2.58)	(-2.56)
R_p^2	0.011	0.018	0.103	0.023	0.027	0.072	0.339	0.204	0.001	0.044	0.000	0.002	0.044	0.008
$\overline{R}_i$	0.032	0.076	-0.045*	0.015	0.012	0.067	0.374*	-0.002	0.006*	0.037	0.276	-0.036	0.001	-0.003
	(0.37)	(0.73)	(-2.45)	(0.29)	(0.34)	(1.20)	(3.02)	(-0.23)	(3.87)	(1.04)	(0.40)	(-0.34)	(0.86)	(-0.12)
R_p^2	0.004	0.029	0.012	0.045	0.005	0.064	0.205	0.003	0.078	0.004	0.015	0.000	0.057	0.032
$\overline{\Delta R}_i$	19.92*	11.18*	0.293	-2.122*	2.468*	1.389	-3.759	-0.120	0.119*	4.684*	6.970	-0.539	0.018	0.822
	(14.0)	(6.70)	(0.99)	(-2.58)	(4.42)	(1.55)	(-1.89)	(-0.79)	(4.83)	(8.08)	(0.62)	(-0.32)	(1.21)	(1.71)
R_p^2	0.629	0.312	0.039	0.041	0.146	0.034	0.005	0.004	0.179	0.366	0.012	0.001	0.039	0.053
$\overline{\lvert\Delta R\rvert}_i$	-0.550	-1.048	-0.002	-0.299	-0.513*	-0.666*	-5.485*	0.171*	-0.042*	-0.426*	0.391	-0.071	-0.007	-0.362*
	(-1.11)	(-1.80)	(-0.02)	(-1.05)	(-2.64)	(-2.13)	(-7.92)	(3.23)	(-4.87)	(-2.11)	(0.10)	(-0.12)	(-1.39)	(-2.17)
R_p^2	0.012	0.031	0.000	0.011	0.065	0.043	0.383	0.094	0.190	0.042	0.000	0.000	0.019	0.045

Descriptive statistics and multiple regression results for country specific estimates or test statistics. Six countries showing outlying average interest rates have been excluded ($N = 108$). Explanatory variables for the multiple regressions are the average level of inflation and interest rates ($\overline{\pi}_i$, $\overline{R}_i$), of its changes ($\overline{\Delta\pi}_i$, $\overline{\Delta R}_i$) and of its absolute changes ($\overline{|\Delta\pi|}_i$, $\overline{|\Delta R|}_i$). $t-$ratios are given in parentheses underneath the coefficient estimates. * indicates significance at the 5% level. R^2 and R_p^2 denote the degree of explanation and the partial degree of explanation, respectively, with the latter obtained after partialling out all but one explanatory variables.

The surface regression characterizing the cross sectional pattern of ADF statistics of inflation yields a degree of explanation of $R^2 = 42.1\%$. From the set of potential factors explaining the overall pattern of ADF statistics for the level of inflation we see that on the one hand the test statistic increases significantly with the average level of inflation and average inflation changes. According to the partial degrees of explanation it turns out that the 'influence' of the level of inflation on ADF statistics is by far stronger ($R_p^2 = 0.279$) in comparison with average inflation changes ($R_p^2 = 0.037$). Thus, over economies which are likely characterized by some period of upward trending inflation it is more likely to decide in favor of a stochastic trend governing inflation. On the other hand, ADF statistics are the smaller the larger is $|\overline{\Delta\pi}|_i$, the unconditional measure of inflation's volatility, with the partial degree of explanation being $R_p^2 = 0.103$. The latter result may reflect the intuition that high volatility of a process could hide its smoothness generated by some stochastic trend. Moreover, Fischer et al. (2002) point out that for high inflation economies periods of accelerating inflation tend to be longer than periods of successfully decelerating policies. For such economies eyeball inspection may diagnose some mean reverting pattern of inflation.

For completeness we also test the unit root null hypothesis for the real interest rates $r_{it} = R_{it} - \pi_{it}$. In case the Fisher hypothesis holds in its original form r_{it} should be stationary. It turns out that for about one half of the cross section (51.9%) the null hypothesis of a stochastic trend governing real interest rates is rejected. Regressing ADF statistics for real interest rates on characteristics of the cross section provides an overall degree of explanation of $R^2 = 0.348$. We obtain that with 5% significance the test statistics increase (decrease) with the average level of interest (inflation) rate changes offering a partial R_p^2 measure of 0.146 (0.066). Thus over economies that experienced large upward movements of interest rates it is more likely to diagnose real interest rates which are integrated of order one. On the one hand this result seems at odds with the arguments in Mishkin (1992) according to which it is more likely to find a stationary real rate over periods of strong trends in inflation and interest rates. Note, however, that other things being unchanged the higher the average level of inflation rate changes the smaller is the real rate's ADF statistic thereby pointing to its stationarity. On the other hand, interest rates and inflation could be disconnected over periods of large positive interest rate changes when agents expect future consumption and output to decline and, hence, ask for higher risk premia to avoid losses in their financial investments. Then, nonstationary risk premia may reveal a stochastic trend governing the real rate.

3.4 Error correction dynamics

In the first place the error correction model in (15) allows to test for cointegration by means of the (inverted) Fisher relation. Given the nonstationarity of interest rates cross section specific parameters $\alpha_i^{(j)}$, $j = 1, 2$, are zero in case of no cointegration or if only one of the two level variables is nonstationary. Secondly and in case of cointegration, parameters $\alpha_i^{(j)}$ give the strength of the interest (inflation) rate's reaction in response to deviations from the long-run equilibrium. Under cointegration one of the two error correction coefficients may be zero owing to weak exogeneity of either interest ($\alpha_i^{(1)} = 0$) or inflation rates ($\alpha_i^{(2)} = 0$). Note that in case of cointegration one of the two variables of the system must respond to violations of the long run equilibrium. Summarizing the latter considerations, one may look at the fraction of error correction estimates $\hat{\alpha}_i^{(j)}$ having $t-$ratios smaller than -1.77, the expected value under the hypothesis of no cointegration (Westerlund 2005b), or smaller than the corresponding 5% critical value (-3.37) to test the null hypothesis of no cointegration. Along similar lines, the frequency of $t-$ratios of parameter estimates which are negative or less than -1.96 indicates if weak exogeneity may be reasonably assumed to hold. Corresponding frequencies for empirical t-ratios of $\hat{\alpha}_i^{(j)}$, $j = 1, 2$, are collected in Table 4.

Table 4: Distribution of estimated t-ratios of error correction coefficients

< 0			< -1.77			< -1.96			< -3.37		
$\hat{\alpha}_i^{(1)}$	$\hat{\alpha}_i^{(2)}$	$\hat{\alpha}_i^{(\bullet)}$	$\hat{\alpha}_i^{(1)}$	$\hat{\alpha}_i^{(2)}$	$\hat{\alpha}_i^{(\bullet)}$	$\hat{\alpha}_i^{(1)}$	$\hat{\alpha}_i^{(2)}$	$\hat{\alpha}_i^{(\bullet)}$	$\hat{\alpha}_i^{(1)}$	$\hat{\alpha}_i^{(2)}$	$\hat{\alpha}_i^{(\bullet)}$
.935	.972	1.00	.599	.778	.944	.556	.750	.954	.176	.333	.462

Relative frequencies of t-ratios of error correction coefficient estimates smaller than 0, -1.77 (expectation of $t_{\hat{\alpha}}$ under the null of no cointegration, Westerlund 2005b), -1.96 (two sided test on weak exogeneity with 5% significance), -3.37 (5% critical value for testing the null of no cointegration). We distinguish dynamic versions of the Fisher relation ($t-$ratios of $\hat{\alpha}_i^{(1)}$) and the inverted Fisher relation ($t-$ratios of $\hat{\alpha}_i^{(2)}$). $\hat{\alpha}_i^{(\bullet)}$ indicates if the postulated inequality holds for at least one $t-$ratio.

For almost every member of the cross section (94.4%) at least one of the two estimators $\hat{\alpha}_i^{(j)}$, $j = 1, 2$, has a $t-$ratio which is smaller than its expected value under a null hypothesis of no cointegration. When discussing the results from ECM based panel unit root tests on cointegration the obtained relative frequencies of single equation $t-$ratios being smaller than -1.77 will explain the strong evidence in favor of cointegration obtained at the pooled level. Note that underscoring the prevalence of cointegration the latter result also implicitly indicates that interest and inflation rates likely share the same order of integration ($I(1)$). Testing the null hypothesis of no cointegration on the basis of single equation ECMs it turns out that it is rejected for 'only' 22 and 40 members of the cross section, when the test is performed by means of the Fisher equation and the inverted Fisher specification, respectively. Owing to the general picture of mostly negative parameter estimates, however, we conjecture that the latter result may be addressed to the presumably low power of single equation ECM based cointegration tests in small samples. When discussing the latter test on the level of pooled series (Westerlund 2005a) such a power deficiency might be overcome.

For both versions of the ECM the estimated error corrections coefficients are negative for more than 93% of the estimated ECMs. The average of estimated error correction coefficients is $\hat{\alpha}_i^{(\bullet)}$ is -0.051 for both ECM specifications, the Fisher relation and the inverted Fisher relation (see Table 3). Under the assumption of cointegration the estimated error correction coefficient is significant at the 5% level for 56.4% (74.5%) of all members of the cross section. In other words, for about 45% (25%) of empirical models the null hypothesis of weak exogeneity of interest (inflation) rates cannot be rejected on the basis of single equation modeling. Weak exogeneity of interest rates is, however, at odds with Fisher's hypothesis according to which inflation drives nominal interest rates. Having almost uniformly negative estimates for the error correction coefficient $\alpha_i^{(1)}$, however, we address the latter finding to power deficiencies involved when estimating the ECM for cross section members having a presumably small time dimension (T_i). At the pooled level estimated error correction dynamics of interest rates are likely significant under the hypothesis of cointegration.

The strength of error correction dynamics or the evidence in favor of cointegration are, other things being unchanged, stronger the smaller is the estimated error correction coefficient $\alpha_i^{(j)}$, $j = 1, 2$, or the respective $t-$ratio. Therefore, it is of interest if the magnitude of parameter estimates or $t-$statistics can be related to particular factors sampled over the cross section. The R^2 of multiple regressions explaining $\hat{\alpha}_i^{(1)}$ and $t_{\alpha^{(1)}}$ is 47.6% and 52.0%, respectively. Regarding the corresponding outcomes from the inverted regression ($R^2 = 18\%$, ($\hat{\alpha}_i^{(2)}$), $R^2 = 23.1\%\,(t_{\alpha^{(2)}})$) reveals that estimates from the latter are more difficult to control by simple cross sectional measures of unconditional economic conditions.

