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ABSTRACT

Falling costs of coordination and communication have allowed firms in rich countries to fragment their production process and offshore an increasing share of the value chain to low-wage countries. Popular discussions about the aggregate impact of this phenomenon on rich countries have stressed either a (positive) productivity effect associated with increased gains from trade, or a (negative) terms of trade effect linked with the vanishing effect of distance on wages. This paper proposes a Ricardian model where both of these effects are present and analyzes the effects of increased fragmentation and offshoring in the short run and in the long run (when technology levels are endogenous). The short-run analysis shows that when fragmentation is sufficiently high, further increases in fragmentation lead to a deterioration (improvement) in the real wage in the rich (poor) country. But the long-run analysis reveals that these effects may be reversed as countries adjust their research efforts in response to increased offshoring. In particular, the rich country always gains from increased fragmentation in the long run, whereas poor countries see their static gains partially eroded by a decline in their research efforts.

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1 Introduction

Technological change has led to a dramatic decline in the cost of communication and in the cost of coordinating activities performed in different locations. This has allowed firms in rich countries to fragment their production process and offshore an increasing share of the value chain to low-wage countries.[1,2] Baldwin (2006) refers to this phenomenon as the "second unbundling." In his words, "rapidly falling transportation costs caused the first unbundling, namely the end of the necessity of making goods close to the point of consumption. More recently, rapidly falling communication and coordination costs have fostered a second unbundling – the end of the need to perform most manufacturing stages near each other. Even more recently, the second unbundling has spread from factories to offices with the result being the offshoring of service-sector jobs." (p. 7).

The purpose of this paper is to explore the welfare consequences of this phemonenon. There has been much discussion recently about this with a specific focus on the impact of offshoring on rich countries. Two popular approaches can be clearly distinguished. They both start from the notion that the unbundling of the production process entails an expansion of the set of tradeable goods and services, but go on to explore different implications. The first approach starts from the premise that trade entails gains for all parties involved, and then concludes that fragmentation and offshoring should be good for all countries. As Gregory Mankiw argued during a press conference in 2004: "More things are tradable than were tradable in the past, and that's a good thing" (Mankiw and Swagel, 2006, p. 9). In contrast, the second approach reasons that increased fragmentation possibilities and lower trade costs would in the limit allow the world to reach an "integrated equilibrium" in which wages for identical workers in different countries would necessarily be equalized. In other words, wages would no longer be affected by the location of workers. For example, in their recent book on offshoring, Hira and Hira (2005) argue that offshoring affects American workers by undermining their "primary competitive advantage over foreign workers: their physical presence in the US." Other noneconomists writing about offshoring have expressed similar concerns.[3]

[1] Jones and Kierzkowski (1990) proposed this way of thinking about technological change, fragmentation and international trade. Yi (2003) develops a Ricardian model of trade to show that trade liberalization may also lead to increased fragmentation (or what he calls vertical specialization) and trade.

[2] See Blinder (2006), Mankiw and Swagel (2006) Grossman and Rossi-Hansberg (2006a), for an analysis of the U.S. data showing that offshoring has grown dramatically over the last years.

[3] See Roberts (2004) and Friedman (2005).

A simple "toy" model may be useful to understand these two approaches to offshoring. Consider first a two-country model with labor as the only factor of production and one final good. For concreteness, let us think of the two countries as the United States (US) and the rest of the world (RW), and assume that the US has higher productivity, which entails higher wages. The existence of a single tradable good implies that there is no trade. But assume that fragmentation becomes feasible, so that some labor services can now be unbundled from the production of the final good. If the productivity in these labor services is the same across the two countries then trade arises, with the US specializing in the production of the final good in exchange for labor services imported from the RW via offshoring operations. It is clear that both countries gain from the new trade made possible by fragmentation, just as in the first of the two approaches discussed above.

Imagine now that there are *two* final goods that can be traded at no cost between the US and the RW, and further assume that the US has a higher productivity in good 1, while productivities are the same in good 2. If the US is not too large relative to the world's demand for good 1, then it will specialize completely in that good and enjoy gains from trade that allow it to sustain higher wages than in the RW. As fragmentation becomes possible, US firms will engage in offshoring to use labor in the RW for part of their production process in good 1. This will effectively enlarge the US supply of good 1, which will worsen its terms of trade. If this process is sufficiently strong, the international relative price of good 1 will converge to the US opportunity cost of this good, at which point the US will no longer benefit from trade and its wage level will become equal to that in the RW.[4] This captures the concerns of the second approach to offshoring mentioned earlier.

Each of these examples highlights an important aspect of the offshoring phenomenon: fragmentation leads both to new trade and to an expansion in the supply of the good in which the advanced country has a comparative advantage. From the point of view of the advanced country, the first effect is positive while the second effect is negative. What is the net effect? To answer this question, one needs to consider a general trade model that is able to capture the roles played by both absolute and comparative advantage. The presence of an overall absolute advantage in the advanced country is a key element, as this is what leads to the wage gap that generates incentives for offshoring. Comparative advantage is also clearly necessary as this is

[4]This effect of fragmentation and offshoring on the rich country is discussed in Deardorff (2001) and in Leamer (2006).

what gives rise to trade in the absence of fragmentation, which is required for the negative terms of trade effect to arise. A general yet parsimonious model in which both absolute and comparative advantage play a role in determining wages and the gains from trade is Eaton and Kortum's (2002) model of Ricardian trade. In this paper I start out with this model and then allow for fragmentation and offshoring to explore their impact on wages in both advanced and poor countries.[5]

Eaton and Kortum model sector-level productivities as being drawn from a distribution that is common across countries except for a technology parameter T. This technology parameter determines the location of the productivity distribution: countries with a higher T have "better" distributions in the sense of first-order stochastic dominance. Apart from T, countries also differ in L, the size of their labor force (which is the only factor of production). Assuming away trading costs for simplicity, wages are determined by the ratio of technology to size, T/L. A high T/L means that the country would have many sectors in which it has absolute advantage relative to its size, leading to a high equilibrium wage. It is interesting to note that, given a fixed technology level, an increase in a country's labor force - caused perhaps by immigration - would lead to a decline in T/L and hence a decline in the country's wage. This is nothing but the classic effect of size on a country's terms of trade in a Ricardian model.[6]

Fragmentation is introduced into the model by assuming that production involves the combination of a continuum of labor services, a share of which may be offshored at no cost and with no loss of productivity.[7] Thus, fragmentation leads firms in high high-wage countries (i.e., countries with a high T/L) to offshore a part of their production process to low-wage countries. This represents new trade, where high T/L countries export final goods in exchange for imports of labor services through offshoring.

This model provides a simple way to study the impact of fragmentation and offshoring on wages in both rich and poor countries. Both effects mentioned above are present: there are

[5] I focus entirely on the impact of offshoring on average wages rather than on the wage distribution or skill premia. In other words, I am interested in understanding the conditions under which the winners from offshoring can compensate the losers, but do not consider the differential impacts on workers with different skill levels or in different activities or industries. Readers interested in this issue can consult Feenstra and Hanson (1996, 1999), Jones and Kierzkowski (2001), Deardorff (2004), Markusen (2004), Blinder (2006), and Grossman and Rossi-Heinsberg (2006a, 2006b), among others.

[6] See Davis and Weinstein (2002) for a recent discussion of the economic impact of immigration in the United States using this basic idea.

[7] The modelling of offshoring as trade in a continuum of labor services is similar to the approach in Grossman and Rossi-Hansberg (2006b), see below.

gains from the new trade that takes place, as well as a movement towards wage equalization that harms the rich countries and benefits the poor countries. The first is a *productivity effect*; it captures the idea that firms experience a decline in their unit costs as they offshore part of their production to low-wage countries. The second is a *terms of trade effect*. Finally, this analysis also reveals the existence of a *world-efficiency effect*, often neglected in discussions of offshoring, which entails a decline in world prices as labor is effectively reallocated from countries with low to countries with high T/L ratios.

From the point of view of poor countries, only the terms of trade and world efficiency effects are present, and both are positive, so these countries always benefit from fragmentation. But rich countries have to deal with the negative terms of trade effect. The analysis reveals that there is always a point beyond which increased fragmentation leads to a negative effect on the real wage in the rich country. In other words, when fragmentation is already high, a further increase in fragmentation generates a negative terms of trade effect that dominates the productivity and world-efficiency effects.[8] More specifically, if the technology gap between rich and poor countries is not too low, then the real wage in rich countries as a function of the level of fragmentation is shaped like an inverted U: initially fragmentation leads to a higher real wage, but this is eventually reversed as fragmentation becomes sufficiently high. In the limit, as we approach a world with complete fragmentation and wage equalization, the real wage in the rich country must necessarily be lower than it would be under no fragmentation.

The result that in rich countries the positive productivity effect of offshoring can be dominated by a negative terms of trade effect is reminiscent of the possibility of "immiserizing growth" for large countries analyzed by Bhagwati (1958). This suggests that in the presence of an optimal tariff or export tax, increased fragmentation would always increase welfare for the rich country. I show that this is indeed the case (at least for a "small economy" for which this can be shown analytically).

The discussion of fragmentation and wages so far takes technology levels as exogenous, and hence can be interpreted as a short-run analysis. But in the long run technology levels are endogenous, determined by research efforts and research productivity. It is conceivable that the resources released by offshoring in the rich countries lead to an increased allocation of resources to research. This would tend to increase the T/L ratio and hence provides a new positive effect

[8] Although clearly related, this is not a simple application of the immiserizing growth possibility studied by Bhagwati (1958). In fact, as discussed below in footnote 12, although higher efficiency in the Eaton and Kortum model leads to declining terms of trade, this would never dominate the direct benefits.

on wages not present in the static analysis.

To explore this possibility, I consider a dynamic model where technology levels are endogenous, as in Eaton and Kortum (2001). In this dynamic model workers choose to work in the production sector or to do research, which leads to new ideas or technologies. When the technology discovered is superior to the state of the art, its owner (or patent holder) earns quasi-rents that provide a return on the opportunity cost of research. The technology parameter T can now be interpreted as the "stock of ideas" in a country, and richer countries are the ones that have a higher stock of ideas per worker. Without fragmentation, the fraction of workers devoted to research turns out to be the same across countries, but countries with a higher research productivity (i.e., a higher rate of arrival of ideas per researcher) can sustain a higher T/L and hence higher wages in steady state. Fragmentation generates the same short-run effects as above, but now we must also take into account the impact on the allocation of workers between production and research in both the rich and poor countries. It will be shown that fragmentation and offshoring induce more people in rich countries to work as researchers, which in the long run increases T/L and wages, counteracting the negative effects mentioned above. In fact, the analysis reveals that in steady state this *research effect* weakens the terms of trade effect to such an extent that it is now always dominated by the productivity effect. The result is that in the long run wages in rich countries always increase with fragmentation.

The long-run effects of fragmentation turn out to be quite different in poor countries. There, as people start to work as providers of labor services through offshoring operations, the fraction of people devoted to research falls, decreasing T/L and wages. This entails a negative research effect that in steady state exactly compensates the positive terms of trade effect. Thus, just like every other country (even the ones that do not participate in offshoring activities), poor countries benefit from fragmentation only through the world-efficiency effect.

In sum, the analysis suggests that increased fragmentation could indeed have negative effects for rich countries, but that these effects dissipate in time, so that the long run effects are always positive for the countries doing the offshoring. In contrast, the long run effects of fragmentation in poor countries are weaker than the corresponding short run effects. For the rich country, the presence of opposite short and long run effects implies that increased fragmentation could be harmful or beneficial. For a special case that can be analytically solved, I show that as long as the speed with which resources can be reallocated across production and research is sufficiently high then the long run effects dominate and the rich country gains from offshoring.

There is a long list of recent papers that have analyzed the possible effects of fragmentation and offshoring on wages in rich countries.[9] Samuelson (2004) stressed the possible negative impact of offshoring if it leads to spillovers that erode the rich countries' technological advantage in exporting sectors, while Deardorff (2001, 2005) showed that, even without such spillovers, rich countries would suffer a deterioration of their terms of trade that could more than compensate any associated gains. Bhagwati et. al. (2004) and Mankiw and Swagel (2006) argued that the terms of trade effect would likely be dominated by the positive productivity effect. The present paper shows that in the short run this is not necessarily the case; in fact, when fragmentation is sufficiently high, further increases in fragmentation (and offshoring) necessarily hurt the rich country. But, again, this applies only in the short run; in the long run, when research efforts have had a time to fully adjust to the new environment, then rich countries are always better off with offshoring than without.

