Heiner, Ronald Asher

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Robust Evolution Of Contingent Cooperation In Pure One-Shot Prisoners' Dilemmas. Part II: Evolutionary Dynamics & Testable Predictions

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ROBUST EVOLUTION OF CONTINGENT COOPERATION IN PURE ONE–SHOT PRISONERS' DILEMMAS

Part I: Vulnerable Contingent Participators Versus Stable Contingent Cooperators

Center for the Study of Law and Economics Discussion Paper 2002-09

Part II: Evolutionary Dynamics & Testable Predictions

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Ronald A. Heiner
James Buchanan Center For Political Economy
George Mason University
MSN 1D3 Carow Hall
Fairfax, VA 22030
email: rheiner@gmu.edu

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ABSTRACT

ROC curves from the signal detection literature are used in an evolutionary analysis of one-shot and repeated prisoners' dilemmas: showing if there is any discounting of future payoffs, or any cost of searching for an additional partner, then cooperative players who contingently participate – in terms of who to play with or when to exit – cannot survive when most other players unconditionally defect; even when contingent participators only interact with themselves by perfectly detecting their own type.

However, quite different results hold for players who act contingently, not in terms of whether to play or exit, but rather in terms of how to act with any given partner. There is a form of contingent cooperation in one-shot prisoners' dilemmas (called CD behavior) that will robustly evolve through any payoff monotonic process, such as replicator dynamics. That is, whenever CD–players can detect their own type better than pure chance, they are guaranteed to evolve from any initial population – eventually to a unique evolutionarily stable population composed entirely of contingent cooperators – provided the fear payoff difference is less than the sum of greed and cooperation payoff differences.

The adaptive capabilities just described hold for *pure* one–shot prisoners' dilemmas : meaning no repeated interactions or pairings in any generation are involved; no information or third party reports about past behavior are involved, all signal information arises only from symptoms detected after two strangers meet for the first time; and no subjective preferences for altruism, fairness, equity, reciprocity, or morality affect the raw evolutionary dynamics.

Testable predictions are also derived that agree with a large body of experimental data built up since the prisoners dilemma was first introduced in 1950. They describe how the CD–players' equilibrium probability of cooperating changes: depending on the relative size of fear, greed, and cooperation payoff differences; and depending on the players' history of communication, especially when face-to-face discussion is involved.

(JEL C60, C62, C72, C92, D80)

KEYWORDS: prisoners' dilemma, cooperation, Nash equilibrium, evolutionary stability, replicator dynamics, signal detection, ROC curves, experiment, testable predictions

ROBUST EVOLUTION OF CONTINGENT COOPERATION IN PURE ONE–SHOT PRISONERS' DILEMMAS, PART II:

Evolutionary Dynamics & Testable Predictions

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Ronald A. Heiner [1]

Part I modified traditional analysis in three ways. The first change was allowing players to detect symptoms from each other: governed by overlapping density functions that are caused by histories determining how each player is programmed to react to symptoms categorized as an x or y signal by the receiving player. The resulting signal probabilities of rightly versus wrongly detecting a player's own type also become weights multiplying the fear, greed, and cooperation payoff differences; producing linear and quadratic expected payoffs shown in Table 1.

The second change was the transformation in players' strategies implied by signal detection: from action strategies C and D into four signal response strategies (CC, CD, DC, DD), plus a randomized action strategy CλD. The always defect strategy DD dominates CC and CλD, as expected from traditional analysis; but is not dominant over both signal contingent strategies CD and DC. Instead, Theorem 2 of Part I implies contingently cooperative CD behavior can be evolutionarily stable plus a strict Nash equilibrium for one–shot prisoners dilemmas.

The third change was using ROC curves to restrict the relationship between CD–players' signal probabilities (r_{CD}, w_{CD}): requiring these probabilities be contained on an ROC curve that describes the CD–players' detection skill, and showing through numerous experiments that CD–players can more or less cautiously detect their own type by shifting (r_{CD}, w_{CD}) toward (0, 0) or (1, 1) along an ROC curve.

The last property about cautious detection along an ROC curve implies a further dynamic result for one–shot prisoners dilemmas: CD–players will robustly evolve through any payoff monotonic process and from any initial population, eventually to a unique evolutionarily stable population composed entirely of CD–players. The upcoming Sections I & II of Part II show why this must happen.

Sections III and IV then derive testable predictions for the CD–players' probability of cooperating. These predictions agree with a large body of experimental data built up over the years since the prisoners dilemma was first introduced in 1950. They involve *normalized* greed and fear payoff differences, meaning *relative* to the cooperation payoff difference; as measured by the ratios (T – R)/(R – P) and (P – S)/(R – P). Provided the fear payoff difference is less than the sum of greed and cooperation payoff differences, the following predictions for example hold: (1) The cooperation probability will rise or fall if normalized greed and fear payoff differences both fall or rise at the same rate, but not necessarily all the way up to 1 nor all the way down to 0. (2) The normalized greed payoff difference will have a greater effect than the normalized fear payoff difference, whenever the cooperation probability exceeds a threshold above ½. This threshold will be exceeded as normalized greed and fear payoff differences decrease sufficiently, and vice versa if these normalized differences increase sufficiently.

Further predictions result from holding normalized payoff differences constant while detection skill changes. For example, doing so implies the probability of cooperation is more sensitive to signals detected through face–to–face discussion; compared to other means of communicating by telephone, email, or video conferencing: because telecommunication degrades access to causal mechanisms involved in eliciting

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verbal/body-language symptoms; thereby shifting the CD–players' NN–ROC curve closer to pure chance detection. This prediction agrees with experiments described by Ostrom (2000) and with experiments back to 1958 analyzed by Sally (1995).[2]

I. ROBUST EVOLUTION OF CD BEHAVIOR AGAINST DD BEHAVIOR

Consider first the subpopulation with only CD and DD players, called the CD/DD subpopulation. To do so, look at the linear expected payoff example (so that T – R = P – S) shown Figure 2a in Part I, and recall the crossover frequency $\xi^0_{CD/CD}$ above which CD players will outperform DD players. Recall also from Part I that shifting (r_{CD}, w_{CD}) closer to (0, 0) means a CD–player is *more cautiously* detecting whether it has been matched with its own type instead of another type. To see what this implies, set the formulas for the CD and DD players' expected payoff lines in Figure 2a equal to each other and solve for the crossover frequency $\xi^0_{CD/DD}$; yielding the formula:

$$x^0_{CD/DD} \;=\; \frac{w_{CD}\,(P-S)}{(r_{CD}-w_{CD})(R-P)} \;=\; \frac{P-S}{\left(\frac{r_{CD}}{w_{CD}}-1\right)(R-P)} \qquad (6)$$

Formula (6) enables a quick answer to what happens to the crossover frequency $\xi^0_{CD/DD}$ as the CD players more cautiously detect their own type. Recall from statement (1) in Part I, that the ratio r_{CD}/w_{CD} rises to infinity as the CD players more cautiously detect: by shifting the signal probabilities (r_{CD}, w_{CD}) closer to (0, 0) along a convex NN-ROC curve that bows above the diagonal dashed-line in Figure 1a of Part I. This corresponds to detecting with any degree of skill beyond the lower limit of pure chance detection.

Formula (6) immediately implies from the above property [$r_{CD}/w_{CD} \rightarrow \infty$ as $(r_{CD}, w_{CD}) \rightarrow (0, 0)$] that the crossover frequency $\xi^0_{CD/DD}$ shifts to 0 as CD–players detect more cautiously, provided they can detect beyond pure chance. This result in turn implies the CD–players can always shift $\xi^0_{CD/CD}$ below their frequency in the population ξ_{CD}, no matter how small as long as ξ_{CD} is positive: *thereby guaranteeing CD–players will outperform DD–players as their population frequency* ξ_{CD} *grows from any positive level all the way to 1.*

Figure 2b shows an example of this implication. The three pairs of solid versus dashed *thin* straight–lines represent the CD versus DD players' expected payoffs implied by holding the CD–players' pair of signal probabilities *constant* at three different points on their ROC curve given by (r^K_{CD}, w^K_{CD}) for K = 1, 2, 3. The solid versus dashed ***bold*** curves represent the CD versus DD players' expected payoffs for population frequencies ξ_{CD} ranging continuously from 0 to 1, including $0 < \xi^1_{CD} < \xi^2_{CD} < \xi^3_{CD} < 1$; but now with the CD–players more cautiously detecting, by shifting (r_{CD}, w_{CD}) closer to (0, 0) as their population frequency ξ_{CD} drops. Notice how doing so causes their expected payoff curve to exceed the DD–players' expected payoff curve for any positive frequency, $\xi_{CD} > 0$. *Consequently, the only stable equilibrium within the CD/DD subpopulation is,* $\xi_{CD} = 1$.

