

NBER WORKING PAPER SERIES

FUND MANAGERS, CAREER CONCERNS, AND ASSET PRICE VOLATILITY

Veronica Guerrieri  
Péter Kondor

Working Paper 14898  
<http://www.nber.org/papers/w14898>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
April 2009

We are grateful to Fernando Alvarez, Cristina Arellano, Péter Benczúr, Emmanuel Farhi, Bernardo Guimaraes, Nobuhiro Kiyotaki, Narayana Kocherlakota, István Kónya, Arvind Krishnamurthy, Guido Lorenzoni, Guillermo Ordonez, Rafael Repullo, Balázs Szentes, Nancy Stokey, Harald Uhlig, Dimitri Vayanos, Laura Veldkamp, Neng Wang, Iván Werning, and seminar participants at Central Bank of Hungary (MNB), Washington University in St. Louis, Northwestern, 2007 SED Annual Meeting in Prague, 2007 EEA-ESEM Annual Meeting in Budapest, CREI-CEPR Conference on Sovereign Risk, University of Chicago, Conference "Beyond Liquidity" at the Chicago GSB, 2008 Minnesota Workshop in Macroeconomic Theory, Wharton School, Philadelphia Federal Reserve Bank, Fuqua, CEMFI, Harvard University. This research was funded in part by the Initiative on Global Markets at the University of Chicago Graduate School of Business. An early version of this paper was circulated under the title "Fund Managers and Defaultable Debt." The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2009 by Veronica Guerrieri and Péter Kondor. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Fund Managers, Career Concerns, and Asset Price Volatility  
Veronica Guerrieri and Péter Kondor  
NBER Working Paper No. 14898  
April 2009  
JEL No. D53,D8,G1

**ABSTRACT**

We propose a model where investors hire fund managers to invest either in risky bonds or in riskless assets. Some managers have superior information on the default probability. Looking at the past performance, investors update beliefs on their managers and make firing decisions. This leads to career concerns which affect investment decisions, generating a positive or negative "reputational premium". For example, when the default probability is high, uninformed managers prefer to invest in riskless assets to reduce the probability of being fired. As the economic and financial conditions change, the reputational premium amplifies the reaction of prices and capital flows.

Veronica Guerrieri  
University of Chicago  
Booth School of Business  
5807 South Woodlawn Avenue  
Chicago, IL 60637  
and NBER  
vguerrie@chicagobooth.edu

Péter Kondor  
Central European University  
Department of Economics  
Nador u. 9.  
Budapest, H-1051  
Hungary  
kondorp@ceu.hu

# 1 Introduction

The financial turmoil started in the summer of 2007 and exploded in 2008 has costily testified that the incentives of financial intermediaries and the performance of the economy are closely related. In this paper, we provide a general equilibrium model of the interaction between investors, financial intermediaries and the users of capital. In particular, we focus on the career concerns of fund managers who take into account the effect of their portfolio choices on the probability of being fired. We show that career concerns can magnify the reaction of asset prices and capital flows to both real and financial shocks.

Although our model can be applied to any risky asset, in this paper we focus on debt instruments. In the few years before the recent subprime crises, many market observers were concerned about a growing “overenthusiasm” for risky investments in debt instruments, including high-yield corporate bonds, mortgage-backed assets and emerging market bonds. One observer notices:

Bonds issued by Ecuador, which is politically very unstable, are among the riskiest bets in the emerging markets. It is hard to predict what will happen there next month, let alone in 10 years time. Yet buyers appear to be ready and willing to line up for a sale by the government of up to Dollars 750m in 10-years bonds, the first international bond offer since the country defaulted in 1999. The issue, [...] is the latest example that the prolonged love affair with emerging market debt is far from over. (December 9, 2005, Financial Times).

A similar observation related to the role of financial intermediaries in the leveraged buy-out deals follows:

The head of one of the biggest commercial lenders in the US describes the amount of leverage on some buy-out deals as “nutty”. Much of the wildest lending is being done by hedge funds awash with cash, he says. “Some funds believe they have to invest the money even if it’s not a smart investment. They think the people that gave them the money expect them to invest it. But it’s madness.” (March 14, 2005, Financial Times)

Figure 1 shows the pattern of the yield spreads of a sample of emerging market bonds, the AAA and the B-graded corporate bonds, and the investment graded commercial mortgage-backed assets, between October 1994 and October 2008. The figure shows at least two periods in which all spreads shrunk to very low levels, close to the AAA corporate spreads: in 1996-1997 and then again from 2005 to the summer of 2007. Observers describe these

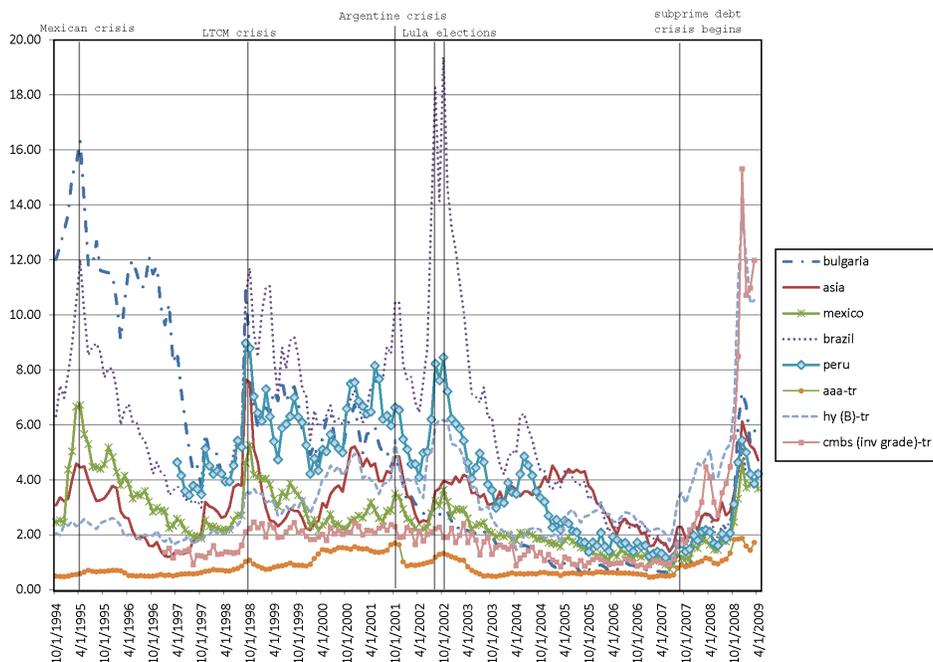


Figure 1: The JPMorgan EMBI+ spread for Asia, Brazil, Bulgaria, Mexico, Peru, the yield spread of AAA corporate bonds and an index of the yield spread of B-graded corporate bonds and investment graded commercial mortgage-backed assets between October 1994 and April 2009. Source: Datastream, St. Louis Fed.

periods as periods of overenthusiasm, which typically occur right before the emergence of a crisis (e.g. Kamin and von Kleist, 1999, Duffie et al., 2003). The figure also shows at least four episodes of high turbulence in which the spreads of many high-risk bonds jump up and capital tends to flow out of these markets, a phenomenon dubbed as flight-to-liquidity or flight-to-quality. Our model is able to rationalize both episodes of overenthusiasm and episodes of flight-to-liquidity.

We propose a model where investors delegate their portfolio decision to risk-neutral fund managers. We assume that fund managers can invest either in risk-less assets or in risky bonds, and differ in the degree of information about the default risk. The core of our model builds on the career concerns of the fund managers. Every period, each investor has a manager working for him. We consider an extreme environment, where a small portion of informed managers can perfectly predict if the risky bond is going to default, and a mass of uninformed managers know only the ex ante default probability. At the end of the period, the investor updates his belief based on his manager's performance, and decides whether to keep him or to hire a new one. The investors' firing decision distorts the investment decision

of uninformed managers who would like to be perceived as informed managers.

One of our main result is that managers' career concerns impose a premium on the price of risky bonds, which may be positive or negative depending on the default probability. Uninformed fund managers try to time the market in order to behave as if they were informed and knew in advance if there would be default or not. Default hurts the reputation of uninformed managers who invest in the risky bond, and no default hurts the reputation of uninformed managers who invest in the riskless asset. Thus, when the probability of default is high, the premium for investing in the risky bond is positive to compensate for the risk of being fired. When instead the default probability is low, the risky bond will trade at a negative premium. A shock either to the ex ante default probability or to the financial market, such as a change in the risk-free rate, may generate a switch in the sign of the premium. This tends to amplify the reaction of the bond price to the shock, in comparison to an economy with no career concerns.

We then consider an extended version of the model where we endogenize the supply of risky bonds and the default rule. In particular, we introduce a continuum of entrepreneurs who have access to a risky project and can issue risky bonds. They take the bond price as given and choose how much to borrow in order to cover their consumption and finance their project. After observing the realization of the project's productivity, they can decide whether to pay back the outstanding debt or to default and suffer a loss. The interaction between managers and entrepreneurs determines the price of the risky bond, the probability of default and the volume of debt in the economy. The investment choice of the managers determines the required bond price for a given probability of default. The default rule of the entrepreneurs determines the bond's ex ante probability of default for a given price. Hence, the equilibrium bond price and the default probability are jointly determined by the financial market and the fundamentals of the risky project.

We show that, once we endogenize the supply side of the model, there is a feedback effect from asset pricing to the real economy, which magnifies further the reaction of prices and capital flows to financial and real shocks. On the financial side, a higher default probability leads to a lower bond price, also because of a larger premium. On the real side, when borrowing is more expensive, entrepreneurs have to make larger repayments and, hence, default with higher probability. In general equilibrium, these two mechanisms reinforce each other and generate excess volatility in bond prices.

**Literature review.** To our knowledge, this is the first paper to show that imperfect information about fund managers' skills leads to amplification of shocks, counter-cyclical premium on risky assets and volatile asset prices. Our work is related to several areas of macroeconomics and finance.

First, our paper is related to herding models, such as Scharfstein and Stein (1990), Zweibel (1995), and Ottaviani and Sorensen (2006), where, as in our paper, decision makers with career concerns make inefficient decisions to convince their clients that they are informed. However, there are two main differences with our work. On the one hand, this literature traditionally concentrates on partial equilibrium models while our focus is on the interaction of career concerns and asset prices. On the other hand, these papers present mechanisms in which each decision maker herds on others' decision because going against the average action is a bad signal about his ability. In our model, at the equilibrium prices, fund managers choose the inefficient action regardless of other managers' decision. That is, there are no strategic complementarities. The closest paper to ours is Rajan (1994), who shows that herding might motivate bank executives to overextend credit in good times by amplifying real shocks. In contrast to our model, Rajan (1994) predicts that in bad times banks provide the right amount of credit while we argue that in bad times managers underinvest in the risky bonds.

Second, there is a growing literature which analyzes the effect of delegated portfolio management on asset prices, including Allen and Gorton (1993), Shleifer and Vishny (1997), Vayanos (2003), Cuoco and Kaniel (2007), He and Krishnamurthy (2008). Unlike our work, most of this literature takes managers' distorted incentives as given. A notable exception is Dasgupta and Prat (2006, 2008) and Dasgupta, Prat and Verardo (2008) who introduce herding into a Glosten-Milgrom type of sequential trading model. They show that reputational concerns can lead to excessive trading, slow revelation of information and (if the market maker has market power) biased prices. They are the first to use the term reputational premium and to point out that the potential trade-off between reputation and trading profits might lead managers to choose bets with negative net present value. However, we have a different focus. We are interested in how career concerns amplify the response of price and capital flows to financial and real shocks. In particular, we build a standard, competitive, asset pricing model to show that career concerns have systematic effects on prices.

Our paper is also related to a large literature on the propagation and amplification of fundamental shocks due to the interaction between asset values and collateralized lending. Seminal papers in this area are Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) on the macro-side, and Gromb and Vayanos (2002) on the finance side<sup>1</sup>. The main difference with our mechanism is that these papers have typically an asymmetric distortion, given that collateral constraints build into the model an external finance premium, usually generating

---

<sup>1</sup>See also Aghion, Banerjee and Piketty (1999), Rampini (2003), Krishnamurthy (2003), Gai, Kondor and Vause (2005), Guerrieri and Lorenzoni (2008) on the macro side and Danielsson, Shin and Zigrand (2004), Morris and Shin (2004), Bernardo and Welch (2004) on the finance side.

underinvestment. In our model, instead, we microfound the financial distortion and we generate a premium that can be either positive or negative.

Finally, our model can be naturally interpreted as a model of sovereign debt. In this sense is related to the vast literature on financial crisis in emerging economies and reversal of capital flows, including Atkenson (1991), Cole and Kehoe (2000), Calvo and Mendoza (2000), Caballero and Krishnamurthy (2003), Benczur and Ilut (2005), Aguiar and Gopinath (2006), Uribe and Yue (2006), and Kovrijnykh and Szentes (2007), Arellano (2008). This strand of literature abstracts from the effects of intermediation in financial markets, and could be interestingly complemented with our mechanism.

The rest of the paper is organized as follows. In Section 2, we introduce an example to illustrate the main mechanism of our model. In Section 3, we describe the model, and define and characterize an equilibrium. In Section 4, we analyze an extended version of the model, where the supply of risky bonds and the default decision are endogenous. In Section 5, we analyze an economy where productivity is persistent and we propose some numerical exercises. Finally, Section 6 concludes. The Appendix includes all the proofs which are not in the text.

## 2 An Example

In this section, we introduce a simple example to show the main mechanism of the model, that is, how prices may be distorted by an incentive scheme which rewards successful fund managers over and above the returns of their investment.

Assume that a large group of risk-neutral fund managers have to decide whether to invest a unit of capital in a risky asset or in a riskless asset. The risky asset has price  $p$  and pays 1 if the good state realizes and 0 if the bad state realizes. The probability of the bad state is equal to  $q$ . The riskless asset pays the safe return  $R < 1/p$ . Just for this example, assume that a manager obtains a *bonus*  $W$  if he succeeds in his investment, that is, if he invests in the risky bond when there is no default or in the riskless asset otherwise. The riskless asset is in infinite supply, while the supply of the risky bond is fixed and smaller than the total capital invested by the managers.