Another group of papers have explored the implications of fragmentation on wages for skilled and unskilled workers in the context of a Hecksher-Ohlin model of trade.[10] Prominent examples are Feenstra and Hanson (1996), Jones and Kierzkowski (2001), Deardorff (2004), Kohler (2004), Markusen (2005), Grossman and Rossi-Hansberg (2006a, 2006b) and Baldwin and Robert-Nicoud (2007). The contribution of Grossman and Rossi-Hansberg (2006b) is particularly relevant to the present paper. In their main specification, fragmentation is seen as the decline in the cost of trading a continuum of unskilled tasks. Focusing on a skilled-labor abundant country, they show that fragmentation leads to increased offshoring of such tasks, a positive productivity effect that increases the wage of unskilled workers, and an *improvement* in the terms of trade that has the usual Stolper-Samuelson implications. In another specification they explore the consequences of an overall decline in the costs of offshoring all tasks (skilled and unskilled). They note that this generates a positive productivity effect, but a deterioration of the country's terms of trade. Offshoring also has these conflicting effects on the rich country in the Ricardian model presented below, but the model has the advantage that these two effects can be compared in such a way that the net result can be fully characterized both in the short run and in the long run.

[9] For recent surveys see Mankiw and Swagel (2006) and Baldwin (2006). See also Baily and Lawrence (2004) for an exploration of the implications of offshoring for the loss of manufacturing jobs in the U.S. over the last decades. For an analysis of the effect of offshoring on unemployment see Mitra and Ranjan (2007).

[10] Another approach is Kohler (2004), who explores the consequences of offshoring in a specific-factors model of a small-open economy and shows conditions under which the presence of non-convexities may lead offshoring to harm the rich country.

The connection between offshoring and innovation has received scant attention in the literature. One exception is Glass and Saggi (2001), who extend Grossman and Helpman's (1991) quality-ladder growth model to a two-country setting with offshoring. Focusing on the steady state, they show that a decline in the cost of offshoring leads to an increase in innovation in the rich country and an increase in the growth rate. However, in their model, the increase in innovation must be accompanied by a decline in the rich country's wage to keep innovation profitable. In contrast, I build on the quasi-endogenous growth framework of Kortum (1997) and Eaton and Kortum (2001), so offshoring has no growth effects and the steady state effect on the rich country's wage is positive thanks to the direct productivity effect and the long run effect of increased research on the rich country's technology distribution.

The rest of the paper is organized as follows. Section 2 introduces the static model and derives the implications of fragmentation on both rich and poor countries participating in offshoring activities. In two extensions I consider the optimal trade policy for the rich country and an alternative way to think of fragmentation as a decline in the cost of offshoring. Section 3 presents a dynamic model where technology levels are endogenously determined by research efforts in each country. In the short-run this model is equivalent to the static model of Section 2. I use this model to explore the implications of fragmentation on long run (steady state) research intensities and wages in the rich and poor countries participating in offshoring. I also study the transition dynamics to understand the net welfare effects of an unexpected increase in fragmentation. Section 4 compares the implications of offshoring to immigration, and Section 5 presents some extensions of the model. Section 6 concludes. All proofs are relegated to the Appendix.

2 The static model

The static model builds on the Eaton and Kortum (2002) model of Ricardian trade under the simplifying assumption of no transportation costs. There are N countries, indexed by $i \in \{1, 2, ..., N\}$, and a continuum of tradable final goods, indexed by $j \in [0, 1]$. Labor is the only factor of production, and is supplied inelastically with measure L_i in country i. Preferences across goods are Cobb-Douglas and symmetric, so that an equal share of income is spent on each good j.

All final goods are produced from a single "common input" whose cost in country i is denoted by c_i. In a standard Ricardian model the common input is labor, so c_i is simply the wage w_i. Here I allow for a more general production structure to introduce fragmentation and offshoring into the model, so c_i may differ from w_i. In particular, I assume that the common input is produced through a Leontief production function from a continuum of "intermediate services" indexed by $k \in [0, 1]$. Formally, letting $x(k)$ represent the quantity of intermediate service k, then output of the common input is $X = \min_k\{x(k)\}$. In turn, $x(k)$ is produced one-to-one from labor. If all intermediate services must be produced directly by the firm, then this collapses to the standard case with $c_i = w_i$. Fragmentation is introduced by allowing firms to costlessly offshore *at most* a certain exogenous share $\beta \in [0, 1[$ of the intermediate services. The assumption that β is exogenous simplifies the analysis considerably but is not essential: as I show in Section 5, the main results go through in a setting where the measure of services that are offshored is endogenous to the costs of trading services as in Kohler (2004) and Grossman and Rossi-Hansberg (2006). Below I refer to the restriction that firms cannot offshore more than a share β of services as the "offshoring restriction."

To simplify the analysis and exposition, I focus on the possibility of offshoring by country 1 from country 2 (country 1 is the rich country), while offshoring is not possible for all the other countries. In Section 5 I extend the analysis to allow for offshoring between three countries. If $w_1 > w_2$ then firms in country 1 would want to exploit all opportunities for offshoring, and hence the unit cost of the common input there would be

$$c_1 = (1 - \beta)w_1 + \beta w_2 \tag{1}$$

More generally, we have $c_1 = \min\{w_1, (1 - \beta)w_1 + \beta w_2\}$ while $c_i = w_i$ for $i \neq 1$.

The common input is converted into final goods through the use of linear technologies that vary in productivity across goods and countries. Letting $z_i(j)$ denote the productivity for good j in country i then country $i's$ unit cost for j is $c_i/z_i(j)$. These linear technologies are available to all firms within a country, so the appropriate market structure is perfect competition. Given the absence of transportation costs, then the price of good j in all countries is simply $\min_i \{c_i/z_i(j)\}$.

As in Eaton and Kortum (2002), the productivities $z_i(j)$ are modelled as the realization of a random variable that is assumed to be independent across goods and countries. In particular, in country i the productivity z_i for each good $j \in [0, 1]$ is drawn from the Frèchet distribution,

$$F_i(z) = \Pr[z_i \leq z] = \exp[-T_i z^{-\theta}] \tag{2}$$

where $T_i > 0$ and $\theta > 1$. The parameter T_i can vary across countries and determines the location of the distribution: a higher T_i implies that the productivity draws are likely to be better. Thus, T_i is country $i's$ technology level and determines the share of goods in which it has absolute advantage relative to other countries across the continuum of goods. The parameter θ (which is common across countries) determines the variability of the draws and hence the strength of comparative advantage: a lower θ implies a stronger comparative advantage.

2.1 Equilibrium with no offshoring

To establish a benchmark, introduce some notation and develop some initial intuition for the results to come, consider first the case with no offshoring, or $\beta = 0$. The unit cost of the common input in country i is then simply w_i (i.e., $c_i = w_i$ for all i).

Given the preferences specified above, the share of total income that each country spends on imports from country i is equal to the share of goods for which country i is the least-cost producer. In turn, this is equal to $\pi_i = T_i w_i^{-\theta}/\Phi$ where $\Phi \equiv \sum_k T_k w_k^{-\theta}$.[11] Note that, given w_i, a higher T_i implies more exports, and the same happens with a lower w_i given T_i.

Wages are determined by the trade-balance conditions, which in this context of no trade costs are simply given by

$$\pi_i Y = w_i L_i \tag{3}$$

where $Y \equiv \sum_k w_k L_k$ is worldwide income. Using country $N's$ labor as numeraire (i.e., $w_N = 1$) it is easy to show that

$$w_i = \delta \left(T_i/L_i\right)^b \tag{4}$$

where $\delta \equiv (T_N/L_N)^{-b}$ and $b \equiv 1/(1+\theta)$. Note that an increase in size L_i holding the technology level T_i constant implies a decline in country $i's$ wage. This happens through a deterioration of country $i's$ terms of trade and is the channel through which increased fragmentation and offshoring could lower country $1's$ income level.[12]

[11] To see this, note that the distribution of the price that country i would charge for a particular good, $p_i = w_i/z$, is $\Pr_i(p_i \leq p) = \Pr_i(z \geq w_i/p) = G_i(p) \equiv 1 - e^{-T_i(w_i/p)^{-\theta}}$. In turn, the distribution of the minimum price across countries $i \in \Gamma$, $p(\Gamma) \equiv \min_{i\in\Gamma}\{p_i\}$, is $G_\Gamma(p) = 1 - \prod_{i\in\Gamma} \Pr_i(p_i \geq p) = 1 - e^{-\Phi(\Gamma)p^\theta}$, where $\Phi(\Gamma) \equiv \sum_{i\in\Gamma} T_i w_i^{-\theta}$. Hence, letting $\Gamma(-i)$ be the set of countries other than i, the probability that country i has the lowest cost is $\pi_i = \int_0^\infty G_{\Gamma(-i)}(p)dG_i(p) = T_i w_i^{-\theta}/\Phi$.

[12] Note, however, that growth cannot be immiserizing in this case. Consider an increase in productivity that is manifested as an increase in "efficiency units" per person (an increase in T would always lead to a higher wage). Total efficiency units are now $L = e \cdot N$, with e being efficiency units per person and N being the level

2.2 Equilibrium with offshoring

Consider now the case in which offshoring is feasible ($\beta > 0$). The cost of the common input in country 1 will differ from the wage there because of the possibility of indirectly using labor at the cheaper cost w_2 in country 2. In particular, if $w_1 > w_2$, then the offshoring restriction will be binding, and c_1 will be given by (1). Moreover, since a share $1 - \beta$ of the total quantity of the common input is produced domestically, then the full employment condition in country 1 entails $(1 - \beta)X = L_1$. The total amount of labor used in country 2 via offshoring, βX, is then equal to αL_1, where $\alpha \equiv \beta/(1 - \beta)$. Since all countries other than 1 do not engage in offshoring then $c_i = w_i$ for all $i \neq 1$. The import shares in equilibrium are now

$$\pi_i = T_i c_i^{-\theta}/\Phi \tag{5}$$

for all i. The trade balance conditions are unchanged for $i \neq 1, 2$, whereas for countries 1 and 2 they are now given by

$$\pi_1 Y = w_1 L_1 + \alpha w_2 L_1 \tag{6}$$

and

$$\pi_2 Y = w_2 L_2 - \alpha w_2 L_1 \tag{7}$$

The term $\alpha w_2 L_1$ is simply the value of intermediate services imported by country 1 from country 2.

Combining (5) for $i = 2$ with (7) yields

$$w_2 = \delta \left(T_2/\widetilde{L}_2\right)^b \tag{8}$$

where

$$\widetilde{L}_2 \equiv L_2 - \alpha L_1 \tag{9}$$

is the number of workers left in country 2 for production given that αL_1 workers are devoted to offshoring services for country 1. Comparing (4) and (8) shows that country $2's$ wage is increased by offshoring, i.e. $w_2'(\alpha) > 0$. The reason for this is that a decline in the number of workers left for production given a fixed technology level increases the ratio $T_2/\widetilde{L}_2$ and thereby improves country $2's$ terms of trade. As intuition would suggest, the effect of offshoring on w_2 is exactly the same as the effect of a reduction in L_2 due to outmigration in country 2.

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of population (or labor force). The wage is now $\delta e(T/eN)^b$ which is increasing in e given that $b < 1$.

Turning to country 1, combining equations (5) for $i = 1$ with (6) implies that

$$(1-\beta)w_1 + \beta w_2 = \delta \left(T_1/\widetilde{L}_1\right)^b \tag{10}$$

where

$$\widetilde{L}_1 \equiv (1+\alpha)L_1 \tag{11}$$

is the "effective" amount of labor devoted to production in country 1 once we take into account the extra labor used through offshoring. Equation (10) shows that, given w_2, offshoring has two opposite effects on the wage in country 1: first, there is an increase in the effective number of workers in production (i.e., $\widetilde{L}_1 > L_1$), which worsens its terms of trade; and second, there is a decline in costs thanks to the use of cheaper labor in country 2 through offshoring (i.e., $w_2 < w_1$). The net impact of these two effects on the equilibrium wage in country 1 is explored below. For now, the task is to fully characterize the equilibrium for all the relevant parameter values.

