

## Energy Saving Innovations, Non-Exhaustible Sources of Energy and Long-Run: What Would Happen if we Run Out of Oil?

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**Abstract.** We formulate and solve a model of factor saving technological improvement considering three factors of production: labor, capital and energy. The productive activities have three main characteristics: first, in order to use capital goods firms need energy; second, there are two sources of energy: non-exhaustible and exhaustible; third, capital goods can be of different qualities and the quality of these goods can be changed along two dimensions—reducing the need of energy or changing the source of energy used in the production process. The economy goes through three stages of development after industrialization. In the first one, firms make use of exhaustible energy and the efficiency in the use of energy is constant. In the second stage, as the price of energy grows the efficiency in its use is increased. In the third stage, the price of exhaustible sources is so high that firms have incentives to use non-exhaustible sources of energy. During this stage the price of energy is constant. In this set up, the end of the oil age has level effects on consumption and output but it does not cause the collapse of the economic system.

**Keywords:** non-exhaustible energy, energy saving innovations, economic growth.

**JEL classification:** 031, 041, Q20.

**Resumen.** Se formula y resuelve un modelo de cambio tecnológico ahorrador de factores de producción que considera tres factores: capital, trabajo y energía. El modelo cuenta con características específicas con respecto a la interacción entre la energía (la cual, de acuerdo a su fuente puede ser renovable y no renovable) y el capital. Una vez esta economía se ha definido, se supone que evoluciona en tres etapas luego de su industrialización, durante las cuales el carácter renovable o no renovable de la energía influye su precio relativo, eficiencia y afecta también el nivel agregado de consumo y producción de la economía, sin que esta evolución lleve al colapso del sistema económico.

**Palabras clave:** energía renovable, innovaciones ahorradoras de energía, crecimiento económico.

**Clasificación JEL:** 031, 041, Q20.

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## 1. Introduction

Natural resources have been an important source of wealth all through the history of the world. Every product we consume comes from at least one natural resource, either directly or indirectly. Scarce natural resources such as hydrocarbons have become one of the greatest concerns of our times as both scholars and journalists have foreseen the end of the oil age. In fact, analysts have forecasted the end of important energy sources such as gas and oil. The cries of oil scarcity heard some decades ago were certainly wrong: the world is not about to run out of hydrocarbons. Thanks to advances in exploration technology, there are more proven reserves of oil today than there were three decades ago (see Watkins, 2006). However, the question of what is going to happen as we approach the end of the oil age deserves attention

The classic supply side effect implies that when the supply of natural resources, especially energy sources, declines their prices raise. This price increase indicates the reduced availability of basic inputs in production, which motivates the use of new forms of energy. Using this logic, in 1932, John R. Hicks introduced the theory of induced innovation, according to which changes in relative factor prices lead to innovations that reduce the need for the relatively expensive factor. This theory has been tested. Kuper and Soest (2003), for example, who found through a panel of sectors of the Dutch economy that energy saving technical progress is particularly significant in periods preceded by high and rising energy prices, whereas the pace of this form of technical change happens to be much slower in periods of low energy prices<sup>1</sup>. David Popp (2002) shows that there is a strong positive correlation between energy prices and innovations. With this evidence in mind, we extend the neoclassical growth models introducing the existence of capital goods that make use of energy and model in it, by a very simplified way, the production of energy.

In the next section, we motivate the need of considering the finite supply of energy in traditional growth models. Then, we describe and solve the model. Finally we conclude.

## 2. Machines, Energy and Growth

Important episodes in the history of capitalism are related to the invention and use of machines. Capital accumulation is the only source of economic growth during the transition in a Solow-like type of model. In endogenous growth models *à la* Romer<sup>2</sup> technology is embodied in capital goods, so capital accumulation generates neutral technological change and, for this reason, long-run growth. Finally, in models of factor saving innovations<sup>3</sup> capital abundance stimulates labor-saving innovations and savings are higher in economies where

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<sup>1</sup>See also Pommeret and Boucekkine (2004).

<sup>2</sup>Romer (1986 and 1990).

<sup>3</sup>See Kennedy (1964), Zeira (1998), Acemoglu (2002), Boldrin and Levine (2002a), Zuleta (2004), Zuleta (2008) and Peretto and Seater (2007).

the technology is more capital-intensive. Therefore, also in this type of models the invention and use of machines is the main source of economic growth. Machines, however, need energy in order to be productive and energy sources today are predominantly finite. Thus, economic growth depends also on the supply of energy<sup>4</sup>.

We consider energy saving innovations and the existence of non-exhaustible sources of energy. Therefore, as exhaustible energetic resources become more expensive, the agents of the economy can adopt technologies that are more efficient in the use of energy or technologies that use non-exhaustible sources of energy.

We assume that technology is embodied in capital goods and capital goods of better qualities are more costly. Also, as economies grow they consume more energy so the reserves of exhaustible sources of energy decrease and their price increases. Therefore, economic growth generates incentives to use non-exhaustible sources of energy in a more intensive way. This implies that in the long run, when oil has become exhausted, we will use only non-exhaustible sources of energy. Along the transition, the efficiency in the use of energy grows as exhaustible sources become more expensive. In the same way in which agents innovate in order to save labor and land when these factors become scarce they devote efforts to reduce the need of fossil combustibles.