It is straightforward that the bond market clears if and only if managers are indifferent between investing in the risky bond and in the riskless asset. Hence, the equilibrium price of the risky bond has to satisfy the following indifference condition

$$(1 - q)(1/p + W) = R + qW. \tag{1}$$

The left-hand side of equation (1) represents the expected payoff of a manager who invests in the risky bond. With probability  $1 - q$  there is no default and the manager gets a return  $1/p$  and the bonus  $W$ . If instead there is default, the manager gets zero revenues and no bonus. Similarly, the right-hand side of equation (1) represents the expected payoff of a manager who invests in the riskless asset. He gets always a return  $R$ , but he obtains the bonus only if there is default.<sup>2</sup>

In order to characterize the price distortion generated by the bonus  $W$ , we define the *premium*  $\Pi$  as the difference between the expected return and the risk free rate

$$\Pi \equiv \frac{1 - q}{p} - R.$$

The indifference condition (1) immediately implies that  $\Pi = 0$  when there is no bonus scheme, that is,  $W = 0$ . In this case, fund managers care only about the expected returns of the bond and the premium is zero. When instead  $W > 0$ , the premium can be negative or positive. In particular, if  $q > 1/2$ , the payoff of the risky bond is skewed to the left as the probability of default is larger than the probability of no default. In this case, investing in the riskless asset has an advantage over the risky bond as this ensures the bonus payment with larger probability. If the expected return of the two assets were equal, all managers would prefer the riskless one, because of this advantage. Thus, in equilibrium there must be a positive premium on the risky bond to induce managers to hold it. Similarly, if  $q < 1/2$  the payoff of the risky bond is skewed to the right. In this case, the risky bond has an advantage and the premium is negative.

This simple example is suggestive, but it clearly calls for some microfoundations behind the bonus scheme. Why investors should ex post reward managers who make successful investment? The story we have in mind is a story of career concerns, which needs both a dynamic environment and some form of heterogeneous information. In the rest of the paper we build a dynamic general equilibrium model of delegated portfolio management with informed and uninformed managers. Investors rationally learn about the type of the fund managers based on their past performance. Uninformed managers' career concerns generate a reputational premium similar to the one described in this example. In particular,  $W$  becomes an equilibrium object equal to the continuation utility of an employed uninformed manager. We show that small shocks to the financial market or to the fundamentals of the risky project may lead to large changes in asset prices and capital flows, due to the presence of the reputational premium. In Section 4, we also endogenize the default probability  $q$  and the supply of the risky bond. We introduce entrepreneurs who issue the risky bond and can

---

<sup>2</sup>The equilibrium price is consistent with the assumption that  $1/p > R$  if  $R > W(1 - 2q)/q$ .

decide to default on their loans. This generates a feed-back effect from asset pricing to the real economy, which further magnifies the reaction of prices to shocks.

## 3 Baseline Model

### 3.1 Set up

Consider an infinite horizon economy with discrete time. There is a mass  $\Gamma$  of risk-neutral investors and two investment possibilities: a riskless asset, in infinite supply, which pays  $R$  units of consumption per unit of capital invested, and a risky bond with fixed supply  $b$  and price in terms of capital  $p_t$ , which pays 1 unit of consumption if there is no default and 0 otherwise. Let  $\chi_{t+1}$  denote the default indicator such that  $\chi_{t+1} = 1$  if there is default and  $\chi_{t+1} = 0$  otherwise. Assume that the ex-ante probability of default  $E(\chi_{t+1})$  is equal to  $q$ .

At any time  $t$ , each investor has one unit of capital and needs a fund manager to invest it. There are two types of risk-neutral fund managers: informed and uninformed.<sup>3</sup> Informed managers know in advance  $\chi_{t+1}$ , that is, they can perfectly predict if there is going to be default.<sup>4</sup> Uninformed managers, instead, expect the bond to default with probability  $q$ . There is a mass  $M^I$  of informed managers and a larger continuum of uninformed managers. Fund managers do not have any capital, and need to be employed by an investor to make any investment decision. Each investor can employ only one fund manager and a fund manager can work only for a single investor. Moreover, investors do not know the managers' type.

At time  $t$  there is a mass  $\Gamma_t^s$  of employed managers of type  $s$ , and all investors have a manager working for them, so that  $\Gamma_t^I + \Gamma_t^U = \Gamma$ . Then, employed managers choose how to invest the unit of capital they manage. Next, investors observe the return of their manager's investment and decide whether to fire him or not. Moreover, they receive a signal that reveals the type of an uninformed manager with probability  $1 - \omega$ .<sup>5</sup> Each manager has a probability  $1 - \delta$  to die, in which case a new manager of the same type is born. Finally, all investors who do not have a manager, either because they have fired him or because he is dead, search for a new one. At the same time, the unemployed managers decide whether to pay a cost  $\kappa$  to search for a job. Let  $N_t^s$  be the mass of managers of type  $s$ , with  $s = I, U$ , who decide to look for a job. The probability for a manager to find a job, denoted by  $\mu_t$ , is

---

<sup>3</sup>Risk neutrality ensures that the demand for the risky bond is perfectly elastic, making the model more tractable. Moreover, it allows us to focus on the premium on risky assets coming exclusively from career concerns and not from risk.

<sup>4</sup>The extreme assumption that informed managers have perfect information is not crucial for the analysis. However, it allows not to keep track of the history of beliefs, making the model more tractable.

<sup>5</sup>As we will discuss later on, this exogenous signal makes the analysis more tractable because it guarantees that we can focus on equilibria where a manager who does succeed in his investment is never fired.

equal to the ratio of the mass of investors searching for a manager to the mass of managers searching for a job.<sup>6</sup>

We look for a stationary equilibrium where the mass of informed and uninformed employed managers,  $\Gamma_t^I$  and  $\Gamma_t^U$ , and the matching probability,  $\mu_t$ , are constant over time. Hence, from now on we can drop the time dependence for these objects. Moreover, for simplicity, we fix the contract between investors and fund managers: fund managers keep a share  $\gamma \geq \kappa/R$  of the revenues and leave the rest to the investors.<sup>7</sup> Both investors and managers fully consume their net revenues in each period.

Any employed manager selects a demand schedule  $d^I(p_t, \chi_{t+1}) \in [0, 1]$  if he is informed and  $d^U(p_t) \in [0, 1]$  if he is uninformed. At the same time, there is a mass  $y_t$  of noise traders, with one dollar each, who demand the bond, where  $y_t$  is a random variable uniformly distributed on the support  $[0, \bar{y}]$ , with  $\bar{y} \in [M^I, b]$ . After collecting all the demand schedules, an auctioneer selects the equilibrium price  $p_t$  and assigns the risky bonds to the managers and the noise traders. The selected price and bond allocation must be consistent with the submitted demand schedules and with the bond market clearing. Let us denote by  $x_t^I$  and  $x_t^U$  the equilibrium fraction of informed and uninformed managers who obtain the bond. The market clearing condition is then

$$\Gamma^I x_t^I + \Gamma^U x_t^U + y_t = p_t b. \quad (2)$$

The right-hand side of the market clearing condition represents the value of the supply of bonds. The left-hand side, instead, represents the demand of bonds, which comes from three different sources: 1) a proportion  $x_t^I$  of informed employed managers, 2) a proportion  $x_t^U$  of uninformed employed managers, and 3) a mass  $y_t$  of noise traders.

At the beginning of time  $t$ , each investor has a manager  $j$  working for him that he believes is informed with probability  $\eta_t^j$ . Let  $\theta_t^j$  be an indicator variable, which is 1 if manager  $j$  is allocated a unit of the risky bond, and 0 otherwise. A law of large number ensures that  $x_t^s$  represents also the probability for an agent of type  $s$  of receiving the risky bond. At the end of time  $t$ , the investor observes  $\theta_t^j$ , gets his share of the realized returns, and learn whether there has been default or not. Moreover, if his manager is uninformed, he discovers his type with probability  $1 - \omega$ . Then, according to the Bayes' Rule, he updates his belief about the type of his manager to  $\eta_{t+1}^j$ . Conditional on his posterior, he chooses his firing strategy  $\phi(\eta_{t+1}^j)$ , that is, whether to keep his manager for next period ( $\phi(\eta_{t+1}^j) = 0$ ), or to fire him and hire a new one ( $\phi(\eta_{t+1}^j) = 1$ ). Clearly, the investor's firing decision is affected by the

---

<sup>6</sup>The job search structure guarantees the existence of a stationary equilibrium.

<sup>7</sup>The assumption that  $\kappa \leq \gamma R$  is sufficient to ensure that it is profitable for an informed manager to search for a job.

probability that a new hire is informed. The key feature of our model is that manager  $j$  knows that his investment decision will affect the investors' firing decision by changing his posterior belief. This generates career concerns affecting the investment strategy that are at the core of our model.

## 3.2 Equilibrium

Let us first introduce the definition of a stationary equilibrium for any  $M^I > 0$ . Next, we will propose the type of stationary equilibrium we are interested in, an *interior equilibrium*, we will characterize it, and show under which assumptions it exists. Finally, we will explore the limit of an interior equilibrium as  $M^I \rightarrow 0$ , which is a very tractable and insightful special case.

**Definition 1** *For a given  $M^I > 0$ , a stationary equilibrium is a demand schedule for informed managers,  $d^I(p_t, \chi_{t+1})$ , a demand schedule for uninformed managers,  $d^U(p_t)$ , a firing strategy for investors,  $\phi(\eta_{t+1}^j)$ , bond allocations for the informed and uninformed managers,  $x^I(\chi_{t+1}, y_t)$  and  $x^U(\chi_{t+1}, y_t)$ , a price  $p(\chi_{t+1}, y_t)$ , a constant mass of employed informed and uninformed managers,  $\Gamma^I$  and  $\Gamma^U$ , a constant matching probability  $\mu$ , and an updating rule  $\eta_{t+1}^j = \zeta(\eta_t^j, \theta_t^j, p_t, \chi_{t+1})$ , such that*

1. *investors maximize their expected utility, taking as given equilibrium price, allocation and strategies of other agents;*
2. *fund managers maximize their expected utility, taking as given equilibrium price, allocation and strategies of other agents;*
3. *bond price and allocations are consistent with the managers' demand schedules and with market clearing;*
4.  *$\Gamma^I$ ,  $\Gamma^U$  and  $\mu$  are consistent with free entry in the labor market for fund managers;*
5. *investors' beliefs are consistent with Bayes' rule.*

## 3.3 Interior Equilibrium

When  $M^I > 0$ , employed uninformed managers face the risk of being fired and, hence, their investment decisions are affected by their expected future utility. Moreover, they can potentially extract some information about the strategy of the informed managers from the equilibrium price. We focus on equilibria where uninformed managers are typically the

marginal traders, that is, are indifferent between investing in the bond and in the risk-free asset whenever prices are not fully revealing. We call this type of equilibrium, an “interior” equilibrium. Let  $z$  represent the total potential demand of informed managers and noise traders, that is,  $z(\chi_{t+1}, y_t) \equiv \Gamma^I (1 - \chi_{t+1}) + y_t$ .

**Definition 2** *A stationary interior equilibrium is an equilibrium where*

(i) *prices are*

$$p(\chi_{t+1}, y_t) = \begin{cases} y_t/b & \text{if } z(\chi_{t+1}, y_t) \in [0, \Gamma^I) \\ p^* & \text{if } z(\chi_{t+1}, y_t) \in [\Gamma^I, \bar{y}] \\ 1/R & \text{if } z(\chi_{t+1}, y_t) \in (\bar{y}, \bar{y} + \Gamma^I] \end{cases}, \quad (3)$$

with  $p^* \in (\bar{y}/b, 1/R)$ ;

(ii) *managers’ demand schedules are*

$$d^I(\chi_{t+1}, p_t) = \begin{cases} 1 - \chi_{t+1} & \text{if } p_t < 1/R \\ \{0, 1\} & \text{if } p_t = 1/R \end{cases}$$

and

$$d^U(p_t) = \begin{cases} 0 & \text{if } p_t \leq \bar{y}/b \\ \{0, 1\} & \text{if } p_t \in (\bar{y}/b, 1/R] \end{cases};$$

(iii) *managers’ bond allocations are*

$$x^I(\chi_{t+1}, y_t) = \begin{cases} 1 - \chi_{t+1} & \text{if } z(\chi_{t+1}, y_t) \in [0, \bar{y}] \\ \frac{b/R - y_t}{\Gamma} & \text{if } z(\chi_{t+1}, y_t) \in (\bar{y}, \bar{y} + \Gamma^I] \end{cases}$$

and

$$x^U(\chi_{t+1}, y_t) = \begin{cases} 0 & \text{if } z(\chi_{t+1}, y_t) \in [0, \Gamma^I) \\ \frac{p^* b - z(\chi_{t+1}, y_t)}{\Gamma - \Gamma^I} & \text{if } z(\chi_{t+1}, y_t) \in [\Gamma^I, \bar{y}] \\ \frac{b/R - y_t}{\Gamma} & \text{if } z(\chi_{t+1}, y_t) \in (\bar{y}, \bar{y} + \Gamma^I] \end{cases}; \quad (4)$$

(iv) *investors’ firing rule requires to fire manager  $j$  if the exogenous signal reveals that  $j$  is uninformed or  $\theta_t^j \neq 1 - \chi_{t+1}$  and  $p(\chi_{t+1}, y_t) < 1/R$ , and keep him otherwise.*

An interior equilibrium is characterized by a mass  $\Gamma^I$  of employed informed managers and an interior price  $p^*$ , which ensure that conditions (i)-(iv) characterize a stationary equilibrium. In particular,  $\Gamma^I$  must be consistent with free entry in the labor market for fund managers, while  $p^*$  must be such that the uninformed managers are indifferent between demanding the risky asset and the risk-free one when their only information is the ex-ante probability of default  $q$ .

