MEASURING THE PERFORMANCE OF TWO-STAGE PRODUCTION SYSTEMS WITH SHARED INPUTS BY DATA ENVELOPMENT ANALYSIS

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ABSTRACT

As a non-parametric technique in Operations Research and Economics, Data Envelopment Analysis (DEA) evaluates the relative efficiency of peer production systems or decision making units (DMUs) that have multiple inputs and outputs. In recent years, a great number of DEA studies have focused on two-stage production systems in series, where all outputs from the first stage are intermediate products that make up the inputs to the second stage. There are, of course, other types of two-stage processes that the inputs of the system can be freely allocated among two stages. For this type of two-stage production system, the conventional two-stage DEA models have some limitations e.g. efficiency formulation and linearizing transformation. In this paper, we introduce a relational DEA model, considering series relationship among two stages, to measure the overall efficiency of two-stage production systems with shared inputs. The linearity of DEA models is preserved in our model. The proposed DEA model not only evaluates the efficiency of the whole process, but also it provides the efficiency for each of the two sub-processes. A numerical example of US commercial banks from literature is used to clarify the model.

Keywords: Data envelopment analysis, Decision making unit, Two-stage, Shared input, Efficiency

1. INTRODUCTION

Data Envelopment Analysis (DEA) was first introduced by Charnes et al. (1978) as a mathematical and linear programming-based technique for measuring the relative efficiency of peer production systems or decision making units (DMU) that have multiple inputs and outputs. DEA has become a popular management tool, because of the following advantages (Banker and Thrall, 1992; Cooper et al., 2000):

- It does not need a functional production relationship between inputs and outputs.
- It allows inputs and outputs to be specified with flexibility.

- It makes a single efficiency score by simultaneously comparing multiple inputs and outputs of comparable units from the observed best practice.
- It identifies inefficient DMUs and causes of inefficiency.
- It evaluates the relative importance of the various performance criteria on an objective basis.
- It assesses each DMU along its own favorable direction.

In recent years, a great number of DEA studies have focused on two-stage production systems, where all outputs from the first stage are intermediate products that make up the inputs to the second stage. For example, Seiford and Zhu (1999) developed a two-stage DEA approach for measuring the efficiency of the profitability and marketability of US commercial banks. Zhu (2000) applied the same two-stage process to assess the financial efficiency of the best 500 companies. Sexton and Lewis (2003) studied the Major League Baseball performance in a two-stage process. Chen and Zhu (2004) developed a linear type DEA model where each stage's efficiency is defined on its own production possibility set. Liang et al. (2006) proposed a model to evaluate the performance of supply chains with two members. Kao and Hwang (2008) developed a different approach where the overall efficiency of the system can be decomposed into the product of the efficiencies of the two-stages. Chen et al. (2009a) presented a model similar to the Kao and Hwang's model, but in an additive form.

Actually, in the real world it may not be possible that a DMU generates its final output only by using intermediate products without any other inputs. For example, a bank uses employees and fixed assets to produce intermediate output, such as deposits in the first stage. Some of employees and fixed assets may flow into second stage and use together with intermediate products as the inputs for the second stage to produce the second stage's output, such as profit. As pointed out by a number of researchers, including Kao and Hwang (2008) and Chen et al. (2009b) these situations impose some limitations e.g. efficiency formulation and linearizing transformation in using conventional two-stage DEA models (See Zha and liang (2010) for more details). The aim of this paper is to develop a relational DEA model to evaluate the performance of a two-stage production system, where the shared inputs can be freely distributed between the two stages. The proposed approach measures the efficiencies of the whole process as well as the two sub-processes. Different to Zha and Liang's approach, the linearity of DEA models is preserved in our model. The top 30 commercial banks in US, whose production process is similar to the two-stage process with shared inputs, are used to illustrate the proposed model. The rest of this paper is organized as follows. Section 2 presents some preliminary considerations. In Section 3, we first present a general two-stage process with sharing inputs and then develop a relational DEA model for measuring the efficiencies of the whole system as well as the two stages. Section 4 applies the new approach to the 30 firms in the banking industry in US. Finally, conclusions are provided in the last section.