Figure 2b & Table 1 About Here

Recall from the end of Section I in Part I, that ROC curves from the signal detection literature show numerous experiments demonstrate the signal detection properties used in the above analysis are the typical result of ordinary people in ordinary circumstances; thereby achievable without any special detection skill nor any special signals like 'secret handshakes', and so on.

So the properties just described mean the following for one-shot interaction in the CD/DD subpopulation: ordinary players under ordinary conditions with no special detection skill can follow a simple strategy of more cautiously interpreting symptoms from their partners (whenever needed if their own frequency in the subpopulation drops); that will guarantee at least a small statistical bias in their favor over DD–players; and likewise guarantee the subpopulation will evolve toward only contingently cooperating CD–players. CD–players thus have the flexibility to shift their behavior at will, closer statistically to pure DD or pure CC behavior; but with a guaranteed statistical bias that will likewise guarantee they will always take over the

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[2] Appendix E describes qualified results when players' fear payoff difference exceeds the sum of greed and cooperation differences. Part II also continues with the indexing used in Part I; which ended with Equation 5, Figure 2a, Table 1, and Theorem 2.

subpopulation. Yet CD players need no unusual detection skills or special signals in order to guarantee their dominance against DD–players.

II. EVOLUTIONARY DYNAMICS WITHIN AN ARBITRARY POPULATION

The analysis so far has been limited to the CD/DD subpopulation. So we need to see whether the same conclusions hold within a full population containing arbitrary frequencies of all five types of players, $\xi = (\xi_{CC}, \xi_{CD}, \xi_{DC}, \xi_{DD}, \xi_{C\lambda D})$. This is shown by using any payoff monotonic process, such as *replicator dynamics*, to analyze the interactions of all five types of players.

A. *Comparing DD–Players Versus DC, CC, & CλD Players*

To analyze payoff monotonic dynamics, we must compare the expected payoff formulas resulting from each of the five player types against their own type, or the other four types. These were shown in Table 1 of Part I, also shown here for convenience. Recall that quadratic terms (with probability multiples such as $(r_{CD})^2$, λ^2, or λw_{DC}) are eliminated if the fear and greed payoff differences are equal. So cases where $(P - S) \neq (T - R)$ imply *quadratic expected payoffs*; and cases where $(P - S) = (T - R)$ imply *linear expected payoffs*.

Next use Table 1 to compare expected payoffs of DD vs DC players when they play each of the five player types. To do so, subtract the formulas in the DC row from the DD row in Table 1. These comparisons reveal a simple result: DD–players always outperform DC–players against all player types, except at the limit were the DC players act just like DD–players by shifting their signal probabilities to the upper limit $(r_{DC}, w_{DC}) = (1, 1)$.

By comparing formulas in the DD row with those in the CC and CλD rows in Table 1, similar results hold: DD–players always outperform CC–players against all player types, and likewise outperform CλD–players against all player types, for any positive $\lambda = p(C) > 0$. When combined with Section II – about CD–players always outperforming DD players by more cautiously detecting when their frequency ξ_{CD} drops – *this is sufficient to prove any payoff monotonic process will only converge to an exclusive population of CD–players; starting from any interior population with positive frequencies of all player types*.

The main text focuses on this result; leaving derivations about 'boundary' cases to Appendix D. For now we note that such cases with a zero frequency of DD–players sometimes allow the CD/DC subpopulation to contain interior equilibria that are unstable within any larger population containing a positive frequency of DD–players.

Recall from experiments discussed in Part I, that besides more cautious detection as their population frequency drops, CD–players will also more cautiously detect as the costs of mistaken detections rises: for example, if the fear and greed payoff differences rise compared to the cooperation payoff difference. So the CD–players' signal probabilities represent "frequency and payoff dependent signal detection".

Such frequency and payoff dependent signal detection will determine the dynamics from any payoff monotonic process, as defined in Part 1 of Appendix D. For player types with positive population frequencies, such dynamics imply the following: *one player type's frequency will grow relative to another type's frequency if and only if the first type's expected payoff across the five player types exceeds that of the second type*. The next section describes an example with one unstable equilibrium in the CD/DC subpopulation.

B. *Evolutionary Dynamics With One Unstable Interior CD/DC Equilibrium*

Figure 3a shows dynamic paths between (CC, CD, DC, DD). The randomizing CλD–players are not shown because their behavior is a statistical combination of both CC and DD; and because Part 3 of Appendix D shows that CλD–players do not affect the qualitative dynamics involving themselves plus the other players.

Dynamic paths are shown by unfolding a four-sided *tetrahedron* whose sides are similar triangles. Each vertex represents a degenerate population composed of only one player type. The CC vertex is shown three times (at the extreme bottom, left and right) because it represents a single vertex that would result from re–folding the three 'outer' triangles upward toward a single CC vertex located vertically above the page on which Figure 3a is printed.

Notice the curved boundary indicated by the three B–points, called the B–boundary. It shows frequency profiles where the CD and DC players' expected payoffs are equal; producing an unstable equilibrium at point B on the edge strictly between the CD and DC vertices. So as dynamic paths cross the B–boundary, the expected payoff of the CD–players changes from 3rd place to alone in 2nd place (still below DD but now above DC and CC).

The other boundary formed by the lines from the DD vertex to the two points labeled 'A' near the CD vertex, is called the 'A–boundary'. This boundary represents the threshold at which the expected payoffs of the DD and CD players become tied for 1st place above the CC and DC players. Once the A–boundary is crossed along any dynamic path, the CD–players are alone in 1st place – outperforming the other types of players, including DD.

Figures 3a & 3b About Here

Notice how some paths from the unstable equilibrium between the CD and DC vertices shift mostly toward the DD vertex, and then turn mostly toward plus converge to the CD vertex. Likewise, other paths arising from the DC and CC vertices initially raise the frequency of DD–players relative to the other three players besides DD: either converging all the way to the DD vertex, or crossing the A–boundary as the frequency of DD–players rises.

If the DD vertex is reached, then arbitrarily small perturbations toward any positive frequency of CD–players inside the A–boundary will trigger payoff monotonic dynamics away from the DD vertex and toward the CD vertex. Alternatively, if the A–boundary is crossed before reaching the DD vertex, then the dynamic path will quickly "turn–the–corner" away from the DD vertex and again toward the CD vertex.

So ultimately, all dynamic paths from any initial population will converge – either directly through any payoff monotonic process, or combined with arbitrarily small perturbations from unstable equilibria – to a unique evolutionarily stable equilibrium composed entirely of CD–players, $\xi_{CD} = 1$.

Concerning this basic result, notice how always defect DD behavior – traditionally thought to be the top performer in one-shot prisoners' dilemmas – does initially attract most dynamic paths toward the DD vertex. Yet this very attraction shifts the population toward the DD/CD subpopulation; where frequency–dependent signal detection will at some point cause payoff monotonic dynamics to shift away from the DD vertex that initially attracted these dynamic paths. For comparison, Figure 3b shows similar dynamic properties when no unstable equilibrium exists between the CD and DC vertices.

C. *Evolutionary Dynamics In One–Shot Prisoners' Dilemmas: General Results*

Given the dynamics shown in Figures 3(a, b), we can now describe the basic properties that must hold in an evolutionary contest between the five types of players. The assumptions under which these properties hold were listed at the start of Part I. [3] They are proved for both linear and quadratic expected payoffs in Parts 2 – 4 of Appendix D.

THEOREM 3 (*Robust Evolution Of Contingent Cooperation In Pure One-Shot Prisoners' Dilemmas*)

PART A Let $\xi^0_{CD/DD}$ (r_{CD}, w_{CD}, Z) equal the crossover threshold in Figure 2a implied from the signal detection probabilities (r_{CD}, w_{CD}) and payoffs Z such that (P – S) < (T – R) + ® – P). Then Theorem D1 in Appendix D shows there exist frequency and payoff dependent signal probability functions, denoted $[r_{CD}(\xi_{CD}, Z), w_{CD}(\xi_{CD}, Z)]$, such that for any $\xi_{CD} > 0$:

$$0 < \xi^0_{CD/DD}[r_{CD}(\xi_{CD}, Z), w_{CD}(\xi_{CD}, Z), Z] < \xi_{CD}$$

CD–players will therefore always outperform DD–players within the CD/DD subpopulation, for any $\xi_{CD} > 0$, as shown in Figure 2b. Part 3 of Appendix D shows a similar result holds about CD–players always outperforming CλD–players within the CD/CλD subpopulation.