We do not model the invention of technologies. We assume that such technologies exist and are costly. Similarly we do not explain the beginning of the industrial era<sup>5</sup>. We assume that the economy starts with a small amount of capital, a given technology, and big reserves of exhaustible sources of energy. Thus, during the first stage of industrialization, firms make use of exhaustible sources of energy and the efficiency in the use of energy is low. In a second stage, the reserves of exhaustible sources of energy are smaller so the price of energy is higher and firms start using more energy-efficient capital goods. Finally, in the third stage the stock of capital becomes big if compared with the reserves of exhaustible sources of energy, so the price of exhaustible sources is high and firms have incentives to use non-exhaustible sources of energy. In the long run, all the energy used in the production process comes from non-exhaustible sources.

Finally, there is only progress in the efficiency with which energy is used, but not in the use of labour and capital. This assumption is made for simplicity. We want to focus on the technological changes related to the use of energy.

The first studies that explicitly modelled the need of energy in the production process were presented by Stiglitz (1974) and Solow (1974). However they didn't consider the possibility of energy saving innovations or the existence of non-exhaustible sources of energy. Non-exhaustible sources of energy were introduced into economic models by Dasgupta and Heal (1974), Heal (1976) and Nordhaus (1979) and more recently by Manne and Richels (1992) and Tahvo-

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<sup>4</sup>Finn (1991) studies the problem in relation with business cycles.

<sup>5</sup>For explanations about the industrial revolution see Galor(2005), Hansen and Prescott (2000) or Zuleta (2006).

nen (1994). However, none of these models consider the possibility of energy saving innovations (Tahvonen and Salo, 2001). Finally, Groth and Schou (2002) and Smulders and Nooij (2003) consider the existence of exhaustible sources of energy but do not consider the existence of non-exhaustible sources of energy.

### 3. The Model

#### 3.1. Consumers

We assume that labor supply is inelastic, that population growth is zero and we normalize labor to one.

The problem of the representative consumer-worker is the standard one,

$$\max_{c_t} \sum_0^{\infty} \log(c_t) \beta^t \quad \text{s.t.} \quad a_{t+1} = a_t (1 + r_t) + w_t - c_t$$

where  $a$  is the amount of assets at time  $t$ ,  $r_t$  is the interest rate,  $w_t$  is the market wage,  $c_t$  is the consumption of the representative agent and  $\beta$  is the discount factor.

From where,

$$\frac{c_{t+1}}{c_t} = \beta (1 + r_{t+1}) \quad (1)$$

#### 3.2. Producers of Final Goods

The production function is a Cobb-Douglas that combines capital and labor. We assume, however, that an energy source ( $e_s$ ) is needed to operate capital goods. There are two sources of energy ( $s$ ), non-exhaustible ( $N$ ) and exhaustible ( $E$ ) differentiated by their cost ( $s \in [N, E]$ ). Capital goods can be also of different qualities, they are differentiated by the source of energy they use,  $e_s$ , and by the efficiency in the use of energy ( $\gamma_s$ ).

Therefore, there is a production function for each quality of capital. Any production function is characterized by  $A [\min(K_{\gamma_s, s}, \gamma_s e_s)]^\alpha L_s^{1-\alpha}$  where  $K_{\gamma_s, s}$  is the amount of capital of quality  $\gamma_s$  designed to operate with energy,  $e_s$ ,  $\frac{1}{\gamma_s}$  is the amount of energy  $e_s$  needed to operate 1 unit of capital, for this reason we also refer to  $\gamma_s$  as the efficiency rate in the use of energy type  $s$ . Finally,  $L_s$  is the amount of labor working with capital goods of type  $K_{\gamma_s, s}$ .

The cost of the firms include the cost of labor  $wL_s$ , the cost of capital goods  $p_{\gamma_s, s} K_{\gamma_s, s}$  and the cost of energy  $p_s e_s$ . Where  $p_{\gamma_s, s}$  and  $p_s$  are the price of capital of quality  $\gamma_s, s$  and the price of energy of type  $s$  respectively.

Firms are price takers and choose the amounts of capital and labor and the

quality of capital ( $\gamma_s$  and  $s$ )

$$\begin{aligned} \max_{K_t, L_t, e_{i,t}} \sum_{t=0}^{\infty} & \left[ \left( \sum_s (A [\min(K_{\gamma_{s,t},s}, \gamma_{s,t} e_{s,t})]^\alpha L_{s,t}^{1-\alpha} \right. \right. \\ & \left. \left. - p_{\gamma_{s,t},s} K_{\gamma_{s,t},s} - p_{s,t} e_{s,t} - w_t L_{s,t}) \right) \left( \frac{1}{1+r_t} \right)^t \right] \\ \text{s.t. } & K_{\gamma_{s,t},s} \geq 0 \end{aligned}$$

Since the production function is of the Leontief type, firms use capital and energy in such a way that  $K_{\gamma_{s,t},s} = \gamma_{s,t} e_{s,t}$ . Thus, for analytical convenience we rewrite the problem in the following way:

$$\begin{aligned} \max_{K_t, L_t, e_{i,t}} \sum_{t=0}^{\infty} & \left( \sum_s (A [K_{\gamma_{s,t},s}]^\alpha L_{s,t}^{1-\alpha} \right. \\ & \left. - p_{\gamma_{s,t},s} K_{\gamma_{s,t},s} - p_{s,t} e_{s,t} - w_t L_{s,t}) \right) \left( \frac{1}{1+r_t} \right)^t \\ \text{s.t. } & 0 = K_{\gamma_{s,t},s} - \gamma_{s,t} e_{s,t}; \quad 0 \leq K_{\gamma_{s,t},s}; \quad \gamma_0 \leq \gamma_{s,t} \end{aligned}$$

From the solution of the problem we find factor prices,

$$w_t = (1 - \alpha) A [k_{\gamma_{s,t},s}]^\alpha \tag{2}$$

$$p_{\gamma_{s,t},s} = \alpha A [k_{\gamma_{s,t},s}]^{\alpha-1} - (\lambda_t + \mu_t) \tag{3}$$

$$p_{s,t} = \lambda_t \gamma_{s,t} \tag{4}$$

$$\frac{\partial p_{\gamma_{s,t},s}}{\partial \gamma_{s,t}} K_{\gamma_{s,t},s} = \lambda_t e_{s,t} - \kappa_t \tag{5}$$

where  $k_t$  is the capital labor ratio,  $\lambda_t$  is the multiplier of the first restriction,  $\mu_t$  is the multiplier of the second restriction and  $\kappa_t$  is the multiplier of the third restriction.