Equations (8) and (10) determine the equilibrium wages in countries 1 and 2 if two constraints are satisfied. First, there is a resource constraint in country 2, which implies that $\alpha L_1 \leq L_2$. Second, wages satisfy $w_1 > w_2$. This is equivalent to $T_1/\widetilde{L}_1 > T_2/\widetilde{L}_2$. Letting $\eta \equiv \frac{T_1/L_1}{T_2/L_2}$, then this inequality can be written as

$$\eta\left(1 - \alpha L_1/L_2\right) > 1 + \alpha \tag{12}$$

From now on I will assume that $\eta > 1$. This is simply a condition that with no offshoring we have $w_1 > w_2$. Given $\eta > 1$ then the inequality in (12) is satisfied for $\alpha = 0$. As α increases the LHS falls, whereas the RHS increases, and there is a level of α such that the two sides become equal, namely

$$\bar{\alpha} \equiv \frac{\eta - 1}{1 + \eta L_1/L_2}$$

Thus, the inequality in (12) is satisfied if and only if $\alpha < \bar{\alpha}$. If this inequality is satisfied, then it is easy to check that the resource constraint in country 2 (i.e., $\alpha L_1 \leq L_2$) is also satisfied. Thus, if $\alpha < \bar{\alpha}$ then the equilibrium is characterized by the solution of equations (8) and (10).

What is the equilibrium if $\alpha \geq \overline{\alpha}$? In this case the equilibrium entails $w_1 = w_2$, the offshoring restriction is not binding, and the equilibrium is characterized by the equations (8) and (10) *but with* $\overline{\alpha}$ *rather than* α. It is important to note that if $\alpha \geq \overline{\alpha}$ then offshoring allows economies 1 and 2 to reach an integrated equilibrium, so factor price equalization (FPE) holds (i.e., $w_1 = w_2$). In the rest of the paper I refer to this case as "full offshoring."

2.3 Wages under Full Offshoring

In this subsection I compare the wage in country 1 under full offshoring with the level that prevails with no offshoring. In the next subsections I turn to a more general comparative-statics analysis to understand the effect of fragmentation on wages in countries 1 and 2.

Since economies 1 and 2 are effectively integrated through offshoring, it is possible to consider them *as if* they were a single region in a world with no offshoring. To explore this further, I now use the index m to refer to the region composed of countries 1 and 2. Letting $T_m \equiv T_1 + T_2$ then the share of world income spent on imports from region m is given by

$$\pi_m = \frac{T_m w_m^{-\theta}}{\Phi}$$

where $\Phi = T_m w_m^{-\theta} + \Phi_{-m}$, and $\Phi_{-m} \equiv \sum_{k \neq 1,2} T_k w_k^{-\theta}$. Letting $L_m \equiv L_1 + L_2$ then total income in region m is $w_m L_m$ and the trade balance condition for this region is now simply $\pi_m Y = w_m L_m$. Just as in the case of no offshoring considered above we now have

$$w_m = \delta \left(T_m / L_m\right)^b$$

The effect of full offshoring on the wage in country 1 can now be determined by comparing w_1 under no offshoring with w_m. It is easy to see that since $\eta > 1$ then $T_m/L_m < T_1/L_1$ and hence $w_m < w_1 \mid_{\alpha=0}$. Intuitively, integration with country 2 through offshoring effectively lowers country $1's$ technology level per worker (T/L) and this leads to a decline in its terms of trade.

This result concerns the effect of full offshoring on the wage in country 1 relative to the wage of the numeraire country. But it is also important to consider the impact on the real wage w_1/P, where P is the price index of a unit of utility. It is straightforward to show that

$$P = \widetilde{\gamma} \Phi^{-1/\theta} \tag{13}$$

where $\widetilde{\gamma} \equiv e^{-\gamma/\theta}$, and γ is Euler's constant.[13] Since $\Phi = \sum_k T_k c_k^{-\theta}$, this expression implies that higher technology levels or lower unit costs lead to lower prices. From this expression it is

[13] To see this, note from footnote 11 that the distribution of the international price is $G_\Gamma(p)$ with Γ being the set of all countries, or $G(p) = 1 - e^{-\Phi p^\theta}$. Therefore, $P = \exp \int_0^\infty \ln(p) dG(p) = e^{-\gamma/\theta} \Phi^{-1/\theta}$, where γ is Euler's constant (i.e., $\gamma \equiv -\int_0^\infty \ln(x) e^{-x} dx$). Readers familiar with Eaton and Kortum (2001) will note that this is slightly different from their result, namely $P = \gamma \Phi^{-1/\theta}$. This difference is due to an inconsequential mistake in Eaton and Kortum (2001).

now easy to establish that P is lower under full offshoring than with no offshoring,[14] a result that reflects the higher efficiency attained when labor effectively reallocates from country 2 to country 1. There are then two opposite effects on the real wage in country 1 as we move from no offshoring to full offshoring: the *terms of trade effect*, which decreases the relative wage w_1, and the *world-efficiency effect*, which lowers the price index P. Note that there is no productivity effect here because there is no longer a wage gap between countries 1 and 2; as a consequence, country 1 does not gain from trading services with country 2. It is shown in the Appendix that the terms of trade effect always dominates the world-efficiency effect, so that w_1/P is necessarily lower under full offshoring than with no offshoring. Recalling that the wage in country 2 increases with offshoring, this result leads to the following proposition:

Proposition 1 *There is full offshoring if $\alpha \geq \bar{\alpha}$. Under full offshoring w_1 and w_1/P are lower and w_2 and w_2/P are higher than with no offshoring.*

2.4 The effect of offshoring on relative wages

Above it was already shown that the wage in country 2 increases with offshoring. I now explore how offshoring affects w_1. Solving for w_1 from (10) yields

$$w_1 = (1+\alpha)\widetilde{w}_1 - \alpha w_2$$

where $\widetilde{w}_1 \equiv \delta \left(T_1/\widetilde{L}_1\right)^b$ is the wage that would prevail in country 1 with no offshoring if its labor supply was $\widetilde{L}_1$. In other words, this would be the equilibrium wage if offshoring only generated a terms of trade effect but no productivity effect. Note that both $\widetilde{w}_1$ and w_2 are affected by α. Differentiating with respect to α and simplifying yields

$$w_1' = (1+\alpha)\widetilde{w}_1' - \alpha w_2' + (w_1 - w_2)/(1+\alpha) \tag{14}$$

The first term on the RHS of (14) captures the *terms of trade effect*. It is negative because $\widetilde{w}_1' = -b\widetilde{w}_1/(1+\alpha) < 0$. Intuitively, as α increases the "effective" supply $\widetilde{L}_1$ increases and this leads to a decline in the wage through a worsening of country $1's$ terms of trade. The second term is negative because, as shown above, w_2 is increasing in α. This is simply a *demand effect*: as offshoring increases, this pushes up country $2's$ wages and this hurts country 1, which uses

[14] This just requires showing that $T_m w_m^{-\theta}$ is higher than $T_1 w_1^{-\theta} + T_2 w_2^{-\theta}$ for the wages w_1 and w_2 that prevail with no offshoring. But using $w_i = \delta\left(T_i/L_i\right)^b$ for $i = 1, 2, m$, then this follows from the concavity of the function $f(x) = x^{b\theta}$.

country $2's$ labor as an input. This second term could also be seen as part of a broader terms of trade effect that takes into account the price that country 1 must pay for imported services. Finally, the third term on the RHS of (14) is the *productivity effect*, which is positive as long as $w_1 > w_2$. This effect captures the idea that by having access to cheaper labor in country 2, country 1 achieves a decline in its costs, and this leads to higher wages there.

To characterize the net marginal effect of offshoring on wages in country 1, i.e. $w_1'(\alpha)$, it is useful to note the following two points: first, the productivity effect depends positively on the wage difference $w_1 - w_2$ which in turn is increasing in the ratio of per capita technology levels in country 1 relative to country 2, or η.[15] Thus, $w_1'(\alpha)$ is more likely to be positive if η is large. In particular, evaluating w_1' at $\alpha = 0$ in (14) yields

$$w_1'(0) = w_2(0)\left[(1-b)\eta^b - 1\right]$$

Thus, $w_1'(0) \gtrless 0$ according to whether $\eta \gtrless (1-b)^{-1/b}$. Second, as α gets close to $\bar{\alpha}$ the wage difference $w_1 - w_2$ goes to zero and the productivity effect vanishes, so $w_1'(\alpha)$ is necessarily negative for α close enough to $\bar{\alpha}$. These two points combined suggest that for $\eta \leq (1-b)^{-1/b}$ the curve $w_1(\alpha)$ is always decreasing, whereas for $\eta > (1-b)^{-1/b}$ this curve is shaped like an inverted U. The next Proposition summarizes these results.

Proposition 2 *If $\eta \leq (1-b)^{-1/b}$ then $w_1(\alpha)$ is decreasing in $\alpha \in [0, \bar{\alpha}[$, whereas if $\eta > (1-b)^{-1/b}$ then $w_1(\alpha)$ is shaped like an inverted U on $\alpha \in [0, \bar{\alpha}[$.*

2.5 The effect of offshoring on real wages

To explore the effects of offshoring on real wages, we need to bring the world-efficiency effect into the analysis. As one would expect, offshoring decreases the price index P. Intuitively, an increase in α effectively implies more possibilities to trade, and this increases worldwide efficiency. The following proposition formalizes this result:

Proposition 3 *The price index P is decreasing in $\alpha \in [0, \bar{\alpha}[$.*

Since $w_2(\alpha)$ is increasing then clearly $w_2(\alpha)/P(\alpha)$ will also be increasing. Similarly, if $w_1(\alpha)$ is increasing then $w_1(\alpha)/P(\alpha)$ will be increasing as well. But what happens when $w_1(\alpha)$

[15] The result that the gains from offshoring are more likely to be positive when the wage gap is higher is also present in Kohler (2004).

is decreasing? The following Proposition shows that the characterization of $w_1(\alpha)/P(\alpha)$ is very similar to the characterization of $w_1(\alpha)$ in Proposition 2.

Proposition 4 *There exists $\hat{\eta}$ such that if $\eta \leq \hat{\eta}$ then w_1/P is decreasing in $\alpha \in [0, \bar{\alpha}[$, while if $\eta > \hat{\eta}$ then w_1/P is shaped like an inverted U in $\alpha \in [0, \bar{\alpha}[$.*

This proposition shows that when fragmentation is sufficiently high, then further increases in fragmentation (and offshoring) necessarily hurt the rich country. This arises because the (positive) productivity and world-efficiency effects are dominated by the (negative) terms of trade and demand effects.

2.6 Export Taxes

As discussed above, the negative impact of offshoring on the rich country takes place through a deterioration of its terms of trade. A natural question is whether an appropriate tariff or export tax could prevent such a negative impact. This section explores this idea, focusing on the case of an export tax; the impact of a tariff would be equivalent. To derive analytical results, I will consider the region composed of countries 1 and 2 as a "small economy," in the Alvarez and Lucas (2005) sense of the limit of a sequence in which the ratios $k_i = T_i/L_i$ for $i = 1, 2$ and L_2/L_1 remain constant but $L_1 \to 0$. The results reveal that, under an appropriate export tax, an increase in fragmentation never makes the economy worse off. This is analogous to the well-known proposition that an optimal tariff or export tax rules out the possibility of immiserizing growth for a large economy (Bhagwati, 1958).

Consider an export tax in country 1 of $\tau - 1$, so that if a firm exports value v, the government collects $(\tau - 1)v$. The price of a good with productivity z that is exported by country 1 would be $\tau c_1/z$: the firm only gets c_1/z, while the government collects the rest, $(\tau - 1)c_1/z$. I assume that the revenue collected from this tax, R, is distributed back to consumers in lump-sum fashion. Thus, income in country 1 is now $Y_1 = w_1 L_1 + R$.

Let π_1^f be the share of spending by foreigners (i.e., consumers in countries other than country 1) on goods from country 1. It is easy to see that

$$\pi_1^f = \frac{T_1(\tau c_1)^{-\theta}}{T_1(\tau c_1)^{-\theta} + \Phi_{-1}} \tag{15}$$

where $\Phi_{-1} \equiv \sum_{i \neq 1} T_i w^{-\theta}$. On the other hand, the share of spending by foreigners on goods from country $i \neq 1$ is

$$\pi_i^f = \frac{T_i {w_i}^{-\theta}}{T_1(\tau c_1)^{-\theta} + \Phi_{-1}} \tag{16}$$

The corresponding spending shares for consumers in country 1, which are denoted by π_i, are the same as in the previous sections. The trade balance conditions for countries $i \neq 1, 2$ are then $\pi_i Y_1 + \pi_i^f Y_{-1} = w_i L_i$, where $Y_{-1} = \sum_{i \neq 1} L_i w_i$ is the income level in the rest of the world. Given the expressions for π_i and π_i^f, it is easy to show that for $i \neq 1, 2$ wages are $w_i = \delta \left(T_i/L_i\right)^b$, just as in (4). Similarly, as long as the offshoring restriction is binding, the wage in country 2 is $w_2(\alpha) = \delta \left(\frac{T_2}{L_2 - \alpha L_1}\right)^b$, as in equations (8) and (9).