[3] The (CC, DD, CλD) players behave independently of signals x or y; so they don't need to meet or talk with their partners to detect such signals. However, all five types of players must meet and talk with their partners, or they will be immediately identified by CD–players as someone who will not react contingently to symptoms that cannot be detected without meeting and talking with them, and thus a ~CD–player. So all five types of players have the same communication set up costs of meeting and talking with their partners, even though (CC, DD, CλD) players will ignore any symptoms detected while meeting and talking with them.

PART B Likewise, the formulas in Table 1 imply DD–players always outperform (CC, DC, CλD) players, for any (r_{DC}, w_{DC}) < (1, 1) and $\lambda > 0$: so any positive frequency of these players is unstable against any positive frequency of DD–players in the population. One or more unstable equilibria may also arise in the interior of the CD/DC subpopulation; as illustrated in Figure 3a, and further described in Theorem D3 of Appendix D.

PART C Parts A and B imply the following general result whenever CD–players can detect their own type with any degree of skill beyond pure chance : [4]

The population profile ξ will evolve from any initial population – either directly through any payoff monotonic process, or combined with arbitrarily small perturbations from unstable equilibria – eventually to a unique evolutionarily stable population composed entirely of CD–players, $\xi_{CD} = 1$.

Recall from Part I, that the signal detection literature shows numerous experiments demonstrate the signal detection properties used in Theorem 3 are the typical result of ordinary people in ordinary circumstances; thereby achievable without any special detection skills nor any special signals like 'secret handshakes', and so on.

So the dynamic properties of Theorem 3 mean the following for one-shot competition between the five player types : ordinary players under ordinary conditions with no special detection skills can follow a simple strategy of more cautiously interpreting symptoms from their partners (whenever needed if their own frequency in the population drops); that will guarantee at least a small statistical bias in their favor over other types of players, including always defecting players; and thereby also guarantee the population will ultimately shift away from other types of behavior toward an exclusive population of contingently cooperating CD players. CD players thus have the flexibility to shift their behavior at will, closer statistically to either pure DD or pure CC behavior; but with a guaranteed statistical bias that will likewise guarantee they will always take over the population.

III. PREDICTING THE EQUILIBRIUM COOPERATION PROBABILITY

Part A of Theorem 3 focuses on CD–players cautiously detecting whether they are matched with other CD–players. However, doing so may not be required once the evolutionary success from cautious detection causes their population frequency ξ_{CD} to rise toward the stable equilibrium, $\xi_{CD} = 1$: thereby allowing CD–players to shift their signal probabilities (r_{CD}, w_{CD}) closer to (1, 1) along an NN-ROC curve.

Since CD–players cooperate if and only if they detect signal x, their probability of cooperating equals the signal probability of rightly detecting (r_{CD}), whenever their partners are other CD–players; which must happen at $\xi_{CD} = 1$. So the next objective is to determine how the cooperation probability varies between 0 and 1; as implied by the signal probability function evaluated at the $\xi_{CD} = 1$ equilibrium, $r_{CD}(\xi_{CD} = 1, Z)$.

To do so, note that random mutations may produce perturbations that cause ξ_{CD} to fluctuate below 1, and thus potentially below the crossover threshold $\xi^0_{CD/DD}$. To guard against this happening, the CD–players must maintain sufficient caution in detecting their own type; in order to maintain a gap $(1 - \xi^0_{CD/DD})$ larger than these perturbations. Otherwise, the $\xi_{CD} = 1$ equilibrium might not remain stable.

The CD–players' cooperation probability has the same qualitative properties for any size of the above gap $(1 - \xi^0_{CD/DD})$ resulting from cautious detection. That is, any degree of cautious detection *except* total caution – so that $\xi^0_{CD/DD} > 0$ holds because (r_{CD}, w_{CD}) > (0, 0) – implies the CD–players' cooperation probability has the same qualitative properties; provided there also is sufficient caution to ensure $\xi^0_{CD/DD} < 1$ still holds by shifting (r_{CD}, w_{CD}) sufficiently below (1, 1) along an ROC curve. To see this, let γ denote a positive frequency bounding $\xi^0_{CD/DD}$ strictly between 1 and 0; so the inequality $(0 < \gamma \le \xi^0_{CD/DD} \le 1 - \gamma < 1)$ holds.

Then look at Figure 4 which shows how these lower and upper bounds on $\xi^0_{CD/DD}$ correspond to different signal probability ratios r_{CD}/w_{CD}. The slope of the *steeper* line, from (0, 0) in unit square, equals the signal probability ratio r_{CD}/w_{CD} corresponding to $\xi^0_{CD/DD}$ equaling the *lower* bound, $\gamma > 0$. The slope of the *flatter* line

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[4] Another paper (Heiner 2002) extends the results of Theorem 3, so that individual CD–players can shift their signal probabilities (r_{CD}, w_{CD}) independently of each other; allowing their individual signal probabilities to differ from each other. Similar results hold in which the CD–players have a dominant strategy to shift their signal probabilities toward (1,1) along their ROC curve; up to the point where shifting any further would allow invasion by DD–players. This corresponds to the same minimum degree of cautious detection implied by Theorem 3 to likewise maintain stability against DD–players.

likewise equals the signal probability ratio r_{CD}/w_{CD} corresponding to $\xi^0_{CD/DD}$ equaling the *upper* bound, $1 - \gamma < 1$.

Figure 4 About Here

Notice the dashed intervals along each of the NN-ROC curves shown in Figure 4, between two points labeled with capital letters 'A' and 'B'. These show, for a given NN-ROC curve, the signal probabilities (r_{CD}, w_{CD}) permitted by the lower and upper bounds γ and $1 - \gamma$. For example, the dashed interval from A'' to B'' shows this range of signal probabilities resulting from near pure chance detection skill; while the dashed interval from A to B shows this range of signal probabilities resulting from near perfect detection skill. And dashed interval from $A^{\diamond}$ to $B^{\diamond}$ shows this range of signal probabilities resulting from perfect detection skill.

Notice what happens to the dashed interval for given payoffs Z = (T, R, P, S) – as the CD–players' detection skill shifts between the limits of pure chance versus perfect detection: the dashed interval converges to the origin (0, 0) as detection skill drops to pure chance detection; while at the other extreme, the dashed interval converges to a *horizontal* interval from $A^{\diamond}$ to $B^{\diamond}$ as detection skill becomes perfect, where $r_{CD} = 1$ along this entire horizontal interval.

So consider the range of r_{CD} probabilities between the A and B points along each dashed interval in Figure 4. The relationships just described imply any r_{CD} probability along a corresponding dashed interval will shift toward 0–% versus 100–% as detection skill shifts between pure chance versus perfect detection. Since CD–players' equilibrium cooperation probability equals $r_{CD}(\xi_{CD} = 1, Z)$, the last conclusion likewise implies the following:

COROLLARY A

Any crossover frequency within the bounds ($0 < \gamma \le \xi^0_{CD/DD} \le 1 - \gamma < 1$) implies the equilibrium probability of cooperation $r_{CD}(\xi_{CD} = 1, Z)$, anywhere along the corresponding dashed interval in Figure 4, will shift toward 0–% versus 100–% as the CD–players' detection skill shifts between pure chance versus perfect detection.

Alternatively, consider what happens to the probability of CD–players cooperating with each other, for any given detection skill corresponding to a given NN-ROC curve, as players' greed and fear payoff differences change relative to the cooperation payoff difference. Part 5 of Appendix D shows that, if the fear and greed payoff differences are not exactly equal, then the two lines in Figure 4 will both become steeper, but still have positive slope as the cooperation payoff difference drops to 0; provided the fear difference in less than the sum of greed and cooperation differences. So we have the following implication:

COROLLARY B

The CD–players' probability of cooperating $r_{CD}(\xi_{CD} = 1, Z)$ will drop but not necessarily all the way to 0, as the cooperation payoff difference drops to 0 relative to positive but unequal fear and greed payoff differences; provided their payoff differences satisfy $(P - S) < (T - R) + (R - P)$.