Equations 2 to 4 tell that the price of labor is equal to its marginal productivity; the price of a capital good (of any quality) is equal to its marginal productivity (equation 3); the price of energy is equal to its marginal productivity (equation 4) and finally, equation 5 tells that for any change in the quality of capital goods, the change in the price multiplied by the units of capital must be equal to the savings generated by the increase in efficiency, namely, the marginal cost of innovations is equal to its marginal productivity.

Note that if  $\kappa_t \neq 0$  then  $\gamma$  is constant. In other words, only if  $\kappa = 0$  the technology becomes more efficient as capital gets accumulated.

Note also that equation 2 implies that when both sources of energy are used the capital labor ratios must be equal, that is,  $k_{\gamma_{E,t},E} = k_{\gamma_{N,t},N}$ . Indeed, we are assuming homogenous labor, perfect mobility and perfect competition, so the

marginal productivity of labor does not depend on the type of energy. Similarly, equation 3 implies that when the capital labor ratio is the same regardless of the type of energy used then the price of capital goods must be the same for the two types of capital,  $p_{\gamma E,t,E} = p_{\gamma N,t,N}$ .<sup>6</sup>

Combining equations 3, 4 and 5 we find that whenever  $K_{\gamma t,s} > 0$ ,

$$p_{\gamma s,t,s} = \alpha A [k_{\gamma s,t,s}]^{\alpha-1} - \frac{p_{s,t}}{\gamma_{s,t}} \quad (6)$$

$$\frac{\partial p_{\gamma s,t,s}}{\partial \gamma_{s,t}} K_{\gamma s,t,s} = \frac{p_{s,t}}{\gamma_{s,t}} e_{s,t} - \kappa_t \quad (7)$$

Since any increase in capital must be accompanied by an increase in energy in order to be productive, the price of a capital good must be equal to its marginal productivity (given  $K_{\gamma s,t,s} \leq \gamma_{s,t} e_{s,t}$ ) minus the cost of the additional needs of energy. In the same way, the cost of a technological improvement that reduces the need for energy, this is, the increase in the price of capital multiplied by the units of capital used in the production process  $\left( \frac{\partial p_{\gamma s,t,s}}{\partial \gamma_{s,t}} K_{\gamma s,t,s} \right)$ , must be equal to the cost of the energy needed to produce with  $K_{\gamma s,t,s}$  units of capital  $\left( p_{s,t} \frac{e_{s,t}}{\gamma_{s,t}} \right)$ .

Note also that if  $\frac{p_{s,t}}{\gamma_{s,t}} > \alpha A [k_{\gamma s,t,s}]^{\alpha-1}$  then  $K_{\gamma s,t,s} = 0$ , this is, if the price of energy of type  $s$  is higher than the marginal productivity of capital  $K_{\gamma s,t,s}$  then no capital of type  $s$  is used. Indeed, from equation 6 it follows that if  $p_{s,t} > \gamma_{s,t} \alpha A [k_{\gamma s,t,s}]^{\alpha-1}$  then final good producers would only demand capital if the price of capital goods of type  $\gamma_{s,t}$  is negative.

Therefore, given the technology, if the price of energy  $N$  is too high then firms do not have incentives to use this type of energy. However, given the price of energy, technological improvements (increases in  $\gamma$ ,  $\alpha$  or  $A$ ) can generate incentives to use non-exhaustible sources of energy. Additionally, an exogenous increase in the price of energy of type  $E$  can generate incentives for the firms to use only non-exhaustible sources.

Finally, *ceteris paribus*, given the technology, an exogenous increase in  $p_{s,t}$  reduces the quantity of capital and, given the stock of capital, an exogenous increase in  $p_{s,t}$  increases the efficiency  $\gamma_s$ .

Summarizing, the price of both types of energy determine whether or not the agents of the economy have incentives to accumulate capital. If the price of exhaustible energy is higher than the marginal productivity of the capital goods that use this energy then it is better not to use this type of capital. However, a way to increase the marginal productivity of capital is buying capital goods which embody more efficient technologies. Therefore, as the price of energy grows, the agents of the economy have incentives to choose more efficient technologies.

<sup>6</sup>Equations 3 and 2 also imply that if  $p_{N,t} > p_{E,t}$  then only exhaustible energy is used in the production process and if  $p_{N,t} < p_{E,t}$  then only non-exhaustible energy is used in the production process

### 3.3. Producers of Capital Goods

Capital goods producers receive assets from the consumers  $a_t$ , pay the interest rate  $r_t$ , build capital goods and receive a price  $p_{\gamma_{s,t},s}$  for any unit of capital of quality  $\gamma_{s,t}$ . The cost of a technology  $\gamma_{s,t}$  can be interpreted in two different ways: (i) the cost of inventing and implementing a technology and (ii) the cost of copying a new technology and building a similar capital good. If we assume that technology is non-rival, then the cost described in (ii) is likely to be smaller than the one described in (i). However, if we assume that technology is embodied in goods and that it is costly to reverse engineer and appropriate, the difference between (i) and (ii) is substantially reduced<sup>7</sup>. We assume that the technology to produce capital goods of different qualities exists (interpretation (ii)), but the cost is increasing in  $\gamma_{s,t}$ .

