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Address
80 Kennedy Avenue
CY-1076 Nicosia, Cyprus

Postal Address
P. O. Box 25529
CY-1395 Nicosia, Cyprus

E-mail
publications@centralbank.gov.cy

Website
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Nonlinearities in the Real Exchange Rate and Monetary Policy: Interest Rate Rules Reconsidered

Konstantinos D. Mavromatis*

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Abstract

Empirical research during the last ten years has found significant evidence in favour of a nonlinear-threshold type behaviour of the real exchange rate. Interest rate rules which include the exchange rate appear to have either an insignificant effect on or generate small coefficients for the real exchange rate. However, the empirical studies do not take into account the nonlinear behaviour of the exchange rate. The inclusion of nonlinearities in the real exchange rate could imply nonlinear behaviour in the interest rate rule, whenever the exchange rate is included. We use a two-country sticky price model to show that nonlinear Taylor-type rules where the exchange rate is included lead to lower variation in output and inflation.

Keywords: Taylor rules, real exchange rate, nonlinearities.
JEL Classification: E52, F41, F42.

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Correspondence: Konstantinos D. Mavromatis, Department of Economics, University of Warwick, CV4 7AL, Coventry, UK. E-mail: k.mavromatis@warwick.ac.uk

1 Introduction

Recent work on monetary economics has focused on the modeling of monetary policy in models of imperfect competition and nominal rigidities. The virtue of such models is that they provide a better insight into the evaluation of alternative monetary policies. Nominal rigidities and market power allow for real effects. Alternative policies concern specifications about the way the Central Bank conducts monetary policy. Moreover, recent work has focused on the evaluation of monetary policy for open economies (Gali and Monacelli, 2005; Benigno, 2004; Monacelli, 2003; Svensson, 2000; Ball, 1998).

Research on monetary policy, however, has focused more on models with closed economies, or on interest rate rules where the target variables are the inflation rate and the output gap (Taylor type rules). This approach relies on the fact that the real or nominal exchange rate need not be included in the rule. One reason is that the exchange rate effect exists indirectly. The exchange rate affects the other two target variables, anyway, through its pass-through effect (Taylor, 1999; Ball, 1998). Another reason is that data do not support its existence in the rule (Clarida et al., 1998)[1]. On the other hand, others tend to argue for the importance of including the exchange rate in a feedback rule[2] (Svensson, 2000).

Exchange rate behavior has been the focus of much research since the early '90s. Rogoff (1996) originally inaugurated a new kind of approach regarding the short run and the long run dynamics of the exchange rate. The 'PPP-Puzzle' put into question the standard linear time series techniques as a way of estimating the horizons needed in order for the exchange rate to mean revert[3]. Simple AR models appeared unable to capture the behavior observed. Additionally, standard linear time series tests could not reject the null of no stationarity, implying that the real exchange rate is a random walk and, thus, invalidating long-run purchasing power parity[4]. Half life estimates appeared to be incorrect as well[5].

The existence of transaction costs in the interantional trade of goods affects the trade volume and hence the behavior of the exchange rate. When transaction costs are high, international trade is less profitable. Consequently, deviations of the real exchange from PPP will be corrected very slowly. On the other hand when transaction costs are low, international trade is profitable and the real exchange rate will inherit a mean reverting property. Therefore, the existence of such costs

[1] Clarida et al. found either very small or statistically insignificant coefficients for the exchange rate in a forward looiking interest rate rule.

[2] Svensson also argues that apart from the exchange rate, foreign fundamental variables appear to be important in the feedback rule.

[3] The 'PPP-Puzzle' states the following: how can one reconcile long-run mean reversion with short-run high volatility.

[4] For a detailed analysis on exchange rate behavior see Coakley, Flood, Fuertes and Taylor (2005), MacDonald and Taylor (1994) and the references therein.

[5] Half life is the number of periods it takes a shock to dissipate by a half.

in international trade imply a threshold (nonlinear) behavior for the real exchange rate[6].

In this paper, we argue that the threshold type behavior of the exchange rate must be taken into account when the latter is included into an interest rate rule. We argue that the nominal interest rate need not react to exchange rate movements when transaction costs are high and the volume of international trade low. The nominal interest rate reacts to the exchange rate movements only when the latter's deviation from PPP is high or when transaction costs are low[7]. If the real exchange rate enters in a linear fashion in the interest rate rule, then inference may not be valid due to model misspecification. Consequently, policy implications will be wrong.

The paper is organized as follows. In section 2 we use data for major economies against the US to rpovide support for the nonlinearities in the interest rate when the real exchange rate is introduced. In section 3 we develop a DSGE two country model with transaction costs to show that the existence of such costs entails a threshold type behavior for the real exchange rate. In section 4 we present the log linearized version of the model. In section 5 we introduce monetary policy by examining how the system behaves under alternative interest rate rules. In section 6 we present the conclusions and policy implications.

2 Empirical evidence and motivation

In this section we use real time series data to estimate a parsimonious VAR model similar to that estimated in Rudebusch (2002). The use of unrestricted VAR models in examining monetary policy has been criticized. However, they can constitute a simple benchmark for the dynamics of a structural model[8]. Rudebusch estimates a near VAR model of three equations, one for output, one for the inflation rate and one for the interest rate. We extend this model by introducing the exchange rate in the VAR. The exchange rate is allowed to have both contemporaneous and lagged effects in the system. In particular, we allow the exchange rate to affect the inflation rate both contemporaneously and with a lag. In the four variable VAR

[6]From a theoretical point of view, modelling a behavior like that described in the 'PPP-Puzzle' is nontrivial. Nominal regidities may not be enough to generate persistence in the real exchange rate. Persistence could be generated by the degree of correlation in monetary shocks as in Chari et al. (2000) and Benigno (2004). However, this finding could be weak in case of a low degree of autocorrelation. Additionally, this approach tries to epxlain the persistence in real exchange rate relying on assumptions concerning exogenous variables, without endogenizing it. Gali and Monacelli (2004), in an attempt to model volatility in the real exchange rate, find that the former is determined by the degree of correlation between productivity and world output. A high positive (negative) correlation between domestic productivity and world ouptut will tend to decrease (increase) the volatility of the nominal and real exchange rates.

[7]In other words we argue that the interest rate should react to exchange rate movement only when international trade is profitable.

