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MONETARY NON-NEUTRALITY IN A MULTI-SECTOR MENU COST MODEL

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ABSTRACT

Empirical evidence suggests that as much as 1/3 of the U.S. business cycle is due to nominal shocks. We calibrate a multi-sector menu cost model using new evidence on the cross-sectional distribution of the frequency and size of price changes in the U.S. economy. We augment the model to incorporate intermediate inputs. We show that the introduction of heterogeneity in the frequency of price change triples the degree of monetary non-neutrality generated by the model. We furthermore show that the introduction of intermediate inputs raises the degree of monetary non-neutrality by another factor of three, without adversely affecting the model's ability to match the large average size of price changes. Our multi-sector menu cost model with intermediate inputs generates variation in real output in response to calibrated aggregate nominal shocks that can account for roughly 23% of the U.S. business cycle.

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1   Introduction

Much applied work in monetary economics relies on models in which nominal rigidities are the key friction that generates monetary non-neutrality. The workhorse models in this literature—e.g., the Calvo (1983) model and the Taylor (1980) model—make the simplifying assumption that the timing of price changes is independent of firms' incentives to change prices. It has been recognized at least since Caplin and Spulber (1987) that models based on this assumption can yield very different conclusions about monetary non-neutrality than models in which nominal rigidities arise due to a fixed cost of changing prices (see also Caballero and Engel, 1991 and 1993; Caplin and Leahy, 1991 and 1997; Danziger, 1999; Dotsey et al., 1999). Golosov and Lucas (2007) calibrate a menu cost model based on newly available micro-data on the frequency and size of price changes and conclude that nominal rigidities due to menu costs yield monetary non-neutrality that is "small and transient".

Given the importance of nominal rigidities as a source of monetary non-neutrality in most models that analyze the transmission of monetary policy, this conclusion poses a serious challenge for monetary economics. If realistically modeled nominal rigidity yields monetary non-neutrality that is small and transient, much of our understanding of the transmission of monetary policy is called into question. It is therefore of great importance for monetary economics to assess whether the implications of highly stylized menu cost models hold up in a richer, more realistic setting.

Monetary economists have long relied heavily on strategic complementarity in price setting to amplify the degree of monetary non-neutrality generated by nominal rigidities. One natural response to Golosov and Lucas's paper is therefore to simply ramp up the degree of strategic complementarity between price setters. However, recent work has cast doubt on this method for amplification in models with nominal rigidities by showing that the introduction of several popular sources of strategic complementarity renders the models unable to match the average size of micro-level price changes for plausible parameter values (Klenow and Willis, 2006; Golosov and Lucas, 2007; Burstein and Hellwig, 2006).

In this paper, we address both of these challenges. We extend a simple benchmark menu cost model to include two features for which there exists particularly clear empirical evidence: 1) heterogeneity across sectors in the frequency and size of price changes; 2) intermediate inputs. We show that when we subject our model to calibrated nominal shocks it generates fluctuations in real

output that can account for 23% of the U.S. business cycle.[1]

This result of our model accords well with the empirical evidence on the importance of nominal shocks for business cycle fluctuations. Shapiro and Watson (1988) attribute 28% of the variation in output at short horizons to nominal shocks.[2] In contrast, the Golosov and Lucas model generates fluctuations of real output that can account for only roughly 2% of the U.S. business cycle. Roughly half of the difference in monetary non-neutrality in our model relative to the model of Golosov and Lucas (2007) is due to the introduction of heterogeneity in the frequency of price change; the remaining half is due to the introduction of intermediate inputs.

Importantly, our model has no trouble matching the average size of price changes even though the introduction of intermediate inputs generates a substantial amount of strategic complementarity in price setting. To explain this, we follow Ball and Romer (1990) and Kimball (1995) in dividing the sources of strategic complementarity into two classes—ω-type strategic complementarity and Ω-type strategic complementarity. We show that models with a large amount of ω-type strategic complementarity are unable to match the average size of price changes, while this problem does not afflict models with a large amount of Ω-type strategic complementarity. The introduction of intermediate inputs increases the amount of Ω-type strategic complementarity. It therefore does not affect the size of price changes or require unrealistic parameter values.

Midrigan (2006) and Gertler and Leahy (2008) discuss several additional mechanisms that raise the degree of monetary non-neutrality generated by menu cost models. Midrigan (2006) argues that the Golosov-Lucas model overstates the strength of the "selection effect". He augments the Golosov-Lucas model by allowing for fat-tailed idiosyncratic shocks and multi-product firms with scale economies in changing prices. He shows that these features mute the selection effect and thereby increase monetary non-neutrality. The empirical importance of these features depends on the extent to which product level heterogeneity—as opposed to fat-tailed shocks—explains the size distribution of price changes. Gertler and Leahy (2008) analyze a model in which labor markets are

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[1]Here we compare the variance of real output generated in the model in response to nominal shocks to the variance of HP-filtered real GDP.

[2]In fact, Shapiro and Watson (1988) refer to these shocks as "demand" shocks. We follow Lucas (2003) in interpreting these shocks as "nominal" shocks. As Lucas (2003) discusses, these shocks capture not only monetary shocks, but also temporary monetary non-neutrality due to real shocks. Monetary shocks themselves are commonly estimated to account for a relatively modest fraction of business cycle variation in output (see, e.g., Cochrane, 1994; Smets and Wouters, 2007). More comprehensive measures of monetary non-neutrality are higher. The estimates of Justiniano and Primiceri (2008a) imply that more than 2/3 of business cycle fluctuations are due to monetary non-neutrality.

assumed to be independent at the sector level. They assume that firms in only a subset of sectors receive idiosyncratic shocks and change their price in each period. The resulting staggering of price changes across sectors generates Ω-type strategic complementarity that amplifies the monetary non-neutrality in their model. However, time series data on the evolution of the frequency of price change in U.S. economy does not support the notion that the frequency of price change in particular sectors varies greatly over time, even for narrowly defined product categories within the same city. Without a large amount of such time series variation, the Gertler-Leahy model does not generate a quantitatively significant degree strategic complementarity.

Our multi-sector model generates three times more monetary non-neutrality than a single sector model calibrated to the mean frequency of price change. We also calculate the degree of monetary non-neutrality generated by a single sector model calibrated to the *median* frequency of price change. This calibration of the single sector model yields a degree of monetary non-neutrality that is quite similar to that of the multi-sector model. This suggests that researchers that seek to calibrate single sector models for the U.S. economy should use the median frequency of price change rather than the mean frequency of price change.

To understand the effect that heterogeneity has on the degree of monetary non-neutrality in our model, consider the response of the economy to a permanent shock to nominal aggregate demand. In the Calvo model, the effect of such a shock on output at any given point in time after the shock is inversely proportional to the fraction of firms that have changed their price at least once since the shock occurred. If some firms have vastly higher frequencies of price change than others, they will change their prices several times before the other firms change their prices once. But all price changes after the first one for a particular firm do not affect output on average since the firm has already adjusted to the shock. Since a marginal price change is more likely to fall on a firm that has not already adjusted in a sector with a low frequency of price change, the degree of monetary non-neutrality in the Calvo model is convex in the frequency of price change and heterogeneity therefore amplifies the overall degree of monetary non-neutrality in the economy relative to a single sector model calibrated to the mean frequency of price change (Carvalho, 2006).

The relationship between the frequency of price change and the degree of monetary non-neutrality is more complicated in a menu cost model since firms are not selected at random to change their prices. In menu costs models, the difference in monetary non-neutrality between two

sectors will depend not only on their relative frequencies of price change but also on what underlying differences cause the sectors to have different frequencies of price change. Caplin and Spulber (1987) analyze an extreme case in which changes in the size of price changes completely offset changes in the frequency of price change and money is completely neutral regardless of the frequency of price change. We show that the degree of amplification due to heterogeneity depends critically on the relationship between the frequency of price change and the size of price changes across sectors in the menu cost model more generally. Intuitively, heterogeneity in characteristics across sectors can cause variation in the strength of the "selection effect" across sectors that can offset variation in the frequency of price change across sectors. We furthermore show that the degree of amplification due to heterogeneity varies with the economy's average frequency of price change.

Our conclusion that heterogeneity amplifies the degree of monetary non-neutrality by roughly a factor of 3 for our multi-sector menu cost model is driven by three features of the U.S. data: 1) the low average level of inflation in the U.S. economy, 2) the fact that the average size of price changes is large and that there is no strong correlation between the size and frequency of price change across sectors, and 3) the relatively low average frequency of price change in the U.S. economy. We perform a number of counterfactual simulations to illustrate these results. Under alternative assumptions about the inflation rate and the size of idiosyncratic shocks (inconsistent with U.S. data) heterogeneity in the frequency of price change yields minimal amplification of monetary non-neutrality. This contrasts with the Calvo model in which heterogeneity in the frequency of price change amplifies monetary non-neutrality irrespective of these other characteristics of the economy.

Bils and Klenow (2002) and Carvalho (2006) investigate the effect of heterogeneity in the frequency of price change in multi-sector Taylor and Calvo models.[3] Bils and Klenow (2002) analyze the Taylor model and find that heterogeneity amplifies the degree of monetary non-neutrality by a modest amount. Carvalho (2006) considers both the Taylor and Calvo model as well as several time-depentent sticky information models. He incorporates strategic complementarity into his model and considers a different shock process than Bils and Klenow (2002). Carvalho (2006) shows that in time-dependent models the effect of heterogeneity rises with the degree of strategic complementarity. In contrast, we find that in our menu cost model the amplification due to heterogeneity is roughly independent of the degree of strategic complementarity. More recently, Bouakez et al.

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[3]See also Aoki (2001).

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(2009a,b) have extended these results to consider heterogeneity along additional dimensions.

The other feature that amplifies the degree of monetary non-neutrality in our model is intermediate inputs. As in earlier models with time-dependent price setting, introducing intermediate inputs amplifies the degree of monetary non-neutrality because the intermediate inputs cause the pricing decisions of different firms to become strategic complements (Basu, 1995; Huang et al., 2004; Huang and Liu, 2004; Huang, 2006). Intuitively, in the model with intermediate inputs, firms that change their price soon after a shock to nominal aggregate demand choose to adjust less than they otherwise would because the prices of many of their inputs have not yet responded to the shock.

Finally, we also consider an extension of our model that incorporates the idea that firms may at times receive opportunities to change their prices at comparatively low costs for various reasons. These additional low cost price changes will increase the degree of price flexibility in the economy. However, since their timing is not chosen optimally, they will induce less price flexibility than the same number of regular price changes. Using this model, we show that introducing a moderate number of time-dependent price changes into a purely state dependent model has little impact on the overall degree of monetary non-neutrality. Conversely, introducing a small number of state-dependent price changes into a purely time-dependent model has a large effect on the overall degree of monetary non-neutrality.

We argue that times at which firms introduce new products are an important example of such low cost price changes. We document that product turnover is by far most important in durable goods sectors such as apparel and automobiles and that in these sectors the appropriate model of product turnover is likely to be different from the appropriate model of price changes for identical items—particularly in sectors such as apparel and automobiles with highly seasonal product cycles. We show that if price changes due to product introduction are time-dependent, rather than state-dependent, they have only a trivial effect on the degree of monetary non-neutrality in the model.

Our analysis builds on the original work on menu cost models in partial equilibrium by Barro (1972), Sheshinski and Weiss (1977), Mankiw (1985), Akerlof and Yellen (1985) and others. The implications of menu costs in general equilibrium have been analyzed analytically in simple models by Caplin and Spulber (1987), Caballero and Engel (1991, 1993), Caplin and Leahy (1991, 1997), Danziger (1999), Dotsey et al. (1999) and Gertler and Leahy (2008). Willis (2003), Burstein (2005),

Golosov and Lucas (2007) and Midrigan (2006) analyze the implications of menu cost models in general equilibrium using numerical solution methods similar to ours. Finally, we build on a long literature in monetary economics on real rigidities and the use of intermediate inputs by Ball and Romer (1990), Basu (1995), Kimball (1995), Woodford (2003) and others.

The paper proceeds as follows. Section 2 contains a description of the multi-sector menu cost model with intermediate inputs. Section 3 discusses our calibration of the model. Section 4 contains our results regarding the effect of heterogeneity on monetary non-neutrality. Section 5 contains our results on the effect of intermediate inputs on the degree of monetary non-neutrality. Section 6 contains our results on the effect of product turnover on price flexibility. Section 7 contains a discussion of the quantitative importance of our results. Section 8 concludes.

2 A Multi-Sector Menu Cost Model

The model we develop is a multi-sector generalization of the model presented by Golosov and Lucas (2007) in which firms use intermediate inputs as well as labor as a factor of production.

2.1 Household Behavior

The households in the economy maximize discounted expected utility given by

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\frac{1}{1-\gamma} C_{t+\tau}^{1-\gamma} - \frac{\omega}{\psi+1} L_{t+\tau}^{\psi+1} \right], \tag{1}$$

where E_t denotes the expectations operator conditional on information known at time t, C_t denotes household consumption of a composite consumption good and L_t denotes household supply of labor. Households discount future utility by a factor β per period; they have constant relative risk aversion equal to γ; the level and convexity of their disutility of labor are determined by the parameters ω and ψ, respectively.

Households consume a continuum of differentiated products indexed by z. The composite consumption good C_t is a Dixit-Stiglitz index of these differentiated goods:

$$C_t = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $c_t(z)$ denotes household consumption of good z at time t and θ denotes the elasticity of substitution between the differentiated goods.

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The households must decide each period how much to consume of each of the differentiated products. For any given level of spending in time t, the households choose the consumption bundle that yields the highest level of the consumption index C_t. This implies that household demand for differentiated good z is

$$c_t(z) = C_t \left(\frac{p_t(z)}{P_t} \right)^{-\theta} \tag{3}$$

where $p_t(z)$ denotes the price of good z in period t and P_t is the price level in period t given by

$$P_t = \left[\int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \tag{4}$$

The price level P_t has the property that $P_t C_t$ is the minimum cost for which the household can purchase the amount C_t of the composite consumption good.

A complete set of Arrow-Debreu contingent claims are traded in the economy. The budget constraint of the households may therefore be written as

$$P_t C_t + E_t[D_{t,t+1} B_{t+1}] \leq B_t + W_t L_t + \int_0^1 \Pi_t(z) dz, \tag{5}$$

where B_{t+1} is a random variable that denotes the state contingent payoffs of the portfolio of financial assets purchased by the households in period t and sold in period $t + 1$, $D_{t,t+1}$ denotes the unique stochastic discount factor that prices these payoffs in period t, W_t denotes the wage rate in the economy at time t and $\Pi_t(z)$ denotes the profits of firm z in period t. To rule out "Ponzi schemes", we assume that household financial wealth must always be large enough that future income suffices to avert default.

The first order conditions of the household's maximization problem are

$$D_{t,T} = \beta^{T-t} \left(\frac{C_T}{C_t} \right)^{-\gamma} \frac{P_t}{P_T}, \tag{6}$$

$$\frac{W_t}{P_t} = \omega L_t^\psi C_t^\gamma, \tag{7}$$

and a transversality condition. Equation (6) describes the relationship between asset prices and the time path of consumption, while equation (7) describes labor supply.

2.2 Firm Behavior

There are a continuum of firms in the economy indexed by z. Each firm belongs to one of J sectors and specializes in the production of a differentiated product. The production function of firm z is

given by,

$$y_t(z) = A_t(z)L_t(z)^{1-s_m}M_t(z)^{s_m}, \tag{8}$$

where $y_t(z)$ denotes the output of firm z in period t, $L_t(z)$ denotes the quantity of labor firm z employs for production purposes in period t, $M_t(z)$ denotes an index of intermediate inputs used in the production of product z in period t, s_m denotes the materials share in production and $A_t(z)$ denotes the productivity of firm z at time t. The index of intermediate products is given by

$$M_t(z) = \left[\int_0^1 m_t(z, z')^{\frac{\theta-1}{\theta}} dz' \right]^{\frac{\theta}{\theta-1}},$$

where $m_t(z, z')$ denotes the quantity of the z'th intermediate input used by firm z.

