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Aggregate Matching Function. The Case of Poland

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Abstract

The main goal of this paper is to analyze the matching function in the Polish labour market in 1994-2008. Matching function is the relationship between outflows from unemployment to employment and the number of unemployed persons and vacancies as well as other variables which affect the efficiency of the matching process directly or indirectly. Such matching function in its augmented form is estimated here for Poland with the use of data from register of unemployed persons.
The results indicate that there is a statistically stronger impact of the unemployed than vacancies on new hires. Furthermore, the institutional conditions of the labour market, the structure of the unemployed and the participants of active labour market programs (ALMP) play a role in the matching process.

Keywords: matching function, matching effectiveness, unemployment duration

JEL Classification: J31, J63, J64.

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1 Introduction

The main goal of this paper is twofold. First, the parameters (elasticities) of the matching function are estimated and, second, the impact of labour market characteristics on outflows from unemployment to employment is investigated.
The matching function is the quantification of the process that matches job seekers to vacancies. In other words, it is the relationship between outflows from unemployment to employment and the number of unemployed people and vacancies, as well as other variables which affect the efficiency of the matching process directly or indirectly.
The efficiency of the matching process impacts the performance of the labour market. A high inefficiency of that process might lead to large structural mismatches between the unemployed and vacant jobs and, as a consequence, a high structural (or natural) unemployment rate. Therefore, analysis of the factors affecting the matching process is particularly significant.
This paper, by using data given in the publication *Registered Unemployment in Poland* (Central Statistical Office, various editions from the years 1994-2007), attempts to estimate the parameters of the above-mentioned matching function.
The structure of the paper is as follows. The theoretical framework of matching functions is discussed in Section 2, and in particular the problem of the mathematical properties of these functions and their homogeneity. Moreover, Section 2 attempts to systemize the knowledge about the factors determining labour market efficiency, which, at the same time, also affect the matching process. Previous research results on the matching process in Poland are summarized in Section 3. The first part of Section 4 is an illustration of the situation of the Polish labour market in the analyzed period (1997-2004). It presents basic indicators of the climate in the labour market, including the unemployment rate, the ratio of long-term unemployment, the rates of inflows to and outflows from unemployment and the number of vacancies per unemployed individual. In the second part of Section 4, the attempt to estimate the parameters of the matching function in Poland is presented. Section 5 provides a conclusion.

2 Theoretical framework

An aggregate matching function is a relationship in which hires (or outflows from unemployment) depend on the number of unemployed and the number of vacancies. The following equation can be used to express this relationship; see Petrongolo, Pissarides (2001); Burda (1993); Stasiak, Tokarski (1995), (1998); Kucharski, Tokarski (2003):

$$O(t) = A(t)F\left[U(t), V(t)\right] \tag{1}$$

where: $O(t)$ - number of new hires at time t; $U(t)$ - number of the unemployed at time t; $V(t)$ - number of vacancies at time t; $A(t)$ - the overall productivity factor (or the labour market efficiency parameter) which describes the efficiency of labour market

processes. If $A(t) = A_0$ then function (1) is called a standard matching function. In the other case, when $A(t) = f(x_{1t}, x_{2t}, \ldots, x_{nt})$ (where $x_{1t}, x_{2t}, \ldots, x_{nt}$ are variables determining the efficiency of the labour market), function (1) is called an augmented matching function.
Function F is assumed to be characterized by the following conditions; see e.g. Petrongolo, Pissarides (2001); Stasiak, Tokarski (1995), (1998); Kucharski, Tokarski (2003); Tokarski (2005):

1. Arguments (U and V) of the function F are not negative.
2. The variables used in Equation (1) are at least twice differentiable with respect to time.
3. $A(t)F[0, V(t)] = A(t)F[U(t), 0] = 0$, so $U(t)$ and $V(t)$ must be positive in order to obtain $O(t) > 0$.
4. $\frac{\partial O(t)}{\partial U(t)} > 0$ and $\frac{\partial O(t)}{\partial V(t)} > 0$, that is, the growth of the number of job seekers (vacant jobs) causes an increase in outflows from unemployment to employment.
5. The marginal outflow is negative $\frac{\partial^2 O}{\partial U^2} < 0$ ($\frac{\partial^2 O}{\partial V^2} < 0$). Moreover, the condition $\frac{\partial^2 O}{\partial U^2} \cdot \frac{\partial^2 O}{\partial V^2} - \frac{\partial^2 O}{\partial U \partial V} > 0$ and the two conditions mentioned above cause a three-dimensional concavity of function F.
6. If $\frac{\partial A(t)}{\partial t} > 0$ ($\frac{\partial A(t)}{\partial t} < 0$), then outflows from unemployment will be higher (lower) in the next period *ceteris paribus*.
7. The function F is homogenous of degree $s > 0$.

The type of a matching function which fulfils conditions 1-7 is a function of the Cobb-Douglas type (usually used in empirical studies):

$$O(t) = A(t)F[U(t)]^{\alpha_U}[V(t)]^{\alpha_V} \tag{2}$$

The parameters of function (2) can be interpreted as follows; see Petrongolo, Pissarides (2001):

- α_U (α_V) is the elasticity of flows from unemployment to employment with respect to job seekers (vacancies);
- $\alpha_U - 1$ is a measure of negative externalities caused by competing job seekers (one vacant job is offered to one person);
- $\alpha_V - 1$ is a measure of negative externalities brought out by firms to each other (one unemployed person may be employed only by one firm).
- The higher the elasticity is (α_U or α_V), the less congestion there is, and the more positive externalities are.

