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Working Paper

Anti-Sharing.

CSLE Discussion Paper, No. 2003-02

Provided in cooperation with:
Universität des Saarlandes (UdS)

Anti-Sharing.

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Center for the Study of Law and Economics
Discussion Paper 2003-02 - Draft of July 2003

Abstract

The paper proposes a mechanism that may implement first-best effort in simultaneous teams. Within the framework of this mechanism, each team members is obliged to make a fixed, non-contingent payment, and chooses his individual effort. After the output is produced, each team member receives a gross payment that equals the actual team output. We demonstrate that a Nash equilibrium exists in which each team member chooses first-best effort. We call this mechanism "Anti-Sharing" since it solves the sharing problem that causes the inefficiency in teams. The Anti-Sharing mechanism requires one player to specialize on the role of an "Anti-Sharer". With an external Anti-Sharer who works on a non-profit base, the mechanism can implement first-best effort. If, however, the Anti-Sharer comes from within the team and desires a positive payoff, then the mechanism may implement not more than second-best effort. The latter version of the model could be interpreted as a new theory of firms and partnerships in the sense of the theory of Alchian and Demsetz (1972).

JEL-Classification: *M 21, D 23, C 72*

Encyclopedia of Law and Economics: *5610, 0700*

Keywords: *Efficient effort in teams, second-best solution, partnerships.*

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1 Introduction

This paper proposes a new mechanism to solve the problem of inefficient effort provision in teams. Even if the team output is deterministic and team members are risk-neutral, the sharing of the output induces the players to spend suboptimal low effort. We call this mechanism "Anti-Sharing". Within its framework, the team members are obliged to make a fixed payment to an "Anti-Sharer". Each team member chooses his effort, which produces the actual team output. This team output is collected by the Anti-Sharer, who finally pays exactly this amount to each of the team members. With an external Anti-Sharer who works on a non-profit basis, this mechanism implements the first-best efforts as a Nash equilibrium.

If, however, one of the team members takes over the role of the Anti-Sharer, only a suboptimal solution can be implemented. We prove that an internal Anti-Sharer must not be productive. Thus, our mechanism can be interpreted as a theory of a firm, since it provides an explanation for the division of labor between inactive (senior) partners and active (junior) partners.

The idea that a team member assumes a specialized role to solve the team problem is very similar to the proposal of Alchian/Demsetz (1972). In their seminal paper, these authors have based their theory of the firm on the idea of team production: if the inputs of the agents are not linear separable, then individual incentive contracts are of no use. According to their proposal, one agent should become the residual claimant and monitor the effort of the other agents. Being the residual claimant motivates the monitor to fulfill his monitoring duties. However, this proposal requires the effort to be observable by the specialized monitor.

However, in the Alchian/Demsetz model specialization in monitoring may raise opportunity costs, since it is likely that the monitoring task may keep this agent from contributing productive effort to the team output. If monitoring is possible without opportunity costs, then a first-best solution can be achieved. Taking the opportunity costs of monitoring into account, the Alchian/Demsetz solution is not first-best, due to two reasons: first of all, the monitor's effort is not contributed to the output. Moreover, this may decrease the other agents' marginal productivity if, as it is usually assumed for team problems, the production function is characterized by positive cross-partials.

Holmstrom (1982) has shown that, within a fairly general framework, budget-balanced sharing schemes are unable to induce a first-best effort. Unbalanced rules would implement first-best effort as a Nash quilibrium. However, such a Nash equilibrium is not subgame perfect, since it would require the agents to waste a part of the output. No agent anticipates such a threat to be actually carried out. Without some outside enforcer who has an interest in enforcing such an ex-post inefficient punishment, the unbalanced sharing scheme introduced by Holmstrom would not work.

Our mechanism is different from the outside enforcer of a non-balanced sharing rule in Holmstrom (1982). In our model, the payments to the Anti-Insurer are not contingent on the observation of an output which is smaller than the efficient one. Our mechanism obliges the agents to make a fixed payment to the Anti-Sharer. In turn, they receive a payment that equals the full actual output (instead of only a share). The fixed payments enable the Anti-Sharer to pay each of the team members. Since these payments are fixed, they are irrelevant for the effort decision of the agents. Therefore, our mechanism is different from "bonding" as mentioned in Holmstrom (1982).