Following Basu (1995), we assume that all products serve both as final output and inputs into the production of other products. This "round-about" production model reflects the complex input-output structure of a modern economy.[4] When the material share s_m is set to zero, the production function reduces to the linear production structure considered by Golosov and Lucas (2007). Basu shows that the combination of round-about production and price rigidity due to menu costs implies that the pricing decisions of firms are strategic complements. In this respect, the round-about production model differs substantially from the "in-line" production model considered, for example, by Blanchard (1983). The key difference is that in the round-about model there is no "first product" in the production chain that does not purchase inputs from other firms. The fact that empirically almost all industries purchase products from a wide variety of other industries lends support to the "round-about" view of production.[5]

Firm z in sector j maximizes the value of its expected discounted profits

$$E_t \sum_{\tau=0}^{\infty} D_{t,t+\tau}\Pi_{t+\tau}(z), \tag{9}$$

where profits in period t are given by

$$\Pi_t(z) = p_t(z)y_t(z) - W_t L_t(z) - P_t M_t(z) - \chi_j W_t I_t(z) - P_t U. \tag{10}$$

Here $I_t(z)$ is an indicator variable equal to one if the firm changes its price in period t and zero otherwise. We assume that firms in sector j must hire an additional χ_j units of labor if they decide

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[4]See Blanchard (1987) for an earlier discussion of a model with "horizontal" input supply relationships between firms. Huang, Liu, and Phaneuf (2004) uses the round-about production model to explain variation in the cyclicality of real wages over the 20th century. Huang and Liu (2004) and Huang (2006) investigate the persistence of monetary non-neutrality in a model with round-about production. These papers all assume staggered price contracts of fixed length.

[5]See Basu (1995) for a detailed discussion of this issue.

to change their prices in period t. We refer to this fixed cost of price adjustment as a "menu cost". Finally, U denotes fixed costs the firm must pay to operate. The level of these costs affect the level of profits of the firm as a fraction of output, making it possible to reconcile large markups estimated in the industrial organization literature with small profits in the national accounts. These fixed costs do not affect the firm's decision problem.

Firm z must decide each period how much to purchase of each of the differentiated products it uses as inputs. Cost minimization implies that the firm z's demand for differentiated product z' is

$$m_t(z, z') = M_t(z) \left(\frac{p_t(z')}{P_t} \right)^{-\theta}. \tag{11}$$

Combining consumer demand—equation (3)—and input demand—equation (11)—yields total demand for good z:

$$y_t(z) = Y_t \left(\frac{p_t(z)}{P_t} \right)^{-\theta}, \tag{12}$$

where $Y_t = C_t + \int_0^1 M_t(z)dz$. It is important to recognize that C_t and Y_t do not have the same interpretations in our model as they do in models that abstract from intermediate inputs. The variable C_t reflects *value-added* output while Y_t reflects *gross* output. Since gross output is the sum of intermediate products and final products, it "double-counts" intermediate production and is thus larger than value-added output. GDP in the U.S. National Income and Product Accounts measures value-added output. The variable in our model that corresponds most closely to real GDP is therefore C_t.

The firm maximizes profits—equation (9)—subject to its production function—equation (8)—demand for its product—equation (12)—and the behavior of aggregate variables. We solve this problem by first writing it in recursive form and then by employing value function iteration. To do this, we must first specify the stochastic processes of all exogenous variables.

We assume that the log of firm z's productivity follows a mean-reverting process,

$$\log A_t(z) = \rho \log A_{t-1}(z) + \epsilon_t(z), \tag{13}$$

where $\epsilon_t(z) \sim N(0, \sigma_{\epsilon,j}^2)$ are independent. Notice that we assume that the variance of firm's idiosyncratic shocks are sector specific.

We assume that the monetary authority targets a path for nominal value-added output, $S_t = P_t C_t$. Specifically, the monetary authority acts so as to make nominal value-added output follow a

random walk with drift in logs:

$$\log S_t = \mu + \log S_{t-1} + \eta_t \tag{14}$$

where $\eta_t \sim \mathrm{N}(0, \sigma_\eta^2)$ are independent. We will refer to S_t either as nominal value-added output or as nominal aggregate demand.[6]

The state space of the firm's problem is infinite dimensional since the evolution of the price level and other aggregate variables depend on the entire joint distribution of all firms' prices and productivity levels. Following Krusell and Smith (1998), we make the problem tractable by assuming that the firms perceive the evolution of the price level as being a function of a small number of moments of this distribution.[7] Specifically, we assume that firms perceive that

$$\frac{P_t}{P_{t-1}} = \Gamma\left(\frac{S_t}{P_{t-1}}\right). \tag{15}$$

To allow for convenient aggregation, we also make use of log-linear approximations of the relationship between aggregate labor supply, aggregate intermediate product output and aggregate value-added output.

Using the function Γ to form expectations about the price level turns out to be highly accurate. Figure 2 plots the perceived law of motion for inflation—i.e., Γ—as well as the actual log inflation rate as a function of $\log(S_t/P_t)$ over a 1000 month simulation of the multi-sector model using our benchmark calibration. Γ is a step function since we solve the model on a grid for $\log(S_t/P_t)$. For over 99% of months, the difference between the perceived law of motion and the actual law of motion is less than one grid point in our discrete approximation of inflation. The approximation errors scale with the size of the grid we use, implying that the errors can be made increasingly small as we raise the number of gridpoints.[8] We have experimented with larger grids and found that this does not affect our results. Krusell and Smith (1998) emphasize the R^2 of a regression of the actual law of motion on the perceived law of motion as a test of accuracy. In our model, the R^2 of a regression of true inflation on perceived inflation is larger then 99%, similar to the results in Krusell and Smith (1998) and Midrigan (2006). Den Haan (2008) advocates going beyond this

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[6]This type of specification for nominal aggregate demand is common in the literature. It can be justified by a model of demand in which nominal aggregate demand is proportional to the money supply and the central bank follows a money growth rule. It can also be justified in a cashless economy (Woodford, 2003). In a cashless economy, the central bank can adjust nominal interest rates in such a way to achieve the target path for nominal aggregate demand. In section 4, we consider a generalization of the model in which S_t follows an AR(1) process in growth rates.

[7]Willis (2003) and Midrigan (2006) make similar assumptions.

[8]This is true up to a point well beyond the grid size needed to get an accurate solution.

test to assess multi-period perception errors by comparing long simulations generated on the one hand entirely from using the perceived law of motion (Γ) and on the other hand entirely from using the actual law of motion (simulating the entire model). In a 1000 period simulation of this type for our model, we find that the root mean squared error for both inflation and output are less then one grid point and again scale with the number of gridpoints in our simulation without affecting our results on monetary non-neutrality. For the model reported in figure 2, the root mean squared error for inflation is less than five hundredths of a percent.

Given these assumptions, firm z's optimization problem may be written recursively in the form of the Bellman equation

$$V\left(A_t(z), \frac{p_{t-1}(z)}{P_t}, \frac{S_t}{P_t}\right) = \max_{p_t(z)} \left\{\Pi_t^R(z) + E_t\left[D_{t,t+1}^R V\left(A_{t+1}(z), \frac{p_t(z)}{P_{t+1}}, \frac{S_{t+1}}{P_{t+1}}\right)\right]\right\}, \qquad (16)$$

where $V(\cdot)$ is firm z's value function, $\Pi_t^R(z)$ denotes firm z's profits in real terms at time t and $D_{t,t+1}^R$ denotes the real stochastic discount factor between time t and $t+1$.[9]

An equilibrium in this economy is a set of stochastic processes for the endogenous price and quantity variables discussed above that are consistent with household utility maximization, firm profit maximization, market clearing and the evolution of the exogenous variables $A_t(z)$ and S_t. We use the following iterative procedure to solve for the equilibrium: 1) We specify a finite grid of points for the state variables, $A_t(z)$, $p_{t-1}(z)/P_t$ and S_t/P_t. 2) We propose a function $\Gamma(S_t/P_{t-1})$ on the grid. 3) Given the proposed Γ, we solve for the firm's policy function F by value function iteration on the grid. 4) We check whether Γ and F are consistent.[10] If so, we stop and use Γ and F to calculate other features of the equilibrium. If not, we update Γ and go back to step 3. We approximate the stochastic processes for $A_t(z)$ and S_t using the method proposed by Tauchen (1986).[11]

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[9]In appendix A, we show how the firm's real profits can be written as a function of $(A_t(z), p_{t-1}(z)/P_t, S_t/P_t)$ and $p_t(z)$.

[10]We do this in the following way: First, we calculate the stationary distribution of the economy over $(A(z), p(z)/P, S/P)$ implied by Γ and F as described in appendix B. Second, we use the stationary distribution and equation (4) to calculate the price index implied by Γ—call it P_Γ—for each value of S/P. Third, we check whether $|P_\Gamma - P| < \xi$, where $|\cdot|$ denotes the sup-norm.

[11]A drawback of numerical methods of the type we employ in this paper is that it is difficult to prove uniqueness. The main feature of our model that potentially could generate non-uniqueness is the combination of strategic complementarity and menu costs (Ball and Romer, 1991). However, the large idiosyncratic shocks that we assume in our model significantly reduce the scope for multiplicity (Caballero and Engel, 1993). In particular, the type of multiplicity studied by Ball and Romer does not exist in our model since the large idiosyncratic shocks prevent sufficient synchronization across firms. In this respect our results are similar to John and Wolman (2004). It is also conceivable that our use of Krusell and Smith's approximation method could yield self-fulfilling approximate equilib-

2.3 The CalvoPlus Model

Much applied work in monetary economics relies on models in which the timing of price changes is independent of firm's incentives to change prices. Such price changes are said to be "time-dependent". In this subsection, we describe an extension of our menu cost model in which a fraction of price changes are largely time-dependent. We use this model to assess the sensitivity of our purely state-dependent benchmark model to the introduction of a small number of time-dependent price changes and, conversely, the sensitivity of time-dependent models to the introduction of a small number of state-dependent price changes.

The most widely used model of time-dependent price changes is the model of Calvo (1983).[12] In this model, price changes are free with probability $(1-\alpha)$ but have infinite cost with probability α. These extreme assumptions make the Calvo model highly tractable. However, they also cause the model to run into severe trouble in the presence of large idiosyncratic shocks or a modest amount of steady state inflation.[13] The reason is that the firm's implicit desire to change its price can be very large and it frequently prefers to shut down rather than continue producing at its pre-set price.

Rather than assuming that price changes are either free or infinitely costly, we assume that with probability $(1-\alpha)$ the firm faces a low menu cost $\chi_{j,l}$, while with probability α it faces a high menu cost $\chi_{j,h}$. These assumptions retain the tractability of the Calvo model. But at the same time they capture the idea that the timing of some price changes is largely orthogonal to the firm's desire to change its price in a more realistic way than the Calvo model does. We refer to this model as the "CalvoPlus" model. The CalvoPlus model has the appealing feature that it nests both the Calvo model and the menu cost model as special cases.[14]

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ria. There is, however, nothing in the economic link between agents beliefs and their pricing decision that suggests such self-fulfilling equilibria. In fact, the actual behavior of the price level in our model is quite insensitive to even relatively large changes in beliefs. The reason for this is that by far the most important factor in agent's decisions is movements in their idiosyncratic productivity levels as opposed to movements in aggregate variables. We solved our model with more sophisticated beliefs (additional moments) and starting our fixed point algorithm at various initial values. In all cases the resulting approximate fixed point is virtually identical.

[12]Examples of papers that use the Calvo model include Christiano et al. (2005) and Clarida et al. (1999). An alternative time-dependent price setting model was proposed by Taylor (1980). This model has been used, e.g., by Chari et al. (2000).

[13]See Bakhshi et al. (2006) for an analysis of the latter issue.

[14]Our CalvoPlus model is related to the random menu cost model analyzed by Dotsey et al. (1999) and Caballero and Engel (2006). It is also related to the model developed by Midrigan (2006). Midrigan augments the Golosov-Lucas model by allowing for fat-tailed idiosyncratic shocks and multi-product firms with scale economies in changing prices. These features imply that the hazard of price change is much less strongly related to the firm's price relative to its desired price, muting the selection effect as in our CalvoPlus model.

3 Calibration

We focus attention on the behavior of the economy for a specific set of parameter values. Table 1 reports our benchmark parameter values. We set the monthly discount factor equal to $\beta = 0.96^{1/12}$. We assume log-utility in consumption ($\gamma = 1$). Following Hansen (1985) and Rogerson (1988), we assume linear disutility of labor ($\psi = 0$). The most important way in which these parameters affect our results is through the elasticity of the real wage with respect to output. Our calibration implies that the elasticity of the real wage with respect to output is equal to one—equation (7). Solon, Barsky, and Parker (1994) estimate that the elasticity of real wages with respect to output in the U.S. is in fact only about 0.6. In our model, a higher elasticity of the real wage reduces monetary non-neutrality. On the face of it, our calibration thus seems tilted towards generating too little monetary non-neutrality. However, we show below that the absence from capital in our baseline model lowers the elasticity of marginal costs by roughly the same factor. Our benchmark model thus implies a degree of monetary non-neutrality similar to a model that incorporates capital and is calibrated to match the cyclicality of real wages.

We set ω such that in the flexible price steady state labor supply is 1/3. We set $\theta = 4$ to roughly match estimates of the elasticity of demand from the industrial organization and international trade literatures.[15] Our choices of $\mu = 0.0028$ and $\sigma_\eta = 0.0065$ are based on the behavior of U.S. nominal and real GDP during the period 1947-2005.[16] Since our model does not incorporate a secular trend in economic activity, we set μ equal to the mean growth rate of nominal GDP less the mean growth rate of real GDP. We set σ_η equal to the standard deviation of nominal GDP growth.

We calibrate the size of the menu cost and the variance of the idiosyncratic shocks in each sector of our model based on empirical evidence on the frequency and size of price changes excluding sales in consumer prices across sectors of the U.S. economy presented in Nakamura and Steinsson (2008).[17] We group goods with similar price change characteristics into 6 sectors, 9 sectors and

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[15] Berry et al. (1995) and Nevo (2001) find that markups vary a great deal across firms. The value of θ we choose implies a markup similar to the mean markup estimated by Berry et al. (1995) but slightly below the median markup found by Nevo (2001). Broda and Weinstein (2006) estimate elasticities of demand for a large array of disaggregated products using trade data. They report a median elasticity of demand below 3. Also, Burstein and Hellwig (2006) estimate an elasticity of demand near 5 using a menu cost model. Midrigan (2006) uses $\theta = 3$ while Golosov and Lucas (2007) use $\theta = 7$. The value of θ affects our calibration of the menu cost—a higher θ imply higher menu costs—and it affects our calibration of the intermediate input share—a higher θ implies lower values for s_m. Holding fixed the frequency of price change, the value of θ does not affect the degree of monetary non-neutrality in our model.

[16] Our results are virtually identical if we set $\mu = 0$ rather than $\mu = 0.0028$.

[17] We have also used the distribution of the frequency of price change including sales. We find that both of these distributions yield a similar results regarding amplification of monetary non-neutrality due to heterogeneity. We do

14 sector. Table 2 presents the mean frequency and mean absolute size of price changes for these sectors.[18] Both the frequency and size of price changes varies enormously across sectors. There is no simple relationship between these two variables (see figure 3). Furthermore, the distribution of the frequency of price change is highly asymmetric. The right tail being much longer than the left tail. This skewness implies that the mean frequency of price change across sectors is much higher than the median frequency of price change—21.1% versus 8.7% for 1998-2005.[19]

Table 3 presents the parameterization of the menu cost and the variance of the idiosyncratic shocks at the sectoral level that allow the model to match the empirical statistics on the frequency and size of price changes presented in table 2. We report the average yearly cost of changing prices in each sector as a fraction of steady state revenue. In all cases, the cost of changing prices is less than 1% of revenue and in most sectors it is less than 0.5%. The cost of changing prices is less than half as large in the model with intermediate inputs as it is in the model without intermediate inputs.

The standard deviation of the idiosyncratic shocks needed to match the size of price changes in the data are quite large. They range from about 3% to about 11%. Figure 4 plots a sample path for a "typical" firm in the model with intermediate inputs. The plot illustrates that the standard deviation of the idiosyncratic shocks is many times larger than the standard deviation of the shocks to nominal aggregate demand. As is emphasized by Golosov and Lucas (2007), this is crucial for generating price changes sufficiently large to match the data. It is also crucial for generating the substantial number of price decreases observed in the data.[20] For computational reasons, we set the speed of mean reversion of the firm productivity process equal to $\rho = 0.7$. This value is close to the value we estimate for ρ in Nakamura and Steinsson (2008).

The parameter s_m denotes the cost share of intermediate inputs in the model. Table 4 contains information from the 2002 U.S. Input-Output Table published by the Bureau Economic Analysis.

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not have an analytical proof of unique identification in the multi-sector model. In numerical simulations, we have found that variation of parameters in one sector has virtually no effect on the size and frequency in other sectors. This implies that the overall model is uniquely identified since the parameters in each sector are uniquely identified.

[18]To be able to aggregate the sectors easily, we calibrate the multi-sector models to the mean frequency and mean absolute size of price change at the sectoral level. The difference between the sectoral mean and median are small.

[19]In Nakamura and Steinsson (2008), we find a similar pattern for finished goods producer prices. In the producer prices case the mean is 24.7% while the median is 10.8%.

[20]Empirical evidence suggests that variation of firm productivity is in fact much smaller than what is implied by our calibration (Abraham and White, 2007). The idiosyncratic productivity shocks should therefore be viewed as a stand-in for a broader class of idiosyncratic shocks that cause variation in firms' desired prices.

