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Does Purchasing Power Parity Hold Sometimes? Regime Switching in Real Exchange Rates

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Does purchasing power parity hold sometimes?

Regime switching in real exchange rates

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Abstract

Real exchange rates are quite persistent. Standard unit root tests are not very powerful in drawing a conclusion regarding the validity of purchasing power parity [PPP]. Rather than asking if PPP holds throughout the whole sample period, we examine if PPP holds sometimes by employing Hamilton-type (1989) Markov regime switching models. There are various reasons that the persistence of real exchange rates changes over time. When at least one of multiple regimes is stationary, PPP holds locally within the regime. Employing 5 real exchange rates spanning more than 100 years, we find strong evidence that the strength of PPP is changing over time. We make comparisons to an early work throughout the article. The new model selection criterion, provided by Smith et al. (2006), called the Markov switching criterion devised especially for discriminating Markov regime switching models, unambiguously indicates a preference for the Hamilton-type Markov regime switching model employed in this article. Also, the evidence for PPP is not much different during the Bretton-Woods and current float periods whether PPP holds or not.

JEL classifications: C22, F31

Key words: Regime switching, real exchange rates, Markov switching criterion, purchasing power parity

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1. Introduction

Does purchasing power parity hold? This question is asked repeatedly in international finance literature. Readers are encouraged to see, for example, Froot and Rogoff (1995), Rogoff (1996), Sarno and Taylor (2002), Taylor and Taylor (2004), and Taylor (2006, 2003) for a survey on the vast literature about the validity of purchasing power parity [PPP]. When real exchange rates are found to be stationary by unit root tests, it is commonly said that PPP holds. Given that real exchange rates are quite persistent, however, unit root tests are not very powerful and one may be forced to conclude that PPP does not hold (see e.g. Engel (2000) and Ng and Perron (2002)). The fact that the unit root null is so hard to reject indicates that, even if they are finite, deviations from PPP can last a long time. More powerful unit root tests and multivariate models are also applied, and data series with longer spans and panels with more countries are compiled to further increase the power of the tests (see e.g. Abuaf and Jorion (1990), Lothian and Taylor (1996), Taylor (2002), and Elliott and Pesavento (2006)). However, no consentaneous answers emerge yet.

Rather than asking if PPP holds all the time, we may consider whether PPP holds *sometimes* by looking at variation in the strength of PPP. There are indeed various mechanisms that could induce regime switches in real exchange rates. For instance, the behavior of real exchange rates is known to depend on nominal exchange rate arrangements (see e.g. Mussa (1986), Baxter and Stockman (1989), and Grilli and Kaminsky (1991)). It would be interesting to test if PPP holds better during fixed or flexible nominal exchange rate regimes. Also, a large real interest rate differential may induce mean-reversion in real exchange rates as shown e.g. in Dumas (1992). Rapid inflation in the U.S. may also weaken the strength of PPP. Further, stochastic central bank interventions in the foreign exchange rate markets may induce regime switching behavior (see e.g. Taylor (2004) and Lee and Chang (2007)). Market fundaments, themselves, that determine the behavior of real exchange rates could be regime-switching (see e.g. Sarno et al. (2004) and Vigfusson (1997)).

To test for the presence of locally stationary regimes where PPP holds, we may apply, for instance, the newly developed procedure by Leybourne et al. (2007) that allows multiple changes in persistence between $I(0)$ and $I(1)$ regimes at *a priori* unknown break dates. Yoon (2006) shows that there is ample evidence for regime changes between stationarity and nonstationarity by employing the procedure to the real exchange rates compiled by Taylor (2002) and extended by Lopez et al. (2005a). Also, Huang and Kuan (2006) propose another approach based on Innovation Regime-Switching model in Kuan et al. (2005).

Also, the Markov regime switching model of Hamilton (1989) would be an ideal tool to find if the strength of PPP varies over time. If at least one of multiple regimes is stationary, it could be said that PPP holds within the locally stationary regime. Indeed, the Markov regime switching model is already widely applied to various exchange rates. For instance, Engel and Kim (1999) estimate a 3-state Markov regime switching model for the US/UK real exchange rate. Kanas (2006) recently applies a 2-state Markov regime switching model to the real exchange rates in the Taylor's (2002) data series. Kanas finds that the stationarity of the real exchange rates is really regime-dependent. Also, Engel and Hamilton (1990), Engel (1994), and Klaassen (2005), among many others, apply Markov regime switching models to nominal exchange rates. While the one employed in Kanas (2006)[1] is a valid Markov regime switching model, it is somewhat different from the standard Hamilton-type (1989) model. It turns out that the standard Hamilton-type Markov regime switching model utilizes more information about the states of real exchange rates, without introducing additional parameters to estimate. It produces quite different dynamic behavior from that found in Kanas06 for the real exchange rates that we studied in our work described here. We show that the Hamilton-type Markov regime switching model, even though it is apparently more complicated,

[1] For an efficiency of exposition, the reference to Kanas (2006) will be denoted Kanas06 from now on.

contains simpler dynamics than the Markov switching model in Kanas06. Also, a new model selection criterion, called Markov switching criterion [*MSC*], devised especially for discriminating Markov regime switching models by Smith et al. (2006), unambiguously implies a preference for the Hamilton-type Markov regime switching model for all the real exchange rates we studied here. Also, we find strong evidence that PPP holds sometimes, even though not always, for the samples spanning more than 100 years. It does not appear, however, that the variation in the strength of PPP is closely related with nominal exchange rate regimes. For instance, for most countries, there is little difference in the stationarity of their real exchange rates over the Bretton-Woods and current float periods. This is true whether PPP holds or not. Overall, the new estimation results from the Hamilton-type Markov regime switching model provide quite an improvement for most countries over those in Kanas06 on behavior of the real exchange rates.

Before progressing any further, we add some remarks for clarification. For the Markov regime switching models we studied, we know that local stationarity does not necessarily imply global stationarity; for instance, even if both regimes are stationary, the overall process could be globally nonstationary. Also, it could be globally stationary even with locally explosive regimes. Francq and Zakoïan (2001) among others provide detailed discussions on the stationarity of Markov-switching ARMA models. However, their conditions for stationarity, which involve the top Lyapunov exponent and the spectral radius associated with the models, are not trivial to apply to the ones estimated in this study. It is not clear yet if our estimated Markov regime switching models are globally stationary or not.[2] Hence, even if, for instance, a locally nonstationary regime is found, the overall process could be weakly stationary, and hence PPP holds globally. Also, the overall process could be strictly, but not weakly, stationary, so that we may argue that PPP is not violated globally in that real exchange rates are not nonstationary.[3] Rather than the global stationarity of Markov

[2] Their conditions for stationarity do not require local stationarity.

[3] One may instead argue that PPP still does not hold because the real exchange rates are not weakly

regime switching models, our study is concerned only with their local stationarity in order to examine the variation in the strength of PPP over time. With this qualification in mind, PPP is said to hold sometimes when there is at least one locally stationary regime. While many other specifications of Markov regime switching models are possible, only two will be discussed in the next section.

2. Two Markov regime switching models for real exchange rates

Kanas06 estimates the following Markov switching model with what he calls a Markov switching augmented Dickey-Fuller [ADF] regression:

$$\Delta q_t = a(s_t) + b(s_t) q_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta q_{t-i} + u_t , \tag{1}$$

where q_t denotes a real exchange rate and $u_t \sim N(0, \ \sigma^2)$. s_t is an unobservable latent variable which follows a first order Markov process with a constant probability of transition from regime *i* to *j*, $p_{ij} = \Pr(s_{t+1} = j \mid s_t = i)$. Only intercept $a(s_t)$ and $b(s_t)$ are assumed to be regime-dependent; while the autoregressive coefficients γ_i and $Var(u_t) = \sigma^2$ are not. When $b(\cdot) < 0$ for a certain regime, q_t is locally stationary and PPP holds within the regime. Also, when $b(\cdot) = 0$, q_t becomes $I(1)$ locally and PPP does not hold in the regime. A similar specification is also employed in e.g. Kanas and Genius (2005) and Hall et al. (1999).[4] Also, when $b(s_t) = 0$ for all *t*, (1) becomes what Clements and Krolzig (2003) call a Markov switching in the intercept model. For

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stationary anyway.

[4] Both allow the autoregressive coefficients γ_i to be regime-dependent as well, $\Delta q_t = a(s_t) + b(s_t) q_{t-1} + \sum_{i=1}^{p} \gamma_i (s_t) \Delta q_{t-i} + u_t$, while the error term u_t might also display regime-dependent heteroskedasticity. Sarno et al. (2004) estimate a multivariate model.

instance, Clements and Krolzig (1998) apply the model to the U.S. GNP along with regime-dependent heteroskedasticity, and Bergman and Hansson (2005) to real exchange rates.