- $\frac{O(t)}{U(t)} = A(t)F\left[U(t)\right]^{\alpha_U - 1}\left[V(t)\right]^{\alpha_V}$ is the average probability of finding a job by an unemployed worker, $\frac{O(t)}{V(t)} = A(t)F\left[U(t)\right]^{\alpha_U}\left[V(t)\right]^{\alpha_V - 1}$ is the average probability of filling a vacant job. The inverses of these probabilities are (in the stationary state) equal to the mean duration of unemployment and vacancies.

The main problem, widely analyzed in the literature, which pertains to a theoretical (as well as empirical) matching function, is the degree of homogeneity of that function. Petrongolo and Pissarides (2001) surveyed the number of matching function studies in European countries. In their opinion *"the stylized fact that emerges from the empirical literature is that there is a stable aggregate matching function of a few variables that satisfies the Cobb-Douglas restrictions with constant returns to scale in vacancies and unemployment"*; see Petrongolo, Pissarides (2001), pp. 397).
Then, Layard, Nickell and Jackman (2005) justify the homogeneity of the first degree as follows: *"Each hiring involves a marriage between a vacancy and an unemployed person. In a marriage market of reasonable size the number of marriages will double as the numbers of men and women double, even if the numbers of each sex are unequal. The same is true in the labour market"*; see Layard, Nickell, Jackman (2005), pp. 217.
We can find also the following reasons for (particularly) increasing returns to scale. First, if there are increasing returns to scale in the matching process, there might be more than one equilibrium due to positive externalities. Increasing returns to scale can support the existence of both high and low activity equilibria even when there are increasing marginal costs to the search effort. In an equilibrium, employers and workers could put more resources into search, pushing up the returns from search available to the other side. In another equilibrium, they could put less effort into the searching process with lower returns from search and higher unemployment. Complementarity between the actions of firms and workers is still present when constant returns to scale exist; see Petrongolo, Pissarides (2001).
Secondly, one can assume that the market is organized in such a way that there are firms with job vacancies and workers looking for a job. Each time they meet, the firms observe the productivity of the match and make a wage offer. Unemployed workers, depending on their reservation wage, may reject or accept that wage offer. If we assume that, due to returns to scale, matches are more productive in a larger market, then the offered wage will also be higher. Then, the workers can see the higher wage offer and choose a higher reservation wage. It is probable that reservation wages rise adequately to offset the impact of a higher offered wage on the matching probability. Consequently, the increasing returns are connected with higher post-unemployment wages but not with a shorter duration of unemployment. However, the size of the market should rather be taken into account in wage equations and not in estimated matching functions; see Petrongolo, Pissarides (2006).
Many empirical studies adopt different specifications and different sets of additional macroeconomic variables (other than unemployed persons and vacant jobs) which may have some impact on labour market conditions and the matching process.

In the commonly used modifications of Equation (1), the efficiency parameter $A(t)$ is explained by including additional variables in the standard matching function; see Petrongolo, Pissarides (2001); Bleakley, Fuhrer (1997); Shimer (2005); Yashiv (2006); Coles, Smith (1996); Hujer, Zeiss (2003); Kaczorowski, Tokarski (1997); Rogut, Tokarski (2000). These additional variables are:

- Time trend (linear or nonlinear); however, the premise about the more and more slowly increasing efficiency of the labour market suggests the use of a nonlinear form of the trend function. Besides, as institutional changes and technological advances (such as computerization of employment offices, job advertising on the Internet, changes in the employment protection rate, increased governmental resources put into subsidized matching, etc.) are hardly measurable. The time trend is supposed to be an approximation of these changes.

- The structure and level of expenditures on labour market programs (both active labour market programs (ALMPs): interventional or public works, vocational training for unemployed workers, retraining, vocational skills development, and passive ones: unemployment benefits, social benefits). In general, the impact of the ALMPs on labour market efficiency as well as on outflows from unemployment is positive (although to a varying degree).

- The structure of the unemployed by unemployment duration (i.e. share of short- and long-term unemployed in the unemployment stock). The long-term unemployed are considered by employers as persons who have lost a significant part of their skills and qualifications (as the human capital theory suggests), and are then less desirable as employees as compared to those with more recent work experience. So, the higher the share of the long-term unemployed in the labour market is, the smaller the matching scale is.

- The minimum wage to average wage ratio and the ratio of unemployment benefits to average wages, which are commonly used measures of labour market tightness (or the generosity of the unemployment insurance system). High values of these ratios may affect the matching process negatively.

- GDP per capita, the pace of economic growth, the sectoral structure of employment and the sectoral structure of value added, the investment rate, the level (and rate) of capital per worker and labour productivity. These variables are used as an approximation of the economic condition of the country.

- Demographic variables (sex, education, place of residence, and age composition of the labour force).

Layard, Nickell, Jackman (2005) propose some modification of the matching function. They suggest taking into account the effectiveness of the unemployed in searching

process. The simplest matching function with this effectiveness (c) can be illustrated as:

$$O(t) = A(t)F\left[cU(t)\right]^{\alpha_U}\left[V(t)\right]^{\alpha_V} \tag{3}$$

The index of effectiveness of unemployed workers can be estimated according to the formula:

$$\widehat{c}(t) = \sum_{d} h_{d\tau} f_{dt} \tag{4}$$

where:

$h_{d\tau}$ is the exit rate at each duration d in any selected period τ;
f_{dt} is the proportion of the unemployed at duration d in period t.