In an interior equilibrium, three possible revelation regimes arise:  $p(\chi_{t+1}, y_t) = 1/R$  reveals that there is going to be default,  $p(\chi_{t+1}, y_t) = \bar{y}/b$  reveals that there is going to be no default, and, thanks to the uniform distribution of  $y$ ,  $p(\chi_{t+1}, y_t) = p^*$  does not reveal any information. If  $p(\chi_{t+1}, y_t) = 1/R$ , the two assets are equivalent and all managers are indifferent between them. If  $p(\chi_{t+1}, y_t) = \bar{y}/b$ , there is revelation of default and all managers demand the riskless asset. Finally, when  $p(\chi_{t+1}, y_t) = p^*$ , informed managers know in advance if there is going to be default or not, and hence can time the market perfectly, by demanding the risky bond if and only if there is not going to be default. The uninformed managers, however, do not have this superior information and  $p^*$  makes them indifferent between obtaining the bond or not. This is the notion of interior solution we refer to when we call this equilibrium an interior equilibrium. It follows that  $\Gamma^I/\bar{y}$  represents the probability that in equilibrium the price is fully revealing. The assumption that  $\bar{y} > M^I$  ensures that prices are not always fully revealing, that is,  $\Gamma^I/\bar{y} < 1$ . According to the demand schedules, the auctioneer determines the fraction of uninformed and informed managers who obtain the bond in order to guarantee that the bond market clears. Finally, investors fire their manager whenever they have an exogenous signal that he is uninformed and, if  $p(\chi_{t+1}, y_t) < 1/R$ , when their manager makes an unsuccessful investment.

In equilibrium, the values of  $\Gamma^I$  and  $p^*$  are jointly determined with the values of the expected utility of an employed uninformed manager,  $W$ , and the probability that an unemployed manager finds a job,  $\mu$ . In particular,  $\Gamma^I$  must be such that the mass of employed informed managers stay constant after  $\delta$  of them die and  $\mu$  of the unemployed ones are hired, that is, it satisfies

$$\Gamma^I = \delta\Gamma^I + \mu(M^I - \delta\Gamma^I). \quad (5)$$

Due to free entry, uninformed managers looking for a job get zero ex ante expected utility, that is,

$$\mu W - \kappa = 0, \quad (6)$$

while the expected utility of an employed uninformed manager  $W$  satisfies

$$W = \gamma R + \delta\omega [\Gamma^I/\bar{y} + q(1 - \Gamma^I/\bar{y})] W. \quad (7)$$

When the price is fully revealing, with probability  $\Gamma^I/\bar{y}$ , an uninformed manager always gains  $\gamma R$ , given that if he learns that there is default he invests in the risk-free asset, while if he learns that there is no default, the bond price will be equal to  $1/R$ . Moreover, he is never fired and gets the continuation utility  $W$ , as long as he does not die and there is no exogenous signal. When instead the price does not reveal any information, with probability

$1 - \Gamma^I/\bar{y}$ , uninformed managers are indifferent in each point in time between investing in the risk-free asset or in the risky bond, and hence their expected utility can be calculated as the value of always investing in the risk-free asset. Again, they will always gain  $\gamma R$ , but, in this case, they will get the continuation utility  $W$  only if there is default, with probability  $q$ , no exogenous signal, with probability  $\omega$ , and no death, with probability  $\delta$ . Finally, the indifference condition for an uninformed manager is given by

$$(1 - q)(\gamma/p^* + \delta\omega W) = \gamma R + q\delta\omega W, \quad (8)$$

This condition is the analogous to condition (1) in the example in Section 2. The left-hand side of equation (8) represents the expected payoff of a manager who invests in the risky bond. With probability  $1 - q$ , there is no default and the manager gets a return  $\gamma/p^*$ . If he survives and there is no exogenous signal, he is not fired and gets expected continuation utility  $W$ . If instead there is default, the manager gets zero revenues, is fired, and gets 0 continuation utility, due to free entry. Similarly, the right-hand side of equation (8) represents the expected payoff of a manager who invests in the risk-free bond. He gets always a return  $\gamma R$ , but only if he survives, there is default, and there is no exogenous signal, he is not fired and gets expected continuation utility  $W$ . Otherwise, the investor learns that he was not informed and fires him. The system of equations (5)-(8) determine the equilibrium values of  $\Gamma^I$ ,  $p^*$ ,  $W$ , and  $\mu$  for a given  $q$ , so defining a map  $P(\cdot)$  from  $q$  to  $p^*$ .

The following assumptions are sufficient to ensure that such a stationary interior equilibrium exists:

$$\frac{M^I + \bar{y}}{b} < P(q) < \min \left\{ \frac{\Gamma - M^I}{b}, \frac{1}{R} \right\}, \quad (A1)$$

and

$$\omega < \frac{1}{1 + \delta}. \quad (A2)$$

Assumption A1 guarantees that there are always some uninformed managers investing in both the risky bond and the riskless asset. Assumption A2 ensures that the proportion of informed managers among those who are searching for a job is sufficiently small that if an uninformed manager does not make a mistake, he is not fired.<sup>8</sup> This last assumption is not crucial, but it makes the analysis simpler.

**Proposition 1** *Suppose A1 and A2 hold. Then an interior equilibrium exists.*

---

<sup>8</sup>See the proof of Proposition 1 in the Appendix.

### 3.4 Limit Equilibrium

Let us now focus on the model with  $M^I \rightarrow 0$ . This limit case is very tractable and insightful at the same time. We show that as  $M^I \rightarrow 0$ , the sequence of interior equilibria converges to a *limit interior equilibrium* where the bond price never reveals any information, and is constant over time. Intuitively, this can be the case because as the fraction of informed manager is infinitesimal, the uninformed managers will have to demand essentially all the bonds supplied and hence won't learn any information from the equilibrium price.

**Definition 3** *When  $M^I \rightarrow 0$ , a limit interior equilibrium is an equilibrium where the price  $p^* \in (\bar{y}/b, 1/R)$  is determined by the indifference condition of the uninformed managers; the informed managers' demand schedule is  $d^I(\chi_{t+1}) = 1 - \chi_{t+1}$  and their bond allocation is  $x^I(\chi_{t+1}) = d^I(\chi_{t+1})$ ; the uninformed managers' demand schedule is  $d^U = \{0, 1\}$  and their bond allocation is  $x^U(y_t) = (p^*b - y_t) / \Gamma$ ; the investors' strategy is to fire manager  $j$  if he receives a negative exogenous signal or if  $\theta_t^j \neq 1 - \chi(a_{t+1})$ , and keep him otherwise.*

In a limit interior equilibrium, career concerns always affect the bond price by generating a reputational premium. The equilibrium behaves in a similar way to the general one, in the case in which the price does not reveal any information. Investors fire the managers who reveal to be uninformed, while informed managers never make a mistake and are never fired. The market clearing condition for the risky bond determines the fraction  $x^U(y_t)$  of uninformed managers who invest in the risky bond. Assumption A1 guarantees that  $x^U(y_t) \in (0, 1)$  for any  $y_t$ , so that there are always some uninformed managers investing in the risky bond and some investing in the risk-free asset. Hence, it must be that the uninformed managers are indifferent between the two investment possibilities. For a given default probability  $q$ , the equilibrium price  $p^*$  is determined by the same indifference condition (8), where the expected continuation utility of an employed uninformed manager  $W$  satisfies condition (7) with  $\Gamma^I \rightarrow 0$ , which yields

$$W = \frac{\gamma R}{1 - \delta \omega q}. \quad (9)$$

Combining (8) and (9) implies that the limit equilibrium is characterized by

$$p^* = P^L(q) \equiv \frac{1 - q}{R} \left[ \frac{1 - \delta \omega q}{1 - \delta \omega (1 - q)} \right].$$

Similarly to section 2, let  $\Pi$  be the difference between the expected repayment and the risk free rate  $R$ , that is,

$$\Pi \equiv \frac{1 - q}{p} - R. \quad (10)$$

We call  $\Pi$  the *reputational premium* because it characterizes the price distortion generated by the career concerns of the uninformed managers.

As a point of comparison, consider a model with  $M^I = 0$ . In this case, all managers are uninformed, so investors will be indifferent between keeping the manager they started with and hiring a new one. Then, there exists an equilibrium where managers are never fired and maximize their period by period profit. In this case, the bond price is determined by the standard no-arbitrage condition

$$\frac{1 - q}{p} = R. \quad (11)$$

We call this equilibrium the *benchmark equilibrium*.

In the benchmark equilibrium, the standard arbitrage condition (11) immediately implies that  $\Pi$  is equal to zero. When instead there is a positive measure of informed managers,  $M^I > 0$ , the reputational premium can be negative or positive. Typically, it is positive when  $q$  is sufficiently large and negative when  $q$  is sufficiently small. Betting on large probability events is especially attractive for an uninformed manager with career concerns, because it increases the chance that he will not make an unsuccessful decision and will not be fired. The equilibrium price reflects this preference for large probability events. Fund managers are willing to get a lower expected return in exchange for a large probability of not being fired. It is interesting to point out that there is a discontinuity at  $M^I = 0$ , given that the benchmark equilibrium disappears as  $M^I > 0$ , even if  $M^I \rightarrow 0$ .

In the benchmark equilibrium the pricing rule is given by the standard no-arbitrage condition (11), that is,  $p^B = P^B(q) = (1 - q)/R$ . Notice that the reputational premium is zero iff  $q = 1/2$  and both  $P^L(q)$  and  $P^B(q)$  are decreasing in  $q$ . This implies that  $\Pi > 0$  iff  $q > 1/2$  and  $\Pi < 0$  otherwise.

Proposition 1 guarantees that under assumptions A1 and A2, a limit interior equilibrium exists. The equilibrium regime is determined jointly by the fundamentals of the risky project and the state of the financial market. Figure 2 represents graphically both the limit equilibrium for an economy with career concerns (L) and the benchmark equilibrium (B) when  $M^I = 0$ .

In this numerical example we set the parameters so that  $q < 1/2$ , and hence  $P^L(q) > P^B(q)$  and the reputational premium is positive.<sup>9</sup> It is immediate to see that if  $q > 1/2$  then  $P^L(q) < P^B(q)$  and the reputational premium is negative. The figure shows that in a limit interior equilibrium, when  $q$  is not too close to 1, prices react more to changes in the fundamentals of the economy, both on the supply and on the demand side. This is simply due to the fact that the pricing rule  $P^L(q)$  is steeper than the benchmark pricing rule  $P^B(q)$

---

<sup>9</sup>For the numerical example illustrated in Figure 2 we set  $u(c) = \log c$  and  $v(c) = c$ .

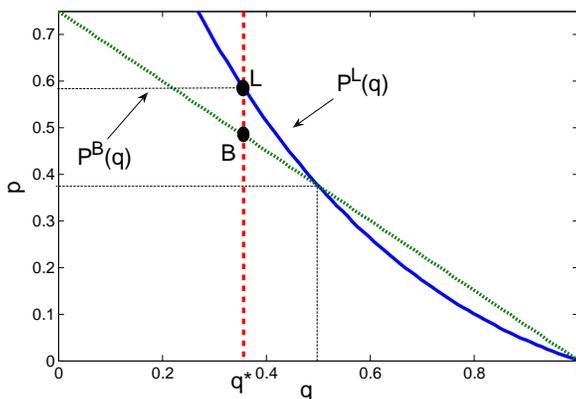


Figure 2: The solid curve and the dotted curve represent the pricing rule in the economy with career concerns and in the benchmark economy, respectively, while the dashed line shows the exogenous probability of default. Points L and B denote respectively the limit interior equilibrium when  $M^I \rightarrow 0$  and the benchmark equilibrium when  $M^I = 0$ .

around the equilibrium. For example if there is an exogenous shock either to the default probability  $q$  or to the return of the safe asset  $R$ , it is easy to see that prices react more in a limit equilibrium than in the benchmark equilibrium. This shows the essence of the amplification result that we will discuss in more detail in Section 4.4, after introducing the extended model with endogenous supply.

## 4 Endogenous Bond Supply

In this section, we extend the model in order to endogenize the supply of risky bonds and their default probability. In particular, we introduce entrepreneurs who issue the risky bond described so far in order to finance a risky project they have access to. We assume that they can decide to default ex post on their loans at a given cost. In our baseline model we have established that the equilibrium bond price is affected by the default probability. This extended model generates a feedback effect from the bond price to the borrowing and default choice. We will see that this feedback effect magnifies the amplification generated by the presence of career concerns.

### 4.1 Entrepreneurs

The set up of the economy is identical to the one presented in Section 3.1 except that now  $b$  and  $q$  are endogenous objects. There are overlapping generations of entrepreneurs who live for two periods. A generation is represented by a continuum of measure 1 of entrepreneurs.

In each period a new generation is born. Consider an entrepreneur born at time  $t$ . When she is young, she can choose to pay a cost  $k > 0$  to invest in a risky project with return  $a_{t+1}$ , distributed according to the cumulative distribution function  $F(a_{t+1})$  with  $a_{t+1} \in [0, \infty)$ , or to enjoy an outside option that gives her utility  $\bar{V}$ . Assume that  $k \geq \bar{y}$  to ensure that the supply of the risky bond is always big enough to cover the demand of the noise traders. If the entrepreneur decides to undertake the risky project, she can borrow by issuing one-period discount bonds. Define  $p_t$  the price of the bonds issued at time  $t$ . The entrepreneur chooses how much to borrow and how much to consume, taking  $p_t$  as given. When she is young, her budget constraint is

$$c_t + k \leq p_t b_{t+1}, \quad (12)$$

where  $c_t$  represents consumption at time  $t$  and  $b_{t+1}$  represents the one-period discount bonds issued at time  $t$ . There is an upper bound  $\bar{b}$  on how much entrepreneurs can borrow.