On the other hand, suppose the fear and greed payoff differences drop to 0 *relative to* a positive cooperation payoff difference; as measured by the *normalized* ratios $(T - R)/(R - P)$ and $(P - S)/(R - P)$ dropping toward 0. Part 5 of Appendix D shows both sloped lines in Figure 4 will rotate downward to the diagonal line in Figure 4, with a slope of 1; corresponding to the ratio inequality $r_{CD}/w_{CD} > 1$. This implies the crossover frequency $\xi^0_{CD/DD}$ will drop to zero for any signal probabilities (r_{CD}, w_{CD}) strictly between (0, 0) and (1, 1); which in turn implies the CD–players will outperform DD–players for any $r_{CD}(\xi_{CD} = 1, Z)$ strictly between 0 and 1.

The last inequality allows $r_{CD}(\xi_{CD} = 1, Z)$ to rise near 1, and still allow CD–players to outperform DD–players. However, it does not require this must happen. For example, CD–players may want to guard against payoff perturbations by following a more risk averse strategy of holding (r_{CD}, w_{CD}) some distance below (1, 1) on their NN-ROC curve. So we have the following implication:

COROLLARY C

If normalized fear and greed payoff differences $(P - S)/(R - P)$ and $(T - R)/(R - P)$ both drop toward 0, then CD–players can raise their equilibrium probability of cooperating $r_{CD}(\xi_{CD} = 1, Z)$ near 1, and remain evolutionarily stable for <u>fixed</u> payoffs Z. But CD–players can also hold $r_{CD}(\xi_{CD} = 1, Z)$ noticeably below 1, if they want to ensure stability against payoff perturbations.

IV. RELATIVE IMPACT OF NORMALIZED GREED & FEAR PAYOFF DIFFERENCES

Consider the next formula giving the minimum signal probability ratio r_{CD}/w_{CD} that must be exceeded in order for the CD–players to outperform DD–players at the $\xi_{CD} = 1$ equilibrium, denoted $r^{\min}_{CD/DD}(x_{CD} = 1)$. It is obtained by setting $\xi_{CD} = 1$ in statement (D5a) of Appendix D, yielding the following condition:

$$\frac{r_{CD}}{w_{CD}} \quad > \quad r^{\min}_{CD/DD}(x_{CD} = 1) \tag{7a}$$

where

$$r^{\min}_{CD/DD}(x_{CD} = 1) \quad = \quad \frac{(R-P) + (T-R)}{(R-P) + (1-r_{CD})[(T-R)-(P-S)]} \ > \ 1 \tag{7b}$$

Then partially differentiate (7b) with respect to both fear and greed payoff differences to compare their relative impact on the minimum signal probability ratio required for CD–players to outperform DD–players at the $\xi_{CD} = 1$ equilibrium. Doing so and rearranging terms yields the following:

$$\frac{\P\left(r^{\min}_{CD/DD}(x_{CD} = 1)\right)}{\P(T-R)} \quad > \quad \frac{\P\left(r^{\min}_{CD/DD}(x_{CD} = 1)\right)}{\P(P-S)} \qquad \text{if and only if} \tag{8a}$$

$$r_{CD}(\xi_{CD} = 1, Z) \quad > \quad \frac{(T-R)+(R-P)+(P-S)}{(T-R) + 2(R-P)+(P-S)} \ > \ \frac{1}{2} \tag{8b}$$

The relationships characterized in statements (8a, b) imply the following predictions:

COROLLARY D

If the cooperation probability $r_{CD}(\xi_{CD} = 1, Z)$ *exceeds* a threshold above ½, the normalized greed payoff difference will have a *greater* impact on this probability than the normalized fear payoff difference. This threshold drops toward ½ as both normalized greed and fear payoff differences drop toward 0. By Corollary C, such reductions also allow $r_{CD}(\xi_{CD} = 1, Z)$ to rise well above ½; and thus above the threshold where the normalized greed payoff difference has a greater impact than the normalized fear payoff difference. Conversely, if normalized fear and greed payoff differences both rise sufficiently, then $r_{CD}(\xi_{CD} = 1, Z)$ will shift *below* ½, thereby causing the normalized greed payoff difference to have a *smaller* impact on this probability than the normalized fear payoff difference.

Next partially differentiate (7b) with respect to the cooperation payoff difference instead of the fear or greed payoff differences separately; to obtain:

$$\frac{\P\left(r^{\min}_{CD/DD}(x_{CD} = 1)\right)}{\P(R-P)} \quad = \quad - \ \frac{[(T-R)r_{CD} + (P-S)](1-r_{CD})]}{\left((R-P) + (1-r_{CD})[(T-R)-(P-S)]\right)^2} \quad < \quad 0 \tag{9}$$

Statement (9) implies the following result:

COROLLARY E

If the cooperation payoff difference changes relative to given fear and greed payoff differences, then the CD–players' probability of cooperating $r_{CD}(\xi_{CD} = 1, Z)$ will <u>rise</u> if the cooperation payoff difference also <u>rises</u>, and vice versa. Similarly, if both fear and greed payoff differences change at the same rate relative to a given cooperation payoff difference, then the CD–players' probability of cooperating will again <u>rise</u> if these payoff differences both <u>fall</u> at the same rate, and vice versa.

V. COMPARING PREDICTIONS WITH EXPERIMENTAL DATA

Corollaries A – E require no ad hoc parameter adjustments in order to derive them from Theorem 3. So they illustrate another objective of this paper: to derive general results like Theorem 3 from experimentally well verified signal detection principles that also lead directly to testable implications without ad hoc parameter adjustments needed to successfully predict experimental data. Accordingly, let us compare the implications of

Theorem 3 described in Corollaries A – E, with data from one–shot prisoners' dilemma experiments.

A. *Sensitivity of CD Behavior To Normalized Fear & Greed Payoff Differences*

Corollaries B – D imply a number of interrelated predictions about how the CD–players' equilibrium probability of cooperating, $r_{CD}(\xi_{CD} = 1, Z)$, is affected by greed, fear, and cooperation payoff differences. For example, these include the following two predictions; provided the fear payoff difference is less than the sum of greed and cooperation payoff differences:

(1) $r_{CD}(\xi_{CD} = 1, Z)$ will rise or fall if the normalized greed and fear payoff differences both fall or rise at the same rate relative to the cooperation payoff difference, but not necessarily all the way up to 1 nor all the way down to zero (from Corollaries B, C, & E)

(2) The normalized greed payoff difference will have a greater effect than the normalized fear payoff difference, whenever $r_{CD}(\xi_{CD} = 1, Z)$ exceeds a threshold above ½; where this threshold is likely to be exceeded as the normalized greed and fear payoff differences both decrease sufficiently, and vice versa if these normalized differences both increase sufficiently (from Corollary D).

Predictions (1) and (2) agree with observed patterns in recent experiments by Ahn–Ostrom–Shmidt–Shupp–Walker (2001). See for example their regression equation (5) discussed on page 151. It measures behavioral sensitivity to normalized fear and greed payoff differences; including incomplete adjustment up to 1 or down to 0, as also permitted by prediction (1).

Predictions (1) and (2) also agree with data accumulated from experiments published since the prisoners' dilemma was introduced 1950 : including Minas, et. al. 1960; Lave 1962, 1965; Rapoport & Chammah 1965; Rapoport 1967; Rapoport, Guyer, & Gordon 1976; Coleman 1983; Liebrand et. al. 1992; Frey 1995; Ledyard 1995; Sajio & Nakamura 1995; Palfrey & Prisbrey 1997; and Bolton & Ockenfels 2000. Such data was not predicted from traditional game theory models. Instead, subjective preferences for altruism, fairness, equity, reciprocity, and so on have been assumed: as for example analyzed by Robert Trivers (1971); Geanakoplos, Pearce, & Stacchetti (1989); Rabin (1993); Falk & Fischbacker (1998); Fehr & Schmidt (1999); and Bolton & Ockenfels 2000.

On the other hand, Corollaries B – E follow directly from payoff monotonic dynamic principles used to derive Theorem 3. So additional subjective factors are no longer required to organize the data. Furthermore, Theorem 3 does not involve 'clustering' or 'assortative' interaction. So it implies CD behavior can evolve in one-shot prisoners' dilemmas without limitation to kin selection in family or ethnic groups, and without assuming subjective preferences favoring 'reciprocal altruism'; as for example analyzed by Hamilton (1963), Greenberg &Frisch (1972), Axelrod & Hamilton (1981), Sethi & Somanathan (1999), and Gintis (2000).