For simplicity, we assume that capital is reversible, that is, people can reduce the existing stock of capital in order to increase consumption.

The amount of resources needed to build one unit of capital is an increasing function of the quality of the capital good,  $K_{\gamma_{s,t},s} = \frac{\varphi}{\gamma_{s,t}}a$ , where  $\varphi$  is the productivity of the capital good producer and is the same for  $E$  and  $N$ , so the profits of the capital producers are given by,  $(p_{\gamma_{s,t},s}K_{\gamma_{s,t},s} - a_t r_t)$ . To keep things simple we assume that each capital producer produces capital goods that use only one type of energy.

Since  $K_{\gamma_{s,t},s} = \frac{a_t \varphi}{\gamma_{s,t}}$ , the free entry condition implies,

$$p_{\gamma_{s,t},s} = \frac{\gamma_{s,t} r_t}{\varphi} \tag{8}$$

Equation 8 tells us that the price of a capital good must be equal to its cost of production. Note also that if  $p_{\gamma_{E,t},E} = p_{\gamma_{N,t},N}$ , then  $\gamma_{E,t} = \gamma_{N,t}$ . Therefore from equations 4 and 8 it follows that if both sources of energy are used then the efficiency in the use of energy is the same regardless the type of energy,  $\gamma_{E,t} = \gamma_{N,t}$ .

### 3.4. Energy Sources

Producers extract  $e_{s,t}$  units of source  $s$  at period  $t$  and receive a price  $p_{s,t}$  for each unit. They pay a cost for the extraction of energy given the amount of reserves  $N_t$  at period  $t$ . The evolution of reserves is given by,

$$R_{s,t+1} = R_{s,t} - \varrho_s e_{s,t}$$

where  $\varrho_E = 1$  and  $\varrho_N = 0$ . For simplicity we assume that the unit cost of extraction is given by

$$C(e_{s,t}, R_{s,t}) = \begin{cases} \phi_s \left(1 + \frac{e_{E,t}}{R_{s,t}}\right) & \text{if } s = E \\ \phi_s & \text{if } s = N \end{cases}$$

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<sup>7</sup>See Boldrin and Levine, 2002b.

where  $\phi_N > \phi_E$ . In other words, the unit cost of extraction of exhaustible sources of energy depends negatively on the amount of reserves and positively on the amount extracted<sup>8</sup>. The unit cost of extraction of non-exhaustible sources is assumed to be constant. We made this assumption for simplicity and does not change the main results of the model. We are aware of the fact that technology in the production of renewable sources of energy has improved substantially in the last decades. However, including this type of progress in the model would accelerate. On the other hand, the marginal costs of aeolian or solar energy could rise as the best sites are used up. Including this type of cost would reduce the steady state capital but would not affect the qualitative results of the model. The profits of the firm are:

$$[p_{s,t} - C(e_{s,t}, R_{s,t})] e_{s,t}$$

Since there is free entry, we know that  $p_{s,t} = C(e_{s,t}, R_{s,t})$  so

$$p_{E,t} = \phi_E \left( 1 + \frac{e_{E,t}}{R_{E,t}} \right) \quad (9)$$

$$p_{N,t} = \phi_N \quad (10)$$

Therefore, the unit cost of extraction must be equal to the price of oil. From section 3.2 we know that when the two sources of energy are used, their prices must be equal,  $p_{E,t} = p_{N,t}$ , so using equations 9 and 10,

$$\phi_E \left( 1 + \frac{e_{E,t}}{R_{E,t}} \right) = \phi_N \quad (11)$$

Note that equation 11 may not hold. Indeed, since  $\phi_E < \phi_N$  when the capital stock of the economy is small the need of energy  $e$  is also small. Then, if the reserves  $R_{E,t}$  are big, only the the exhaustible source of energy  $E$  is used.

Therefore, for early stages of development, only exhaustible sources of energy are used. However, once the amount of assets reaches a minimum level, agents have incentives to use non-exhaustible sources too. Moreover, when the reserves  $R_{E,t}$  decrease the extraction of exhaustible energy  $e_{E,t}$  is reduced and both technologies are used. In particular, from equation 11 it follows that the supply of exhaustible energy is given by,

$$e_{E,t} = R_{E,t} \left( \frac{\phi_N}{\phi_E} - 1 \right) \quad (12)$$

From where,

$$\frac{\Delta e_E}{e_E} = \frac{\Delta R_{E,t}}{R_{E,t}} \quad \text{and} \quad \frac{\Delta K_{E,t}}{K_{E,t}} - \frac{\Delta \gamma_{E,t}}{\gamma_{E,t}} = \frac{\Delta R_{E,t}}{R_{E,t}}$$

<sup>8</sup>High oil prices stimulates investment in exploration and improvement in exploration technology. However, this efforts have no long run effects so, for the sake of simplicity, we can assume away new discoveries.



So, as the reserves of exhaustible sources of energy decrease, the supply of this type of energy is reduced as well as the amount of capital goods that use this type of energy. This means that as the economy grows the amount of capital goods that make use of exhaustible energy decreases and in the very long run all capital goods will make use of non-exhaustible energy.