[8]For a more detailed analysis on the weaknesses and the criticism on unrestricted VARs in monetary policy analysis see Rudebusch (1998) and the references therein.

four lags of the inflation rate, output and the nominal interest rate are introduced. Specifically, the VAR interest rate equation regresses the nominal interest rate i_t on four lags of each variable and on the contemporaneous values of the inflation and the output gap. The inflation equation regresses the CPI rate π_t on four lags of each variable as well as the contemporaneous value of the output gap and the exchange rate. The output gap[9] equation regresses the output gap y_t on four lags of all the variables. Finally, the exchange rate equation regresses the exchange rate q_t on four lags of each variable as well as the contemporaneous value of the nominal interest rate. We allowed for the latter due to the fact that interest rate movements are likely to affect in the short-run the spot rate, and, thus, due to nominal rigidities, the real one. The near VAR representation is defined as

$$\Delta X_t = A_1 \Delta X_{t-1} + A_2 \Delta X_{t-2} + A_3 \Delta X_{t-3} + A_4 \Delta X_{t-4} + + I\Omega_t$$

where A_i, $i = 1, 2, 3, 4$ are 4×4 matrices specified in Appendix 1 and $X_t = (i_t, q_t, y_t, \pi_t)'$. I is a 4×4 identity matrix andΩ_t is a 4×1 matrix of *i.i.d.* errors.

2.1 Data

We assume a two country model. US is assumed to be the foreign country. Japan, Germany, France and the United Kingdom are assumed to be the home country in each case.

Quarterly data were gathered from the IMF International Financial Statistics for the CPI of each country, the end of period spot exchange rate of the Japanese Yen, the German Mark, the French franc and the UK pound against the US dollar respectively. The financing bill rate for Japan, the Treasury Bill rate for France, Germany, the UK and the US were used as proxies for the nominal interest rate. Data for Japan and the United Kingdom cover the period from Q1 1970 through Q2 2009, whereas data for Germany and France cover the period from Q1 1970 though Q4 1998.

2.2 Linearity tests

As a next step, we performed system and equation specific linearity tests. The testing procedure is the same as that described in Van Dijk (1999)[10]. The linearity

[9]The output gap was proxied using the hp filter. The latter's accuracy in capturing the actual output gap has been criticized. One reason is that the natural rate of output is proxied by a deterministic trend. However, the former may be a function of technology, monetary and demand shocks, and thus, more volatile. For a more detailed survey on the criticism on the output gap measures see Gali (2002), Gali and Gertler (1999), Sbordone (1999), Gertler, Gali and Lopez-Salido (2000) and the references therein.

[10]For a more detailed description of the testing procedure for both univariate and multivariate models see Terasvirta (1994), Granger and Terasvirta (1993) and Van Dijk (1999) and the references therein.

tests were performed by estimating an auxiliary regression that is consisted by a linear and a nonlinear part. The linear part is the same as that described by the VAR representation above. Assuming that the real exchange rate is the transition variable the auxiliary model[11] is specified as

$$\Delta X_t = \sum_{i=1}^{4} \Gamma_i \Delta X_{t-i} q_{t-1}^{i-1} + I\Omega_t$$

where q_t is the real exchange rate[12].

The system linearity test[13] tests the null of $\Gamma_2 = \Gamma_3 = \Gamma_4 = 0$, that is, all elements of the three matrices are equal to zero. However, for the sake of power and, hence, better inference we also performed equation specific linearity tests[14]. In all cases the system linearity tests reject the null of linearity. The equation specific linearity tests reject linearity in the interest rate equation in each country. Additionally, linearity is rejected in the inflation equation, and, unsurprisingly, in the real exchange rate equation. The interesting result is that in none of the cases could linearity be rejected in the output gap equation. The results of the tests are presented at Tables 1 and 2 below.

Table 1: Linearity tests (threshold variable: q_{t-1})

	France	Germany	Japan	UK
Δi_t	0.000	0.000	0.000	0.000
Δq_t	0.000	0.000	0.030	0.001
Δy_t	0.620	0.500	0.083	0.183
$\Delta \pi_t$	0.000	0.245	0.097	0.001

Notes: $P - values$ from equation specific tests reported.

[11] The auxiliary regressions correspond to a smooth transition autoregressive model with a zero threshold, that is a two regime model. The adjustment can be either symmetric or asymmetric. The testing procedure, though, is robust to both modes of adjustment.

[12] The real exchange rate is lagged one period in the auxiliary regression. This is the delay parameter which, in our case, implies that it takes the real exchange rate one period (e.g. quarter) to switch from one regime to the other.

[13] System linearity tests were performed as $F - tests$. They could have also been performed as $LM - tests$. However, Terasvirta (1994) shows that the $F - version$ of the test is better sized, especially when the sample is small and the number of restrictions large.

[14] System linearity tests are expected to have greater power as it is enough that in only one of th\sum_{i=1}^{4}e four equations the null of linearity is rejected, so that to reject linearity in the system. Consequently, relying only on the system linearity tests, one cannot derive secure inference about which equations in the system have nonlinear terms.

Table 2: Linearity tests (threshold variable: π_{t-1})

	France	Germany	Japan	UK
Δi_t	0.000	0.000	0.000	0.000
Δq_t	0.000	0.000	0.004	0.064
Δy_t	0.917	0.952	0.181	0.473
$\Delta \pi_t$	0.014	0.627	0.097	0.001

Notes: $P - values$ from equation specific tests reported.

As a next step, a three variate VAR as in Rudebusch (2002) was estimated without the real exchange rate. Linearity tests were performed. At first, we used the real exchange rate as the threshold variable. Only for the UK could we reject the null of linearity in the interest rate equation. The results are shown at table 3 below.

Table 3: Linearity tests (threshold variable: q_{t-1})

	France	Germany	Japan	UK
Δi_t	0.821	0.204	0.062	0.000
Δy_t	0.578	0.399	0.412	0.109
$\Delta \pi_t$	0.437	0.073	0.091	0.000

Notes: $P - values$ from equation specific tests reported.

The null of linearity is not rejected at 5% significance level for France, Germany and Japan. However, linearity is rejected in the interest rate and the inflation equation for the UK in the trivariate system as well. As a second step, the above procedure was repeated using the inflation rate as the threshold variable[15]. The results are shown at table 4 below.

Table 4: Linearity tests (threshold variable: π_{t-1})

	France	Germany	Japan	UK
Δi_t	0.861	0.000	0.471	0.000
Δy_t	0.384	0.615	0.271	0.293
$\Delta \pi_t$	0.655	0.561	0.000	0.000

Notes: $P - values$ from equation specific tests reported.

When the inflation rate is the threshold variable, linearity is rejected for the German and the UK treasury bill rate equation. It is not rejected for France and Japan. Linearity in the interest rate equation is rejected in all four cases in the four variable VAR, independently of what the threshold variable is. On the other hand, in the trivariate VAR, linearity is rejected only for the UK interest rate equation when the threshold variable is the real exchange rate. When the inflation rate is the threshold variable, linearity in the interest rate equation is rejected for the UK and Germany in the trivariate VAR. Finally, in the four variable VAR

[15]In this case we assume that it takes the inflation rate one quarter to start its adjustment, implying a delay parameter equal to one.

linearity in the exchange rate equation is always rejected. Therefore, non linearity in the interest rate is always rejected whenever the real exchange rate is introduced in the system. For France and Japan, the data show that non linearity in the interest rate setting is caused by the real exchange rate exclusively. The same holds for Germany when the real exchange rate rate is the threshold variable in the trivariate system. However, the data show that interest rate setting is always nonlinear (or of a threshold type) for the UK. Consequently, they provide evidence in favor of a threshold type interest rate rule whenever the real exchange rate is introduced as a target variable.