Even though (1) is a valid Markov switching model and seems a natural generalization of ADF regressions, see (5) below, it is somewhat different from the standard Hamilton-type (1989) Markov regime switching models. In our work, we consider the following Markov regime switching model:

$$\Delta q_t - a(s_t) = b(s_t) q_{t-1} + \sum_{i=1}^{p} \gamma_i \left\{ \Delta q_{t-i} - a(s_{t-i}) \right\} + u_t , \tag{2}$$

which is more closely related with the one used in e.g. Hamilton (1989) and Kim and Nelson (1999), except for the $b(s_t) q_{t-1}$ term. The extra term intends to capture the regime-dependent persistence in q_t. While it appears more complicated than (1), no additional parameters are needed to be estimated in (2). In fact, the following argument shows that model (2) has a simpler dynamic structure than model (1). It can be easily shown that model (1) implies

$$\Delta q_t = \gamma^{-1}(L) a(s_t) + \gamma^{-1}(L) b(s_t) q_{t-1} + \gamma^{-1}(L) u_t , \tag{3}$$

where L is the lag operator and $\gamma(L) = 1 - \sum_{i=1}^{p} \gamma_i L^i$. Instead, model (2) implies that

$$\Delta q_t = a(s_t) + \gamma^{-1}(L) b(s_t) q_{t-1} + \gamma^{-1}(L) u_t \tag{4}$$

Hence, model (1) contains more dynamics for Δq_t due to the persistent effects of regime switches reflected in $\gamma^{-1}(L) a(s_t)$. The following discussion further highlights the difference between the two Markov switching models. For simplicity, assume now that $b(s_t) = 0$ for all t in (1) and (2). With $\gamma(L) = 1 - \sum_{i=1}^{p} \gamma_i L^i$, (3) or (1) becomes

$$\Delta q_t = \gamma^{-1}(L) a(s_t) + \gamma^{-1}(L) u_t , \tag{1$'$}$$

whereas (4) or (2) becomes

$$\Delta q_t = a(s_t) + \gamma^{-1}(L)u_t. \qquad (2)'$$

$(2)'$ clearly has a simpler structure for Δq_t. In addition to the (lagged) contributions of the error terms, $(2)'$ could exhibit discrete regime switches in Δq_t depending on the state of the variable at time *t*, while $(1)'$ could display more persistent effects of regime changes on Δq_t. For instance, employing the maximum likelihood estimation results reported below in Table 5 for Australia, the two Markov regime switching models $(1)'$ and $(2)'$ are simulated. One particular realization from the two models is plotted in Figure 1. The vertical lines indicate regime changing dates. The figure clearly shows that while model $(1)'$ exhibits more persistent effects of regime switches; model $(2)'$, shown with a broken line, exhibits a step-function like behavior in Δq_t. It also appears that model $(1)'$ responds more in magnitude to regime changes than model $(2)'$ does, at least under the parameter values employed here for these simulations. Clearly, a particular model would not always produce better inference results than another, but some features, such as e.g. data frequency, should determine the proper regime switching model between (1) and (2). For the annual real exchange rates we studied, however, as adjustments to regime changes are expected to be made within a time span less than a year, model (2) should yield improved inference results over model (1).[5] When $p = 0$, the two models (1) and (2) are equivalent. Clearly, when $a(s_t)$ and $b(s_t)$ are not regime-dependent so that $a(s_t) = a$ and $b(s_t) = b$ for all *t*, both (1) and (2) become a standard ADF regression,

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[5] Hamilton (1993, p.237) notes that model (2) "will provide a more promising description of many economic and financial time series." Both models are also compared in Hansen (1992) for the U.S. GNP growth rate. He employs his (nonstandard) likelihood ratio tests.

$$\Delta q_t = a' + b q_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta q_{t-i} + u_t , \tag{5}$$

where $a' = a$ in (1) and $a' = \gamma(1) a$ in (2).

While model (1) is a valid Markov switching model, our study shows that, in general, model (2) yields more reliable inference results for the 5 real exchange rates studied here. For instance, the maximized log likelihood values of model (2) are usually higher, sometimes very much, than those of model (1) for most countries. Clearly, their log likelihood functions are not the same. For model (1),

$$\log\left(l\left(\Delta q_t \mid I_t, S_t\right)\right) = -\frac{1}{2}\log\left(2\pi\sigma^2\right) - \frac{\left(\Delta q_t - a(s_t) - b(s_t) q_{t-1} - \sum_{i=1}^{p}\gamma_i \Delta q_{t-i}\right)^2}{2\sigma^2} , \tag{6}$$

while for model (2),

$$\log\left(l\left(\Delta q_t \mid I_t, S_t, S_{t-1}, \dots, S_{t-p}\right)\right) = -\frac{1}{2}\log\left(2\pi\sigma^2\right) - \frac{\left(\Delta q_t - a(s_t) - b(s_t) q_{t-1} - \sum_{i=1}^{p}\gamma_i \left(\Delta q_{t-i} - a(s_{t-i})\right)\right)^2}{2\sigma^2} \tag{7}$$

where $I_t = \{\Delta q_t, \Delta q_{t-1}, \dots, \Delta q_{t-p}\}$. Compared to (6), (7) shows clearly that model (2) utilizes more information about the states of Δq_t than model (1) does. In fact, expressed into the formulation of model (1), model (2) becomes

$$\begin{aligned}\Delta q_t &= \left\{ a(s_t) - \sum_{i=1}^{p}\gamma_i a(s_{t-i}) \right\} + b(s_t) q_{t-1} + \sum_{i=1}^{p}\gamma_i \Delta q_{t-i} + u_t \\ &= \gamma(L) a(s_t) + b(s_t) q_{t-1} + \sum_{i=1}^{p}\gamma_i \Delta q_{t-i} + u_t ,\end{aligned}$$

and its intercept depends on the states of the current and past p periods, while it depends only on the current state in model (1). Additionally, along with $p = 2$, only 2 states Markov regime switching models are considered in this study. Also, of the two regimes, the less persistent, or more stationary regime, is called the first regime, and the nonstationary or $I(1)$ regime the second regime.

Finally, even though not treated here, there is an interesting extension of model (2), and for that matter of model (1). Assuming further that

$$E\left[b(s_t)\right]=0, \tag{8}$$

q_t now has a unit root only on average, while it is still subject to Markov regime changes. Under the restriction (8), the model might be called a Markov regime switching stochastic unit root process, MRS-STUR for short. It would be an interesting extension of the STUR processes discussed at length in e.g. Granger and Swanson (1997), Leybourne et al. (1996), and Leybourne et al. (1996), and also see Hu (2005). In the next section, a rationale on the choice of some of the real exchange rates from the data series studied in Kanas06 is given and preliminary data analysis results are also provided.

3. Real exchange rate data and some preliminary results

Taylor (2002) compiles real exchange rate series using consumer price deflators or, when they are not available, GDP deflators, for 20 countries for the set of years from 1850 to 1996. Lopez et al. (2005a) extend the data series by adding two more years, through 1998, for 16 countries to focus on developed countries. As the sample periods span over different nominal exchange rate arrangements, it would be crucial to examine regime changing behavior in the real exchange rates. This is a unique data set that could provide interesting answers about the behavior of real exchange rates or about the validity of PPP (see e.g. Lopez et al. (2005b), Kanas06, Papell and Prodan (2006), and Yoon (2006, 2007), who also employ the same data set). The data series are available at the data archive of the Journal of Money, Credit and Banking.