Rasmusen (1987) has demonstrated that, in the case of risk-averse team-members, a budget balanced Sharing contract may implement the first-best effort. The mechanism is based on random punishment in case of underperformance. Strausz (1999) has derived a simple sharing rule for sequential teams that is budget-balanced (thus credible) in equilibrium, and which implements first-best efforts if earlier inputs are observable by those team members who move later during the game.[1] In our model, we focus on simultaneous teams that consist of risk-neutral agents. Hence, the proposals of Rasmusen and Strausz are not applicable.

Finsinger/Pauly (1990) have applied the idea of Holmstrom to Law and Economics[2] and demonstrated that an optimal liability rule would require burdening both parties with the full damage. A budget balanced liability rule, however, leads to an over-insurance problem. At least one party (or even both) does not have to bear the full damages, and thus enjoys too much insurance. E.g., strict liability and no liability imply that one party bears the full risk, whereas the respective other party is fully insured. Under negligence rules as well as under liability rules that require each party to bear a part of the damage, there is no situation in which each party has to bear the full damage simultaneously.

The idea of Cooter/Porat (2002) proposes "Anti-Insurance" as a solution to the over-insurance problem. If, in a tort case with two parties, a third

[1] See Lülfesmann (2001) for a generalization of this result.

[2] See already Brown (1973), Coase (1960). See also Görke (2002).

person plays the role of an "Anti-Insurer", then both parties can be exposed to the full risk, which provides incentives to spend efficient effort. Under the strict liability rule, e.g., the Anti-Insurer initially pays an amount that equals the damage to the insured party. If an accident occurs, the uninsured party bears the damage, whereas the insured party has to pay the same amount to the Anti-Insurer. Thus, both parties have to pay full damages in case of an accident. The ex-ante payment by the Anti-Insurer is required to satisfy the participation constraints of the two parties. If the liability rule distributes the damage among the parties, then the ex-ante payment from the Anti-Insurer and the ex-post payments to the Anti-Insurer have to be adjusted accordingly.[3] This mechanism is different from the one in Varian (1994), where each agent pays the other agents for their effort, and simultaneously demands from all agents a compensation for his own effort. Varian's mechanism requires each of the n team members to announce $2n$ prices, whereas in the simplest version of the Anti-Insurance mechanism, only one payment of each team member to the Anti-Insurer has to be computed.

The paper by Cooter/Porat (2002) is closely related to ours, since the authors have already pointed out that their idea can also be applied to uncertain gains. A difference between their paper and ours is that our model can be applied to teams with deterministic output, while their paper focuses on stochastic outcomes. Note furthermore that the Anti-Insurer in Cooter/Porat (2002) is introduced as an additional player, while we also analyze Anti-Sharers who come from within the team. Insofar, our model could be interpreted as an explanation of the division of labor between (inactive) senior partners and (productive) junior partners. Partnerships are defined by Farrell/Scotchmer (1988) as coalitions that divide their output equally. Their concept differs from ours since they expressly exclude team production in the sense of Alchian/Demsetz (1972). Consequently, they focus on the optimal team size, while moral hazard within the partnership plays no role in their analysis. This is also true for the model in Lang/Gordon (1995), who explain partnership as a risk-pooling device. In our model, risk is excluded.

In section 2, we set up our model and notation by repeating the results of team production under a Sharing contract as demonstrated by Holmstrom (1982). We maintain the assumption that individual efforts are unobservable. This excludes the application of monitoring - as in Alchian/Demsetz (1972) - or sequential teams - as in Strausz (1999).

Section 3 demonstrates the implementation of first-best effort by an external Anti-Sharing contract, offered to the team members by a zero-profit Anti-

[3]See also Polinsky/Rubinfeld (2003) with a related idea.

Sharer. The introduction of an Anti-Sharer addresses the sharing problem, as expressed by Lang/Dordon (1995): "Pooling of profits denies a partner the entire value of his marginal effort and therefore reduces the effort". Under an (external) Anti-Sharing contract, the complete value of each team member's effort appears in his respective yield function.

However, the case of an external Anti-Sharer who operates on a zero-profit base is not the most realistic one. More relevant appears to be a scenario in which one of the team members takes over the role of the Anti-Sharer. In section 4 we therefore endogenize the Anti-Sharing. If a team that consists of n agents faces the problem that the spontaneous interaction leads to an equilibrium with inefficient effort, then one out of these n agents may serve as Anti-Sharer. An internal Anti-Sharing contract may implement cannot implement first-best efforts among the remaining $(n-1)$ agents, since it is necessary to keep the agent who plays the role of the Anti-Sharer from contributing productive effort. However, we demonstrate the conditions under which an internal Anti-Sharing contract leads to a Pareto-improvement for the team members, compared to the situation under a Sharing contract.

