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Working Paper

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CSLE Discussion Paper, No. 2001-06

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Suggested citation: Haucap, Justus; Kirstein, Roland (2001) : Government Incentives when Pollution Permits are Durable Goods, CSLE Discussion Paper, No. 2001-06, <http://hdl.handle.net/10419/23066>

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# Government Incentives when Pollution Permits are Durable Goods<sup>†</sup>

Justus Haucap\*      Roland Kirstein\*\*

Center for the Study of Law and Economics  
Discussion Paper 2001-06

## Abstract

This paper analyzes the incentive effects of pollution taxes versus pollution permits for a revenue maximizing Government that also pursues environmental objectives. In our model, pollution permits are analyzed as durable goods, and the leasing of pollution permits is seen as an equivalent to a pollution tax. We show that environmental policy based on durable pollution permits can be welfare superior to a pollution tax regime. The intuition is that a monopolistic Government would, in order to maximize its revenues, try to restrict the permit sales below the welfare maximizing level.

While a pollution tax or leasing charge allows the Government to credibly commit to a monopoly level of pollution in future periods, a system based on durable permits weakens the monopolistic Government's ability to credibly restrict future sales. Therefore, a pollution tax regime may be better for the environment and simultaneously increase Government revenues, but social welfare is larger with pollution permits.

Hence, a regime where the Government cannot commit to monopoly quantities may be preferable from a welfare economic perspective. This argument in favor of durable permits complements more traditional arguments based on information asymmetries and innovation incentives.

*JEL classification:* D7,H2, K3

*Encyclopedia of Law and Economics:* 2300, 2500

*Keywords:* Emissions Permits, Pollution Tax, Time Inconsistency, Durable Goods

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<sup>†</sup>We would like to thank Bouwe Dijkstra, Mathias Erlei, Alexander Neunzig, Dieter Schmidtchen and Peter Strobel for helpful comments and discussions. We also received valuable suggestions from participants at the 2001 Conference of the European Public Choice Society at Paris and seminar participants at the Hamburg Institute for International Economics (HWWA) and the University of the Saarland. Of course, remaining errors are our responsibility for which the authors blame each other.

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# 1 Introduction

How to combat global warming is one of the most vigorously debated issues of environmental policy today. Among economists, much of the discussion in this area has circled around the question which policy instruments are best suited to achieve greenhouse emission objectives efficiently. There appears to be some agreement that, in general, a regime based on tradable emissions permits is preferable to a pollution tax system (see, e.g., Tietenberg, 1985). The main argument in favor of tradable emission permits is that they ensure that pollution is reduced where it is least costly. Tradable permits, therefore, rank relatively highly in terms of allocative (and productive) efficiency. Furthermore, permits have also been argued to lead to relatively high degrees of dynamic efficiency, as they carry relatively strong incentives for innovation regarding clean products and technologies (see Jung, Krutilla and Boyd, 1996). This view, however, has recently been challenged by Requate and Unold (1997) who show that in their model permits do never provide higher incentives to adopt advanced abatement technologies.<sup>1</sup>

While the incentives that permits and taxes carry for polluting firms are reasonably well understood today, there has been comparatively little analysis of the incentives the two different policy instruments carry for policy makers. There is some normative analysis of Governments' optimal policy choice, given certain environmental objectives, but this literature provides no further insights regarding the incentives different policy instruments contain for policy makers.<sup>2</sup> An exception is the paper by Boyer and Laffont (1999) who focus on the emergence of sophisticated market-based or incentive mechanisms of environmental regulation in a model where politicians pursue the private agendas of their electoral base. Furthermore, there are a few papers which explicitly address policy makers' incentives to introduce pollution taxes only, analyzing the political economy of ecological taxation schemes from a positive point of view (see, e.g., Fredriksson, 1997; Gawel and Schneider, 1997).

Our paper adds another aspect to this stream of work, as its focus is on the credibility aspects associated with pollution taxes versus emissions permits. More specifically, we ask what policy instruments different types of Government are

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<sup>1</sup>Further papers focussing on the dynamic efficiency aspects of different regulatory approaches are Milliman and Prince (1989), Requate (1995), Gersbach and Glazer (1996), Laffont and Tirole (1996) and Denicolo (1999). While Milliman and Prince (1989), Requate (1995) and Gersbach and Glazer (1996) all argue in favor of permits to induce firms to invest in new abatement technologies, Denicolo (1999) obtains the opposite result, namely that taxes carry higher incentives to invest in pollution reducing technologies. Similarly, Laffont and Tirole (1996) show that permits may actually induce over-investment. They argue for an approach based on pollution options which lead to superior welfare results in their model.

Another difference between permits and taxes is the financial impact on the regulated industry: Permits, once issued, leave the average cash flow unaffected, whereas taxes persistently extract money from the industry (see Dewees, 1998, 598).

<sup>2</sup>For example, Dewees (1998) points out that Governments should prefer permits over taxes if they have certain quantity objectives because permits implement certain quantities of pollution reduction (but at an uncertain price), whereas pollution taxes can, at best, implement certain prices, but not certain quantities or quantity reductions.

likely to prefer and, given the Government's incentives, how environmental policy should be designed.

Pollution permits can be viewed as durable goods sold by a monopoly supplier, namely the Government. As we know from Coase (1972), a profit maximizing durable goods monopolist has incentives to put additional stock on the market in every single period. While the monopolist maximizes its first period profit by selling the monopoly quantity, its incentives in the second period are to put more stock out on the market, on top of the quantity already sold in the past. This carries through to any future period until the monopolist reaches the point where the additional revenues from putting out more stock equal the marginal costs of provision. If buyers are not myopic, however, they foresee the monopolist's incentives to put out more stock in the future, which in turn makes them more reluctant to buy the durable good today. Put differently, the monopolist faces a credibility problem - which prevents them from exerting market power - if they cannot commit not to sell more stock in the future. In the extreme case where buyers have no time preference the monopolist is only left with selling the product at a price which equals marginal costs right from the start.

A solution to the monopolist's credibility problem can be leasing instead of selling the product. If the monopolist is allowed to lease out the durable good, consumers only pay a leasing charge or royalty per period. This eliminates the monopolist's incentive to put out more stock in the future, as any deviation from the monopoly quantity in any single period will only reduce profits.<sup>3</sup> Similar incentives exist for a revenue maximizing Government which sells pollution permits. As the revenues of today's license sale will be sunk tomorrow, a revenue maximizing Government is tempted to sell additional permits in period 2, and then again in period 3 and so on.