3 Structure of the Model

A stochastic model is specified as in Benigno (2004), Obstfeld and Rogoff (1998, 1999). Prices adjust in a sticky way as in Calvo (1983). Each country exports and imports goods. There are shipping costs (iceberg type) in transporting goods from one country to the other. Transaction costs are modelled as in Dumas (1992), Sercu, Van Hulle and Uppal (1995) and Coeurdacier (2006)[16]. As a result this implies that trade will not always take place. Only when the price of the exported good is such that makes trade profitable, will each country be involved in international trade. When the real exchange rate lies inside certain bands then trade is not profitable and, hence, each economy will consume only domestically produced goods. Hence, the degree of openness of an economy depends on where the real exchange rate lies (i.e. within or outside the thresholds that determine international trade).

Monetary policy is conducted by the Central Bank which uses the short term nominal interest rate as its instrument. In the present model, the Central Bank must take into account whether the home country is involved in international trade or not. The threshold behavior of the real exchange rate, implies a threshold behavior for the instrument, once the former is introduced into the rule. Consequently, the interest rate rule will be regime dependent.

3.1 Households

There are two countries where a continuum of goods is produced in each and agents have identical preferences. Home country residents constitute the interval [0, n], while foreign residents (n, 1]. The population size is set equal to the number of the goods produced. Hence, home country produces goods on the interval [0, n], while foreign country produces goods on (n, 1]. Home agents and goods are indexed by

[16] Coeurdacier introduces transaction costs in the price aggregator assuming that the price of the imported good will be $(1+\tau)p_j$. We follow the same approach.

h, while foreign agents and goods by f. Home agent's consumption at date t is denoted by C_t^h, real money holdings by $\frac{M_t^h}{P_t}$ and labor supply by L_t^h. Home agent maximizes her separable utility function which is given as follows:

$$U_t^h = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{(C_s^h)^{1-\sigma}}{1-\sigma} + \chi \log \frac{M_s^h}{P_S} - \frac{L_s^{1+\gamma}}{1+\gamma} \right] \tag{1}$$

where σ is the degree of relative risk aversion. C_t^h is a composite consumption index described as

$$C_t^h = \left[\delta^{\frac{1}{\rho}} C_{H,t}^{\frac{\rho-1}{\rho}} + (1-\delta)^{\frac{1}{\rho}} C_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \qquad \rho > 0 \tag{2}$$

where ρ captures the intratemporal elasticity of substitution between home and foreign goods. $\delta \geq \frac{1}{2}$ is a parameter of home bias in preferences.C_H is the home consumption index. C_F is the foreign consumption index. Both indices are defined as

$$C_H = \left[\left(\frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n c(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \; C_F = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 c(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

Money deflator is given by the aggregate consumption price index, specified as

$$P_t = \left[\delta P_{H,t}^{1-\rho} + (1-\delta) \left[(1+\tau_t) P_{F,t} \right]^{1-\rho} \right]^{\frac{1}{1-\rho}} \tag{4}$$

where P_H and P_F prices indices for home goods and foreign goods, expressed in the domestic currency and τ_t captures the time varying transaction cost assumed to follow an $AR(1)$, $\tau_t = \varphi_\tau \tau_{t-1} + \nu_t$, $\nu_t \backsim N(0, \sigma^2)$. The price indices are defined as

$$P_H = \left[\left(\frac{1}{n} \right) \int_0^n p(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \; P_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{5}$$

Agents in both economies hold home country's one period bond. Home resident's bond holdings at date t are denoted by B_t. Foreign resident's bond holdings are denoted by $\frac{B_t}{\varepsilon_t}$, where ε_t denotes the nominal exchange rate, defined as the domestic price of the foreign currency. The home agent maximizes her utility subject to the period budget constraint

$$P_t C_t + M_t^h + B_t = (1 + i_t)B_t + W_t + M_{t-1} + S_t \tag{6}$$

where W_t is the nominal wage, S_t are nominal transfers the individual receives from the government and i_t is the nominal interest rate.

Government's balanced budget requires the following

$$\int_0^n (M_t - M_{t-1})\, di = \int_0^n S_t di \tag{7}$$

3.2 First order conditions

Maximizing the utility function (1) subject to the budget constraint (6) yields the following first order conditions

$$E_t C_{t+1} = \left[\beta(1 + i_{t+1}) \frac{P_t}{E_t P_{t+1}}\right]^{\frac{1}{\sigma}} C_t \tag{8}$$

$$L_t = C_t^{-\frac{\sigma}{\gamma}} w_t^{\frac{1}{\gamma}} \tag{9}$$

$$\frac{M_t}{P_t} = \chi C_t^{\sigma} \left(\frac{1 + i_{t+1}}{i_{t+1}}\right) \tag{10}$$

where the first equation is the usual Euler equation, the second determines the labor supply schedule and the third the demand for real money balances.

Individual demands for each good z produced in the home and in the foreign country respectively are expressed as

$$c_{h,t}(z) = \left(\frac{p_t^h(z)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \delta C_t \tag{11}$$

$$c_{f,t}(z) = \left(\frac{p_t^f(z)}{P_{F,t}}\right)^{-\theta} \left(\frac{(1+\tau_t)P_{F,t}}{P_t}\right)^{-\rho} (1-\delta)C_t \tag{12}$$

3.3 Real exchange rate and transaction costs

The law of one price does not hold continuously in the model. If transaction costs in international goods markets were ignored, then international goods trade would eliminate any deviation from the law implying, thus, a mean reverting behavior of the real exchange rate. In this model, however, transaction costs imply that goods will not always be traded internationally. Only when the total costs of shipping the good are such that profitable opportunities arise, will the international trade volume be high. Otherwise, each country will consume domestically produced goods more. In other words, the presence of transaction costs generate an area where international trade volume is low. In the absence of transaction costs, international trade will occur until price differentials are decreased in such a level where no profitable opportunities will exist any longer.