The table provides information about both the share of intermediate inputs in the gross output of each sector (column 1) and about how intensively the output of each sector is used as an intermediate input in other sectors (column 2). The revenue share of intermediate inputs varies from about 1/3 to about 2/3. It is highest in manufacturing and lowest in utilities. The use of different sectors as intermediate inputs (column 2) is closely related to their weight in gross output (column 4). In particular, services are used heavily as an intermediate input (accounting, legal, consulting, financial, marketing). The main deviations from this pattern is that the output of manufacturing is used somewhat more intensively as intermediate inputs than its weight in gross output would suggest while the output of the government sector and the construction sector are used less.

The weighted average revenue share of intermediate inputs in the U.S. private sector using CPI expenditure weights was roughly 52% in 2002. The cost share of intermediate inputs is equal to the revenue share times the markup. Our calibration of θ implies a markup of 1.33. Our estimate of the weighted average cost share of intermediate inputs is therefore roughly 70%.

This calibration depends on a number of assumptions. Alternative assumptions yield estimates of the intermediate inputs share that are either lower or higher. On the one hand, we employed CPI weights as we do elsewhere in the paper. Using gross output weights would yield a slightly lower number (63% rather than 70%) since services have a higher weight in gross output than in the CPI. However, increasing the weight of services would also lower the mean frequency of price change and increase the skewness of the frequency distribution. A higher value for the elasticity of demand would also yield a lower intermediate input share. For example, Golosov and Lucas (2007) use $\theta = 7$. This would yield and intermediate input share equal to 60% rather than 70%.

On the other hand, we have assumed that intermediate inputs make up the same fraction of marginal costs as they do average variable costs. With a more general production structure, this is not necessarily the case. Materials might be disproportionately important at the margin, in which case the share of intermediate inputs in marginal costs would be higher than we estimate. Also, the constant intermediate inputs share that matches the behavior of an economy with heterogeneity in the use of intermediate inputs across sectors is slightly higher than the average s_m across sectors (see discussion in section 5). Given the uncertainty associated with these factors, we report results for a range of different values for s_m from 0.5 to 0.9 in table 8 below.[21]

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[21]Basu (1995) and Bergin and Feenstra (2000) argue for values of the parameter s_m between 0.8 and 0.9. Huang et al. (2004) favor a value of 0.7. Rotemberg and Woodford (1995), Chari et al. (1996) and Woodford (2003, ch. 3)

The assumption of round-about production implicitly assumes that prices are rigid to all customers whether they are consumers or firms. Direct evidence on producer prices from Carlton's (1986) work on the Stigler-Kindahl dataset as well as Blinder et al.'s (1998) survey of firm managers supports the view that price rigidity is an important phenomenon at intermediate stages of production. In Nakamura and Steinsson (2008), we present a more comprehensive analysis of producer prices based on the micro-data underlying the producer price index and find that the rigidity of producer prices is comparable to the rigidity of non-sale consumer prices. The median frequency of price change of finished goods and intermediate goods producer prices is 10.8% and 14.3%, respectively, while the median frequency of price change of consumer prices is 8.7%. Moreover, we document a high correlation between the frequency of non-sale consumer price changes and the frequency of producer price changes at a very disaggregated level. This evidence is reproduced in table 5. Over the 153 matches, the correlation between the frequency of price change for producer prices and consumer prices excluding sales is 0.83.

Our baseline model abstracts from capital accumulation. However, in appendix C we develop a model with capital to assess the effect that capital has on our results. The main way in which introducing capital into our model affects our results is by affecting the variability of marginal costs and thus the degree of real rigidity in the model. In the baseline model with intermediate inputs, the elasticity of marginal costs with respect to output is equal to $1 - s_m = 0.3$. In appendix C, we derive an upper bound of 0.38 for the elasticity of marginal costs with respect to output in the model with capital. The empirical results of Solon, Barsky, and Parker (1994) on the cyclicality of real wages suggest that for the U.S. economy the elasticity of real wages with respect to output is in fact only about 0.6. Our baseline calibration assumes a unit elasticity of the real wage with respect to output, somewhat overstating the response of real wages relative to empirical evidence. If we incorporate Solon, Barsky and Parker's empirical estimate for the wage elasticity into our model with capital we get an elasticity of marginal cost of 0.28. This is almost exactly equal to the elasticity of 0.3 that we assume in our baseline model. In other words, we have adopted a baseline specification that implies an elasticity of marginal costs similar to what is implied by a

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use values closer to $s_m = 0.5$. The lower values of s_m are based on much lower calibrations of the markup of prices over marginal costs than we use. These low markups are meant to match the fact that pure profits are a relatively small fraction of GDP in the U.S.. We base our calibration of the markup of prices over marginal costs on evidence from the industrial organization and international trade literature. These high markups are consistent with small pure profits if firms have fixed costs and/or if firm entry involves sunk investment costs that must be recouped with flow profits post-entry as in our model (e.g., Dixit and Pindyck, 1994; Ryan, 2006).

model with capital and calibrated to match the empirical evidence presented in Solon, Barsky, and Parker (1994). We discuss this in detail in apprendix C.

4 Heterogeneous Price Rigidity and Monetary Non-Neutrality

Our primary interest is the degree of monetary non-neutrality generated by the menu cost model. Table 6 presents estimates of this for a number of different calibrations of the model. We measure the degree of monetary non-neutrality as the variance of real value-added output when the model is simulated with purely nominal aggregate shocks.[22] We first consider the behavior of the menu cost model with the intermediate input share set to zero. We will consider the effect of introducing intermediate inputs in section 5.

The first column of table 6 presents results for our 6, 9 and 14 sector models as well as two calibrations of a single-sector version of our model. The degree of monetary non-neutrality is sharply increasing in the number of sectors. The 14 sector model generates roughly three times as much monetary non-neutrality as the single-sector model that is calibrated to match the mean frequency of price change.[23] The table also reports results for the single-sector model calibrated to match the median frequency of price change. This calibration of the single-sector model yields a degree of monetary non-neutrality that is more similar to the multi-sector model than does the single-sector model calibrated to match the mean frequency of price change.

Why does heterogeneity in the frequency of price change amplify the degree of monetary non-neutrality? A simplifying feature of the model without intermediate inputs is that the pricing decisions of different firms are virtually independent. This is due to a combination of two features of our model. First, firms face a constant elasticity of demand which implies that their static desired price is a constant markup over marginal costs. Second, firms' marginal costs are $MC_t(z) = W_t/A_t(z)$ and the wage is given by $W_t/P_t = \omega L_t^\psi C_t^\gamma = \omega C_t$ where the second equality is due to our choice of preference parameters (see table 1). This implies that $W_t = \omega P_t C_t = \omega S_t$ and $MC_t(z) = \omega S_t/A_t(z)$. So, firm z's marginal costs are exogenous and therefore independent of other

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[22]This measure of monetary non-neutrality has been used, e.g., by Midrigan (2006). An alternative measure of monetary non-neutrality is the cumulative impulse response (CIR) of real value-added output to a permanent shock to nominal aggregate demand. If our model were log-linear and delivered an AR(1) response of real output to a permanent shock to nominal aggregate demand these measures would be proportional. We have calculated the CIR for all cases presented in the paper and the results are practically identical using this alternative measure.

[23]We considered models with more than 14 sectors. They yielded very similar results to the 14 sector model.

firm's prices.

In this case, the degree of monetary non-neutrality in the economy is approximately a weighted average of the monetary non-neutrality in each sector viewed independently. Heterogeneity in the frequency of price change across sectors, therefore, increases the overall degree of monetary non-neutrality in the economy if the degree of monetary non-neutrality in different sectors of the economy is a convex function of each sector's frequency of price change (Jensen's inequality).

The simplest model in which to study the relationship between heterogeneity in the frequency of price change and monetary non-neutrality is the Calvo model since in that model the firms that change their price in each period are a random sample of all firms. Carvalho (2006) shows that in the Calvo model the degree of monetary non-neutrality is highly convex in the frequency of price change. The intuition for this is simple. Consider the response to a permanent shock to nominal aggregate demand. In the Calvo model, the effect of the shock on output at any given point in time after the shock is inversely proportional to the fraction of firms that have changed their price at least once since the shock occurred. If some firms have vastly higher frequencies of price change than others, they will change their prices several times before the other firms change their prices once. But all price changes after the first one for a particular firm do not affect output on average since the firm has already adjusted to the shock. Since a marginal price change is more likely to fall on a firm that has not already adjusted in a sector with a low frequency of price change, the degree of monetary non-neutrality in the Calvo model is convex in the frequency of price change.

In the menu cost model, firms are not selected at random to change their prices. The relationship between the frequency of price change and the degree of monetary non-neutrality in different sectors of the economy is therefore more complicated in a menu cost model. It depends crucially on the nature of the differences between the sectors that give rise to the differences in the frequency of price change. Consider two sectors—A and B—in the menu cost model. One reason why sector A may have a lower frequency of price change than sector B is that firms in sector A face larger menu costs than firms in sector B. Another possible reason is that firms in sector A may face smaller idiosyncratic shocks but face menu costs of the same size. These two cases will give rise to different implications regarding the relative degree of monetary non-neutrality in the two sectors. We can empirically distinguish between there two cases in our model because they have different implications about the size of price changes.

The degree of monetary non-neutrality generated by each sector in a menu cost model is governed by the degree to which the distribution of relative prices of different firms in that sector changes in response to monetary shocks. To build intuition, it is instructive to consider the model analyzed by Caplin and Spulber (1987). They consider a continuous time model with no idiosyncratic shocks and a process for aggregate demand that always increases. In this setting, firms raise their relative price to a level S whenever it hits a level s. If the initial distribution of relative prices is uniform it will continue to be uniform at all future dates and money will thus be neutral for any frequency of price change. The Caplin-Spulber economy is the limiting case of our model when inflation is high, idiosyncratic shocks small and the time period short.[24]

For an economy with many sectors, the relationship between the frequency of price change and monetary non-neutrality depends critically on the relationship between the frequency of price change and the size of price changes across the different sectors. This is illustrated in figure 5. Each of the solid lines in this figure plots the degree of monetary non-neutrality in a sector for a given variance of idiosyncratic shocks as the size of the menu cost changes. The top line has the largest idiosyncratic shocks and the bottom line the smallest. For each level of the variance of idiosyncratic shocks, the frequency of price change falls and the degree of monetary non-neutrality increases as the size of menu costs increase. But the level of monetary non-neutrality at a given frequency of price change is different depending on the variance of the idiosyncratic shocks. This occurs because the "selection effect" becomes stronger as the size of the idiosyncratic shocks is reduced at a given frequency of price change since the average inflation rate becomes a more and more important determinant of price changes relative to the idiosyncratic shocks. Intuitively, reducing the size of idiosyncratic shocks moves the economy towards the Caplin-Spulber extreme.

In actual economies, the variance of idiosyncratic shocks may vary greatly across sectors. This implies that the different sectors in a particular economy need not lay on the same line. The dashed

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[24]Caballero and Engel (1991, 1993) generalize the results of Caplin and Spulber (1987) to a case with idiosyncratic shocks. To derive their neutrality results, Caballero and Engel must maintain the assumption that the dynamic optimal price always increases. They must therefore effectively assume that the idiosyncratic shocks are either very small or one-sided. Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008) present evidence that a sizable fraction of price changes are price decreases. The data are thus inconsistent with Caballero and Engel's assumptions. An alternative setting in which money is neutral in the presence of idiosyncratic shocks is a slight variation of the one analyzed by Danziger (1999). Danziger (1999) assumes uniformly distributed idiosyncratic shocks large enough that all firms have an equal probability of exiting the inaction band regardless of the initial price. The resulting frequency of price change is much higher than what we observe in the data. Danziger's model implies monetary neutrality if the menu cost takes the same form as in our model. Thus, the parameterizations of the menu cost model that generate monetary neutrality require either very small or very large idiosyncratic shocks and are inconsistent with the data.

line illustrates this by connecting four sectors of a hypothetical economy which has one sector on each line. In this example, the relationship between the size of menu costs and the variance of idiosyncratic shocks across sectors is such that the sector with the lowest frequency of price change has the lowest degree of monetary non-neutrality and the relationship between the frequency of price change and monetary non-neutrality is concave as opposed to the convex shape of each of the solid lines. A wide range of relationships between the frequency of price change and the degree of monetary non-neutrality are possible by connecting points on different lines.

Another determinant of the degree of amplification of monetary non-neutrality due to heterogeneity is the level of the overall frequency of price change. Table 7 illustrates this using a number of simple one and two sector models. Holding fixed the spread between the frequency of price change in the two sectors of the two sector economy, as we raise the average frequency of price change, the degree of amplification relative to a single sector model with the same average frequency of price change diminishes. Specifically, the first row presents results for a two sector economy in which half of firms have a frequency of price change equal to 10% and the other half have a frequency of price change of 20%. This economy is compared to a single-sector economy with a frequency of price change equal to 15%. The two-sector economy yields 14% more monetary non-neutrality. Rows 2 through 4 show that the degree of amplification falls steeply for similar comparisons as the overall frequency of price change rises. A comparison of rows 1, 5 and 6 shows that amplification arises from heterogeneity in the frequency of price change across sectors, not other features such as its skewness.

Now that we have established what *can* happen in the model, let's consider what *does* happen for parameter values calibrated to U.S. data. The darker line in figure 6 plots the variance of real output as a function of the frequency of price change for our calibration of the U.S. economy. It shows that the relationship between the degree of monetary non-neutrality and the frequency of price change in our model is highly convex. This yields the large amount of amplification documented in table 6. The convexity in our baseline calibration is a consequence of three features of the U.S. data: 1) the low average level of inflation in the U.S. economy, 2) the fact that the average size of price changes is large and that there is no strong correlation between the size and frequency of price change across sectors, and 3) the relatively low average frequency of price change in the U.S. economy. The lighter line in figure 6 plots a counterfactual calibration of our model in which we

have assumed that the yearly inflation rate in the U.S. is 12% rather than 3.5% and the variance of the idiosyncratic shocks that affect firm's marginal costs are roughly 4 times smaller than in our baseline calibration. In this case, the relationship between the degree of monetary non-neutrality and the frequency of price change is almost linear and heterogeneity implies little amplification of monetary non-neutrality.

Our baseline model assumes that nominal output follows a random walk with drift. Empirically, the growth rate of U.S. nominal GDP over the period 1947-2005 is better described as an AR(1) with an autoregressive coefficient of roughly 0.5 at a quarterly frequency. For robustness, we have solved a version our model in which nominal output growth follows an AR(1) calibrated to the data.[25] The model with AR(1) growth in nominal output yields somewhat higher monetary non-neutrality than the baseline model.

As Midrigan (2006) emphasizes, the effects of persistence in the money growth process are quite different in the menu cost model than the Calvo model.In the menu cost model, firms may optimally delay price changes after they see a shock to nominal output in anticipation of further movements in the same direction in the near future. This desire to optimally time price changes may lead to wider sS bounds in the AR(1) case than the random walk case. In the Calvo model, firms are not able to time their price changes. Those firms that have an opportunity to change their price immediately after a persistent shock respond preemptively to future expected movements in nominal output since they can't know when they will again get to change their price, raising the responsiveness of prices. Midrigan (2006) notes that firms with state-dependent pricing policies do not have the same incentive to front-load since they are able to choose the timing of their next price adjustment after a shock.

To evaluate the sensitivity of our results to the introduction of time-dependent price changes we consider simulations of the CalvoPlus model introduced in section 2.3. The degree of monetary non-neutrality in the CalvoPlus model is highly sensitive to the fraction of price changes that occur in the low cost state. Figure 7 plots the variance of output in a single sector version of the CalvoPlus model as the fraction of price changes in the low menu cost state varies from zero to one. In this experiment, we set $1 - \alpha$ equal to the median frequency of price change in the economy

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[25]We choose the autocorrelation and innovation variance for monthly nominal output growth in such a way that the resulting process—when time-aggregated to a quarterly frequency—has the same autocorrelation and unconditional variance as quarterly nominal GDP growth in the data.

and $\sigma_\epsilon = 0.0425$. We vary χ_h and χ_l so that the model matches the median frequency of price changes and a particular fraction of price changes in the low menu cost state. The figure shows that the degree of monetary non-neutrality drops off rapidly as the fraction of price changes in the low cost state falls below 100%. When 85% of price changes occur in the low menu cost state, the variance of output is less than half of what it is when all of price changes occur in the low cost state. When 50% of price changes occur in the low menu cost state, the variance of output is close to identical to the value in the constant menu cost model. Figure 7 therefore suggests that the relatively large amount of monetary non-neutrality generated by the Calvo model is quite sensitive to even a modest amount of selection by firms regarding the timing of price changes.