Next consider the equilibrium in country 1. Foreigners spend $\pi_1^f Y_{-1}$ on goods from country 1, firms there earn $\pi_1^f Y_{-1}/\tau$ as revenue on those exports, and the government collects $\tau - 1$ times this amount. Thus, government revenues in country 1 are

$$R = \frac{(\tau - 1)\pi_1^f Y_{-1}}{\tau} \tag{17}$$

and the trade balance condition for country 1 is now

$$\pi_1^f Y_{-1} = (1 - \pi_1) Y_1 + \alpha w_2 L_1 \tag{18}$$

The LHS of (18) is total export revenue, while the RHS is total spending on imports, including services. Recalling that $Y_1 = w_1 L_1 + R$, using (17) and noting that $Y_{-1} = w_2(\alpha) L_2 + Y_{-2}$ (for $Y_{-2} = \sum_{i \neq 1,2} L_i w_i$) then

$$\pi_1^f \left(w_2(\alpha) L_2 + Y_{-2}\right) = (1 - \pi_1) \Big(w_1 L_1 + (1 - 1/\tau)\pi_1^f \left(w_2(\alpha) L_2 + Y_{-2}\right)\Big) + \alpha w_2(\alpha) L_1 \tag{19}$$

Given τ, the solution to this equation yields the equilibrium wage in country 1 as long as $w_1 \geq w_2$. Otherwise, the equilibrium entails "full offshoring," with the extent of offshoring given by $\bar{\alpha}(\tau)$ defined implicitly by the previous equation with $w_1 = w_2(\alpha)$.

Equation (19) has an analytic solution in w_1 only for the case in which the region composed of countries 1 and 2 is a "small economy," i.e. in the limit as $L_1 \to 0$ (with k_i for $i = 1, 2$ and L_2/L_1 constant along the sequence). The results derived above for countries $i \neq 1, 2$ imply that $\Phi_{-2} \equiv \sum_{i \neq 1,2} T_i w^{-\theta}$ and Y_{-2} do not depend on the export tax, and do not change as countries 1 and 2 are getting small along the sequence that we consider below (with $L_1 \to 0$). Moreover, since $w_2(\alpha) = \delta \left(\frac{k_2 L_2/L_1}{L_2/L_1 - \alpha}\right)^b$, then w_2 is also constant along the sequence and so is Y_{-1}. Thus, taking the limit as $L_1 \to 0$ in (19), using hats over variables to denote the limits, and recalling that $c_1 = (1-\beta)w_1 + \beta w_2$, then

$$\widehat{c}_1(\alpha,\tau)\tau = \delta \left[\frac{k_1}{1+\alpha}\right]^b \tag{20}$$

whereas $\widehat{w}_1(\alpha,\tau) = (1+\alpha)\widehat{c}_1(\alpha,\tau) - \alpha w_2(\alpha)$.[16] This implies that as long as the export tax does not affect the extent of offshoring (see below), then the only effect of this policy is to decrease c_1 in such a way that $c_1\tau$ remains constant. This happens through a decline in the wage that exactly offsets the increase in the export cost caused by the tax, leaving total export revenues constant.

If the tax is high enough, however, then the extent of offshoring will be affected. To see this, note that for high enough τ the wage $\widehat{w}_1(\tau,\alpha)$ would become lower than $w_2(\alpha)$, and the equilibrium would then be characterized by full offshoring, with the extent of offshoring $\bar{\alpha}(\tau)$ determined implicitly by $\widehat{w}_1(\tau,\bar{\alpha}) = w_2(\bar{\alpha})$, and given by

$$\bar{\alpha}(\tau) = \frac{\eta - \tau^{1/b}}{\tau^{1/b} + \eta L_1/L_2} \tag{21}$$

It is clear that $\bar{\alpha}(\tau)$ is decreasing in τ.[17] Moreover, if τ is so high that $\tau > \eta^b$ then offshoring would vanish. Thus, if α and τ are such that $\widehat{c}_1(\alpha,\tau) \geq w_2(\alpha)$, the equilibrium entails offshoring up to the offshoring restriction given by α; otherwise, the equilibrium depends on whether $\bar{\alpha}(\tau) \gtrless 0$: if $\bar{\alpha}(\tau) \geq 0$ then the extent of offshoring is $\bar{\alpha}(\tau)$ and wages are equalized in countries 1 and 2, whereas if $\bar{\alpha}(\tau) < 0$ then there is no offshoring and the wage in country 1 is lower than in country 2 (i.e., $\widehat{w}_1(0,\tau) < w_2(0)$).[18] These results are stated formally in the following lemma:

Lemma 1 *Let* $\alpha_o(\tau,\alpha) \equiv \max\{\min\{\alpha,\bar{\alpha}(\tau)\},0\}$. *Relative to the wage in country N, the equilibrium wages in countries $i \neq 1,2$ are $w_i = \delta\,(T_i/L_i)^b$, whereas (in the limit as $L_1 \to 0$) wages in countries 1 and 2 are given by $\widehat{w}_1(\alpha_o(\alpha,\tau),\tau)$ and $w_2(\alpha_o(\alpha,\tau))$.*

The total effect of the export tax on country 1 depends on the impact of τ on $\widehat{Y}_1(\alpha,\tau)$. But it turns out that $\lim Y_1(\alpha,\tau)/L_1 = \widehat{w}_1(\alpha,1) = w_1(\alpha)$.[19] This implies that, given α, the decline

[16] To derive this result, first divide boths sides of equation (19) by π_1^f, and then take the limit of both sides. Simplifying the resulting expression and then noting from $w_i = \delta\,(T_i/L_i)^b$ for $i \neq 1,2$ that $Y_{-2}/\Phi_{-2} = \delta^{1/b}$ yields (20).

[17] Also note that $\bar{\alpha}(1) = \bar{\alpha} = \frac{\eta-1}{1+\eta L_1/L_2}$, as defined in subsection 2.3.

[18] I am implicitly assuming here that it is not possible for country 2 to import services from country 1 through offshoring. The extension to consider this possibility is straightforward.

[19] To see this, simply note that $\lim Y_1(\alpha,\tau)/L_1 = \widehat{w}_1(\alpha,\tau) + \lim R/L_1$. Taking the limit on the RHS and simplifying yields the result.

in the wage generated by the export tax is exactly matched by the revenue collected by the export tax. The effect of the tax on total income in country 1 is then easy to characterize. Let α_M be the level of α at which $w_1(\alpha)$ is maximized.[20] If $\alpha \leq \alpha_M$ then the optimal export tax is zero, whereas if $\alpha_M < \alpha$ the optimal export tax is given implicitly by $\bar{\alpha}(\tau^*) = \alpha_M$. Imagine that the tax is set at its optimal level given α. Then it is clear that total income in country 1 is always weakly increasing in α. The following proposition states this formally:

Proposition 5 *Let α_M be the level of α at which $w_1(\alpha)$ is maximized and let τ^* be defined implicitly by $\bar{\alpha}(\tau^*) = \alpha_M$. For a small economy, the optimal export tax is zero if $\alpha \leq \alpha_M$ and given by τ^* for $\alpha > \alpha_M$. Under the optimal export tax policy, the extent of offshoring increases with α until α_M and remains constant thereafter, and total income in country 1 is weakly increasing in α.*

3 The Full Dynamic Model

The previous section analyzed the effects of offshoring in a static model where technology levels are fixed. This section explores how these results are affected when technology levels are endogenous in a full dynamic model. The "short run" of this dynamic model will be equivalent to the static model analyzed above.

Technological progress is modeled as in Eaton and Kortum (2001). Workers choose to do research or work in the productive sector. Recall that in the previous section we used L_i to denote the number of workers engaged in production (including producing intermediate services as part of offshoring operations for other countries). Letting L_{it}^F be the total labor force and R_{it} be the number of people working as researchers in country i at time t, then the full employment condition is $R_{it} + L_{it} = L_{it}^F$. I assume that L_{it}^F grows at a constant rate g_L that is common across countries. Also, I assume that the reallocation of workers between production and research is sluggish. This implies that L_{it} will be a state variable, and hence fixed in the short run (as in the previous section and subsection 3.1 below). To simplify the analysis, I assume that people are born as producers or researchers in proportion to the current population, and then at each point in time people get a chance to switch sectors at a constant and exogenous probability υ_P (υ_R) for those in production (research). For future purposes, note that in steady state people

[20] Just as in the previous subsection, the curve $\widehat{w}_1(\alpha, 1)$ is either downward sloping or behaves like an inverted U, so α_M is well defined.

will be happy to stay where they are born, so υ_P and υ_R will not be relevant for the steady state analysis in subsection 3.2. The size of υ_P and υ_R will affect the transition path after the economy is hit by a shock that changes the steady state allocation of people between research and production, as I consider in subsection 3.3.

A researcher in country i draws technologies or "ideas" at a Poisson rate ϕ_i. This parameter reflects research productivity and may vary across countries. Letting T_{it} be the total number of ideas that have been generated in country i up to time t, then $\dot{T}_{it} = \phi_i R_{it}$ and

$$T_{it} = \phi_i \int_0^t R_{is} ds \tag{22}$$

Each idea has two characteristics: first, the good $j \in [0, 1]$ to which it applies, and, second, its productivity q. Each of these characteristics is modeled as the realization of a random variable: j is distributed uniformly over the interval $[0, 1]$, while q is distributed Pareto with parameter $\theta > 1$. Formally, for $q \geq 1$ it is assumed that

$$H(q) = \Pr[q' \leq q] = 1 - q^{-\theta}$$

It can be shown that the distribution of $z_{it}(j)$ (which will be independent across goods and countries) has the Frèchet form, as in (2), with T_{it} given by (22).[21] In other words, the process for the arrival of ideas specified here leads to the Frèchet productivity frontier postulated in the static model, with the parameter θ in the Frèchet distribution coming from the parameter θ in the Pareto distribution of the quality of ideas, and the parameter T_i growing over time and being equal to the stock of ideas in country i at time t.

Researchers sell their ideas to firms that engage in Bertrand competition with other firms in the worldwide market for consumer goods. Consider the competition for a particular good. Only the firms holding the best idea for this good within some country have a chance of surviving the competition in the international market. Letting $z_{it}(j)$ be the maximum q over ideas that apply to good j in country i at time t, then the country that captures the worldwide market for good j at time t is given by $\arg\min_i \{c_i/z_{it}(j)\}$.

[21] To derive this result, note that the number of ideas k that have arrived for any good at time t is distributed Poisson with parameter T_{it}, so $\Pr(k' = k) = e^{-T_{it}} T_{it}^k / k!$. Hence, $\Pr(z'_{it} \leq z) = \sum_{k=0}^{\infty} \left(e^{-T_{it}} T_{it}^k / k!\right) H(z)^k$, which given $\sum_{k=0}^{\infty} x^k/k! = e^x$ implies $\Pr(z'_{it} \leq z) = F_{it}(z) = \exp[-T_{it} z^{-\theta}]$ for $z \geq 1$. Note that since $H(q)$ is defined for $q \geq 1$ then this distribution is defined for $z \geq 1$, whereas the distribution in (2) is defined for $z \geq 0$. But, as discussed in footnote 9 of Eaton and Kortum (2001), this difference gets arbitrarily small as the $T's$ get large, so one can safely ignore this difference.

The firm that captures the worldwide market for a good will make positive quasi-profits by charging a mark-up that depends on the second-least unit cost. Eaton and Kortum (2001) show that this mark-up is also distributed Pareto with parameter θ, or $m \sim H(m)$.[22] This is the distribution for the mark-up charged by firms from any country, and is constant through time. Letting Y_t denote worldwide income at time t, then (given the assumed preferences) this is also the worldwide expenditure on every good. Hence, if a firm charges a mark-up m, then its profits are $Y_t(1 - 1/m)$, and total worldwide profits are

$$Y_t \int_1^\infty (1 - 1/m) dH(m) = bY_t$$

where $b \equiv 1/(1+\theta)$. Since country i captures the worldwide market for a share $\pi_{it} = T_{it} {c_{it}}^{-\theta}/\Phi_t$ of goods, its income is $\pi_{it} Y_t$ and its total profits are a share b of that.