Regarding the ECM implementations of the Fisher equation it turns out that the evidence in favor of cointegration is weaker over economies showing a relatively large unconditional level of interest rate changes. Among the set of employed economic indicators $\overline{\Delta R_i}$ obtains the largest or second largest partial degree of explanation for $t_{\alpha^{(1)}}$ ($R_p^2 = 0.366$) and $\alpha_i^{(1)}$, ($R_p^2 = 0.179$), respectively. Central banks in economies with a relatively high 'empirical drift' in interest rates are likely to suffer from credibility problems which could be accompanied by trending price levels. Then, to stabilize the development of prices it is tempting to import external stability via some form of exchange rate policy. In turn, a domestic link between inflation and interest rates could be distorted.

Moreover, it is more likely to find evidence in favor of cointegration over countries having a relatively high level of unconditional interest rate risk. Error correction coefficient estimates ($R_p^2 = 0.19$) or their $t-$ratios ($R_p^2 = 0.042$) decrease in interest rate risk. In economies showing high interest rate uncertainty central banks, on the one hand, may want to reduce the costs of interest rate instability in the sense that they try to keep the real interest rate stationary. On the other hand, investors want to earn a real interest from their financial investments, which are typically written in nominal terms.

As a further measure of economic conditions the mean level of inflation rate changes appears to go along with the evidence in favor of an existing long run linkage between interest rates and inflation. Small changes in the inflation rate will hardly affect real earnings whereas large positive changes may potentially imply negative real rates. For this reason, investors have a high preference for nominal interest rates responding rather quickly to changes in inflation over such economic states.

With regard to the specification of the inverted Fisher relation we find that the $t-$ratio of $\hat{\alpha}_i^{(2)}$ and, thus, evidence against cointegration, increases with the average level of inflation ($R_p^2 = 0.085$) and declines with interest rate ($R_p^2 = 0.045$) or inflation risk ($R_p^2 = 0.008$). Regarding these three factors the third one appears to be of only marginal importance and the second is well in line with the results obtained for the Fisher relation discussed above. With respect to the impact of $\overline{\pi}_i$ on $t_{\alpha^{(2)}}$ it might be the case on the one hand that the implicit null hypothesis tested is not the failure of cointegration but weak exogeneity of inflation. Then, owing to credibility problems faced by the central bank of high inflation economies price adjustments are likely to respond to outstanding dynamics which are not captured by the equilibrium relation between inflation and interest rates. On the other hand, economies with high average inflation rates are likely to have experienced phases of hyper inflation which in turn may involve a change of monetary policy that could disturb the long run linkage between interest and inflation rates. For example, in 1990 Argentina faced an annual inflation rate of 2300% and in 1991 it introduced a currency board system. Similarly, Bulgaria experienced an annualized inflation of about 2500% in the first quarter 1997 against the last quarter 1996. As a consequence a currency board system was installed in June 1997.

3.5 Dynamic OLS

Evaluating the Fisher coefficient by means of the DOLS regression in (16) obtains that, on average, the impact of inflation on interest rates is 0.453, and, owing to a corresponding standard error of 0.05 it is clearly different from unity. This evidence is in line with panel cointegration estimates in Crowder (2003) and is in contrast to the Taylor principle mentioned in Section 2.2. The latter results imply that even in case an equilibrium relation between interest rates and inflation exists, still, on average, the real interest rate is driven by a stochastic trend. Regarding particular quantiles obtained over the cross section we find, for instance, that the 75% quantile is 0.682, which still is far below unity. With regard to our unconditional measures of risk it turns out that the estimated Fisher coefficient increases significantly with $\overline{|\Delta R|}_i$ ($R_p^2 = 0.094$) and is decreasing in $\overline{|\Delta \pi|}_i$ ($R_p^2 = 0.204$). From an evaluation of the parameter estimates we obtain that the implied long run parameter approaches unity in case average absolute interest rate variations exceed the corresponding inflation based quantity by a factor of about 1.5. Intuitively, the real interest rate's variability is likely to increase in both risk measures $\overline{|\Delta R|}_i$ and $\overline{|\Delta \pi|}_i$. Then, it is noteworthy that the empirically observed positive impact of interest rate risk on the long run parameter is at odds with the theoretical arguments in Mishkin (1992) detailed in Section 2.2.

The estimated constant $\hat{c}_i^{(1)}$ varies over the cross section such that it is significantly larger for economies having a high average level of interest rates ($R_p^2 = 0.205$), showing relatively low average interest rate changes ($R_p^2 = 0.383$) or relatively high average inflation rate changes ($R_p^2 = 0.339$). Moreover, the estimated constant is adversely related to the average inflation changes ($R_p^2 = 0.112$). As outlined before the intercept term in the Fisher equation may cover implied risk premia (Crowder and Hoffman 1996). Apparently from (8) this premium increases in the variance of expected inflation which could be approximated by $\overline{|\Delta \pi|}_i$. Thus, the marginal impact detected via the cross sectional regression is in so far supportive for the model in (8). In addition, the risk premium is negatively related to the conditional covariance between inflation and consumption growth. Regarding $\Delta \pi_{it}$ to be negatively correlated with Δc_t, however, our empirical results are in contrast to the theoretical model. Measuring uncertainty by means of the average absolute changes of inflation we have that over the cross section higher unconditional inflation uncertainty commands a higher risk premium. This result may support the model in Crowder and Hoffmann (1996) formalized in equation (8).

In response to our measures of unconditional inflation and interest rate risk the estimated constant $\hat{c}_i^{(1)}$ varies over the cross section in the opposite direction. Countries with a relatively high level of absolute inflation changes have a lower Fisher coefficient in comparison with economies showing lower average absolute inflation changes. This is in contrast to the hypothesis that higher inflation risk implies a clearer evidence for the Fisher relationship. We might infer such a result when looking at the interest rate risk measure. It seems that the effect of this latter risk measure mirrors that financial investors realize, on average, a positive real interest.

Regarding DOLS estimates obtained from the inverted Fisher relation we find on average an inverted Fisher coefficient rather close to unity (0.982). Although our results on the relative magnitudes of $\theta_i^{(1)}$ and $\theta_i^{(2)}$ are in line with Crowder (2003) it is worthwhile to point out that, on average, the parameter estimates obtained from both relations cannot be retrieved from each other. Given the superconsistency of single equation DOLS estimators this result appears counterintuitive but one should note that 'invertibility' of long run parameter estimates holds only in case of fully explained dependent variables ($R^2 = 1$, Engle and Granger 1987). Moreover, the parameter $\theta_i^{(2)}$ appears to be homogeneous over the cross section in the sense that the factors used to uncover structural features of the parameter estimates are insignificant at the 5% level, throughout.

3.6 A note on heterogeneity

So far, our descriptive analyses indicate that the relation between interest rates and inflation likely lacks homogeneity over the cross section. In addition, it appears that economic factors govern key parameters of the econometric model. However, the cross sectional analysis gives only a first view at the link between economic states on the one hand and the relation between interest rates and inflation one the other hand. Owing to the large time dimension investigated in this work it is likely that economic states observed for a specific economy change over time. Therefore we will continue to disaggregate the sample information and follow a state and country dependent semiparametric approach in Section 4 and Section 5. For completeness and for the purpose of comparison with related empirical studies the following paragraph will discuss results from panel data models that rely on parameter homogeneity and were recently applied in analyzing the Fisher relation.

3.7 Results from panel data models

Panel unit root tests are reviewed, for example, by Banerjee (1999) or Hurlin and Mignon (2004). In this empirical study all panel unit root tests are applied which have been implemented in EViews 5.0. Integration properties are tested for the nominal interest and inflation rates as well as their first differences. In addition, panel unit root tests are performed for the real interest rate. Panel unit root test results are provided in Table 5. Regarding the LLC and Breitung test (Breitung 2000) the null hypothesis of a unit root is not rejected for the interest rate whereas its first difference is found to be stationary. Testing the null hypothesis of stationarity the Hadri statistic (Hadri 1999) turns out to be significant for both interest rates and changes thereof. In contrast, using the the IPS and Maddala and Wu (MW) tests (Maddala and Wu 1999) the null hypothesis of nonstationarity is rejected for the interest rate. With respect to inflation rates the results from most panel unit root tests also fail a consistent interpretation. In this case the null hypothesis of a unit root is not rejected when using the LLC test. It is rejected according to the Breitung, IPS, and MW statistic. Using the Hadri test the null hypothesis of stationary inflation is rejected. For the first difference of inflation all tests obtain a rejection of their respective null hypotheses. Turning to the real interest rate the null hypothesis of a unit root is not rejected when applying the LLC test, whereas the Breitung, IPS, and MW tests indicate stationarity of the real interest rate at the pooled level. In sum, with the LLC test being the only exception the applied panel unit root tests do not allow any consistent conclusion. In particular the outcome of the Hadri test has to be interpreted with care since single equation ADF tests discussed above provided almost uniform evidence in favor of stationarity of both ΔR_{it} and $\Delta \pi_{it}$.

Recently, Westerlund (2005b) uses equation (15) to derive new tests for cointegration in panel data building on appropriately centered and rescaled single equation $t-$ratios of estimated error correction coefficients. Similar as e.g. the derivation of the asymptotic distribution of the IPS statistic it can be shown that the ECM based panel test statistic is asymptotically Gaussian under the null hypothesis of no cointegration. The Westerlund statistic applied to the Fisher relation is -14.7, such that the null hypothesis of no cointegration is clearly rejected at any conventional significance level. The latter result is robust if cross sectional subsamples are investigated. For this purpose, we order the cross section according to the average level of inflation. Over the ordered sample we apply moving windows containing 25 cross section members and determine the Westerlund statistic recursively. The results obtained from this exercise are displayed in the left hand side panel of Figure 2. Apparently, the null hypothesis

Table 5: Panel unit root tests ($N = 114$).

Variable	LLC	Breit	IPS	MW-ADF	MW-PP	Hadri
R_{it}	2.323	0.863	-5.473	-4.841	-7.192	24.234
ΔR_{it}	-35.923	-44.584	-73.397	-55.080	-86.537	18.433
π_{it}	6.281	-7.760	-12.183	-11.824	-18.628	32.576
$\Delta\pi_{it}$	-13.372	-33.436	-51.202	-48.960	-108.40	4.908
r_{it}	16.658	-7.630	-14.149	-13.994	-21.608	9.868

LLC: Levin, Lin, Chu (2002) using modified AIC, Breit: Breitung (2000), IPS: Im, Pesaran, Shin (2003) using modified AIC, MW-ADF: Maddala-Wu (1999) ADF-test using modified AIC, MW-PP: Maddala-Wu (1999) Phillips-Perron-test using Bartlett window with Newey-West truncation lag estimator, Hadri: Hadri (1999) generalized KPSS-test. LLC, IPS, Breit, MW-ADF, MW-PP have the null hypothesis of a unit root at the pooled level. The null hypothesis of no unit root is tested by Hadri. All tests are one sided and, asymptotically, the test statistics have an asymptotic standard normal distribution under the null hypothesis.

of no cointegration is rejected for all considered cross sections at the 5 % significance level. Being well in line with Westerlund (2005a) the latter results strongly support the prevalence of cointegration at the pooled level.