The assumption of complete markets and identical preferences implies that the real exchange rate will be given by the ratio of the marginal utilities of consumption of the foreign and home residents[17]. Given agents in both countries hold only Home country's bonds, foreign agent's budget constraint will be given by

$$P_t^* C_t^* + M_t^* + \frac{B_{t+1}}{\varepsilon_t} = (1+i_t)\frac{B_t}{\varepsilon_t} + W_t^* + M_{t-1}^* + S_t^* \tag{13}$$

Therefore, the Euler equation from the foreign agent's maximization problem is

[17] Moreover, we have assumed that only traded goods exist. The introduction of non traded goods would not change the results presented here, as we focus on developed countries. In these countries the percentage of the total volatility of the real exchange rate that is explained by variations in the prices of nontradables is very small anyway (Berka, 2005).

$$C_t^* = \left[\beta(1+i_t)\frac{\varepsilon_0}{\varepsilon_t}\frac{P_0^*}{E_t P_t^*}\right]^{\frac{1}{\sigma}} C_0^* \tag{14}$$

substituting for the degree of patience $\beta(1+i_t)$ from (8), I receive the following expression

$$\left(\frac{C_t^f}{C_t^h}\right)^{-\sigma} = \varpi q_t \tag{15}$$

where $\varpi \equiv \left(\frac{C_0^f}{C_0^h}\right)^{-\sigma} \frac{P_0}{\varepsilon_0 P_0^*}$ depends on initial conditions and $q_t = \frac{\varepsilon_t P_t^*}{P_t}$ is the real exchange rate.

The ratio of foreign to home marginal utility of consumption is found to be bounded below and above by thresholds whose value is determined by the shares of consumption, the transaction cost and the price levels:

$$\left(\frac{P_{F,t}^*}{(1+\tau_t)P_{F,t}}\right)^{\frac{\rho\sigma}{\rho\sigma+1}} \left(\frac{(1-\delta)\,C_{F,t}^*}{\delta C_{F,t}}\right)^{\frac{\sigma}{\rho\sigma+1}} \leq \frac{\partial U/\partial C}{\partial U^*/\partial C^*} \leq \left(\frac{(1+\tau_t)P_{H,t}^*}{P_{H,t}}\right)^{\frac{\rho\sigma}{\rho\sigma+1}} \left(\frac{\delta C_{H,t}^*}{(1-\delta)\,C_{H,t}}\right)^{\frac{\sigma}{\rho\sigma+1}} \tag{16}$$

by (15) the bands for the real exchange rate are determined as

$$\left(\frac{P_{F,t}^*}{(1+\tau_t)P_{F,t}}\right)^{\frac{\rho\sigma}{\rho\sigma+1}} \left(\frac{(1-\delta)\,C_{F,t}^*}{\delta C_{F,t}}\right)^{\frac{\sigma}{\rho\sigma+1}} \leq \varpi q_t \leq \left(\frac{(1+\tau_t)P_{H,t}^*}{P_{H,t}}\right)^{\frac{\rho\sigma}{\rho\sigma+1}} \left(\frac{\delta C_{H,t}^*}{(1-\delta)\,C_{H,t}}\right)^{\frac{\sigma}{\rho\sigma+1}} \tag{17}$$

where P_t^* and P_t are the price levels of the foreign and home country.

Once the real exchange rate touches the upper threshold then home country increases its exports, whereas once it touches the lower threshold the home country increases its imports of foreign goods. Therefore, the behavior of the real exchange rate when international trade takes place is summarized as

$$q_t = \begin{cases} \frac{1}{\varpi}\left[\left(\frac{(1+\tau_t)P_{H,t}^*}{P_{H,t}}\right)^{\frac{\rho\sigma}{\rho\sigma+1}} \left(\frac{\delta C_{H,t}^*}{(1-\delta)C_{H,t}}\right)^{\frac{\sigma}{\rho\sigma+1}}\right] \\ \\ \frac{1}{\varpi}\left[\left(\frac{P_{F,t}^*}{(1+\tau_t)P_{F,t}}\right)^{\frac{\rho\sigma}{\rho\sigma+1}} \left(\frac{(1-\delta)C_{F,t}^*}{\delta C_{F,t}}\right)^{\frac{\sigma}{\rho\sigma+1}}\right] \end{cases} \tag{18}$$

The thresholds will be symmetric as in Sercu, Van Hulle and Uppal (1995), if equal shares are assumed and if relative prices of goods consumed in the foregn country are equal to one. When the real exchange rate lies within the above thresholds, then the volume of trade will be low due to high transaction costs. Since transaction costs are high in the middle regime, traders are not interested in trading internationally due to the absence of profitable opportunities. Additionally, the Central Bank is not interested in intervening in the face of real exchange rate movements. Therefore, the exchange rate will behave either as a random walk, or as an autoregressive process with a high degree of persistence. The larger the deviations from the law of one price, the higher the profits from international trade, and the larger the volume of trade. Consequently, the speed of mean reversion of the real exchange rate will be higher, the farther away it is from the thresholds. The speed of mean reversion will be decreased as it moves closer to the bands.

3.4 Price setting

Prices are sticky with a price setting behavior *a ℓa* Calvo (1983). At each date, each firm changes its price with a probability $1-\omega$, regardless of the time since it last adjusted its price. The probability of not changing the price, thus, is ω. The probability of not changing the price in the subsequent s periods is ω^s. Consequently, the price decision at time t determines profits for the next s periods. The price level for home goods at date t will be defined as

$$P_{H,t} = \left[\omega P_{H,t-1}^{1-\theta} + (1-\omega)\widetilde{p_t}(h)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{19}$$

Dividing by $P_{H,t-1}$:

$$\Pi_{H,t}^{1-\theta} = \omega + (1-\omega)\left(\frac{\widetilde{p_t}(h)}{P_{H,t-1}}\right)^{1-\theta} \tag{20}$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$.

Similarly, for the foreign goods consumed in the home economy:

$$\Pi_{F,t}^{1-\theta} = \omega + (1-\omega)\left(\frac{\widetilde{p_t}(f)}{P_{F,t-1}}\right)^{1-\theta} \tag{21}$$

The aggregate price level dynamics are specified, thus, as

$$\Pi_t^{1-\rho} = \delta\left[\left(\frac{P_{H,t-1}}{P_{t-1}}\right)\Pi_{H,t}\right]^{1-\rho} + (1-\delta)\left[(1+\tau_t)\left(\frac{P_{F,t-1}}{P_{t-1}}\right)\Pi_{F,t}\right]^{1-\rho} \tag{22}$$

Firm's maximization problem comprises of two decisions. The one concerns the price for the domestic market and the other the price charged in the foreign market, when it exports.

A continuum of firms is assumed for home economy indexed by $z \in [0, n]$. Each firm produces a differentiated good, with a technology

$$Y_t(z) = A_t L_t(z) \tag{23}$$

where A_t is a country specific productivity shock at date t which is assumed to follow a log-difference stationary process $\Delta\alpha_t = \rho_{\alpha_t}\alpha_{t-1} + \upsilon_t$, where υ_t is an $i.i.d.$ process.