Contrast this with a pollution tax system: Under the assumption that the Government has set its revenue maximizing pollution tax rate in period 1, there is nothing it can do in order to raise further revenues in period 2 but to charge the same revenue maximizing tax rate again. The level of environmental pollution remains at the same low (monopoly) level in both periods.

Starting from the idea that pollution taxes can be seen as a leasing solution to preserve the monopoly power of a revenue-oriented Government, the following model will analyze the efficiency characteristics of durable pollution permits versus taxes and identify circumstances under which either policy instrument is preferable from an efficiency point of view, given policy makers' incentives. The credibility problem may possibly also help to explain why Governments, even if they could fairly be judged as revenue maximizers, have failed to generate positive revenues from durable pollution permits.<sup>4</sup> In Germany, for ex-

<sup>3</sup>More elaborate analyses of the durable-goods monopoly are Stokey (1981), Bulow (1982), Bagnoli, Salant and Swierzbinski (1989) and von der Fehr and Kühn (1995).

<sup>4</sup>As Norregaard & Reppelin-Hill (2000, 9ff.) state, tradable permits have not been a favored policy tool for pollution control worldwide, with the exception of the United States. The vast majority of countries currently favors taxes over permits.

ample, pollution rights have traditionally been issued in the form of durable, non-tradable permits granted by authorities for approved investment projects. Nowadays, the German Government shows a tendency towards pollution taxes, whereas tradable permits play virtually no role.

The idea that license-based regimes create time consistency or credibility problems for policy makers, be it regulatory agencies or Governments, is not entirely new, since it has also been discussed by Biglaiser, Horowitz and Quiggin (1995) and Laffont and Tirole (1996). However, both papers model time consistency problems for a neoclassical benevolent despot-type Government when firms invest in pollution-eliminating technologies. In contrast, we explore the incentives for different types of Government that not only have environmental concerns, but also revenue objectives. Furthermore, we also explicitly compare a tax based regime with a system where pollution permits are sold.

The rest of our paper is organized as follows: Section 2 presents and analyzes our general model which allows us to compare durable permit sales and permit leases or pollution taxes. In this more general approach, we will not distinguish yet between different types of Government, but simply assume that the Government has both welfare and budget concerns in its objective function.

Section 3 analyzes four different types of Government which are modeled as special cases of the general model presented in section 2. We parameterize our general model in order to derive the specific results for four different types of Government: a benevolent dictator, a pure Leviathan, a business-friendly, and a green Government. Section 4 briefly draws policy conclusions.

## 2 Pollution Permits as Durable Goods

### 2.1 Outline of the Model

Consider an industry which produces a good involving the emission of a fixed quantity of some harmful substance per unit of output. All firms in the industry are symmetric, i.e. all firms employ the same production technology. Also suppose that the Government,  $G$ , requires all producers to hold a pollution permit for every unit of output, and that  $G$  is the monopoly supplier of these emissions permits. Let us also assume that there are two periods in the game and that emissions permits are valid for the entire two periods; i.e. they are durable goods. In accordance with Coase (1972) we assume that the Government as the monopoly seller cannot commit itself not to issue and sell additional permits in period 2; i.e. the Government faces a time consistency or credibility problem.<sup>5</sup>

We assume that  $G$  determines the number of permits to be sold in both periods. Firms are assumed to compete for these permits in Bertrand fashion. Hence, in order to simplify the analysis let  $F$  represent the entire industry, where  $F$ , as

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<sup>5</sup>The two-period model, drawing on Rasmusen (1991, 276ff.), is sufficient to analyze our idea. In a model with an infinite time horizon, the effects we analyze would be even stronger, see Coase (1972).

the buyer of pollution permits, has no market power. The timing of the game then is as follows:

- $G$  sets the number of permits to be sold in period 1, denoted as  $q_1$ .
- $F$  offers a price per permit for this quantity, denoted as  $p_1$ .
- First-period payoffs are realized.
- $G$  sets the number of additional permits to be sold, denoted as  $q_2$ .
- $F$  offers a price,  $p_2$ , for any additional permits,
- Second-period payoffs are realized, and the game ends.

$F$ 's payoffs are  $B_1 - p_1q_1 + B_2 - p_2q_2$ , where  $B_t$  denotes  $F$ 's total gross benefit in period  $t$ . The benefit,  $B_t$ , is derived from  $F$ 's marginal utility function  $R = R(q_t)$ , where  $q_t$  is the number of permits that the industry holds in period  $t$ . Hence,  $R$  gives the inverse industry demand function for pollution rights and is assumed to be linear and given by

$$R(q_t) = a - q_t \quad (1)$$

with  $a > 0$ .<sup>6</sup> Thus, given quantity choices  $q_1$  and  $q_2$ ,  $F$ 's total gross benefit in the two periods is denoted as  $B = B_1 + B_2$  with

$$B_1(q_1) = \int_0^{q_1} R(q_t) dq = aq_1 - \frac{q_1^2}{2} \quad (2)$$

and

$$B_2(q_1, q_2) = \int_0^{q_1+q_2} R(q_t) dq = a(q_1 + q_2) - \frac{(q_1 + q_2)^2}{2}. \quad (3)$$

We assume that the the social costs of pollution are  $c$  per period per unit of output, with  $a > c > 0$ . Thus, the total cost of pollution as  $C = (2q_1 + q_2)c$ . The total revenues of the Government are denoted as  $T = T_1 + T_2$ , where  $T_t = p_tq_t$ , and the net benefits to the industry or to other permit buyers are given by  $N = N_1 + N_2$  with  $N_t = B_t - T_t$ ,  $t \in \{1; 2\}$ .<sup>7</sup> The Government's main objective is to maximize its surplus from selling permits, but we assume that the Government may also have environmental objectives and product market concerns. Therefore, the Government's payoff is modeled by  $\Pi$ , with

$$\Pi = T + \beta N - \gamma C \quad (4)$$

<sup>6</sup>Our results can also be derived without limiting the the slope of this function to  $-1$ . Our approach, however, simplifies the exposition.