This study examines only the following subset of the 16 countries studied in Kanas06: Australia, Finland, Italy, Norway, and Switzerland. Figure 2 shows their real exchange rates against the U.S. dollar in levels and first differences. The starting years are not the same. These years are listed in Table 1. We decide to work only on the above 5 countries for the following reason. For 13 countries in total, including the 5 countries, Kanas06 finds evidence for Markov switching; his Markov switching ADF regression (1) yields significantly higher log likelihood values than the liner model (5) does without regime switching, so that the null of no Markov switching is rejected by the Davies's (1987) test. For the 5 countries in particular, however, their transition probabilities between two regimes,

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{pmatrix},$$

where $p_{ij} = \Pr\left[s_{t+1} = j \mid s_t = i\right]$ for $i, j = 1, 2$, are quite peculiar. They are reproduced here in the fifth column in Table 4.[6] The probability p_{ii} is almost 0 for some countries and $p_{ii} < 0.5$ for $i = 1$, $i = 2$, or both, for all of the 5 countries. For instance, in the case of Italy, its estimated transition probabilities are $\begin{pmatrix} 0.001 & 0.999 \\ 0.332 & 0.668 \end{pmatrix}$, so that it is hard to say that its first regime really exists. The expected duration of the first regime, D_1, is only $E\left(D_1\right) = \dfrac{1}{1-p_{11}} = 1.001$. Indeed, the filtered probabilities for the country shown in Figure 1 in Kanas06 reveal that its real exchange rate is in the second regime most of the time, with only few brief temporary observations in a stationary regime. The results for Norway are also quite similar, with Norway's transition probabilities $\begin{pmatrix} 0.001 & 0.999 \\ 0.311 & 0.689 \end{pmatrix}$. A similar phenomenon is also observed for Australia and Finland. Their p_{22} values are very small, 0.037 and 0.131, respectively, their filtered probabilities reveal that their second regimes contain just a few outliers. Also, for Switzerland, with $p_{11} = 0.245$ and

[6] Clearly, the peculiar results on the transition probabilities are not due to the Markov switching ADF regression (1) employed in Kanas06.

$p_{22} = 0.263$, the two states are expected to change regimes quite randomly. Switzerland's filtered probabilities show that it is really hard to figure out what is which regime and when. Klaassen (2005, p.91) notes that "if both p_{11} and p_{22} are small, $p_{11} + p_{22}$ is close to 1, which implies that the current regime is almost independent of the previous one."[7] All in all, it is hard to argue that these are meaningful regime switching findings. The empirical results on the 5 real exchange rates deserve further reexamination, as the dynamic behavior of the real exchange rates critically depends on the estimation results. In this study, quite different, but more plausible, results are found with Hamilton-type Markov regime switching model (2). The new results are unambiguously preferred by the *MSCs* of Smith et al. (2006).

Some preliminary data analysis results for the 5 real exchange rates are reported in Tables 1 ~ 3. First, various unit root tests are in Table 1. All the four tests agree that Finland and Italy are (trend) stationary, or $I(0)$, and that Switzerland is $I(1)$. For Australia and Norway, the results are not consistent. No definite conclusion is possible.[8] Second, in Table 2, similar results are found with the feasible exact local Whittle estimates of the long memory parameter, *d*, by Shimotsu and Phillips (2005). Finland and Italy appear to be stationary with $d < 0.5$, while other remaining countries are nonstationary with $d > 0.5$. For Norway, the null that $d = 1$ is not rejected when $m = T^{0.7}$ frequencies are employed where T denotes the sample size. See Yoon (2007) for more details. The unit root tests and exact local Whittle estimators all assume that the persistence of the data series is not changing throughout the sample period. Third, a new testing procedure by Leybourne et al. (2007) allowing multiple changes in persistence between $I(0)$ and $I(1)$ at *a priori* unknown dates is also applied. The results are presented in Table 3. Interestingly, more

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7 See also Engel and Hamilton (1990), who test if $p_{11} + p_{22} = 1$.

8 The Breitung's test is known to be robust to misspecification and structural breaks in the short-run components. The KPSS test has the stationarity null against the nonstationarity alternative.

evidence for stationarity is found during the fixed nominal exchange rate regimes except for Norway. Also, Finland is found to be $I(0)$ throughout the whole sample period. Finally, Kanas06 uses James Davisdon's TSMod program (version 4.03), now called TSM, to estimate his Markov switching ADF regression model. All of the calculations in this article are carried out using GAUSS, along with the OPTIMUM application module, for maximum likelihood estimation.[9]

4. Main results

Before presenting the maximum likelihood estimation results of two Markov regime switching models (1) and (2), we, first, discuss how to compare and rank the estimation results. Several criteria are currently available in the literature: for example the highest Bayes factor (Koop and Potter, 1999), the highest marginal likelihood (Chib, 1998), the ability to forecast (Hamilton and Susmel, 1994); or the tests by Hansen (1992), Garcia (1998), Carrasco et al. (2004), and Marmer (2007). Also, see Hamilton (2005).[10] In this study, we employ the newly suggested *MSC* of Smith et al. (2006), which is discussed in the next subsection.

4.1. Model selection based on the Markov switching criterion

Using the Kullback-Leibler divergence between the true and candidate models, Smith et al. (2006) devise the following *MSC* to select the number of states and variables simultaneously:

$$MSC = -2\log L + \sum_{i=1}^{N} \frac{T_i\left(T_i + \lambda_i K\right)}{\delta_i T_i - \lambda_i K - 2} \tag{9}$$

[9] The TSM program is available at http://www.timeseriesmodelling.com/. A review of the program (version 4.03) is in the paper from Fuertes et al. (2005). The BFGS algorithm is used in numerical optimization both in the TSMod and OPTIMUM module from GAUSS. It should be noted that the TSM program can also estimate model (2).

[10] Psaradakis and Spagnolo (2006) provide some simulation results on the performance of various model selection criteria within Markov regime switching models.

where N is the number of the states and K is the number of the regressors in the model. Also, T_i is the sum of smoothed probabilities of being in the regime *i*. $\sum_{i=1}^{N} T_i = T$, where *T* is the number of the effective observations used in estimation. The second term in (9) is a penalty for the complexity of a model. Smith et al. (2006) recommend setting $\delta_i = 1$ and $\lambda_i = N$; also $\lambda_i = 1$ or $\lambda_i = N^2$ is another possible choice. The *MSC* with these values is called $MSC_{\lambda=N}$, $MSC_{\lambda=1}$, and $MSC_{\lambda=N^2}$, respectively. To ensure a positive and meaningful penalty in (9), the denominator $\left(\delta_i T_i - \lambda_i K - 2\right)$ should exceed 1 for each regime. Otherwise, the *MSC* is not defined. Figure 3 shows different penalty terms for various values of δ_i and λ_i with $N = 2$, $K = 4$, and $T = 100$. On the horizontal axis, it is T_1. It can be shown easily that when $N = 2$, as assumed in our work, the penalty term is minimized at $T_1 = T_2 = \frac{T}{2}$, when $\delta_1 = \delta_2$ and $\lambda_1 = \lambda_2$. Given that there is little *a priori* reason to put different δ_i and λ_i for different regimes, the penalty term in the *MSC* is symmetric at $\frac{T}{2}$. Also, its penalty is typically much larger than that of the Akaike information criteria [*AIC*], $2\left(NK + N^2\right)$. For instance, for models (1) and (2) with $p = 2$, $N = 2$ and $K = 4$, the penalty of the *AIC* is 24.[11] All the penalty values of the *MSC*, as shown in Figure 3 with $T = 100$, are much higher than 24. Hence, the *MSC*, in general, will choose smaller models than the *AIC* selects. Confirming simulation results are available in Smith et al. (2006). Figure 3 also shows that the penalty terms of the *MSC* are much higher at the extreme values of T_1 or T_2. These terms indicate that the *MSC* prefers models with "similar" distributions of the states across different

[11] Other Markov regime switching models could also be compared to those estimated in our work. To be compatible with his results, however, only model (2), with the same numbers of explanatory variables and regimes as in Kanas06, is treated here.

regimes in the sense that T_1 and T_2 are not much different from each other. Also, the penalty terms appear to be flat in the middle regions. In this study, two Markov regime switching models (1) and (2) are compared with *MSCs*. According to Smith et al. (2006), a difference of the *MSCs* larger than 10 indicates that essentially no empirical evidence exists in favor of the model with a higher *MSC*; see remark 8 in Smith et al. (2006). Additionally, it should be mentioned that in deriving the *MSCs*, Smith et al. (2006) assume that all the *K* regressors, as well as error variances, are regime-dependent, while the autoregressive coefficients γ_i and error variances in models (1) and (2) are not. Hence, a refinement of the *MSC* could estimate the Kullback-Leibler distance better. This interesting extension is left for future research.[12]

4.2. The maximum likelihood estimation results

The maximum likelihood estimation of Markov regime switching models is not trivial because log likelihood functions could be ill-behaved with numerous local maxima. We try to replicate the estimation results in Kanas06 for his Markov switching ADF regression (1) with various starting values. Unfortunately, none of the estimation results, for instance on the transition probabilities, is close to those reported in Kanas06. However, the newly estimated smoothed probabilities, shown in the second panel in Figures 4~8 for each country and the filtered probabilities, not shown here to save space, allow for a much easier interpretation. To fully appreciate the difference, we need to take a look at Figure 1 in Kanas06 for the 5 countries. Also, the newly estimated transition probabilities of model (1), reported in Table 4, are also very different from those found in Kanas06. The new transition probabilities seem to be more plausible as both regimes are now equally persistent in the sense that the diagonal elements of the transition probability matrices are much higher and similar in magnitude. Furthermore, in terms of the maximized log likelihood values, the new estimation results in this article produce higher values except for Finland and Norway (see

[12] We thank Aaron Smith for this point.