Note also that equation 11 implies that in the long run the price of energy is constant so the analysis of the long run becomes simpler.

Finally, recall that  $K_{E,t} = \gamma_{E,t}e_{E,t}$  so, equation 11 implies that,

$$\gamma_{E,t} = \frac{K_{E,t}}{R_{E,t}} \left( \frac{\phi_E}{\phi_N - \phi_E} \right)$$

differentiating,

$$\frac{\Delta\gamma_{E,t}}{\gamma_{E,t}} = \frac{\Delta K_{E,t}}{K_{E,t}} - \frac{\Delta R_{E,t}}{R_{E,t}}$$

So, when exhaustible sources of energy are used, the growth rate of the efficiency of capital goods (energy saving technological change) is equal to the growth rate of capital minus the growth rate of reserves of exhaustible sources of energy. In the long run only non-exhaustible sources are used so

$$\frac{\Delta R_{E,t}}{R_{E,t}} = 0 \quad \text{and} \quad \frac{\Delta\gamma_{E,t}}{\gamma_{E,t}} = \frac{\Delta K_{E,t}}{K_{E,t}} = 0$$

### 3.5. Equilibrium: Beginning, Transition and Long Run

In this section we use the results obtained in the previous sections in order to characterize the equilibrium. We first summarize the most important results in a formal way in propositions 1, 2, 3 and 4. Then, we show how this results relate to the evolution of the industrialized economies.

Using equations 6 and 9 we get the price of capital goods and the interest rate when only exhaustible sources of energy are used and technology is constant,

$$p_{\gamma_0} = \alpha A [k_t]^{\alpha-1} - \frac{\phi_E}{\gamma_0} \left( 1 + \frac{e_{E,t}}{R_{E,t}} \right) \tag{13}$$

$$r_t = \frac{\varphi}{\gamma_0} \left( \alpha A [k_t]^{\alpha-1} - \frac{\phi_E}{\gamma_0} \left( 1 + \frac{\phi_E}{\gamma_0} \frac{K_t}{R_{E,t}} \right) \right) \tag{14}$$

It is straightforward that  $\frac{\partial r_t}{\partial k_t} < 0$  and  $\frac{\partial r_t}{\partial R_t} > 0$ . Therefore, for early stages of development, the interest rate decreases as the economy grows. Now, from equation 1, the behavior of the interest rate translates in to a decreasing trend in the growth rate of consumption. Note also that in the absence of energy-saving innovations or alternative sources of energy the economy would collapse. Indeed, if the energy used in the production process is exhaustible, then the amount of reserves decreases period by period and the unitary cost of extraction

grows as well as the price of this type of energy (equation 9). Now, once the cost of extraction of exhaustible sources equals the cost of extraction of non-exhaustible sources the two sources of energy are used. This results are formally presented in propositions 1 and 2.

**Proposition 1.** *In Equilibrium: (i) If  $p_{N,t} > p_{E,t}$  then only exhaustible energy is used in the production process, (ii) if  $p_{N,t} < p_{E,t}$  then only non-exhaustible energy is used in the production process and (iii) if  $p_{N,t} = p_{E,t}$  both sources of energy are used.*

Proposition 1 follows from equations 2, 6 and 8 and states that only economies where the price of non-exhaustible energy is relatively low have incentives to use this type of energy. If the price of the two sources of energy is the same, both sources are used in the production process.

The capital labor ratio must be the same regardless the type of energy used, otherwise the marginal productivity of labor and the wages would depend on the type of energy used (equation 2). If the capital labor ratio is the same, the marginal productivity of capital is also equal and so is the price of the capital goods (equation 4). Similarly, if the price of capital goods is equal then the efficiency on the use of energy is the same regardless the type of energy (equation 8). Finally, if the price of capital, the efficiency in the use of energy and the capital labor ratio are the same then the price of energy must be the same (equation 6).

**Proposition 2.** *Let's define the amount of assets  $a$  as the sum of capital goods:  $a_t = K_{E,t} + K_{N,t}$ . Given the amount of reserves of exhaustible energy  $R_E$ , there exists a critical level of assets  $\tilde{a}$  such that for any  $a < \tilde{a}$  only exhaustible energy is used.*

Proposition 2 follows from section 3.4 and Proposition 1 and implies that only a capital abundant economy has incentives to use non-exhaustible energy. In early stages of development the stock of capital is small and the amount of reserves is big so the price of exhaustible energy is relatively low. For this reason if the assets of the economy are small, only exhaustible energy is used. Now, as the economy develops the stock of capital grows and the amount of reserves decrease so the price of exhaustible energy grows until the point where it is profitable to use non-exhaustible sources of energy.

Now, let's recall that the economy has two ways to face the scarcity of exhaustible sources of energy, one is by using non-exhaustible sources of energy and the other one is through energy-saving innovations. We already described the conditions under which the use of non-exhaustible sources is convenient. In the following lines we refer to the incentive to undertake energy saving innovations.