<sup>7</sup>Thus,  $B = N + T$ .

where  $\beta \in [0, 1]$  represents the intensity of the Government's product market interests, while  $\gamma \in [0, 1]$  is the weighting factor for the Government's environmental concern.<sup>8</sup> Let us denote as  $\Pi_t$  the payoff of G in period  $t$ . Social welfare, denoted as  $W$ , is given by<sup>9</sup>

$$W = B - C = B_1(q_1) + B_2(q_1, q_2) - (2q_1 + q_2)c. \quad (5)$$

## 2.2 Optimal Sale of Durable Permits

Let us now solve the game by backward induction and first analyze  $F$ 's optimal choice of  $q_2$ , given the previous choices of  $p_1, q_1$  and  $p_2$ . At the beginning of period 2, the firms already own a number of  $q_1$  permits. If they purchase an additional  $q_2$  permits, the allowed level of pollution or production in the second period amounts to  $q_1 + q_2$ . Hence,  $F$ 's residual willingness to pay is

$$p_2 = R(q_1 + q_2) = a - (q_1 + q_2), \quad (6)$$

which is the effective inverse demand function for additional permits in the second period. This inverse demand function has a lower intercept than the marginal utility function  $R(q_t)$ , but the same slope. We now derive the optimal price  $p_2$  set by  $G$  in order to maximize  $\Pi_2 = T_2 + \beta N_2 - \gamma C_2$ . Using (6), the optimization problem is<sup>10</sup>

$$\max_{q_2} \Pi_2 = p_2 q_2 + \beta [B_2(q_1, q_2) - p_2 q_2] - \gamma c q_2 \quad (7)$$

which yields the first-order condition  $a - q_1 - \gamma c \stackrel{!}{=} (2 - \beta)q_2$ . Hence, the optimal monopoly quantity of additional permits  $q_2$ , denoted as  $q_2^P$ , is<sup>11</sup>

$$q_2^P(q_1) \stackrel{!}{=} \frac{a - q_1 - \gamma c}{2 - \beta}. \quad (8)$$

Making again use of (6), this leads to the optimal monopoly price (denoted as  $p_2^P$ )

$$p_2^P(q_1) = \frac{(1 - \beta)(a - q_1) + \gamma c}{2 - \beta}. \quad (9)$$

Now we turn to  $F$ 's willingness to pay at the beginning of period 1, given  $G$ 's earlier choice of  $q_1$ , and anticipating the later choices of  $p_2$  and  $q_2$ . To buy a permit in period 1 brings two different benefits for  $F$ : Firstly, the permit can

<sup>8</sup>Our results do not depend on the limitation of these parameters on values between zero and one. This limitation only simplifies the exposition.

<sup>9</sup>Note that  $W$  does not contain  $T$  as a separate figure, as  $T$  is included in the gross benefits generated on the product market.

<sup>10</sup>Recall that  $B_2$  depends on  $(q_1 + q_2)$ , whereas  $T_2$  only refers to  $q_2$ , the quantity purchased in period 2.

<sup>11</sup>The index  $P$  henceforth stands for outcomes under the permit sales regime.

be used and provides some benefit in that very period, and secondly, it also saves purchasing costs at the beginning of the next period if one already owns a permit. Hence, in the first period  $F$ 's maximum willingness to pay is the sum of the current benefit and the price in period 2,  $R(q_1) + p_2$ . If permits were more expensive in period 1, firms would be better off waiting until the second period. Using (9), the inverse demand in the first period is

$$p_1^P(q_1) = R(q_1) + p_2^P(q_1) = \frac{(3 - 2\beta)(a - q_1) + \gamma c}{2 - \beta}. \quad (10)$$

Given  $q_2(q_1)$ ,  $p_2(q_1)$  and  $p_1(q_1)$ , the last step of our analysis is to derive  $q_1^P$ , the optimal amount of permits chosen by  $G$  at the very beginning of the game.

The Government knows that, due to the lack of commitment opportunities, it will sell  $q_2^P(q_1)$  at a price  $p_2^P(q_1)$  in period 2. Furthermore,  $G$  knows that it can sell permits at a price of  $p_1^P(q_1)$  now. Thus, the Government chooses  $q_1$  so as to solve the following maximization problem:

$$\max_{q_1} T(q_1, q_2) + \beta[N_1(q_1) + N_2(q_1, q_2)] - \gamma(2q_1 + q_2)c$$

with  $p_1 = p_1^P(q_1)$ ,  $p_2 = p_2^P(q_1)$  and  $q_2 = q_2^P(q_1)$ . After rearranging,<sup>12</sup> this yields the following first-order condition:

$$q_1^P \stackrel{!}{=} (2 - \beta) A, \quad (11)$$

where  $A = (a - \gamma c)/(5 - 6\beta + 2\beta^2)$ . Accordingly, the number of permits sold in period 2 is given by

$$q_2^P = \frac{3 - 5\beta + 2\beta^2}{2 - \beta} A. \quad (12)$$

The respective permit prices are

$$p_2^P = a - \frac{7 - 9\beta + 3\beta^2}{2 - \beta} A$$

in period 2 and

$$p_1^P = 2a - \frac{11 - 13\beta + 4\beta^2}{2 - \beta} A$$

in period 1. The Government's overall revenue from the sale of permits, given by  $T^P(q) = p_1 q_1 + p_2 q_2$ , then is

$$T^P = \frac{a(9 - 21\beta + 16\beta^2 - 4\beta^3) + (13 - 16\beta + 5\beta^2)\gamma c}{(2 - \beta)^2} A, \quad (13)$$

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<sup>12</sup>The necessary re-arrangements are demonstrated in the Appendix.



while the total costs of pollution are

$$C^P = (2q_1 + q_2)c = \frac{11 - 13\beta + 4\beta^2}{2 - \beta} cA. \quad (14)$$

Total welfare is given by  $W^P = B_1 + B_2 - C^P$  or

$$W^P = \frac{(a - \gamma c)(31 - 58\beta + 37\beta^2 - 8\beta^3) - (44 - 74\beta + 42\beta^2 - 8\beta^3)(1 - \gamma)c}{2(2 - \beta)^2} A. \quad (15)$$

The Government's net payoff from selling permits as durable goods is given by  $\Pi^P = T^P + \beta(B_1 + B_2 - T^P) - \gamma C^P$ , or

$$\Pi^P = \frac{(9 - 10\beta + 3\beta^2)(a - \gamma c)}{2(2 - \beta)} A. \quad (16)$$

### 3 Leasing of Permits: Pollution Taxes

Permit sales usually grant pollution rights, if not forever, at least for a number of periods. Another way of implementing pollution control is to tax emissions. Conceptually, a pollution tax can be thought of as the equivalent to a leasing fee for the right to pollute. If the Government would decide to lease out pollution rights for just one period instead of selling them, then its optimal leasing fee would be just the same as the optimal pollution tax, at least in our model of complete and certain information and without innovation.