The Westerlund statistic obtained when modeling the inverted Fisher realtion is -16.8, such that again the null hypothesis of no cointegration is clearly rejected. The latter result is robust if cross sectional subsamples obtained from moving windows covering 25 members of the cross section are investigated. Recursive Westerlund statistics obtained along these lines are shown in the right hand side panel of Figure 2. Apparently, at the 5% significance level the null hypothesis of no cointegration is rejected for all considered subsamples of the cross section.

4 Functional coefficient models

From the foregoing discussion one may conjecture that the relation between interest and inflation rates not merely exhibits cross sectional specialities but could also be driven by economic factors changing over time. Our empirical study exploits a rather large data set comprising time series over the majority of the world's economies. For the reasons discussed before we refrain from a-priori assuming cross sectional homogeneity but will rather indicate to which extent key model parameters could ex-post be regarded as homogeneous over the cross section. In addition, we allow key parameters of the econometric model to be time varying. The empirical approach outlined in this section is semiparametric and thought to provide a state dependent framework generalizing the restrictive dynamic specifications in (15) and (16). Although the adopted semiparametric approach might be extended to formally test both time and cross sectional homogeneity we will not rely on formal significance tests but rather provide a more descriptive analysis of the dynamics linking interest and inflation rates. The adopted empirical models fit into the general class of functional coefficient models (Cai, Fan and Yao 2000) and are briefly motivated in the next subsection. Since the reader may not be that familiar with this model class we comment below on a few issues that are relevant when implementing the semiparametric estimator. We will sketch model representation, bandwidth selection and parameter estimation in turn. Empirical results obtained from the semiparametric models will then be discussed in Section 5.

4.1 Motivation

Estimating long run relationships between nonstationary variables or, accordingly, error correction dynamics typically proceeds from the assumption that the underlying model parameters are invariant over time. Similar to parameter invariance, moreover, (log) linearity often postulated in parametric models may be regarded as a strong restriction. Recent contributions to the econometric analysis of cointegrated systems allowing for nonlinearity of the equilibrium relationship or adjustment towards the latter are e.g. Park and Phillips (2001), Chang, Park and Phillips (2001) or Balke and Fomby (1997), Conradi, Swanson and White (2000), Granger (2001), respectively.

4.2 Specification and estimation

So far our empirical analysis focusses on models which are structurally invariant over time. A semiparametric model should allow to detect deviations from the common (log) linear model. Apart from allowing cross sectional heterogeneity we now regard the key parameters of the econometric models in (15) and (16) namely $\alpha_i^{(j)}$ and $\theta_i^{(j)}$, $j = 1, 2,$ to be time varying. Motivated by our discussion of cross sectional variations of parameter estimates, we relate the long run and error correction coefficients to some factor characterizing the state of the economy. As potential factors one may consider the level of inflation, changes of inflation or absolute changes of inflation. In addition, potential factors could be derived from the state of interest rates, namely the level of short term interest rates, interest rate changes or absolute changes. For notational convenience we refer to any factor as $\widetilde{w}_{it}$ and will clarify its explicit choice when discussing estimation results. Providing a cross sectionally uniform discussion of factor impacts on the long run parameters or error correction coefficients we will consider standardized factors $w_t = (\widetilde{w}_{it} - \bar{w}_i)/\sigma_i(\widetilde{w})$, where $\bar{w}_i$ and $\sigma_i(\widetilde{w})$ denote the empirical time mean, $\bar{w}_i = 1/T_i \sum_{t=1}^{T_i} \widetilde{w}_{it}$, and the cross section specific standard error of $\widetilde{w}_{it}$, respectively. Note that to obtain w_t cross section specific first and second order moments are used. Such cross sectional dependence is, however, not expressed by the notation to improve the readability of the remainder of the paper.

To concentrate on time variation of the parameters of interest, $\alpha_i^{(j)}$ in (15) and $\theta_i^{(j)}$ in (16), we presume that the remaining parameters in these regression models are time invariant. For this reason we will first apply partial regression techniques to isolate the functional relation of interest. Then, in a second step the latter is generalized towards a model fitting into the general framework of functional coefficient models outlined in Cai, Fan and Yao (2000). For each member of the cross section both models (15) and (16) may be given compactly in matrix form as

$$y_i = Z_i \gamma_i + X_i \vartheta_i + e_i, \tag{17}$$

where γ_i contains the model parameter(s) of interest and ϑ_i collects the parameters that are time invariant by assumption. Rewriting the model in (16) this way, we have e.g. $Z_i = (z_i)$ and $X_i = (\mathbf{1}, \Delta z_{i,+2}, \Delta z_{i,+1}, \Delta z_{i,-1}, \Delta z_{i,-2})$, where $\mathbf{1}$ is a unit vector and $\Delta z_{i,\bullet}$ are columnvectors collecting for the i-th cross section the differenced variables Δz_{it} at the indicated lag or lead. Regarding the ECM in (15) we will set alternatively $Z_i = (y_{i-}, z_{i-})$ and $Z_i = (y_{i-})$, where y_{i-} (z_{i-}) is short for a column vector collecting the lagged levels of y_{it} (z_{it}). Thus, the former choice ($Z_i = (y_{i-}, z_{i-})$) allows to model time variation of $\alpha_i^{(j)}$ building on the assumption that also $\theta_i^{(j)}$ is also factor dependent. The latter choice of Z_i ($Z_i = (y_{i-})$) a-priori builds on the assumption that the parameters $\theta_i^{(j)}$ are time invariant. Before specifying the factor model we apply partial regression techniques to remove X_i from the model in (17). Along these lines we obtain an intermediate specification

$$\tilde{y}_{it} = \tilde{z}_{it}' \gamma_i + \tilde{e}_{it}. \tag{18}$$

In (18) $\tilde{y}_{it}, \tilde{z}_{it}$ and $\tilde{e}_{it}$ are typical elements of

$$\tilde{y}_i = M_i y_i,\ \tilde{Z}_i = M_i Z_i \text{ and } \tilde{e}_i = M_i e_i, \tag{19}$$

respectively, where $M_i = (I_i - X_i(X_i'X_i)^{-1}X_i')$ and I_i denotes the $(T_i \times T_i)$ identity matrix. Although (18) is an equivalent representation of the model in (17) the former is more intuitive, when generalizing it towards a nonlinear or factor dependent relationship reading as:

$$\tilde{y}_{it} = \tilde{z}_{it}'\gamma_i(w) + \tilde{\epsilon}_{it}. \tag{20}$$

To estimate the parameter(s) of interest in $\gamma_i(w)$ we proceed similar to the Nadaraya Watson estimator (see e.g. Härdle 1990) providing the following weighted sums of cross products of observations:

$$\mathcal{Z}_i(w) = \sum_{t=1}^{T} \tilde{z}_{it}\tilde{z}_{it}'K_h(w_t - w),\ \mathcal{Y}_i(w) = \sum_{t=1}^{T} \tilde{z}_{it}\tilde{y}_{it}K_h(w_t - w),\ w_t = (\widetilde{w}_{it} - \bar{w}_i)/\sigma_i(\tilde{w}). \tag{21}$$

In (21) we denote $K_h(u) = K(u/h)/h$, where $K(\cdot)$ is a kernel function and h is the so-called bandwidth parameter. From the moments given in (21) the semiparametric estimator is obtained as

$$\hat{\gamma}_i(w) = \mathcal{Z}_i^{-1}(w)\mathcal{Y}_i(w). \tag{22}$$

Note that $\hat{\gamma}_i(w)$ in (22) is identical to the Nadaraya Watson estimator in case of choosing $\widetilde{w}_{it} = \tilde{z}_{it}$.

4.3 Implementation

Cai, Fan and Yao (2000) refer to models like (20) as 'functional coefficient regression models' and illustrate how prominent time series models as, for instance, threshold autoregressions (Tong 1990) or autoregressive models with random coefficients (Nicholls and Quinn 1982) may be embedded in this general model class. In particular, functional coefficient models are motivated as a means to address the potential of parameter variation when analyzing longitudinal data. From the perspective of a well developed statistical theory addressing estimation of and inference within functional coefficient models, it is appealing to employ such a flexible framework to indicate the efficiency loss or a case of spurious results involved when falsely applying structurally invariant econometric models.

Cai, Fan and Yao (2000) discuss asymptotic properties of the local linear Kernel estimator (Fan 1993) employed to estimate functional coefficient models and also provide a cross validation criterion to choose a particular state or factor variable out of a set of possible candidates. As typical in nonparametric regression local estimates suffer from a trade-off between bias and variance. Owing to the use of a kernel function the functional relation formalized in (20) is always evaluated in a neighborhood of the factor w. Therefore, nonparametric estimates may be seen as local averages of the underlying function such that e.g. $\hat{\gamma}_i(w)$ is essentially an estimate of a smoothed version of $\gamma_i(w)$. Since we are interested in the overall behavior of functional coefficients over states of, say, lower vs. higher inflation, our main conclusions will not be affected by (small) local biases. The theoretical results in Cai, Fan and Yao (2000) are derived for (auto)regression designs with stationary variables. As it is known from parametric cointegration modeling estimates of the error correction coefficient will behave as coefficient estimates in stationary models whenever the involved levels of interest rates and inflation are both stationary or both nonstationary but cointegrated. In the latter case the linear combination $ec_{it}^{(j)} = y_{it} - \theta_i^{(j)} z_{it},\ j = 1, 2,$ will be stationary. Therefore we presume the theoretical framework outlined in Cai, Fan and Yao (2000) to cover the considered ECM in (15). With respect to the dynamic OLS regression employed to estimate the (inverted) Fisher coefficient $\theta_i^{(j)},\ j = 1, 2,$ stationarity is clearly an unsuitable assumption. However, from recent work on the statistical properties of nonparametric regression with integrated processes we regard the functional coefficient estimates $\hat{\theta}_i^{(j)}(w)$ to obey the usual properties of nonparametric estimates. In particular, owing to results in Park (2005) $\hat{\theta}_i^{(j)}(w)$ converges to $\theta_i^{(j)}(w)$ and will show some (local) bias in case of a finite bandwidth parameter.