B. *Incremental Benefits From Psychological, Cultural, & Institutional Factors*

Prediction (1) implies CD–players are more likely to cooperate as the greed and fear payoff differences both get smaller relative to the cooperation difference. The relative size of these payoff differences may also be affected by subjective factors involving cultural norms, fairness, equity, or reciprocal altruism. So the last two paragraphs of the preceding section do not necessarily imply CD players will prevent these subjective factors from impacting their behavior. Rather, prediction (1) can be combined with these subjective factors to produce a synergistic effect that magnifies their impact. That is, prediction (1) implies these factors can now *incrementally* stimulate more observed cooperation, *without* necessarily reversing the fear and greed payoff differences in order to do so.

So a whole variety of psychological and institutional factors can now incrementally stimulate observed cooperation in one-shot prisoners' dilemmas. This implication strengthens work by Fehr, Güth, Schmidt, Yamagishi, and others who assume preferences exist for fairness and reciprocity that are strong enough to actually reverse either (T – R) or both (P – S) and (T – R) into *negative* differences, and thereby switch the evolutionary arena from prisoners' dilemmas into assurance games or coordination games. However, prediction (1) implies such strong subjective preferences are *not* necessary. Instead, subjective preferences that only narrow the fear and greed payoff differences will incrementally stimulate more observed cooperation; provided the CD–players can detect their own type (now also including these subjective preferences) better than pure chance.[5]

[5] On switching to assurance games, see Kiyonari, Tanida, & Yamagishi (2000); and on switching to coordination games, see Fehr & Schmidt (1999), Fehr & Fischbacher (2002), and Güth & Kliemt (1994).

So the payoff monotonic dynamics in Figures 3(a, b) imply the co–evolution of contingently cooperating CD–players who also subjectively prefer fairness, reciprocity, and so on – yet the raw evolutionary dynamics enabling both CD behavior and these subjective preferences to evolve in the first place, depend only on objective material consequences devoid of any subjective component.

C. *Investing In More Reliable Signal Detection Through Face-to-Face Communication*

Suppose different types of communication can affect players' skill at detecting signals from each other, such as whether face-to-face communication is possible or not. Corollary A then implies there will be a correlation between observed cooperation in one-shot prisoners' dilemmas being preceded by possibly costly resources devoted to enhancing opportunities for face-to-face interviewing between the persons involved.

For example, corporate executives may orchestrate costly "summit meetings" before starting a new cooperative venture. These meetings will not necessarily be relinquished in favor of cheaper means of long range "tele-communication", such as with telephone conference calls, live video hookups, email and internet messaging, and so on. Why ? Because tele-communication degrades the kind of combined (verbal-plus-body-language) signals that are more readily observable through the give and take face-to-face discussion.

The general principle is that types of 'preplay' communication that strengthen causal links to another player's internal motivations will thereby strengthen their ability to detect signals correlated with CD versus non-CD behavior. So Corollary A implies the frequency of observed cooperation will vary systematically with the type of communication involved. This principle agrees with the pattern reported by Ostrom (2000): that the frequency of cooperation versus defection in prisoners' dilemma experiments is quite sensitive to the history of communication when *face-to-face* discussion is involved, but not to other types of pre-play communication.[6] It also agrees with experiments back to 1958 analyzed by Sally (1995).

VI. CONCLUSION

Parts I and II used signal detection principles and ROC curves in an evolutionary analysis of prisoners' dilemmas. Doing so implies the following : if there is any discounting of future payoffs, or any cost of searching for an additional partner, then cooperative players who contingently participate (in terms of who to play with or when to exit) cannot survive in a population containing mostly players who always defect. This vulnerability holds even when contingent participators only interact with themselves by perfectly detecting their own type.

However, quite different results hold for players who act contingently, not in terms of whether to play or exit, but rather in terms of how to act with a any given partner : there is a form of contingent cooperation in one-shot prisoners' dilemmas (called CD behavior) that will evolve from any initial population through any payoff monotonic process, and remains stable thereafter to attempted invasion by other players, including always defecting players. Such behavior follows a simple contingent rule for two players 'Adam' and 'Eve' :

> Adam cooperates instead of defects if and only if he detects a signal (x instead of y in Figure 1b of Part I) that is positively correlated with his partner Eve behaving in a similar contingent manner to a signal she detects from Adam, that is likewise positively correlated with his reacting contingently to his signal detected from her. Since Adam and Eve are randomly matched strangers, such correlation does not arise from any information about each other's past behavior other than symptoms elicited while communicating with each other for the first time.

The CD behavior just described is not accidently successful. Rather, the properties described in Theorem 3 imply CD–players can adapt to a wide range of different circumstances : for example, arbitrary shifts in the initial population mix don't matter, including arbitrary perturbations from any current population mix; nor do arbitrary changes in payoffs so long as their ordinal ranking is preserved and the fear payoff difference remains below the sum of greed and cooperation payoff differences; nor do any potential changes in the DC and CλD players' signal and action probabilities, nor do arbitrary changes in the CD or DC players' detection skill so long as the CD–players can still detect their own type better than pure chance; and so on.

The robust adaptive capabilities just described are strict mathematical implications not dependent on

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[6] Other factors may enhance the reliability of face-to-face communication. For example, players with a common educational, cultural, or ethnic experience (including fluency in a common language) may communicate more reliably.

computer simulations. They also apply to *pure* one–shot prisoners' dilemmas: meaning no repeated interactions nor repeated pairings in any generation are involved; no memory from past encounters nor reports from third parties are involved; all signal correlations arise only from symptoms detected after two randomly matched strangers meet for the first time; and no subjective preferences for fairness, altruism, reciprocity, equity, morality, and so on affect the raw evolutionary dynamics.[7]

Recall also from Section I in Part I, that ROC curves from the signal detection literature show that numerous experiments demonstrate the signal detection properties used in the analysis are the typical result of ordinary people in ordinary circumstances; thereby achievable without any special detection skills nor any special signals like 'secret handshakes', and so on.

So the dynamic properties of Theorem 3, along with testable predictions implied by Corollaries A – E, mean the following for one-shot prisoners' dilemmas: ordinary players under ordinary conditions with no special detection skills can follow a simple strategy of more cautiously interpreting symptoms from their partners (whenever needed if their own frequency in the population drops, or normalized fear and greed payoff differences rise); that will guarantee at least a small statistical bias in their favor over other types of players, including always defecting players; and thereby also guarantee the population will ultimately shift away from other types of behavior toward an exclusive population of contingently cooperating CD players. CD players thus have the flexibility to shift their behavior at will, closer statistically to either pure DD or pure CC behavior; but with a guaranteed statistical bias that will likewise guarantee they will always take over the population. Yet CD players need no unusual detection skills or special signals in order to guarantee their eventual dominance in the population.

APPENDIX D (*Proofs To Establish Theorem 3 & Corollaries A – E*)

Players' strategies are indexed from 1 to 5 by $(s^1, s^2, s^3, s^4, s^5)$ = (CC, CD, DC, DD, CλD). The five player types' population frequencies are given by the profile $\xi = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5)$. So ξ_m is the frequency of s^m–players in the population; where ξ_m 0 for each m, and $\xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 = 1$. Payoffs are denoted by Z = (T, R, P, S).

Recall that (r_{CD}, w_{CD}) denotes a pair of signal probabilities for the s^2 = CD players along an NN–ROC curve in Figure 1a. The other type of contingent DC–players also detect an x versus y signal from their partners, with signal probabilities, denoted (r_{DC}, w_{DC}). The signal probabilities for the CD and DC players, plus the action probability λ of the CλD players are denoted, $p = (r_{CD}, w_{CD};\ r_{DC}, w_{DC};\ \lambda)$.

Let $E(s^m\ s^k, p, Z)$ denote an s^m–player's expected payoff when it plays against an s^k player; depending on the signal and action probabilities p, and depending on payoffs Z. Player s^m's "*total*" expected payoff across the five types of players is denoted, $E(s^m\ \xi, p, Z) = \sum_{k=1}^{k=5} E(s^m / s^k, p, Z)\, x_k$ for m = 1, ... , 5.