Letting d_{it} be the probability of a random idea from country i having a market at time t, then the expected profits of a random idea from country i are $bd_{it}Y_t$. Thus, the expected discounted value of a random idea from country i at time t is given by

$$V_{it} = b \int_t^\infty e^{-\rho(s-t)} \left(P_t/P_s\right) d_{is} Y_s ds$$

where ρ is the discount rate in consumers' intertemporal utility function, $u_t = \int_0^\infty e^{-\rho(s-t)} U_s ds$ and where P_t is the price index in (13).[23]

Eaton and Kortum (2001) show that $d_{it} = \pi_{it}/T_{it}$.[24] To understand this result, recall that π_{it} is the share of worldwide spending devoted to purchases from country i and also the probability that country i is the least-cost producer for a particular good. For an idea in country i to have a market it must be the best idea in country i *and* it must beat the competition from all other countries. The probability that a random idea is the best idea in country i is simply $1/T_{it}$ whereas the probability that the idea beats the foreign competition is π_{it}.

[22] To see this, recall from footnote 13 that the distribution of prices is $G_t(p) = e^{-\Phi_t p^{-\theta}}$. Thus, the probability that an entrepreneur with an idea of quality q in country i can charge a mark-up at least as high as m is $1 - G_t(mw_i/q)$. Hence, the probability that an idea of *unknown quality* from country i can charge a mark-up of at least m is $d_{it}(m) = \int_1^\infty \left[1 - G_t(mw_i/q)\right] dH(q) \approx (mw_i)^{-\theta}/\Phi_t$, where the approximation is arbitrarily acurate as the $T's$ get large (see Eaton and Kortum (2001), footnote 9). Conditional on selling at all, the distribution of the mark-up is then $\Pr[M \leq m \mid M \geq 1] = \frac{d_{it}(1) - d_{it}(m)}{d_{it}(1)} = H(m)$. This is independent of source and time, hence this is also the distribution of the mark-up across all firms in the world.

[23] The linearity assumption is made to simplify the analysis. The short-run and steady state results are clearly independent of this assumption. As to the transition dynamics in subsection 3.3, the same results would obtain under a more general specification of intertemporal preferences as long as countries 1 and 2 were able to access international capital markets. See footnote 32.

[24] Formally, note from footnote 22 that the probability that an idea of unknown quality from country i is competitive (i.e., $m \geq 1$) is simply $d_{it} \equiv d_{it}(1) = {w_{it}}^{-\theta}/\Phi_t = \pi_{it}/T_{it}$.

3.1 Short run analysis

At any point in time both L_{it} and T_{it} are fixed, just as in the static model. Thus, the only difference between the full dynamic model and the static model of the previous section regarding the short-run implications of offshoring is the market structure: in the static model there is perfect competition, whereas in the dynamic model technologies are owned by firms that engage in Bertrand competition. It turns out, however, that the existence of mark-ups and profits under Bertrand competition has no effect on any of the comparative statics results derived under perfect competition. This is because, as explained above, the profit share is common across countries.

To see this formally, note that trade balance now requires that exports of goods and offshoring services plus domestic sales be equal to wages plus imports of offshoring services *plus profits*. Since the value of exports and domestic sales of goods is $\pi_{it}Y_t$ and profits are a share b of this value, then we can equivalently state that trade balance requires $(1-b)\pi_{it}Y_t$ plus exports of offshoring services to equal wages paid to domestic and foreign workers (through offshoring). Thus, the trade balance conditions in the static model in equations (3), (6) and (7) are simply adjusted by multiplying Y_t by $1-b$. All the results for wages in (4), (8) and (10) are not affected, and the comparative statics results of the previous section remain valid.

3.2 Steady state analysis

In steady state R_{it}/L_{it}^F will be constant and equal to r_i, so the growth rate of the stock of ideas T_{it} will be $\dot{T}_{it}/T_{it} = g_L$ and its level will be

$$T_{it} = (\phi_i r_i / g_L) L_{it}^F \tag{23}$$

The choice of country N's labor as the numeraire implies that steady-state wages will be constant, $w_{it} = w_i$, so from (13) we can see that P_t falls at a rate equal to θg_L, so $P_s = P_t e^{-(g_L/\theta)(s-t)}$. In steady state π_{it} is also constant (and simply denoted by π_i). Moreover, equality between sales and expenditures, or trade balance, entails $\pi_i Y_s = Y_{is}$. These results imply that

$$V_{it} = b \int_t^\infty e^{-(\rho - g_L/\theta)(s-t)} \left(Y_{is}/T_{is} \right) ds \tag{24}$$

Consider country 1. Total expenditures are equal to wages paid, the cost of offshoring, and profits,

$$Y_{1t} = w_1 L_{1t} + w_2 \alpha L_{1t} + b Y_{1t} \tag{25}$$

Using $L_{1t} = (1 - r_1)L_{1t}^F$, and solving for Y_{1t} in (25), plugging the resulting expression for Y_{1t} into (24), using (23) and assuming $\theta\rho > g_L$ yields

$$V_1 = w_1 \left[1 - r_1 + \alpha(1 - r_1)\frac{w_2}{w_1}\right] \left(\frac{g_L}{\phi_1 r_1}\right) \frac{1}{\theta\rho - g_L} \tag{26}$$

Turning to country 2, we have

$$Y_{2t} = w_2 L_{2t} - w_2 \alpha(1 - r_1)\varphi L_{2t}^F + bY_{2t} \tag{27}$$

where $\varphi \equiv L_{1t}^F / L_{2t}^F$. A similar procedure as above yields

$$V_2 = w_2 \left[1 - r_2 - \alpha(1 - r_1)\varphi\right] \left(\frac{g_L}{\phi_2 r_2}\right) \frac{1}{\theta\rho - g_L} \tag{28}$$

For all the rest of countries ($i \neq 1, 2$) the corresponding expected value of an idea can be derived from the previous results by simply plugging in $\alpha = 0$, hence

$$V_i = w_i(1 - r_i) \left(\frac{g_L}{\phi_i r_i}\right) \frac{1}{\theta\rho - g_L} \tag{29}$$

In equilibrium the expected payoff to research must be equal to the wage in every country. This entails, $\phi_i V_i = w_i$. For countries $i \neq 1, 2$ this can be solved to yield

$$r_i = r \equiv g_L / \theta\rho \tag{30}$$

This implies that differences in ϕ_i do not affect the proportion of workers engaged in research.[25] For countries 1 and 2 the equilibrium conditions are (after some simplification)

$$r_1 / r = 1 + \alpha(1 - r_1) w_2 / w_1 \tag{31}$$

and

$$r_2 / r = 1 - \alpha(1 - r_1)\varphi \tag{32}$$

Given the wage ratio w_2/w_1, these two equations determine the research intensities in countries 1 and 2.

Using (23) and $L_{it} = (1 - r_i)L_{it}^F$ yields

$$\frac{T_{is}}{L_{is}} = \frac{\phi_i r_i}{g_L(1 - r_i)} \tag{33}$$

[25] Also note that r does not depend on country size or openness. This is because although larger markets lead to higher profits for successful innovators, stronger competition reduces the probability of being successful, and in this model these two effects exactly offset each other (see Eaton and Kortum, 2001).

Thus, from (4) and (30), we see that for $i \neq 1, 2$ the steady-state equilibrium wage is

$$w_i = (\phi_i/\phi_N)^b \tag{34}$$

This is the same as in Eaton and Kortum (2001) and implies that wages differ only because of differences in research productivity ϕ_i. Notice that with no offshoring (i.e., $\alpha = 0$) wages in countries 1 and 2 are also given by (34). Thus, the condition that $w_1 > w_2$ in steady state with no offshoring is that $\phi_1 > \phi_2$, which I assume henceforth. (This is the long-run counterpart to the condition $\eta > 1$ in the previous section.)

I now turn to the determination of steady state wages in countries 1 and 2 when $\alpha > 0$. As long as the resource constraint $\alpha(1-r_1)L_{1t}^F \leq L_{2t}^F$ is satisfied, steady stage wages in countries 1 and 2 are determined by equations (8), (9), (10), and (11) together with $L_{it} = (1-r_i)L_{it}^F$ and equations (31) and (32).[26] Consider first the determination of w_2. From (8) we can get

$$w_2 = \left(\frac{\phi_2 r_2}{\phi_N r/(1-r)} \frac{1}{1 - r_2 - \alpha(1-r_1)\varphi} \right)^b$$

Using (32) then

$$w_2 = (\phi_2/\phi_N)^b \tag{35}$$

which is the same as in the case of no offshoring. The reason for this result is that the decline in $\widetilde{L}_2$ generated by increased offshoring in the static model is now exactly compensated by a decline in T_2 caused by a decline in r_2 (see below).

Turning to w_1, recall from (10) that $((1-\beta)w_1 + \beta w_2) = (T_{Ns}/L_{Ns})^{-b} (T_{1s}/\widetilde{L}_{1s})^b$. With endogenous research the ratio $T_{1s}/\widetilde{L}_{1s}$ now depends on research efforts as well as the extent of offshoring. In fact, from (31), (33) and (11) we get

$$T_{1s}/\widetilde{L}_{1s} = \left(\frac{T_{Ns}/L_{Ns}}{\phi_N} \right) \left(\frac{\phi_1}{w_1} \right) ((1-\beta)w_1 + \beta w_2)$$

The equilibrium steady state wage in country 1 is then determined by

$$(1-\beta)w_1 + \beta w_2 = \left(\frac{\phi_1/\phi_N}{w_1} \right)^b ((1-\beta)w_1 + \beta w_2)^b \tag{36}$$

[26] For this steady state analysis it is no longer necessary to worry about the possibility of factor price equalization and the outsourcing constraint becoming non–binding. The reason is that - as will be shown below - $w_2(\alpha)$ is constant whereas $w_1(\alpha)$ is increasing. Thus, since $w_1(0) > w_2(0)$ by assumption, then $w_2(\alpha) > w_1(\alpha)$ for all $\alpha > 0$.

The LHS is the unit cost of the common input, whereas the RHS is proportional to $\left(T_{1s}/\widetilde{L}_{1s}\right)^b$ and captures the impact of offshoring and research on country $1's$ terms of trade. It is easy to show that given our assumption that $\phi_1 > \phi_2$ the level of w_1 determined by equation (36) is higher than w_2.[27] But this implies that offshoring lowers the unit cost of the common input (i.e., LHS is increasing in β). This represents the productivity effect discussed above. Turning to the RHS, note that an increase in β decreases this term, a reflection of the negative terms of trade effect discussed above. Which effect dominates? Since $b < 1$ then the productivity effect always dominates, so w_1 is increasing in β (or α).[28,29]

I have so far ignored the resource constraint in country 2 that the amount of labor used for exporting services to country 1 must be lower than its total labor force, namely $\alpha(1-r_1)L_{1t}^F \leq L_{2t}^F$. In fact, it can be shown from the results above that if $r > \frac{\phi_1}{\phi_1+\phi_2/\varphi}$ then the resource constraint is satisfied for all α. Otherwise, there exists a level of α, $\hat{\alpha}$, such that the resource constraint is binding for $\alpha > \hat{\alpha}$. In this case the equilibrium entails wage equalization, with all workers in country 2 employed in offshoring operations for country 2.

Again, the previous results relate to wages in countries 1 and 2 relative to some third country N. But it can be shown that the price index P will decline with offshoring, as the efficiency gains in the static model are only expanded in this dynamic model as offshoring allows a reallocation of labor towards the activity where they have comparative advantage (research in country 1 and production in country 2). The following proposition summarizes these results:

Proposition 6 *As long as the resource constraint in country* 2 *is non-binding, an increase in offshoring (i.e., an increase in α) increases the wage in country* 1, *whereas the wage in country* 2 *is not affected. The real wages w_i/P increase in all countries.*

What happens to r_1 and r_2 as α increases? Equation (31) implies

$$r_1 L_{1t}^F = r\left[L_{1t}^F + \alpha(1-r_1)L_{1t}^F w_2/w_1\right] \tag{37}$$

[27] To see this, note that this is equivalent to saying that the LHS of (36) is lower than the RHS of this same equation if w_1 were equal to w_2, or $w_2^{1-b} < (\phi_1/w_2\phi_N)^b$, but this is equivalent to $\phi_2 < \phi_1$.

[28] Formally, from (36) we get $[(1-\beta)w_1 + \beta w_2]^{1-b} = \left(\frac{\phi_1/\phi_N}{w_1}\right)^b$. The LHS is increasing in w_1 while the RHS is decreasing, and since $w_1 > w_2$ then an increase in β implies a decline in the LHS, and hence an increase in the equilibrium w_1.

[29] A natural question is whether country 1 would also want to offshore research to country 2. This would require $w_1/\phi_1 > w_2/\phi_2$. But it can be shown (35) and (36) that this is never satisfied for any $\beta \in [0,1]$.