As typical in nonparametric regression the choice of the bandwidth parameter is of crucial importance for the factor dependent estimates detailed in (21) and (22) (Härdle, Hall and Marron 1988). Choosing too small a bandwidth may result in a wiggly pattern of the semiparametric estimates. Choosing h 'prohibitively' large the estimator $\hat{\gamma}_i(w)$ will degenerate to time invariant standard ECM or DOLS parameter estimates. In this sense the adopted semiparametric approach nests the common cointegration analysis and it is interesting to see if a more flexible framework implemented via local averaging will support the view that the relationship between interest and inflation rates is as restrictive as presumed by a parametric model. Given the large number of empirical models employed in this work a data driven bandwidth selection such as cross validation is infeasible to implement. Note that cross validation would in any case deliver a homogeneous bandwidth parameter which could be inappropriate if the density of the respective factors is very heterogenous over the factor support. To solve the trade-off between feasibility and optimality of bandwidth selection for this empirical study we choose the bandwidth parameter locally as

$$h_i(w) = c\hat{f}_i(w)^{-a},$$

where $\hat{f}_i(w)$ is a kernel density estimate for w implemented with factor observations w_t for the i-th member of the cross section. The parameters c, a are chosen as $T^{-0.25}$ and 0.25, respectively. A general treatment of Kernel smoothing with local bandwidth selection is delivered by Jennen-Steinmetz and Gasser (1988) showing in particular that the latter choice of the local bandwidth roughly corresponds to spline smoothing. To implement density estimation for the factor w_t we use $h = 1.06$ as the bandwidth parameter (Silverman 1986) since the empirical standard deviation of w_t is unity by construction. With regard to the Kernel function we use the quartic Kernel, $K(u) = 15/16(1-u^2)^2 I(|u| < 1)$, throughout.

In empirical practice and for a given member i of the cross section the estimator in (22) is typically evaluated for all $w = w_t$, $t = 1, \ldots, T$. Since we are interested in a comparison of $\gamma_i(w)$ over the cross section we will estimate the functional model for each member of the cross section over a grid

$$w_k = -1.5 + 0.03k,\ k = 0, 1, 2, \ldots, 100. \tag{23}$$

Owing to the use of centered and standardized factor variables the empirical results given in Section 5 will be representative for a support of w covering 3 standard errors of the respective empirical distributions of w_t or $\widetilde{w}_{it}, t = 1, \ldots, T$. Thus, under unconditional normality of the particular factors our reported estimation results are informative for approximately 85% of the available sample information.

4.4 Inference for functional coefficient panel models

To illustrate the precision of semiparametric estimates pointwise confidence bands for $\hat{\gamma}_i(w)$ in (22) could be obtained from quantiles of the Gaussian distribution and some variance estimate $\hat{\zeta}_i^2(w)$ or from resampling techniques as outlined in Neumann and Kreiss (1998). For a given member of the cross section the construction of confidence bands for estimates $\hat{\gamma}_i(w)$, $i = 1, \ldots, N$, may provide a number of interesting characteristics of the relation between interest rates and inflation. Along such lines the analyst would gain some guidance in evaluating the adequacy of linear models as specified in (15) or (16). Common panel cointegration models may provide an adequate econometric framework if it were possible to find a (set of) constant(s) covered jointly by all the confidence intervals over the entire support of a particular factor variable w. A candidate constant may be a parametric estimate obtained from standard (panel) cointegration analysis. Alternatively, particular parametric restrictions obtained from economic theory as e.g. the postulated Fisher coefficient of unity may undergo such an empirical test of the parametric model at the pooled level.

As mentioned one merit of the semiparametric model in (20) and its local implementation is that it might give valuable information on the accuracy of the restrictive nature of the parametric model. Instead of characterizing significance of semiparametric estimates by country we will rather base inferential issues on the variation of semiparametric estimates over the cross section similar to concepts that have been introduced under the notion of mean group estimation or inference (Pesaran and Smith, 1995). In particular we will exploit the panel structure of the sample to characterize the mean level of the parameters of interest, $\theta_i^{(j)}$ and $\alpha_i^{(j)}$, $j = 1, 2$. Moreover, we indicate to which extent the parametric estimates obtained from (15) or (16) find support by the overall shape of the more flexible semiparametric estimator. The latter issues of inference are now discussed in turn:

i. Inference in case of cross sectional homogeneity: In the spirt of Phillips and Moon (1999) or Pesaran and Smith (1995) country specific estimates $\hat{\gamma}_i(w)$ may be seen as estimates of one particular true parameter featuring the dynamic relationship between interest and inflation rates. Although the cross sectional homogeneity assumption which is explicit in this argument is likely not supported by the investigated panel it is tempting to illustrate the degree of cross sectional parameter heterogeneity. As a possible statistical quantity characterizing the heterogeneity one may consider for a given value $w = w_k$ of the factor the standard deviation of mean estimates, $\sigma(\bar{\gamma}(w_k))$, $\bar{\gamma}(w_k) = 1/N \sum_{i=1}^{N} \hat{\gamma}_i(w_k)$, as a convenient means to assess the prevalence of parameter heterogeneity. Note that under parameter homogeneity and consistent estimation of $\gamma(w_k)$ pointwise confidence bands like $\bar{\gamma}(w_k) \pm 2\sigma(\bar{\gamma}(w_k))$ cover the true parameter with a probability of approximately 95%.

ii. Inference against time invariance: Under the presumption of a time invariant model we have $\gamma_i(w_k) = \gamma_i(w) = \gamma_i$. Note that in this case the bias of the Kernel estimator in (22) will vanish since local averaging, outlined in (21), will take place over a homogeneous function. As detailed in (23) we evaluate the functional coefficient model for each member of the cross section over a grid consisting of 101 equidistant points providing the support of w. Thus, the quantity

$$\hat{\gamma}_i^{\Delta}(w_k) = \hat{\gamma}_i(w_k) - \frac{1}{101} \sum_{k=1}^{101} \hat{\gamma}_i(w_k) \tag{24}$$

will measure for the i-th member a local deviation of $\gamma_i(w)$ from the time invariant model at $w = w_k$. The idea to describe on systematic over- or underestimation of the true relation

$\gamma_i(w) = \gamma_i$ implied by a parametric model, then, exploits the cross sectional variation in $\hat{\gamma}_i^{\Delta}(w_k)$. If the true model is time invariant the empirical mean of $\hat{\gamma}_i^{\Delta}(w_k)$, i.e.

$$\bar{\gamma}^{\Delta}(w_k) = \frac{1}{N}\sum_{i=1}^{N}\hat{\gamma}_i^{\Delta}(w_k)$$

should be insignificantly different from zero for all grid points w_k. To detect significance of $\bar{\gamma}^{\Delta}(w_k)$ at a particular grid point common confidence bands for sample means may be used. Note that a test of a parametric model along such lines is essentially a test against the 'overall shape' of $\gamma_i(w)$. Significance of $\bar{\gamma}^{\Delta}(w_k)$ is likely in case an underlying factor dependence of γ_i shows a 'similar shape' over the entire cross section.

5 Empirical results from functional coefficient models

In this section we will discuss the results obtained when generalizing the parametric models (15) and (16) towards semiparametric relations. Firstly, we make some preliminary remarks on the applied factors, on a few details of model implementation and the selection of empirical results to be discussed below. Then, we address the two issues raised throughout the paper, namely the significance and/or time invariance of error correction dynamics and the magnitude of the (inverted) Fisher coefficient.

5.1 Preliminary remarks

State dependent modeling of the parameters of interest requires the choice of some factor. As potential factors we regard economic states to be related to the level of interest ($\widetilde{w}_{it} = R_{it}$) or inflation rates ($\widetilde{w}_{it} = \pi_{it}$), their corresponding changes ($\widetilde{w}_{it} = \Delta R_{it}$ or $\widetilde{w}_{it} = \Delta\pi_{it}$) and their absolute changes ($\widetilde{w}_{it} = |\Delta R_{it}|$ or $\widetilde{w}_{it} = |\Delta\pi_{it}|$) the latter of which we regard as an approximation of interest rate or inflation risk. In addition, we also provide some more standard semiparametric estimates where we use the right hand side variable itself as a factor. For instance, when investigating factor dependence of the error correction coefficient in the Fisher relation we formalize a specification

$$\alpha_i^{(1)} = \alpha_i^{(1)}(w),\ \widetilde{w}_{it} = \tilde{z}_{it},$$

where $\tilde{z}_{it}$ is obtained from R_{it-1} by partialling out the effects of variables having time invariant coefficients in (15) by assumption.

As mentioned above we implemented functional coefficient analysis for the error correction dynamics under two presumptions concerning the long run parameter, namely state dependence and time invariance. It turned out that estimation results obtained under both presumptions where qualitatively almost identical and quantitatively very similar. For the latter reason we will show only state dependent error correction estimates obtained under the assumption of a time varying (inverted) Fisher coefficient.

Owing to the use of standardized factors it is feasible to display functional estimates $\hat{\gamma}_i(w)$ against w jointly for all members of the cross section. Given the large dimension of the latter, however, we provide a descriptive statistical measure characterizing the distribution of $\hat{\gamma}_i(w)$ over the cross section for given $w = w_k$. To be precise we will give cross sectional means and corresponding confidence bands to indicate factor dependence of $\gamma_i(w)$. When estimating cross sectional averages of $\hat{\gamma}_i(w_k)$ it turned out that in the boundaries of the support of w_k some cross section specific functional estimates were quite unstable. This result in turn mirrors that over the cross section for some empirical models and particular states w_k only a small number of observations entered the Kernel estimation (22) thereby obtaining unstable functional estimates. Therefore we decided to use "local sample sizes" N_k to implement averaging of estimates $\hat{\gamma}_i(w_k)$ over the cross section such that only those cross section members enter the displayed mean estimates where at least ten observations are used to obtain the cross section specific moments in (21) determining $\hat{\gamma}_i(w_k)$. As one might expect estimated averages $\bar{\gamma}(w_k) = \frac{1}{N_k}\sum_{i=1}^{N_k}\hat{\gamma}_i(w_k)$ are likely based on the full cross section the closer $\hat{\gamma}_i(w_k)$ is determined to the center of the empirical distribution of w.

To illustrate the number of cross section members entering the empirical means $\bar{\gamma}(w_k)$ Figure 3 displays the relevant sample sizes N_k used when averaging estimates $\hat{\theta}_i^{(1)}(w_k)$ and $\hat{\alpha}_i^{(1)}(w_k)$ (left hand side panel) and $\hat{\theta}_i^{(2)}(w_k)$ and $\hat{\alpha}_i^{(2)}(w_k)$ (right hand side panel), respectively. As the underlying factors $\widetilde{w}_{it} = \pi_{it}$ (Fisher relation) and $\widetilde{w}_{it} = R_{it}$ (inverted Fisher relation) are used. Similar results on grid point specific sample sizes N_k are obtained when employing alternative factors as changes or absolute changes of inflation or interest rates. Apparently, in case w_k is close to zero, i.e. a particular factor realization is drawn in the center of its empirical distribution all cross section members enter the arithmetic means $\bar{\gamma}(w_k)$ and the respective confidence bands. In the boundaries of the factor's empirical distribution, however, the relevant number of cross section members entering the empirical averages could shrink to a minimum of 98 economies. It is worthwhile to point out that even a cross section dimension of about $N_k = 100$ is rather large for inference on arithmetic means. Each panel in Figures 4 to 7 displays an empirical average, e.g. $\bar{\gamma}(w_k) = \sum_{i=1}^{N_k} \hat{\gamma}_i(w_k)$, and a confidence band around the latter having a width of ± 2 times the corresponding standard error.