Part 1 *Payoff Monotonic Dynamics Plus Frequency & Payoff Dependent Signal & Action Probabilities*

Payoff monotonic dynamics are described by differential equations showing the time rate of change of the ratio of each player type's population frequency relative another type's frequency (see Weibull 1997, page 73):

$$\frac{d}{dt}\left[\frac{x_m}{x_k}\right] = M^+\Big(E(s^m/x,p,Z) - E(s^k/x,p,Z)\Big)\frac{x_m}{x_k} \qquad \text{for any } m, k = 1, \dots, 5 \text{ such that } \xi_k > 0 \qquad \text{(D1)}$$

where M^+ denotes any monotonic strictly increasing function

[7] The adaptive ability of CD behavior raises the question of whether it can "rational" according to a player's "self–interest". Traditional game theory implies that only always defect behavior can be rational in one-shot prisoners' dilemmas. However, the signal correlations in the above description of CD behavior arise from causal relationships; and traditional game theory was developed without incorporating causal principles. This question is analyzed in Heiner, Schmidtchen, & Albert (2002): showing how to combine rationality with causal principles into a theory of "rationally caused" behavior; and showing CD behavior is rationally caused in one-shot prisoners' dilemmas from pure self–interest alone.

The above equation implies one player type's population frequency will rise relative to another type's frequency if and only if its total expected payoff exceeds that of the other type, provided their two frequencies are positive, $\xi_m > 0$ and $\xi_k > 0$. *Replicator* dynamics is the special case where the function M^+ is the identity function.

Next recall that more cautious detection requires CD–players to reduce their signal probabilities as their frequency drops, or the costs of mistaken detections rise. So the CD–players' signal probabilities represent '*frequency & payoff dependent signal detection*'. The signal probabilities (r_{CD}, w_{CD}) are thus functions of the CD–players' frequency ξ_{CD} and the set of payoffs Z, denoted:

$$[\, r_{CD}(\xi_{CD}, Z), w_{CD}(\xi_{CD}, Z)\,] \quad \text{for any} \quad 0 \le \xi_{CD} \le 1 \quad \text{and} \quad Z = (T, R, P, S) \qquad \text{(D2a)}$$

Note the CD–players' signal probabilities in (D2a) are only functions of their own population frequency ξ_{CD} relative to the sum of other players' frequencies, $1 - \xi_{CD} = \xi_{CC} + \xi_{DC} + \xi_{DD} + \xi_{C\lambda D}$. This is assumed to make explicit that CD–players can cautiously detect their own type without any information about the sub–composition of the remaining population besides themselves.[8]

The signal probabilities of the DC–players may also be frequency and payoff dependent; represented by, $[r_{DC}(\xi, Z), w_{DC}(\xi, Z)]$. These can be arbitrary functions of the full profile of population frequencies ξ and payoffs Z. Likewise, the randomizing CλD–players might vary their action probability λ toward choosing C, also as an arbitrary function of ξ and Z; represented by the function $\lambda(\xi, Z)$. Such arbitrary signal and action probability functions are assumed in order to develop results about the evolution of CD–players that are robust to any type of detection behavior or randomizing behavior of the (DC, CλD) players.

Using the above definitions, the full profile of frequency and payoff dependent signal and action probabilities across the (CD, DC, CλD) players is represented by,

$$p(\xi, Z) = [\, r_{CD}(\xi_{CD}, Z),\ w_{CD}(\xi_{CD}, Z);\ r_{DC}(\xi, Z),\ w_{DC}(\xi, Z);\ \lambda(\xi, Z)\,] \qquad \text{(D2b)}$$

The objective is to analyze payoff monotonic dynamics with frequency and payoff dependent signal and action probabilities; by substituting functions (D2b) into (D1) to obtain :

$$\frac{d}{dt}\left[\frac{x_m}{x_k}\right] = M^+\Big(E[s^m\ \ \xi, p(\xi, Z), Z] - E[s^k\ \ \xi, p(\xi, Z), Z]\Big)\, \frac{x_m}{x_k} \quad \text{for any } m, k = 1, \ldots, 5 \text{ such that } \xi_k > 0 \qquad \text{(D2c)}$$

PART 2 *Proving CD–Players Always Outperform DD–Players*

Recall the crossover threshold in Figure 2a of Part I, $\xi^0_{CD/DD}$: where the CD–players' expected payoff will exceed the DD–players' expected payoff within the CD/DD subpopulation if and only if their frequency exceeds this threshold. To represent this more generally, let an s^m–player's expected payoff when playing against players of only the s^k and s subpopulations be denoted,

$$E_{k/}(s^m\ \ \xi, p, Z) = \{\xi_k E(s^m\ \ s^k, p, Z) + \xi\ E(s^m\ \ s\ , p, Z)\}/(\xi_k + \xi\ ) \quad \text{for} \quad m, (k \ne\ \ ) = 1, \ldots, 5 \qquad \text{(D3a)}$$

Then define the difference in these expected payoffs, of s^m–players compared to s^n–players when they both play against players from only the s^k and s subpopulations:

$$\Delta_{k/}(s^m, s^n\ \ \xi, p, Z) = E_{k/}(s^m\ \ \xi, p, Z) - E_{k/}(s^n\ \ \xi, p, Z) \quad \text{for} \quad (m \ne n), (k \ne\ \ ) = 1, \ldots, 5 \qquad \text{(D3b)}$$

Next consider the expected payoff formulas at the endpoints of the two expected payoff lines in Figure 2a of Part I, and substitute the quadratic formula from Table 1 (in the CD row and CD column) for the linear formula shown on the right-hand vertical scale. Then use these four expected payoff formulas to solve for the crossover threshold $\xi^0_{CD/DD}$; but now calculated with quadratic expected payoffs for a CD–player against another CD–player. The resulting formula is given by the function:

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[8] Subjects in experiments also do not need to know the frequency of events they are trying to detect, such as the CD–players' frequency ξ_{CD}; because their own detections, though imperfect, provide enough information to produce shifts toward greater or lesser caution as the actual frequency of events shifts down or up respectively. For example, animal subjects cannot be told about changing event frequencies or payoffs. Yet as they experience new conditions with different event frequencies or payoffs, their (r, w) probabilities systematically shift along an ROC curve.

$$\xi^{0}_{CD/DD}\,(r_{CD}, w_{CD}, Z) \quad = \quad \frac{w_{CD}\,(P-S)}{(r_{CD})^{2}[(P-S)\,-\,(T-R)] + (r_{CD} - w_{CD})[(T-P)-(P-S)]} \quad > \quad 0 \qquad \text{(D4a)}$$

Then for any positive frequency of CD–players in the CD/DD subpopulation ($\xi_{CD} > 0$) , $\xi^{0}_{CD/DD}(r_{CD}, w_{CD}, Z)$ has the following property :

$$\Delta_{CD/DD}(CD, DD\ \xi, p, Z) \ > \ 0 \quad \text{if and only if} \quad \xi_{CD/DD} \ > \ \xi^{0}_{CD/DD}(r_{CD}, w_{CD}, Z) \qquad \text{(D4b)}$$

Next rearrange inequality (D4b) into an equivalent inequality involving the signal probability ratio r_{CD}/w_{CD} :

$$\frac{r_{CD}}{w_{CD}} \quad > \quad r^{\,0}_{CD/DD}(x_{CD}, r_{CD}, Z) \quad = \quad \frac{x_{CD}\,(T-P) + (1-x_{CD})(P-S)}{x_{CD}[(R-P) - \nabla(1-r_{CD})]} \qquad \text{(D5a)}$$

where = (P – S) – (T – R) (D5b)

The crossover threshold $\xi^{0}_{CD/DD}(r_{CD}, w_{CD}, Z)$ will lie strictly below the CD–players' current population frequency ξ_{CD}, if and only if the signal probability ratio r_{CD}/w_{CD} exceeds the ratio formula in (D5a) by some positive amount, denoted $\mu > 0$. That is, the following relationship must hold :

$$\xi^{0}_{CD/DD}(r_{CD}, w_{CD}, Z) \ < \ \xi_{CD} \quad \text{if and only if} \qquad \text{(D5c)}$$

$$\frac{r_{CD}}{w_{CD}} \ = \ r^{\,0}_{CD/DD}(x_{CD}, r_{CD}, Z) \ + \ m \ > \ 1 \qquad \text{for some} \ \ \mu \ > \ 0 \qquad \text{(D5d)}$$

Figure 5 shows how the above equation geometrically determines the signal probability ratio $r_{CD}{}^{0}/w_{CD}{}^{0}$: by drawing a line from the (0, 0) origin in the unit square with a slope equaling the sum of the ratio formula (D5a) plus μ, also shown on the right–hand side of (D5d). The ratio $r_{CD}{}^{0}/w_{CD}{}^{0}$ is determined by the intersection of this line with the CD–players' NN-ROC curve.