The contour line of the matching function (1) is a Beveridge curve. The Beveridge curve depicts a negative relationship between U and V in the case when inflows into unemployment are equal to outflows from employment (a steady state situation) (see i.e. Yashiv (2006)). The Beveridge curve may be interpreted (by analogy to the production function) as a curve of the same outflows from unemployment. After calculating the total differential of function (1), we arrive at:

$$dO(t) = \frac{\partial O(t)}{\partial U(t)} \cdot dU(t) + \frac{\partial O(t)}{\partial V(t)} \cdot dV(t) \tag{5}$$

and comparing to zero, we have $\frac{\partial U}{\partial V} = -\frac{\partial O}{\partial V} / \frac{\partial O}{\partial U}$.
The above formula implies that there exists a nonlinear and negative relationship between unemployment and vacancy rates.
Thus, the Beveridge curve shows all possible combinations of unemployment and vacant rates relating to a given efficiency (or flexibility) of the labour market. Moreover, the position of the curve itself may result from changes in market clearing, institutional changes or changes in the sectoral structure of employment. Hence, shifts in the Beveridge curve are a sufficient but not necessary condition for the equilibrium unemployment to change. According to Solow (1998), the distance of the Beveridge curve from the origin of (U, V) space can be treated as a measure of labour market flexibility. His proposal allows for comparing changes on the local labour markets between countries and during different periods.

3 Empirical evidence for the matching function in the Polish economy

Empirical analyses of the matching function in the Polish economy have been attempted since the mid-1990s, due to a lack of reliable data before. Some authors have tried to estimate the parameters of the matching function and analyze the impact of the ALMP on outflows from unemployment to employment.
Kwiatkowski, Tokarski (1997) using quarterly data (1st quarter, 1992 - 1st quarter,

1996, for 49 "old" voivodeships in Poland) estimated the parameters of an augmented matching function (similar estimates can be found in the analysis of Stasiak, Tokarski (1998)) , but in their research the ALMP variables were not taken into consideration). The parameter of the effectiveness of matching was endogenized. The variables describing its variability were: a linear time trend, expenditures on labour market programs (on interventional works, public works, loans to firms and unemployed workers and expenditure on retraining for the unemployed). Moreover, the authors divided the unemployed into 2 groups: short-term unemployed and long-term ones. Basic conclusions from the analyses of Kwiatkowski, Tokarski (1997) are as follows:

- Both the number of vacancies and the number of the unemployed determine outflows from unemployment to employment in a statistically significant manner.
- The impact of the short-term unemployed on outflows is stronger than the impact of those who remained unemployed for longer than 12 months. It confirms that the long-term unemployed are likely to lose their employability through the loss of skills.
- The time trend is positive and statistically significant, which confirms the strong tendency of diminishing the unemployment stock (particularly of the structural type) in Poland in the 1990s.
- Analyzing the estimated influence of labour market programs on outflows from unemployment it appears that it is only expenditures on interventional works per unemployed person that affect outflows in a significant way. Besides, spending on public works (per unemployed person) has a statistically significant and negative impact on the outflow rate.
- The sum of elasticities of outflows from unemployment with respect to U and V is almost equal 1, although no restrictions have been imposed.

Analyses provided by Kaczorowski, Tokarski (1998) are a continuation of those mentioned above; also see Kaczorowski, Tokarski (1997). Kaczorowski and Tokarski include additional variables describing the efficiency of the matching process. Apart from ALMP variables, they take into consideration variables illustrating the industrial changes in the Polish economy such as: the number of small private entities, the share of the employed in the service sector (or in non-agricultural sectors) and the private sector investment rate. The main conclusion coming from these analyses is the influence on outflows from unemployment by: small private entities, expenditures on interventional works per unemployed worker and the time trend. The positive time trend is explained by the improvement of the searching process and functioning of the labour offices (more vacancies were registered in the analyzed period). However, the negative impact of expenditures on training unemployed workers on outflows from unemployment is surprising. The authors justify it by the incompatible structure of

this training in terms of labour demand and by addressing them to the lowest qualified unemployed workers (or those with a small chance of finding a job). Moreover, the impact of ALMPs on the outflow rate was analyzed by Puhani (1999) and conclusions coming from his research are consistent with those mentioned above.
The other augmented matching function was presented in works by Rogut and Tokarski (2000, 2002). The efficiency ratio was explained by the real GDP growth rate, time trend and sectoral structure of employment (measured by the distance - in Euclidean space - from a corresponding structure in the G7 group of countries). Using cross-sectional data (49 "old" voivodeships in 1992-1998) the authors indicate that:

- The time trend determines outflows from unemployment to employment in a positive and statistically significant way and indicates an improvement in the labour market situation. The impact of the GDP growth rate is also positive.
- The distance of the structure of employment from the G7 structure matters. The more similar structure of employment in a given region is to that observed in the G7 countries, the higher outflows from unemployment are.
- The estimated elasticities of outflows with respect to U and V are 0.64-0.67 and 0.22-0.33, respectively. These estimates indicate the homogeneity of the matching function and constant returns to scale of the matching process.

The parameters of the augmented matching function of the Cobb-Douglas type (exploring regional data at the NUTS2 (NUTS - Nomenclature of Units for Territorial Statistics) level for Poland were estimated by Kucharski, Tokarski (2003) and Tokarski (2005). Apart from U and V, the real GDP growth rate and time trend were used as additional explanatory variables. The analysis provided by Kucharski, Tokarski (2005) seems to be particularly interesting because of using the error correction model. Their research indicates that the elasticity of outflows from unemployment with respect to vacancies is lower than the corresponding elasticity with respect to unemployment. Moreover, the long-term elasticities of outflows with respect to the unemployed (in an error correction model of matching) are higher (about 0.2-0.3) than short-term ones (0.49-0.65). Hence, in the long term the number of the unemployed comes into prominence in the matching process at the cost of vacancies.
Monthly data (January 1992 - January 2006) were used in a study by Gałecka (2007). The author used the quadratic trend as an additional variable. The main results of that paper are as follows: the elasticity of outflows with respect to unemployed workers (vacant jobs) is 0.4-0.6 (0.6) and a positive time trend impact is observed.