The term $\alpha(1-r_1)L_{1t}^F w_2/w_1$ is the number of workers indirectly hired by country 1 from country 2 through offshoring, adjusting for the wage ratio. Thus, this equation says that the number of people doing research in country 1 is a proportion r of the total labor force in country 1 including the workers indirectly working in country 1 through offshoring (adjusting for wages). Thus, r_1 is necessarily higher with offshoring than without offshoring. Moreover, it can be shown that $\alpha(1-r_1)w_2/w_1$ is increasing in α, so it is also the case that as offshoring increases the research intensity r_1 in country 1 increases.

Turning to country 2, rearranging equation (32) we get

$$r_2 L_{2t}^F = r\left(L_{2t}^F - \alpha(1-r_1)L_{1t}^F\right)$$

Analogously to the result for country 1, this expression says that the number of people doing research in country 2 is a proportion r of its total labor force excluding the workers producing services for export through offshoring operations. This implies that $r_2 < r$ as long as $\alpha > 0$. More generally, it can be shown that r_2 is decreasing in α. Formally,

Proposition 7 *The research intensity r_1 in country 1 increases while the research intensity r_2 in country 2 decreases as α increases.*

3.3 Transition dynamics

Imagine an unexpected increase in fragmentation at time t_0. We know from the previous section that if the increase in α is large enough, it would lead to a decline in the real wage in country 1 at time t_0. As time goes by, however, workers in country 1 would switch from production to research, increasing T_{1t}/L_{1t} and improving country 1's terms of trade. In the new steady state, the real wage in country 1 would be higher than it was before the increase in α. There are then two opposite effects of a large (and unexpected) increase in fragmentation: a negative short-run effect and a positive long-run effect. What is the net effect for utility at time t_0?

To answer this question I now analyze the transition dynamics after a positive and unexpected shock to β to show that if the speed with which people can switch between production and research (i.e., υ_R) is sufficiently high then the net effect is positive. I restrict the analysis to the limiting case in which the region composed of countries 1 and 2 is vanishingly small (as in Section 2.6). This assumption implies that the rest of the world (i.e., countries $i \neq 1, 2$) is not affected by anything that happens in countries 1 and 2, and that P_t continues to fall at rate g_L/θ even after a shock to β. This implies that the expression for V_{it} in equation (24) remains valid during the transition for countries 1 and 2. Differentiating this expression yields the no-arbitrage condition

$$\dot{V}_{it}/V_{it} = \rho(1-r) - b\left(Y_{is}/w_{it}T_{is}\right)\left(w_{it}/V_{it}\right)$$

Substituting for Y_{it} from (25) and (27), and then for L_{it}/T_{it} from equations (8) – (11) yields

$$\dot{V}_{1t}/V_{1t} = \rho(1-r) - \left(\delta^{1/b}\phi_1/\theta\right)\left(\left(1-\beta\right)w_{1t} + \beta w_{2t}\right)^{-\theta}\left(\frac{1}{\phi_1 V_{1t}}\right) \tag{38}$$

and

$$\dot{V}_{2t}/V_{2t} = \rho(1-r) - \left(\delta^{1/b}\phi_2/\theta\right)\left(1/w_{2t}\right)^{1/b}\left(\frac{w_{2t}}{\phi_2 V_{2t}}\right) \tag{39}$$

Letting $x_{it} \equiv T_{it}/L^F_{it}$ and recalling that $L_{it} = (1-r_{it})L^F_{it}$, then from the short-run equilibrium conditions we can get the following expressions for the unit cost c_{1t} in country 1 and the wage w_{2t} in country 2,

$$c_{1t} = (1-\beta)w_{1t} + \beta w_{2t} = \delta\left(\frac{x_{1t}}{(1+\alpha)(1-r_{1t})}\right)^b \tag{40}$$

and

$$w_{2t} = \delta\left(\frac{x_{2t}}{1-r_{2t}-\alpha\varphi(1-r_{1t})}\right)^b \tag{41}$$

Moreover, simple differentiation reveals that x_{it} evolves according to

$$\dot{x}_{it}/x_{it} = \phi_i(1-r_{it})/x_{it} - g_L \tag{42}$$

Finally, the laws of motion of r_{it} are governed by whether $\phi_i V_{it} \lesseqgtr w_{it}$. If $\phi_i V_{it} > w_{it}$ then all those workers in production who get a chance to switch will do so, and

$$\dot{r}_{it}/r_{it} = \frac{g_L R_{it} + \upsilon_P L_{it}}{R_{it}} - g_L = \upsilon_P(1-r_{it})/r_{it}$$

On the other hand, if $\phi_i V_{it} < w_{it}$ then researchers who get a chance to leave research will do so, and

$$\dot{r}_{it}/r_{it} = \frac{g_L R_{it} - \upsilon_R R_{it}}{R_{it}} - g_L = -\upsilon_R$$

Of course, if $\phi_i V_{it} = w_{it}$ then anything in between these values for $\dot{r}_{it}/r_{it}$ would be compatible with equilibrium. These results are summarized as follows:

$$\dot{r}_{it}/r_{it}\begin{cases} = \upsilon_P(1-r_{1t})/r_{1t} & if \quad \phi_i V_{it} > w_{it} \\ \in [-\upsilon_R, \upsilon_P(1-r_{1t})/r_{1t}] & if \quad \phi_i V_{it} = w_{it} \\ = -\upsilon_R & if \quad \phi_i V_{it} < w_{it} \end{cases} \tag{43}$$

An equilibrium adjustment path after an unexpected shock to α in countries 1 and 2 is a path for $w_{1t}, w_{2t}, r_{1t}, r_{2t}, V_{1t}, V_{2t}, x_{1t}, x_{2t}$ that satisfies (38), (39), (40), (41), (42) and (43) and converges to the steady state in which $w_{it} = w_i = \phi_i V_i$, $\dot{r}_{it} = \dot{x}_{it} = 0$ and $r_{it} = r_i$, where w_i and r_i are as determined in the previous section.

Inspection of equations (38) – (41) reveals that for values of x_{1t} and x_{2t} that are not too far below their steady state, we can always find values for r_{1t} and r_{2t} such that V_{it} and w_{it} are at their steady state, with $\phi_i V_i = w_i$ for $i = 1, 2$.[30] To understand this, note that – other things equal – a low value for the technology parameter T_{1t} implies a low x_{1t} and low wage w_{1t}. But this would not occur if the low x_{1t} is accompanied by a high research intensity r_{1t}, because this would nullify the negative effect of a low T_{1t} on the ratio T_{1t}/L_{1t}, which determines wages in the short run. More generally, if r_{1t} and r_{2t} were free values (and not predetermined as they are in this model) then they could always accommodate (small) temporary deviations of x_{1t} and x_{2t} from their steady state and thereby keep wages at their steady state values. In fact, this is the key to understand the main result of this section, namely that if frictions in the adjustment of people between research and production were very small, then an increase in α would necessarily benefit country 1, because the (possible) short run loses identified in Sections 2 and 3.1 would be rapidly reversed thanks first to an decrease in L_{1t} (increase in r_{1t}) and then to the increase in T_{1t} brought about by the increased research efforts. Of course, for country 2 this works in exactly the opposite direction: the short-run gains identified above vanish rapidly as the research intensity there declines.

To simplify the analysis I make two assumptions on the parameters υ_P and υ_R which govern the speed of adjustment: first, I assume that they are sufficiently large that r_{it} can adjust in the speed required for the RHS of (40) and (41) to remain constant given that x_{it} is moving according to (42). This implies that in the last stage of the adjustment process wages in countries 1 and 2 will be at their steady states values, with r_{1t} and r_{2t} adjusting accordingly (as explained in the previous paragraph). Second, I assume that exit from the research sector is easier than entry into that sector. Formally, this entails assuming that υ_R is large relative to υ_P.

Under the previous assumptions, the equilibrium adjustment after an unexpected increase in β has three stages. In the first stage $\phi_1 V_{1t} > w_{1t}$ and $\phi_2 V_{2t} < w_{2t}$, so there is maximal entry

[30] Simple algebra reveals that plugging $\phi_i V_i = w_i$ and the steady state levels of w_1 and w_2 calculated in the previous section into equations (38) and (39) yields $\dot{V}_{it} = 0$.

into research in country 1 and maximal exit from research in country 2. This stage ends when w_{2t} reaches w_2.[31] In the second stage, $\phi_1 V_{1t} > w_{1t}$ and $\phi_2 V_{2t} = w_{2t} = w_2$, so maximal entry into research continues in country 1 while the constraint on exit from research in country 2 is no longer binding. This stage ends when w_{1t} reaches w_1. The third stage is as explained above: it entails $\phi_i V_{it} = w_{it} = w_i$ for $i = 1, 2$, so that wages in countries 1 and 2 are at their steady state values, and r_{1t} and r_{2t} adjust in response to the continued movement of x_{1t} and x_{2t} towards their steady state values.

In these conditions, it is clear that if υ_P and υ_R are very high, then the first two stages of the adjustment process will be very short, and the adjustment will entail wages being at their new steady state values most of the time. Since an increase in α brings about an increase in the steady state wage of country 1, then this country must benefit from such a shock even if it experiences some losses in the short run. Country 2 also experiences a positive welfare effect, because wages are momentarily higher there after the shock, although they rapidly converge to the same level as before the shock.[32]

The analysis so far has focused on the effects of an unexpected shock in fragmentation. If the shock is anticipated, then the previous analysis suggests that the effects should be even more positive for country 1, as it can start reallocating its labor from production to research even before the shock and in that way lessen the terms of trade deterioration. The opposite occurs for country 2, where the temporary increase in its terms of trade may vanish if the shock is anticipated. Although there are no clear statistics that one could use to measure fragmentation, it is reasonable to assume that it is a gradual and somewhat anticipated process rather than a sudden shock. In this case, the analysis suggests that as long as reallocation between production and research is not too sluggish, the net effect should be positive for rich countries and small (but positive) for poor countries.

[31] This necessarily happens before c_{1t} reaches its steady state thanks to the assumption that υ_R is large relative to υ_P.

[32] These results are valid under the assumption that intertemporal preferences are linear. When the intertemporal elasticity of substitution is low and there are no international capital flows, my conjecture is that even with pefect mobility of people between research and the production sector, a large unexepected increase in fragmentation would decrease utility, as people would not be willing to decrease their consumption to allow for a large increase in research efforts to accelerate the transition.

4 Offshoring and immigration

This kind of analysis can also be used to shed light on the effects of migration, which in turn may allow us to gain some intuition about the effects of offshoring just described.[33] Consider again countries 1 and 2, with $w_1 > w_2$ thanks to $\eta > 1$ and no offshoring, and imagine that a restricted share ι of people from country 2 can costlessly migrate to country 1. As ι increases, there is a short-run (with constant $T's$) decline in η, which leads to a decline in w_1 and an increase in w_2. This captures the idea put forth by Davis and Weinstein (2002) that immigration leads to losses to the host country due to a deterioration of its terms of trade.

But, again, this is only in the short run: in the Eaton and Kortum (2001) model with endogenous technology levels, immigration leads to an expansion of research in country 1 and a contraction of research in country 2 in such a way that (in steady state) T_1/L_1 and T_2/L_2 remain constant because $T_i/L_i = \phi_i r/(1-r)g_L$ doesn't depend on L_i^F. Wages w_1 and w_2 are not affected, and the only effect is a decline in prices thanks to the increased efficiency generated by migration towards countries with higher research productivities (i.e., the long-run world efficiency effect). Thus, in the long run all countries gain equally, and the main beneficiaries of migration are the migrants themselves, who experience an increase in wages from w_2 to w_1.[34]

Let's compare these results of migration with those of offshoring in the long run. As shown in the subsection 3.3, in steady state offshoring does not affect wages in country 2, but wages in country 1 experience an increase. Thus, focusing on the long run implications, offshoring is better for country 1 than immigration. The reason for this is that with migration the receiving country ends up paying the high country 1 wage to immigrants, whereas with offshoring country 1 firms pay the low country 2 wage to workers who remain in country 2. Thus, whereas with migration the main beneficiaries are the migrants, with offshoring the main beneficiaries are workers in country 1, whose wage can now increase thanks to the efficiency gains from offshoring.[35]

[33] Baldwin and Robert-Nicaud (2007) relate the effects of offshoring to what they call "shadow migration."

[34] The result that the effect of immigration on wages is more beneficial in the long run than in the short run can also be obtained in the Hecksher-Ohlin model as well as in models that allow for capital accumulation (see Klein and Ventura, 2007, and Ottaviano and Peri, 2006).

[35] The point that offshoring leads to larger gains than migration because of the difference in the wage paid to the extra labor made available is discussed in Jones (2005).