Notice as the frequency ξ_{CD} gets smaller, (D5d) implies the slope of the line in Figure 5 rises, while also causing $(r_{CD}{}^{0}, w_{CD}{}^{0})$ to shift closer to (0, 0) along an ROC curve. Note also the ratio formula in (D5a) is positive and finite for any positive ξ_{CD} ; provided the bracketed-expression [(R – P) – (1 – r_{CD})] remains positive as r_{CD} converges to 0. The latter must hold if the inequality (T – P) > (P – S) holds; because it implies < (R – P) must hold, which in turn implies the preceding bracketed-expression is positive for all $r_{CD} \in [0, 1]$.

So it doesn't matter how large the finite and positive ratio formula in (D5a) might become as ξ_{CD} drops closer to 0; because the signal probability ratio r_{CD}/w_{CD} rises to infinity along a convex NN-ROC curve [from statement (1) of Part I] and so can always be shifted to satisfy statement (D5d) no matter how close ξ_{CD} gets to 0.

Figure 5 About Here

These relationships imply the variables in (D5d) uniquely determine a pair of signal probabilities along the CD–players' ROC curve, that will in turn cause $0 < \xi^{0}_{CD/DD} < \xi_{CD}$ to hold for any positive $\mu > 0$ and positive $\xi_{CD} > 0$; provided the payoff inequality (T – P) > (P – S) also holds. So there exists a unique pair of signal probabilities denoted by the functions,

$$r_{CD}{}^{0} \ = \ r_{CD}(\mu, \xi_{CD}, Z) \quad \text{and} \quad w_{CD}{}^{0} \ = \ w_{CD}(\mu, \xi_{CD}, Z); \qquad \text{(D6a)}$$

such that when substituted into statement (D4a), will cause the crossover threshold function $\xi^{0}_{CD/DD}(r_{CD}, w_{CD}, Z)$ resulting from those these signal probabilities to fall below the CD–players' frequency in the population ξ_{CD}. So the following theorem holds :

Theorem D1 (*Signal Probabilities Enabling CD–Players To Outperform DD-Players*)

For any $0 < \mu$, any $0 < \xi_{CD} \le 1$, and any payoffs Z such that P – S < T – P; there exists a unique pair of signal probabilities along a convex NN-ROC curve, denoted $r_{CD}(\mu, \xi_{CD}, Z)$ and $w_{CD}(\mu, \xi_{CD}, Z)$, such that the formula for $\xi^{0}_{CD/DD}(r_{CD}, w_{CD}, Z)$ in statement (D4a) implies :

$$0 \ < \ \xi^{0}_{CD/DD}[r_{CD}(\mu, \xi_{CD}, Z), w_{CD}(\mu, \xi_{CD}, Z), Z] \ < \ \xi_{CD} \qquad \text{(D6b)}$$

PART 3 *Instability of CC and CλD Players Against CD and DD Players*

Next analyze payoff monotonic dynamics with CC and CλD players also in the population. First compare CC– players against DD–players. To do so, compare the expected payoff formulas across the CC and DD rows of Table 1; and notice that DD–players outperform CC–players no matter what type of player they play against. So the DD–players' total expected payoff always exceeds that of the CC–players; which in turn implies any positive frequency of CC–players is unstable within a larger population containing a positive frequency of DD–players.

Also let the next expression denote the expected payoff difference between player s^m versus s^n when they both play against player s^k:

$$\Delta(s^m, s^n \;\; s^k, p, Z) \;=\; E(s^m \;\; s^k, p, Z) \;-\; E(s^n \;\; s^k, p, Z) \qquad \text{for } m \neq n, k = 1, \ldots, 5 \qquad \text{(D7a)}$$

Using the above notation, CD–players are also guaranteed to outperform CC–players in the CD/CC subpopulation, as implied by the following difference formulas obtained from the expected payoffs in Table 1; because both formulas are strictly positive for any $(0, 0) \leq (r_{CD}, w_{CD}) < (1, 1)$ along an NN ROC curve:

$$\Delta(CD, CC \;\; CC, p, Z) \;=\; (T - R)(1 - w_{CD}) \qquad \text{(D7b)}$$

$$\Delta(CD, CC \;\; CD, p, Z) \;=\; (P - S)[1 - w_{CD} - r_{CD}(1 - r_{CD})] + (T - R)r_{CD}(1 - r_{CD}) + (R - P)(r_{CD} - w_{CD}) \qquad \text{(D7c)}$$

These two formulas combined with payoff monotonic dynamics (D2c) imply any positive frequency of CC–players is unstable when there is also a positive frequency of CD–players in the population, provided CD–players don't act just like CC–players by shifting (r_{CD}, w_{CD}) up to (1, 1). This in turn implies there do not exist any unstable equilibria in the interior of the CD/CC subpopulation.

Next compare the remaining case of CλD–players who randomize according to an external signal; where λ versus 1 – λ represent the probability of choosing C versus D. As above with the CC–players, compare the expected payoff formulas across the CλD and DD rows of Table 1; and notice that DD–players outperform CλD–players no matter what type of player they might play against, provided λ > 0. So λ > 0 implies the DD–players' total expected payoff always exceeds that of the CλD–players; which in turn implies any positive frequency of CλD–players is unstable within a larger population containing a positive frequency of DD–players.

Recall the probability λ may depend on the profile of population frequencies, ξ, as well as on the payoffs Z; represented by the function, $\lambda = \lambda(\xi, Z)$. Now compare CD and CλD players to see whether there might be any unstable equilibria within the CD/CλD subpopulation. To do so, use Table 1 to compare the four expected payoff formulas: E(CD|CD, p, Z), E(CλD|CD, p, Z),E(CD|CλD, p, Z), E(CλD|CλD, p, Z); and then use definition (D3b) to calculate the average of these differences within the CD/CλD subpopulation, $\Delta_{CD/C\lambda D}(CD, C\lambda D \;\; \xi, p, Z)$. Then rearrange the resulting formula to obtain the following population threshold for CD–players outperforming CλD–players within the CD/CλD subpopulation:

$$\Delta_{CD/C\lambda D}(CD, C\lambda D \;\; \xi, p, Z) \;>\; 0 \text{ if and only if} \qquad \text{(D8a)}$$

$$\xi_{CD} \;>\; x^{0}_{CD/ClD} \;=\; \frac{(w_{CD} - l)k(l, Z)}{(w_{CD} - l)k(l, Z) + (P - S)e(p_{CD}, l) + (T - R)f(p_{CD}, l) + (R - P)(r_{CD} - w_{CD})} \qquad \text{(D8b)}$$

where

$$e(p_{CD}, \lambda) \;=\; \lambda(1 - w_{CD}) \;-\; r_{CD}(1 - r_{CD}) \qquad p_{CD} \;=\; (r_{CD}, w_{CD}) \qquad \text{(D9a)}$$

$$f(p_{CD}, \lambda) \;=\; r_{CD}(1 - r_{CD}) \;-\; w_{CD}(1 - \lambda) \qquad \text{(D9b)}$$

$$k(\lambda, Z) \;=\; \lambda(T - R) + (1 - \lambda)(P - S) \qquad \text{(D9c)}$$

Then rearrange inequality (D8b) to derive an equivalent formula for a lower bound on the CD–players' signal probability ratio r_{CD}/w_{CD}; in order to determine how close they must shift (r_{CD}, w_{CD}) to (0, 0) – to outperform CλD–players (so that $\Delta_{CD/C\lambda D}(CD, C\lambda D \;\; \xi, p, Z) > 0$) for population frequencies $\xi_{CD} \;\; [0, 1]$. Doing so obtains,

$$\frac{r_{CD}}{w_{CD}} \;>\; r^{0}_{CD/ClD}(l, x_{CD}, r_{CD}, Z) \qquad \text{(D10a)}$$

$$- \;\; \frac{l\left((T - R)\, l\,(1 - x_{CD}) + (P - S)[1 - l\,(1 - r_{DC})]\right)}{x_{CD}[(R - P) - \nabla(1 - r_{CD})]w_{CD}} \qquad \text{(D10b)}$$

where

$$r^{0}_{CD/C1D}(l, x_{CD}, r_{CD}, Z) \quad = \quad \frac{x_{CD}[(R-P)+l(P-S)+(1-l)(T-R)]+(1-x_{CD})[l(T-R)+(1-l)(P-S)]}{x_{CD}[(R-P)-\nabla(1-r_{CD})]} \qquad \text{(D10c)}$$

Notice the limit where CλD–players become equivalent to DD–players; by holding their action probability constant at λ = λ(ξ, Z) 0 as ξ shifts over time. The last identity implies the ratio formula in (D10b) drops to zero, and the formula in (D10c) reduces to the same formula derived in equation (D5a), for the lower bound on the CD–players' signal probability ratio within the CD/DD subpopulation : so $\rho^0_{CD/C\lambda D}(\lambda = 0, \xi_{CD}, r_{CD}, Z) = \rho^0_{CD/DD}(\xi_{CD}, r_{CD}, Z)$.