In section 5, we discuss the constitutional aspects of internal Anti-Sharing: if the team members are homogenous with respect to their productivity, then an equilibrium requires the ex-ante payoff of the Anti-Sharer to equal those of the remaining team mambers. In a case of a heterogenous agents, the agent with the lowest productivity should assume the role of the Anti-Sharer. Section 6 concludes the paper.

2 Teams without "Anti-Sharing"

2.1 Basic notation

Consider a group of n agents who may spend effort $e_i, i = 1..n$ to produce an output $Y(e_1..e_n)$.[4] The individual efforts are assumed to be unobservable, thus not contractible.[5] The agents are assumed to dislike effort. We denote

[4]The subscript i denotes player $i = 1..n$. We also use the subscript i to denote the derivative of Y with respect to player i's effort e_i.

[5]It is not possible to infer the individual effort from the observed output if $n > 2$, or if output is stochastic, i.e., when a random variable influences the output. An example for the latter would be a situation where the agents' efforts increase the probability of success, whereas the project value is fixed. However, even if an agent could infer another agent's effort, this still does not imply that efforts are verifiable.

the effort cost of agent i as $c_i(e_i)$,[6] and assume that individual costs are independent of other players' effort choices.[7] We assume furthermore that players' utility is separable in wealth and effort cost - the players maximize their income minus their effort cost.

To simplify the notation, we employ the following convention for effort vectors. While a lower case letter e_i denotes the individual effort of player i, a capital letter E represents a vector of individual efforts, with the following subscripts:

- E is the effort vector of all n players: $E = (e_1..e_n)$.
- $E_{(-i)}$ is the effort vector of all n players except player i: $E_{(-i)} = (e_1..e_{i-1}, e_{i+1}..e_n)$. Consequently, $E_{(-i,-j)}$ denotes the effort vector without the contributions of players i and j.
- For convenience, we write $E = (E_{(-i)}, e_i) = (E_{(-i,-j)}, e_i, e_j)$.

The production function $Y(E)$ is twice differentiable, continuous, and increasing in individual efforts, but with diminishing marginal returns: $Y_i > 0 > Y_{ii}$. We assume $Y_{ij} > 0$; thus, efforts are not linear separable and a team production problem exists in the sense of Alchian/Demsetz (1972).[8]

2.2 Optimal and equilibrium efforts

The socially optimal effort (E^*) solves

$$E^* = \arg\max Y(E) - \sum_{i=1}^{n} c_i(e_i)$$

and therefore satisfies the following first order conditions:[9]

[6] Note that we do not assume the agents to be homogeneous. Individual productivity and effort cost may be different.

[7] Different in Strausz (1999), who introduces individual cost functions that are decreasing in other agents' effort. We follow an alternative way to model a team problem and assume an output function with positive cross derivatives, see below.

[8] We exclude the case of negative cross-partials.

[9] Due to the assumptions we have made concerning the second derivatives, we can neglect the second-order conditions.

$$Y_i(E^*_{(-i)}, e_i) \stackrel{!}{=} \frac{dc_i(e_i)}{de_i} \;\forall\; i = 1..n \tag{1}$$

All players should choose their individual effort such that their marginal productivity equals their marginal cost, given that all other players have chosen optimally.

However, as Holmstrom (1982) has demonstrated, with a budget-balanced Sharing contract the players will not have an incentive to do so: s_i denotes the share player i receives from the output Y. A Sharing contract is called "budget-balanced" if $\sum s_i = 1$.

Let e'_i denote the actual choice of player i under a Sharing contract. In equilibrium, i's actual choice solves the following maximization problem:

$$e'_i = \arg\max s_i Y(E'_{(-i)}, e_i) - c_i(e_i).$$

Thus, the first-order conditions for the individually optimal choices are:

$$s_i Y_i(E'_{(-i)}, e_i) \stackrel{!}{=} \frac{dc_i(e_i)}{de_i} \;\forall\; i = 1..n \tag{2}$$

Note that the right hand side of equation (1) equals the right hand side of equation (2). The difference between socially optimal and individually rational behavior lies in the respective left hand side. If, given a budget-balanced Sharing contract, a player exists whose share is greater than zero, then at least one other player exists whose share is smaller than one. Therefore, at least for some player (if not for all of them) the left hand side of equation (2) must be smaller than in equation (1), even if all other players were choosing efficient effort. This leads to two different disincentives:[10]

1. Since at least some players acquire a share smaller than one, they have an incentive to choose $e'_i < e^*_i$.
2. If other players exist who choose $e'_j < e^*_j$, and $Y_{ij} > 0$, player i's marginal productivity decreases, which motivates him to further decrease his effort.