5 Extensions

I have assumed thus far that increasing offshoring is made possible by the raising capability to fragment the production process and thereby arrange to have more intermediate services performed abroad. Alternatively, as in Kohler (2004) and Grossman and Rossi-Hansberg (2006), the expansion of offshoring could be seen as the consequence of a decline in the cost of importing these services. In fact, the model presented above could be interpreted in this light by assuming that a share β of services can be offshored at no cost, whereas the rest entail an infinite cost of offshoring. A question is whether the results derived under this set-up generalize to other ways of modeling such costs. In this section I present an extension of the model to explore this question, and then I extend the analysis to offshoring among three countries.

I assume that importing labor service k entails an iceberg cost $\zeta(k) \geq 1$. The cost of labor service k offshored to country 2 is then $\zeta(k)w_2$. Country 1 will procure service k from country 2 if $\zeta(k)w_2 < w_1$ and may do so also if $\zeta(k)w_2 = w_1$. Following the stochastic approach of Eaton and Kortum (2002), I assume that $\zeta(k)$ (for $k \in [0,1]$) is independently drawn from an exponential distribution with parameter λ and a mass point at 1. Formally, for any k and $\zeta_0 \geq 1$ we have $\Pr(\zeta \leq \zeta_0) = F(\zeta_0, \lambda) \equiv 1 - \exp(-\lambda\zeta_0)$, where $\lambda > 0$. Note that a *higher* λ implies *lower* average offshoring costs and that as $\lambda \to 0$ then offshoring necessarily goes to zero. This stochastic approach will be useful below when I extend the analysis to more than two countries.

The goal is to understand the short-run and long-run effects of an increase in λ. Consider first the short run. Analogously to the results of Section 2, when λ is sufficiently high there is full offshoring, with wages in both country 1 and 2 equal to $w_m = \delta(T_m/L_m)^b$ (recall that $T_m = T_1 + T_2$ and $L_m = L_1 + L_2$). The critical value for λ, λ_m, is implicitly defined by $F(1, \lambda_m) = \bar{\beta}$, where $\bar{\beta} \equiv \bar{\alpha}/(1+\bar{\alpha})$ ($\bar{\alpha}$ was defined in Section 2). In other words, λ needs to be high enough that the share of services for which there are no transportation costs ($\zeta(k) = 1$) is at least as high as the share of services that must be offshored for there to be full offshoring ($\bar{\beta}$). It is clear that the wage under full offshoring is higher than the wage that would prevail under no offshoring in country 2, but lower than the corresponding wage in country 1. Formally, letting $w_i(0) \equiv \lim_{\lambda\to 0} w_i(\lambda)$, we have $w_1(0) > w_m > w_2(0)$.

If $\lambda < \lambda_m$, the equilibrium entails $w_1 > w_2$. I now characterize this equilibrium. Let $C_1(k) \equiv \min\{w_1, \zeta(k)w_2\}$ represent the cost of service k for country 1. (Since all services are homogenous, from now on I supress the index k unless necessary to avoid confusion.) The cost C_1 can never be lower than w_2 and is lower than c_1 for $c_1 \in [w_2, w_1[$ if $\zeta w_2 < c_1$. This implies that for $c_1 \in [w_2, w_1[$, $\Pr(C_1 \leq c_1) = \Pr(\zeta w_2 \leq c_1) = F(c_1/w_2, \lambda) = F(c_1, \lambda/w_2)$, so the distribution of the variable C_1 is given by

$$\Pr(C_1 \leq c_1) = \begin{cases} 0 & if \quad c_1 < w_2 \\ F(c_1, \lambda/w_2) & if \quad c_1 \in [w_2, w_1[\\ 1 & if \quad c_1 \geq w_1 \end{cases}$$

The unit cost of the common input in country 1 is simply the expectation of C_1 for this distribution. I will use $c_1(\mathbf{w})$ (with $\mathbf{w} \equiv (w_1, w_2)$) to denote this unit cost as a function of wages. This is simply

$$c_1(\mathbf{w}) = w_2 F(1, \lambda) + \int_{w_2}^{w_1} x dF(x, \lambda/w_2) dx + w_1(1 - F(w_1, \lambda/w_2))$$

On the other hand, the unit cost of the common input in country 2 is simply w_2. Also, note that if $w_1 > w_2$ then the share of services offshored to country 2 is $s(\mathbf{w}) = F(w_1, \lambda/w_2)$. Thus, following the same analysis as in Section 2, the equilibrium conditions for $\lambda < \lambda_m$ are $c_1(\mathbf{w}) = \delta(T_1/\widetilde{L}_1)^b$ and $w_2 = \delta(T_2/\widetilde{L}_2)^b$ (as in equations (10) and (8)) but with

$$\widetilde{L}_1 = L_1/(1 - s(\mathbf{w}))$$

and

$$\widetilde{L}_2 = L_2 - \Gamma(\mathbf{w}) s(\mathbf{w}) L_1/(1 - s(\mathbf{w}))$$

where $\Gamma(\mathbf{w})$ is the average transportation cost ζ associated with the offshored services and is given by

$$\Gamma(\mathbf{w}) \equiv \frac{F(1, \lambda) + (1/w_2) \int_{w_2}^{w_1} x dF(x, \lambda/w_2) dx}{s(\mathbf{w})}$$

Note that in these definitions of $\widetilde{L}_1$ and $\widetilde{L}_2$, the term $s(\mathbf{w})$ substitutes for α in equations (11) and (9).

Several of the results in Section 2 remain essentially unchanged in this case. First, there is full offshoring as long as $\lambda \geq \lambda_m$. Second, $w_2(0) < w_m < w_1(0)$, so that the rich (poor) country attains a lower (higher) relative wage under full offshoring than with no fragmentation. This result clearly also extends to real wages, so we can say that the rich (poor) country is worse (better) off under full offshoring than with no fragmentation. Third, as shown in the Appendix, the sign of the partial derivative of w_1 with respect to λ converges to $(w_1(0)/w_2(0))^2 (1 - 2b) - 1$

as $\lambda \to 0$. This implies that, since $b < 1/2$ (because $\theta > 1$), the curve $w_1(\lambda)$ will be upward sloping for low λ if $w_1(0)/w_2(0) = \eta^b$ is sufficiently high.

Turning to the long run equilibrium, the result that w_2 is fixed at $(\phi_2/\phi_N)^b$ remains valid: offshoring costs affect the quantity of labor used in country 2 to export services to country 1, but the steady state wage in country 2 is unaffected by this. As for country 1, the wage is determined as in (36), but with $c_1(\mathbf{w})$ instead of $(1-\beta)\,w_1 + \beta w_2$,

$$c_1(\mathbf{w})^{1-b} = \left(\frac{\phi_1/\phi_N}{w_1}\right)^b$$

Simple derivation (see Appendix) reveals that the curve $c_1(\mathbf{w})$ decreases with λ, hence the steady state w_1 is increasing with λ. This implies that, similar to the results in Section 3, in the long run the rich country gains from a decline in the cost of offshoring whereas the poor country gains but just from the world efficiency effect.

Finally, I extend the model to allow for offshoring among countries 1, 2, and 3, where country 1 is the rich country, and country 2 is the poor country. Formally, I assume that $T_1/L_1 > T_3/L_3 > T_2/L_2$ and $\phi_1 > \phi_3 > \phi_2$. The (iceberg) costs of offshoring service k to country $i = 1, 2, 3$ are $\zeta_i(k)$, which are independently drawn from the same distribution, assumed to be exponential with a mass point at 1 and parameter λ. The Appendix contains the full characterization of equilibrium in the short run and the long run for this case. Here I present a short discussion and the results of a numerical simulation.

Just as in the two country case, there is full offshoring among the three countries if λ is higher than some critical level. Under full offshoring $w_1(0) > w_m > w_2(0)$.[36] The middle-income country is better off under full offshoring than with no offshoring if $T_m/L_m > T_3/L_3$. To gain some additional understanding about the behaviour of wages in relation to λ, I simulated the equilibrium for $\theta = 8$ (the central value of θ in Eaton and Kortum, 2002) with $L_1 = L_2 = L_3$ and $T_1 = 1$, $T_2 = 0.04$, and $T = 0.2$. Figure 1 shows the resulting wages as λ goes from 0 to the value of λ under which there is full offshoring. As in Section 2, w_2 is always increasing while w_1 behaves like an inverted U. (This last result no longer holds when T_1/L_1 is close to T_2/L_2, in which case w_1 is decreasing in λ.) The wages of countries 2 and 3 converge for a level of λ near 0.34. For $\lambda \leq 0.34$, w_3 behaves like an inverted U, but it is increasing thereafter. This behavior of w_3 with respect to λ is not a general result: under alternative parameters, $w_3(\lambda)$ can behave more like $w_1(\lambda)$ (if T_3/L_3 is close to T_1/L_1) or like $w_2(\lambda)$ (if T_3/L_3 is close to T_2/L_2).

[36] As above, we have $w_m = \delta(T_m/L_m)^b$, but now $T_m = T_1 + T_2 + T_3$ and $L_m = L_1 + L_2 + L_3$.

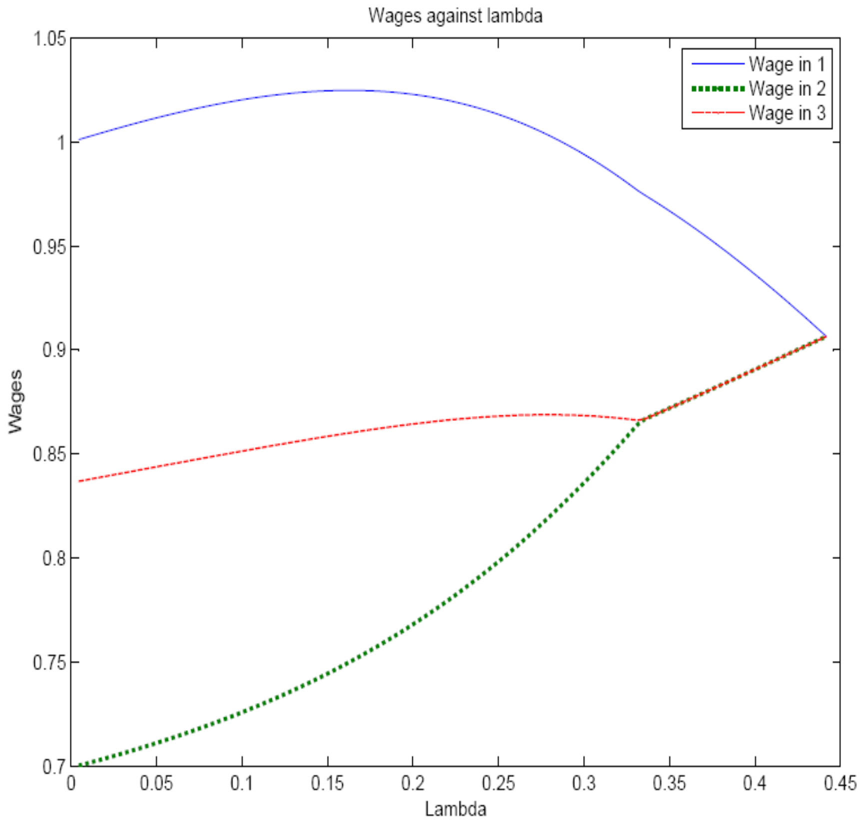

Figure 1: This simulation uses $\delta = 1$, $\theta = 8$, $L_1 = L_2 = L_3 = 1$ and $T_1 = 1$, $T_2 = 0.04$, and $T_3 = 0.2$.

Turning to the long run analysis, it is easy to show that the steady state wage in country 2 is constant, whereas the steady state wage in country 3 is increasing in λ. The reasoning here is exactly the same as for the result that the steady state wage in country 1 is increasing in λ in the two-country case. As to the wage in country 1, there are two opposite effects from the increase in λ: on the one hand, this increases w_3, which has a negative effect on country 1, but on the other hand, there is a direct and positive effect on w_1. In the numerical simulation for $\phi_1/\phi_N = 1$, $\phi_2/\phi_N = 0.04$, and $\phi_3/\phi_N = 0.2$, illustrated in Figure 2, w_1 is increasing in λ. This result holds also in all numerical simulations that I have performed.