The degree of amplification due to heterogeneity is very similar in the CalvoPlus model to what it is in the pure menu cost model. To illustrate this, we consider results for the CalvoPlus model calibrated so that roughly 75% of price changes occur in the low menu cost state in the third column of table 6. The overall level of monetary non-neutrality is about three time higher in this calibration of the CalvoPlus model.[26] However, the degree of amplification due to heterogeneity is very similar to what it is in the pure menu cost model. In both cases, allowing for heterogeneity in the frequency of price change roughly triples the degree of monetary non-neutrality. The same qualitative results hold for other calibrations of the CalvoPlus model.

5   Intermediate Inputs and Monetary Non-Neutrality

5.1   Intermediate Inputs as a Source of Amplification

We now incorporate intermediate inputs into the model. In section 3, we argue that an empirically plausible level for the intermediate input share is 0.7. The second column of table 6 presents results for the menu cost model with this intermediate inputs share. This calibration yields roughly triple the amount of monetary non-neutrality that the model without intermediate inputs does. Table 8 presents results for several additional values of the intermediate inputs share.

As is well known, the presence of intermediate inputs amplifies the degree of monetary non-neutrality because it causes the pricing decisions of firms in the model to become strategic com-

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[26]For this exercise, we set $1 - \alpha$ equal to the frequency of price change in each sector. We set $\chi_l = \chi_h/40$ and subject to this choose χ_h and σ_ϵ to match either the frequency and size of price changes across sectors. This parameterization implies that roughly 75% of price changes occur in the low menu cost state.

plements.[27] In the model with intermediate inputs, firm's marginal costs are a weighted average of the wage the firm faces and the cost of its inputs. Specifically, the firm's marginal costs are given by

$$MC_t(z) = \frac{W_t^{1-s_m} P_t^{s_m}}{A_t(z)} = \frac{\omega S_t^{1-s_m} P_t^{s_m}}{A_t(z)},$$

where the later equality follows from the definition of S_t and the fact that $W_t/P_t = \omega L_t^{\psi} C_t^{\gamma} = \omega C_t$ given our calibration of $\psi = 0$ and $\gamma = 1$. Since the prices of the firm's inputs are the prices of the other goods in the economy, the firm's marginal costs depends directly on the prices of the other goods in the economy. This is the source of strategic complementarity in the model with intermediate inputs. Since the prices of other goods in the economy respond sluggishly to an increase in S_t when firms face menu costs, the firm's marginal costs rise by less than one-percent in response to a one-percent increase in S_t when $s_m > 0$. As a consequence, firms that change their price soon after a shock to S_t choose a lower price than they would if labor was their only input. In other words, firms choose not to change their prices as much as they otherwise would because the price of many of their inputs have not yet responded to the shock.[28]

An important qualitative difference between our menu cost model and time-dependent models is the way in which heterogeneity in the frequency of price chance and intermediate inputs interact. In our menu cost model, the amplification of monetary non-neutrality due to intermediate inputs is virtually identical in the multi-sector model as in the single-sector model. In other words, these two sources of amplification are roughly independent of each other. In contrast, Carvalho (2006) emphasizes the importance of the interaction between these two features in models with time-dependent price changes. Our CalvoPlus model confirms this interaction. In the Calvo model with strategic complementarity and heterogeneity in the frequency of price change, the firms in high frequency of price change sectors are influenced by the non-response of firms in the low frequency of price change sectors. However, firms in the low frequency of price change sectors are much less influenced by firms in the high frequency of price change sectors because so many of them don't

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[27]This point was first made by Basu (1995). Important additional contributions have been made by Huang and Liu (2004) and Huang (2006). Huang and Liu (2004) show that intermediate inputs increase the persistence of output in response to monetary shocks in the presence of staggered prices but not in the presence of staggered wages. Huang (2006) studies a model with both intermediate inputs and specific factors and argues that the presence of these two factors together generates a negative interaction effect that weakens the degree of strategic complementarity.

[28]The firm's profit function in our model simply implies that a fraction $1 - s_m$ of costs are proportional to S_t while a fraction s_m are proportional to P_t. In the derivation of this equation, we assume that the "flexible" input is labor and the "sluggish" input is intermediate inputs. However, this profit function is consistent with other models in which, e.g., wages are sluggish (Burstein and Hellwig, 2006) and other inputs are flexible.

respond at all. In the language of Haltiwanger and Waldman (1991), the high frequency of price change firms are "responders", while most of the low frequency of price change firms are exogenously determined to be "non-responders". This asymmetry implies that the equilibrium increasingly becomes disproportionately affected by the low frequency of price change sectors as the degree of strategic complementarity increases. In the Calvo model, there is thus an interaction between strategic complementarity and heterogeneity in the frequency of price change as in Haltiwanger and Waldman (1991).

In contrast, in the menu cost model, the extensive margin of price change allows the low frequency of price change firms to be influenced by the presence of the high frequency of price change firms to a much greater extent than in the Calvo model. In particular, when a shock occurs, some firms that would otherwise not have changed their prices do change their prices because firms in the high frequency of price change sector are changing their prices. This implies that there is not as sharp a distinction between responders and non-responders and mutes the interaction between strategic complementarity and heterogeneity in the frequency of price change.

How does the degree of real rigidity in our model compare to the degree of real rigidities in recent quantitative monetary business cycle models such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007)? The relative volatility of marginal costs and output is a measure of (Ω-type) real rigidity both in our model and in these other models. We can directly compare the degree of real rigidity in our model to these other models by calculating the relative standard deviation of marginal costs and output in our model and comparing it to this same statistic in a quantitative DSGE model. Justiniano and Primiceri (2008b) analyze a state-of-the-art quantitative DSGE model that builds heavily on the models of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). In their model, the relative standard deviation of marginal costs to output is 0.36.[29] In our model, this statistics is equal to s_m. In our baseline calibration with intermediate inputs, it is thus 0.3. This shows that our calibration implies a very similar degree of real rigidity as the degree of real rigidity estimated by Justiniano and Primiceri (2008b). Were we to calibrate $s_m = 0.64$, our model would have the exact same amount of real rigidity as theirs. This change of calibration would not materially affect our results.

The model we consider above makes the simplifying assumption that s_m is the same for all

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[29]We thank Alejandro Justiniano and Giorgio Primiceri for producing this statistic for us. This statistic is for the time-invariant version of their model.

sectors and also that all sectors use the same mix of intermediate inputs. We have analyzed an extension where we allow s_m to differ across sectors. We set the sectoral s_m based on data from the U.S. Input-Output data for 2002 presented in table 4. Qualitatively, allowing for this type of heterogeneity effects our results through two channels. First, the degree of monetary non-neutrality in a sector is a convex function of s_m, other things equal. Jensens' inequality thus implies that an economy with heterogeneous s_m will have more monetary non-neutrality than an economy in which all sectors have the average s_m. Second, the degree of amplification due to heterogeneity is affected by the correlation between s_m and the frequency of price change across sectors. Empirically, s_m in a sector is positively correlated with the frequency of price change in the sector. This leads to further amplification of monetary non-neutrality since the price level in relatively flexible sectors is held back by heavy use of intermediate inputs from sticky sectors. Quantitatively, extending our model to allow for heterogeneity in s_m across sectors raises the degree of monetary non-neutrality, but the magnitude of the effect is quite small.

It is much harder computationally to allow different sectors to use different mixes of intermediate inputs in our menu cost model since this would require the inflation rates of all 14 sectors to be state variables in the model. However, Bouakez, Cardia, and Ruge-Murcia (2009a) have been able to carefully analyze these issues in the context of the multi-sector Calvo model. Thy incorporate detailed evidence on sectoral input-output tables into a multi-sector Calvo model. They find that the amplification of monetary non-neutrality associated with sectoral heterogeneity carries over to this more general environment.

5.2 A Comparison with Other Sources of Strategic Complementarity

Strategic complementarity has long been an important source of amplification of nominal rigidities (Ball and Romer, 1990; Woodford, 2003). However, recent work has cast doubt on strategic complementarity as a source of amplification in menu cost models with idiosyncratic shocks by showing that the introduction of strategic complementarity can make it difficult to match the large observed size of price changes for plausible values of the menu cost and the variance of the idiosyncratic shocks. Klenow and Willis (2006) show that a model with demand-side strategic complementarity of the type emphasized by Kimball (1995) requires massive idiosyncratic shocks and implausibly large menu costs to match the size of price changes observed in the data. Golosov

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and Lucas (2007) note that their model generates price changes that are much smaller than those observed in the data when they consider a production function with diminishing returns to scale due to a fixed factor of production. Burstein and Hellwig (2006) use supermarket scanner data to calibrate a model with a fixed factor of production and both demand and supply shocks. They find that even with large demand shocks, a substantial amount of strategic complementarity requires large menu costs to match the micro data on the size of price changes.

The challenge first emphasized by Klenow and Willis (2006) that commonly used sources of monetary non-neutrality cannot match the size of price changes for reasonable parameter values is a serious one given the extent to which many monetary business cycle models rely on these mechanisms to amplify monetary non-neutrality. However, it is not clear from Klenow and Willis (2006), what the scope of this problem is. Does it apply to all sources of strategic complementarity of just some? If if only applies to some, to which ones does it apply? The goal of this subsection if to clarify this issue.

Strategic complementarity generated by firms' use of intermediate inputs does not affect the size of price changes or require unrealistically large menu costs and idiosyncratic shocks (see table 3). The reason for this difference can be illustrated using a dichotomy developed by Ball and Romer (1990) and Kimball (1995). A firm's period t profit function may be written as $\Pi(p_t/P_t, S_t/P_t, \tilde{A}_t)$, where p_t/P_t is the firm's relative price, S_t/P_t denotes real aggregate demand and $\tilde{A}_t$ denotes a vector of all other variables that enter the firms period t profit function. The firm's desired price under flexible prices is then given by $\Pi_1(p_t/P_t, S_t/P_t, \tilde{A}_t) = 0$, where the subscript on the function Π denotes a partial derivative. Notice that

$$\frac{\partial p_t}{\partial P_t} = 1 + \frac{\Pi_{12}}{\Pi_{11}}. \tag{17}$$

Pricing decisions are strategic complements if $\zeta = -\Pi_{12}/\Pi_{11} < 1$ and strategic substitutes otherwise.[30] Following Ball and Romer (1990), we can divide mechanisms for generating strategic complementarity into two classes: 1) those that raise $-\Pi_{11}$, and 2) those that lower Π_{12}. We refer to these two classes as ω-type strategic complementarity and Ω-type strategic complementarity, respectively.[31] Mechanisms that generate ω-type strategic complementarity include non-isoelastic demand and fixed factors of production. Mechanisms that generate Ω-type strategic complemen-

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[30] At the equilibrium $\Pi_{11} < 0$ and $\Pi_{12} > 0$.

[31] These names are based on the notation used by Kimball (1995).

tarity include real wage rigidity and sticky intermediate inputs. Notice that $\partial p_t / \partial \tilde{A}_t = -\Pi_{13}/\Pi_{11}$. This implies that ω-type strategic complementarity mutes the response of the firm's desired price to other variables such as idiosyncratic shocks, while Ω-type strategic complementarity does not. Models with a large amount of ω-type strategic complementarity therefore have trouble matching the large size of price changes seen in the micro-data, while this problem does not arise in models with a large amount of Ω-type strategic complementarity.

The key difference between the two types of strategic complementarity is that strategic complementarity due to intermediate inputs only affects the firm's response to aggregate shocks while strategic complementarity due to a fixed factor or non-isoelastic demand mutes the firm's response to both aggregate shocks and idiosyncratic shocks. In the model with a fixed factor, the firm's marginal product of labor increases as its level of production falls. The firm's marginal costs therefore fall as it raises its price in response to a fall in productivity, since a higher price leads to lower demand. This endogenous feedback of the firm's price on its marginal costs counteracts the original effect that the fall in productivity had on marginal costs and leads the firms desired price to rise by less than it otherwise would. In the model with intermediate inputs, the firm's marginal cost is not affected by its own pricing decision. The strategic complementarity in the model with intermediate inputs arises because of the rigidity of other firms' prices rather than because of endogenous feedback on marginal costs from the firm's own pricing decision.

Gertler and Leahy (2008) explore an alternative menu cost model with strategic complementarity that does not affect the size of price changes. Their model has sector specific labor markets in which firms receive periodic idiosyncratic shocks. They assume that in each period firms in only a fraction of sectors receive idiosyncratic shocks and change their prices. This staggering of price changes across sectors generates strategic complementarity that amplifies the monetary non-neutrality in their model. The fact that the labor market is segmented at the sectoral level rather than the firm level avoids endogenous feedback on marginal costs from the firms' own pricing decisions and allows their model to match the size of price changes without resorting to large shocks or large menu costs.

The Gertler-Leahy model assumes that in each period their are entire sectors in which no firm changes prices and other sectors where a large fraction of firms change prices. Time series data on the evolution of the frequency of price change in different sectors of the U.S. economy does not

support the notion that the frequency of price change within narrowly defined categories varies greatly from month to month, even within city. In principle, a similar effect arises if one assumes only that the frequency of price change varies across sectors. We have simulated a 6-sector menu cost model with sector specific labor markets in which the frequency and size of price change is calibrated to match the mean of these statistics in different sectors of the U.S. economy. This model does not generate a quantitatively significant degree of strategic complementarity.

5.3 Intermediate Inputs and Sectoral Comovement

Another important difference between the model with intermediate inputs and the model without intermediate inputs is the difference in the behavior of sectoral output. The relatively modest response of aggregate value-added output to aggregate demand shocks in the model without intermediate inputs masks much larger responses of output in individual sectors. Figure 8 plots the response of aggregate output and sectoral output to an expansionary demand shock in our 14 sector model without intermediate inputs. The sectoral responses vary greatly. Output in the sectors with most price rigidity rises by several times as much as aggregate output, while output in the sectors with most price flexibility falls sharply. Figure 9 is the corresponding plot for the model with intermediate inputs. In contrast to the model without intermediate inputs, output in all sectors rises sharply in response to an expansionary demand shock and the differences between sectors are relatively modest.

In the model without intermediate inputs, the desired price of all firms rises approximately one-for-one in percentage terms with nominal aggregate demand and is approximately independent of the prices charged by other firms. As a consequence, the sectoral price index in sectors with a high frequency of price change—such as gasoline—quickly rises proportionally to the shock, while the sectoral price index in sectors with more rigid prices adjusts more slowly. This causes a large change in relative prices across sectors which leads consumers to shift expenditures toward the sectors in which prices are lower (the sticky price sectors). In contrast, in the model with intermediate goods, a firm's desired price is heavily dependent on the prices of other firms. This implies that even the flexible price firms don't react strongly to the shock and relative price differences are much smaller.

A key characteristic of business cycles is that virtually all sectors of the economy comove strongly (Lucas, 1977; Stock and Watson, 1999). The lack of comovement across sectors in the model without

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intermediate inputs is therefore grossly at odds with the data.[32] This lack of comovement across sectors in models with heterogeneity in the degree of price flexibility has been noted and analyzed by several recent papers including Bils et al. (2003), Barsky et al. (2007) and Carlstrom and Fuerst (2006). The analysis above shows that allowing for intermediate goods substantially increases the comovement between different sectors of the economy.[33] This result also holds in our extended model with heterogeneity in intermediate input use. Barsky et al. (2007) discuss a number of other mechanisms for ameliorating this "comovement problem".[34]

6   Extending the Model to Incorporate Product Turnover

The baseline model we have adopted in this paper is one in which price rigidity arises because firms face a fixed cost of changing their prices. In this setting, firms optimally choose the timing of price changes as well as the new price they set. We have identified regular price changes in the data as price changes that are timed optimally by firms. However, in addition to always having the option of incurring a fixed cost to change their price, firms may also receive occasional opportunities to change their prices at comparatively low cost. The CalvoPlus model we introduce in section 2.3 captures this possibility. These additional low cost price changes will affect the degree of price flexibility in the economy. However, since their timing is not chosen optimally, they will induce less price flexibility than the same number of regular price changes.

An important example of instances in which firms may receive opportunities to change their prices at low cost is the times at which firms introduce new products. Product turnover is quite rapid in certain sectors of the economy. And when a firm introduces a new product, it must necessarily set a new price for this product. Rapid product turnover can therefore affect the degree

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[32]It is easy to show that aggregate productivity shocks lead to similar lack of comovement across sectors.

[33]Hornstein and Praschnik (1997), Dupor (1999) and Horvath (2000) discuss the effects of input-output linkages for comovement in a real business cycle framework.