Table 4). Hence, we do not think that his model is put to a disadvantageous position in our work by not being able to replicate his original estimation results.

Table 5 compares the new maximum likelihood estimation results of models (1) and (2).[13] In terms of the maximized log likelihood value, model (1) yields a higher value only for Australia: 154.72 vs. 141.42. Also, its *AIC* is lower: -285.45 vs. -258.85. However, in terms of the *MSC*, none of model (1) beats the Markov regime switching model (2), regardless of the values of λ_i, for all countries, including Australia. In fact, for the estimation results of model (1), no *MSC*s are defined for Australia and Italy. Also, at least one of the recommended *MSC*s by Smith et al. (2006) is not defined, except for Switzerland. This is due to the fact that only one particular regime is very dominant in the estimation results of model (1) and in a sense the *MSC*s put a penalty of infinity for such estimation results. The results for Switzerland are most comparable between the two Markov regime switching models; however, the difference in the *MSC*s is larger than 17, which indicates that there is essentially no empirical evidence in favor of model (1). In sum, model (2) is preferred to model (1) in terms of the *MSCs* for all real exchange rates studied here. To understand the results on the magnitude of the *MSCs*, the last two rows of Table 5 list the sums of the smoothed probabilities for each regime. For model (1), T_1 and T_2 are quite different, indicating that only one particular regime is dominant; whereas for model (2), the magnitudes of T_1 and T_2 are not much different. This shows that both regimes are indeed present and properly identified. A similar finding could be reached by comparing the last two panels in Figures 4~8. These panels display the smoothed probabilities from the two Markov regime switching models. All in all, the penalty term of the *MSCs* is much higher for model (1) and the Hamilton-type Markov regime switching model (2) is preferred. The ensuing discussion on the estimation results is only about model (2).

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[13] As no estimation results are reported in Kanas06 about the parameters of his model (1), comparisons to his original estimation results are not possible.

4.3 Varying strength of PPP

From the estimation results for model (2) listed in Table 5, we find strong evidence that the strength of PPP varies over time. All the estimates of $b(s_t)$ in the first regimes are negative and significant, while those of $b(s_t)$ in the second regimes are positive, except for Australia with -0.0592, which is not significant. Note that the "*t*-statistics" associated with $b(s_t)$ do not follow the Dickey-Fuller distribution. Hall et al. (1999) and Kanas06 e.g. employ bootstrap methods to calculate the *p*-values of the *t*-statistics. In this work, as the estimates are quite distinct, we just assume that the first regime is stationary and the second nonstationary. [14] Overall, in the first regimes, the real exchange rates are stationary, in which PPP holds locally; while in the second regimes, the real exchange rates are nonstationary, and PPP does not hold. The smoothed probabilities of being in the first regime plotted in the last panels of Figures 4~8 also show ample evidence for regime changes in the real exchange rates. This behavior of changing persistence in the real exchange rates would not be revealed with standard unit root tests which assume that their persistence is the same through the sample periods.

We now ask whether PPP holds better during a fixed or flexible nominal exchange rate arrangement. In Table 6, the starting and ending years are listed across different nominal exchange rate regimes, where the smoothed probabilities of being in the first regime are higher than 0.5. As the smoothed probabilities are quite distinct, it does not make much difference if other criteria are employed. Overall, it appears that there is little evidence that PPP depends significantly on the nominal exchange rate regimes. For instance, for Finland and Italy, PPP holds during the Bretton-

[14] The "*t*-values" of $b(s_t)$ in the first regimes are -4.219, -14.286, -14.721, -3.269, and -9.628 for the 5 countries; while in the second regimes, they are -1.833, 2.304, 3.435, 1.627, and 4.973, respectively (see Table 5).

Woods periods and the current float. Also, for Australia and Norway, PPP does not hold during the same periods, while PPP holds for both countries during the gold standard. Only for Switzerland, there is some evidence that PPP holds better during the fixed nominal exchange rate regimes of the gold standard and the Bretton-Woods periods. There are only short and temporary stationary regimes during the interwar and the current float. Noticeably, Switzerland exhibits quite frequent regime changes during the same periods.[15] In sum, during the Bretton-Woods and current float periods, the evidence for PPP is not much different, possibly except for Switzerland, whether PPP holds or not. These results are very different from those that could be gleaned from Figure 1 in Kanas06. Also, the estimated stationary regimes in Table 6 are somewhat different for most countries from those shown in Table 3 with the testing procedure by Leybourne et al. (2007). The results for Finland are closest; for which their procedure finds it to be $I(0)$ throughout the whole sample period. The result is also consistent with that of the unit root tests for the country in Table 1.

5. Conclusions

There are various reasons to believe why the persistence of real exchange rates is changing over time. For instance, real exchange rates are known to have different dynamics across fixed and flexible nominal exchange rate regimes. Also, market fundamentals that determine the behavior of real exchange rates would be, themselves, regime dependent. Rather than asking if real exchange rates are persistent or not throughout the whole sample period, this study examines if their persistence is changing over time by employing a Hamilton-type Markov regime switching model. When at least one of multiple regimes is stationary, real exchange rates are stationary locally and PPP holds within the regime. Given that real exchange rates are very persistent, standard unit root tests do not reveal much about their dynamics. Throughout the study described in this article,

[15] The results are very different from those in Kanas06. He finds no stationary regimes for the country.

comparisons to the Markov switching ADF regression in Kanas (2006) are also made. We also show that our model specification utilizes more information about the states of the real exchange rates without introducing additional parameters to estimate. The two Markov regime switching models yield quite different dynamic behavior for the real exchange rates examined here. Furthermore, the new Markov switching criteria of Smith et al. (2006), developed especially for discriminating Markov regime switching models, unambiguously gives preference to the Hamilton-type Markov regime switching model. In sum, the evidence presented in this article strongly supports that PPP holds sometimes, though not always, for all of the 5 real exchange rates spanning more than 100 years. Hence, the answer to the question posed as the title of this paper should be in the affirmative: PPP holds sometimes.

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Table 1: Various unit root test results for the 5 real exchange rates

	Sample	ADF_{GS}	$DF-GLS_{MAIC}$	*Breitung*	*KPSS*
Australia	1870~1998	-2.63*	-2.30**	0.042074	0.83***
Finland	1881~1998	-6.01***	-4.52***	0.005387***	0.22
Italy	1880~1998	-4.29***	-3.37***	0.002800***	0.08
Norway	1870~1998	-3.72***	-1.37	0.017100	0.39*
Switzerland	1892~1998	-1.49	-0.76	0.061428	0.93***

*, **, and *** denote that the test statistic is significant at the 10, 5, and 1% significance level, respectively. A constant is included in the tests.

ADF_{GS} : ADF tests using the general-to-specific procedure as in Lopez et al. (2005a)

$DF-GLS_{MAIC}$: Elliott et al. (1996) tests with lags selected by the modified *AIC* of Ng and Perron (2001)

Breitung: Breitung's (2002) unit root tests

KPSS: Kwiatkowski et al's (1992) stationarity tests with the lag truncation parameter $l_8 = \left\lfloor 8(T/100)^{1/4} \right\rfloor$ where T is the sample size.

Table 2: The exact local Whittle estimates of long memory parameters

	Sample	$m=T^{0.5}$		$m=T^{0.6}$		$m=T^{0.7}$	
		$\hat{d}$	$\hat{d}_\tau$	$\hat{d}$	$\hat{d}_\tau$	$\hat{d}$	$\hat{d}_\tau$
Australia	1870~1998	0.537	0.390	0.603	0.606	0.774	0.707
Finland	1881~1998	0.045	-0.031	0.206	0.136	0.330	0.291
Italy	1880~1998	0.236	0.224	0.455	0.450	0.752	0.751
Norway	1870~1998	0.637	0.435	0.638	0.502	0.893	0.896
Switzerland	1892~1998	0.744	0.558	0.746	0.610	0.823	0.791

Feasible exact local Whittle estimates of the long memory parameter, *d*, by Shimotsu and Phillips (2005) are reported for m frequencies, for the original ($\hat{d}$), and linearly detrended ($\hat{d}_\tau$) series. Standard errors of the estimates are $\frac{1}{2}m^{-0.5}$. The entities for which the null that $d_0=1$ is not rejected at the 5% significance level are highlighted. T denotes the sample size. The results are taken from Yoon (2007).