2. PRELIMINARIES

Let x_{ij}, (i=1,…,m) and y_{rj}, (r=1,…,s) represent the ith input, and rth output of DMU$_j$, (j=1,…,n). It is assumed that all inputs and outputs are non negative, but at least one of the inputs and outputs are positive. If v_i and u_r be the known multipliers or prices associated with inputs i and outputs r, then the relative efficiency score of DMU$_o$ $(o \in \{1,...,n\})$, E_o, can be expressed as

$$E_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}. \tag{1}$$

In the absence of known multipliers, Charnes et al. (1978) proposed a fractional programming problem (CCR model) to derive appropriate multipliers for a given DMU. The CCR model for evaluating the relative efficiency of DMU$_o$ under the assumption of constant returns to scale (CRS), is indicated as follows:

$$
\begin{aligned}
E_o^* = \ & \max \ \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\
& \text{s.t.} \ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \ j = 1,\dots,n, \\
& u_r, v_r \ge \varepsilon, \ r = 1,\dots,s, i = 1,\dots,m,
\end{aligned}
\tag{2}
$$

where ε is non-Archimedean small value designed to impose strict positivity on the multipliers. E_o is the efficiency score of DMU$_o$, that $E_o^* = 1$ indicates efficiency and $E_o^* < 1$ for inefficiency.

3. A TWO-STAGE DEA MODEL WITH SHARED INPUTS

Now, consider a production system composed of a two-stage process with shared inputs as shown in Fig. 1. Suppose we have n DMUs, that each DMU$_j$ (j=1,…,n) has m inputs x_{ij} (i=1,…,m) that can be freely allocated in any of the two stages. Also, there are p intermediate products z_{dj} (d=1,…,p) that are the outputs of stage 1 as well as the inputs of stage 2. The outputs from the second stage are y_{rj} (r=1,…,s).

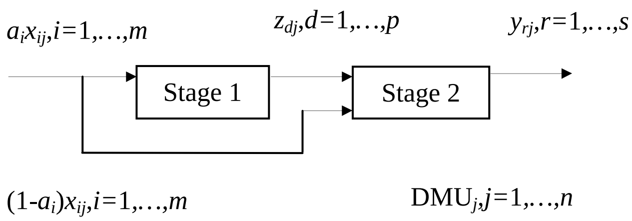

Fig. 1. Two-stage production system with shared inputs

We suppose that $a_i x_{ij}, i = 1,\dots,m$ and $(1-a_i)x_{ij}, i = 1,\dots,m$ are the amounts of shared input i distributed to the stage 1 and stage 2, respectively.

Consider, DMU$_o$ $(o \in \{1,\dots,n\})$ be the DMU under evaluation. Based on the CCR model, the efficiency scores of the whole process and the two individual stages can be calculated as follows:

$$
\begin{aligned}
E_o &= \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i a_i x_{io} + \sum_{i=1}^{n} \tilde{v}_i (1-a_i) x_{io}}, \\
E_o^1 &= \frac{\sum_{d=1}^{p} \eta_d z_{do}}{\sum_{i=1}^{m} v_i a_i x_{io}}, \\
E_o^2 &= \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{p} \tilde{\eta}_d z_{do} + \sum_{i=1}^{n} \tilde{v}_i (1-a_i) x_{io}},
\end{aligned}
\tag{3}
$$

where $v_i, \tilde{v}_i, u_r, \eta_d$ and $\tilde{\eta}_d$ are unknown non-negative multipliers. Note that η_d can be equal to $\tilde{\eta}_d$.
In an effort to estimate the overall efficiency of the DMU$_o$, taking into account the series relationship among two stages, we formulate the following fractional program:

$$
\begin{aligned}
E_o^* = \max\ & E_o \\
\text{s.t.}\quad & E_j^1 \le 1,\ \ j = 1,\dots,n. \\
& E_j^2 \le 1,\ \ j = 1,\dots,n, \\
& \eta_d = \tilde{\eta}_d,\ d = 1,\dots,p.
\end{aligned} \tag{4}
$$

Note that, similar to Kao and Hwang (2008) and Chen et al. (2009a), we have assumed that every intermediate product has the same multiplier, no matter whether it plays the role of input or output.
Let $v_i a_i = \pi_i$ and $v_i(1 - a_i) = \omega_i$ then model (4) is equivalent to the following model:

$$
\begin{aligned}
E_o^* = \max\ & \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} \pi_i x_{io} + \sum_{i=1}^{n} \omega_i x_{io}} \\
\text{s.t.}\quad & \frac{\sum_{d=1}^{p} \eta_d z_{dj}}{\sum_{i=1}^{m} \pi_i x_{ij}} \le 1,\ \ j = 1,\dots,n. \\
& \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{p} \eta_d z_{dj} + \sum_{i=1}^{m} \omega_i x_{ij}} \le 1,\ \ j = 1,\dots,n, \\
& \pi_i, \omega_i, u_r, \eta_d \ge \varepsilon, i = 1,\dots,m, r = 1,\dots,s, d = 1,\dots,p.
\end{aligned} \tag{5}
$$