For all parameters of interest ($\theta_i(w_k)^{(j)}$, $\alpha_i(w_k)^{(j)}$, $j = 1, 2$) Table 6 provides unconditional empirical variances evaluated over the cross section and the support of the factor $w_k, k = 1, \ldots, 101$. As it is well known empirical variances allow a decomposition as the sum of inner and outer variance. When measuring parameter variation over two dimensions, the cross section and the factor support, a-priori two ways of decomposing the empirical variance could be considered. Indicating unconditional means with a 'bar' notation the following results hold for the variation of parameter estimates $\hat{\gamma}_i(w_k)$:

$$S^2 = \overline{(\hat{\gamma}_i(w_k) - \bar{\gamma})^2} = \overline{(\hat{\gamma}_i(w_k) - \bar{\gamma}(w_k))^2} + \overline{(\bar{\gamma}(w_k) - \bar{\gamma})^2} \tag{25}$$

$$= \overline{(\hat{\gamma}_i(w_k) - \bar{\gamma}_i(w))^2} + \overline{(\bar{\gamma}_i(w) - \bar{\gamma})^2}, \tag{26}$$

where

$$\bar{\gamma}(w_k) = \frac{1}{N} \sum_i \hat{\gamma}_i(w_k), \; \bar{\gamma}_i(w) = \frac{1}{101} \sum_k \hat{\gamma}_i(w_k),$$

and $\bar{\gamma}$ is the overall average of estimates $\hat{\gamma}_i(w_k)$.

For both, the long run parameter and the error correction coefficient, relating the overall variation of parameter estimates to the inner variation measured over the cross section powerfully underscores the case of cross sectional parameter heterogeneity. About 99% of the observed variation goes back to inner variation measured over the cross section. The latter result holds for the empirical models of the Fisher relation and, as well, for implementations of the inverted specification. Moreover, it is robust over all employed factors inflation or interest rate based economic states or the respective (linearly transformed) explanatory variables. Measuring the inner variation over the support of the employed factor variables the fraction of inner variation is considerably smaller on the one hand but it is also substantial on the other hand. Note that under a presumption of parameter invariance the latter fraction would be zero. It turns out that for the long run parameter depending on the adopted factor inner variation accounts for up to 40% (19.3%) of the overall parameter variation when estimating the parameter by means of the Fisher relation (inverted Fisher relation). With respect to the estimated error correction parameter the corresponding fractions are even higher, obtaining that up to 75% of the overall variation may be interpreted as internal variation measured over the factor support. The variance decompositions concerning the error correction coefficient are also indicative for the

Table 6: Variance decompositions for functional coefficient estimates ($\hat{\gamma}_i(w_k)$)

	Fisher relation						Inverted Fisher relation					
γ_i	w	S^2	S^2_{in}	S^2_{out}	S^2_{in}	S^2_{out}	w	S^2	S^2_{in}	S^2_{out}	S^2_{in}	S^2_{out}
	Inflation based factors						Interest rate based factors					
			114 × 101		101 × 114				114 × 101		101 × 114	
$\theta^{(\bullet)}$	π	234.5	.998	.002	.301	.699	R	3558.5	.998	.002	.154	.846
	$\Delta\pi$	211.0	.997	.003	.356	.644	ΔR	1468.2	.996	.004	.040	.960
	$\lvert\Delta\pi\rvert$	185.7	.999	.001	.173	.827	$\lvert\Delta R\rvert$	1413.0	.998	.002	.030	.970
$\alpha^{(\bullet)}$	π	2.118	.995	.005	.600	.400	R	0.929	.973	.027	.648	.352
	$\Delta\pi$	2.649	.990	.010	.729	.271	ΔR	0.727	.992	.008	.612	.388
	$\lvert\Delta\pi\rvert$	2.231	.992	.008	.689	.311	$\lvert\Delta R\rvert$	0.578	.996	.004	.567	.433
$\tilde{\alpha}^{(\bullet)}$	π	1.288	.989	.011	.561	.439	R	0.603	.986	.014	.574	.426
	$\Delta\pi$	2.322	.991	.009	.694	.306	ΔR	0.705	.991	.009	.631	.369
	$\lvert\Delta\pi\rvert$	2.172	.991	.009	.651	.349	$\lvert\Delta R\rvert$	0.558	.997	.003	.552	.448
	Factor: explanatory variable											
$\theta^{(\bullet)}$	$\tilde{\pi}$	290.2	.993	.007	.400	.600	$\tilde{R}$	4012.9	.999	.001	.193	.807
	$\Delta\tilde{\pi}$	243.1	.997	.003	.183	.817	$\Delta\tilde{R}_-$	1441.0	.997	.003	.067	.933
	$\lvert\Delta\tilde{\pi}\rvert$	175.6	.999	.001	.099	.901	$\lvert\Delta\tilde{R}_-\rvert$	1613.5	.996	.004	.080	.920
$\alpha^{(\bullet)}$	$\tilde{R}_-$	2.622	.980	.020	.737	.263	$\tilde{\pi}_-$	1.493	.990	.010	.747	.253
	$\Delta\tilde{R}_-$	1.903	.984	.016	.706	.294	$\Delta\tilde{\pi}_-$	0.553	.990	.010	.616	.384
	$\lvert\Delta\tilde{R}_-\rvert$	1.350	.989	.011	.702	.298	$\lvert\Delta\tilde{\pi}_-\rvert$	0.593	.995	.005	.593	.407
$\tilde{\alpha}^{(\bullet)}$	$\tilde{R}_-$	2.395	.989	.011	.733	.267	$\tilde{\pi}_-$	1.434	.995	.005	.695	.305
	$\Delta\tilde{R}_-$	1.466	.987	.013	.587	.413	$\Delta\tilde{\pi}_-$	0.628	.989	.011	.634	.366
	$\lvert\Delta\tilde{R}_-\rvert$	1.229	.988	.012	.622	.378	$\lvert\Delta\tilde{\pi}_-\rvert$	0.553	.995	.005	.606	.394

S^2 is the empirical variance of functional coefficient estimates $\hat{\gamma}_i(w_k)$ multiplied by 100. S^2_{in} and S^2_{out} denote the fractions of the corresponding inner and outer empirical variances, respectively. $\theta^{(\bullet)}$ and $\alpha^{(\bullet)}$ are used to indicate variance decompositions for the estimated long run and error correction parameters. $\tilde{\alpha}^{(\bullet)}$ indicates estimation results obtained for the error correction coefficient under the assumption that the underlying long run parameter is time invariant. $\tilde{R}$, $(\tilde{R}_-)$, $\tilde{\pi}$, $(\tilde{\pi}_-)$ are short for factors obtained from (lagged) interest and inflation rates after regressing out remaining variables in the respective ECM or DOLS regression models.

impression that these functional coefficient estimates are only mildly affected when conditioning their estimation on the assumption of a state dependent or state invariant long run parameter. Note that the latter impression is well in line with the relatively small fraction of inner variation (about 30% or 20%, respectively) obtained for the long run parameter over the factor support.

5.2 Fisher relation

5.2.1 Error correction dynamics

The left hand side panels of Figure 4 provide factor dependent estimates of the error correction coefficient in the Fisher relation where inflation ($\widetilde{w} = \pi$, panel A), its changes ($\widetilde{w} = \Delta\pi$, B) and absolute changes ($\widetilde{w} = |\Delta\pi|$, C) are used to describe the economic states. Note that over all employed factors the estimated functional coefficient has an unconditional level of about -0.05 which, in turn, corresponds to the average parametric estimate given in Table 3. In this sense the functional coefficient model may be seen as a general framework nesting the restrictive parametric ECM. Whereas the error correction parameters are rather stable over the support of $\widetilde{w} = \pi$ it appears that the evidence in favor of cointegration (i.e. significance of $\bar{\alpha}^{(1)}(w)$) is weaker over scenarios of relatively high changes of inflation (panel B). Note that a similar impact has already been detected by means of the linear cross sectional regressions discussed in Section 3 (see also Table 3). Given the magnitude of the investigated cross section one may expect some members where either interest rates and inflation fail to cointegrate or interest rates are weakly exogenous. Both latter scenarios may explain functional estimates close to zero. High (positive) changes of inflation may be seen to indicate that a central bank fails to control the development of inflation. Then, the response of interest rates is likely insufficient or ineffective to invoke convergence to some target level of inflation. Moreover, Fischer et al. (2002) illustrate that periods of very high inflation are associated with bad macroeconomic performance due to instability of real interest rates. In turn, very high inflation is likely to go along with large positive changes of inflation ($w > 1.5$).

Conditioning the analysis on inflation risk (Panel C), on average, highest absolute values of the error correction parameter estimates are obtained over economic states showing a level of inflation risk which is about one standard error of the risks unconditional distribution. Over states of even higher uncertainty associated with the development of inflation the interest rate's average reaction to violations of the long run equilibrium does not differ significantly from zero anymore. Note that the obtained U-shaped pattern of state dependence is impossible to detect by means of the linear cross sectional regressions discussed in Section 3. On the one hand, over states of high inflation risk an increasing and potentially nonstationary risk premium may disconnect the equilibrium relation between inflation and interest rates. On the other hand, a high level of $|\Delta\pi|$ characterizes states of high positive inflation changes discussed before (Panel B) and in the same time successful disinflation policies. As argued before and in Section 3 both latter scenarios may disturb the long run relationship.

The upper left panel (A) in Figure 5 shows error correction dynamics conditional on the lagged level of interest rates after partialling out all variables having time invariant parameters by assumption ($\widetilde{w} = \tilde{y}_{-}$) . On average, significant error correction dynamics are clearly diagnosed for states with relatively low levels of lagged interest rates. For the average lagged interest rate ($w \approx 0$) the loading coefficient obtains the lowest values. Over states of high lagged interest rates $\bar{\alpha}^{(1)}(w)$ differs only insignificantly from zero and, thus, interest rates either fail to adjust in response to violations of the long run relationship (weak exogeneity) or the latter does not exist. The latter arguments are well in line with the discussion in Section 3.4. Moreover, a high interest rate may be interpreted has an indicator of a monetary policy stance. If a central bank tries to fight against an unstable price development it is inevitable to gain confidence of market participants. This could be achieved, for instance, by relating the own exchange rate to some reference unit (see Mishkin, 2003 p. 511). If market participants expect the central bank to succeed in inflation fighting the expected inflation declines. In turn, the monetary authority may reduce the interest rate without spurring inflation. Along these lines the traditional view at the relation between inflation and interest rates is weakened as is the evidence for cointegration.