[10]The first inefficiency effect is taken into account in Holmstrom (1982), but not he second.

In a team according to the Alchian/Demsetz (1972) terminology, i.e., with a positive cross derivative, the moral hazard problem is even worse than predicted by the Holmstrom model. Without some additional institution, the n agents will produce $Y' = Y(E')$ as their output, which is smaller than the efficient output $Y^* = Y(E^*)$.

The individual payoff in equilibrium amounts to $\pi_i' = s_i Y' - c_i(e_i')$. Assuming positive cross-partials, this is for all agents smaller than the individual payoff if all agents choose efficiently, i.e., $\pi_i^* = s_i Y^* - c_i(e_i^*)$. Even an agent who is entitled to a share $s_i = 1$ would receive less than Y^*, due to $Y_{ij} > 0$ and $e_j' < e_j^* \forall\ j \neq i$. The difference between the efficient and the equilibrium payoff leaves room for a Pareto-improvement, even if the institution that implements a higher output level raises transaction costs.

3 First-best Anti-Sharing

In this section we add an external Anti-Sharer to a team of n members. The Anti-Sharer thus is introduced as a player $(n + 1)$. Moreover, we assume the external Anti-Sharer to be competitive, thus to work without profit. We demonstrate that, under these circumstances, first-best effort is a Nash equilibrium. Let us denote the actual effort of player i as e_i''; thus, $e_i'' = e_i^*$ is to be proven.

The contract between the team members and the external zero-profit Anti-Sharer consists of two components. The contract obliges

- each of the n team members to make a fixed payment $\frac{n-1}{n} Y(E^*)$ to the Anti-Sharer.
- the team members to deliver the actual output $Y(E'')$ to the Anti-Sharer (who becomes residual claimant).
- the Anti-Sharer to pay $Y(E'')$ to each of the team members.

We will prove the following[11]

> **Proposition 1:** If an external, zero-profit Anti-Sharer offers the above contract to the n team-members, then

[11] A parallel result has been demonstrated in Cooter/Porat (2002).

a) To choose the efficient effort e_i^* is a Nash equilibrium for all team members $i = 1..n$.

b) If the team members choose efficient effort, then the mechanism obeys the zero-profit condition.

The proof of part a) is based on the following maximization problem that each team member solves by choosing his actual effort:

$$e_i'' = \arg\max Y(E''_{(-i)}, e_i) - c(e_i) - \frac{n-1}{n} Y^*.$$

The first-order condition for e_i'' thus is

$$\frac{\partial Y(E''_{(-i)}, e_i)}{\partial e_i} = \frac{dc_i(e_i)}{de_i} \ \forall\ i = 1..n \tag{3}$$

which is identical to equation (1), the condition for first-best effort. Thus, we have established that it is a Nash-equilibrium for all team members to choose first best effort under this mechanism. It is individually rational to choose the efficient effort if all other team-members do the same.

To prove part b) of proposition 1, we calculate the Anti-Sharer's net payoff. He collects $(n-1)Y^* + Y(E'')$ and pays out $nY(E'')$. Obviously, his net payoff is zero if $E'' = E^*$; q.e.d.

The mechanism is not budget-balanced if the team members do not choose first-best efforts. Without Anti-Sharer, a Sharing contract that is not budget-balanced imposes a credibility problem (the parties would be motivated to spend efficient effort, if they were credibly committed to throw away some part of the output in case of inefficient effort, but they anticipate that they would not do so). However, the existence of an Anti-Sharer avoids this credibility problem, since he may claim the agents' payments and only has to pay out their actual achievement. If the parties choose less than the efficient effort, then the Anti-Sharer's net payoff accrues to $(n-1)Y^* - (n-1)Y(E'')$ which is positive.

Individual rationality keeps the team-members form investing more than the efficient effort, since they had to bear the effort costs. Thus, the Anti-Sharer does not face the problem to have to pay out more than he collects. The only weakness of this mechanism is that the efficient equilibrium is not unique. If some team players expect their colleagues to spend less than efficient effort, it is individually rational to spend lower effort as well (due to the positive cross-partial), which makes the expectation self-confirming.