As we know from Coase (1972), leasing can be one way to overcome the durable good monopolist's time-inconsistency problem described above. As tax rates or leasing fees are set for one period of time, firms do not have to take into account the future value of their pollution rights, because they are valid for one period only, when deciding about their output and thereby about the level of pollution. A leasing arrangement separates the demand for pollution over the two periods. In each period, the inverse demand function is given through the marginal benefit schedule,  $R(q_t) = a - q_t$ . Hence, the Government's maximizing problem is now

$$\max_{q_t} \Pi = p_t(q_t)q_t + \beta N_t(q_t) - \gamma c q_t \quad (17)$$

for both  $t = 1$  and  $t = 2$ . Solving for the first-order condition we obtain the optimal leasing quantity,  $q_t^L$ , which is for both periods given by

$$q_t^L = \frac{a - \gamma c}{2 - \beta}. \quad (18)$$

Accordingly, the leasing fee or pollution tax rate is given by

$$p_t^L = \frac{(1 - \beta)a + \gamma c}{2 - \beta}, \quad (19)$$

and the Government's overall revenue from taxing pollution (i.e., leasing out pollution rights) is

$$T^L = \frac{2((1 - \beta)a + \gamma c)(a - \gamma c)}{(2 - \beta)^2}, \quad (20)$$

while the total costs of pollution are

$$C^L = \frac{2(a - \gamma c)}{2 - \beta}c. \quad (21)$$

Total welfare under the tax or leasing system amounts to

$$W^L = (a - \gamma c) \frac{(3 - 2\beta)a - (4 - 2\beta - \gamma)c}{(2 - \beta)^2}, \quad (22)$$

and the Government's payoff is given by

$$\Pi^L = \frac{(a - \gamma c)^2}{2 - \beta}. \quad (23)$$

## 4 Incentives for Different Types of Government

### 4.1 Evaluation of the General Model

In the general model, which we have presented in the previous section, the Government pursues both budget and welfare objectives. This type of Government is comparable with Williamson's (1963) model of discretionary managerial behavior, where a firm's managers use their discretion to pursue both the firm's profit (here: welfare) and their own perks (here: tax revenues). Let us now analyze the welfare effects of durable permit sales and annual permit leases (pollution taxes) as well as the Government's general incentives for choosing one or the other policy instrument.

#### 4.1.1 Welfare

We start with the analysis of the social welfare impacts of the two policy instruments. A durable permit system can be welfare superior compared to a leasing or tax system because durable permits pose a time-consistency or credibility problem for the Government which in fact leads in fact to a welfare improvement. This is due to the additional permits the Government is tempted to issue in period 2, which reduce the welfare loss from monopoly. As a pollution tax overcomes the Government's credibility problem, it simultaneously reduces product market efficiency as it implements the monopoly outcome.

Social welfare is given by total product market benefits net of the total cost of pollution:  $W^S = B^S - C^S$ , where the superscript  $S \in \{P; L\}$  represents the durable permit system ( $S = P$ ) or the leasing system ( $S = L$ ). As introduced

above, for any  $\beta$  and  $\gamma$  the Government's objective function is given by equation (4).

Comparing equations (15) and (22), note that social welfare is lower under a leasing (or pollution tax) system than with a durable-good permit sale if  $W^P > W^L$  or, equivalently:

$$\frac{a-c}{c}(1-\beta)^2 > (3-\beta)(1-\beta)(1-\gamma) \quad (24)$$

Recall that  $a > c > 0$  and  $\beta, \gamma \in [0, 1]$ . For given values of  $a, c$  and  $\gamma$ , the parameter  $\beta$  has the following impact on the above condition: For  $\beta = 1$  both the left- and the right-hand side of condition (24) reduce to 0; hence, there is no welfare difference between permit sales and a tax (leasing) system. For  $\beta < 1$ , condition (24) can be simplified to

$$\frac{a-c}{c} > \frac{(3-\beta)(1-\gamma)}{1-\beta}. \quad (25)$$

For  $\gamma < 1$  the right-hand side of the above expression is increasing in  $\beta$ ; thus condition (24) is more likely to hold the lower the Government's product market concern. The right-hand side of condition (24) is also decreasing in  $\gamma$ , and for  $\gamma = 1$  the condition is obviously always fulfilled (for  $a > c$ ). Thus, a permit system is the more likely to be welfare superior the larger the Government's environmental concern and the less it is concerned about product market benefits. Accordingly, a pollution tax or leasing system can only be welfare superior if  $\beta$  is sufficiently large or  $\gamma$  sufficiently small. Since a higher  $\beta$  means that the Government puts more emphasis on product market outcomes, the Government's inability to credibly commit to the level of pollution with a durable permit system may reduce welfare if the social costs of pollution are sufficiently high.

#### 4.1.2 Government's incentives

Apart from their welfare implications, it is of interest which of the policy instruments is preferred by the Government. Its policy choice is determined by its payoff function. Comparing the Government's payoff under the two systems (equations (16) and (23)), we find that a pollution tax or leasing system will never lead to a smaller payoff for the Government than a regime with permit sales, i.e  $\Pi^P \leq \Pi^L$  always holds.

Two cases are to be distinguished:  $\Pi^L > \Pi^P$  and  $\Pi^P = \Pi^L$ .  $\Pi^P > \Pi^L$  is equivalent to

$$\frac{9-10\beta+3\beta^2}{2(5-6\beta+2\beta^2)} \cdot \frac{(a-\gamma c)^2}{2-\beta} < \frac{(a-\gamma c)^2}{2-\beta}, \quad (26)$$

which can be reduced to  $9-10\beta+3\beta^2 < 10-12\beta+4\beta^2$ . This is equivalent to  $2\beta < 1+\beta^2$  or  $(1-\beta)^2 > 0$ . This condition holds for all  $\beta < 1$ , while for  $\beta = 1$  we obtain  $\Pi^P = \Pi^L$ .

To summarize, a Government with  $\beta < 1$  will always strictly prefer a leasing/tax system over permit sales, while a Government with  $\beta = 1$  is indifferent between the two policy instruments. No Government with  $\beta \leq 0$  will ever prefer permit sales over pollution taxes.

## 4.2 Four Stylized Types of Government

As special cases of the general model outlined above, we want to distinguish four types of government:<sup>13</sup>

1. a benevolent dictator, whose objective is to maximize social welfare, regardless of the collected budget. In our model, this type is parameterized as  $\beta = 1, \gamma = 1$ .
2. a pure Leviathan type of Government, which is only concerned with tax revenues ( $\beta = 0, \gamma = 0$ ).
3. a green Government, which is interested in both tax revenues and pollution ( $\beta = 0, \gamma = 1$ );<sup>14</sup> and
4. a business friendly Government, which is not only interested in tax revenues, but also shares product market concerns ( $\beta = 1, \gamma = 0$ ).