From equations 7 and 9 it follows that if the amount of reserves of exhaustible sources of energy is rather big, then the gain derived from energy

saving innovations is lower than the cost. Formally, if

$$R_t > \frac{\phi_E}{\gamma_0} \frac{K_t}{\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E}$$

then  $\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 > \phi_E \left(1 + \frac{\phi_E K_t}{\gamma_0 R_t}\right)$  and  $\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 > p_{E,t}$  so  $\frac{\partial p_{\gamma_0}}{\partial \gamma_0} K_t > \frac{p_{E,t}}{\gamma_0} e_t$ . Therefore, from equation 7 it follows that there are no incentives to increase the efficiency in the use of energy. However, as the economy develops, the amount of reserves decreases until the point where the incentives for energy-saving innovations appear, this is,

$$R_t < \frac{\phi_E}{\gamma_0} \frac{K_t}{\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E}$$

As the economy develops and the amount of reserves decreases the economy either begins to use non-exhaustible sources of energy or to undertake energy saving innovations. But which one happens first? This question can be answered using equations 7 and 9 and Proposition 3:

**Proposition 3.** *If*

$$\frac{\phi_E}{\gamma_0} \frac{K_t}{\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E} > R_t > K_t \left( \frac{\phi_E}{\phi_N - \phi_E} \right)$$

*then only exhaustible energy is used and the efficiency in the use of energy grows over time. If*

$$\frac{\phi_E}{\gamma_0} \frac{K_t}{\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E} < R_t < K_t \left( \frac{\phi_E}{\phi_N - \phi_E} \right)$$

*then non-exhaustible energy is used and the efficiency in the use of energy grows with time. Therefore, if  $\left(\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E\right) \gamma_0 > \phi_N - \phi_E$  then energy saving innovations are adopted over the use of non-exhaustible sources of energy and if  $\left(\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E\right) \gamma_0 < \phi_N - \phi_E$  then energy saving innovations are never adopted.*

Proposition 3 implies that if the efficiency in the use of energy is initially very high, the marginal cost of non-exhaustible sources of energy is relatively low, and if the marginal cost of increasing the efficiency in the use of energy is high then energy saving innovations are not adopted. Now, taking into account that all through the XXth Century many technological innovations were actually energy savers (see Kuper and Soest, 2003 or Pommeret and Boucekine, 2004) it is reasonable to assume that  $\left(\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E\right) \gamma_0 > \phi_N - \phi_E$ .

We already showed that the reduction in the amount of reserves of exhaustible sources of energy generates incentives either to use non-exhaustible

sources of energy or to undertake energy saving innovations. In this setting, the price of exhaustible energy grows up to the point when it becomes equal to the cost of extraction of non-exhaustible sources. While the use of exhaustible energy initially grows, then decreases and finally vanishes.

**Proposition 4.** *Given  $A$  and  $\varphi$  there exists a steady state where the capital labor ratio is given by  $k^* = \left( \frac{\alpha A}{2} \left( \frac{\varphi}{\phi_N} \frac{\beta}{1-\beta} \right)^{\frac{1}{2}} \right)^{\frac{1}{1-\alpha}}$ .*

Proposition 4 follows equation 14 and implies that in the long run the income depends on TFP  $\frac{\partial k^*}{\partial A} > 0$ , the capital share  $\frac{\partial k^*}{\partial \alpha} > 0$  and the discount factor  $\frac{\partial k^*}{\partial \beta} > 0$  (as it is usual in growth models). Additionally, the steady state stock of capital also depends on the steady state price of energy  $\phi_N$  and on the productivity in the production of capital goods  $\varphi$ . The productivity of capital goods' production positively affects the steady state stock of capital ( $\frac{\partial k^*}{\partial \varphi} > 0$ ). Finally, note that if either  $A$  or  $\varphi$  grow, for any given reason, then there is long run growth. In other words, if we extend the model assuming constant growth rate for  $A$ , then in the long run this model would look like the standard exogenous growth model.

### 3.5.1. The History

The industrial revolution was characterized by the generalization of capital intensive technologies. In terms of our model, the industrial revolution would be the invention of capital goods and it is natural to think that the technology embodied in the earliest capital goods developed was characterized by a positive  $\gamma$ , that is, even if the first technology was not very efficient in the use of energy, it was not completely inefficient. Therefore, the first years of the industrial age were characterized by a big amount of reserves  $R$ , a small amount of capital and a given technology  $\gamma$ . Under such circumstances exhaustible resources were likely to be cheap so there were no incentives neither to use non-exhaustible sources of energy nor to increase the efficiency in the use of energy. However, the amount of reserves  $R$  decreased as the economy was producing, so the price of exhaustible energy grew generating incentives to adopt energy saving innovations<sup>9</sup>. Finally, the amount of reserves decreased despite the energy-saving innovations so the price of exhaustible energy kept growing up to the point where the use of non-exhaustible sources of energy became profitable.

Under such conditions, the economy went through three stages of development after industrialization. In the first one, reserves of exhaustible sources of energy are big compared with the stock of capital, firms make use of exhaustible sources of energy and the efficiency in the use of energy is constant. In the second stage, as the price of energy grows, the efficiency in the use of

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<sup>9</sup>The assumption we make about new discoveries does not affect the qualitative results of the model.

energy gets increased. In the third stage the stock of capital becomes rather big once compared with the reserves of exhaustible sources of energy, so the price of exhaustible sources is so high that firms have incentives to use non-exhaustible sources of energy. Along this stage the price of energy is constant.

#### 4. Conclusions and Discussion

Throughout this paper, we formulate and solve a classical growth model of factor saving technological improvement, considering three factors of production, labor, capital and energy. The productive activities were characterized as follows: first, in order to use capital goods firms need energy; second, there are two sources of energy: one non-exhaustible and the other exhaustible; third, capital goods can be of different qualities and the quality of such goods can be changed in two dimensions, either by reducing the need of energy or by changing the source of energy used in the production process.