When conditioning on lagged changes of interest rates the average error correction estimates (panel D of Figure 5) show a hump shaped pattern such that the strength of a consistent response of interest rates to violations of the long run equilibrium is the highest over states of small (w=-1.0) or relatively large ($w = 0.8$) interest rate changes. Although obtained over a huge cross section it is worthwhile to note that the latter result is similar to the country specific analyses in Mishkin (1992) and Mishkin and Simon (1995) detecting cointegration *locally* over periods of trending variables. Over states with $w \approx 0$ interest rates are stable and weak, if any, adjustment dynamics are at work. Over states of more extreme interest rate changes ($w \geq 1.2$) the average coefficient estimate increases such that the evidence in favor of a cointegration relationship is weakened. The latter impression, however, mirrors the evidence in Panel A of Figure 5 when noting that states of high (positive) interest rate changes are likely to coincide with states of high interest rates.

The upper line of Figure 6 (Panels A and C) provides empirical mean estimates and a corresponding 2 standard error band obtained when testing the factor dependent model against the time invariant parametric specification. The factors used are inflation ($\widetilde{w} = \pi$) and changes of inflation ($\widetilde{w} = \Delta\pi$). Corresponding results obtained when using $\widetilde{w} = y_{-}$ and $\widetilde{w} = |\Delta y_{-}|$ are given in the right hand side panels of Figure 5 (E and F). If the parametric model were, on average, an adequate representation of the Fisher relation the displayed mean estimates should not differ from zero. It turns out, however, that the mean parametric estimate, $\bar{\alpha}^{(1)} = -0.05$ say, on average, overestimates factor specific error correction dynamics over states of small changes of inflation ($w = -1.0$, Figure 6, Panel D), medium levels of lagged interest rates ($w = -0.1$, Figure 5, Panel E) and relatively larger degrees of lagged interest rate risk ($w = 0.4$, Figure 5, Panel F). In all these cases the response of interest rates to violations of the equilibrium relation is stronger according to the state specific model in comparison with a parametric unconditional evaluation.

5.2.2 The Fisher coefficient

A factor dependent evaluation of the cointegration coefficient (Figure 4, Panels D,E and F) reveals that on average $\theta^{(1)}(w)$ is greater than zero irrespective of the actual inflation rate ($\widetilde{w} = \pi$, Panel D), its changes ($\widetilde{w} = \Delta\pi$, E) or inflation risk ($\widetilde{w} = |\Delta\pi|$, F). The confidence bands constructed around mean estimates $\bar{\theta}^{(1)}(w)$ do not cover the zero line. The latter impression may be seen as a strong descriptive hint at the existence of long run link between interest rates and inflation. In case of spurious regression and or diverging degrees of integration of the two variables the average parameter estimate is likely not significantly different from zero. Moreover, over most considered conditioning scenarios the provided confidence bands fail to cover unity. Some exceptions are medium states of inflation ($w \approx 0.0$, Panel D) or scenarios where inflation risk is somewhat above its mean unconditional level ($w = 0.8$, F). Note that the estimated functional coefficient appears to vary around an unconditional level of about 0.60 which, in turn, is the parametric estimate provided in Table 3. Using $\widetilde{w} = \Delta\pi$ to indicate alternative economic states eyeball inspection reveals that the average long run parameter estimate increases with changes of inflation. For the largest factor values ($w \geq 1.0$) the average cointegration parameter is not significantly different from unity. Given a failure of cointegration over states of high (positive) inflation changes as discussed above (see also Figure 4, Panel B), however, the latter result might be spurious rather than indicating some tendency towards a one to one relationship between inflation and interest rates.

To illustrate time dependence of the Fisher coefficient from some other perspective the lower line of Figure 6 (Panels B and D) displays the outcome of statistics contrasting the factor dependent specification against the time invariant model. Indicating economic states by means of the changes of inflation it appears that the true underlying functional shape is indeed increasing as is the average difference from the time invariant model. According to the overall coverage of the provided confidence bands the latter trend is significant. Moreover, as shown in the left hand side panels of Figure 5 (Panels B and C) it appears that the time invariant parametric estimator $\hat{\theta}^{(1)}$ is too large on average over states of medium changes of inflation ($\widetilde{w} = \Delta z, w = 0.0$, Panel B) and lower or higher states of inflation risk ($\widetilde{w} = |\Delta z|, w = -1.0, w = 0.8$, C). In contrast, the parametric estimator underestimates the Fisher coefficient over states of medium inflation risk ($\widetilde{w} = |\Delta z|, w = 0.0$, C).

5.3 Inverted Fisher relation

To complete the analysis selected factor specific estimates obtained when modeling the inverted Fisher relation are provided in Figure 7. Economic states are described by the level of interest rates and changes thereof. Over the considered support of interest rates the estimated error correction coefficients are significantly negative on average. For the long run coefficient mean functional estimates conditioned upon the level of interest rates appear to support the conclusion from the parametric model documented in Table 3 that the corresponding true parameter is likely around unity. In fact, conditioning upon the level of interest rates $\bar{\theta}_w^{(2)}$ almost parallels the unit line. It is worth to note that this finding is in line with estimation result documented by Crowder (2003). The average estimate of the long run parameter turns out to be remarkably stable not just when using interest rates as factor but also when regarding other factors as the explanatory variables in the DOLS regression (16) or states defined by means of interest rate risk. For space considerations we do not provide corresponding Figures but refer to the discussion of the results in Table 3 (Section 3.5) and Table 6 at the beginning of this Section.

Opposite to the long run coefficient the average level of the error correction parameter is likely state dependent (Figure 7, Panel C). The largest adjustment of the inflation rate in response to the violation of the equilibrium relationship between interest rates and inflation is obtained over states of relatively low interest rates. A priori one may regard a low level of interest rates to indicate a successful monetary policy that enables the monetary authorities to achieve price stability.

A similarly increasing pattern of average estimates is also obtained when comparing the factor dependent model with an time invariant parameterization of the model. Here the average parametric estimate given in Table 3 ($\bar{\alpha}^{(2)} \approx -0.05$) underestimates (overestimates) in absolute value the state specific dynamics whenever interest rates are relatively low (high) (Figure 7, Panel B). Similarly, as shown in the lower right panel of Figure 7 the time invariant model underestimates state specific error correction dynamics over states of medium changes of inflation (Figure 7, Panel D).

6 Conclusions

This study investigates the link between inflation and nominal interest rates for a cross section of 114 economies over a period covering at most 43 years of monthly observations. Analyzing the implications of the Fisher hypothesis we focus on common econometric approaches to integration and cointegration testing. Cross section specific results indicate that key parameters of the econometric model are hardly homogeneous over the cross section. As a consequence, nonstationary panel models fail to allow consistent conclusions. In addition, cross sectional regressions give rise to the conjecture that empirical results may depend on specific economic features like the level of inflation, interest rates, their changes or absolute changes. Since the latter states change over time we adopt a general country and time specific semiparametric approach to estimate the Fisher coefficient and the error correction parameter. The applied functional coefficient models support the view that both error correction dynamics and the long run coefficient are heterogeneous over the time and cross section dimension.

Overall, our results show that the Fisher coefficient is likely less than unity. Interest rates and inflation are found to exhibit a long run equilibrium relation for numerous economic states. However, in states of, for instance, large positive changes of inflation, high inflation risk or high interest rates, a long run equilibrium relation, as presumed by Fisher (1930), may not exist. Hence, the analysis allows to characterize economic conditions where a basic economic relationship like the postulated stability of real interest rates fails.

Appendix: List of countries

Algeria, Argentina, Armenia, Austria, Bahamas, Bahrain, Bangladesh, Barbados, Belgium, Benin, Bolivia, Botswana, Brazil, Bulgaria, Burkina Faso, Burundi Cameroon, Canada, Central African Republic, Chad, Chile, China Hong Kong, Colombia, Democratic Republic of Congo, Republic Congo, Costa Rica, Cte d'Ivoire, Croatia, Cyprus, Czech Republic, Denmark, Dominica, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Fiji, Finland, France, Gabon, Gambia, Georgia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea Bissau, Guyana, Haiti, Honduras, Hungary, Iceland, India, Indonesia, Israel, Italy, Jamaica, Japan, Jordan, Kazakhstan, Kenya, Korea, Kuwait, Laos People's Democratic Republic, Luxembourg, Macedonia, Mali, Malta, Malaysia, Mauritius, Mexico, Mongolia, Myanmar, Namibia, Nepal, Netherlands, Netherlands Antilles, Nicaragua, Niger, Nigeria, Norway, Pakistan, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Senegal, Seychelles, Sierra Leone, Singapore, Slovak Republic, Slovenia, Solomon Islands, South Africa, Spain, Sri Lanka, Swaziland, Sweden, Switzerland, Tanzania, Thailand, Togo, Trinidad, Tunesia, Turkey, Uganda, United Kingdom, United States of America, Uruguay, Zambia, Zimbabwe.

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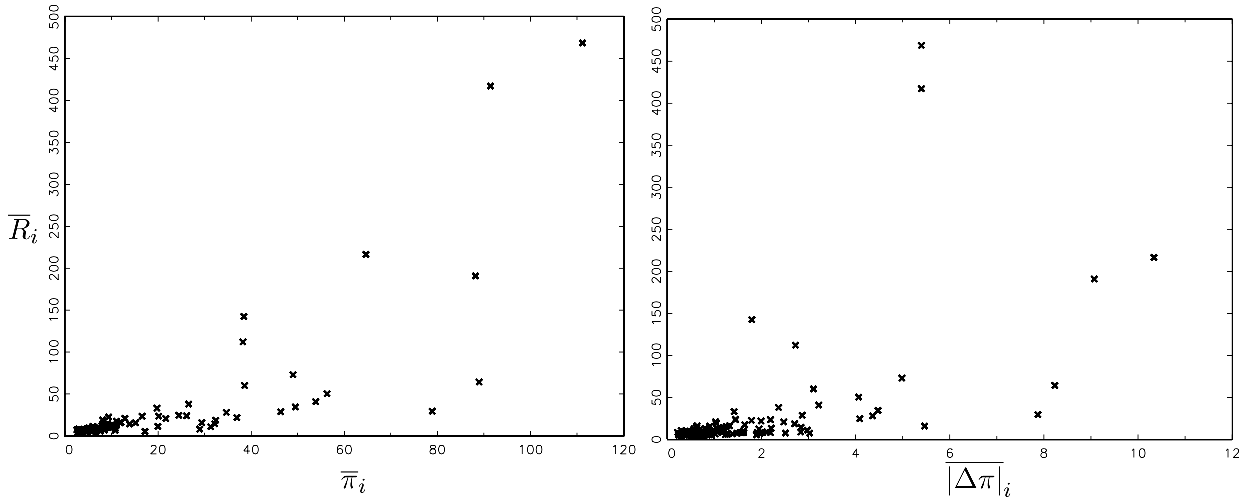

Figure 1: Scatter plot of mean of interest rates $\overline{R}_i$ against mean inflation rates $\overline{\pi}_i$ (left hand side panel) and (right hand side panel) and average absolute changes of inflation ($\overline{|\Delta\pi|}_i$) (inflation risk measure). The cross section dimension is $N = 114$.