4 Empirical Matching Function for Poland

4.1 Data consideration and trend

Analyses provided in this paper are based on data from *Registered Unemployment in Poland* (CSO, various editions from 1994-2008).
The main problem in empirical analysis based on the Beveridge curve concept is the reliability of data concerning vacant jobs. However, data concerning vacant jobs are downward biased in some way, and it needs to be stressed that vacancies are strongly procyclical. It means that they reveal changes in economic activity and make it possible to analyze the directions of these changes. Registered vacancies are not complete, because employers do not convey all information about vacant jobs. Usually these vacancies are related to less attractive jobs which are difficult to fill. Moreover, it is not known how many vacant jobs are registered with the labour offices. The problem could be corrected if the scale of bias would be the same in each particular period.
The disadvantage of the data coming from the labour offices is that they are not fully comparable in time. During the period of analysis some changes in the definition of unemployed workers and their entitlements were introduced. Thus, the stock of registered unemployed includes workers not eager to work or those who are changing their jobs. Besides, that stock includes formally unemployed workers who actually do not seek a job and have a job in the black market; see Góra (2005).
A different methodology is the reason why the number and the structure of unemployment given by the labour offices is not the same as that provided by Labour Force Surveys.
Figure 1 illustrates the unemployment stock and the unemployment rate, respectively, in Poland in 1994-2007 (quarterly data). One can see that changes in the Polish labour market in 2004-2007 were not homogenous. Growth in the unemployment rate was observed in 1992-1994 (in the 1^{st} quarter of 1994 the unemployment rate reached the highest value of 16.7%, according to the register of unemployed persons and 15.9% according to LFS).
In the subsequent period, the unemployment rate was decreasing. In 1995-1998 the lowest value of that rate was equal to 10% (LFS data) or 9.5% (data from the register). The following period (1999-2003) was a time of increasing unemployment (the unemployment rate grew to over 20% in the 1^{st} quarter of 2004). In the last period (2004-2007), a decrease in the unemployment rate was observed and the lowest rate (at about 8.5% (LFS) or 11.4% (register)) in the 4^{th} quarter of 2007 was recorded.
Changes in long-term unemployment are a good indicator of the labour market situation. The long-term unemployed are those whose unemployment duration exceeds 12 months. During a recession, those unemployed have a rather small chance to get a job and flow out of unemployment and either the average duration of being unemployed or the long-term unemployment stock increases.
Figure 2 illustrates the long-term unemployment stock and the share of that stock in unemployment. The numbers of the long-term unemployed were increasing in

1999-2003. It is worth noting that the share of the long-term unemployed in the unemployment stock was above 50% in 2002-2007. Among the long-term unemployed, those who remained unemployed for longer than 24 months constituted above 60%.
Analysis of the unemployment level (or unemployment rate) in consecutive periods does not show changes in the labour market. During one period, participants may flow in or out of unemployment. Thus, a constant level of unemployment may indicate that there is either stagnation or dynamism in the labour market. Therefore analyses of flows are essential.
Changes in the unemployment stock depend on both total inflows (from employment and from economic inactivity) and outflows (to the employment stock and to the inactivity stock; see Kucharski (2002)). The net increase in the unemployment stock is the difference between total inflows to unemployment and total outflows from unemployment. The situation when the difference equals zero can indicate two different cases: high or low levels of the above-mentioned inflows or outflows. If the unemployed register with unemployment offices frequently, then they remain briefly in the unemployment stock (i.e. they do not lose their qualifications and skills). As previously mentioned, in stable economies the long-term unemployed are regarded as employability losers because of their human capital depreciation. Hence, if outflows to and inflows from unemployment are low then the chance of dismissing is small but the probability of exit from the unemployment stock and reemployment is slight. Then, the mean duration of unemployment becomes relatively high and unemployment is stagnant; see Kucharski (2002).
Looking at the numbers (Figure 3), one can see that the average quarterly inflow to or outflow from the unemployment stock was about 600 000 in the analyzed period (i.e. 2 400 000 yearly).
From the point of view of flows in the labour market, it is essential to define an inflow to and outflow from unemployment rates. An inflow to the unemployment rate is the relationship of the newly registered unemployed at time t to the economically active population at time t. Hence, an outflow from the unemployment rate is the number of unemployed persons removed from unemployment at time t with respect to the unemployment stock at the end of time $t-1$. Rates defined as above are illustrated in Figure 4.
The following conclusions can be drawn from Figure 4. Firstly, the outflow rate was increasing in the years 1994-1998. Secondly, a growing tendency in outflow and inflow rates has been present since 2002. That increasing mobility might indicate an improvement in the flexibility of the Polish labour market.
Moreover, the rate of outflows from unemployment to employment grew in the years 1994-1998 (see Figure 4). In the following years (i.e. until 2003), that rate diminished. That tendency changed at the beginning of 2003. This can imply an improvement in the situation in the labour market and the growth of employment growth. As we mentioned before, it is worth noting that outflow and inflow rates can be biased because they are based on registration data. Then, analysis of the above-mentioned