Each firm chooses a price for the home economy and a price for the foreign economy, in order to maximize the expected discounted value of its profits

$$maxE_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s}\left\{\widetilde{p_t}(h)y_{t+s}^h(h) + \varepsilon_t\widetilde{p_t}(h)^* y_{t+s}^f(h) - W_{t+s}^h L_{t+s}^h\right\} \tag{24}$$

where $y_t^i(h)$, $i = h, f$ is the demand for the home good for home and foreign agents.

When no trade takes place, the firm chooses one price, that for the home economy only and its objective collapses to the following maximization problem

$$maxE_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s}\left\{p_t(h)y_{t+s}^h(h) - W_{t+s}^h L_{t+s}^h\right\}$$

The demand for the Home good from the Home and Foreign country is specified as

$$y_t^h(p_t(h)) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \delta C_t, \tag{25}$$

$$y_t^f(p_t^*(h)) = \left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\theta} \left(\frac{(1+\tau_t)P_{H,t}^*}{P_t^*}\right)^{-\rho} (1-\delta)C_t^* \tag{26}$$

The firm maximizes its objective function (24) subject to (25) in order to find the optimal price for the Home good in the Home economy. It maximizes subject to (26), in order to find the optimal price for the Home good in the Foreign economy. The firm chooses a price for the Home good in the Home economy that satisfies the first order condition

$$E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}(p_t(h)) \left\{ p_t(h) - \frac{\theta}{\theta-1} MC_{t+s} \right\} = 0$$

where $MC_{t+s} = \frac{W_{t+s}}{A_{t+s}}$ denotes the nominal marginal cost and $\frac{\theta}{\theta-1}$ captures the optimal markup. The optimal price, thus, for the Home good in the Home economy is specified as

$$p_t(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^h(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^h(p_t(h))} \tag{27}$$

Respectively, the optimal price for the Home good in the Foreign country is specified as

$$p_t^*(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^f(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^f(p_t(h)) \varepsilon_{t+s}} \tag{28}$$

4 Log linearized model

A log linearized version of the relationships found in the previous section serves in providing us with a way to deal with the problem of no closed form solution. Additionally, this is a way to end up in a state space form which can be estimated using real time series data.

4.1 Supply side

We use a first order Taylor approximation around the steady state of zero inflation rate. Log linearized variables are denoted with a hat.

After loglinearizing the first order condition (9), the price level equations (21) and (22), the production function (23) the demand schedules faced by each firm (25) and (26) and optimal price setting rules (27) and (28), we receive the two relations describing the domestically consumed home goods inflation rate and the respective of the home goods consumed in the Foreign country as in Benigno (2004)

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + b_{\pi_H}\pi_{H,t} + b_{\pi_H^*}\pi_{H,t}^* + b_C\hat{C}_t + b_T\hat{T}_t + b_{T^*}\hat{T}_t^* + b_q\hat{q}_t + b_\tau\hat{\tau}_t + \varepsilon_{H,t} \quad (29)$$

$$\pi_{H,t}^* = \beta E_t \pi_{H,t+1}^* + b_{\pi_H}^*\pi_{H,t} + b_{\pi_H^*}^*\pi_{H,t}^* + b_C^*\hat{C}_t + b_T^*\hat{T}_t + b_{T^*}^*\hat{T}_t^* + b_q^*\hat{q}_t + b_\tau\hat{\tau}_t + \varepsilon_{H,t}^* \quad (30)$$

where $\varepsilon_{H,t}$ and $\varepsilon_{H,t}^*$ are *i.i.d.* cost push shocks. $T_t = \frac{P_{F,t}}{P_{H,t}}$ and $T_t^* = \frac{P_{H,t}^*}{P_{F,t}^*}$ captures the terms of trade for the Home and Foreign country respectively.

The log linearized aggregate price level relation (22) is specified as

$$\pi_t = \pi_{H,t} + (1-\delta)(\pi_{F,t} - \pi_{H,t} + \hat{\tau}_t) \quad (31)$$

Note that equations (29) and (30) describe the home goods inflation rate in the Home and the Foreign country when the exchange rate is outside the middle regime given in (18), that is, when the Home country is exporting or importing. In this case the aggregate inflation rate is given by (31).

The Home country inflation dynamics[18] are defined as in (31) which is analyzed further as

$$\pi_t = \beta E_t \pi_{t+1} + \alpha_1\pi_t + \alpha_2\pi_t^* + \alpha_3\hat{C}_t + \alpha_4\hat{T}_t + \alpha_5\hat{T}_t^* + \alpha_6\hat{q}_t + \alpha_7\hat{\tau}_t + \eta_t \quad (32)$$

where η_t is consisted by two *i.i.d.* terms.

[18]The aggregate inflation dynamics are specified here under the assumption that firms face the same degree of price stickiness within the same country.

4.2 Demand side

In this section we proceed to the loglinearization of the Euler equation

$$C_t = \kappa(i_t - E_t\pi_{t+1}) + E_tC_{t+1} \tag{33}$$

where $\kappa = -\frac{1}{\sigma}$.

Goods market clearing assumes the following two conditions

$$Y = Y_H + Y_H^* \quad and \quad Y^* = Y_F + Y_F^*$$

Using the demand schedules as in (25) and (26), and then loglinearizing using the goods market equilibrium conditions, we end up to the following expressions for consumption in the Home country

$$\hat{C}_t = \left(\frac{1}{2\delta - 1}\right)\left(\delta\hat{Y}_t - (1-\delta)\hat{Y}_t^*\right) + \left(\frac{\delta(1-\delta)\rho}{2\alpha - 1}\right)\left(\hat{T}_t^* - \hat{T}_t\right) - (1-\rho)(1-\delta)\hat{\tau}_t - \varepsilon_{td} + \left(\frac{1-\delta}{\delta}\right)\varepsilon_{td}^* \tag{34}$$

Therefore, combining equations (33) and (34), we derive the aggregate demand equation:

$$\hat{Y}_t = E_t\hat{Y}_{t+1} + \lambda(i_t - E_t\pi_{t+1}) - \left(\tfrac{1-\delta}{\delta}\right)\left(E_t\Delta\hat{Y}_{t+1}^*\right) + (1-\delta)\rho\left(E_t\Delta\hat{T}_{t+1}^* - E_t\Delta\hat{T}_{t+1}\right) -$$

$$\left(\frac{(1-\rho)(1-\delta)(2\delta-1)}{\delta}\right)E_t\Delta\hat{\tau}_{t+1} + (1-\rho_\varepsilon)\varepsilon_{td} + \left(\frac{1-\delta}{\delta}\right)(\rho_{\varepsilon^*} - 1)\varepsilon_{td}^* \tag{35}$$

where $\lambda = -\frac{(2\delta-1)}{\delta\sigma}$.