When she is old, she collects the project pay-off  $a_t$  and has the option to default on her debt  $b_{t+1}$ . If she defaults she does not repay the debt, but she suffers an output loss of  $(1 - \theta) a_t$ , that is, she keeps only  $\theta a_t$  of the return on the project. If she does not default, she has to repay her debt and consume the rest. Her budget constraint when old is

$$c_{t+1} \leq a_{t+1} - (1 - \chi_{t+1}) b_{t+1} - \chi_{t+1} (1 - \theta) a_{t+1}, \quad (13)$$

where  $\chi_{t+1} = \chi(p_t, a_{t+1}) : [\underline{p}, 1/R] \times \mathbb{R}_+ \mapsto \{0, 1\}$  denotes the default decision that the agent is making at time  $t + 1$  taking as given the price  $p_t$  and after observing the realization of  $a_{t+1}$ . Hence, the default probability  $q$  which was exogenous in the previous section is now endogenous with  $q_t \equiv E_t[\chi_{t+1}]$ .

The problem for an active entrepreneur born at  $t$  is to maximize her utility

$$u(c_t) + \beta \mathbb{E}[v(c_{t+1}) | p_t],$$

subject to (12) and (13), taking  $p_t$  as given. We assume that  $u(\cdot)$  and  $v(\cdot)$  are increasing and concave and have continuous first and second derivative. Moreover,

$$-cu''(c)/u'(c) \geq 1. \quad (A3)$$

The problem can be rewritten as

$$V(p_t) = \max_{b_{t+1} \leq \bar{b}, \chi_{t+1}} u(p_t b_{t+1} - k) + \beta E[v\{a_{t+1} - (1 - \chi_{t+1}) b_{t+1} - \chi_{t+1} (1 - \theta) a_{t+1}\} | p_t] \quad (14)$$

Ex ante, an entrepreneur will choose to undertake the risky project if and only if  $V(p_t) \geq \bar{V}$ . We denote the aggregate supply of bonds issued by entrepreneurs at a given price by  $B(p_t)$ , which corresponds to  $b$  in the previous section.

In order to have a well-behaved problem we also need to make the following assumption.

**Assumption A4.** For any  $p \in (\underline{p}, 1/R)$ , assume that the function

$$\Phi(p, b) \equiv u(pb - k) + \beta \int_0^{\frac{b}{1-\theta}} v(\theta a) dF(a) + \beta \int_{\frac{b}{1-\theta}}^{\infty} v(a - b) dF(a)$$

is quasi-concave in  $b$  for  $b \in [0, \bar{b}]$  and there exists an optimum  $V(p) = \max_{b \in [0, \bar{b}]} \Phi(p, b)$ .

Assumption A5 is satisfied for many different parametric assumptions. For example, it is satisfied if we assume  $u(c) = \log(c)$ ,  $v(c) = c$  and  $1 - F(a) = \underline{a}^\gamma a^{-\gamma}$ , with  $\underline{a} > 1/\beta(1 - \theta)$  and  $\gamma < 1$ , as in Figure 3. Intuitively, it rules out the possibility that the marginal cost of default is not large enough compared to the advantage of additional borrowing. In that case, the entrepreneur would always like to borrow more and default more often, so that problem (14) could not have a finite solution.

## 4.2 Interior Equilibrium

In this section we characterize the natural extension of a stationary interior equilibrium for our baseline model as defined in Definition 2. In the extended model, informed managers have superior information about the project productivity  $a_{t+1}$ , from which they can infer the entrepreneurs' default decision  $\chi(p_t, a_{t+1})$ . Again, there are going to be three regimes, two with full revelation with default and no default respectively, and one where no information is revealed. The last regime is the more interesting one, where uninformed managers are the marginal traders, that is, are indifferent between investing in the bond and in the risk-free asset. A sufficient assumption to ensure that prices are not always fully revealing is now  $\bar{y} \geq 2M^I$ . In the extended model, the equilibrium prices, the managers' demand schedules, the bond allocation, and the investors' firing rule are the same as in conditions (i)-(iv) in Definition 2, with the caveat that borrowing and default are now endogenous, that is,  $b = B(p_t)$  and  $\chi_{t+1} = \chi(p_t, a_{t+1})$ , and that  $\underline{p}$  substitutes  $y_t/b$  for any  $y_t$ , where  $\underline{p}$  is such that  $V(\underline{p}) = \bar{V}$ . Moreover, now the entrepreneurs' borrowing and default decisions are also equilibrium objects. In particular, the equilibrium borrowing strategy is

$$b(p_t, y_t) = \begin{cases} \bar{b} \text{ with pr. } y_t/(\underline{p}\bar{b}) \text{ and } 0 \text{ otherwise} & \text{if } p_t = \underline{p} \\ b^* & \text{if } p_t = p^* \\ \tilde{b} & \text{if } p_t = 1/R \end{cases}, \quad (15)$$

where  $b^* > \tilde{b}$ , both in  $[0, \bar{b}]$ , and the equilibrium default rule is  $\chi(p_t, a_{t+1}) = 1$  if and only if  $a_{t+1} < \hat{a}(p_t, y_t)$ , where  $\hat{a}(p_t, y_t) = b(p_t, y_t) / (1 - \theta)$ .

An interior equilibrium for the extended model is characterized by equilibrium values for  $\Gamma^I$ ,  $\underline{p}$ ,  $p^*$ ,  $b^*$ ,  $\tilde{b}$ , which ensure that the adapted versions of conditions (i)-(iv) in Definition 2, condition 15, and the default rule  $\hat{a}(p_t, y_t) = b(p_t, y_t) / (1 - \theta)$  characterize a stationary equilibrium.

Entrepreneurs do not have superior information about  $a_{t+1}$  and also learn information from the equilibrium price only when it is fully revealing. If  $p_t = \underline{p}$ , entrepreneurs know that all managers think that there is going to be default and that they are going to invest in the risk-free asset. Hence, they do not have any incentive not to default and if they are able to issue some bonds, they choose to borrow as much as they can,  $\bar{b}$ , and always to default. However, in this case, the demand for bonds comes only from noise traders and is equal to  $y_t$ . Hence, in order to have an equilibrium it must be that the entrepreneurs are indifferent between borrowing  $\bar{b}$  and defaulting with probability 1 and get their outside option  $\bar{V}$ , that is,  $\underline{p}$  must be exactly such that  $V(\underline{p}) = \bar{V}$ . At that price a fraction  $y_t / (\underline{p}\bar{b})$  of entrepreneurs becomes active and borrows from noise traders, while the others stay out of the market and get their outside option. If  $p_t = p^*$ , entrepreneurs do not have any additional information on  $a_{t+1}$  and choose  $b^*$  to maximize their expected utility. Finally, if  $p_t = 1/R$ , they know that all the managers believe that there is not going to be default and choose to borrow  $\tilde{b}$  and default whenever  $a_{t+1} < \tilde{b} / (1 - \theta)$ , which is never the case. Hence, in equilibrium it must be that  $\tilde{b} < b^*$ .

For a given  $q$  and  $p$ , the equilibrium value of  $\Gamma^I$  is jointly determined with  $W$  and  $\mu$  by equations (5)-(7), where (7) is modified to take into account that the probability of full revelation is now endogenous.<sup>10</sup> Then, equation (8) defines a pricing map  $P(\cdot)$  that gives a price for any given default probability  $q$ . Also, we can define a repayment map  $Q(\cdot)$  which assigns a default probability to any given price  $p$ . This map is given by  $Q(p) \equiv F(B(p) / (1 - \theta))$  where  $B(p)$  is implicitly defined by the optimality condition of problem (14) in the case of no information revelation, that is,

$$pu'(pB(p) - k) - \beta \int_{\frac{B(p)}{1-\theta}}^{\infty} v'(a_{t+1} - B(p)) dF(a_{t+1}) = 0. \quad (16)$$

Hence, an interior equilibrium, in the regime of no revelation, is characterized by a price  $p^*$  and a default probability  $q^*$  that solve  $p^* = P(q^*)$  and  $q^* = Q(p^*)$ .

In order to ensure that an interior equilibrium exists, we still need assumption A2 together

---

<sup>10</sup>See equation (23) in the Appendix.

with a modified version of A1, that is,

$$M^I + \bar{y} < \underline{p}B(\underline{p}) \text{ and } \Gamma - M^I > \frac{1}{R}B\left(\frac{1}{R}\right), \quad (\text{A1}')$$

which ensures that there are always some uninformed managers investing in both the risky bond and the riskless asset. In the next proposition, we state sufficient conditions for the existence of a stationary equilibrium. Let us define

$$\tilde{q} \equiv F\left(\frac{\tilde{b}}{1-\theta}\right) \text{ and } \bar{q} \equiv F\left(\frac{\bar{b}}{1-\theta}\right).$$

**Proposition 2** *For a given  $M^I > 0$ , such that assumptions A1', A2, A3, and A4 hold and*

$$P(\tilde{q}) < 1/R \text{ and } P(\bar{q}) > \underline{p}, \quad (17)$$

*an interior equilibrium exists.*

### 4.3 Limit Equilibrium

As in the baseline case, we focus on a limit interior stationary equilibrium with  $M^I \rightarrow 0$ , which is the natural extension of the equilibrium described in Definition 3.

**Definition 4** *A limit interior equilibrium with  $M^I \rightarrow 0$ , is an equilibrium where  $p^* \in (\bar{y}/b, 1/R)$  is determined by the indifference condition of the uninformed managers; the informed managers' demand function is  $d^I(\chi_{t+1}) = 1 - \chi_{t+1}$  and their bond allocation is  $x^I(\chi_{t+1}) = d^I(\chi_{t+1})$ ; the uninformed managers' demand correspondence is  $d^U = \{0, 1\}$  and their bond allocation is  $x^U(y_t) = (p^*b - y_t) / \Gamma$ ; the investors' strategy is to fire manager  $j$  if he receives a negative exogenous signal or  $\theta_t^j \neq 1 - \chi(a_{t+1})$ , and keep him otherwise; the borrowers' strategy is  $b^* \in [0, \bar{b}]$  and  $\chi(p_t, a_{t+1}) = 0$  if  $a_{t+1} \geq \hat{a}^*$  and  $\chi(p_t, a_{t+1}) = 1$  otherwise, with  $\hat{a}^* = b^* / (1 - \theta)$ .*

As in the baseline model, the limit equilibrium behaves in a similar way to an interior equilibrium with  $M^I > 0$ , in the case in which the price does not reveal any information. In particular, it can be characterized by the constant bond price and default probability,  $p^*$  and  $q^*$ , which solve the fixed point problem defined by  $p^* = P^L(q^*)$  and  $q^* = Q(p^*)$ . On the one hand, the pricing rule  $P^L(\cdot)$  is exactly the same as in the baseline model, that is,

$$P^L(q) \equiv \frac{(1-q)(1-\delta\omega q)}{[1-\delta\omega(1-q)]R}.$$

On the other hand the repayment rule is the same as described in the general model, that is,  $Q(p) \equiv F(B(p)/(1-\theta))$  with  $B(p)$  solving condition (16).

Next lemma establishes some important properties of entrepreneurs' optimal behavior.<sup>11</sup>

**Lemma 1** *In a limit equilibrium, as  $p$  increases, (i) the face value of debt  $B(p)$  decreases, (ii) the probability of default  $Q(p)$  decreases, and (iii) the value of the bonds  $pB(p)$  increases.*

As borrowing becomes cheaper, entrepreneurs need to borrow less and, hence, there is higher chance that they can repay their debt. This implies that the probability of default decreases as a function of the bond price, that is,  $q$  is downward sloping in the space  $(p, q)$  as shown in Figure 3 below. Moreover, thanks to assumption A3, the value of the borrowing  $pb$  increases, because entrepreneurs want to smooth consumption between the two periods of their life and, hence, they decrease  $b$  less than proportionally with respect to the initial increase of  $p$ .

Once again, as a point of comparison, consider the benchmark equilibrium, where entrepreneurs behave exactly as described above, but all managers are uninformed,  $M^I = 0$ , investors never fire any manager, and managers maximize their period by period profit. Such an equilibrium can also be characterized by a fixed point  $(\hat{a}^B, p^B)$ , where  $q^B = Q(p^B)$ , but the pricing rule is given by the standard no-arbitrage condition (11), that is,  $p^B = P^B(q^B) \equiv (1 - q^B)/R$ .

Proposition 2 guarantees that, under assumptions A1' and A2-A4, a limit equilibrium exists. The equilibrium regime is determined jointly by the fundamentals of the risky project and the state of the financial market. Figure 3 represents graphically both the limit equilibrium for an economy with career concerns (L) and the benchmark equilibrium (B). The prices in the two equilibria, respectively,  $p^*$  and  $p^B$ , correspond to the intersections of the repayment rule  $Q(p)$  and the corresponding pricing rule, that is,  $P^L(q)$  and  $P^B(q)$ , graphed in the space  $(p, q)$ .

Notice that the premium is zero iff  $q^* = q^B = 1/2$  and  $p^* = p^B = 1/2R$ . Moreover, both  $P^L(q)$  and  $P^B(q)$  are decreasing in  $q$ . This proves the following proposition.

**Proposition 3** *In equilibrium, one of the following reputational regimes arises: (i) if  $q^* = 1/2$ , then  $\Pi^* = 0$ , (ii) if  $q^* < 1/2$ , then  $\Pi^* < 0$ ; (iii) if  $q^* > 1/2$ , then  $\Pi^* > 0$ .*

In the baseline numerical exercise we assume that  $u(c) = \log c$ ,  $v(c) = c$  and  $F(a) = 1 - \underline{a}^\gamma a^{-\gamma}$ . In Figure 3, we choose  $\underline{a} = 1.5$ ,  $\gamma = .6$ , and  $k = .45$ . Such parameters implies that

---

<sup>11</sup>Note that the same Lemma applies to the general equilibrium when the economy is in a not revealing regime.