rates should be based on directions (not on values).
The number of vacant jobs (as part of labour demand) is a significant feature of the labour market. Figure 5 illustrates the number of vacancies (at the end of the quarter) and the number of unemployed persons per one vacant job.
As mentioned before, the data concerning vacancies do not cover the total number of vacant jobs in the Polish economy. Usually, the registered vacancies are less attractive and more difficult to fill. Moreover, the registration of vacancies is cyclical. Figure 5 shows that during a boom, when labour demand grows and filling vacant jobs with well-qualified workers is difficult, the number of registered vacancies increases. Then, during a recession, a labour supply surplus is present and filling a vacant job ceases to be difficult. Firms do not have to use the labour office service and do not register their vacancies; see Kwiatkowski, Lehman, Schaffer (1992).
Furthermore, the number of vacancies per unemployed person is a measure of labour market tightness. The inverse of that measure is illustrated in Figure 5. A significant growth in the number of unemployed persons per vacant job was noticed in 1999-2001. Since 2002, an increase in that ratio has been observed, so the condition of the Polish labour market started to improve.

4.2 Matching function for Poland 1994-2008

The following initial augmented matching equation is assumed:

$$O_t = A_t V_t^{\alpha} U_t^{\beta} \tag{6}$$

where:

O_t is an outflow from unemployment to employment at time t; A_t is the efficiency of the labour market (which depends on i.e. institutional changes) at time t; V_t is the number of registered vacancies at time t; U_t is the number of unemployed workers at time t;
α (β) is the elasticity of outflows from unemployment with respect to vacancies (unemployed workers).

In the next step, the index of effectiveness of unemployed workers (c_t) is taken into account (see (4)). Index c_t can be interpreted as the probability of not leaving the unemployment stock by the unemployed who were unemployed in a given period. Calculating that index, strong assumptions were made. First, it is explicitly assumed that an unemployed person loses their job at the beginning of the quarter; second, that unemployment leavers stay employed; third, that the long-term unemployed do not have a chance to find a job and the probability of remaining unemployed in the next period equals 1 and the modified matching function is as follows:

$$O_t = A_t V_t^{\alpha} (c_t U_t)^{\beta} \tag{7}$$

Parameters of the following three equations were estimated:

$$\ln(O_t) = Constant + \beta \ln(U_t) + \alpha \ln(V_t) + structural\, variables \tag{8}$$

$$\ln(O_t) = Constant + \delta \ln(c_t U_t) + \alpha \ln(V_t) + structural\,variables \tag{9}$$

and assuming constant returns to scale (i.e. $\alpha + \delta = 1$)

$$\ln\left(\frac{O_t}{U_t}\right) = Constant + \gamma \ln(c_t) + \alpha \ln\left(\frac{V_t}{U_t}\right) + structural\,variables \tag{10}$$

Equations 8-10 were estimated using OLS (The stationarity of all of the variables was tested with the KPSS test). General-to-specific model selection procedures were used. The constant returns to scale condition was tested using the Wald test (condition $\alpha + \beta = 1$ in (8) or $\alpha + \delta = 1$ in (9)).
The labour market efficiency parameter (A_t) was represented by structural variables. These include: a linear or non-linear trend, the share of the unemployed entitled to benefits in the unemployment stock, a ratio of minimum wage to average wage in the Polish economy, the share of the unemployed entitled to unemployment benefits, the number of interventional and public workers in relation to total unemployment. The estimates of the parameters are presented in Tables 1-3.
The following conclusions can be drawn from the results presented in Table 1:

- The estimated elasticity of outflows from unemployment with respect to V is about 0.51-0.55. It means that an increase in vacant jobs by 1% caused a growth in outflows from unemployment to employment of about 0.51-0.55% *ceteris paribus* (see Akaike's and Schwarz's comparative criteria).

- The estimated elasticity of outflows (O) with respect to the unemployed (U) equals to 0.54 and slightly exceeds the corresponding elasticity for vacancies. This might mean that the number of the unemployed plays a more important role in the matching process than that of vacant jobs.

- Furthermore, the minimum wage affects positively outflows from unemployment. It seems that usually the level of post-unemployment wages is related to the minimum wage. Consequently, a higher value of the minimum wage with respect to the average wage may induce the unemployed to more effectively search and leave the unemployment stock. An increase in the MIN WAGE variable of about 1 percentage point caused, *ceteris paribus*, a 0.03% increase in unemployment outflows.

At the next stage, the analyzed equation has been augmented by new explanatory variables describing the effectiveness of the unemployed. Table 2 shows estimates of Equation 9. Looking at the estimates of Equation 9, it can be concluded that:

- Inclusion of the index of the effectiveness of unemployed persons led to increased elasticity with respect to the effectively unemployed (in comparison to estimates of Equation 8).

- The proportion of the unemployed entitled to benefits in the previous quarter negatively affects current outflows from unemployment. A possible explanation is that the unemployed lack the motivation to search for a job while they do not remain without any income. Thus, those entitled to an unemployment benefit 2 quarters ago determine in a positive way current outflows from unemployment. It seems that the prospect of loss of benefits make the unemployed take up a job.

- The negative estimate of linear trend parameter can be explained by the increasing effectiveness of the unemployed reflected in parameter c and institutional changes (i.e. changes in the labour law). The changes in the labour law regulate the relationship between employees and employers. In the analyzed period, those changes restricted the freedom of action of labour market participants (for example - the third job contract - with a given company - had to be concluded for an indefinite period of time).

- An increase in the number of ALMP participants (public and interventional workers) at time $t-1$ caused an increase in outflows from unemployment at time t. Thus, the increase of ALMP participants at time $t-2$ led to a fall in those outflows.