For each type of Government, we will now derive equilibrium prices and quantities as well as Government revenues, total pollution costs, and social welfare for both permit sales and leasing (taxes) under the various types of Government. These figures allow us to compare the two policy instruments under different types of Governments, as we can measure social welfare and/or environmental outcomes. Furthermore, the Government's payoff function should tell us which policy instrument would be preferred by a given type of Government. Thereby, we can endogenously explain the choice of environmental policy from a political economy point of view.

### 4.2.1 A Benevolent Dictator: $\beta = 1, \gamma = 1$

Using the model from the previous section and inserting  $\beta = 1$  and  $\gamma = 1$  into the respective formulas, it is straightforward that social welfare, i.e. total product market benefits less total pollution costs ( $B - C$ ), is maximized by setting the permit price equal to the marginal cost of pollution:  $p_1^P = 2c$ . The quantity sold in period 1 is  $q_1^P = (a - c)$ , and there are no additional permits sold in period 2.

Alternatively, a benevolent social planner could set the pollution tax rate at  $p_t = c$ . In both cases, total pollution amounts to  $C = 2(a - c)c$ , which equals

<sup>13</sup>As explained above, the model would also allow us to analyze cases beyond  $\beta, \gamma \in \{0; 1\}$ . However, we feel that this would unnecessarily complicate the exposition of results.

<sup>14</sup>The case of a Government that is only interested in reducing pollution costs leads to a rather trivial result: its optimal choice would be  $q_t = 0$ .

the amount of Government revenues, i.e.  $T = C = 2(a - c)c$ . Total welfare is  $W = B_1 + B_2 - C$ , which equals  $(a - c)^2$ .

Table 1: Results under a Benevolent Social Planner ( $\beta = 1, \gamma = 1$ )

Policy instrument	S	P	L
Government's revenues	$T^S$	$2(a - c)c$	$2(a - c)c$
Pollution costs	$C^S$	$2(a - c)c$	$2(a - c)c$
Total welfare	$W^S$	$(a - c)^2$	$(a - c)^2$
Government's payoff	$\Pi^S$	$(a - c)^2$	$(a - c)^2$

#### 4.2.2 A Pure Leviathan Government: $\beta = 0, \gamma = 0$

In the general model, it was assumed that the Government is partially benevolent in the sense that it cares about product market benefits and also gives weight to the social cost of pollution when making its decisions. However, it has been sometimes argued that a green or pollution tax is not really promoting environmental goals, but rather an attempt to raise revenues.<sup>15</sup> Hence, let us briefly explore what happens if both the costs of pollution and the benefits raised on the product market are external to the Government's objective function and, therefore, do not enter into the Government's decision making. This kind of Government only cares for monetary revenues.<sup>16</sup>

As can be easily verified from (11) to (16), under a permit selling regime the revenue maximizing Government will sell  $q_1^P = 0.4a$  permits in period 1 at a price of  $p_1^P = 0.9a$  and another  $q_2^P = 0.3a$  permits in period 2 for  $p_2^P = 0.3a$ . Total Government revenues accrue to  $T^P = 0.45a^2$ , which equals the Leviathan Government's payoff  $\Pi^P$ . The social costs of pollution are  $C^P = 1.1ac$ , and total welfare is given by  $W^P = 0.775a^2 - 1.1ac$ .

If the Leviathan Government uses a pollution tax, its revenue maximizing leasing charge is  $p_t^L = 0.5a$ , and the leased quantity is  $q_t^L = 0.5a$  in each period. Government revenues under the leasing model are  $T^L = 0.5a^2$ , which is also the Government's payoff  $\Pi^L$ . Pollution costs amount to  $C^L = ac$ , and total welfare is  $W^L = 0.75a^2 - ac$  (see (18) to (23)).

The comparison these results, as summarized in Table 2, reveals that under a pure Leviathan Government a tax or leasing system may actually be welfare superior when compared to a durable permit sale. More specifically, this is the case for  $a < 4c$ ,<sup>17</sup> i.e. if the costs of pollution are sufficiently high or the market for the good whose production causes the pollution is relatively small. If, on

<sup>15</sup>See, e.g., Schneider (1999, 27).

<sup>16</sup>The Leviathan is a representative of the traditional, profit maximizing durable-good monopolist that has been analyzed in the literature so far, see Rasmusen (1991, 276) or Coase (1972).

<sup>17</sup>The same result can be derived from condition (24).

Table 2: Results under a Leviathan Government ( $\beta = 0, \gamma = 0$ )

Policy instrument	S	P	L
Government's revenues	$T^S$	$0.45a^2$	$0.5a^2$
Pollution costs	$C^S$	$1.1ac$	$ac$
Total welfare	$W^S$	$0.775a^2 - 1.1ac$	$0.75a^2 - ac$
Government's payoff	$\Pi^S$	$0.45a^2$	$0.5a^2$

the other hand, the costs of pollution are relatively low or if the demand on the respective product market is relatively large ( $a > 4c$ ), a permit system results in higher welfare than a pollution tax.

In both cases, however, the Leviathan Government prefers the leasing solution, since  $\Pi^L > \Pi^P$ . Hence, if the social cost of pollution is not too high, the Government's preferred policy instrument (a tax or leasing charge) is suboptimal from a welfare point of view. The Leviathan Government's tax regime also leads to less pollution than the durable permit system ( $C^L < C^P$ ), even though the Leviathan Government does not pursue environmental goals. The leasing or pollution tax solution provides higher revenues:  $T^L > T^P$ .

#### 4.2.3 A Green Government: $\beta = 0, \gamma = 1$

A Green Government is interested in both tax revenues and pollution costs. The easiest way to parameterize this type of Government is setting  $\beta = 0$  and  $\gamma = 1$ .<sup>18</sup> The green Government's objective function hence is  $\Pi^S = T^S - C^S$ , where  $S \in \{P; L\}$  represents either the durable permit system ( $S = P$ ) or the leasing system ( $S = L$ ).