The mechanism implements first-best efforts since each team member calculates with the total actual output $Y(E'')$ - and not with a share $s_i Y(E'')$ - in his individual yield function. Therefore, the total marginal return of his effort is internalized. Of course, the actual effort can be distributed only once, but the difference between nY'' (what the n team members are heading for) and Y'' (what is actually distributed among them) is covered by their fixed payment. This payment is fixed, since it is independent of their actually chosen effort.

However, from the viewpoint of institutional economics this approach is not satisfying. It leaves the question unanswered where the player $(n+1)$ comes from. If he does not appear out of the blue, but has already been present in society before the team chooses to employ the mechanism under scrutiny, then the question arises what this agent has been doing before. If this was a productive activity, then being employed by the team may keep him from pursuing this alternative activity.

4 Internal Anti-Sharing

4.1 Setup

In the previous section, the first-best solution was generated by introducing an additional player, the Anti-Sharer. This leaves two questions unanswered:

- Where does this $(n+1)^{st}$ player come from?
- What are the opportunity costs (in the sense of omitted productive activity) of being Anti-Sharer?

In this section, we analyze the case when one of the n team members plays the role of the Anti-Sharer. Without loss of generality, we assume agent number n to specialize in Anti-Sharing for the other $(n-1)$ agents who remain productive. An Anti-Sharing contract requires the productive $(n-1)$ agents to make a lump-sum payment to the Anti-Sharer.[12] After the $(n-1)$

[12] We disregard liquidity constraints, since the lump-sum payment does not actually have to be made; it is sufficient for the mechanism to work that the payment is subtracted from the respective agent's payoff after the output has been produced.

agents have chosen their efforts, each of them receives a payment that equals the actual output from the Anti-Sharer. Note that these payments do not necessarily have to be made in chronological order; it is sufficient for the players to be obliged by the Anti-Sharing contract to make these payments. We therefore assume the fixed payments and the actually produced output to be contractible.

The lump-sum payment may consist of two components: a fee for player n and the necessary payment that enables player n to pay out the actual output $(n-1)$ times. The fee can be zero or positive. Let us describe the Anti-Sharing mechanism that implements (second-best) efficiency by describing the necessary contractual provisions in more detail:

1. The $(n-1)$ productive agents pay an amount of

$$\frac{n-2}{n-1}\hat{Y} + T_i$$

   where $\hat{e}_i$ describes the efficient effort of player $i = 1...(n-1)$ and $\hat{Y} = Y(\hat{e}_1..\hat{e}_{(n-1)}, 0)$ represents the efficient output, given that player n contributes $e_n = 0$. Note that both components of this payment are fixed amounts of money which are independent of the actually chosen effort of either player.

2. Each player $i = 1..(n-1)$ chooses his actual effort, denoted as $\tilde{e}_i$, which leads to an actual output

$$\tilde{Y} = Y(\tilde{e}_1..\tilde{e}_{n-1}, 0)$$

3. Player n acquires the actual output.

4. Player n pays $\tilde{Y}$ to each of the other $(n-1)$ players.

4.2 Optimal efforts with internal Anti-Sharing

Since the agent n abstains from productive activity, the highest output that can be achieved by making use of an internal Anti-Sharer is smaller than the first-best output.[13] Moreover, the lack of effort exerted by agent n decreases the marginal productivity of all other agents $1..(n-1)$. Hence, the maximum output under the Anti-Sharing mechanism is smaller than the first-best

[13] In a situation where the agents' efforts are perfect complements, Anti-Sharing thus is useless.

output $Y(E^*)$. Let us now derive the condition for optimal efforts under the assumption $e_n = 0$. The optimal efficient efforts $\hat{e}_i, i = 1..(n-1)$ maximize

$$Y(\hat{E}_{(-n)}, 0) - \sum_{i=1}^{n-1} c_(\hat{e}_i)$$

and satisfy the first-order conditions (we again assume second-order conditions to be satisfied):

$$Y_i(\hat{E}_{(-n)}, 0) \stackrel{!}{=} \frac{\partial c_i}{\partial e_i} \;\forall\; i = 1..(n-1) \tag{4}$$

4.3 Equilibrium efforts with Anti-Sharing

Now we prove the following

> **Proposition 2:** If in a team of n members, player n assumes the role of an internal Anti-Sharer and offers the above contract to the other $n-1$ team-members, then:
>
> a) It is a Nash-equilibrium to choose the efficient effort $\hat{e}_i$ for all team members $i = 1..(n-1)$.
>
> b) If the team members choose efficient effort and $\sum T_i = 0$, then the mechanism obeys the zero-profit condition.