- Moreover, the strength of ALMP participants' impact on outflows from unemployment differs. The impact of interventional workers is stronger. It may result from the fact that public works are financed with public expenditures (employers do not bear any costs) and the main target of those works is to activate the long-term unemployed who stand a small chance on finding a job by themselves.

- Furthermore, the impact of MIN WAGE on outflows from unemployment decreased, probably because of taking into consideration the effectiveness of the unemployed.

In the next step constant returns to scale were assumed. The estimates of the parameters of the equation (10) with this restriction are presented in table 3.
The following conclusions can be drawn from the estimates of equation (10):

- The elasticity of the outflows to employment rate $\left(\frac{O}{U}\right)$ with respect to vacant jobs is about 0.4.

- The index of the effectiveness of the unemployed is statistically significant and its impact on the outflow rate is positive. An increase in that index of about 1% led to an increase in the outflow rate of about 0.1-0.2%, *ceteris paribus*.

To sum up the estimation results, the empirical matching function for Poland in 1994-2008 shows that the elasticities of outflows from unemployment to employment with respect to vacancies and the unemployed are equal to 0.4 and 0.6, respectively. It

seems that the unemployed play a crucial role in the matching process in Poland. Moreover, the unemployed structure concerning unemployment duration matters - the index of the effectiveness of the unemployed influences the matches positively.

5 Conclusions

In this paper the matching function for Poland was considered. The matching function is a relationship between the stock of vacancies and unemployed persons and outflows from unemployment to employment.
The trends of major labour market categories for 1994-2008 in Poland indicate some significant shifts. Analyzing outflow and inflow rates one can observe an increasing trend in 1994-1998 (but only in the case of the outflow rate) and since 2002 (for both of them). That increasing mobility might indicate an improvement in the Polish labour market flexibility.
Also, the number of vacancies per unemployed worker is a measure of labour market tightness. A significant growth in the number of unemployed persons per vacant job was noticed in 1999-2001. Since 2002, an increase in that ratio has been observed, so also that indicator shows an improvement in the Polish labour market.
The analyses confirm the existence of an aggregate matching function with constant returns to scale in the Polish economy. The elasticity of substitution equals 1 with the weights of 0.6 and 0.4 on unemployment and vacancies, respectively. Taking into account the index of effectiveness of the unemployed caused an increase in the matching elasticity with respect to the effectively unemployed.
The institutional changes (approximated by the time trend coefficient) in the labour market slightly lowered the outflow from unemployment rate in the years 1994-2008. On the other hand, the increasing efficiency of the unemployed led to a growth in that rate in the analyzed period.
A positive impact of the minimum to average wage ratio on outflows from unemployment is also confirmed. It seems that a higher value of the minimum to average wage ratio might encourage the unemployed to effectively search for jobs and leave the unemployment stock.

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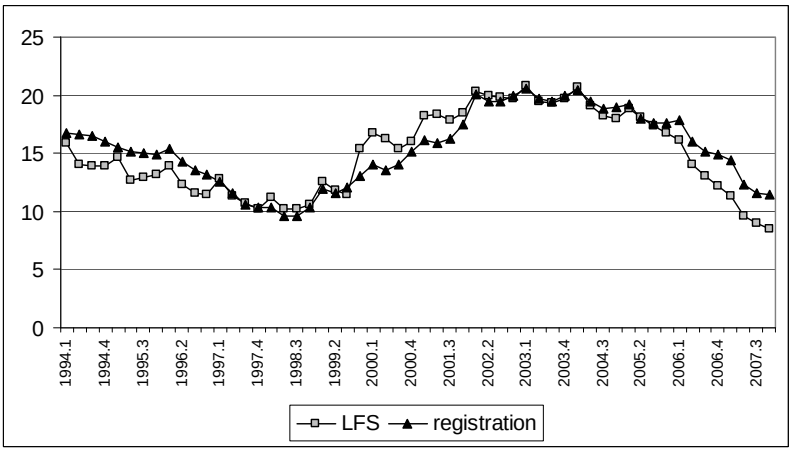
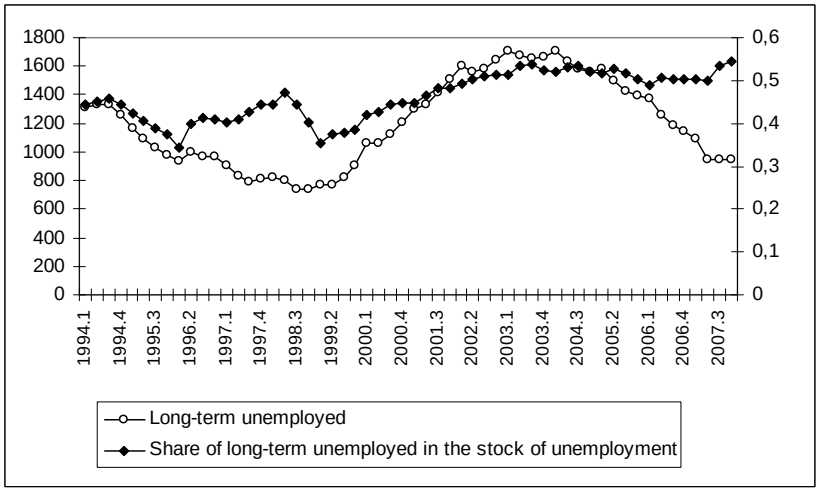
Appendix - Tables and Figures

Figure 1: Unemployment rate according to LFS and register of unemployed persons in Poland in 1994-2007 (in %)