Under a durable permit system, a Green Government sells  $q_1^P = 0.4(a - c)$  licenses for  $p_1^P = 0.9a + 1.1c$  in Period 1. In the second period, the quantity sold is  $q_2^P = 0.3(a - c)$  at a price of  $p_2^P = 0.3a + 0.7c$ . Hence, the Government collects  $T^P = 0.45a^2 + 0.2ac - 0.65c^2$ . The total cost of pollution then is  $C^P = 1.1c(a - c)$ , and the total welfare accrues to  $W^P = 0.775(a - c)^2$ . The Government's payoff is  $\Pi^P = 0.45(a - c)^2$ .

Under a leasing or tax system, the Green Government would sell  $q_t^L = 0.5(a - c)$  at a price of  $p_t^L = 0.5(a + c)$  in both periods. Thereby, it collects  $T^L = 0.5(a^2 - c^2)$ . This leads to pollution costs of  $C^L = c(a - c)$ , and total welfare is  $W^L = 0.75(a - c)^2$ . The Government's payoff then is  $\Pi^L = 0.5(a - c)^2$ .

These results are summarized in Table 3. The durable permit system is welfare superior ( $W^P > W^L$ ). However, we expect a green Government to prefer a leasing or pollution tax system, since  $\Pi^L > \Pi^P$ .<sup>19</sup>

<sup>18</sup>While we could also derive results for more complicated cases with  $\gamma \neq 1$ , we feel the additional insight we gain from this exercise is rather limited.

<sup>19</sup>Also note that tax revenues are always larger under a Green Government's permit sale than under a green tax system.  $T^P \geq T^L$  holds as long as  $a \geq c$ , which must hold for a

Table 3: Results under a Green Government ( $\beta = 0, \gamma = 1$ )

Policy instrument	S	P	L
Government's revenues	$T^S$	$0.45a^2 + 0.2ac - 0.65c^2$	$0.5(a^2 - c^2)$
Pollution costs	$C^S$	$1.1c(a - c)$	$c(a - c)$
Total welfare	$W^S$	$0.775(a - c)^2$	$0.75(a - c)^2$
Government's payoff	$\Pi^S$	$0.45(a - c)^2$	$0.5(a - c)^2$

Figure 1 demonstrates the different pollution levels under a Green Government's pollution tax system. The upper diagonal line represents the marginal utility of pollution,  $R(q) = a - q$ , and the lower diagonal line represents the marginal increase in Government revenues ( $a - 2q$ ). The firms choose  $q^0$  if the Government fails to take any action, i.e. sets a zero price for pollution rights. From a social welfare point of view the optimal pollution level is reached at  $q^*$ ; the marginal utility of pollution  $R(q^*)$  equals the pollution cost  $c$ . Hence,  $q^0$  is clearly too high.

However, the Government's budget-orientation induces it, as the monopoly supplier, to charge too high a price for pollution rights, as shown in the sections above. If the Green Government (with  $\gamma = 1$ ) introduces a leasing system, it sets the monopoly price where marginal revenues equal pollution costs. If the green government would sell durable permits instead of charging a pollution tax, the average amount of pollution per period <sup>20</sup> would be  $q^P = 1.1q^L$ , which is still suboptimal low, but at least the welfare loss is smaller than under the leasing system. In Figure 1, social welfare is represented by the difference of the  $R$ -line and the  $c$ -line. Compared to the socially optimal pollution level,  $q^*$ , the welfare loss caused by the durable permit system is the small triangle between  $q^*$  and  $q^P$ , labeled as 1. The additional welfare loss of a pollution tax system is the square between  $q^L$  and  $q^P$ , labeled as 2.

#### 4.2.4 A Business-friendly Government: $\beta = 1, \gamma = 0$

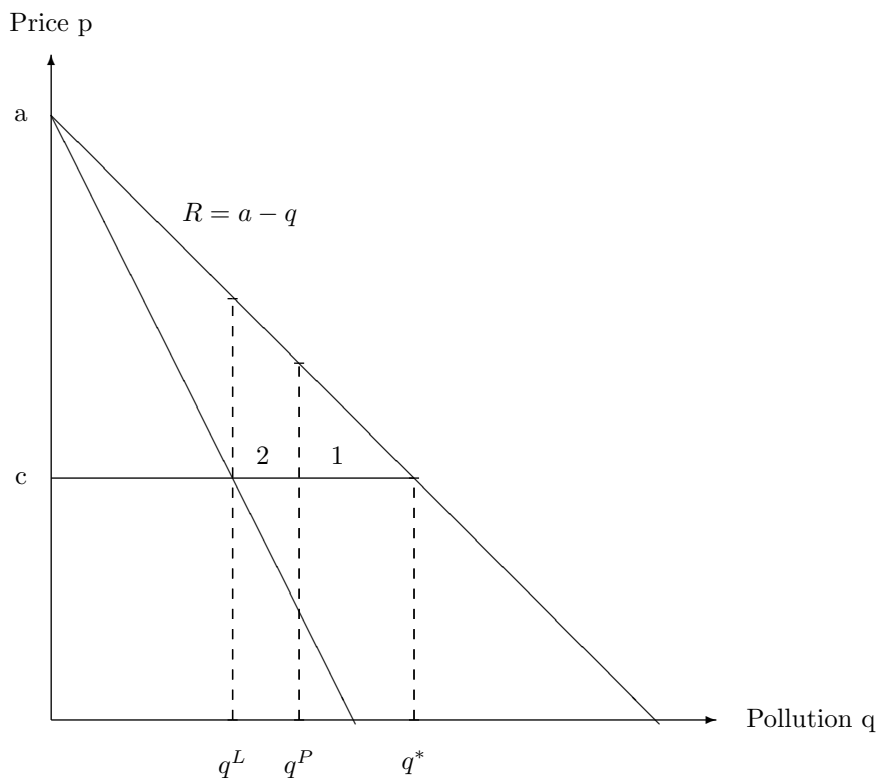
A business-friendly Government can be characterized by the parameter constellation  $\beta = 1$  and  $\gamma = 0$ , as we assume that the Government is primarily interested in product market benefits,<sup>21</sup> while neglecting the social cost of pollution. While for such a Government, permits are nothing but a source of income, the Government is still concerned about product market performance which contrasts with the pure Leviathan Government. The Government's ob-

product market to bring social benefits at all. From  $a \geq c$  it follows that  $a^2 - 4ac + 3c^2 \leq 0$ , which implies  $T^P \geq T^L$ .

<sup>20</sup>The average pollution amount  $q^P$ , as indicated in figure 1, equals  $0.5(2q_1^P + q_2^P)$ .

<sup>21</sup>Note that in our model all product market benefits will accrue to consumers if firms compete in Bertrand fashion. In this case, our entire analysis does never contain a double-marginalisation problem.