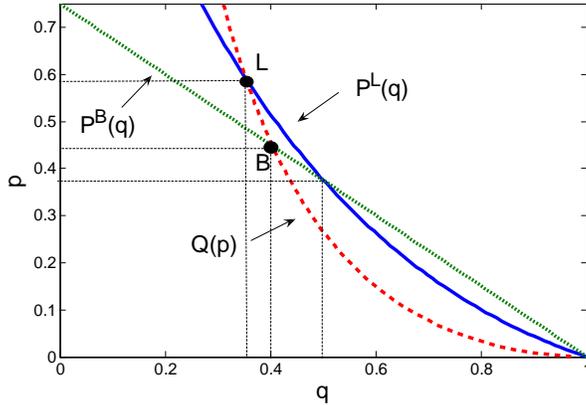


Figure 3: The solid curve and the dotted curve represent the pricing rule in the economy with career concerns and in the benchmark economy, respectively, while the dashed line shows the probability of default, that is now endogenous. Points L and B denote respectively the limit interior equilibrium when  $M^I \rightarrow 0$  and the benchmark equilibrium when  $M^I = 0$ .

$q^* < 1/2$  and then  $p^* > p^B$ , that is, that the premium is positive. Changing the parameters, we can easily obtain the analogous figure where  $p^* < p^B$  and the premium is negative. For example, this is the case if we decrease the lower bound for the productivity process  $\underline{a}$  to 1.

#### 4.4 Amplification

Next, we analyze some interesting properties of the limit equilibrium. In particular, we are interested in the reaction of the economy to shocks both to financial markets and to the fundamentals of the risky project. The first type of shocks affect the pricing rule and we refer to them as demand-side shocks; the second type affect the repayment rule and we label them supply-side shocks. Our main result is that there is an amplification effect that magnifies the reaction of the equilibrium to both types of shocks, in comparison to the benchmark equilibrium. The mechanism behind this result is that both types of shocks can move the economy from one reputational regime to the other, generating a natural amplification in the price and in the default probability.

Let us focus on the vector of parameters  $\sigma = (R, \alpha)$ , where  $R$  is the return on the risk-free asset and  $\alpha$  represents a parameter affecting the distribution of the productivity shock  $a$ , such that if  $\alpha'' > \alpha'$ , then  $F(a|\alpha'') < F(a|\alpha')$ . Assume that  $\lim_{\alpha \rightarrow -\infty} F(a|\alpha) = 1$ . A change in  $R$  represents a typical demand-side shock, that is, a change in the return of alternative investment opportunities. A change in  $\alpha$ , instead, represents a first-order stochastic shift of the productivity distribution of the risky project, that is, a typical supply-side shock.

With a slight abuse of notation, let us denote by  $(q^*(\sigma), p^*(\sigma))$  and  $(q^B(\sigma), p^B(\sigma))$  the

equilibrium default probability and price, respectively in the limit equilibrium and in the benchmark equilibrium, when the parameters are  $\sigma$ . When there are multiple equilibria, let us focus on the equilibrium with the highest bond price.

Next proposition states our main amplification result.

**Proposition 4** *Suppose there exists a pair  $(\sigma', \sigma'')$  such that  $q^B(\sigma') < 1/2$ ,  $q^B(\sigma'') > 1/2$ , and  $q^*(\sigma'') > 1/2$ . Then, there is amplification, that is,  $q^*(\sigma'') - q^*(\sigma') > q^B(\sigma'') - q^B(\sigma')$  and  $p^*(\sigma'') - p^*(\sigma') > p^B(\sigma'') - p^B(\sigma')$ .*

Proposition 4 shows that if there is a change in the parameters such that the equilibrium switches regime from a positive premium to a negative premium, then both prices and default probabilities respond more than in the benchmark model. Suppose, for example, we start from a regime where the premium is negative. As the outside investment opportunities improve, that is,  $R$  increases, the bond price decreases making borrowing more expensive and default happening more often. If the shock is big enough, it can generate a shift in the sign of the premium and a switch of regime. Alternatively, the economy can move from a regime to another because of a change in the parameters on the supply-side of the model. For example, a big enough decrease in  $\alpha$  can increase the default probability enough to make the premium negative. The effect on both prices and quantities is amplified in comparison to the benchmark model.

Next proposition shows that when, for a given set of parameters, there exists a unique interior equilibrium with a negative reputational premium, it is possible to change  $R$  or  $\alpha$  enough to generate a shift in the reputational regime.

**Proposition 5** *Suppose there exists a unique interior equilibrium for any  $\sigma = (R, \alpha)$ , with  $R \geq 0$  and  $\alpha \in [\underline{\alpha}, \bar{\alpha}]$  for some  $\underline{\alpha}$  and  $\bar{\alpha}$ . Take a  $\sigma' = (R', \alpha')$  with  $q^*(\sigma') < 1/2$ . Then,*

1. *there exists  $\hat{R} > R'$  such that for any  $R'' > \hat{R}$ ,  $q^*(\sigma'') > 1/2$ , where  $\sigma'' = (R'', \alpha')$ ;*
2. *if  $\underline{\alpha}$  is sufficiently low, there exists  $\hat{\alpha}$  with  $\underline{\alpha} < \hat{\alpha} < \alpha'$  such that for any  $\alpha''$  with  $\underline{\alpha} < \alpha'' < \hat{\alpha}$ ,  $q^*(\sigma'') > 1/2$ , where  $\sigma'' = (R', \alpha'')$ .*

Propositions 4 and 5 show that as the financial environment or the fundamentals of the risky project change, the economy can switch from a regime with low bond spreads (high  $p$ ) and high level of capital invested in the risky bond market (high  $pb$ ) to a regime with high bond spreads (low  $p$ ) and low level of capital invested (low  $pb$ ). The first type of regimes are frequently described as regimes of *abundant liquidity* or with *traders reaching for yield*. To describe phenomena where the economy switch to the second type of regime,

common terms are *flight-to-quality*, *flight-to-liquidity*, *disappeared liquidity*, or *drop in risk appetite*. In our model, phenomena of this type can arise even if fund managers are risk-neutral and their aggregate funds are constant. We argue that abrupt changes in prices can be caused by managers' career concerns. In good times, when the default probability of credit instruments is low, it is very attractive for uninformed fund managers to invest in these instruments because if they prefer less risky investment opportunities, they are likely to produce lower returns, lose reputation, and, hence, funds. If suddenly a negative shock hits either the demand or the supply side of the market, the probability of default increases, and investing in the risky asset increases the probability of losing their reputation. Hence, prices increase not only because of the higher probability of default, but also because of an additional premium coming from career concerns. This generates the amplification result we have discussed.

It is well known in finance that the premium on risky assets is time-varying: the difference between the expected return on risky asset and riskless assets is lower in good times than in bad times. The most established explanations connect this fact to time-varying marginal utility of consumption due to habit formation, to time-varying probability of disasters or to a slow-moving component in consumption risk.<sup>12</sup> In our model the premium changes, because the time-varying risk that a given investment results in large fund outflows. In contrast to the established explanations, our mechanism implies that in good times some managers are willing to take risky bets without the sufficient compensation in returns. This argument is consistent with several empirical observations across various markets. For example, regarding emerging market bonds, Duffie et al. (2003) document that the implied short spread of Russian bonds was spectacularly low during the first 10 months of 1997. Moreover, their estimation shows that in one short interval in 1997, bond prices were so high that the implied risk-neutral default adjusted short spread was negative. Although this implication relies heavily on their specific term structure model, it worth to point out that this is inconsistent with most established explanations of time-varying premium, but consistent with our model.

Similarly, Coval, Jurek and Stafford (2008) argue that in the period 2004- September 2007 collateralized debt obligations (CDOs) provided too little compensation for risk compared to portfolios of securities of the same pay-off structure. As CDOs are traded only by institutional investors, our mechanism is a good candidate to explain this phenomenon.

Finally, Brunnermeier and Nagel (2004) argues that hedge funds were investing heavily in technological stocks during the dotcom bubble. Furthermore, they also argue that fund managers seemed to be aware of the mispricing. Our model suggests that part of the reason of the willingness of hedge funds to buy technological stocks at highly inflated prices was

---

<sup>12</sup>See Cochrane (2006) for a detailed review.

their fear of loss of funds if they miss out on the opportunity. This is consistent with the additional finding of Brunnermeier and Nagel (2004) that the largest hedge fund (Tiger Fund) which refused to invest in technology stocks experienced severe fund outflows in 1999 compared to its main competitor who did invest in technology stocks (Quantum Fund).

## 5 Persistent Productivity Shock

In this section, we generalize the model to allow for persistency in the productivity process of the risky project. In particular, assume that  $a_{t+1}$  is distributed according to a first-order Markov process with cumulative density function  $G(a_{t+1}|a_t)$ . The environment is a natural generalization of the one with *i.i.d.* shock, where  $a_t$  represents an additional state variable. We look for a Markovian equilibrium, which extends the notion of interior equilibrium described in definition 3.

**Definition 5** *A Markovian interior equilibrium with  $M^I \rightarrow 0$  is an equilibrium where  $p^*(a_t) \in (0, 1/R)$  is determined by the indifference condition of the uninformed managers; the informed managers' bond allocation is  $x^I(y_t, a_{t+1}) = 1 - \chi(y_t, a_{t+1})$ ; the uninformed managers' allocation is  $x^U(y_t, a_t) = (p^*(a_t)b^*(a_t) - y_t)/\Gamma$ ; the investors' strategy is to fire manager  $j$  if he receives a negative exogenous signal or  $\theta_t^j \neq 1 - \chi(y_t, a_{t+1})$ , and keep him otherwise; the borrowers' strategy is  $b^*(a_t) > 0$  and  $\chi(y_t, a_{t+1}) = 0$  if either  $a_{t+1} \geq \hat{a}^*(a_t)$ , and  $\chi(y_t, a_{t+1}) = 1$ , otherwise.*

When the process for  $a_t$  is not *i.i.d.*, the expected utility of the uninformed managers at time  $t$  depends on the realization of  $a_t$ , because they can use the past information to update the distribution of  $a_{t+1}$ , that is,

$$W(a_t) = \gamma R + \delta \int_0^{\hat{a}(a_t)} W(a_{t+1}) dF(a_{t+1}|a_t).$$

Moreover, their indifference condition becomes

$$(1 - q(a_t)) \frac{\gamma}{p(a_t)} + \delta \int_{\hat{a}(a_t)}^{\infty} W(a_{t+1}) dF(a_{t+1}|a_t) = \gamma R + \delta \int_0^{\hat{a}(a_t)} W(a_{t+1}) dF(a_{t+1}|a_t).$$

This condition implicitly defines the equilibrium price as a function of the state  $a_t$ ,  $P(a_t, q(a_t)) = p(a_t)$ , and the default rule  $\hat{a}(a_t)$  where  $\hat{a}(a_t) = F^{-1}(q(a_t))$ .

Also borrowers update their expectation of the distribution of  $a_{t+1}$ , conditional on  $a_t$ . Their default rule is  $Q(a_t, p(a_t)) = F(B(a_t, p(a_t)) / (1 - \theta) | a_t)$ , where  $B(a_t, p(a_t)) = b(a_t)$

is implicitly defined by

$$p(a_t) u'(p(a_t) b(a_t) - k) - \beta \int_{\frac{b(a_t)}{1-\theta}}^{\infty} v'(a_{t+1} - b(a_t)) dF(a_{t+1}|a_t) = 0.$$

Hence, a Markovian interior equilibrium is characterized by a fixed point such that  $p^*(a_t) = P(a_t, q^*(a_t))$  and  $q^*(a_t) = Q(a_t, p^*(a_t))$ .

Here we have performed some numerical exercises to illustrate the dynamic properties of our equilibrium when productivity shocks are persistent. In particular, we show how career concerns can magnify the reaction of the economy to shocks, hence, increasing the volatility of prices.

Let us first consider the equilibrium behavior in the benchmark economy. As a bad shock hits, the financial market realizes that, even for a given default rule, the probability of default is higher and requires a lower bond price. As borrowing becomes more expensive, borrowers increase their default cut-off, magnifying the reduction in the bond price. A lower bond price also decreases the amount of capital entrepreneurs borrow, so capital flows out from the market of risky bonds. Hence, for low realizations of productivity, the default cut-off will be higher and the bond price and the dollar value of outstanding bonds lower. Now, consider the economy with career concerns and suppose the default probability is high enough that the premium is positive. In this case, the financial market will require a bond price even lower than the benchmark economy because of the negative reputational premium. Given that productivity is persistent, a bad realization of the shock will further increase the probability of default, increasing the fear of the uninformed managers of being fired and pushing the bond price further down. This implies that the premium itself is higher after bad shocks and can even switch sign.

We report the numerical results for an example similar to the one illustrated in Figure 3 with  $u(c) = \log c$  and  $v(c) = c$ . However, now  $\log(a_t)$  follows an AR(1) process.<sup>13</sup> Figure 4 shows how the premium varies in equilibrium with the different realizations of the productivity shocks.

Now, consider an economy that at time zero is hit by a shock. Figure 5 shows how the equilibrium prices react in expected terms to a bad and to a good shock, both with and without career concerns. The figure shows our amplification result: the economy with career concerns reacts more to the shocks than the benchmark economy. Moreover, notice that in the economy considered, the premium would be negative in expected terms and a bad shock can actually make the economy shift regime.

---

<sup>13</sup>Figures (4) and (5) use the following process for  $a_t$ :  $\log(a_{t+1}) = (1 - \rho)\mu + \rho \log(a_t) + \varepsilon_t$  with  $\rho = .7$ ,  $\mu = 2.8$  and  $\varepsilon_t \sim N(0, 2)$ . Moreover,  $\beta = .75$ ,  $\delta = .5$ ,  $\gamma = 1$ ,  $k = .4$ , and  $\theta = .1$ .