[34]Bils et al. (2003) argue that the relative price of goods in sectors with flexible prices do not increase after an expansionary monetary policy shock as sticky-price business cycle model imply they should. Boivin et al. (2009) point out that the empirical model used by Bils et al. (2003) gives rise to a substantial "price-puzzle" in response to monetary shocks with inflation falling for several quarters after an expansionary shock. Boivin et al. (2009) argue that this suggests misspecification of the monetary shocks. They analyze the behavior of sectoral output after a monetary shock in a factor augmented VAR that does not give rise to a price puzzle and show that their estimates regarding relative prices of flexible priced goods line up will with sticky-price model. Using a different empirical strategy, Mackowiak et al. (2009) find that prices in sectors with flexible prices respond more rapidly to aggregate shocks than price in sectors with sticky prices. Bouakez et al. (2009) provide further evidence that sectors with more frequent price changes respond more rapidly to monetary policy shocks using a structural estimation approach.

of price flexibility in the economy. Furthermore, since firms can often anticipate future product turnover—e.g., fall-spring turnover in apparel—they may decide not to incur the fixed cost needed to change the price of an existing product.

Table 2 reports the frequency of product substitution for the sectors in our multi-sector models.[35] It reveals that product substitution is a frequent occurrence in several categories of durable goods— Apparel, Transportation Goods (Cars), Household Furnishing and Recreation Goods—but less frequent for other products. A number of these categories—especially Apparel—have a very low frequency of price change but a substantial frequency of product turnover.

Many factors influence a firm's decision about the timing of new product introduction including seasonality, development cycles, innovation and random shifts in consumer tastes. Figure 10 plots the frequency of product substitution across different months of the year for the four categories for which product substitution is most frequent. In Apparel, seasonal variation in tastes is a dominant factor in the timing of product introduction. The main determinant of the timing of product entry and exit is the timing of the fall and spring clothing seasons. In the automobile industry, product introduction is heavily influenced by a yearly development cycle with new models being introduced in the fall of each year.

This evidence suggests that in these product categories—where product turnover is relatively important—the timing of product turnover may be largely orthogonal to a firm's desire to change its price and to macroeconomic conditions. A computationally tractable way of modeling this type of event is to consider a model in which new products arrive according to an exogenous Poisson process. This model is equivalent to the CalvoPlus model where $\chi_l = 0$ and $1 - \alpha$ in each sector is equal to the frequency of product substitution.[36] In this calibration of the CalvoPlus model, the menu cost in the high cost state is set so that the frequency of high cost price changes in the model matches the frequency of price change for identical items in the data for each sector. In

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[35]Ideally we would have a measure of the rate of product introduction since pricing decisions are made when new products are introduced. However, the BLS does not track the introduction of new products. When a product that the BLS has been tracking becomes permanently unavailable, the BLS agent is instructed to substitute to the most similar existing product. In most cases this product will have existed for some time. If the hazard of product exit is upward sloping, the frequency of product substitution is therefore an upward biased measure of the frequency of product introduction.

[36]One could also consider a "TaylorPlus" model, i.e., a model in which product introduction was on a fixed schedule as in Taylor (1980). Such a TaylorPlus model is much less tractable computationally since the months of the year are state variables in that model. However, the crucial element is not whether or not agents anticipate the specific date on which they will introduce new products but rather whether or not product introductions arise from firms' desire to change their prices as opposed to other factors such as seasonality, the new model year or product innovation.

other words, all price changes for identical items are viewed as state dependent as in our baseline menu cost model. However, now we consider an additional dimension of flexibility in the form of price changes due to product turnover.[37]

Table 9 shows that product turnover associated with factors unrelated to the firms' pricing decisions have little effect on the monetary non-neutrality implied by the model. This is because the "selection effect" applies only to the regular price changes. While new fashion items are priced to keep up with inflation, they are not (in this model) introduced *because* the old fashion items were mispriced. For comparison purposes, table 9 also presents results for a calibration of the menu cost model where we treat product introductions as if they were the same as regular price changes. In this case, "product flexibility" would have a much larger effect on monetary non-neutrality. In either case, the inclusion of product substitutions in the model has little effect on the amplification effect associated with heterogeneity.

7 Do Menu Costs Generate Sizable Monetary Non-Neutrality?

In the context of a simple menu cost model, Golosov and Lucas (2007) argue that the amount of monetary non-neutrality generated by nominal rigidities is "small and transient". An important question is whether this conclusion holds up in a richer, more realistic setting. To answer this question, we compare the variance of real output generated by our multi-sector model with intermediate inputs in response to calibrated aggregate nominal shocks to the variance of HP-filtered log U.S. real GDP.

Table 10 reports the results of this comparison. The variance of HP-filtered log U.S. real GDP for the period 1947-2006 is 2.72×10^{-4}. The menu cost model is simulated with nominal aggregate shocks that are calibrated as described in section 3 to match the behavior of log U.S. nominal GDP over the period 1947-2005, less the growth rate of log real GDP. The variance of real output in response to these nominal aggregate shocks in our multi-sector model with intermediate inputs is 0.63×10^{-4}. Our model is therefore able to account for 23% of the U.S. business cycle. This result

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[37]Broda and Weinstein (2007) argue that product introduction is pro-cyclical. However, the variation in product turnover at business cycle frequencies is an order of magnitude smaller than the seasonality we document in figure 10. They study product turnover for non-durable goods for which the average rate of product turnover is quite low. The assumptions we make here abstract from this for simplicity. One could easily extend our model to consider intermediate cases where the timing of some product introductions but not others are exogenous to the firm's desire to change prices.

of our model accords well with empirical evidence on the importance of nominal shocks for business cycle fluctuations. Lucas (2003) argues that the shocks that Shapiro and Watson (1988) refer to as "demand" shocks should be interpreted as "nominal" shocks. These shocks capture not only the effect of monetary disturbances but also temporary monetary non-neutrality due to real shocks. Shapiro and Watson (1988) attribute 28% of the variation in output at short horizons to these nominal shocks. In contrast, a single-sector version of our model without intermediate inputs—a model that is virtually identical to the Golosov and Lucas (2007) model—yields variation in real output that can account for only 2% of the U.S. business cycle.[38]

Our model does not incorporate aggregate real shocks. It is therefore not able to match the behavior of real output. The absence of aggregate real shocks in our model also means that we must abstract from any relationship between real shocks and movements in nominal aggregate demand. In a richer model with both real and nominal aggregate shocks, it would be possible to allow nominal aggregate demand to respond both to real shocks and nominal shocks. It would then be possible to "turn off" the nominal shocks and assess how large a fraction of business cycle fluctuations in output they cause. This type of exercise would arguably yield a preferable estimate of importance of monetary non-neutrality in business cycle dynamics to the one we present above. Carrying out this exercise is, however, beyond the scope of this paper.

8   Conclusion

Recent work on state-dependent pricing models suggests that these models generate only a "small and transient" amount of monetary non-neutrality (Golosov and Lucas, 2007). Given the importance of nominal rigidities as a source of monetary non-neutrality in most models that analyze the transmission of monetary policy, this conclusion poses a serious challenge for monetary economics. We extend a simple benchmark menu cost model to include two features for which there exists particularly clear empirical evidence: 1) Heterogeneity across sectors in the frequency and size of price changes; 2) Intermediate inputs. We show that when we subject our model to calibrated nominal shocks it generates fluctuations in real output that can account for 23% of the U.S. business cycle. This accords well with Shapiro and Watson's (1988) result that 28% of variation in output at short

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[38]Midrigan (2006) identifies two other mechanisms that raise the degree of monetary non-neutrality in a menu cost model: fat-tailed idiosyncratic shocks and multi-product firms with scale economies in changing prices.

horizons is due to nominal shocks.

Our multi-sector model generates three times as much monetary non-neutrality as does a single-sector model calibrated to the mean frequency and size of price changes. This amplification due to heterogeneity is driven by three features of the U.S. data: 1) the low average level of inflation in the U.S. economy, 2) the fact that the average size of price changes is large and that there is no strong correlation between the size and frequency of price change across sectors, and 3) the relatively low average frequency of price change in the U.S. economy. A single-sector menu cost model calibrated to match the median frequency of price change yields a degree of monetary non-neutrality that is similar to the multi-sector model.

The introduction of intermediate inputs raises the degree of monetary non-neutrality by another factor of three. Intermediate inputs amplify the degree of monetary non-neutrality because they generate a substantial amount of strategic complementarity in the pricing decisions of different firms. Importantly, the model can fit both the size and frequency of price change. In contrast, other popular sources of strategic complementarity—such as fixed factors of production and non-isoelastic demand curves—yield price changes that are much too small on average for reasonable parameter values. Following Ball and Romer (1990) and Kimball (1995), we divide the sources of strategic complementarity into two classes—ω-type strategic complementarity and Ω-type strategic complementarity. We show that models with a large amount of ω-type strategic complementarity are unable to match the average size of price changes, while this problem does not afflict models with a large amount of Ω-type strategic complementarity. An empirically realistic intermediate input share can generate a substantial amount of Ω-type strategic complementarity. Sector specific labor markets, however, do not generate a substantial amount of such strategic complementarity unless price adjustments are heavily staggered across sectors; something we do not observe in the data.

A   Profit Function

Cost minimization by firm z implies that labor demand and demand for the composite intermediate input be governed by

$$\frac{W_t}{P_t} = (1 - s_m) A_t L_t(z)^{-s_m} M_t(z)^{s_m} \Omega_t(z),$$

$$1 = s_m A_t L_t(z)^{1-s_m} M_t(z)^{s_m-1} \Omega_t(z),$$

where $\Omega_t(z)$ denotes the marginal costs of firm z at time t. Combining these two equations yields

$$\frac{W_t}{P_t} = \frac{1 - s_m}{s_m} \frac{M_t(z)}{L_t(z)}. \tag{18}$$

The real value of firm z's profits in period t are

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t} \right) y_t(z) - \left(\frac{W_t}{P_t} \right) L_t(z) - M_t(z) - \chi \left(\frac{W_t}{P_t} \right) I_t(z).$$

Using this equation (18) we can rewrite these profits as

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t} \right) y_t(z) - \frac{1}{1 - s_m} \left(\frac{W_t}{P_t} \right) L_t(z) - \chi \left(\frac{W_t}{P_t} \right) I_t(z).$$

Combining the production function—equation (8)—and equation (18) yields

$$L_t(z) = \left(\frac{y_t(z)}{A_t(z)} \right) \left(\frac{s_m}{1 - s_m} \right)^{-s_m} \left(\frac{W_t}{P_t} \right)^{-s_m}.$$

Using this equation, we can rewrite profits as

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t} \right) y_t(z) - (1 - s_m)^{s_m-1} s_m^{-s_m} \left(\frac{W_t}{P_t} \right)^{1-s_m} \left(\frac{y_t(z)}{A_t(z)} \right) - \chi \left(\frac{W_t}{P_t} \right) I_t(z) - U. \tag{19}$$

Using the firm's demand curve—equation (12)—and the labor supply curve—equation (7)—we can rewrite profits as

$$\Pi_t^R(z) = Y_t \left(\frac{p_t(z)}{P_t} \right)^{1-\theta} - (1 - s_m)^{s_m-1} s_m^{-s_m} \omega^{1-s_m} L_t^{\psi(1-s_m)} C_t^{\gamma(1-s_m)} \left(\frac{1}{A_t(z)} \right) Y_t \left(\frac{p_t(z)}{P_t} \right)^{-\theta}$$
$$-\chi \omega L_t^{\psi} C_t^{\gamma} I_t(z) - U.$$

Finally, log-linear approximations of $Y_t = C_t + \int_0^1 M_t(z) dz$, the production function and labor supply around the steady state with flexible prices yield $\hat{Y}_t = a_1 \hat{C}_t$ and $\hat{L}_t = a_2 \hat{C}_t$. Here $\hat{Y}_t = \log(Y_t/Y)$ and Y denotes the steady state of Y_t with flexible prices. $\hat{C}_t$ and $\hat{L}_t$ are defined analogously. Using these log-linear approximations and the fact that $C_t = S_t/P_t$, we can rewrite profits as a function of $(A_t(z), p_{t-1}(z)/P_t, S_t/P_t)$ and $p_t(z)$.

B Stationary Distribution

We solve for the stationary distribution over the state space of the firm's problem using the following algorithm:

0. Start with an initial distribution $Q(A(z), p_{-1}(z)/P, S/P)$. We use a uniform distribution as our initial distribution.

1. Map $Q(A(z), p_{-1}(z)/P, S/P)$ into $Q(A(z), p(z)/P, S/P)$ using the policy function F.

2. Map $Q(A(z), p(z)/P, S/P)$ into $Q(A_{+1}(z), p(z)/P, S/P)$ using the transition probability matrix for the technology process.

3. Map $Q(A_{+1}(z), p(z)/P, S/P)$ into $Q(A_{+1}(z), p(z)/P, S_{+1}/P)$ using the probability transition matrix for the nominal aggregate demand process.

4. Map $Q(A_{+1}(z), p(z)/P, S_{+1}/P)$ into $Q(A_{+1}(z), p(z)/P_{+1}, S_{+1}/P_{+1})$ using the function Γ.

5. Check whether $|Q(A_{+1}(z), p(z)/P_{+1}, S_{+1}/P_{+1}) - Q(A(z), p_{-1}(z)/P, S/P)| < \xi$ where $|\cdot|$ denotes a sup-norm. If so, stop. If not, go back to step one.

C   A Model with Capital

Consider an extension of the model presented in section 2 in which firms use capital as well as labor and intermediate inputs to produce goods. The presence of capital affects the equilibrium behavior of this type of model primarily by affecting the cyclicality of marginal costs. If the marginal product of capital is highly variable over the cycle, this will raise the cyclicality of firms' marginal costs and thereby reduce the amount of monetary non-neutrality generated by the model. In the language of section 5, capital may generate Ω-type strategic substitutability.

Capital adjustment costs make the capital stock adjust sluggishly to variations in the marginal product of capital. Such adjustment costs thus increase the variability of the marginal product of capital and the variability of firms' marginal costs (Christiano et al. 2005). The capital stock being fixed is a limiting case as capital adjustment costs become large. Other things equal, the effect that introducing capital into our model has to reduce monetary non-neutrality is thus maximized if the aggregate capital stock in the economy is fixed. To simplify our analysis, we assume that the aggregate capital stock is fixed and analyze the effect that introducing capital has on the cyclicality of marginal costs. We interpret our results as an upper bound on the effect that capital would have on the cyclicality of marginal costs. A model with smaller adjustment costs would inply a smaller response of marginal cost to output and thus greater monetary non-neutrality.

C.1   Household Behavior

Households own the capital stock and rent it to firms each period in a competitive capital market. Since capital is fixed, households make no choices regarding capital. The household budget constraint becomes

$$P_t C_t + E_t[D_{t,t+1} B_{t+1}] \leq B_t + W_t L_t + P_t R_t K + \int_0^1 \Pi_t(z) dz, \tag{20}$$

where R_t denotes the real rental rate on capital and K denotes the fixed amount of capital owned by the households. Households behave as before.

C.2   Firm Behavior

The production function of firm z is given by

$$y_t(z) = A_t(z)(L_t(z)^\alpha K_t(z)^{1-\alpha})^{1-s_m} M_t(z)^{s_m}. \tag{21}$$

Cost minimization by firms implies

$$\frac{W_t}{P_t} = (1 - s_m)\alpha L_t(z)^{\alpha(1-s_m)-1} K_t(z)^{(1-\alpha)(1-s_m)} M_t(z)^{s_m} \Omega_t(z),$$

$$1 = s_m A_t (L_t(z)^\alpha K_t(z)^{1-\alpha})^{1-s_m} M_t(z)^{s_m-1} \Omega_t(z),$$

$$R_t = (1 - s_m)(1 - \alpha) L_t(z)^{\alpha(1-s_m)} K_t(z)^{(1-\alpha)(1-s_m)-1} M_t(z)^{s_m} \Omega_t(z),$$

where $\Omega_t(z)$ denotes the marginal costs of firm z at time t. Eliminating $\Omega(z)$ from these three equations yields

$$\frac{W_t/P_t}{R_t} = \frac{\alpha}{1-\alpha} \frac{K_t(z)}{L_t(z)}, \tag{22}$$

$$\frac{W_t}{P_t} = \alpha \frac{1 - s_m}{s_m} \frac{M_t(z)}{L_t(z)}. \tag{23}$$

These two equations imply that all firms have the same capital-labor ratio and the same materials-labor ratio.