Note that model (5) is a fractional programming problem that can be converted into a linear programming problem by applying the Charnes-Cooper transformation (Charnes and Cooper, 1962) as follows:

$$
\begin{aligned}
E_o^* = \max\ & \sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t.}\quad & \sum_{i=1}^{m} (\pi_i + \omega_i) x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} (\pi_i + \omega_i) x_{ij} \le 0,\ j = 1,\dots,n, \\
& \sum_{d=1}^{p} \eta_d z_{dj} - \sum_{i=1}^{m} \pi_i x_{ij} \le 0,\ j = 1,\dots,n, \\
& \sum_{r=1}^{s} u_r y_{rj} \text{ - } \Big(\sum_{i=1}^{m} \omega_i x_{ij} + \sum_{d=1}^{p} \eta_d z_{dj}\Big) \le 0,\ j = 1,\dots,n, \\
& \pi_i, \omega_i, u_r, \eta_d \ge 0, i = 1,\dots,m, r = 1,\dots,s, d = 1,\dots,p.
\end{aligned} \tag{6}
$$

This model will be solved for n times, once for each DMU, to evaluate the overall efficiency score of the system. On optimality, the efficiency scores of two stages of each DMU$_o$ (o=1,…,n), can be calculated as follows:

$$E_o^1 = \frac{\sum_{d=1}^{p} \eta_d^* z_{do}}{\sum_{i=1}^{m} \pi_i^* x_{io}},$$
$$E_o^2 = \frac{\sum_{r=1}^{s} u_r^* y_{ro}}{\sum_{i=1}^{m} \omega_i^* x_{io} + \sum_{d=1}^{p} \eta_d^* z_{do}}, \quad (7)$$

where $u_r^*, \pi_i^*, \omega_i^*$ and η_d^* are the optimal multipliers calculated from model (6). Using the model (6), we can evaluate the overall efficiency of the DMU_o in a way that takes into account the operations of its two stages. Also, using equation (7) we are able to recognize the inefficient sub-processes and make later improvements.

4. NUMERICAL EXAMPLE

In this section, the new approach is applied to the top 30 commercial banks in US as studied by Zha and Liang (2010).

Table 1
Data set

DMU	Inputs			Intermediate products		Outputs		
	Employees	Assets	Equity	Revenue	Profits	Market Value	Earnings	Returns
1	85300	256853.0	19581.0	31690.0	3464.0	33221.7	7.21	66.1
2	95288	232446.0	20222.0	20386.0	2664.0	27148.6	6.49	69.4
3	58322	187298.0	12801.0	16298.0	1950.0	20295.9	7.13	59.7
4	39078	182926.0	11912.0	14884.0	1805.0	16971.3	6.73	70.5
5	15600	184879.0	10451.0	13838.0	1296.0	15003.5	6.42	49.4
6	33365	121173.0	9134.0	11336.0	1165.0	12616.4	5.76	82.4
7	35328	122002.0	8450.0	10681.0	1150.0	12351.1	3.45	50.0
8	44536	131879.9	9043.1	10582.9	1430.2	16815.0	5.04	39.9
9	46900	90454.0	8197.5	8970.9	1277.9	14807.4	2.91	54.9
10	14000	104000.0	5000.0	8600.0	215.0	5252.4	2.03	28.3
11	30800	84432.2	6364.8	7919.4	610.0	10428.7	1.57	31.8
12	45404	72134.4	5312.1	7582.3	956.0	12268.6	2.76	45.5
13	26757	73404.0	5768.0	6389.5	408.1	9938.2	1.19	61.4
14	28905	66339.1	5152.5	6054.0	825.0	8671.2	3.45	51.6
15	17881	47397.0	3751.0	5410.6	541.0	5310.1	4.55	84.7
16	19700	50316.0	4055.0	5409.0	1032.0	11342.5	20.37	52.8
17	15850	53685.0	5223.0	5327.0	914.0	10101.5	4.57	69.9
18	27200	58071.0	4154.0	4827.5	885.1	12138.0	11.02	108.5
19	24300	40129.0	4106.0	4514.0	691.0	7476.7	4.50	83.8
20	15996	44981.3	3773.8	3755.4	602.5	7623.6	3.50	46.9
21	19415	46471.5	4269.6	3740.3	565.5	7922.5	4.94	46.9
22	20175	41553.5	3272.2	3680.0	533.3	5774.9	5.30	59.0
23	20767	36199.0	2921.0	3449.9	465.1	4912.2	3.03	33.9
24	13231	33874.0	2725.0	3328.3	568.1	8304.0	4.19	54.3
25	13500	35469.9	2607.7	3112.6	413.4	4537.0	3.54	71.7
26	17023	33703.8	2928.1	2996.1	418.8	4997.0	3.25	57.3
27	14081	31794.3	2617.0	2897.3	329.0	4865.1	2.09	66.8
28	13598	29620.6	2379.4	2868.0	452.2	5788.0	3.22	52.0
29	4900	43881.6	3007.8	2859.6	288.6	3218.0	4.66	41.1
30	11171	13228.9	1265.1	2565.4	353.1	6543.3	1.54	60.7