Table 3: Changes in persistence of real exchange rates across nominal exchange rate regimes

	Sample	Gold standard: 1870~1914	Interwar period: 1914~1945	Bretton-Woods: 1946~1971	Current float: 1971~1998
Australia	1870~1998	1904~1920		1947~1976	
Finland	1881~1998	1881~1998			
Italy	1880~1998	1881~1895 1902~1912		1944~1970	
Norway	1870~1998	1871~1920	1928~1945		1986~1997
Switzerland	1892~1998			1935~1959	

The beginning and ending years of identified $I(0)$ regimes with the procedure proposed by Leybourne et al. (2007) are reported.

Table 4: A comparison of Markov regime switching models estimation results

Country	*logL* of (1) in Kanas06	New *logL* of (1)	*logL* of (2)	Transition probabilities of (1) in Kanas06	New transition probabilities of (1)	Transition probabilities of (2)
Australia	145.973	154.72	141.42	$\begin{pmatrix} 0.926 & 0.074 \\ 0.963 & 0.037 \end{pmatrix}$	$\begin{pmatrix} 0.7551 & 0.2449 \\ 0.0111 & 0.9889 \end{pmatrix}$	$\begin{pmatrix} 0.9533 & 0.0467 \\ 0.1022 & 0.8978 \end{pmatrix}$
Finland	90.844	80.70	95.82	$\begin{pmatrix} 0.857 & 0.143 \\ 0.869 & 0.131 \end{pmatrix}$	$\begin{pmatrix} 0.4937 & 0.5073 \\ 0.0392 & 0.9608 \end{pmatrix}$	$\begin{pmatrix} 0.9543 & 0.0457 \\ 0.0734 & 0.9266 \end{pmatrix}$
Italy	77.369	94.91	115.37	$\begin{pmatrix} 0.001 & 0.999 \\ 0.332 & 0.668 \end{pmatrix}$	$\begin{pmatrix} 0.5838 & 0.4162 \\ 0.0110 & 0.9890 \end{pmatrix}$	$\begin{pmatrix} 0.9902 & 0.0098 \\ 0.0198 & 0.9802 \end{pmatrix}$
Norway	149.114	130.10	162.07	$\begin{pmatrix} 0.001 & 0.999 \\ 0.311 & 0.689 \end{pmatrix}$	$\begin{pmatrix} 0.9926 & 0.0074 \\ 0.0473 & 0.9527 \end{pmatrix}$	$\begin{pmatrix} 0.7995 & 0.2005 \\ 0.0563 & 0.9437 \end{pmatrix}$
Switzerland	112.404	114.89	125.64	$\begin{pmatrix} 0.245 & 0.755 \\ 0.737 & 0.263 \end{pmatrix}$	$\begin{pmatrix} 0.9917 & 0.0083 \\ 0.0080 & 0.9920 \end{pmatrix}$	$\begin{pmatrix} 0.7757 & 0.2243 \\ 0.1546 & 0.8454 \end{pmatrix}$

The results in the second and fifth columns are copied from Kanas06. *logL* stands for the value of the maximized log likelihood. The first regime is the one with smaller estimated values of $b(s_t)$ in (1) and (2).

Table 5: Maximum likelihood estimation results of Markov regime switching models (1) and (2)

	Australia		Finland		Italy		Norway		Switzerland	
	Model (1)	Model (2)	Model (1)	Model (2)	Model (1)	Model (2)	Model (1)	Model (2)	Model (1)	Model (2)
p_{11}	0.7551	0.9533	0.4937	0.9543	0.5838	0.9902	0.9926	0.7995	0.9917	0.7757
	(2.4031)	(0.0187)	(0.2391)	(0.0159)	(1.3183)	(0.0058)	(0.0400)	(0.1395)	(0.3224)	(0.0564)
p_{22}	0.9889	0.8978	0.9608	0.9266	0.9890	0.9802	0.9527	0.9437	0.9920	0.8454
	(0.1090)	(0.0404)	(0.0302)	(0.0278)	(0.0355)	(0.0123)	(0.2502)	(0.0202)	(0.3126)	(0.0337)
$a(1)$	0.0036	0.0970	-0.8109	-1.1503	-0.2253	-4.4325	-0.9702	-0.3421	-0.1739	-0.2226
	(0.0005)	(0.0729)	(0.0948)	(0.0794)	(0.3363)	(0.2734)	(0.1982)	(0.1113)	(0.0345)	(0.0177)
$a(2)$	-0.0165	-0.1506	2.3569	0.3139	-0.7237	0.9384	-0.0348	0.3037	0.1277	0.1700
	(0.0084)	(0.0542)	(0.3180)	(0.1303)	(0.7551)	(0.2625)	(0.0956)	(0.1531)	(0.0345)	(0.0206)
γ_1	0.1600	0.4274	0.2110	0.4397	0.1302	0.2139	0.4585	0.4081	0.4610	0.4466
	(0.0675)	(0.0700)	(0.0616)	(0.0298)	(0.0612)	(0.0175)	(0.1371)	(0.0511)	(0.0836)	(0.0286)
γ_2	-0.1302	0.3434	-0.0938	-0.1609	-0.0503	-0.1244	-0.0345	-0.0393	-0.2229	-0.1997
	(0.0697)	(0.0528)	(0.0673)	(0.0166)	(0.0653)	(0.0113)	(0.0823)	(0.0728)	(0.0825)	(0.0208)
$b(1)$	-0.1231	-0.2101	-0.4721	-0.5043	-0.0327	-0.5373	-0.4494	-0.0912	-0.1499	-0.1502
	(0.0298)	(0.0498)	(0.0556)	(0.0353)	(0.0448)	(0.0365)	(0.1117)	(0.0279)	(0.0346)	(0.0156)
$b(2)$	0.0329	-0.0592	1.3356	0.1302	-0.0001	0.1106	-0.0152	0.0711	0.0815	0.0935
	(0.1813)	(0.0323)	(0.1969)	(0.0565)	(0.1088)	(0.0322)	(0.0422)	(0.0437)	(0.036)	(0.0188)
σ^2	0.0036	0.0017	0.0118	0.0030	0.0090	0.0040	0.0061	0.0018	0.0043	0.0006
	(0.0005)	(0.0004)	(0.0017)	(0.0006)	(0.0012)	(0.0006)	(0.0010)	(0.0003)	(0.0007)	(0.0002)
$\log L$	154.72	141.42	80.70	95.82	94.91	115.37	130.10	162.07	114.89	125.64
$MSC_{\lambda=N}$	n.a.	-113.81	n.a.	-31.88	n.a.	-71.24	-31.44	-155.33	-77.44	-100.52
$MSC_{\lambda=N^2}$	n.a.	-60.23	n.a.	29.89	n.a.	-16.14	n.a.	-102.86	9.63	-27.09
$MSC_{\lambda=1}$	n.a.	-137.68	17.47	-54.08	n.a.	-92.44	-104.77	-176.03	-102.28	-124.14
AIC	-285.45	-258.85	-137.40	-167.64	-165.81	-206.74	-236.20	-300.13	-205.78	-227.28
T_1	120.99	52.99	107.61	74.97	113.00	57.67	12.77	65.30	75.21	70.94
T_2	5.01	73.01	7.39	40.03	3.00	58.33	113.23	60.70	28.79	33.06

T_i is the sum of smoothed probabilities of regime *i*. Three initial observations are used for initial values in estimation for each country.