The real value of firm z's profits in period t are

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t}\right) y_t(z) - \left(\frac{W_t}{P_t}\right) L_t(z) - M_t(z) - R_t K_t(z) - \chi\left(\frac{W_t}{P_t}\right) I_t(z) - U.$$

Using equations (22)-(23) we can rewrite these profits as

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t}\right) y_t(z) - \frac{1}{\alpha}\frac{1}{1 - s_m}\left(\frac{W_t}{P_t}\right) L_t(z) - \chi\left(\frac{W_t}{P_t}\right) I_t(z) - U.$$

Combining equations (21)-(23) yields

$$L_t(z) = \left(\frac{y_t(z)}{A_t(z)}\right) \left(\frac{\alpha}{1-\alpha}\right)^{(1-\alpha)(1-s_m)} \left(\alpha\frac{1 - s_m}{s_m}\right)^{s_m} \left(\frac{R_t}{W_t/P_t}\right)^{(1-\alpha)(1-s_m)} \left(\frac{W_t}{P_t}\right)^{-s_m}.$$

Combining these last two equations yields

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t}\right) y_t(z) - \Psi\left(\frac{y_t(z)}{A_t(z)}\right) \left(\frac{W_t}{P_t}\right)^{1-s_m} \left(\frac{R_t}{W_t/P_t}\right)^{(1-\alpha)(1-s_m)} - \chi\left(\frac{W_t}{P_t}\right) I_t(z) - U, \tag{24}$$

where

$$\Psi = \left(\frac{1}{\alpha}\frac{1}{1 - s_m}\right) \left(\frac{\alpha}{1-\alpha}\right)^{(1-\alpha)(1-s_m)} \left(\alpha\frac{1 - s_m}{s_m}\right)^{s_m}.$$

Equation (24) is close to being identical to equation (19). There are two differences. First, the constant Ψ is different from the corresponding constant in equation (19). Second, the second term in equation (24) has an additional piece involving the ratio of the rental rate and the real wage. Notice that the average real marginal cost is pinned down by the assumed markup.

The difference in the elasticity of marginal cost between the model with capital and the model without capital stems from the potential cyclicality of

$$\left(\frac{R_t}{W_t/P_t}\right)^{(1-\alpha)(1-s_m)}.$$

If R_t is more cyclical than W_t/P_t, the model with capital will have more cyclical marginal costs than the model without capital.

Combining equations (7) and (22) and adopting the our calibration of $\gamma = 1$ and $\psi = 0$ yields

$$\omega C_t = \frac{\alpha}{1-\alpha} R_t \frac{K_t(z)}{L_t(z)}.$$

If we log-linearize this equation, aggregate the resulting equation and use the fact that aggregate capital is fixed, we get $\hat{C}_t + \hat{L}_t = \hat{R}_t$.

Log-linearizing equations (7) and (21)-(23) and solving for the relationship between output and labor supply yields

$$\hat{L}_t = \frac{1 - s_m}{(1-s_m)\alpha + s_m/\theta}\hat{C}_t \equiv a_2\hat{C}_t,$$

Combining this equation with $\hat{C}_t + \hat{L}_t = \hat{R}_t$ yields $\hat{R}_t = (1 + a_2)\hat{C}_t$. Since, the real wage in our model has a unit elasticity with respect to output, this shows that the rental rate is more cyclical than the real wage.

The equations above imply that the overall elasticity of marginal cost with respect to output in the model with capital is $(1 - s_m)(1 + a_2(1 - \alpha))$. If we assume that the capital share is 1/3 and the intermediate input share is 0.7, then the elasticity of marginal cost is 0.38. Adding capital to the model thus increases the cyclicality of marginal costs from 0.3 to 0.38. The empirical results of Solon, Barsky, and Parker (1994) on the cyclicality of real wages suggest that for the U.S. economy the elasticity of real wages with respect to output is in fact only about 0.6. Our calibration without capital thus somewhat overstates the elasticity of real wages with respect to output. If we redo the elasticity calculation for the model with capital using the real wage elasticity from Solon, Barsky, and Parker (1994), we get an elasticity of marginal cost of 0.28. This is almost exactly equal to the elasticity of 0.3 that we assume in our baseline model. In other words, we have adopted a baseline specification that implies an elasticity of marginal costs similar to what is implied by a model with capital and calibrated to match the empirical evidence presented in Solon, Barsky, and Parker (1994). The addition of these two features—capital and a realistic value for the elasticity

of real wages with respect to output—thus roughly cancel each other out and yield a model with the same amount our real rigidities as our benchmark model.

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Table 1: Benchmark Parameters

Discount factor	$\beta = 0.96^{1/12}$
Coefficient of relative risk aversion	$\gamma = 1$
Inverse of Frisch elasticity of labor supply	$\psi = 0$
Elasticity of demand	$\theta = 4$
Steady state labor supply	$L = 1/3$
Intermediate inputs share in production	$s_m = 0.7$
Speed of mean reversion of idiosyncratic productivity	$\rho = 0.7$
Mean growth rate of nominal aggregate demand	$\mu = 0.0028$
St. deviation of the growth rate of nominal aggregate demand	$\sigma_\eta = 0.0065$

Table 2: Sector Characteristics for Multi-Sector Models

Name	Weight	Freq.	Abs. Size	Subs
	(%)	(%)	(%)	(%)
Panel A: 6 Sector Model				
Vehicle Fuel, Used Cars	7.7	91.6	4.9	8.9
Transportation Goods, Utilities, Travel	19.1	35.5	10.9	4.5
Unprocessed Food	5.9	25.4	15.9	1.3
Processed Food, Other Goods	13.7	11.9	11.4	2.0
Services (excl. Travel)	38.5	8.8	8.3	2.0
Household Furnishings, Apparel, Recreation Goods	15.1	5.2	11.1	7.9
Panel B: 9 Sector Model				
Vehicle Fuel, Used Cars	7.7	91.6	4.9	8.9
Transportation Goods, Utilities, Travel	19.1	35.5	10.9	4.5
Unprocessed Food	5.9	25.4	15.9	1.3
Services(1)	9.2	19.7	4.6	2.1
Processed Food, Other Goods	13.7	11.9	11.4	2.0
Services(2)	9.6	7.6	7.2	3.7
Services(3)	10.0	5.5	8.1	1.3
Household Furnishings, Apparel, Recreation Goods	15.1	5.2	11.1	7.9
Services(4)	9.7	3.2	12.8	0.9
Panel C: 14 Sector Model				
Vehicle Fuel, Used Cars	7.7	91.6	4.9	8.9
Utilities	5.3	49.4	6.4	0.6
Travel	5.5	43.7	18.4	1.8
Unprocessed Food	5.9	25.4	15.9	1.3
Transportation Goods	8.3	21.3	8.9	8.8
Services (1)	7.7	21.7	4.0	2.2
Processed Food, Other Goods	13.7	11.9	11.4	2.0
Services (2)	7.5	8.4	6.7	4.4
Household Furnishing	5.0	6.5	10.1	5.0
Services (3)	7.8	6.2	8.8	1.7
Recreation Goods	3.6	6.1	10.2	5.9
Services (4)	7.6	4.9	8.1	0.9
Apparel	6.5	3.6	12.4	11.3
Services (5)	7.9	2.9	13.5	1.0

This table presents the weighted mean frequency and log absolute size of price changes as well as the frequency of product substitution for US consumer prices over the period 1998-2005 for divisions into 6, 9, and 14 sectors. These statistics are calculated using the methodology described in Nakamura and Steinsson (2008), based on the individual price quotes underlying the US consumer price index (CPI). The weighted means are calculated using CPI expenditure weights for entry level items (ELI's). "Weight" gives the total expenditure weight for the category, "Freq." gives the weighted mean frequency of price change for the category, "Abs. Size" gives the weighted mean absolute size of log price changes for the category. "Subs" gives the weighted mean frequency of product substitution. See Nakamura and Steinsson (2008) for more details on how these statistics are constructed. In the 9 and 14 sector models, the Service sector is divided equally into 4 and 5 groups respectively, where the ELI's are sorted into different groups according to the frequency of price change in the ELI.

Table 3 : Parameter Values for Multi-Sector Models

	Menu Cost Model				CalvoPlus Model			
	$s_m = 0$		$s_m = 0.7$		$s_m = 0$		$s_m = 0.7$	
	Δp cost	σ_ε	Δp cost	σ_ε	Δp cost	σ_ε	Δp cost	σ_ε
	$x10^{-2}$	$x10^{-2}$	$x10^{-2}$	$x10^{-2}$	$x10^{-2}$	$x10^{-2}$	$x10^{-2}$	$x10^{-2}$
Panel A: 6 Sector Model								
Vehicle Fuel, Used Cars	0.004	5.00	0.001	5.10	0.007	5.99	0.001	5.00
Transp. Goods, Utilities, Travel	0.309	6.90	0.087	6.85	0.399	8.63	0.110	8.50
Unprocessed Food	0.667	9.10	0.194	9.20	0.967	12.40	0.268	12.30
Processed Food, Other Goods	0.331	5.70	0.091	5.70	0.516	9.20	0.129	8.69
Services (excl. Travel)	0.165	3.90	0.046	4.05	0.269	6.75	0.075	6.90
Hh. Furn., Apparel, Rec. Goods	0.271	5.46	0.070	5.40	0.446	9.85	0.107	9.00
Panel B: 9 Sector Model								
Vehicle Fuel, Used Cars	0.004	5.30	0.002	5.40	0.007	5.20	0.001	4.98
Transp. Goods, Utilities, Travel	0.307	6.90	0.091	7.00	0.399	8.63	0.115	8.70
Unprocessed Food	0.667	9.00	0.185	9.00	0.963	12.30	0.234	11.60
Services(1)	0.059	2.40	0.019	2.65	0.099	3.76	0.022	3.40
Processed Food, Other Goods	0.340	5.80	0.093	5.70	0.545	9.41	0.132	8.80
Services(2)	0.137	3.50	0.035	3.45	0.220	6.10	0.065	6.50
Services(3)	0.156	3.80	0.042	3.90	0.261	6.75	0.070	7.20
Hh. Furn., Apparel, Rec. Goods	0.306	5.80	0.076	5.40	0.451	9.77	0.121	9.50
Services(4)	0.340	6.50	0.083	6.39	0.474	11.31	0.122	11.60
Panel C: 14 Sector Model								
Vehicle Fuel, Used Cars	0.005	5.20	0.002	5.20	0.007	5.39	0.002	5.30
Utilities	0.095	4.65	0.027	4.80	0.112	5.28	0.032	5.30
Travel	0.636	11.10	0.210	12.00	0.931	14.00	0.265	14.00
Unprocessed Food	0.724	9.40	0.198	9.00	0.969	12.40	0.266	12.20
Transportation Goods	0.244	5.20	0.060	4.71	2.758	6.80	0.082	6.80
Services (1)	0.069	2.70	0.023	2.97	0.057	3.00	0.019	3.20
Processed Food, Other Goods	0.322	5.60	0.095	5.75	0.508	9.00	1.286	8.90
Services (2)	0.117	3.20	0.034	3.30	0.200	5.70	0.056	5.70
Household Furnishing	0.228	4.80	0.065	4.69	0.405	8.80	0.107	8.70
Services (3)	0.177	4.10	0.049	4.10	0.298	7.40	0.085	7.60
Recreation Goods	0.239	4.80	0.064	4.80	0.413	8.80	0.112	8.90
Services (4)	0.159	3.80	0.047	4.00	0.307	7.60	0.073	7.20
Apparel	0.331	6.05	0.087	5.99	0.537	10.50	0.131	10.50
Services (5)	0.388	7.01	0.102	6.82	0.528	11.50	0.158	12.00

This table presents the cost of changing prices and the volatility of idiosyncratic shocks for the multi-sector menu cost model and CalvoPlus model both with and without intermediate goods. "Δp cost" denotes the average cost of changing prices in a year as a fraction of steady state revenue. In the menu cost model this is equal to $f(\theta-1)/\theta \ \chi/Y_{SS}$ where f denotes the frequency of price change and Y_{SS} is steady state output under flexible prices. In the CalvoPlus model it is calculated analogously but the high menu cost is applied to the price changes that occure in the high menu cost state and the low menu cost to the price changes that occure in the low menu cost state. σ_ε is the variance of shocks to the log of the idiosyncratic productivity shocks. s_m is the fraction of marginal costs accounted for by intermediate goods. In the CalvoPlus model, the fraction of time spent in the "low menu cost" state is set at $1-\alpha$ = freq. for each sector in all cases.

Table 4: Intermediate Inputs in the U.S. Economy in 2002

	% Int. Inputs	% Used	% Gross Y	% GDP	% CPI
Agriculture and Mining	55.1	5.5	2.4	1.9	0.0
Utilities	36.8	2.6	1.7	2.0	5.3
Construction	46.8	1.5	4.8	4.6	0.0
Manufacturing	64.9	28.8	20.5	12.9	51.2
Trade	31.7	6.2	10.4	12.8	0.0
Services	39.3	53.0	48.7	53.0	43.5
Government	37.9	0.9	11.5	12.8	0.0

These data (except the last column) are from the 2002 "Use" table of the U.S. Annual Input-Output Accounts published by the Bureau of Economic Analysis. The last column is taken from Nakamura and Steinsson (2008). "% Int. Inputs" denotes the fraction of intermediate inputs in each sectors gross output. "% Used" denotes the fraction of all intermediate inputs in the economy that come from each sector. "% Gross Y" denotes each sector's weight in gross output. "% GDP" denotes each sector's weight in GDP. "% CPI" denotes each sector's weight in the CPI.

Table 5: Frequency of Price Change: Comparison of CPI and PPI

Category	Num. of Matches	Frequency of Price Change CPI	Frequency of Price Change PPI
Processed Food	32	10.5	7.2
Unprocessed Food	24	25.9	67.9
Household Furnishings	27	6.5	5.6
Apparel	32	3.6	2.7
Recreation Goods	16	6.8	6.1
Other Goods	13	23.2	17.1

This table presents a comparison between the frequency of price change for consumer prices excluding sales and producer prices over the 1998-2005 period. These statistics are from Nakamura and Steinsson (2008), and are based on the individual price quotes underlying the US consumer price index (CPI) and producer price index (PPI). These statistics are constructed by matching Entry Level Items (ELI's) in the CPI to 4, 6 or 8-digit commodity codes within the PPI. "Num. of Matches" denotes the number of such matches that were possible within the Major Group. "Frequency of price change" denotes the median frequency across categories among the matches found. See Nakamura and Steinsson (2008) for more details on how these statistics are constructed.

Table 6: Heterogeneity and Monetary Non-Neutrality

	Menu Cost Model		CalvoPlus Model	
	$s_m = 0$	$s_m = 0.7$	$s_m = 0$	$s_m = 0.7$
Monetary Non-Neutrality: Var(C_t)				
1 Sector Model (Mean)	0.055	0.182	0.173	0.461
6 Sector Model	0.136	0.470	0.458	1.492
9 Sector Model	0.143	0.576	0.495	1.563
14 Sector Model	0.188	0.627	0.520	1.709
1 Sector Model (Median)	0.261	0.658	0.625	1.480

This table presents estimates of the variance or real value-added output for the multi-sector menu cost model and the multi-sector CalvoPlus model for two values of the intermediate inputs share (s_m). The variance of real value added output is multiplied by 10^4. The first two columns present results for the menu cost model. The third and fourth columns present results for the CalvoPlus model. See Table 4 for the menu cost and variance of idiosyncratic shocks assumed in these models. These statistics are presented for versions of the menu cost model with 1, 6, 9 and 14 sectors. In the CalvoPlus model, the fraction of time spent in the "low menu cost" state is set at $1-\alpha$ = freq. for each sector in all cases.

Table 7: Amplification due to Heterogeneity

	Two Sector Models			One Sector Models		Amp.
	Freq 1	Freq 2	Var(C_t)	Freq	Var(C_t)	Factor
(1)	0.10	0.20	0.1194	0.15	0.1050	1.137
(2)	0.20	0.30	0.0395	0.25	0.0360	1.098
(3)	0.30	0.40	0.0154	0.35	0.0152	1.014
(4)	0.40	0.50	0.0060	0.45	0.0059	1.010
(5)	0.10	0.30	0.0889	0.20	0.0620	1.433
(6)	0.10	0.40	0.0702	0.25	0.0360	1.949

The table presents estimates of monetary non-neutrality for 6 two sector models and 6 one sector models. Each row in the table presents a two sector model in which the two sectors have different frequencies of price change and a one sector model with a frequency of price change equal to the average frequency of price change in the two sector model in that row. "Freq 1" denotes the frequency of price change in sector 1 of the two sector model. "Freq 2" denotes the frequency of price change in the other sector. "Freq" denotes the frequency of price change in the one sector model. "Var(C_t)" denotes the variance of output multiplied by 10^4. "Amp. Factor" denotes the factor by which monetary non-neutrality is higher in the two-sector model than in the one sector model.

Table 8: Intermediate Inputs and Monetary Non-Neutrality

		Frequency of Price Change	
	Interm. Input Share	21.1%	8.7%
Monetary Non-Neutrality: Var(C$_t$)			
(1)	0.00	0.055	0.261
(2)	0.50	0.109	0.443
(3)	0.60	0.133	0.518
(4)	0.70	0.182	0.658
(5)	0.80	0.276	0.844
(6)	0.90	0.471	1.346

This table presents estimates of the variance or real value-added output for a single-sector version of the menu cost model for several values of the intermediate inputs share, sm. In all cases, the model is calibrated to match the median size of price changes of 8.5%. For the first column of results, the model is calibrated to match the weighted mean frequency of price change of 21.1%, while for the second column of results it is parameterized to match the weighted median frequency of price change of 8.7%.