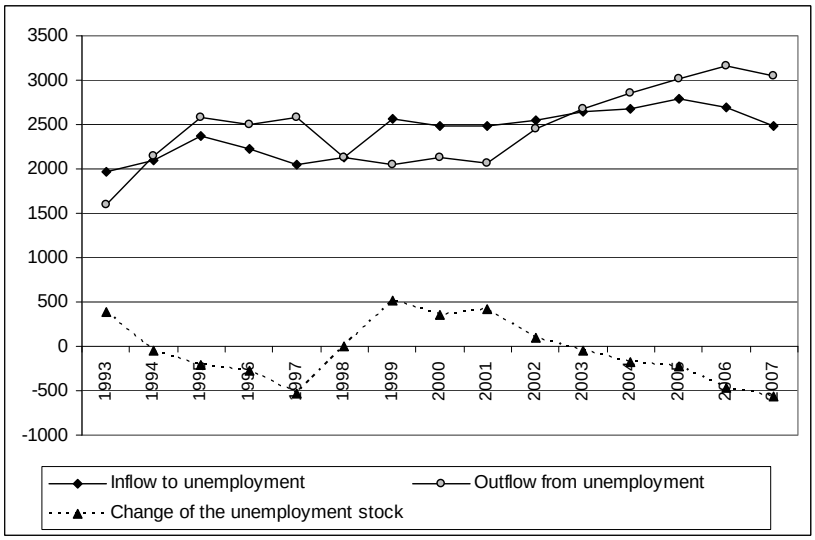
Source: *Labour Force Survey in Poland*, various editions from 1994-2008 and *Registered Unemployment in Poland*, various editions from 1994-2008

Figure 2: Number of long-term unemployed (in thousands, left scale) and share of long-term unemployed in unemployment stock (right scale) in Poland in 1994-2007



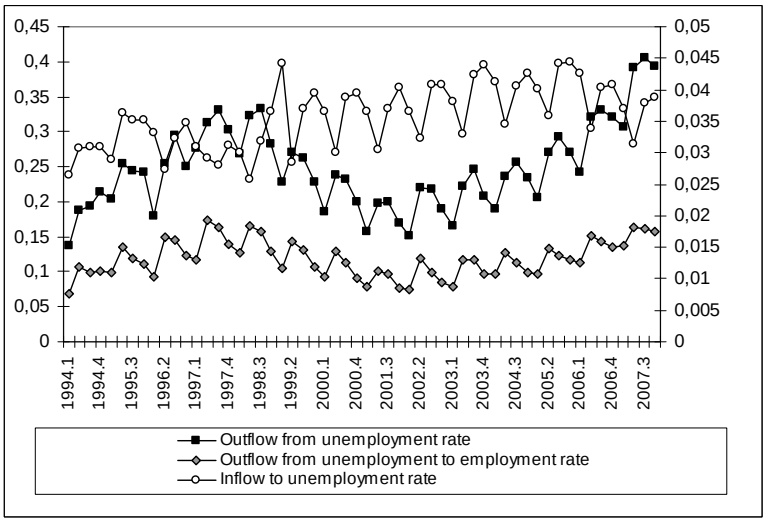
Source: as in figure 1.

Figure 3: Inflow to unemployment, outflow from unemployment and the change of unemployment stock in Poland in 1994-2007 (in thousands of persons)



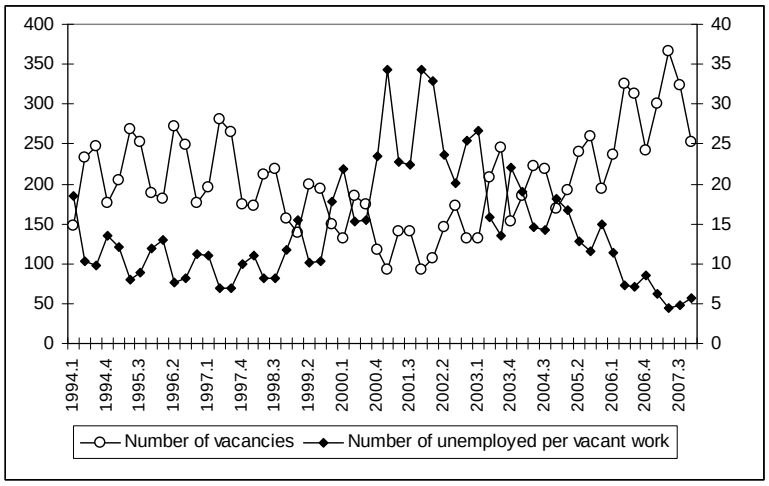
Source: as in figure 1.

Figure 4: Outflow from unemployment rate, outflow from unemployment to employment rate (left scale) and inflow to unemployment rate (right scale) in Poland in 1994-2007



Source: as in figure 1.

Figure 5: Number of vacancies (in thousands, right scale) and number of unemployed per vacant job (left scale) in Poland in 1994-2007



Source: as in figure 1.