According to this model, the economy goes through three stages of development after industrialization. In the first one, reserves of exhaustible sources of energy are big compared with the stock of capital, firms make use of exhaustible sources of energy and the efficiency in the use of energy is constant. In the second stage, the reserves of exhaustible sources of energy are smaller so the price of energy is bigger and firms start using more efficient capital goods. During this stage, as the price of energy grows the efficiency in the use of energy gets increased. Finally, in the third stage, the stock of capital becomes big if compared with the reserves of exhaustible sources of energy so the price of exhaustible sources is so high that firms have incentives to use non-exhaustible sources of energy. Along this stage the price of energy is constant.

The existence of long-run growth does not depend on the reserves of exhaustible sources of energy but on the technological progress in the production of capital goods, the cost of extraction of non-exhaustible energy and the total factor productivity in the production of final goods.

There are many important issues regarding the relation between sources of energy and economic growth that remain unaddressed. First, we are ignoring the negative externality that produces the use of exhaustible sources of energy. Second, we are assuming competitive markets whereas oil reserves are geographically concentrated and the biggest part of the extraction business is managed by few firms<sup>10</sup>. Third, we are assuming away improvements in the extraction technology as well as new discoveries. All these issues deserve attention and may be topics for further research. However, in order to keep things simple we decide not to address them here. Additionally, the main conclusions of our work would remain the same regardless of the simplifying assumptions.

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<sup>10</sup>25% of the world's proven reserves of oil sit under the parched deserts of Saudi Arabia. Russia, by contrast, sits atop barely 5% of the world's reserves. Iraq controls about 10%.

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## Appendix

*Proof of Proposition 1. In Equilibrium: (i) If  $p_{N,t} > p_{E,t}$  then only exhaustible energy is used in the production process, (ii) if  $p_{N,t} < p_{E,t}$  then only non-exhaustible energy is used in the production process and (iii) if  $p_{N,t} = p_{E,t}$  both sources of energy are used.*

**Proof.** (a) From equation 2 it follows that when both sources of energy are used the capital labor ratios must be equal, that is,  $k_{\gamma_{E,t},E} = k_{\gamma_{N,t},N}$ .

(b) From equation 6 it follows that when  $k_{\gamma_{E,t},E} = k_{\gamma_{N,t},N}$  the price of the two capital goods must be equal, that is,  $p_{\gamma_{N,t},N} = p_{\gamma_{E,t},E}$ .

(c) From equation 8 it follows that when  $p_{\gamma_{N,t},N} = p_{\gamma_{E,t},E}$  the price of the two sources of energy must be equal, that is,  $p_{N,t} = p_{E,t}$ .

From (a), (b) and (c) it follows that if  $p_{\gamma_{N,t},N} \neq p_{\gamma_{E,t},E}$  then only one source of energy is used.

Now, suppose that for some initial conditions  $p_{E,t} < p_{N,t}$ . From equation 6 it follows that the producers of final goods are willing to pay more for the capital goods that make use of exhaustible energy. However, the cost of production is identical for the two types of capital so in equilibrium only capital goods that use exhaustible energy are used (a similar argument can be made for an increase in  $p_{E,t}$ ).  $\square$

*Proof of Proposition 2. Define the amount of assets  $a$  as the sum of capital goods:  $a_t = K_{E,t} + K_{N,t}$ . Given the amount of reserves of exhaustible energy  $R_E$  and the efficiency in the use of energy  $\gamma$ , there exists a critical level of assets  $\tilde{a}$  such that for any  $a < \tilde{a}$  only non non-exhaustible energy is used.*

**Proof.** Define  $\tilde{a} = \left[ \gamma R_E \left( \frac{\phi_E}{\phi_N} - 1 \right) \right]$ ,

(a)  $a < \tilde{a}$  implies  $K_{E,t} \leq \gamma R_{E,t} \left( \frac{\phi_N}{\phi_E} - 1 \right)$

(b)  $K_{E,t} \leq \gamma R_{E,t} \left( \frac{\phi_N}{\phi_E} - 1 \right)$  implies  $e_{E,t} < R_{E,t} \left( \frac{\phi_N}{\phi_E} - 1 \right)$

(c)  $e_{E,t} < R_{E,t} \left( \frac{\phi_N}{\phi_E} - 1 \right)$  implies  $\frac{e_{E,t}}{R_{E,t}} > \frac{\phi_N}{\phi_E} - 1$  and  $\phi_E \left( 1 + \frac{e_{E,t}}{R_{E,t}} \right) > \phi_N$

(d)  $\phi_E \left( 1 + \frac{e_{E,t}}{R_{E,t}} \right) > \phi_N$  implies  $p_{N,t} > p_{E,t}$

Finally, from Proposition 1 it follows that if  $p_{N,t} > p_{E,t}$  then only exhaustible energy is used in the production process.  $\square$

*Proof of Proposition 3. If  $\left( \frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E \right) \gamma_0 < \phi_N - \phi_E$  then energy-saving innovations are never adopted.*

**Proof.** (a) From the proof of Proposition 2 it follows that if  $R_t > \frac{K_t}{\gamma_t} \left( \frac{\phi_E}{\phi_N - \phi_E} \right)$  then  $\phi_E \left( 1 + \frac{e_{E,t}}{R_{E,t}} \right) < \phi_N$  and  $p_N > p_E$ , so there are no incentives to use non-exhaustible sources of energy.

(b) From equations 7 and 9 it follows that  $R_t > \frac{\phi_E}{\gamma_0} \frac{K_t}{\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E}$ , then  $\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 > \phi_E \left( 1 + \frac{\phi_E K_t}{\gamma_0 R_t} \right)$  and  $\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 > p_{E,t}$  so  $\frac{\partial p_{\gamma_0}}{\partial \gamma_0} K_t > \frac{p_{E,t}}{\gamma_0} e_t$ , so from equation 7 it follows that there are no incentives to increase the efficiency in the use of energy. Therefore,

(c) If  $\frac{\phi_E}{\gamma_0} \frac{K_t}{\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E} > R_t > K_t \left( \frac{\phi_E}{\phi_N - \phi_E} \right)$  then only exhaustible energy is used and the efficiency in the use of energy grows with time.