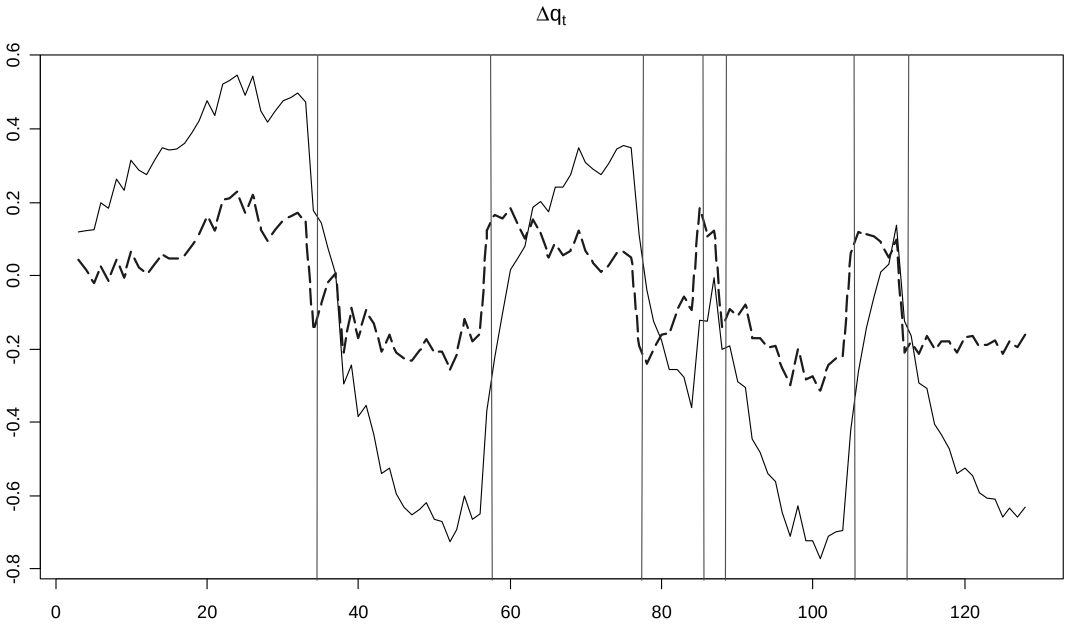
n.a. indicates that the *MSC* is not defined for the estimation results. Standard errors are reported in the parentheses.

Table 6: Identified stationary real exchange rate regimes over different nominal exchange rate arrangements

	Effective sample	Gold standard: 1870~1914	Interwar period: 1914~1945	Bretton-Woods: 1946~1971	Current float: 1971~1998
Australia	1873~1998	1873~1878 1881~1918	1921~1922 1933~1934	1950~1952	1987~1988 1994
Finland	1884~1998		1919~1946	1948~1988	
					1991~1998
Italy	1883~1998		1919~1922	1946~1998	
Norway	1873~1998	1873~1914	1919~1924 1932	1946~1949	1981~1984 1991~1998
Switzerland	1895~1998	1897~1914	1919~1920 1923~1932 1935~1939	1946~1970	1975~1976 1979~1984 1988~1989 1996~1997

The beginning and ending years of identified stationary regimes from the Markov regime switching model (2) are reported in each cell.

Figure 1: Simulated Markov regime switching series from models (1) and (2)



The data generating processes are (1) and (2), respectively, with 129 observations. The broken line is simulated from model (2), and the vertical lines indicate regime switching dates. The parameter values employed in the simulations are in the third column of Table 5 for Australia, with $b(s_t)=0$ for all t.

Figure 2: Real exchange rate data series in levels and first differences

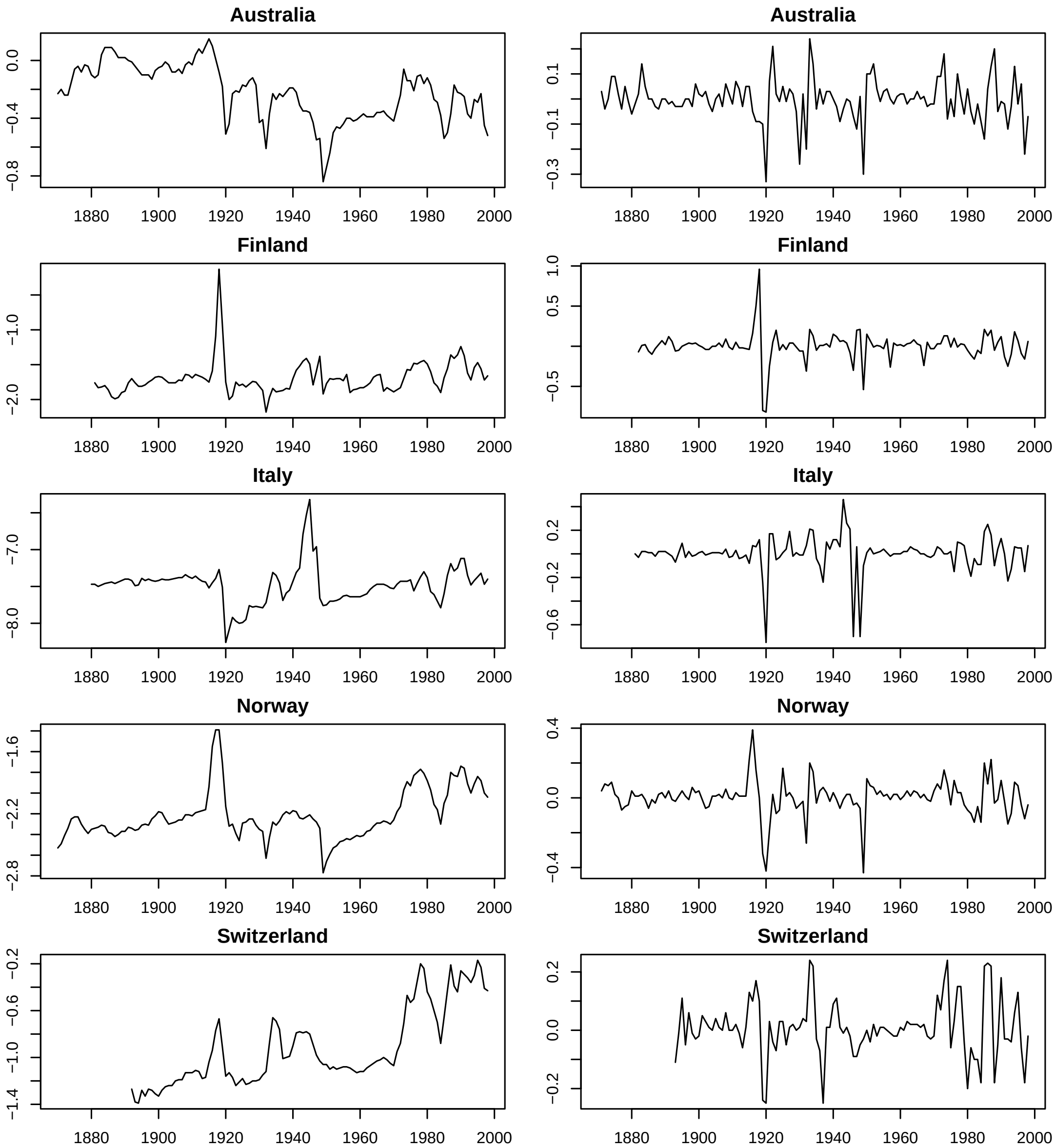
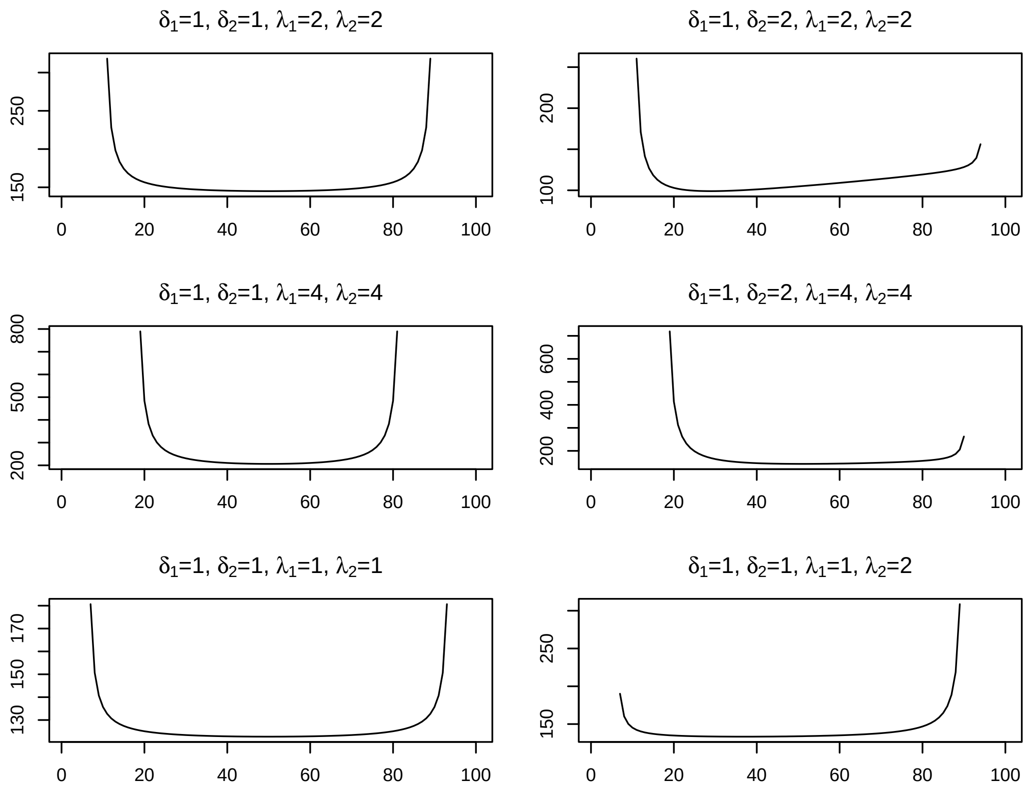
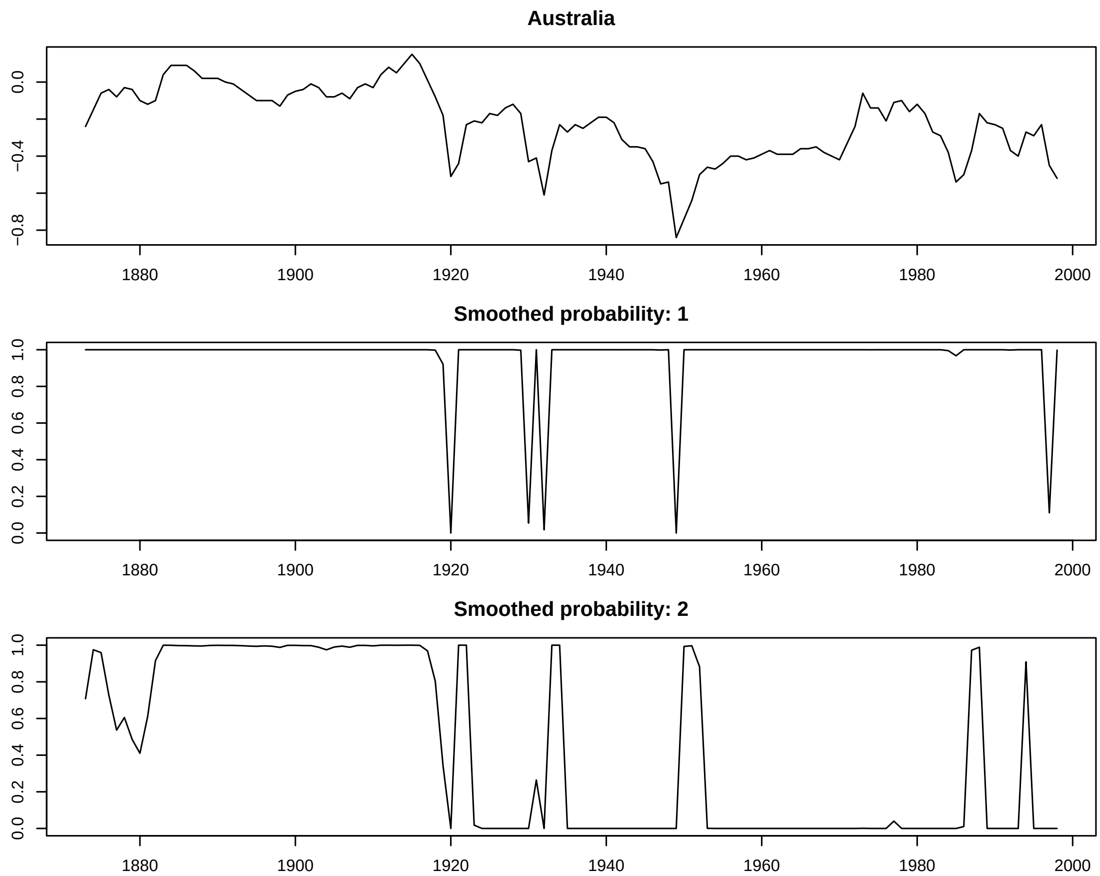