We denote the individually rational efforts of player $1..(n-1)$, given that player n chooses $e_n = 0$, as $\tilde{e}_i$. We have to demonstrate that $\forall\; i = 1..(n-1)$: $\tilde{e}_i = \hat{e}_i$. The equilibrium effort of agent $i = 1..(n-1)$ is determined by

$$\tilde{e}_i = \arg\max Y(\tilde{E}_{(-i,-n)}, e_i, 0) - \frac{n-2}{n-1}\hat{Y} - T_i - c_i(e_i)$$

and satisfies the following first-order conditions:

$$Y_i(\tilde{E}_{(-i,-n)}, e_i, 0) \stackrel{!}{=} \frac{dc_i}{de_i} \;\forall\; i = 1..(n-1) \tag{5}$$

If all of the $(n-1)$ productive players choose their optimal reply on $e_n = 0$, then equations (5) and (4) are identical. These efforts hence constitute a Nash equilibrium.[14]

Part b) of the proposition requires to show that, with $\sum T_i = 0$, this mechanism is budget balanced in the efficient Nash equilibrium. The Anti-Sharer receives $(n-1)$ times the lump-sum payment, which amounts to

$$(n-2)\hat{Y} + \sum_{i=1}^{n-1} T_i$$

If the players $i = 1..(n-1)$ choose $\tilde{e}_i = \hat{e}_1$, then the actual output is $\hat{Y}$. Thus, the net payment of player n is

$$(n-2)\hat{Y} + \sum_{i=1}^{n-1} T_i + \hat{Y} - (n-1)\hat{Y}$$

which equals $\sum T_i$. If the productive agents choose optimal efforts, then the payoff of player n equals the sum of fees. Hence, if $\sum T_i = 0$, then the mechanism is budget balanced in this Nash equilibrium.

If the other $n-1$ players choose suboptimal efforts, then the mechanism is not balanced. Even tough, the mechanism is credible, since the Anti-Sharer may retain the remainder. It is in his vital interest to carry out the mechanism.

4.4 Is a first-best solution possible?

In this section we allow for player n, who assumes the role of the Anti-Sharer, to contribute effort. We show that, even though all n team members may contribute effort, Anti-Sharing is unable to implement the first-best outcome among the n agents.

In this setting, the Anti-Sharing contract requires the $n-1$ other agents to make a lump-sum payment $\check{T}_i = Y^*/n$ to agent n, who becomes the only residual claimant. Having made this arrangements, all n agents choose their effort. And finally, agent n pays their shares of the actual output to his $n-1$

[14] This is one Nash equilibrium among others; the mechanism turns the initial Prisoners' Dilemma into a coordination game. E.g., $e_i = 0 \ \forall \ i = 1..(n-1)$ is a Nash equilibrium as well. We disregard here the problem of equilibrium selection. According to the selection criterion of Pareto-dominance the parties would play the efficient equilibrium.

partners. We assume zero transaction costs and free competition among the n agents for the position of the Anti-Sharer. Thus, this position does not allow agent n to generate an individual rent; he only receives a share due to his productive effort choice. The outcome is described by the following

> **Proposition 3:** If a team member offers an external zero-profit Anti-Sharing contract to each team member of the team, including himself, then all team members choose less than optimal effort.

We label the equilibrium efforts chosen under this scheme as $\check{E}$. In formal notation, this proposition claims $\check{T} = \frac{n-1}{n} Y(E^*) \Rightarrow \check{E} < E^*$. The proof requires us to analyze the maximization problems of the agents $1..(n-1)$ and the internal Anti-Sharer n separately. We start with the agents $j = 1..(n-1)$: each one of these chooses his effort such that

$$\check{e}_j = \arg\max \frac{n-1}{n} Y^* + Y(\check{E}_{(-j)}) - c_j(e_j).$$

Thus, $\check{e}_j$ satisfies the first-order condition

$$\frac{\partial Y(\check{E}_{(-j)}, e_j)}{\partial e_j} = \frac{dc_j}{de_j}$$

which is equivalent to equation (1) above, the condition for a first-best outcome, provided that all players (including n) do the same.