(d) If  $\frac{\phi_E}{\gamma_0} \frac{K_t}{\frac{\partial p_{\gamma_0}}{\partial \gamma_0} (\gamma_0)^2 - \phi_E} < R_t < K_t \left( \frac{\phi_E}{\phi_N - \phi_E} \right)$  then non-exhaustible energy is used and the efficiency in the use of energy grows with time.



From (c) and (d) it follows that as  $K$  grows and  $R$  decreases and at some point the inequalities change sign, that the adoption energy saving innovations or the use of non-exhaustible energy become profitable. What happens first?

- If  $\left(\frac{\partial p_{\gamma_0}}{\partial \gamma_0}(\gamma_0)^2 - \phi_E\right) \gamma_0 < \phi_N - \phi_E$ , inequality (c) changes its sign first and firms start using non-exhaustible sources without energy saving innovations. Note that in this case the price of energy is constant and equal for renewable and exhaustible sources. So if given the price of energy there are no incentives to undertake energy saving innovations then the efficiency in the use of energy remains constant along the transition.
- If  $\left(\frac{\partial p_{\gamma_0}}{\partial \gamma_0}(\gamma_0)^2 - \phi_E\right) \gamma_0 > \phi_N - \phi_E$ , inequality (d) changes sign first and firms undertake energy saving innovations without using non-exhaustible sources of energy. However, as long as capital goods use exhaustible sources of energy  $R$  decreases so sooner or later inequality 1 will also change sign. □

*Proof of Proposition 4.*

**Proof.** *Claim 1: If  $p_{N,t} < p_{E,t}$  and  $p_{N,t}$  and  $r$  are constant then  $\gamma_{N,t}$ ,  $p_{\gamma_{N,t},N}$  and  $k_{\gamma_{N,t},N}$  are constant.*

From equation 3 if  $r$  is constant then  $p_{\gamma_{s,t},s}$  is a linear function of  $\gamma_{s,t}$ . Defining  $\Omega = \frac{r_t}{\varphi}$ , the function is given by  $p_{\gamma_{s,t},s} = \Omega \gamma_{s,t}$ . Now, from equation 7  $\frac{\partial p_{\gamma_{s,t},s}}{\partial \gamma_{s,t}} = \frac{p_{s,t}}{\gamma_{s,t}} \frac{e_{s,t}}{K_{\gamma_{s,t},s}}$  so  $\frac{p_{s,t}}{\gamma_{s,t}} \frac{e_{s,t}}{K_{\gamma_{s,t},s}} = \Omega$ . Rearranging,  $p_{s,t} \frac{e_{s,t}}{K_{\gamma_{s,t},s}} = \Omega \gamma_{s,t}$ . Since  $e_{s,t} \gamma_{s,t} = K_{\gamma_{s,t},s}$  we get  $\gamma_{s,t} = \left(\frac{p_{s,t}}{\Omega}\right)^{\frac{1}{\alpha}}$  and  $p_{\gamma_{s,t},s} = \Omega \left(\frac{p_{s,t}}{\Omega}\right)^{\frac{1}{\alpha}}$ . Therefore, if  $p_{N,t}$  and  $r_t$  are constant then  $\gamma_{N,t}$  and  $p_{\gamma_{N,t},N}$  are constant. Finally, from equation 6 it follows that  $\gamma_{s,t} \left(\alpha A [k_{\gamma_{s,t},s}]^{\alpha-1} - \gamma_{s,t} \Omega\right) = p_{s,t}$ , rearranging,  $k_{\gamma_{s,t},s} = \left(\frac{\alpha A}{\gamma_{s,t}} (p_{s,t} + \gamma_{s,t}^2 \Omega)\right)^{\frac{1}{1-\alpha}}$ . Therefore, if  $p_{N,t}$  and  $r_t$  are constant then  $k_{\gamma_{N,t},N}$  is constant.

*Claim 2: the interest rate decreases as the capital stock grows and the reserves of energy sources are consumed.*

Using equations 6 and 9 we get the price of capital goods and the interest rate,

$$p_{\gamma_0} = \alpha A [k_t]^{\alpha-1} - \frac{\phi_E}{\gamma_0} \left(1 + \frac{e_{E,t}}{R_{E,t}}\right)$$

$$r_t = \frac{\varphi}{\gamma_0} \left(\alpha A [k_t]^{\alpha-1} - \frac{\phi_E}{\gamma_0} \left(1 + \frac{\phi_E}{\gamma_0} \frac{K_t}{R_{E,t}}\right)\right)$$

It is straightforward that  $\frac{\partial r_t}{\partial k_t} < 0$  and  $\frac{\partial r_t}{\partial R_t} > 0$ .

From the solution of the consumer problem, equation 1, if there exists a steady state is must be characterized by

$$(1 + r) = \frac{1}{\beta}$$

We have showed that in the long run the price of energy is given by  $\phi_N$  so it is possible to characterize the steady state using equations 10 and 13,

$$k^* = \left( \frac{\alpha A}{2} \left( \frac{\varphi}{\phi_N} \frac{\beta}{1 - \beta} \right)^{\frac{1}{2}} \right)^{\frac{1}{1 - \alpha}} \quad \square$$