Figure 3: The penalty of the *MSC*



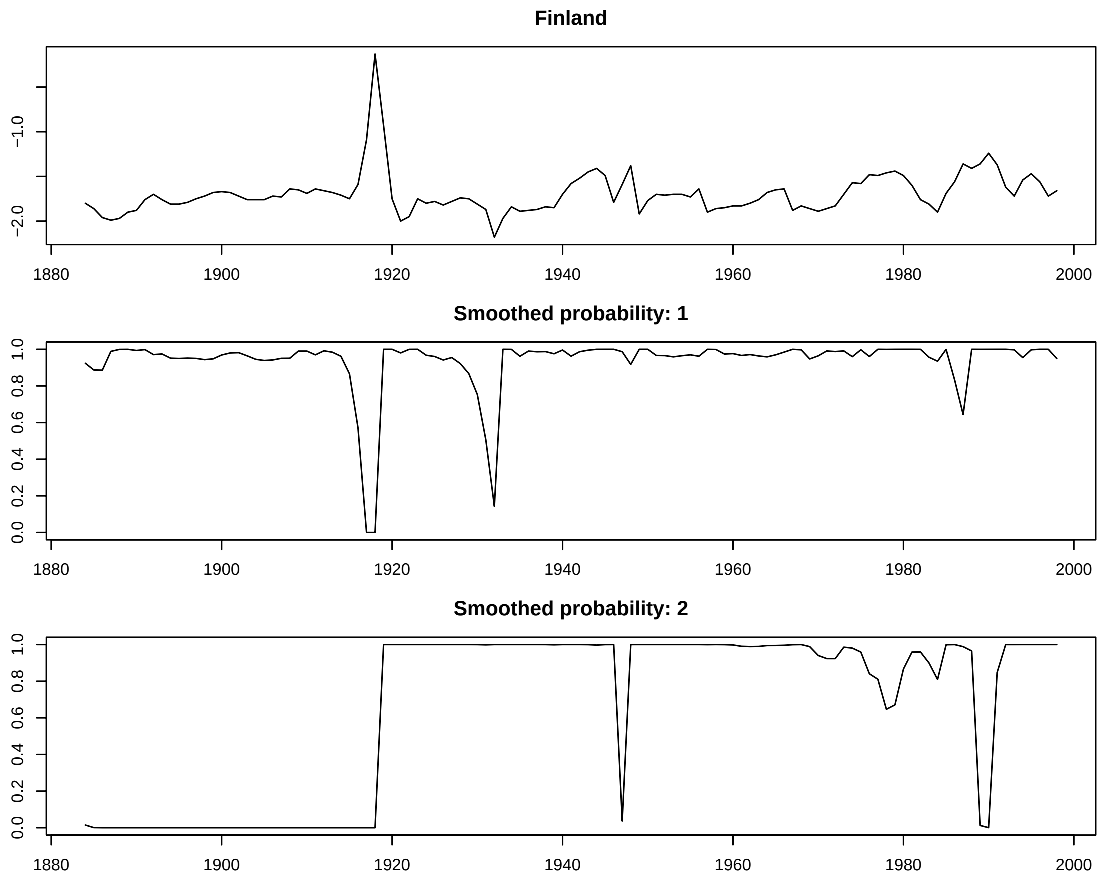
T=100, $N=2$, and $K=4$ are employed in drawing the penalty = $\sum_{i=1}^{N} \frac{T_i\left(T_i+\lambda_i K\right)}{\delta_i T_i-\lambda_i K-2}$ of the *MSC*.