Table 1: Estimation results: equation (8)

Explanatory variable	Dependent variable		
	$\log(O)$		
Constant	-0.743 (-1.384)	-0.811 (-1.956)	-0.930 (-2.595)
$\log(V)$	0.314 (7.973)	0.508 (11.608)	0.545 (17.558)
$\log(U)$	0.455 (11.578)	0.536 (13.735)	0.541 (15.319)
ZAS_{-1}	-0.196 (-0.633)	-0.341 (-1.336)	-
ZAS_{-2}	0.579 (1.993)	0.502 (1.997)	-
$MIN\ WAGE$	2.651 (3.186)	-	-
T	0.0067 (3.815)	-0.0045 (-2.555)	-0.0064 (-6.104)
$\frac{1}{T}$	-	-4.777 (-5.279)	-5.119 (-7.206)
$Q1$	-0.180 (-7.222)	-0.162 (-7.364)	-0.158 (-8.492)
$Q3$	-0.137 (-7.299)	-0.100 (-6.197)	-0.095 (-6.053)
$Q4$	-0.173 (-7.205)	-0.076 (-3.251)	-0.064 (-3.154)
$d2002$	0.169 (3.400)	0.172 (4.001)	0.181 (4.153)
$d1996$	-	-	0.031 (0.712)
R^2	0.953	0.965	0.961
Adj. R^2	0.943	0.957	0.953
AIC	-3.175	-3.458	-3.418
S.C.	-2.774	-3.057	-3.056
DW	1.762	2.058	1.871
F	90.030	120.907	126.604
Jarque-Bera (Probability)	3.526 (0.171)	1.837 (0.399)	0.521 (0.771)
Constant return to scale (Wald test)	NO	YES	YES
Sample No. of observations	1994q1-2008q1 55		

U - number of unemployed workers, V - number of vacant jobs, O - outflow from unemployment to employment; $d1996$ - dummy variable with 1 in 2nd quarter 1996; $d2002$ - dummy variable with 1 in 1st quarter 1996; $MIN\ WAGE$ - a ratio of minimum wage to average wage in Polish economy; ZAS - a share of unemployed entitled to benefit in unemployment stock, INT - a number of interventional workers in relation to total unemployment; $PUBL$ - a number of public workers in relation to total unemployment; $Q1$, $Q3$, $Q4$ - seasonal dummies with 1 in 1st, 3rd and 4th quarter respectively; T - a time trend; AIC - Akaike Information Criterion; S.C. - Schwartz Information Criterion; DW - Durbin-Watson statistic; t-Student statistics in parentheses.

Table 2: Estimation results: equation (9)

Explanatory variable	Dependent variable $\log(O)$			
Constant	-3.138 (-4.313)	-1.473 (-2.583)	-2.900 (-4.268)	-2.481 (-3.564)
$\log(V)$	0.398 (10.450)	0.401 (8.450)	0.313 (5.581)	0.312 (7.460)
$\log(cU)$	0.757 (9.724)	0.736 (9.054)	0.602 (7.450)	0.622 (7.858)
$MIN\ WAGE$	1.950 (2.302)	-	1.963 (2.432)	1.375 (1.681)
ZAS_{-1}	-	-1.081 (-2.452)	-	-
ZAS_{-2}	-	0.579 (3.087)	-	-
INT_{-1}	-	-	13.485 (3.324)	-
INT_{-2}	-	-	-15.836 (-4.598)	-
$PUBL_{-1}$	-	-	-	9.147 (3.923)
$PUBL_{-2}$	-		-	-7.676 (-3.272)
T	-0.007 (-4.350)	-0.005 (-2.188)	-0.006 (-4.220)	-0.005 (-3.995)
$\log(O_{-1})$	0.175 (2.505)	-	0.400 (4.690)	0.330 (4.628)
$d2002$	0.298 (3.747)	0.238 (2.848)	0.313 (4.671)	0.331 (4.841)
$d1996$	0.382 (4.238)	0.383 (3.980)	0.190 (2.112)	0.286 (3.400)
R^2	0.862	0.847	0.909	0.905
Adj. R^2	0.841	0.824	0.890	0.886
AIC	-2.177	-2.073	-2.504	-2.464
S.C.	-1.883	-1.779	-2.132	-2.092
DW	1.936	1.992	2.177	1.988
F	41.153	36.439	47.820	45.750
Jarque-Bera (Probability)	1.416 (0.492)	0.831 (0.660)	0.473 (0.790)	0.070 (0.966)
Constant return to scale (Wald test)	YES	YES	YES	YES
Sample No. of observations	1994q1-2008q1 54			

Explanations as to table 1.

Table 3: Estimation results: equation (10)

Explanatory variable	Dependent variable		
	$\log\left(\frac{O}{U}\right)$		
Constant	-1.165 (-7.093)	-2.050 (-9.293)	-2.808 (-7.563)
$\log(c)$	0.106 (1.781)	0.154 (2.115)	0.207 (2.855)
$\log\left(\frac{V}{U}\right)$	0.429 (19.869)	0.527 (13.495)	0.477 (11.285)
$MIN\ WAGE$	0.743 (1.590)	2.355 (4.335)	3.577 (5.008)
ZAS_{-1}	-0.815 (-3.248)	-1.231 (-3.483)	-0.913 (-2.540)
ZAS_{-2}	0.913 (3.525)	1.221 (3.351)	1.203 (3.478)
T	-	-	0.004 (2.471)
$\log\left(\frac{O_{-1}}{U_{-1}}\right)$	-	-0.241 (-3.965)	-0.242 (-4.196)
$Q1$	-0.155 (-8.305)	-	-
$Q3$	-0.093 (-5.094)	-	-
$Q4$	-0.092 (-4.660)	-	-
$d2002$	0.178 (4.024)	0.234 (3.915)	0.234 (4.139)
R^2	0.972	0.943	0.950
Adj. R^2	0.966	0.935	0.941
AIC	-3.408	-2.791	-2.882
S.C.	-3.040	-2.497	-2.550
DW	1.872	2.196	2.264
F	167.197	109.365	107.079
Jarque-Bera (Probability)	2.906 (0.233)	0.388 (0.824)	1.471 (0.479)
Sample No. of observations	1994q1-2008q1 54		

Explanations as to table 1.