Table 9: Multi-Sector Models with Product Flexibility

	Menu Cost		CalvoPlus Subs		Menu Cost Subs	
	$s_m = 0$	$s_m = 0.7$	$s_m = 0$	$s_m = 0.7$	$s_m = 0$	$s_m = 0.7$
Monetary Non-Neutrality: Var(C$_t$)						
1 Sector Model (Mean)	0.055	0.182	0.173	0.461	0.036	0.157
6 Sector Model	0.136	0.470	0.458	1.492	0.092	0.332
9 Sector Model	0.143	0.576	0.495	1.563	0.104	0.412
14 Sector Model	0.188	0.627	0.520	1.709	0.111	0.410

This table presents estimates of the cumulative impulse response (CIR) and the variance or real value-added output for three calibrations of our multi-sector models and two values of the intermediate inputs share (s_m). The CIR is measured in percent. The variance of real value added output is multiplied by 10^4. The first two columns present results for the menu cost model calibrated to match the frequency of price change across sectors. The third and fourth columns present results for the CalvoPlus model with $\chi_l = 0$, $1 - \alpha$ = freq. of substitutions and χ_h calibrated so that that frequency of price change in the high cost state equals the frequency of price change in the data. The fifth and sixth columns present results for the menu cost model calibrated to match the frequency of price change plus the frequency of substitutions across sectors.

Table 10: Nominal Rigidities and the Business Cycle

	Var(C_t) (10^{-4})	Frac. Tot. (%)
HP-filtered U.S. GDP 1947-2005	2.72	100
Multi-Sector Model with s_m=0.7	0.63	23
Multi-Sector Model with s_m=0	0.19	7
Single-Sector Model with s_m=0.7	0.18	7
Single-Sector Model with s_m=0	0.05	2

This table reports the variance of HP-filtered U.S. real GDP for 1947-2005 as well as estimates of the variance of real value-added output for a single-sector and the 14-sector versions of our menu cost model for two values of the intermediate inputs share (s_m). It also reports the fraction of the variance of HP-filtered U.S. real GDP that each of the models can account for.

Figure 1: The Distribution of the Frequency of Price Change for U.S. Consumer Prices
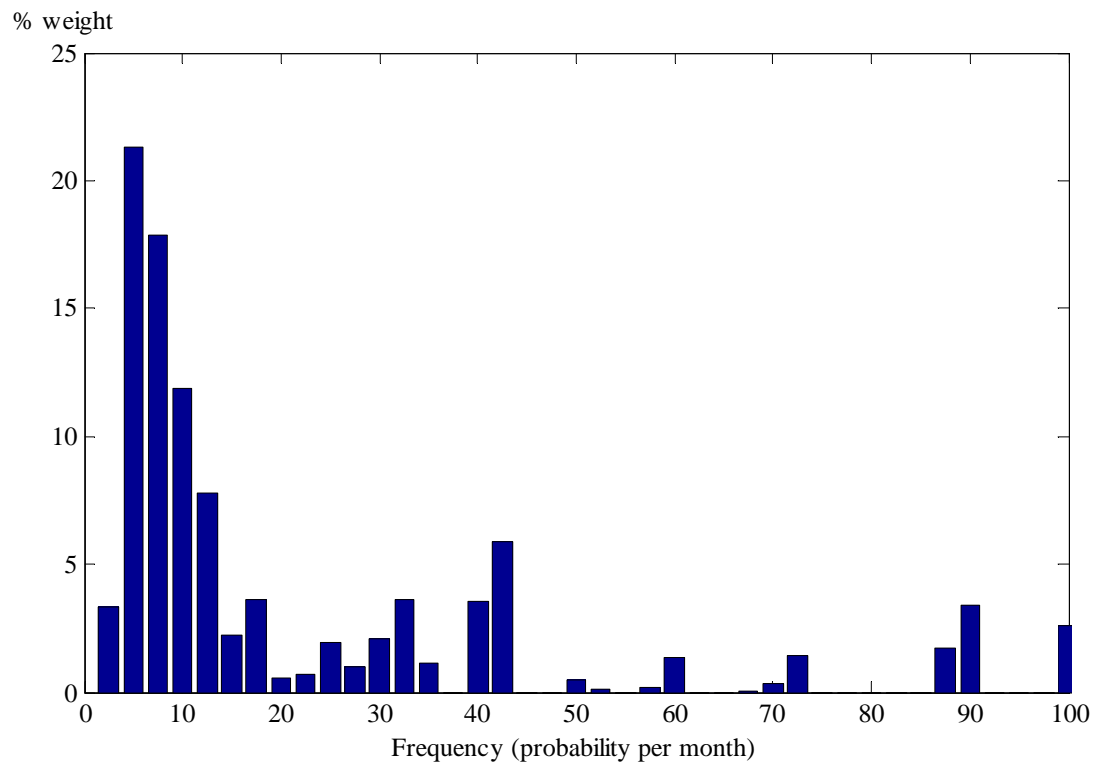
This figure presents a histogram of the cross-sectional distribution of the frequency of non-sale price changes in U.S. consumer prices for the period 1998-2005 (percent per month). The figure is based on the statistics in Nakamura and Steinsson (2008). It is based on the individual price quotes underlying the US CPI. The figure shows the expenditure weighted distribution of the frequency of price changes across entry level items (ELI's) in the CPI.

Figure 2: Actual Inflation and Perceived Inflation for the Multi-Sector Menu Cost Model
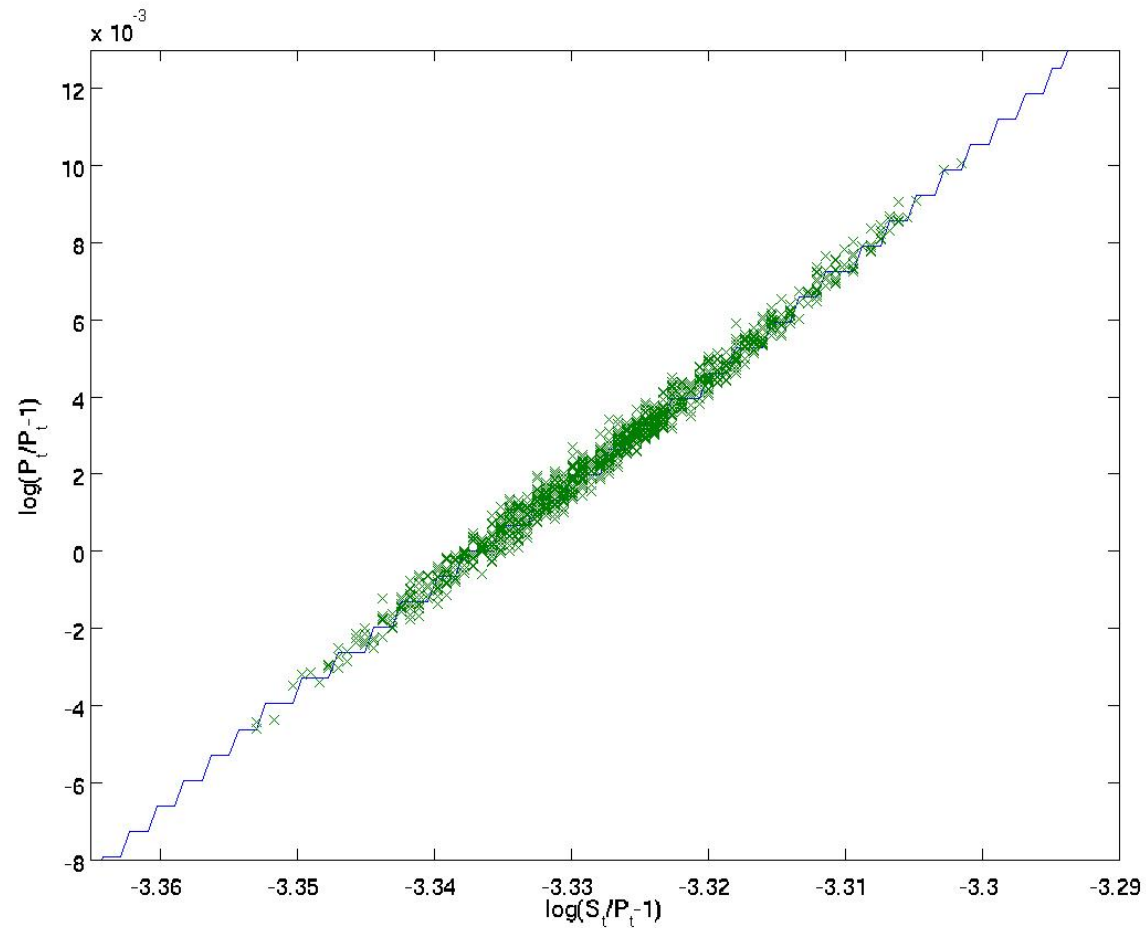
This figure compares actual log inflation (crosses) and perceived log inflation (line) for the multi-sector menu cost model with intermediate inputs. More specifically, the line represents the perceived aggregate law of motion of $\log(P_t/P_{t-1})$ as a function of $\log(S_t/P_{t-1})$, i.e., the function Γ. The crosses denote actual log inflation as a function of $\log(S_t/P_{t-1})$ in a 1000 period simulation of our model.

Figure 3: The Frequency and Size of Price Changes across Different Sectors
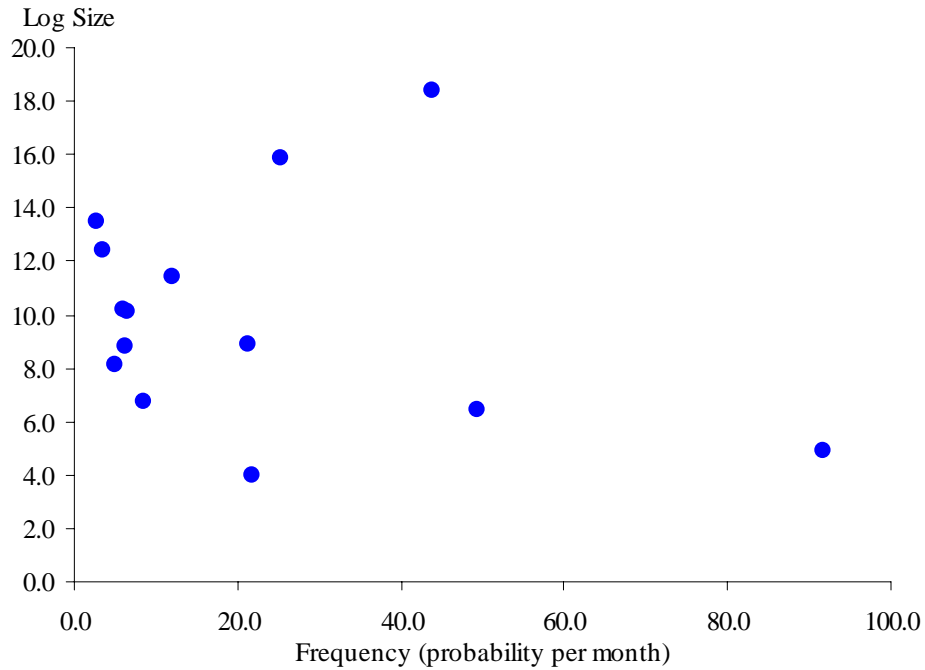
The figure plots the average frequency and size of price changes for each sector in our 14 sector model. See table 3 for the underlying data.

Figure 4: A Sample Path from a Typical Price
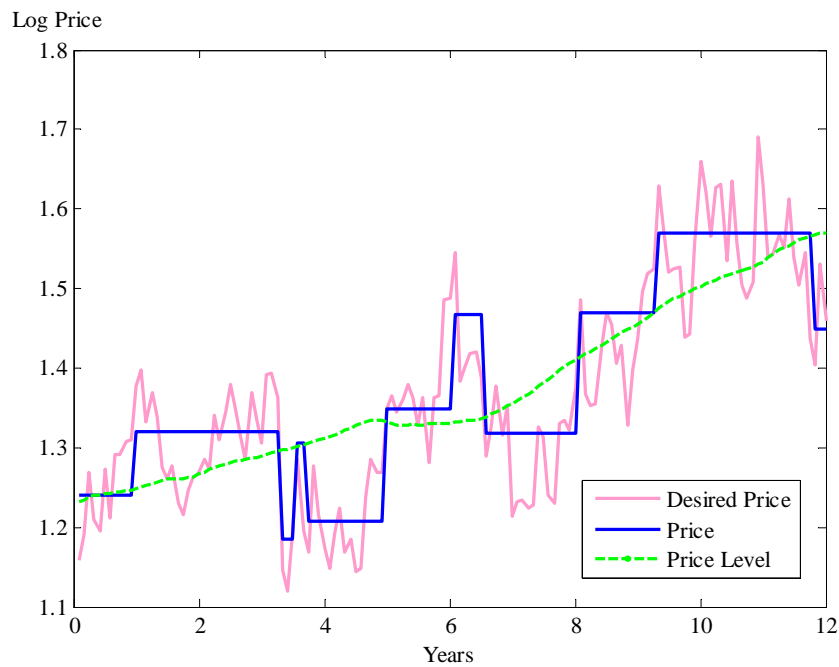
This figure plots a sample path of the price for a single firm in the model with intermediate inputs. The menu cost and variance of idiosyncratic shocks for the firm are set to match the median frequency and size of price changes. It also plots the price level and the firm's static desired price.

Figure 5: Menu Costs, Idiosyncratic Shocks and Monetary Non-Neutrality

VarC (10^{-4})

Frequency of Price Change (probability per month)
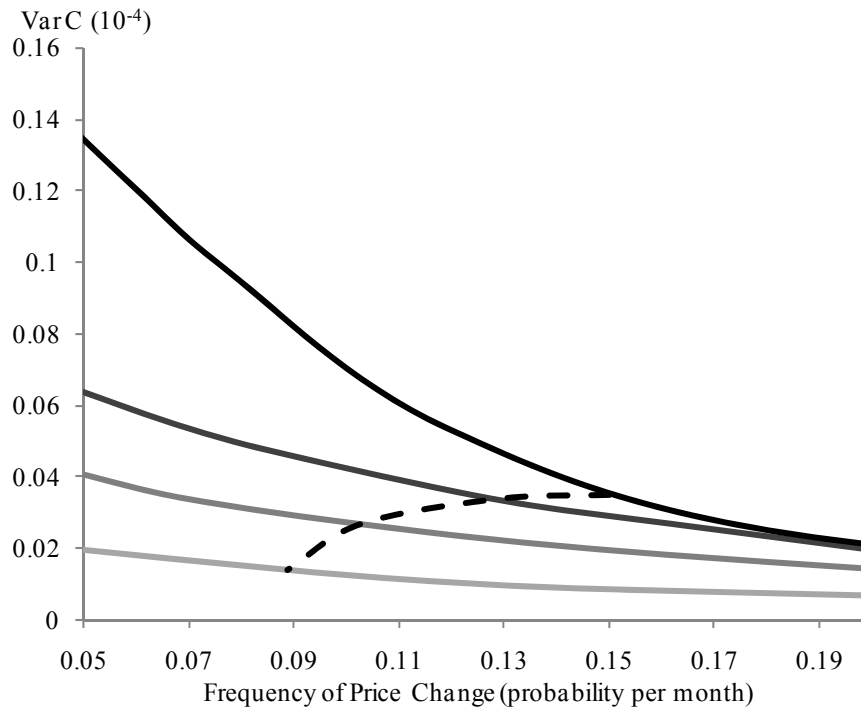
This figure illustrates how the degree of monetary non-neutrality at a given frequency of price change depends on the size of menu costs and the size of idiosyncratic shocks. Each of the solid lines plots the degree of monetary non-neutrality in a sector for a given level of idiosyncratic shocks as the size of the menu cost changes. From top to bottom, the standard deviation of the idiosyncratic shocks (σ_ε) is 0.045, 0.02, 0.015, 0.01. For all the lines, μ=0.01 and σ_η. The dashed line connects the four sectors of an economy with one sector on each line.