4.3 Real exchange rate behavior

As already mentioned the real exchange rate exhibits regime switching behavior depending on whether trade takes place or not. The larger the deviation from the bands (or absolute PPP in case of a two regime model), the higher the volume of trade, and, thus, the faster the real exchange rate reverts back to the thresholds. When no trade occurs, the real exchange rate depends highly on its lagged values. Additionally, under the assumption of identical preferences, frictionless financial markets and the even stronger assumption of the same degree of price stickiness across the two countries, the real exchange rate behaves as a random walk within the thresholds. Such a behavior is consistent with empirical literature on exchange rate, where smooth transition autoregressive models seem to solve the so-called 'PPP-Puzzle'[19]. In particular, Taylor, Peel and Sarno (2001), Sarno and Taylor (2001), Taylor and Peel (2000), Lothian and Taylor (1997), Mac Donald and Taylor (1994) and Sarno, Taylor and Chowdhury (2004) using either quarterly or monthly data for major currencies found significant evidence in favor of threshold (or nonlinear) behavior of the real exchange rate. Combining (15), (34) and the respective equations for the foreign country, we derive the equation describing the real exchange rate dynamics

$$\Delta\hat{q}_{t+1} = \left(\tfrac{\sigma\lambda}{2\delta-1}\right)\left(i_t - E_t\pi^*_{t+1}\right) - \left(\tfrac{\sigma\lambda}{2\delta-1}\right)\left(i_t - E_t\pi_{t+1}\right) - \left(\frac{\sigma(1-\delta)}{\alpha(2\delta-1)}\right)\left(E_t\Delta\hat{Y}_{t+1} - E_t\Delta\hat{Y}^*_{t+1}\right) + \left(\frac{2\sigma(1-\delta)^2\rho}{2\delta-1}\right)\left(E_t\Delta\hat{T}_{t+1} - E_t\Delta\hat{T}^*_{t+1}\right) \tag{36}$$

4.4 Terms of trade

The terms of trade determine the competitive advantage of each of the two countries. For the home country the terms of trade variable is defined as $T_t = \frac{P_{F,t}}{P_{H,t}}$, whereas for the foreign country are defined as $T^*_t = \frac{P^*_{H,t}}{P^*_{F,t}}$. We can write the following two expressions for the two terms of trade.

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t}, \quad \hat{T}^*_t = \hat{T}^*_{t-1} + \pi^*_{H,t} - \pi^*_{F,t}$$

Using the log linearized first order condition of the firm's maximization problem, first order condition (9), the consumption equation (34), the output equation (35) and the respective equations for the foreign country, the home country terms of trade is specified as

$$\hat{T}_t = \delta_1\hat{T}_{t-1} + \delta_2 E_t\pi_{t+1} + \delta_3 E_t\pi^*_{t+1} + \delta_4\hat{Y}_t + \delta_5\hat{Y}^*_t + \delta_6\hat{T}^*_t + \delta_7\hat{q}_t + \delta_8\hat{\tau}_t + \delta_9\alpha_t \tag{37}$$

[19] Rogoff (1996) questions the effectiveness of standard linear autoregressive models in capturing the short run high volatility of the exchange rate with a slow long run mean reversion. As a result, half life estimates, using linear models, appeared to be incorrect.

where the parameters $\delta_i,\ i = 1, \ldots, 9$ are defined in the appendix.

4.5 Flexible price equilibrium

At the flexible price equilibrium firms adjust their prices at each period. Each firm will set its marginal cost equal to the optimal marginal cost (i.e. $log\left(\frac{\theta}{\theta-1}\right)$) which is constant over time and equal across firms. Since firms adjust their prices every period, monetary policy will not have any real effects into the economy. The real marginal cost is specified by the following equations

$$mc_t = -log\left(\frac{\theta}{\theta-1}\right) = \mu$$

$$mc_t = w_t - \alpha_t - \nu$$

where w_t is the real wage, α_t (log) productivity and ν a subsidy to labor. Identical prices and demand conditions imply that the quantities produced and consumed will be the same. Solving for the case with flexible prices, we receive the following set of equations describing the equilibrium processes for output, consumption, labor, real interest rate and real exchange rate, given by:

$$\bar{y}_t = \left(\tfrac{\gamma+1}{\gamma\delta+\sigma} + (1-\delta)\left(\tfrac{\sigma\psi_{a^*}}{\gamma\delta+\sigma}\right)\right)\alpha_t - \left(\tfrac{\sigma(1-\delta)\psi_\alpha}{\gamma\delta+\sigma}\right)\alpha_t^* + \left(\tfrac{\sigma}{(\gamma\delta+\sigma)\delta}\left((1-n)\,\psi_\tau - \rho\right)\right)\tau_t +$$

$$\left(\frac{1}{\gamma\delta+\sigma} + \frac{\sigma(1-\delta)}{\gamma\delta+\sigma}\psi_\zeta\right)\zeta + \left(\frac{\sigma}{(\gamma\delta+\sigma)\delta}\left(\frac{1}{\delta} + (1-\delta)\psi_{\varepsilon^*}\right)\right)\varepsilon_{td} - \left(\frac{\sigma(1-\delta)}{\gamma\delta+\sigma}\psi_\varepsilon\right)\varepsilon_{td}^* \tag{38}$$

$$\bar{c}_t = \psi_\zeta\zeta + \psi_\alpha\alpha_t + \psi_{\alpha^*}\alpha_t^* + \psi_\tau\tau_t - \psi_\varepsilon\varepsilon_{td} + \psi_{\varepsilon^*}\varepsilon_{td}^* \tag{39}$$

$$\bar{l}_t = \left[\left(\tfrac{\gamma+1}{\gamma\delta+\sigma} + (1-\delta)\left(\tfrac{\sigma\psi_{a^*}}{\gamma\delta+\sigma}\right)\right) - 1\right]\alpha_t - \left(\tfrac{\sigma(1-\delta)\psi_\alpha}{\gamma\delta+\sigma}\right)\alpha_t^* + \left(\tfrac{\sigma}{(\gamma\delta+\sigma)\delta}\left((1-\delta)\,\psi_\tau - \rho\right)\right)\tau_t +$$

$$\left(\frac{1}{\gamma\delta+\sigma} + \frac{\sigma(1-\delta)}{\gamma\delta+\sigma}\psi_\zeta\right)\zeta + \left(\frac{\sigma}{(\gamma\delta+\sigma)\delta}(1 + (1-\delta)\psi_{\varepsilon^*})\right)\varepsilon_{td} - \left(\frac{\sigma(1-\delta)}{\gamma\delta+\sigma}\psi_\varepsilon\right)\varepsilon_{td}^* \tag{40}$$

$$\bar{r}_t = -\left[\tfrac{(\gamma+1)+(1-\delta)\sigma\psi_a}{\lambda(\gamma\delta+\sigma)} + \left(\tfrac{\sigma(1-\delta)\psi_{a^*}}{\lambda\delta}\right)\right]\rho_a\Delta\alpha_t + \left(\tfrac{(\gamma+1)-\sigma\psi_{a^*}}{\lambda\delta} + \tfrac{\sigma\psi_a}{\lambda}\right)(1-\delta)\rho_{a^*}\Delta\alpha_t^* -$$