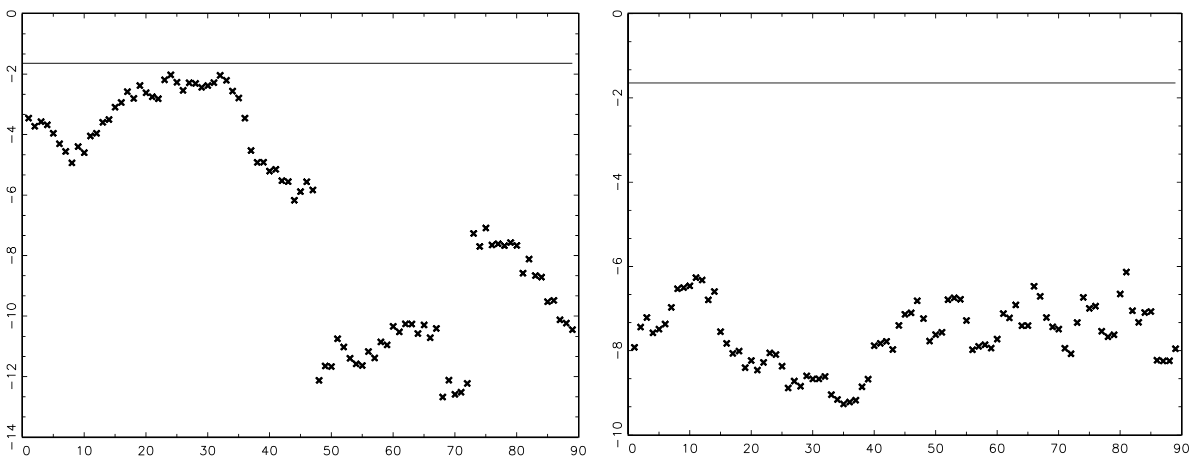

Figure 2: Recursively obtained ECM based panel statistics (Westerlund test) to test the null hypothesis of no cointegration. Each implementation of the test includes 25 members of the cross section which are ordered according to the average level of inflation. Test results from modeling the Fisher relation and the inverted specification are shown in the left and right hand side panel, respectively.

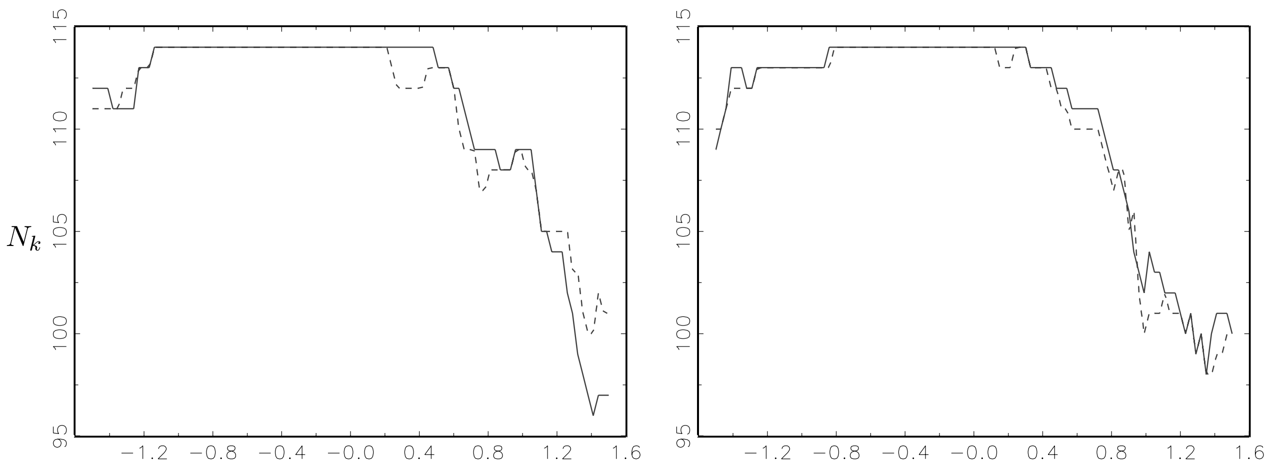

Figure 3: Number of cross section members N_k entering average factor dependent estimates within the Fisher relation (left hand side panel, factor $\widetilde{w}_{it} = \pi_{it}$) and the inverted Fisher relation (right hand side panel, $\widetilde{w}_{it} = R_{it}$) evaluated at $w = w_k$. The solid curve corresponds to $\gamma_i(w_k) = \theta_i^{(1)}$ and $\gamma_i(w_k) = \theta_i^{(2)}$, whereas the dashed line provides N_k for functional error correction estimates $\gamma_i(w_k) = \alpha_i^{(1)}$ and $\gamma_i(w_k) = \alpha_i^{(2)}$.

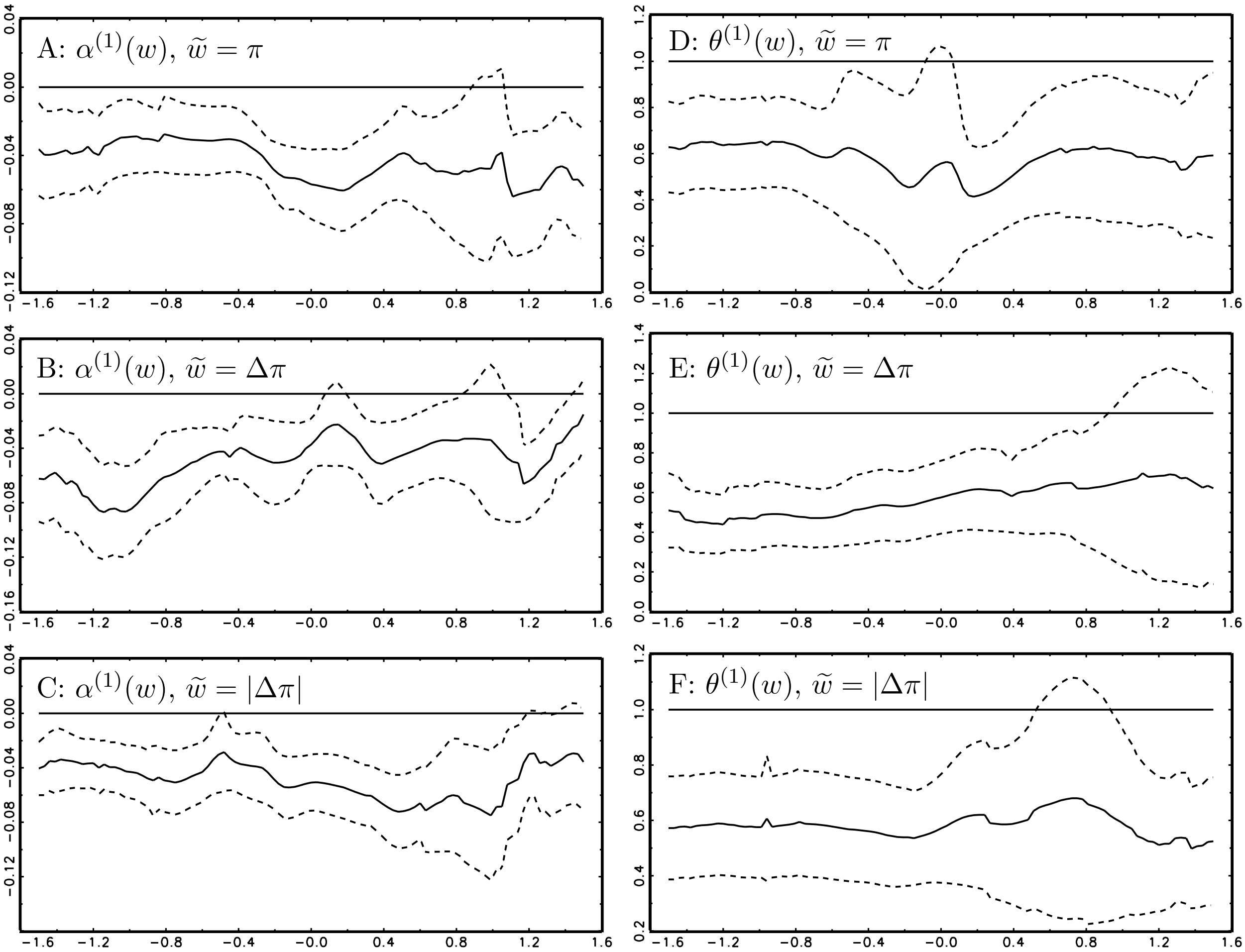

Figure 4: Empirical analyses of the Fisher relation (I): Functional coefficient estimates ($\alpha^{(1)}$) obtained from ECMs (left hand side panels) and DOLS estimates ($\theta^{(1)}$) (right hand side panels) conditional of inflation based factors. $\widetilde{w}$ and w indicate the observed and standardized factor, respectively. The solid line gives mean estimates whereas the dashed curves display confidence bands covering the mean $\pm$ 2 times its standard error.