Figure 6: Variance of Output as a Function of the Frequency of Price Change

Var C (10^{-4})

Frequency of Price Change (Probability per month)
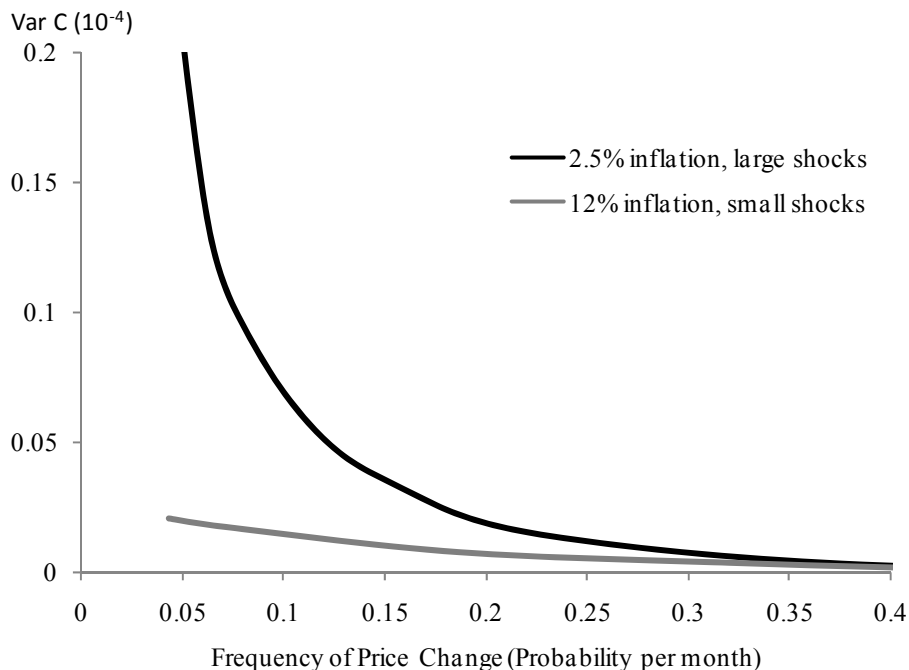
This figure plots the variance of value-added output as a function of the frequency of price change for two calibrations of our menu cost model without intermediate inputs. First, we present our benchmark calibration of μ=0.002, σ_η=0.0037 and σ_ε=0.0425 (dark line). Second, we present a calibration in which μ=0.01, σ_η=0.0037 and σ_ε=0.01 (light line).

Figure 7: Monetary Non-Neutrality in the CalvoPlus Model

VarC (10^{-4})

Fraction of Price Changes in the Low Menu Cost State
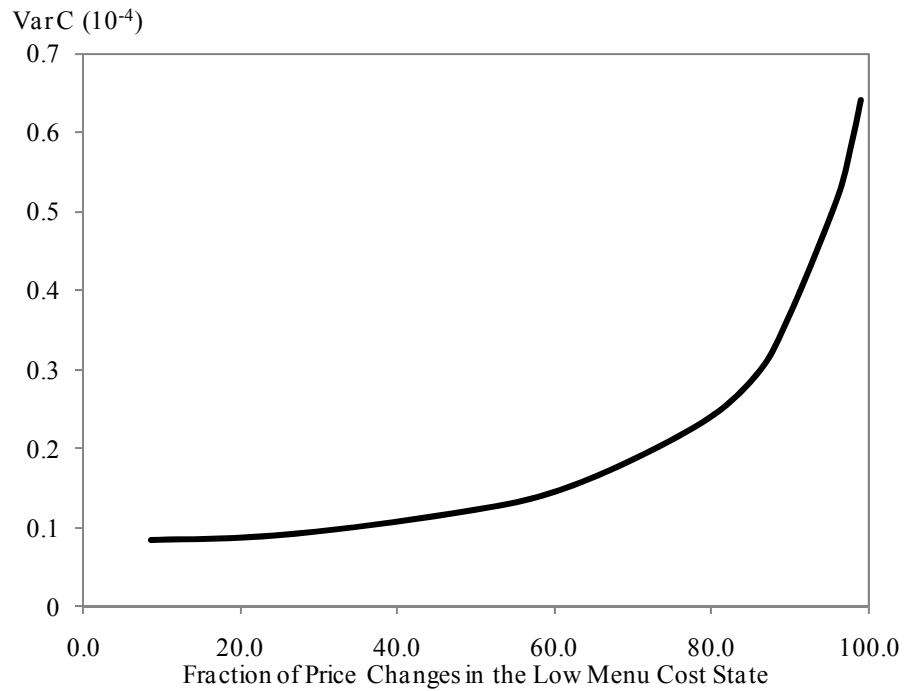
This figure presents the variance of value-added output in the single-sector CalvoPlus model without intermediate inputs as a function of the fraction of price changes in the low menu cost state. The variance of the idiosyncratic shocks is fixed at σ_{ε} =0.0425 (the same value as in the single-sector menu cost model without intermediate goods). The menu costs in the high and low menu cost states are calibrated to match the weighted median frequency of price change 8.7% and the fraction of price changes in the low menu cost state. The fraction of time spent in the low cost state $1-\alpha$=8.7%.

Figure 8: Response of Aggregate Output and Sectoral Output without Intermediate Inputs
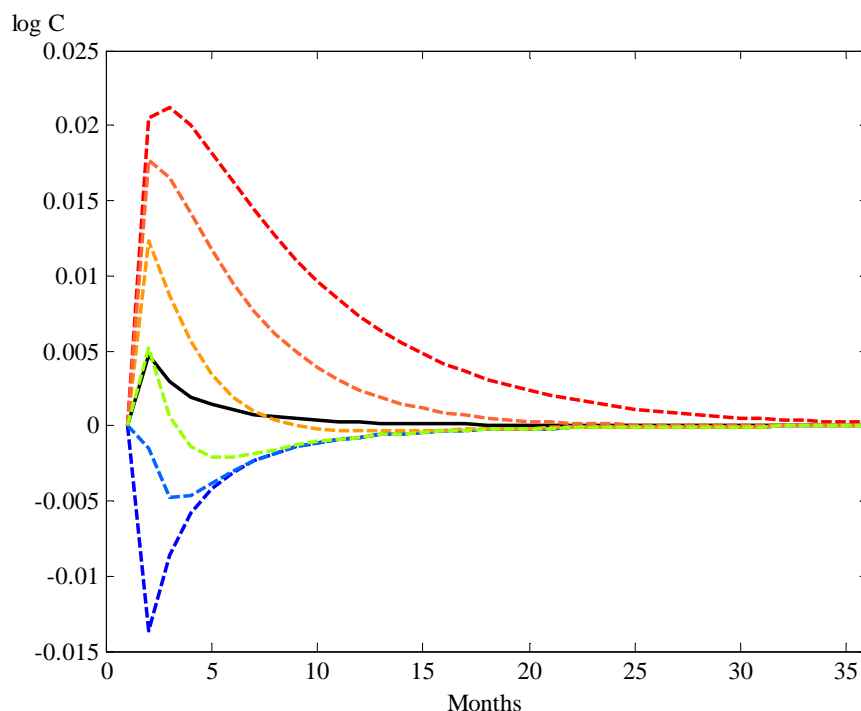
This figure plots the response of aggregate real value-added output (solid line) and sectoral output for several sectors of the 14 sector model without intermediate inputs to a 1% permanent increase in nominal aggregate demand. From top to bottom the sectors that are plotted are: Services(5), Apparel, Services(3), Transportation Goods, Utilities and Vehicle Fuel and Used Cars.

Figure 9: Response of Aggregate Output and Sectoral Output with Intermediate Inputs
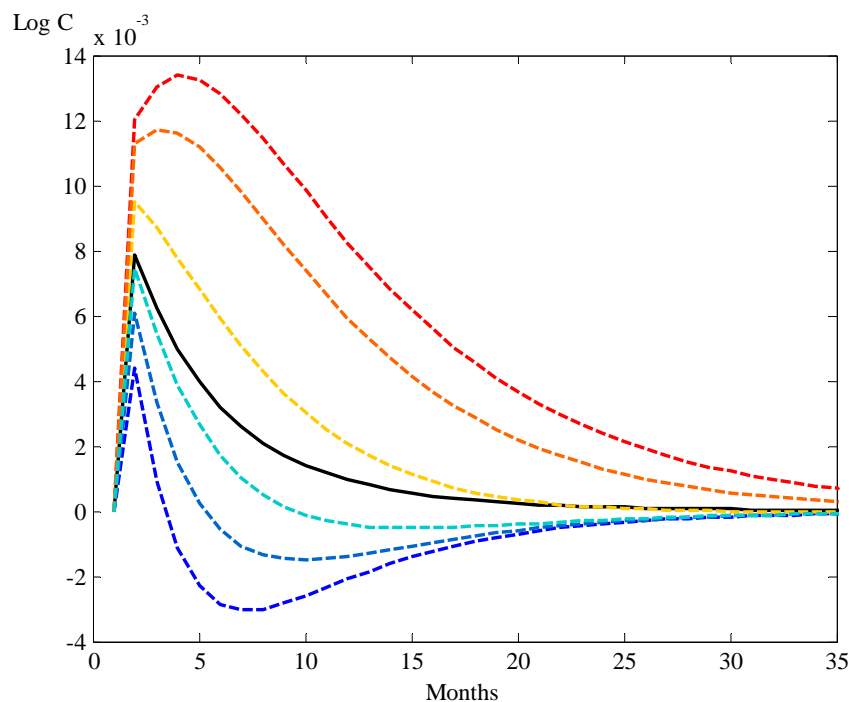
This figure plots the response of aggregate real value-added output (solid line) and sectoral output for several sectors of the 14 sector model with intermediate inputs to a 1% permanent increase in nominal aggregate demand. From top to bottom the sectors that are plotted are: Services(5), Apparel, Services(3), Transportation Goods, Utilities and Vehicle Fuel and Used Cars.
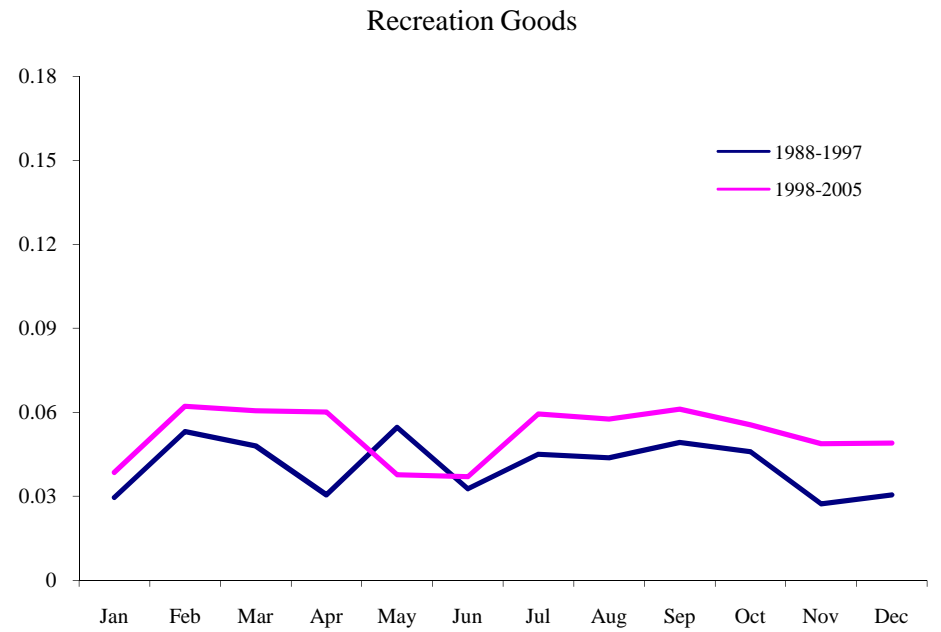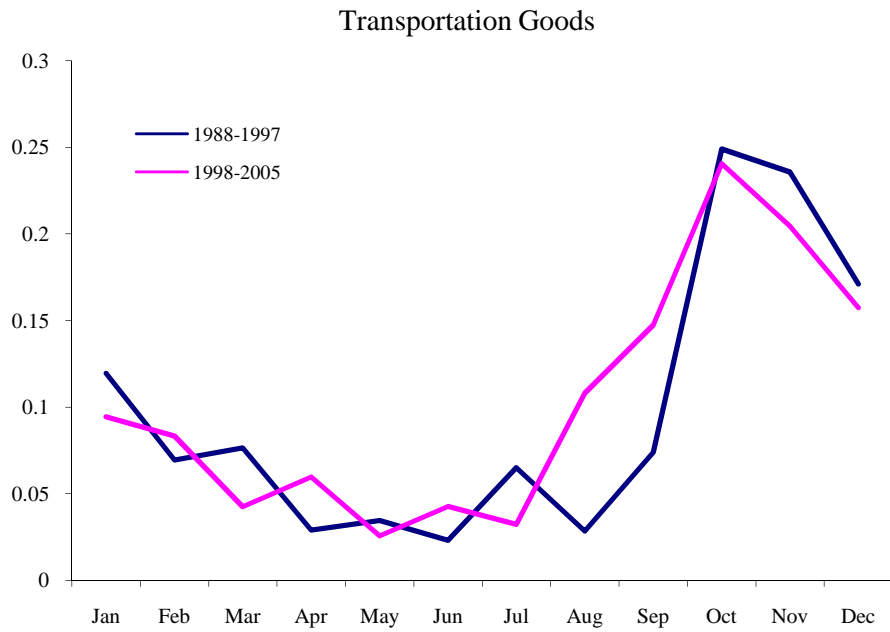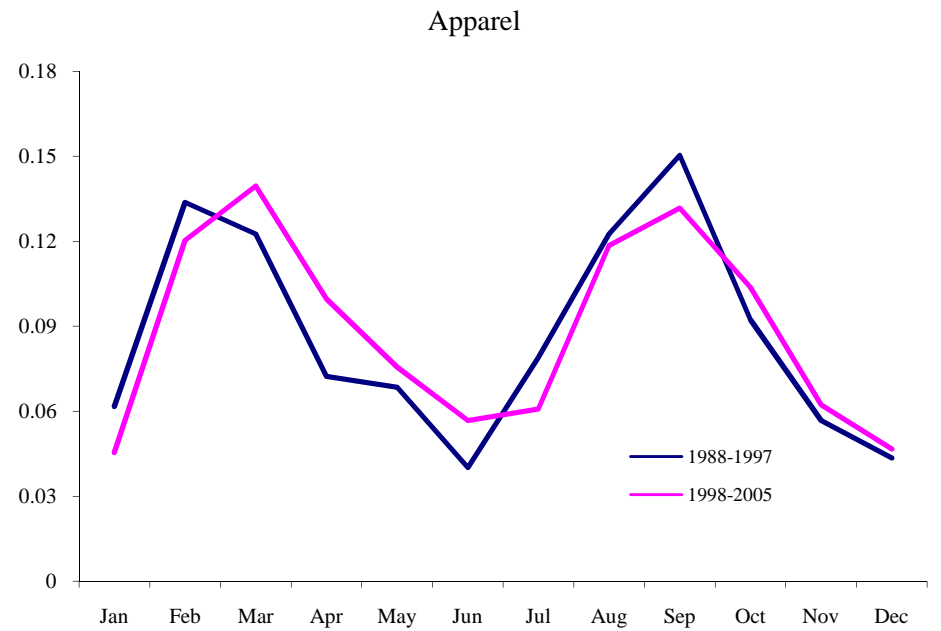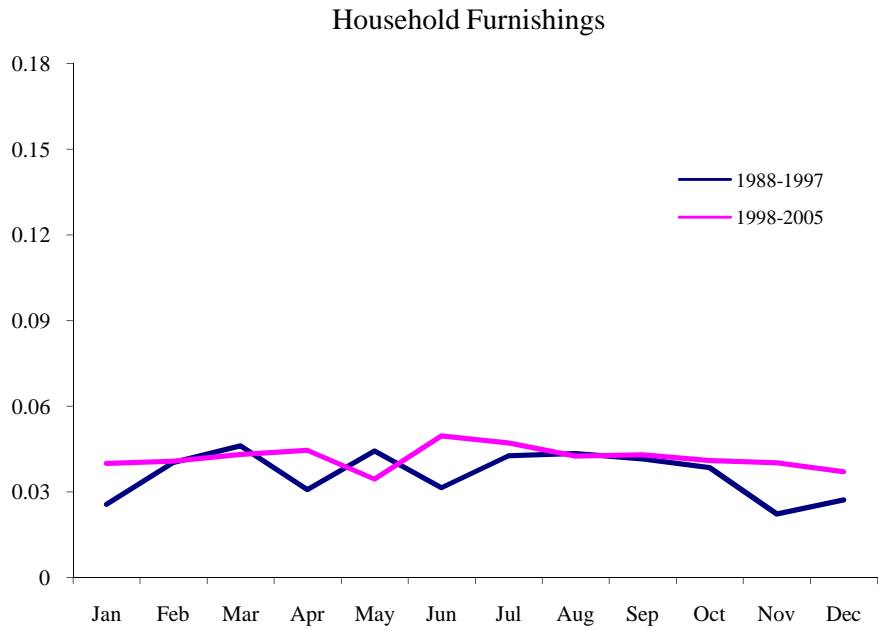
Figure 10: Seasonality in Product Substitution