However, player n remains to be analyzed: he receives the lump-sum payments from the other players and the actual outcome. He has to pay out $n-1$ times the actual outcome and to bear his own effort costs. Thus, he chooses

$$\check{e}_n = \arg\max (n-1) Y^* + Y(\check{E}_{(-n)}, e_n) - (n-1) Y(\check{E}_{(-n)}, e_n) - c_n(e_n)$$

to satisfy the following first-order condition:

$$(2-n) \frac{\partial Y(\check{E}_{(-n)}, e_n)}{\partial e_n} - \frac{dc_n}{de_n} \stackrel{!}{=} 0 \tag{6}$$

which yields $\check{e}_n < e_n^*$. Note that $n > 2$ implies that the left-hand side of equation (6) is negative, thus $\check{e}_n = 0$. All the other players hence face a situation in which, due to the assumption $Y_{ij} > 0$, the marginal productivity of their input is smaller than in the case where all n players choose efficient effort. Therefore, $\forall\ i = 1..n : \check{e}_i < e_i^*$; q.e.d.

The fact that player n collects the actual product and has to pay it out $(n-1)$ times distorts his incentives to contribute effort. Therefore, the successful application of the Anti-Sharing mechanism requires the Anti-Sharer to be unproductive. He is supposed to be (or to become) an outsider rather than (to remain) a team member.

5 Constitutional economics of Anti-Sharing

This section compares two contractual arrangements: the Sharing contract and the internal Anti-Sharing contract. Consider a team of n members that operates under a sharing contract. Assume that no player $n+1$ is available to play the role of an external Anti-Sharer. In this situation, the team will find it beneficial to switch to an Anti-Sharing contract only if this is Pareto-superior. This problem is analyzed in section 5.1.

Pareto-superiority, however, leaves the question unanswered which team member shall assume the role of the Anti-Sharer. If the productivity of the team members is homogenous, then a constitutional equilibrium requires the payoffs for both roles (Anti-Sharer and productive team member) to be equal. This is discussed in section 5.2. If agents are heterogenous with respect to their marginal productivity or their cross derivatives, then the role of the Anti-Insurer should be assigned in order to minimize the productivity loss.

5.1 Efficiency

This section derives the condition under which internal Anti-Sharing implements a better solution than the Sharing contract analyzed in Holmstrom (1982). Moreover, we discuss under which conditions the introduction of Anti-Sharing is even Pareto-superior for the team members.

Under a budget-balanced Sharing contract, the agents $1..n$ spend efforts E' and generate the output $Y' = Y(E')$, yielding the total net benefit $Y(E') - \sum_{i=1}^{n} c_i(e_i)$. The net benefit under Anti-Sharing is greater if, and only if,

$$Y(\hat{E}_{(-n)}, 0) - \sum_{i=1}^{n-1} c_i(\hat{e}_i) > Y(E') - \sum_{i=1}^{n} c_i(e'_i)$$

which is equivalent to

$$\hat{Y} - Y(E') > \sum_{i=1}^{n-1} [c_i(\hat{e}_i) - c_i(e'_i)] - c_n(e'_n).$$

For simplicity, we have written $\hat{Y} = Y(\hat{E}_{(n-1)}, 0)$. The improvement in output is required to be greater than the increase in effort costs, net of the effort costs saved by player n (who assumes the role of the Anti-Sharer). This condition is more likely to hold, the smaller the cross partials $\partial^2 Y / \partial e_i \partial e_n$ for $i = 1..(n-1)$, and the smaller the marginal productivity of player n.

If this condition holds, and the only available institutions are Sharing contracts and internal Anti-Sharing contracts, then the latter is Kaldor-Hicks efficient. The more interesting question is whether Anti-Sharing is Pareto-superior, compared to the Sharing contract. This requires each individual payoff (for player $i = 1..n$) to be greater when an Anti-Sharer is employed, which can be guaranteed by side payments, as the following proposition states:

> **Proposition 4:** If a team of n members can only choose between a Sharing contract and an internal Anti-Sharing contract, and the latter is Kaldor-Hicks-efficient, then a vector of lump-sum payments $(T_1, T_2, ..., T_{(n-1)})$ exists such that the internal Anti-Sharing contract is Pareto-superior.

For the proof of this proposition, recall that Kaldor-Hicks-efficiency implies

$$\Delta Y > \sum_{i=1}^{n-1} \Delta c_i + c_n(e'_n)$$

where ΔY denotes the increase in output ($\Delta Y = \hat{Y} - Y'$, whereas Δc_i represents the increase in costs that player i bears: $\Delta c_i = c_i(\hat{e}) - c_i(e')$. Pareto-superiority of the internal Anti-Sharing contract requires two conditions to be fulfilled:[15]

[15] At least one of the conditions a) and b) needs to be strictly fulfilled.