They divided the production process of the banking industry into two stages: profitability and marketability. The inputs to the first stage are the number of employees, Assets ($millions) and equity ($millions). Some of the employees, assets and equity may flow in the second and act as the inputs of the second stage. The outputs of the second stage are market value ($millions), earning per share ($) and returns to the investors (%). There are also two intermediate products between the two stages, namely revenue ($millions) and profit ($millions). Table 1 reports the data set.

By using the proposed relational two-stage DEA model, model (6), the overall efficiencies of the 30 banks are calculated as shown in the second column of Table 2. The efficiency scores of the two sub-processes for each DMU were calculated by using Equation (7) and the results are reported in Table 3 under the heading "Process efficiency."
From Table 2, it can be seen that only three DMUs, namely DMU 5, DMU 16 and DMU 30, perform efficiently in the whole production system and in both stages. Also, DMUs 17, 24 and 29 have the overall efficiency scores greater than 0.9. It also can be seen that DMU 17 is efficient only in stage 1, stage of profitability, and DMU 24 and DMU 29 are efficient only in stage 2 namely stage of marketability. Results also indicate that for some DMUs, the low overall efficiency is because of the low performance in the first stage and for others is because of the low performance in the second stage.

Table 2
Results

DMU	System efficiency (E_o)	Process efficiency	
		Stage 1 (E_o^1)	Stage 2 (E_o^2)
1	0.444	0.726	0.494
2	0.361	0.567	0.427
3	0.454	0.751	0.498
4	0.537	0.581	0.598
5	1.000	1.000	1.000
6	0.541	0.608	0.842
7	0.498	0.546	0.540
8	0.541	0.698	0.587
9	0.498	0.642	0.526
10	0.477	0.754	0.499
11	0.519	0.580	0.555
12	0.476	0.680	0.614
13	0.561	0.568	0.614
14	0.477	0.695	0.499
15	0.793	0.846	0.806
16	1.000	1.000	1.000
17	0.960	1.000	0.960
18	0.861	0.696	0.898
19	0.657	0.745	0.676
20	0.739	0.753	0.756
21	0.655	0.589	0.675
22	0.605	0.656	0.626
23	0.401	0.603	0.414
24	0.991	0.825	1.000
25	0.899	0.715	0.918
26	0.622	0.634	0.641
27	0.805	0.648	0.826
28	0.713	0.765	0.725
29	0.988	0.818	1.000
30	1.000	1.000	1.000

5. CONCLUSIONS

In the DEA literature, a great number of DEA studies have focused on two-stage production systems, where all outputs from the first stage are intermediate products that make up the inputs to the second stage. However, in some situations DMUs have a two-stage structure that the inputs of the system can be freely allocated among two stages. Efficiency formulation and linearizing transformation may be two important issues when we use the conventional two-stage DEA models. In this paper, a relational DEA model is introduced, taking into account the series relationship between two stages, to measure the overall efficiency of the system as well as the efficiencies of two sub-processes. By using the proposed model, we are able to recognize the inefficient sub-processes and make future improvements.

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