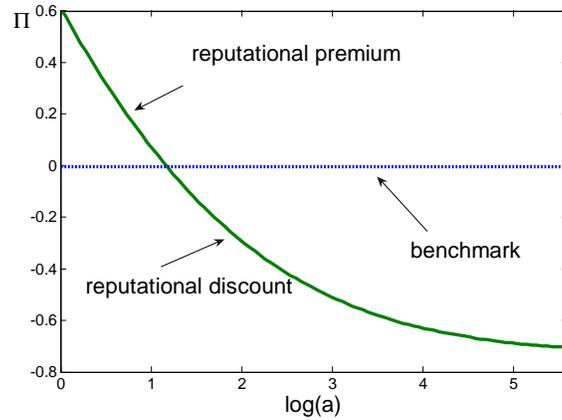


Figure 4: The figure shows the reputational premium as a function of the realization of  $\log(a)$ . The solid line is the premium with career concerns and the dotted line shows the premium in the benchmark case.

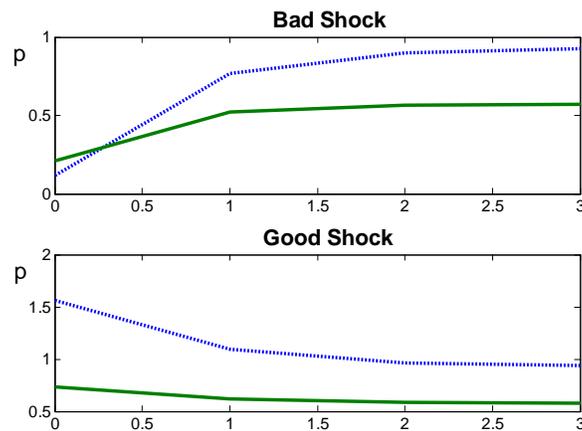


Figure 5: The two panels show the reaction of the equilibrium prices to a bad and a good shock, respectively. The dotted line represents the price in the benchmark economy, and the solid line the price in the economy with career concerns. At time zero productivity drops to the lowest possible realization in the first case and rises to the highest possible one in the second case.

## 6 Conclusion

In this paper, we have proposed a general equilibrium model of delegated portfolio management where career concerns distort asset prices. In particular, risky bonds trade with a reputational premium, which may be positive or negative depending on the equilibrium default probability. When the probability of default is sufficiently high, fund managers prefer to invest in safe bonds even at a lower expected return to reduce the probability of being fired. As the economic and financial conditions change, the reputational premium can switch sign, amplifying the reaction of prices and capital flows.

For future research, it would be interesting to introduce alternative risky assets in the portfolio choice of the managers. In this case, our mechanism would generate contagion. Imagine that there are two risky bonds and a riskless asset. The cost of investing in the risk-less asset depends on the default probability of both the risky bonds. If none of them defaults, the manager who invests in the risk-less asset loses his reputation. Thus, if the probability of default of any of the risky bonds decreases, the risk-less asset will be less attractive, and the prices of both bonds will have to increase in order to make uninformed managers indifferent between different investment opportunities.

Finally, an interesting application of our model is to sovereign debt. A large literature on business cycle characteristics of emerging markets<sup>14</sup> highlights that emerging market bond spreads are very volatile. In particular, the magnitude of volatility of interest rates is hard to reconcile with models where bond prices are determined by the standard no-arbitrage condition. Our model provides an appealing framework to think about this excess volatility. It would be interesting to calibrate our model to quantify how much of the volatility of specific emerging markets bonds can be explained with our mechanism.

## Appendix

### Proof of Proposition 1

Here we show that there exists a unique interior stationary equilibrium, as characterized in definition 2, where  $\Gamma^I$ ,  $p^*$ ,  $W$ , and  $\mu$  solve the system of equations (5)-(8). The proof proceeds in four steps. We first show that the managers' flows are consistent with the labor market for fund managers and that there exist unique values for  $\Gamma^I$ ,  $W$ , and  $\mu$  that solve (5)-(7); second we show that the demand schedules chosen by the managers, (ii) in definition 2, are optimal; third we prove that the bond's allocation, (iii), ensures that the bond's market

---

<sup>14</sup>See Neumeyer and Perri (2005), Uribe and Yue (2006), Arellano (2006), Aguiar and Gopinath (2006), Longstaff et al (2007)

clears and that there exists a unique  $p^*$  that satisfies (8); and finally we show that the firing decision of the investors, (iv), is optimal.

**Step 1.** First we show that, given conditions (i)-(iv) in definition 2, there exist three constant values  $\mu$ ,  $\Gamma^I$ , and  $\Gamma^U$  consistent with free entry in the job market for fund managers. Thanks to free entry and the assumption that there is a large continuum of uninformed managers, condition (6) holds and an uninformed managers looking for a job gets zero ex ante utility. On the contrary, there is a constant mass  $M^I$  of informed managers, who are never fired in equilibrium, and get positive expected utility when searching for a job. It follows that they all search for a job at any  $t$ , and hence  $N_t^I = M^I - \delta\Gamma_t^I$ , where  $\Gamma^I$  satisfies condition (5). Conditions (5) and (6) together with condition (7) determine unique values for  $\mu$ ,  $\Gamma^I$ , and  $W$ . By combining the three equations we obtain an equation in  $\mu$  only  $g(\mu) = 0$ , where

$$g(\mu) = \frac{\kappa}{\mu} - \gamma R \left\{ 1 - \delta\omega \left[ q + \frac{M^I(1-q)}{\bar{y} \left( \delta + \frac{1-\delta}{\mu} \right)} \right] \right\}^{-1}.$$

Notice that  $\lim_{\mu \rightarrow 0} g(\mu) = \infty$ ,  $\lim_{\mu \rightarrow 1} g(\mu) < 0$  thanks to the assumption that  $\kappa < \gamma R$ , and  $g'(\mu) < 0$  by inspection. It immediately follows that there exists a unique  $\mu \in (0, 1)$  such that  $g(\mu) = 0$ . Given  $\mu$ , one can use equation (5) to solve for a unique  $\Gamma^I < M^I$ , and equation (7) to solve for a unique  $W$ . Finally,  $\Gamma^U$  must be such that the mass of employed uninformed managers stay constant after  $1 - \delta$  of them die,  $1 - \omega\xi_t$  of them are fired and  $\mu$  of the unemployed uninformed are hired, that is,

$$\Gamma^U = \delta\Gamma^U - (1 - \omega\xi_t)\delta\Gamma^U + \mu N_t^U, \quad (18)$$

where  $\xi_t$  denotes the proportion of uninformed managers who are successful, that is,

$$\xi_t = \begin{cases} (1 - x_t^U)\chi_{t+1} + x_t^U(1 - \chi_{t+1}) & \text{if } z_t \in [\Gamma^I, \bar{y}] \\ 1 & \text{if } z_t \notin [\Gamma^I, \bar{y}] \end{cases}. \quad (19)$$

Hence, conditions (18) and (19) together with  $\Gamma^U = \Gamma - \Gamma^I$  determine the value of  $N_t^U$  for any  $t$ , changing over time together with  $\xi_t$ , that guarantees that  $\Gamma^U$  is uniquely determined and the equilibrium is stationary.

**Step 2.** Next, we show that the managers' demand schedules characterized in condition (ii) of definition 2 are optimal, taking as given conditions (i), (ii) and (iv). First, notice that at any  $t$  there are three possible regimes. Recall that  $y_t$  is uniformly distributed on  $[0, \bar{y}]$ , and hence  $z_t$  must be in  $[0, \bar{y} + \Gamma^I]$ . From the equilibrium pricing condition (i), if  $p_t \leq \bar{y}/b$ ,

then  $z_t \in [0, \Gamma^I)$  and the uninformed managers learn that  $\chi_{t+1} = 1$ . If, instead,  $p_t = 1/R$ , then  $z_t \in (\bar{y}, \bar{y} + \Gamma^I]$  and they learn that  $\chi_{t+1} = 0$ . In both these cases, the price is fully revealing. When instead  $p_t = p^*$ , then  $z_t \in [\Gamma^I, \bar{y}]$  and the uninformed managers update their beliefs about the probability of default as follows:

$$\Pr(\chi_{t+1} = 1 | p_t = p^*) = \frac{\Pr(\chi_{t+1} = 1, z_t \in [\Gamma^I, \bar{y}])}{\Pr(\chi_{t+1} = 1, z_t \in [\Gamma^I, \bar{y}]) + \Pr(\chi_{t+1} = 0, z_t \in [\Gamma^I, \bar{y}])}.$$

Notice that the assumption that  $\bar{y} \geq \Gamma^I$  ensures that this regime exists, that is, that  $z_t$  can be in  $[\Gamma^I, \bar{y}]$ . This happens in two cases: when  $\chi_{t+1} = 1$  and  $y_t \in [\Gamma^I, \bar{y}]$  and when  $\chi_{t+1} = 0$  and  $y_t \in [0, \bar{y} - \Gamma^I]$ . Given that  $y_t$  is uniformly distributed, the first case arises with probability  $q(\bar{y} - \Gamma^I) / \bar{y}$  and the second with probability  $(1 - q)(\bar{y} - \Gamma^I) / \bar{y}$ . It follows that  $\Pr[\chi_{t+1} = 1 | p = p^*] = q$ . This shows that, thanks to the uniform distribution of  $y_t$ , the price  $p^*$  does not reveal any information.

When  $p_t = 1/R$  and there is full revelation of no default, the risky bond pays for sure  $1/R$  and is equivalent to the risk-free asset. Hence, both the informed and the uninformed managers are indifferent between the two bonds and  $d^I(p_t, \chi_{t+1}) = d^U(p_t) = \{0, 1\}$  if  $p_t = 1/R$ . When instead  $p_t < 1/R$ , informed managers will always choose to invest in the risk-free asset if there is going to be default, and in the bond if there is not going to be default, that is,  $d^I(p_t, \chi_{t+1}) = 1 - \chi_{t+1}$ . This is optimal for them given that investors fire managers who do not do that. Also, they do not have any incentive to deviate, given that the bond price is smaller than  $1/R$ . However, the uninformed managers cannot follow the same strategy because they do not know  $\chi_{t+1}$ . When  $p_t \leq \bar{y}/b$ , there is full revelation and they learn that there is going to be default. Hence, it is optimal to demand  $d^U(p_t) = 0$  to avoid to be fired. If, instead,  $p_t = p^*$ , there is no information revelation and they are indifferent between the two assets, that is,  $d^U(p_t) = \{0, 1\}$ . By combining equations (7) and (8) we immediately obtain the not revealing price as a function of the default probability  $q$ , that is, we obtain the following *pricing rule*:

$$p^* = P(q) \equiv \frac{(1 - q)(1 - \delta[\Gamma^I/\bar{y} + q(1 - \Gamma^I/\bar{y})])}{R[1 - \delta(1 - q)(1 + \Gamma^I/\bar{y})]}, \quad (20)$$

where  $\Gamma^I$  is jointly determined by equations (5), (6), and (7). Given that in step 1 we have shown that there exists a unique triple  $(\Gamma^I, W, \mu)$  that solve (5), (6), and (7), it is immediate to see that for any  $q$ , there exists a unique  $p^*$  that satisfies equation (20).

**Step 3.** Given the equilibrium prices (i) and the equilibrium conditions (ii) and (iii) in Definition 2, it is easy to show that the bond allocation defined in (iii) is consistent

with the managers' demand and ensures that the bond market clears. If  $z_t \in [0, \Gamma^I)$ , all managers demand the riskless asset and the auctioneer must allocate risky bonds only to the noise traders  $y_t$ . Given that in this case the price is equal to  $y_t/b$ , the market clearing condition is automatically satisfied. If  $z_t \in (\bar{y}, \bar{y} + \Gamma^I]$ , all managers are indifferent between the two assets, the price is  $1/R$  and the auctioneer can randomly allocate the risky bond to the managers. In particular, the allocation  $x^I(\chi_{t+1}, y_t) = x^U(\chi_{t+1}, y_t) = (b/R - y_t)/\Gamma$  guarantees that the bond market clears. If instead  $z_t \in [\Gamma^I, \bar{y}]$ , there is no information revelation. Given that the auctioneer's allocation must be consistent with the managers' demand schedule, the informed managers obtain the risky bond if and only if they demand it, that is,  $x^I(\chi_{t+1}, y_t) = 1 - \chi_{t+1}$ , while the uninformed managers are indifferent between the two assets, and the allocation  $x^U(\chi_{t+1}, y_t) = (bp^* - z_t)/\Gamma^U$  guarantees that the market clearing condition is satisfied. Moreover, assumption A1 guarantees that  $p^* \in (\bar{y}/b, 1/R)$  and ensures that  $(bp^* - z_t)/\Gamma^U \in (0, 1)$ , so that the allocation is internally consistent.

**Step 4.** Finally, we show that the investors' firing rule (iv) in definition (2) is optimal, given conditions (i)-(iii). First, notice that investors are indifferent between an informed and an uninformed manager when  $p_t = 1/R$  or  $p_t \leq \bar{y}/b$ , and all managers choose the same demand schedule. However, when  $p_t = p^*$ , the expected return of an informed manager is higher. Hence, investors would like to have informed managers investing their capital, and in order to make their firing decisions, they assess the probability that employed managers are informed. At the end of time  $t$ , each investor observes the investment realization of his manager  $\theta_t^j$  and the default realization  $\chi_{t+1}$ . Then if  $p_t < 1/R$  and either  $\chi_{t+1} = 0$  and  $\theta_t^j = 1$ , or  $\chi_{t+1} = 1$  and  $\theta_t^j = 0$ , he realizes that his manager is not informed, that is,  $\eta_{t+1}^j = 0$ . In this case, the investor fires him, since there is always a positive probability that a new hire is informed, denoted by  $\varepsilon_{t+1}$ , and the probability of finding a new manager is always equal to 1. By definition  $\varepsilon_{t+1}$  satisfies

$$\varepsilon_{t+1} = \frac{M^I - \delta\Gamma^I}{M^I - \delta\Gamma^I + N_t^U} > 0. \quad (21)$$

For the same reason, the investor also fires an uninformed manager whose type is revealed by an exogenous signal with probability  $1 - \omega$ . On the other hand, if the manager does not make a mistake, that is, if  $\theta_t^j = 1 - \chi_{t+1}$  and/or  $p_t = 1/R$ , so that he does not reveal to be uninformed, then he is not fired if and only if his updated belief  $\eta_{t+1}^j$  is higher than  $\varepsilon_{t+1}$ . When manager  $j$  realizes  $\theta_t^j = 1 - \chi_{t+1}$  and/or  $p_t = 1/R$ , the investor's belief is updated as follows  $\eta_{t+1}^j = \eta_t^j / [\eta_t^j + \omega\xi_t(1 - \eta_t^j)]$ , where  $\xi_t$ , defined in (19), represents the proportion of uninformed managers who make the same investment decision of the informed managers. Next, we show that assumption A2 is sufficient to make sure that in equilibrium  $\eta_{t+1}^j \geq \varepsilon_{t+1}$

for any  $\xi_t$  and  $\eta_t^j > 0$ .