Figure 1: Welfare comparison between permits and tax (green Government)



jective function now is  $\Pi^S = T^S + N^S = B^S$ ,  $S \in \{P; L\}$ .

Under a durable permit system ( $S = P$ ), the Government offers  $q_1^P = a$  permits for a price of zero ( $p_1^P = 0$ ) in the first period, and no further permits in the second period ( $q_2^P = 0$ ,  $p_2^P = 0$ ). Obviously there are no Government revenues ( $T^P = 0$ ) whereas the total pollution costs are  $C^P = 2ac$ . Total welfare accrues to  $W^P = a^2 - 2ac$ , and the Government's payoff is  $\Pi^P = a^2$ . The same results hold for the tax or leasing system.

These results are summarized in Table 4.

### 4.3 Welfare Comparison for the Stylized Types

Now that we have derived the outcome for the four different types of Government (benevolent, Leviathan, green, and business-friendly) and for two different policy instruments which determine how the Government assigns the pollution



Table 4: Results under a Product Market-oriented Government ( $\beta = 1, \gamma = 0$ )

Policy instrument	S	P	L
Government's revenues	$T^S$	0	0
Pollution costs	$C^S$	$2ac$	$2ac$
Total welfare	$W^S$	$a^2 - 2ac$	$a^2 - 2ac$
Government's payoff	$\Pi^S$	$a^2$	$a^2$

rights (durable permits vs. leasing as an equivalent to a pollution taxes), let us compare our results in a comprehensive manner.

Durable permits lead, under all of the Government types, to higher or equal emission level than a leasing or tax solution; therefore, total pollution costs under the permit system exceed the pollution costs under a tax or leasing regime. This difference would amount to 10 percent under a green or Leviathan Government. The level of pollution is, however, still suboptimal with both a Leviathan and a green Government when compared to the social welfare maximizing level as given in Table 1, while pollution is too high with a business friendly Government which grants pollution rights for free or does not tax pollution.

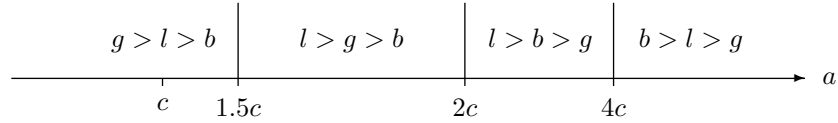
Another welfare comparison can be performed between the four types of Government. All of them prefer (at least weakly) the leasing solution. Given this policy choice, which type of Government is the one that best promotes social welfare? That is: Under which Government is  $W^L$  highest? The natural candidate for the top position is the benevolent Government. If we omit this trivial option, then we limit our view to the candidates for a second-best Government system. The comparison of  $W_t^L, t \in \{b, g, l\}$  (where  $b$  stands for business-friendly,  $g$  for green,  $l$  for Leviathan) yields the following insights:

- A business friendly Government is welfare superior to a Leviathan Government ( $W_b^L > W_l^L$ ) if  $a > 4c$ .
- A business friendly Government is welfare superior to a green Government ( $W_b^L > W_g^L$ ) if  $a > 3c$ ; and
- A Leviathan Government is welfare superior to a green Government ( $W_l^L > W_g^L$ ) if  $a > 1.5c$ .

Figure 2 demonstrates these results. For  $a > c$ , this figure distinguishes the four relevant cases (while the worst of the Government types is omitted respectively):

- $a < 1.5c$ : The green Government is second best, followed by the Leviathan, with the business friendly Government in the last position.
- $1.5c < a < 3c$ : Leviathan is second best, then green, then business-friendly.

Figure 2: Welfare comparison between second-best Government types



- $3c < a < 4c$ : Leviathan is second best, then business-friendly, then green.
- $a > 4c$ : Now the business-friendly government is second best, followed by the Leviathan, finally the green Government.

Hence, a green Government is the second-best type of Government in welfare terms if  $a < 1.5c$ , which may be the case if the cost of pollution is relatively high or the product market concerned relatively small. For  $4c < a < 1.5c$  a Leviathan Government is preferable, while for  $a > 4c$  a business friendly Government brings the highest welfare of these three types of Government.

## 5 Policy Implications and Conclusion

All of the Government types we have analyzed in the previous section (namely the Leviathan, the Green Government and the business-friendly one) prefer a leasing or pollution tax regime over durable permit sales in our model. For the Leviathan, this result is far from being counterintuitive, since the monopoly power secured by the leasing system provides higher revenues. Whereas the impact of leasing (instead of durable permit sales) on pollution cost and revenues are ambiguous with a Green Government, the leasing regime clearly provides a higher payoff to the Government.

Environmental policy based on durable pollution permits can yet be welfare superior to a pollution tax. This is caused by the credibility problem a revenue maximizing Government faces. This credibility problem weakens the Government's ability to commit to the monopoly quantity in the long run. Thereby, durable permits can reduce the allocative inefficiency induced by a monopolistic, revenue maximizing Government. In contrast, a pollution tax or leasing charge allows the Government to credibly commit not to alter the allowed level of pollution in the future in order to raise additional revenues.

In fact, a pollution tax or leasing charge may align the Government's fiscal and environmental objectives in the long run, as a durable permit system leads to both lower Government revenues and higher levels of pollution when compared to a pollution tax or leasing regime.

While a pollution tax regime may be better for the environment and for the Government's revenues, we have shown that it is harmful for overall welfare if the Government also has a revenue objective. In fact, from a social welfare perspective it may be desirable that the Government cannot commit to monopoly quantities. Or to put it differently, it may be desirable to introduce constitutional constraints on the choice of environmental policy instruments.<sup>22</sup> Our argument for a permit regime is completely different, however, from more traditional arguments favoring tradable permit regimes which are usually based on information asymmetries and innovation incentives.

To be provocative, the recent Kyoto protocol may, in light of the above results, be seen as a monopolistic attempt to fix quantities in order to achieve monopoly outcomes, as international treaties and international law may provide another commitment device which we have not explored in this paper.

Finally, the above model may also help to explain from a political economy point of view why many different types of Governments appear to favor pollution taxes over permit regimes. A tax system may simply generate higher Government revenues than pollution permits in the long run, as it serves as a commitment device for a profit maximizing monopolist.

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<sup>22</sup>The idea that the politicians' power should be limited through constitutional constraints on the choice of policy instruments has also been put forward by Boyer and Laffont (1999) in a model where politicians have private information about the true cost of some policy measure.