a) $\forall i = 1..(n-1) : \frac{\hat{Y}}{n-1} - s_i Y' \geq \Delta c_i + T_i$

b) $\sum_{i=1}^{n-1} \geq s_n Y' - c_n(e')$

If a vector of lump-sum fees T_i fulfills condition a), then this implies

$$\sum_{i=1}^{n-1} T_i \leq \hat{Y} - Y' \sum_{i=1}^{n-1} s_i - \sum_{i=1}^{n-1} \Delta c_i.$$

The right-hand side of this inequality denotes the incremental rent generated by the efforts of agents $i = 1..(n-1)$ by switching to the internal Anti-Sharing contract. Kaldor-Hicks-efficiency implies that this rent is greater than the payoff of player n under a Sharing contract:

$$\hat{Y} - Y' \sum_{i=1}^{n-1} s_i - \sum_{i=1}^{n-1} \Delta c_i \geq s_n Y' - c_n(e'_n) = \pi'_n.$$

Thus, lump-sum fee vector the components of which add up to less than the complete rent, but more than π'_n can satisfy condition a) and b) simultaneously.

5.2 Constitutional equilibrium

We have assumed that the activity of the Anti-Sharer can be carried out without cost. Thus, the Anti-Sharer's payoff is $\sum_{i=1}^{n-1} T_i$, whereas the other players' receive

$$\hat{\pi}_i = \frac{1}{n-1}\hat{Y} - T_i - c_i(\hat{e}_i)$$

each.

Consider now the constitutional stage of this partnership: If the n partners assign the roles of the Anti-Sharer and the productive agents through negotiations, a constitutional equilibrium would require all individual payoffs to be equal. Otherwise, on of the roles would be more attractive than the other. In algebraic terms, the constitutional equilibrium condition is

$$\forall\ i = 1..n : \sum_{i=1}^{n-1} T_i \stackrel{!}{=} \frac{1}{n-1}\hat{Y} - T_i - c_i(\hat{e}_i)$$

which is equivalent to

$$\forall\ i = 1..n : T_i \stackrel{!}{=} \frac{1}{n-1}\hat{Y} - \sum_{i=1}^{n-1} T_i - c_i(\hat{e}_i).$$

Given the assumption of a symmetric solution, the total fee collected by the Anti-Sharer is

$$\forall\ i = 1..n : \sum_{i=1}^{n-1} T_i = \hat{Y} - (n-1)\sum_{i=1}^{n-1} T_i - \sum_{i=1}^{n-1} c_i(\hat{e}_i).$$

This is equivalent to

$$n\sum_{i=1}^{n-1} T_i = \hat{Y} - \sum_{i=1}^{n-1} c_i(\hat{e}_i)$$

which implies

$$\sum_{i=1}^{n-1} T_i = \frac{1}{n}[\hat{Y} - \sum_{i=1}^{n-1} c_i(\hat{e}_i)]. \tag{7}$$

Note that the left hand side of equation (7) shows the n^{th} player's equilibrium payoff, i.e., the sum of fees. The right hand side represents the cooperation rent if players $1..(n-1)$ choose optimal effort, which they do in equilibrium. The equation demonstrates that, in an equilibrium among the n agents with player n specializing into the role of the Anti-Sharer while the other $(n-1)$ remain productive, the Anti-Sharer receives the n^{th} part of the optimal cooperation rent. This implies that all of the n players share the cooperation rent evenly.

Interesting question: Given XAS is Kaldor-Hicks efficient. Do the constitutional equilibrium condition and the payment scheme that implements Pareto-efficient exclude each other or are they consistent?

6 Conclusions

To implement efficiency requires the Anti-Sharer not to take part in the productive activity. Thus, if one player specializes on the role of the Anti-Sharer, the lack of his effort leads to a productivity loss. However, this division of labor between him and his $(n-1)$ colleagues leads to a second-best effort choice of the latter. If the resulting cooperation rent exceeds the one under spontaneous team work (which is necessarily below the first best cooperation rent), then each can be made better off by an equal split.

Even though this mechanism is not budget-balanced, it is credible, since it is in the Anti-Sharer's interest to retain the lump-sum payment if the other agents fail to provide efficient effort. The necessary ex-ante and ex-post payments are contractible, as long as the actual output is verifiable. In case the n^{th} agent remains productive, this rule does not implement the first-best outcome.

We think that this mechanism provides a new theory of the firm, at least as far it concerns partnerships of risk-neutral teams. Apart from the theory of the firm in Alchian /Demsetz (1972) and from the sequential team mechanism in Strausz (1999), our theory does not require the n^{th} agent to be able to monitor the other agents' effort choices.

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