Figure 4: A comparison of the smoothed probabilities for Australia



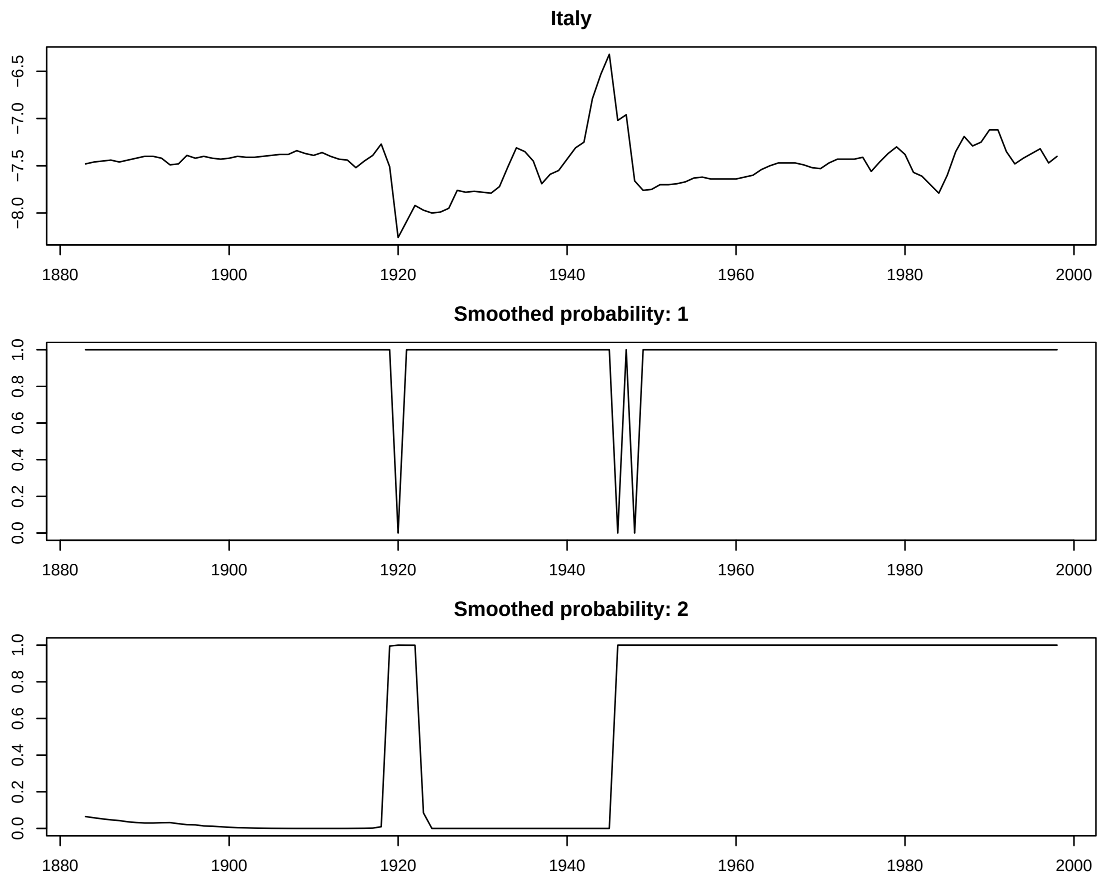
The first panel shows the real exchange rate against the U.S. dollar. The remaining panels show the smoothed probabilities of the first regime from model (1) and model (2), respectively.

Figure 5: A comparison of the smoothed probabilities for Finland



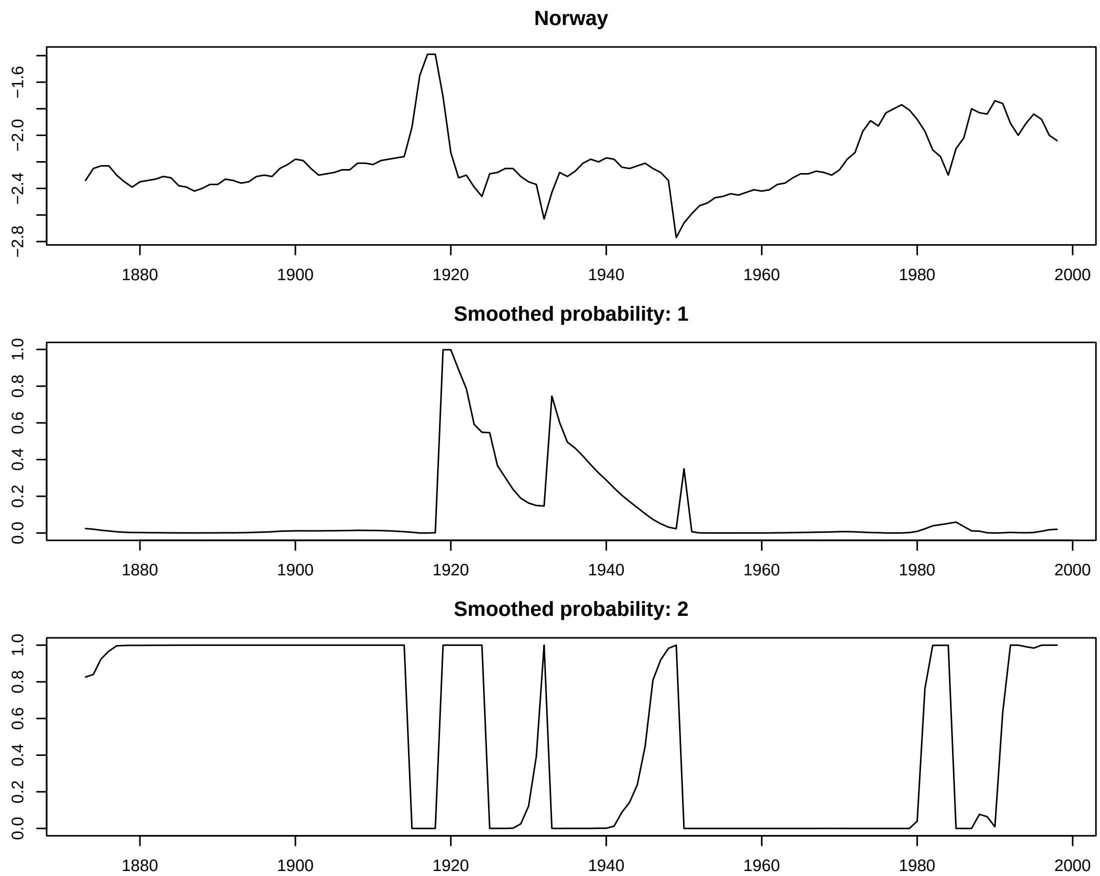
See Figure 4 for more details.

Figure 6: A comparison of the smoothed probabilities for Italy



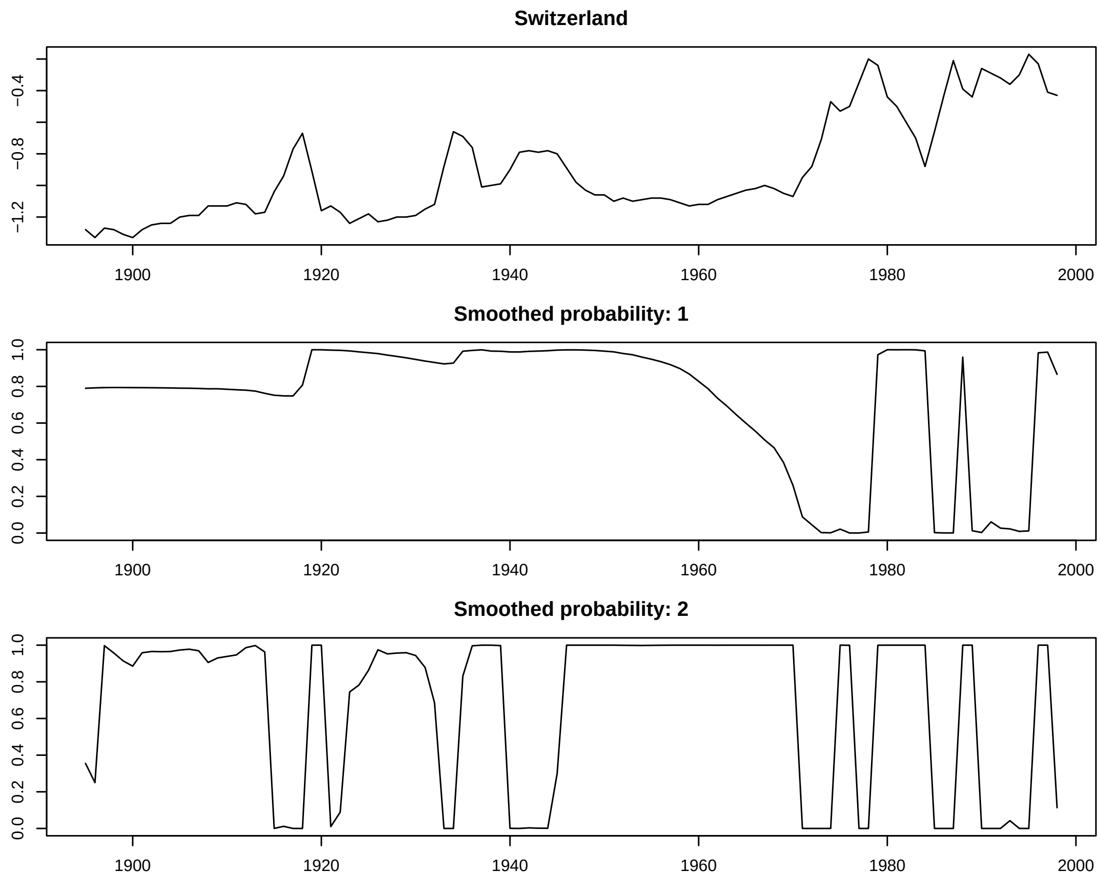
See Figure 4 for more details.

Figure 7: A comparison of the smoothed probabilities for Norway



See Figure 4 for more details.

Figure 8: A comparison of the smoothed probabilities for Switzerland



See Figure 4 for more details.