Notice also that the formula for $\rho^0_{CD/C\lambda D}(\lambda, \xi_{CD}, r_{CD}, Z)$ remains finite for any given $\xi_{CD} > 0$; and so can always be exceeded by shifting (r_{CD}, w_{CD}) close enough to (0, 0) as ξ_{CD} shifts closer to 0 : because r_{CD} / w_{CD} rises to infinity as (r_{CD}, w_{CD}) shifts to (0, 0), by statement (1) of Part I, and shown above in Figure 5. Plus notice that (D10b) ***subtracts*** a ratio formula that is always non-negative for any ξ_{CD} (0, 1].

Hence, keeping r_{CD} / w_{CD} above $\rho^0_{CD/C\lambda D}(\lambda, \xi_{CD}, r_{CD}, Z)$ is always sufficient to guarantee CD–players outperform CλD–players for any positive frequency, $0 < \xi_{CD} \leq 1$. This in turn guarantees by sufficient caution in detecting their own type, CD–players can also guarantee there never exist equilibria in the interior of the CD/CλD subpopulation – no matter how the CλD–players might vary their action probabilities as the population frequency profile ξ evolves over time. Summarizing these implications yields :

Theorem D2 (*Instability & No Interior Equilibria In The CD/CC & CD/CλD Subpopulations*)

(1) Any positive frequency of CC or CλD players is unstable whenever the whole population contains a positive frequency of DD–players, provided λ > 0. (2) Sufficiently cautious detection according to (D10a–c) guarantees there never exist equilibria in the interior of the CD/CC or CD/CλD subpopulations; for arbitrary variation in the CλD–players' action probability λ as the population frequency profile ξ changes over time.

PART 4 *Main Theorem About CD Behavior Robustly Evolving From Any Initial Population*

Given Theorems D1 and D2 compare (DD, CD, CC, CλD) players, the remaining comparison involves DC players. Recall that subtracting the DC row from the DD row in Table 1 implies the DD–players always outperform DC players against any type of player, provided DC–players don't act just like DD players by holding (r_{DC}, w_{DC}) constant at (1, 1). This implies for payoff monotonic dynamics (D2c), that any CD/DC subpopulation with a positive frequency of DC–players is unstable for any positive frequency of DD players in the whole population; which will thereby always shift the whole population toward the CD/DD subpopulation. For boundary cases where $\xi_{DD} = 0$, Appendix F gives a full characterization of any unstable equilibria that might arise in the interior of the CD/DC subpopulation. [9]

So we can now state the following theorem; whose Part B describes unstable CD/DC equilibria noted in Theorem 3 in Part II.

THEOREM D3 (*Robust Evolution Of CD Behavior In Pure One–Shot Prisoners' Dilemma*s)

Assume the fear payoff difference is less than the sum of greed & cooperation payoff differences, (P – S) < (T – R) + (R – P); and let the frequency profile ξ evolve according to the frequency & payoff dependent plus payoff monotonic dynamic equations given in (D2c). Then the following results hold:

PART A Theorem D1 implies CD–players will outperform DD–players within the CD/DD subpopulation for any positive frequency, $\xi_{CD} > 0$. Theorem D2 implies any positive frequency of CC–players or CλD–players with λ > 0, is unstable whenever there exists a positive frequency of DD players in the whole population; plus no unstable equilibria exist in the interior of the CD/CC or CD/CλD subpopulations.

PART B The formulas in Table 1 also imply any positive frequency of DC–players with (r_{DC}, w_{DC}) < (1, 1) is unstable for any positive frequency of DD–players in the population. However, ξ_{DD}= 0 may allow one or more unstable equilibria in the interior of the CD/DC subpopulation, as described in Part 3 of Appendix F ; which implies the following properties hold relative to a threshold pair of signal probabilities along the DC–players' ROC curve, denoted (r^0_{DC}, w^0_{DC}):

(1) If [$r_{DC}(\xi, Z)$, $w_{DC}(\xi, Z)$] stays below the (r^0_{DC}, w^0_{DC}) threshold; then *zero* unstable equilibria

[9] Appendix F is available on request from the author.

exist in the interior of the CD/DC subpopulation, and the resulting payoff monotonic dynamics evolve according to Figure 3b in Part II.

(2) If $[r_{DC}(\xi, Z), w_{DC}(\xi, Z)]$ is held constant above (r^0_{DC}, w^0_{DC}) as ξ changes over time; then *one* unstable equilibrium exists in the interior of the CD/DC subpopulation, and the resulting payoff monotonic dynamics evolve according to Figure 3a in Part II.

(3) If $[r_{DC}(\xi, Z), w_{DC}(\xi, Z)]$ fluctuates as ξ changes over time (as shown in Figures 7c–d, and described in Statement C in Appendix F), then *multiple* unstable equilibria may exist in the interior of the CD/DC subpopulation.[10]

PART C Parts A and B imply the following whenever the CD–players can detect their own type with any degree of skill beyond pure chance:

The population profile ξ will evolve from any initial population – either directly through any payoff monotonic process according to (D2c), or combined with arbitrarily small perturbations from unstable equilibria – eventually to a unique evolutionarily stable population composed entirely of CD–players, $\xi_{CD} = 1$.

PART 5 (*Behavior Of The Two Sloped Lines In Figure 4 Of Part II*)

The slopes the two lines in Figure 4 can be obtained from the formula for the minimum signal probability ratio r_{CD}/w_{CD} given in statement (D5a) above; by substituting γ and 1 – γ for ξ_{CD} into this formula. Doing so gives the following formulas for the slopes of these two lines:

$$r^{0}_{CD/DD}(x_{CD} = g\,,\, Z) \;=\; \frac{g\,(T-P) + (1-g\,)(P-S)}{g\,[(R-P) - \nabla(1-r_{CD})]} \qquad \text{(D11a)}$$

$$r^{0}_{CD/DD}(x_{CD} = 1-g\,,\, Z) \;=\; \frac{(1-g\,)(T-P) + g\,(P-S)}{(1-g\,)[(R-P) - \nabla(1-r_{CD})]} \qquad \text{(D11b)}$$

where $\rho^0_{CD/DD}(\xi_{CD} = \gamma, Z) \;>\; \rho^0_{CD/DD}(\xi_{CD} = 1-\gamma, Z) \;>\; 1 \quad \text{for} \quad 0 < \gamma < ½$ (D11c)

Then assume the fear and greed payoff differences are *not* equal (so that ≠ 0), and notice the condition (P – S) < (T – P) = (T – R) + (R – P) implies (T – R) – (P – S) > 0 must hold as (R – P) drops to 0 for given (T – R) and (P – S). This inequality implies the ratio formulas in (D11a–b) must remain positive and finite as (R – P) drops to 0 for given (T – R) and (P – S); plus at the limit (R – P) = 0, the same ranking of slope formulas as in (D11c) must hold for 0 < γ < ½, namely:

$$\infty \;>\; \frac{g\,(T-R) + (1-g\,)(P-S)}{g\,[(T-R) - (P-S)](1-r_{CD})} \;>\; \frac{(1-g\,)(T-R) + g\,(P-S)}{(1-g\,)[(T-R) - (P-S)](1-r_{CD})} \;>\; 1 \qquad \text{(D11d)}$$

The two finite slope formulas in (D11d) imply the CD–players need *not* shift to total caution in detecting their own type – so (r_{CD}, w_{CD}) can remain above (0, 0) – as the cooperation payoff difference drops to 0 relative to unequal fear and greed payoff differences; thereby establishing Corollary B in Section III.

Finally notice that, as both the fear and greed payoff differences drop to 0 relative to a given positive cooperation payoff difference, both ratio formulas in (D11a–b) converge to a common limit,

$$\frac{g\,(R-P)}{g\,(R-P)} = \frac{(1-g\,)(R-P)}{(1-g\,)(R-P)} = 1 \qquad \text{(D12)}$$

This result corresponds to the two lines in Figure 4 likewise having a common slope of 1; which thus establishes the result needed to derive Corollary C, also in Section III.

[10] Figure 8 in Appendix F shows the resulting dynamics with *two* unstable equilibria in the interior of the CD/DC subpopulation.

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