First, consider an investor who has just hired manager  $j$  and hence, by definition, has prior belief  $\eta_t^j = \varepsilon_t$ . In this case, if  $\theta_t^j = 1 - \chi_{t+1}$ , then  $\eta_{t+1}^j = \varepsilon_t / [\varepsilon_t + \omega \xi_t (1 - \varepsilon_t)]$ . Next, we want to show that  $\eta_{t+1}^j \geq \varepsilon_{t+1}$ . This condition can be rewritten as

$$\frac{1 - \varepsilon_{t+1}}{\varepsilon_{t+1}} \geq \left( \frac{1 - \varepsilon_t}{\varepsilon_t} \right) \omega \xi_t, \quad (22)$$

Using the expression (21) for  $\varepsilon_t$ , we have that  $(1 - \varepsilon_{t+1})/\varepsilon_{t+1} = N_t^U / (M^I - \delta \Gamma^I)$ , and, hence, condition (22) can be rewritten as  $N_t^U / N_{t-1}^U \geq \omega \xi_t$ , where  $N_t^U = (1 - \omega \xi_t \delta) \Gamma^U / \mu$ . Hence, in order for (22) to be satisfied it must be that  $1 - \delta \omega \xi_t > (1 - \delta \omega \xi_{t-1}) \omega \xi_t$ , which is ensured by assumption A2.

Let us now consider managers who were working for an investor for longer than 1 period. First, notice that the investors' beliefs about any manager who is still working but was hired at time  $t - \tau$  with  $\tau \in [0, t]$  must be higher than the initial belief  $\varepsilon_{t-\tau}$ , given that if he was not fired he never made any mistake, that is,  $\eta_t^j \geq \varepsilon_{t-\tau}$ . Hence, the belief about a manager who was hired at time  $t - \tau$  and did not make a mistake at time  $t$  is

$$\eta_{t+1}^j = \frac{\eta_t^j}{\eta_t^j + \omega \xi_t (1 - \eta_t^j)} \geq \frac{\varepsilon_{t-\tau}}{\varepsilon_{t-\tau} + \omega \xi_{t-\tau} (1 - \varepsilon_{t-\tau})}.$$

It follows that a sufficient condition for this manager not being fired is

$$\frac{1 - \varepsilon_{t+1}}{\varepsilon_{t+1}} \geq \left( \frac{1 - \varepsilon_{t-\tau}}{\varepsilon_{t-\tau}} \right) \omega \xi_{t-\tau},$$

which, by the same argument, is satisfied when assumption A2 holds. For the same reason, when  $p_t = 1/R$  no manager is fired, given that there is no information in their action, completing the proof.

## Proof of Proposition 2

Here we show that there exists a unique interior stationary equilibrium, as characterized in section 4.2. The proof proceeds in two steps: first we show that the proof of existence for the baseline model goes through for the extended model to show that managers investment decision and investors' firing rule are optimal, and that the bond's allocation ensures that the bond's market clears, moreover we show that entrepreneurs decision are optimal as well; second we show that there exists a unique fixed point that determines the pair  $(p^*, q^*)$ .

**Step 1.** First, the equilibrium prices are identical to the ones in the baseline model,

except for the price in the regime with full revelation of default, which is now  $\underline{p}$  instead of  $\bar{y}/b$ , where  $\underline{p}$  is such that  $V(\underline{p}) = \bar{V}$ . The proof that the managers' demand schedule and the investors' firing rule are actually optimal, taking as given all the other equilibrium objects is analogous to the one for the baseline model above, with the only caveat that now  $\chi_{t+1} = \chi(a_{t+1}, y_t)$ ,  $b = b(p_t, y_t)$  and  $\underline{p}$  substitutes  $\bar{y}/b$ . With the same modifications, the bond allocation is the same of the baseline model and follows from the bond market clearing as we have shown above. The new part of the proof is to show that the borrowing and default decisions of the entrepreneurs are actually optimal, taking as given the other equilibrium objects. That is, that, in equilibrium, condition (15) is satisfied, with  $\tilde{b} < b^*$  in  $[0, \bar{b}]$ , and  $\chi(p_t, a_{t+1}) = \mathbf{1}\{a_{t+1} < \hat{a}(p_t, y_t)\}$ , where  $\hat{a}(p_t, y_t) = b(p_t, y_t) / (1 - \theta)$ .

Active entrepreneurs choose their default rule and how much to borrow and to consume in order to solve problem (14), taking  $p_t$  as given. Let us first consider the default decision of an old entrepreneur. For a given realization of the shock  $a_{t+1}$ , she will default if and only if  $a_{t+1} - b_{t+1} < \theta a_{t+1}$ . Then  $\chi(p_t, a_{t+1}) = 1$  if  $a_{t+1} \leq \hat{a}(p_t, y_t)$  and  $\chi(p_t, a_{t+1}) = 0$ , otherwise, with  $\hat{a}(p_t, y_t) = b(p_t, y_t) / (1 - \theta)$ . Given that  $y_t$  is uniformly distributed between 0 and 1, the signal of the uninformed  $z_t = \Gamma^I (1 - \chi_{t+1}) + y_t$  must be in  $[0, \bar{y} + \Gamma^I]$ . If  $p_t = 1/R$ , then  $z_t \in (\bar{y}, \bar{y} + \Gamma^I]$ , and the uninformed managers learn that  $\chi(p_t, a_{t+1}) = 0$ . If instead  $p_t = \underline{p}$ , then  $z_t \in [0, \Gamma^I)$ , and they learn that  $\chi_{t+1} = 1$ . If instead  $p_t = p^*$ , as in the baseline model, there is no information revealed.

In the first case,  $p_t = 1/R$  and there is full revelation that  $\chi(p_t, a_{t+1}) = 0$ . Define  $\hat{a}^e$  the default rule expected by the managers. Given that the entrepreneurs learn that  $a_{t+1} \geq \hat{a}^e$ , in this case, problem (14) can be written as

$$\begin{aligned} b\left(\frac{1}{R}\right) &= \arg \max_b u\left(\frac{b}{R} - k\right) + \frac{\beta}{1 - F(\hat{a}^e)} \int_{\hat{a}^e}^{\min\{\frac{b}{1-\theta}, \hat{a}^e\}} v(\theta a_{t+1}) dF(a_{t+1}) \\ &\quad + \frac{\beta}{1 - F(\hat{a}^e)} \int_{\min\{\frac{b}{1-\theta}, \hat{a}^e\}}^{\infty} v(a_{t+1} - b) dF(a_{t+1}). \end{aligned}$$

It follows that  $b(1/R) = \tilde{b}$  where  $g(\tilde{b}) = 0$  with

$$g(b) \equiv \frac{1}{R} u'\left(\frac{b}{R} - k\right) - \frac{\beta}{1 - F\left(\frac{b}{1-\theta}\right)} \int_{\frac{b}{1-\theta}}^{\infty} v'(a_{t+1} - b) dF(a_{t+1}).$$

Hence, in order for this to be an equilibrium it must be that  $\hat{a}^e = \tilde{b} / (1 - \theta)$ , so that the default rule is consistent with the managers' expectations. Moreover, we have to check that if prices are not revealing, that is,  $p_t = p^*$ , entrepreneurs would default for any  $a_{t+1} < \hat{a}^e$ ,

that is, that  $b^* \geq \tilde{b}$ , where  $b^*$  solves  $\tilde{g}(b^*) = 0$ , with

$$\tilde{g}(b) \equiv p^* u'(p^* b - k) - \beta \int_{\frac{b}{1-\theta}}^{\infty} v'(a_{t+1} - b) dF(a_{t+1}).$$

Hence, it is enough to show that  $\tilde{g}(\tilde{b}) > 0$ . This comes straight from assumption A3.

In the second case,  $p_t = \underline{p}$  and there is full revelation that  $\chi(p_t, a_{t+1}) = 1$ . Let again denote by  $\hat{a}^e$  the default rule expected by the managers. Then, it must be that  $a_{t+1} < \hat{a}^e$  and the entrepreneurs problem (14) can be written as

$$\begin{aligned} b(\underline{p}) &= \arg \max_b u(\underline{p}b - k) + \frac{\beta}{F(\hat{a}^e)} \int_0^{\min\{\frac{\bar{b}}{1-\theta}, \hat{a}^e\}} v(\theta a_{t+1}) dF(a_{t+1}) \\ &\quad + \frac{\beta}{F(\hat{a}^e)} \int_{\min\{\frac{\bar{b}}{1-\theta}, \hat{a}^e\}}^{\hat{a}^e} v(a_{t+1} - b) dF(a_{t+1}). \end{aligned}$$

In order to have an equilibrium it must be that  $\hat{a}^e = \bar{b}/(1-\theta)$  and hence it is immediate that  $b(\underline{p}) = \bar{b}$ . It follows that if prices are not revealing entrepreneurs would not default for any  $a_{t+1} \geq \hat{a}^e$  given that by construction  $b^* < \bar{b}$ . Moreover,  $\underline{p}$  must be such that

$$\bar{V} = u(\underline{p}\bar{b} - k) + \frac{\beta}{F\left(\frac{\bar{b}}{1-\theta}\right)} \int_0^{\frac{\bar{b}}{1-\theta}} v(\theta a_{t+1}) dF(a_{t+1}).$$

This, guarantees that they can choose a mixed strategy to ensure that the bond market clears. When  $p_t = \underline{p}$  the demand for the risky bond is given by  $y_t$ , so that to have market clearing it must be that the entrepreneurs choose to borrow with probability  $y_t/(\underline{p}\bar{b})$ , and take their outside option and borrow 0 with probability  $1 - y_t/(\underline{p}\bar{b})$ .

Finally, when  $p_t = p^*$ , then  $z_t \in [\Gamma^I, \bar{y}]$  and no information is revealed. This implies that the entrepreneurs have no additional information on the realization of  $a_{t+1}$ . After substituting the default decision  $\chi(p_t, a_{t+1})$ , for any  $p \in (\underline{p}, 1/R)$  problem (14) becomes  $\max_{b \leq \bar{b}} \Phi(p, b)$ . Let  $B(p)$  denote the optimal borrowing policy for a given non fully revealing price  $p \in (\underline{p}, 1/R)$  and, thanks to assumption A4, is implicitly defined by the first order condition (16).

**Step 2.** To complete the proof we need to show that there exists a pair  $(p^*, q^*)$  that solves the fixed point defined by  $q^* = Q(p^*)$  and  $p^* = P(q^*)$ . Recall that  $Q(p) = F(B(p)/1-\theta)$ , where  $B(p)$  solves (16). Equation (7) is now replaced by

$$W = \gamma R + \delta \omega [\nu(q) + q(1 - \nu(q))] W, \tag{23}$$

where  $\nu(q)$  is the endogenous probability of full revelation and is equal to  $\Gamma^I [\bar{q} + (1 - \tilde{q})] / \bar{y}$ , with  $\bar{q} = F(\bar{b}/(1 - \theta))$ ,  $\tilde{q} = F(\tilde{b}/(1 - \theta))$ , and  $\tilde{b}$  defined above. Using equations (5) and (6),  $\nu(q)$  can be implicitly defined by

$$\nu(q) = \psi(\nu(q); q), \quad (24)$$

where

$$\psi(\nu(q); q) \equiv \frac{M^I [\bar{q} + (1 - \tilde{q})]}{\bar{y}} \left[ \delta + \frac{(1 - \delta) \gamma R}{\kappa \{1 - \delta \omega [\nu(q) + q(1 - \nu(q))]\}} \right]^{-1}.$$

Combining (23) together with (5), (6), and (8), one obtains

$$P(q) = \frac{(1 - q)(1 - \delta \omega [\nu(q) + q(1 - \nu(q))])}{[1 - \delta \omega (1 - q)(1 + \nu(q))] R}.$$

First, assumption A4 ensures that  $B(p)$  is uniquely defined by equation (16) for any  $p \in (\underline{p}, 1/R)$  and the proof of Lemma 1 below shows that  $B'(p) < 0$ . Then,  $Q(p)$  is decreasing on the interval  $(\underline{p}, 1/R)$ , given that it is immediate that  $Q'(p) \propto B'(p)$ .

Next, we show that there is a unique  $\nu(q)$  that solves equation (24). First, notice that  $\partial \psi(\nu(q); q) / \partial \nu(q) < 0$  and that  $\psi(0; q) > 0$ . Then, we only need to show that  $\psi(1; q) < 1$ , where

$$\psi(1; q) = \frac{M^I [\bar{q} + (1 - \tilde{q})]}{\bar{y}} \left[ \delta + \frac{(1 - \delta) \gamma R}{(1 - \delta \omega) \kappa} \right]^{-1}.$$

This follows from the following chain of inequalities:

$$\frac{M^I}{\bar{y}} [1 + \bar{q} - \tilde{q}] < 2 \frac{M^I}{\bar{y}} < 1 < \delta + (1 - \delta) \frac{\gamma R}{(1 - \delta \omega) \kappa},$$

where the first one follows from the fact that  $\bar{q}$  and  $\tilde{q}$  are in  $[0, 1]$ , the second one comes from assumption A1', and the third one follows from the assumption that  $\gamma R > \kappa$ .