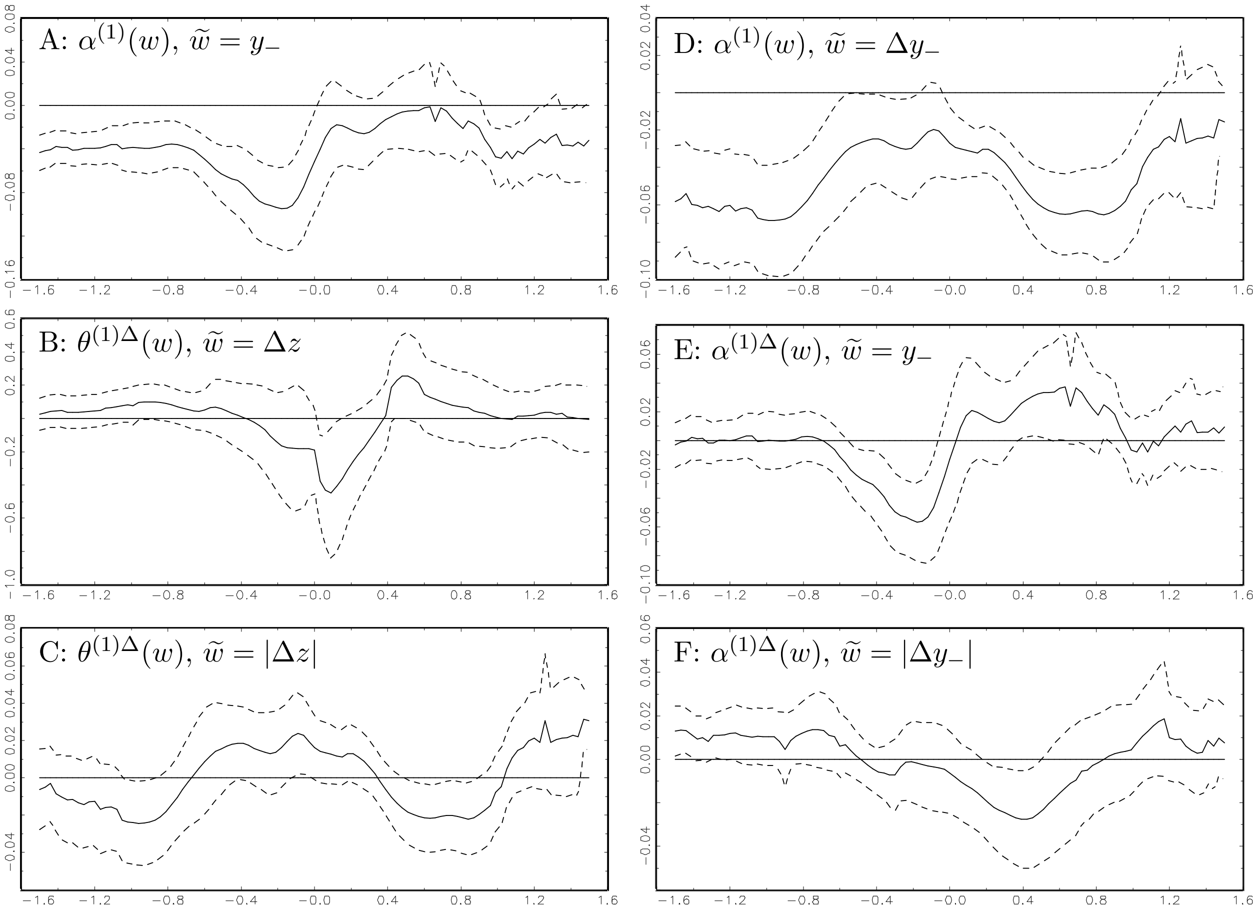

Figure 5: Empirical analysis of the Fisher relation (II): Error correction parameter estimates (upper line) and tests of parameter invariance (medium and bottom line). The respective factors are derived from the respective regression model (16) or (15) after partialling out variables having time invariant parameters by assumption. $\widetilde{w}$ and w indicate the observed and standardized factor, respectively. The solid line gives mean estimates whereas the dashed curves display confidence bands covering the mean $\pm$ 2 times its standard error.

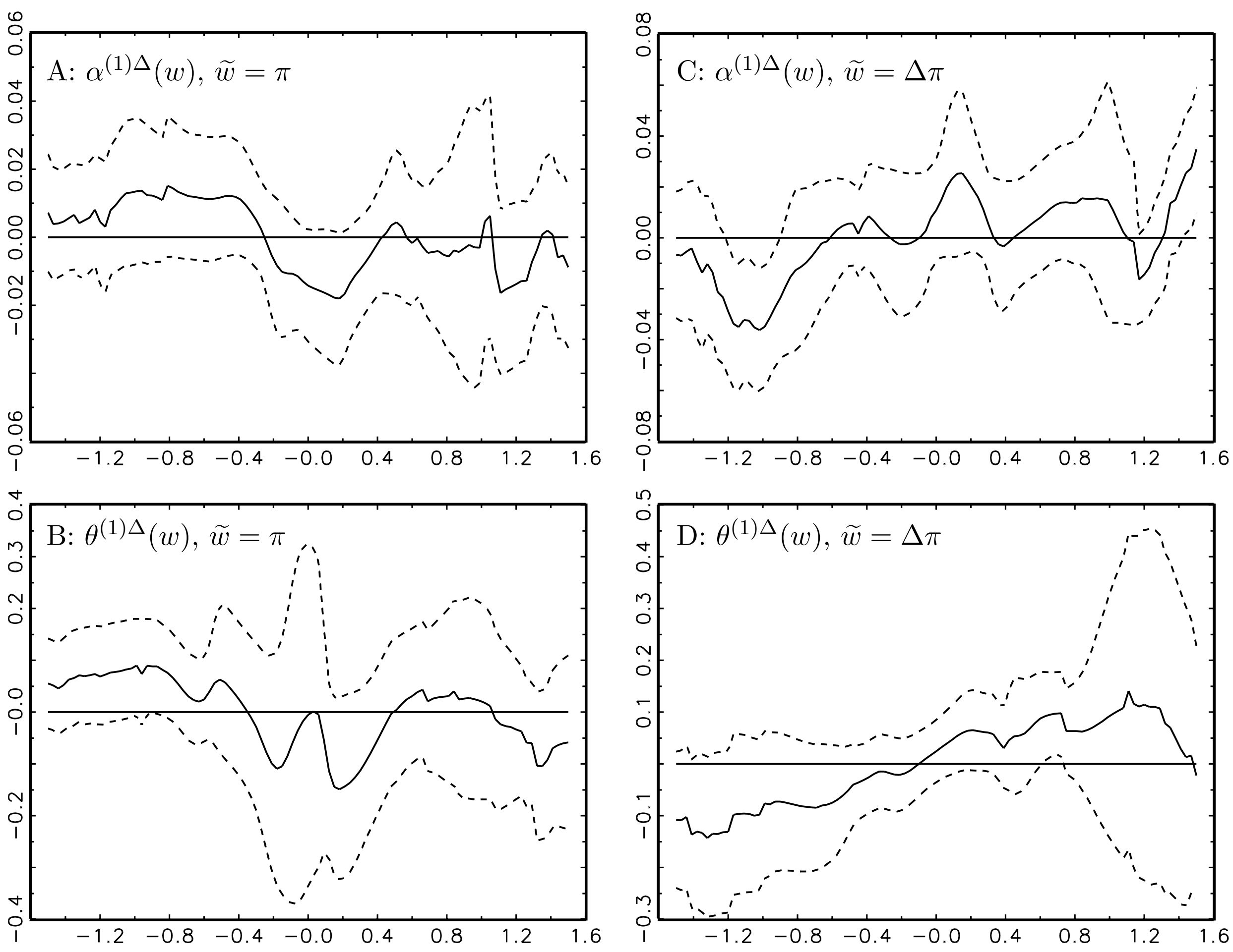

Figure 6: Empirical analysis of the Fisher relation (III): Testing parameter invariance over factors derived from inflation. Error correction parameter estimates $\alpha^{(1)\Delta}$, upper line) and DOLS based estimates ($\theta^{(1)\Delta}$, lower line) are distinguished. $\widetilde{w}$ and w indicate the observed and standardized factor, respectively. The solid line gives mean estimates $\bar{\gamma}^{\Delta}(w_k)$ whereas the dashed curves display confidence bands covering the mean $\pm$ 2 times its standard error.

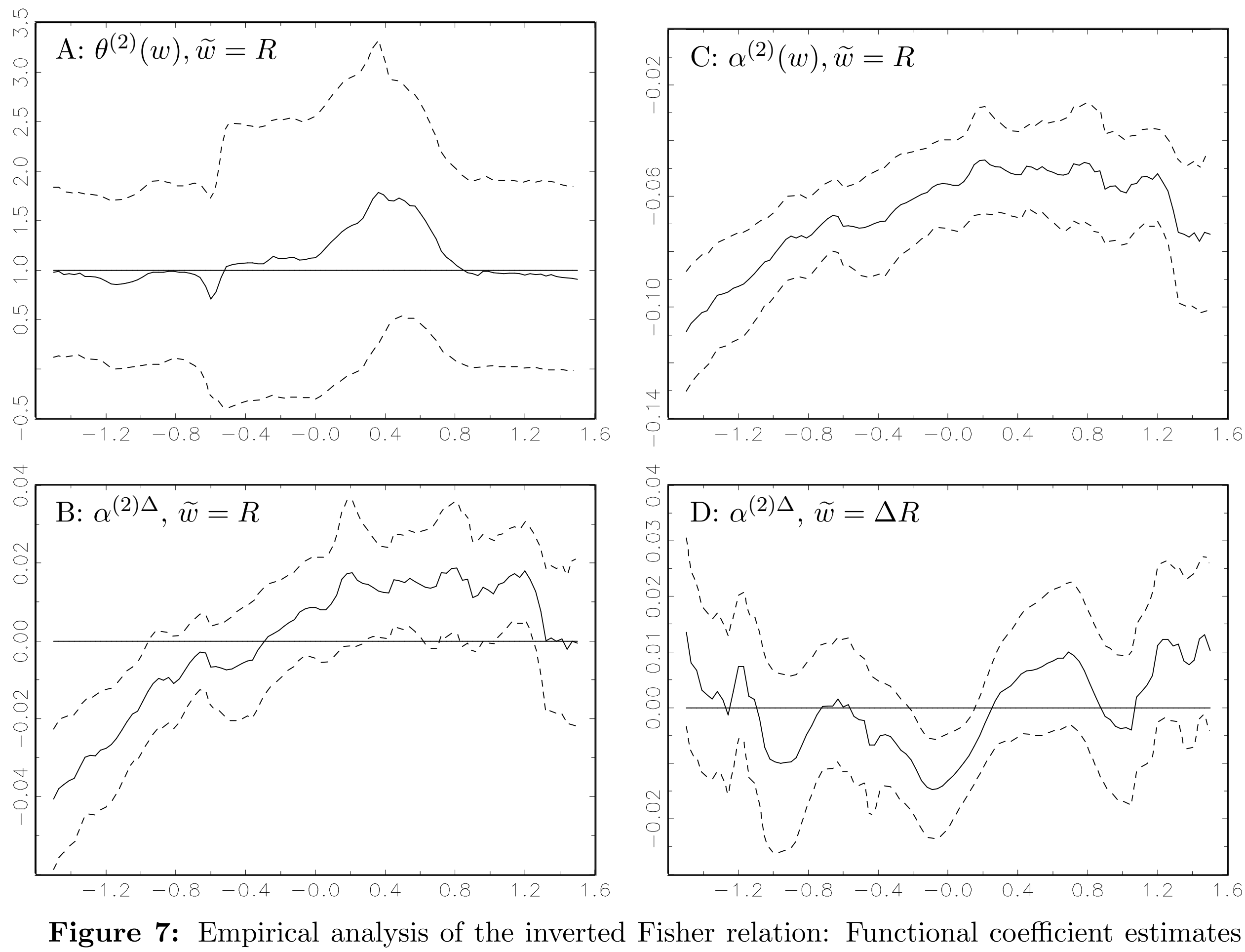

Figure 7: Empirical analysis of the inverted Fisher relation: Functional coefficient estimates (upper line) and test of parameter invariance with factors derived from the state of interest rates. $\widetilde{w}$ and w indicate the observed and standardized factor, respectively. The solid line gives mean estimates whereas the dashed curves display confidence bands covering the mean $\pm$ 2 times its standard error.