## References

- Bagnoli, M.; Salant, S.W. & Swierzbinski, J.E. (1989). Durable-goods monopoly with discrete demand. *Journal of Political Economy* 97, 1459-1478.
- Biglaiser, G.; Horowitz, J.K. & Quiggin, J. (1995). Dynamic pollution regulation. *Journal of Regulatory Economics* 8, 33-44.
- Boyer, M. & Laffont, J.J. (1999). Toward a political theory of the emergence of environmental incentive regulation *RAND Journal of Economics* 30, 137-157.
- Bulow, J.I. (1982). Durable-goods monopolists. *Journal of Political Economy* 90, 314-332.
- Coase, R. (1972). Durability and monopoly. *Journal of Law and Economics* 15, 143-149.
- Denicolo, V. (1999). Pollution-reducing innovations under taxes and permits. *Oxford Economic Papers* 51, 184-199.
- Deweese, D. (1998). Tradeable pollution permits, p. 596-601 in: Newman, P. (ed.), *The New Palgrave of Dictionary of Economics and Law*. Macmillan: London and New York.
- Gawel, E. & Schneider, F. (1997). Implementation problems of eco-taxation: A political-economy analysis. Discussion paper no. 9719, July 1997, University of Linz.
- Gersbach, H. & Glazer, A. (1999). Markets and regulatory hold-up problems. *Journal of Environmental Economics and Management* 37, 151-164.
- Fredriksson, P.G. (1997). The political economy of pollution taxes in a small open economy. *Journal of Environmental Economics and Management* 33, 44-58.
- Jung, C.; Krutilla, K. & Boyd, R. (1996). Incentives for advanced pollution abatement technology at the industry level: An evaluation of policy alternatives. *Journal of Environmental Economics and Management* 30, 95-111.
- Laffont, J.J. & Tirole, J. (1996). Pollution permits and environmental innovation. *Journal of Public Economics* 62, 127-140.
- Milliman, S.R. & Prince, R. (1989). Firm incentives to promote technological change in pollution control. *Journal of Environmental Economics and Management* 17, 247-265.
- Norregaard, J. & Reppelin-Hill, V. (2000). *Controlling pollution. Using taxes and tradable permits*. International Monetary Fund, Economics Issues Vol. 25: Washington, D.C.

- Rasmusen, E. (1991). *Games and Information*. Blackwell: Cambridge, MA.
- Requate, T. (1995). Incentives to adopt new technologies under different pollution-control policies. *International Tax and Public Finance* 2, 295-317.
- Requate, T. & Unold, W. (1997). On the incentive to adopt advanced abatement technology - will the true ranking please stand up? Discussion paper no. 251, July 1997, University of Heidelberg.
- Schneider, F. (1999). Induzieren ökologische Steuerreformen einen Lenkungseffekt oder nur volle Staatskassen? Einige volkswirtschaftliche Überlegungen. Discussion paper no. 9919, November 1999, University of Linz.
- Stokey, N.L. (1981). Rational expectations and durable goods pricing. *Bell Journal of Economics* 12, 112-128.
- Tietenberg, T.H. (1985). Emissions trading. *Resources for the Future*: Washington, D.C.
- von der Fehr, N.H. & Kühn, K.U. (1995). Coase versus Packman: Who eats whom in the durable-goods monopoly? *Journal of Political Economy* 103, 785-812.
- Williamson, O.E. (1963). Managerial discretion and business behavior. *American Economic Review* 53, 1030-1057.
- Wirl, F. & Dockner, E. (1997). Leviathan Governments and carbon taxes: Costs and potential benefits. *European Economic Review* 39 (6), 1215-1236.

## Appendix

### Re-arrangements for equation (11)

The Government's payoff function is  $T + \beta N - \gamma C$ . Substituting  $T = q_1 p_1 + q_2 p_2$  and  $N = B - T$  yields

$$\max_{q_1} p_1 q_1 + p_2 q_2 + \beta[(B_1(q_1) - p_1 q_1) + (B_2(q_1, q_2) - p_2 q_2)] - \gamma(2q_1 + q_2)c$$

Substitution of  $q_2 = q_2^P(q_1)$ ,  $p_2 = p_2^P(q_1)$  and  $p_1 = p_1^P(q_1)$  leads to the following expression:

$$\begin{aligned} & \frac{(3-2\beta)(a-q_1) + \gamma c}{2-\beta} q_1 + \frac{(1-\beta)(a-q_1) + \gamma c}{2-\beta} \frac{a-q_1-\gamma c}{2-\beta} \\ & + \beta[(B_1(q_1) - \frac{(3-2\beta)(a-q_1) + \gamma c}{2-\beta} q_1)] \\ + \beta[B_2(q_1, q_2) - \frac{(1-\beta)(a-q_1) + \gamma c}{2-\beta} \frac{a-q_1-\gamma c}{2-\beta}] - \gamma(2q_1 + \frac{a-q_1-\gamma c}{2-\beta})c \end{aligned}$$

Substitution of  $B_1$  then leads to

$$\begin{aligned} & \frac{(3-2\beta)(a-q_1) + \gamma c}{2-\beta} q_1 + \frac{(1-\beta)(a-q_1) + \gamma c}{2-\beta} \frac{a-q_1-\gamma c}{2-\beta} \\ & + \beta[aq_1 - \frac{q_1^2}{2} - \frac{(3-2\beta)(a-q_1) + \gamma c}{2-\beta} q_1] \\ + \beta[(a(q_1 + \frac{a-q_1-\gamma c}{2-\beta}) - \frac{(q_1+q_2)^2}{2} - \frac{(1-\beta)(a-q_1) + \gamma c}{2-\beta} \frac{a-q_1-\gamma c}{2-\beta}] \\ & - \gamma(2q_1 + \frac{a-q_1-\gamma c}{2-\beta})c \end{aligned}$$

which is to be maximized. The derivative with respect to  $q_1$  of the latter expression is

$$\begin{aligned} & -\frac{10q_1 + 4\gamma c - 4a + 5a\beta - 12q_1\beta - 5\gamma c\beta - 2a\beta^2 + 4q_1\beta^2 + 2\gamma c\beta^2}{(-2+\beta)^2} \\ & + (4\beta - 4\beta^2 + \beta^3) \frac{5q_1 - \gamma c + a - a\beta - 6q_1\beta + \gamma c\beta + 2q_1\beta^2}{(-2+\beta)^4} \end{aligned}$$

Setting this equal to zero yields

$$q_1 = (2-\beta) \frac{a-\gamma c}{5-6\beta+2\beta^2} = (2-\beta)A$$

as the solution, see equation (11).