This implies that equation (24) has a unique interior solution for  $\nu(q)$ . Then there exists a fixed point  $(q^*, p^*)$  with  $Q(1/R) < q^* < Q(\underline{p})$ , given that by assumption  $P(q_1) < 1/R$  and  $P(q_2) > \underline{p}$ .

## Proof of Lemma 1

First, recall that  $B(p) \equiv \arg \max_b \Phi(p, b)$ . Assumption A4 implies that there exists a unique  $B(p)$ , that  $\Phi_b(p, B(p)) = 0$ , and  $\Phi_{bb}(p, B(p)) < 0$ . By total differentiating  $\Phi_b(p, B(p)) = 0$ ,

we obtain  $\Phi_{bp}(p, B(p)) + \Phi_{bb}(p, B(p)) B'(p) = 0$ , and then

$$B'(p) = -\frac{\Phi_{bp}(p, B(p))}{\Phi_{bb}(p, B(p))}. \quad (25)$$

Given that  $\Phi_{bb}(p, B(p)) < 0$ , to show that  $B'(p) < 0$ , it is enough to show that  $\Phi_{bp}(p, B(p)) < 0$ . We can calculate that

$$\Phi_{bp}(p, B(p)) = u'(p_t b_{t+1} - k) + p_t b_{t+1} u''(p_t b_{t+1} - k), \quad (26)$$

and hence  $\Phi_{bp}(p, B(p)) < 0$  given A3 and the assumption that  $k > 0$ , proving condition (i). Moreover, from this, it is immediate that

$$\frac{dF(\hat{a})}{dp} = \frac{dF(\hat{a})}{\hat{a}} \left( \frac{1}{1-\theta} \right) B'(p) \leq 0.$$

proving condition (ii).

Finally, we need to prove condition (iii). Notice that  $d(pb)/dp = b + pB'(p)$ , where  $B'(p)$  satisfies equation (25), with  $\Phi_{bp}(p, B(p))$  given in equation (26) and, given assumption A4,

$$\Phi_{bb}(p, B(p)) = p^2 u''(pb - k) + \beta v' \left( \frac{\theta}{1-\theta} b \right) + \beta \int_{\frac{b}{1-\theta}}^{\infty} v''(a - b) dF(a) < 0. \quad (27)$$

Then, after some algebra, we obtain

$$\frac{d(pb)}{dp} = \frac{b}{\Phi_{bb}(p, B(p))} \left\{ \beta \left[ v' \left( \frac{\theta}{1-\theta} b \right) + \int_{\frac{b}{1-\theta}}^{\infty} v''(a - b) dF(a) \right] - \frac{p}{b} u'(p_t b_{t+1} - k) \right\}.$$

Hence  $d(pb)/dp \geq 0$  iff the term in parenthesis is non-positive. We can show that this is the case combining condition (27) and the fact that  $-cu''(c)/u'(c) \geq 1$ , hence completing the proof.

## Proof of Proposition 4

Take a pair  $(\sigma', \sigma'')$  such that  $q^B(\sigma') < 1/2$ ,  $q^B(\sigma'') > 1/2$ , and  $q^B(\sigma'') > 1/2$ . With a slight abuse of notation define  $Q(p; \sigma)$ ,  $P^L(q; \sigma)$ , and  $P^B(q; \sigma)$  the repayment map and the pricing maps, in the limit and the benchmark equilibrium respectively, as a function of parameters  $\sigma$ . Notice that  $p^*(\sigma) = P^L(Q(p^*(\sigma); \sigma); \sigma)$  and  $p^B(\sigma) = P^B(Q(p^B(\sigma); \sigma); \sigma)$ .

By construction, we know that  $P^L(q; \sigma) > (<) P^B(q; \sigma)$  for all  $q < (>) 1/2$ . Moreover, given that if there are multiple equilibria we always focus on the equilibrium with the highest

price, it must be true that  $P^L(Q(p); \sigma) < p$  for all  $p > p^*(\sigma)$  and  $P^B(Q(p); \sigma) < p$  for all  $p > p^B(\sigma)$ . Also, we assume that an equilibrium exists for both  $\sigma'$  and  $\sigma''$  and hence, that  $P^L(\tilde{q}; \sigma) < 1/R$  and  $P^L(\tilde{q}) > \underline{p}$  for  $\sigma = \sigma', \sigma''$ .

First, we show that if  $q^B(\sigma') < 1/2$ , then  $p^*(\sigma') > p^B(\sigma')$ . Assume by contradiction that  $p^B(\sigma') \geq p^*(\sigma')$ . Then it must be that  $P^L(Q(p^B(\sigma')); \sigma') < p^B(\sigma') \equiv P^B(Q(p^B(\sigma')); \sigma')$ , which is a contradiction given that by assumption  $q^B(\sigma') < 1/2$ . Second, we show that if  $q^*(\sigma'') > 1/2$ , then  $p^*(\sigma'') < p^B(\sigma'')$ . Suppose by contradiction that  $p^B(\sigma'') < p^*(\sigma'')$ , then  $P^B(Q(p^*(\sigma'')); \sigma'') < p^*(\sigma'') \equiv P^L(Q(p^*(\sigma'')); \sigma'')$ , which is a contradiction given that by assumption  $q^*(\sigma'') > 1/2$ .

These two steps immediately imply that  $p^*(\sigma') > p^B(\sigma') > 1/(2R) > p^B(\sigma'') > p^*(\sigma'')$ , and hence  $p^*(\sigma'') - p^*(\sigma') > p^B(\sigma'') - p^B(\sigma')$ . Moreover, for a given  $\sigma$ , Lemma 1 implies that  $Q(p; \sigma)$  is decreasing in  $p$ , and hence  $q^*(\sigma') < q^B(\sigma') < 1/2 < q^B(\sigma'') < q^*(\sigma'')$ , implying that  $q^*(\sigma'') - q^*(\sigma') > q^B(\sigma'') - q^B(\sigma')$ . This completes the proof.

## Proof of Proposition 5

**Claim 1.** Let us fix  $\sigma' = (\alpha', R')$ . We know that  $P^L(1; \sigma) = 0$  for any  $\sigma$ . Moreover we can calculate

$$\frac{dP^L(q; \sigma')}{dq} = -\frac{1}{R'} \frac{\delta\omega(1-q)(1-\delta\omega(1-q)) + (1-\delta q)}{(1-\delta\omega(1-q))^2} < 0,$$

$dP^L(q; \sigma')/dR > 0$ , and  $\lim_{R \rightarrow \infty} dP^L(q; \sigma)/dR = 0$ . Given that the equilibrium exists, it must be that  $P(\tilde{q}) > \underline{p}$  and hence  $Q(\tilde{q}; \sigma') > 1/2$ . This together with  $q^*(\sigma') < 1/2$  implies that there exists  $\hat{R}$ , such that if  $R'' > \hat{R}$  then  $q^*(R'') > 1/2$ .

**Claim 2.** Again fix  $\sigma' = (\alpha', R')$ . Recall that  $P^B(1/2) = 1/(2R)$ . With a slight abuse of notation define  $Q(1/(2R); \alpha) = F(B(1/(2R); \alpha) / (1-\theta) | \alpha)$ , where  $B(p; \alpha)$  is implicitly defined by  $\Phi_b(B(p; \alpha), p; \alpha) = 0$

$$\Phi_b(b, p; \alpha) \equiv pu'(pb - k) - \beta \int_{\frac{b}{1-\theta}}^{\infty} v'(a - b) dF(a | \alpha).$$

Also,  $Q(1/(2R); \alpha)$  is continuous in  $\alpha$  and  $\lim_{\alpha \rightarrow -\infty} F(a | \alpha) = 1$  implies  $\lim_{\alpha \rightarrow -\infty} Q(1/(2R); \alpha) = 1$ . Thus, there exists at least one  $\alpha$  such that  $Q(1/(2R); \alpha) = 1/2$ . If there are multiple  $\alpha$  such that  $Q(1/(2R); \alpha) = 1/2$ , call  $\hat{\alpha}$  the smallest one. If  $\underline{\alpha}$  is sufficiently small so that  $\underline{\alpha} < \hat{\alpha}$ , this completes the proof.

## References

- Allen, F., Gorton, G., 1993. Churning bubbles, *Review of Economic Studies*, 60, 813-836.
- Aghion, P, Banerjee, A, Piketty, T., 1999. Dualism and macroeconomic volatility, *Quarterly Journal of Economics*, 114(4), 1359-97.
- Aguiar, M., G. Gopinath, 2006. Defaultable debt, interest rates, and current account, *Journal of International Economics*, 69, 64-83.
- Arellano, C., 2008. Default risk and income fluctuations in emerging economies, *American Economic Review*, 98 (3), 690-712.
- Atkenson, A., 1991. International lending with moral hazard and risk of repudiation, *Econometrica*, 59, 1069-89.
- Benczur, P. and C. Ilut, 2005. Determinants of Spreads on Sovereign Bank Loans: The Role of Credit History, CEU, mimeo.
- Bernanke, B., Gertler, M., 1989. Agency costs, net worth and business fluctuations, *American Economic Review*, 79(1), 14-31.
- Bernardo, A., Welch, I., 2004. Liquidity and financial market runs, *Quarterly Journal of Economics* 119(1), 135-158.
- Brunnermeier, M., Nagel, S., 2004. Hedge Funds and the Technology Bubble, *Journal of Finance*, 2004, 59(5), 2013-2040.
- Caballero, R. J., Krishnamurthy, A., 2003. Excessive dollar debt: Financial development and underinsurance, *Journal of Finance*, 58(2), 867-893.
- Calvo, G. A., Mendoza, E., 2000. Capital-markets crises and economic collapse in emerging markets: An informational-Frictions approach, *American Economic Review*, Papers and Proceedings, 90(2).
- Cochrane, J. H., 2006. Financial Markets and the Real Economy, Volume 18 of the International Library of Critical Writings in Financial Economics, Ed. Cochrane, J. H. , London: Edward Elgar.
- Cole, H., Kehoe, T., 2000. Self-Fulfilling Debt Crises, *Review of Economic Studies*, 67, 91-116.
- Coval, J., Jurek, J., Stafford, E., 2008. Economic Catastrophe Bonds, *American Economic Review*, forthcoming.
- Cuoco, D., Kaniel, R., 2001. Equilibrium prices in the presence of delegated portfolio management, University of Pennsylvania, mimeo.
- Danielsson, J., Shin, H .S., Zigrand, J-P., 2004, The impact of risk regulation on price dynamics, *Journal of Banking and Finance* 28, 1069-108.
- Dasgupta, A., Prat, A., 2006. Financial equilibrium with career concerns, *Theoretical*

*Economics*, 1, 67-93.

Dasgupta, A., Prat, A., 2008. Information aggregation in financial markets with career concerns, with Andrea Prat, *Journal of Economic Theory*, 143, 83-113, December 2008.

Dasgupta, A., Prat, A., Verardo, M., 2008. The Price Impact of Institutional Herding, London School of Economics, mimeo.

Duffie, D., Singleton, K., Pedersen, L., 2003. Modeling sovereign yield spreads: A case study of Russian debt, *Journal of Finance*, 55, 119-159.

Gai, P., Kondor, P., Vause, N., 2005, Procyclicality, collateral values and financial stability, Bank of England Working Paper 304.

Gromb, D., Vayanos, D., 2002. Equilibrium and welfare in markets with financially constrained arbitrageurs, *Journal of Financial Economics* 66, 361-407.

Guerrieri, V., Lorenzoni, G. 2008, Liquidity and trading dynamics, NBER Working Paper No. w13204.

He, Zhiguo, Krishnamurthy, A., 2008. Intermediary Asset Pricing, Northwestern University, mimeo.

Kamin, S., von Kleist K, 1999. The evolution and determinants of emerging markets credit spreads in the 1990s, BIS Working Papers No 68.

Kovrijnykh, N., Szentes, B., 2007. Equilibrium default cycles, *Journal of Political Economy*, 115, 403-446.

Kiyotaki, N., Moore, J., 1997. Credit cycles, *Journal of Political Economy*, 105(2), 211-48.

Krishnamurthy, A., 2003. Collateral constraints and the amplification mechanism, *Journal of Economic Theory*, 111(2), 277-292.

Morris, S., Shin, H. S., 2004. Liquidity black holes, *Review of Finance* 8, 1-18.

Neumeyer, P., Perri, F. 2005. Business cycles in emerging economies: The role of interest rates. *Journal of Monetary Economics*, 52(2), 345-380.

Ottaviani, M. Sorensen, P. 2006. The Strategy of Professional Forecasting, *Journal of Financial Economics*, 81(2), 441-466.

Rajan, R., 1994, Why Bank Credit Policies Fluctuate: A Theory and Some Evidence, *Quarterly Journal of Economics*, 109, 399-442.

Rampini, A., 2004. Entrepreneurial activity, risk, and the business cycle, *Journal of Monetary Economics*, 51, 555-573.

Scharfstein, D. S., Stein, J. C. 1990. Herd behavior and investment, *American Economic Review*, 80, 465-79.

Shleifer, A., Vishny, R. W., 1997. The limits of arbitrage, *Journal of Finance*, 52(1), 35-55.

Uribe, M. ,Yue, V. Z., 2006, Country Spreads and Emerging Countries: Who Drives Whom?, *Journal of International Economics*, 69(1), 6-36.

Vayanos, D., 2003. Flight to quality, flight to liquidity, and the pricing of risk, LSE, mimeo.

Zwiebel, J. 1995. Corporate conservatism and relative compensation, *Journal of Political Economy*, 103(1), 1-25.