$$\left((1-\delta)(2\delta-1)+\tfrac{\sigma(1-\delta)\psi_{\tau}}{(\gamma\delta+\sigma)}+\tfrac{\sigma}{(\gamma\delta+\sigma)}\right)\left(\tfrac{\rho(1-\rho_{\tau})}{n\lambda}\right)\tau_t+\left(\tfrac{\sigma(1-\delta)^2\psi_{\varepsilon}-\sigma(1-\delta)\psi_{\varepsilon^*}+1/\delta}{\lambda(\gamma\delta+\sigma)\delta}-\tfrac{1}{\lambda}\right)(1-\rho_{\varepsilon})\,\varepsilon_{td}+$$

$$\left[\frac{\sigma(1-\delta)}{\lambda\,(\gamma\delta+\sigma)\,\delta}\left(\psi_{\varepsilon}+\frac{1}{\delta}+\frac{(1-\delta)}{\delta}\psi_{\varepsilon^*}\right)-\frac{1-\delta}{\lambda\delta}\right](1-\rho_{\varepsilon^*})\,\varepsilon^*_{td} \tag{41}$$

$$\bar{q}_t=-\sigma\psi_a\left(\alpha^*_t-\alpha_t\right)+\sigma\left(\psi_{\varepsilon}+\psi_{\varepsilon^*}\right)\left(\varepsilon^*_{td}-\varepsilon_{td}\right) \tag{42}$$

where $\psi_a=\frac{(\gamma+1)(\gamma\delta+\sigma)}{\delta(2\gamma\delta(\gamma\delta^2+\sigma)+\sigma^2)}$, $\psi_{a^*}=\frac{(1-\delta)(\gamma+1)\gamma}{(2\gamma\delta(\gamma\delta^2+\sigma)+\sigma^2)}$, $\psi_{\tau}=\frac{(\sigma\rho-\rho\delta(\gamma\delta+\sigma))(\gamma\delta^2+\sigma)}{\delta^2(2\gamma\delta(\gamma\delta^2+\sigma)+\sigma^2)}$, $\psi_{\zeta}=\frac{(\gamma\delta^2+\sigma)}{\delta(2\gamma\delta(\gamma\delta^2+\sigma)+\sigma^2)}$, $\psi_{\varepsilon}=\frac{\gamma(\gamma\delta+\sigma)}{(2\gamma\delta(\gamma\delta^2+\sigma)+\sigma^2)}$, $\psi_{\varepsilon^*}=\frac{\gamma^2\delta(1-\delta)}{2\gamma\delta(\gamma\delta^2+\sigma)+\sigma^2}$, $\zeta=\mu+\nu$.

From the interest rate equation (41) it is evident that the interest rate response to different shocks, and especially to shocks that affect it independently of the regime the real exchange rate lies (e.g. domestic productivity, Home demand shock and transaction costs), changes depending on the volume of international trade. In particular, the interest rate response to domestic productivity shocks is smaller when the real exchange rate lies between the thresholds. The same result holds for the interest rate response to changes in transaction costs. The degree of response, however, to domestic demand shocks is ambiguous. Given $\lambda<0$, the interest rate response to domestic productivity shocks is positive, whereas it is negative with respect to foreign productivity shocks. From (46) one observes that the real exchange rate persistence and volatility depends on the degree of correlation of the demand shocks, the productivity shocks and the cross correlations between the productivity differentials and the demand shocks, the Home productivity shock and the Home demand shock, the Home productivity and the Foreign demand shock, the foreign productivity and the Home demand shock, and lastly, on the correlation between the Foreign productivity shock and the Foreign demand shock. A unit serial correlation between the two demand shocks and the two productivity shocks implies absolute PPP (zero persistence and low volatility). Positive cross correlations between the productivity differential and the demand shock differential implies lower volatility for the real exchange rate. The same holds for the rest of cross correlations among the two kinds of shocks. The volatility of the real exchange rate can be specified as

$$var(\bar{q}_t)=\sigma^2[\psi^2_{\alpha}\left(\sigma^2_{a^*}+\sigma^2_a\right)+\left(\psi_{\varepsilon}+\psi_{\varepsilon^*}\right)^2\left(\sigma^2_{\varepsilon}+\sigma^2_{\varepsilon^*}\right)-2\psi^2_a\varrho_{a^*,a}\sigma_{a^*}\sigma_a-2\left(\psi_{\varepsilon}+\psi_{\varepsilon^*}\right)^2\varrho_{\varepsilon^*,\varepsilon}\sigma_{\varepsilon^*}\sigma_{\varepsilon}-$$

$$2\psi_a\left(\psi_{\varepsilon}+\psi_{\varepsilon^*}\right)\left(\varrho\sigma_{\varepsilon^*-\varepsilon}\sigma_{a^*-a}+\varrho_{\varepsilon^*,a}\sigma_{\varepsilon^*}\sigma_a+\varrho_{\varepsilon,a}\sigma_{\varepsilon}\sigma_a+\varrho_{\varepsilon^*,a^*}\sigma_{\varepsilon^*}\sigma_{a^*}+\varrho_{\varepsilon,a^*}\sigma_{\varepsilon}\sigma_{a^*}\right)$$

where $\varrho_{\varepsilon^*,\varepsilon}=\frac{cov(\varepsilon_{td},\varepsilon^*_{td})}{\sigma_{\varepsilon}\sigma_{\varepsilon^*}}$, $\varrho_{a^*,a}=\frac{cov(a_t,a^*_t)}{\sigma_{a_t}\sigma_{a^*_t}}$, $\varrho_{\varepsilon^*,a}=\frac{cov(a_t,\varepsilon^*_{td})}{\sigma_a\sigma_{\varepsilon^*}}$, $\varrho_{\varepsilon,a}=\frac{cov(a_t,\varepsilon_{td})}{\sigma_a\sigma_{\varepsilon}}$, $\varrho_{\varepsilon^*,a^*}=\frac{cov(a^*_t,\varepsilon^*_{td})}{\sigma_{a^*}\sigma_{\varepsilon^*}}$, $\varrho_{\varepsilon,a^*}=\frac{cov(a^*_t,\varepsilon_{td})}{\sigma_{a^*}\sigma_{\varepsilon}}$ and $\varrho=\frac{cov((a^*_t-a_t),(\varepsilon^*_{td}-\varepsilon_{td}))}{\sigma_{a^*-a}\sigma_{\varepsilon^*-\varepsilon}}$.

Finally, the natural levels of output, consumption, labor, real interest rate and real exchange rate vary not only according to the exogenous processes of the transaction costs, technology and demand shocks, but also according to where the real exchange rate lies (i.e. within or outside the thresholds)[20].

5 Monetary Policy

Monetary policy is conducted through nominal interest rate rules by the Central Bank. The rules considered in this paper are various and serve the main goal. The latter is whether a nonlinear (or a threshold type) interest rate rule is the optimal policy rule, when the real exchange rate is introduced in it. That question relies on the real exchange rate literature that supports the view of threshold type, or more generally, nonlinear behavior of the real exchange rate. Therefore, this raises the question of whether non linearity inherent in the real exchange rate is the source of nonlinearities in the interest rate rule.

Open economy monetary policy literature has rejected the importance of the exchange rate in the interest rate feedback rules, either because it is argued that its effect is already there, indirectly through its pass through on prices and then in inflation (Ball, 1999; Taylor, 1999), or because data do not support its significance[21] (Clarida, Gali and Gertler, 1998). However, a weakness of that literature is that it does not take into account the potential nonlinear behavior of the real exchange rate, or alternatively, the existence of transaction costs either in the goods, or in financial markets. As already shown the existence of transaction costs determines the trade volume internationally, and, thus, the way the real exchange rate behaves. The threshold behavior of the real exchange rate (and the nominal exchange rate due to nominal rigidities) implies a threshold type feedback rule whenever the latter is introduced.

After the introduction of the real exchange rate in the interest rate feedback rules, more generalized ones are considered. The latter allow for foreign fundamentals in the rule[22]. Svensson (2000) considers variants of Taylor type feedback rules. Simulation results exhibit a non-negligible weight of the exchange rate on the interest rate rule. Moreover, foreign fundamentals appear to be an important component in the rule. However, focusing only on the coefficients of the additional variables in the rule is not a sufficient condition for choosing the optimal policy. The focus must, rather, be on the extent to which the overall variation in output or inflation, or both (depending on the objectives of the Central Banker) is altered across the different policy rules.

[20] The thresholds defined in (18) change appropriately in the flexible price case since $P = P^*$. Therefore, they are determined only exogenously by the transaction costs process.

[21] This argument is supported through either very small, or statistically insignificant coefficients, or both.

[22] Taking into account the fundamental equations found in our model, it is evident that optimal control policies allow for foreign fundamentals to be one of the determinants of the nominal interest rate.

5.1 Policy rules

In this section we focus on different policy rules. We characterize optimal the rule that leads to the lowest variation in output and inflation. Each rule leads to a different system of equations and, thus, different conditions that are necessary for determinacy. In each case we provide those conditions. The rules considered will be of a standard Taylor form to more generalized ones.

5.1.1 Forward versus contemporaneous rules

Interest rate rules which rely on current values of both the output gap and inflation have been used widely because of their simplicity and their ability to induce determinate equilibria[23] . However, beyond the simplicity in their use in a system of equations, rules where the interest rate is determined by the current values of the output gap and inflation have been criticized as unrealistic. McCallum (1999) has criticized such rules as unrealistic due to the fact that policymakers lack information about the output gap and inflation the quarter they form their policy. Bullard and Mittra (2002) note that with such a specification of the interest rate rule tension is introduced, because the monetary authority is reacting to time t information on inflation and the output gap. Consequently, the monetary authority has more information than the private sector. The latter forms its expectations at date t using all the information available until date $t-1$. For that reason, forward looking rules will be considered as well[24]. In the cases where expectational variables are considered, we assume that the private sector and the monetary authority have the same information set and form their expectations at the same time. We consider rules where the interest rate is determined by the expectations about the output gap and inflation solely, and rules where the expectation about the real exchange rate is added. Those two rule are specified as

$$i_t = \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1} + \varepsilon_{1t} \tag{43}$$

$$i_t = \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1} + \phi_q E_t q_{t+1} + \varepsilon_{2t} \tag{44}$$

where x_t is an output gap measure and ε_{1t}, ε_{2t} are monetary *i.i.d.* shocks.

[23]The conditions necessary for determinacy using simple Taylor rules in a closed economy framework see Bullard and Mittra (2002), Gali (2002) and the references therein. Additionally for a more detailed analysis of the conditions for stability in a system of difference equations see Blanchard and Khan (1980).

[24]The pitfalls of the interest rate rules using current values of the target variables may jeopardize the credibility of the monetary authority, and, thus, cause an inflation bias in the long-run.

5.1.2 Lagged data rules

An alternative way to deal with the above criticism is to use lagged data rules, instead of froward looking ones[25]. The only difference with the previous case is the conditions necessary determinacy. in this case the policy rules that will be considered are summarized as

$$i_t = \phi_\pi \pi_{t-1} + \phi_x x_{t-1} + \varepsilon_{3t} \tag{45}$$

$$i_t = \phi_\pi \pi_{t-1} + \phi_x x_{t-1} + \phi_q q_{t-1} + \varepsilon_{4t} \tag{46}$$

5.1.3 Inflation targeting rules

The natural rate of output, specified in (42), is determined by Home productivity and demand shocks. Additionally, when the real exchange rate lies on the thresholds (or beyond them), the natural rate of output depends on the Foreign country productivity and demand shocks and the transaction cost as well. That is, the natural rate of output is determined by exogenous processes. Moreover, the central bank or the private sector cannot form expectations or even observe the future and the current behavior of those processes. This causes the problem of a good and reliable measure for the output gap. Using detrended series to approximate the output gap might be risky, since the natural rate of output, and hence the output gap, is likely to be more volatile[26]. Consequently, the problem of measurement error arises, which may lead to the wrong policy responses and, hence, to instability. Gali (2000), McCallum and Nelson (1999) and Rotemberg and Woodford (1999) argue that using detrended output in the interest rate rule may cause inefficiencies, especially when shocks to fundamentals call for large changes in output. The output gap induced by such a policy will, in turn, lead to unnecessary fluctuations in inflation (Gali, 2002)[27]. Therefore, rules where no weight is placed on the output stabilization are considered as well. Orphanides (1999) refers to the advantages of such rules, as they decrease the risks related to large and persistent measurement errors in the output gap.

[25] Bullard and Mittra (2002) argue that such rules are closer to the reality of central bank practice.

[26] The volatility of the natural rate of output has the same sources as the real exchange rate plus the transaction cost and its correlation with the other shocks.

[27] Gali stresses the practical difficulties in implementing such rules associated with the measurement of variables like total factor productivity.

6 Conclusions (preliminary)

We used a DSGE model to show that when transaction costs exist in international trade the real exchange rate exhibits threshold behavior. Such costs determine the volume of trade. When the exchange rate is introduced in the interest rate rule then the nominal interest rate appears to be nonlinear as supported by the data. Therefore, nonlinear interest rate rules are the correct model specification when the exchange rate is taken into account. Additionally, such rules lead to lower output gap and inflation variance.

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