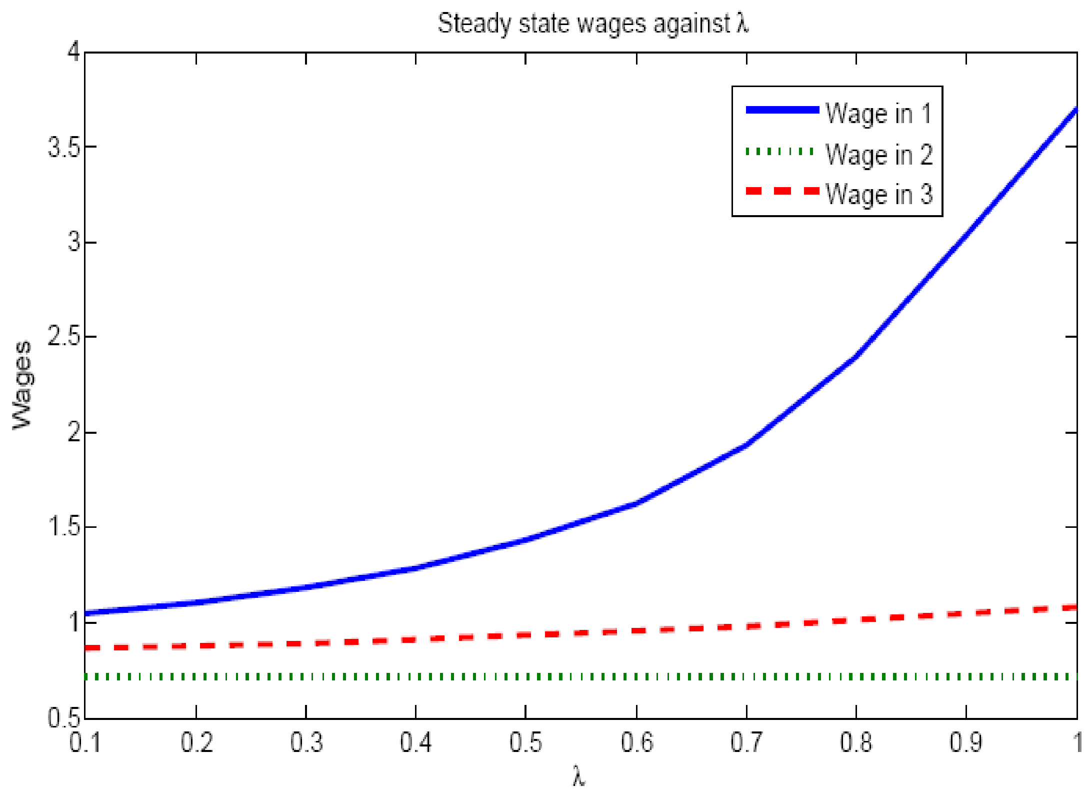

Figure 2: This simulation uses $\theta = 8$, $\phi_1/\phi_N = 1$, $\phi_2/\phi_N = 0.04$, and $\phi_3/\phi_N = 0.2$.

6 Conclusion

Over the last years there has been much discussion about the possible effects of increased offshoring on rich countries. Those with a favorable view have focused on the productivity gains associated with increasing trade in services, while the critics have emphasized the negative implications for rich-country wages of what some have called "the death of distance" (Cairncross, 1997). In this paper I have presented a model that captures both of these effects. A main result is that a large and unexpected increase in fragmentation necessarily harms the rich country and benefits the poor country in the *short run*. But this also triggers a reallocation of resources towards research in the rich country and towards production in the poor country. Such reallocations weaken the terms of trade effects of offshoring and imply that the *long run* effect of increased fragmentation is always positive for the rich country. In contrast, the poor country derives no direct gains from offshoring and benefits only from the improvement in world efficiency that arises from increased trade, just as third countries that do not participate at all in offshoring.

The implications of offshoring for rich countries turn out to be closely related to those of immigration. In both cases there is a short-run decline in the terms of trade and reallocation of resources from production to research that weakens this effect in the long run. But there is a key difference: whereas workers that export services through offshoring are paid the wages prevailing in poor countries, migrants earn rich-country wages. As a result, rich countries stand to gain more from increased fragmentation and offshoring than from immigration.

Coming back to the effects of offshoring, the presence of opposite short and long run effects implies that the net effect of increased fragmentation for intertemporal utility in the rich country could be positive or negative. This depends on the speed with which resources can be reallocated across production and research: if this is sufficiently fast, then the long run effects dominate and the rich country gains from offshoring. More generally, if there is a gradual process of increasing fragmentation, the rich country gains as long as the intersectoral reallocation of resources is not too sluggish relative to the pace at which fragmentation is increasing.

Blinder (2007) has expressed concerns that the future increase of offshoring in services will generate large costs for the U.S. during a prolonged transition. One way to interpret this concern in light of the model presented here is that the process of deepening fragmentation will be too fast in relation to the country's ability to reallocate resources from production to research. The model suggests one way to prevent these transitory costs: by imposing an optimal tariff or export tax the rich country would eliminate the possibility that increased fragmentation and offshoring harms the rich country even in the short run. Of course, such a policy presents many potential dangers, so a better (but more difficult) approach would be the implementation of education and other policies to facilitate the reallocation of people from production to research, or from simple tradable tasks to the development of "new processes, new products, and entirely new industries" (Blinder, 2007, p. 28).

A final issue worth discussing concerns the result that fragmentation does not directly benefit the poor countries engaged in offshoring in the long run. This seems inconsistent with the impression of large gains from increasing service exports by some poor countries, particularly India. It could be argued that, as in the model, these are merely *short run* gains that will dissipate in the long run, but this seems unlikely. One explanation for long run gains is the existence of knowledge spillovers triggered by offshoring. The modeling of such spillovers and the estimation of their quantitative importance is certainly an important issue for future research.[37] Alternatively, India's current prosperity could be seen as resulting from its innovative provision of services that permit firms in rich countries to fragment and offshore part of their production process. According to this view, the increase in β that causes increased offhoring in the model above is actually the result of innovations by Indian firms. Such firms would then capture some of the productivity gains that in this paper have been assumed to go entirely to rich countries. This too seems a worthwile topic for further exploration.

[37] See Trefler (2005) for a broad discussion of this issue in the context of service offshoring.

Appendix

This Appendix presents the proofs of Propositions 1, 2, 3, and 4.

Proof of Proposition 1

We want to show that

$$\left(\frac{T_m}{L_m}\right)^b \left(T_m w_m^{-\theta} + \Phi_{-m}\right)^{1/\theta} \geq \left(\frac{T_1}{L_1}\right)^b \left(T_1 w_1^{-\theta} + T_2 w_2^{-\theta} + \Phi_{-m}\right)^{1/\theta}$$

Since $\left(\frac{T_1}{L_1}\right)^b > \left(\frac{T_m}{L_m}\right)^b$, it is enough to prove the inequality for $\Phi_{-m} = 0$. Thus, using $w_i = \delta \left(T_i/L_i\right)^b$ for $i = 1, 2, m$ then we need to show that

$$\begin{aligned} \left(\frac{T_m}{L_m}\right)^b \left(T_m \delta^{-\theta} \left(\frac{L_m}{T_m}\right)^{b\theta}\right)^{1/\theta} &= \frac{\left(T_m\right)^{1/\theta}}{\delta} \\ &\geq \left(\frac{T_1}{L_1}\right)^b \left(T_1 \delta^{-\theta} \left(\frac{L_1}{T_1}\right)^{b\theta} + T_2 \delta^{-\theta} \left(\frac{L_2}{T_2}\right)^{b\theta}\right)^{1/\theta} \\ &= \left(\frac{T_1}{L_1}\right)^b \frac{\left(T_1 \left(\frac{L_1}{T_1}\right)^{b\theta} + T_2 \left(\frac{L_2}{T_2}\right)^{b\theta}\right)^{1/\theta}}{\delta} \end{aligned}$$

We have

$$\left(\frac{T_1}{L_1}\right)^b \frac{\left(T_1 \left(\frac{L_1}{T_1}\right)^{b\theta} + T_2 \left(\frac{L_2}{T_2}\right)^{b\theta}\right)^{1/\theta}}{\delta} \geq \left(\frac{T_1}{L_1}\right)^b \frac{\left(T_1 \left(\frac{L_1}{T_1}\right)^{b\theta} + T_2 \left(\frac{L_1}{T_1}\right)^{b\theta}\right)^{1/\theta}}{\delta} = \frac{\left(T_m\right)^{1/\theta}}{\delta}.$$

Q.E.D.

Proof of Proposition 2

We have

$$\begin{aligned} w_1(\alpha) &= (1+\alpha)\tilde{w}_1(\alpha) - \alpha w_2(\alpha) \\ &= \delta \left(\frac{T_1}{L_1}\right)^b (1+\alpha)^{1-b} - \delta \left(T_2\right)^b \frac{\alpha}{\left(L_2 - \alpha L_1\right)^b} \end{aligned}$$

This implies that

$$w_1'(\alpha) = \delta \left(\frac{T_1}{L_1}\right)^b \frac{(1-b)}{(1+\alpha)^b} - \frac{\delta \left(T_2\right)^b}{\left(L_2 - \alpha L_1\right)^b} \left(1 + \frac{\alpha b L_1}{L_2 - \alpha L_1}\right)$$

Obviously $\delta\left(\frac{T_1}{L_1}\right)^b\frac{(1-b)}{(1+\alpha)^b}$ is decreasing, while $\frac{\delta(T_2)^b}{(L_2-\alpha L_1)^b}\left(1+\frac{\alpha b L_1}{L_2-\alpha L_1}\right)$ is increasing in α. To determine the sign of $w_1'(\alpha)$ on $[0,\bar{\alpha})$ we should then compare $w_1'(0)$ and $w_1'(\bar{\alpha})$ with zero. Focusing first on $w_1'(\bar{\alpha})$, using the definition of $\bar{\alpha}$, we get

$$w_1'(\bar{\alpha}) = \frac{bT_2^b\delta\left(1+\eta L_1/L_2\right)^b}{\left(L_2+L_1\right)^b}\left(-1-\frac{(\eta-1)\,L_1}{L_2+L_1}\right)<0$$

Turning to $w_1'(0)$, note that

$$\begin{aligned} w_1'(0) &= \delta\left(\frac{T_1}{L_1}\right)^b(1-b)-\frac{\delta T_2^b}{L_2^b} \\ &= \delta\left(\frac{T_2}{L_2}\right)^b\left(\eta^b\,(1-b)-1\right)>0 \iff \eta>(1-b)^{-1/b} \end{aligned}$$

Thus, if $\eta\leq(1-b)^{-1/b}$, then $w_1(\alpha)$ is always decreasing on $[0,\bar{\alpha})$. If $\eta>(1-b)^{-1/b}$, then $w_1(\alpha)$ is shaped like an inverted U on $[0,\bar{\alpha})$. **Q.E.D.**

Proof of Proposition 3

Recall that $\Phi\equiv\sum_k T_k c_k^{-\theta}$. Thus, it is useful to use

$$\Phi = T_1c_1^{-\theta}+T_2w_2^{-\theta}+\Phi_{-m}$$

where Φ_{-m} is not affected by α. We know that $c_1=\delta\left(T_1/\tilde{L}_1\right)^b$ and $w_2=\delta\left(T_2/\tilde{L}_2\right)^b$, so

$$T_1c_1^{-\theta}+T_2w_2^{-\theta}=\delta^{-\theta}\left(T_1^bL_1^{\theta b}(1+\alpha)^{\theta b}+T_2^b(L_2-\alpha L_1)^{\theta b}\right)$$

This implies that

$$\left(T_1c_1^{-\theta}+T_2w_2^{-\theta}\right)'_\alpha=\delta^{-\theta}\theta bL_1\left[\left(T_1/L_1\right)^b(1+\alpha)^{-b}-T_2^b(L_2-\alpha L_1)^{-b}\right]$$

We need to compare $f(\alpha)\equiv\left(\frac{T_1}{L_1}\right)^b(1+\alpha)^{-b}-T_2^b(L_2-\alpha L_1)^{-b}$ with zero on $[0,\bar{\alpha})$. Obviously, $f(0)=\left(\frac{T_1}{L_1}\right)^b-\left(\frac{T_2}{L_2}\right)^b>0$, while simple algebra reveals that $f(\bar{\alpha})=0$. Since $f'(\alpha)<0$, then $f(\bar{\alpha})=0$ implies that $f(\alpha)>0$ for any $\alpha\in[0,\bar{\alpha})$. This means that $\left(T_1c_1^{-\theta}+T_2w_2^{-\theta}\right)'_\alpha>0$, or $\Phi'_\alpha>0$. But given $P=\gamma\Phi^{-1/\theta}$ then this implies that $P'_\alpha<0$. **Q.E.D.**

Proof of Proposition 4

We know that the sign of $\left(\frac{w_1}{P}\right)'_\alpha$ is the same as the sign of $\frac{w_1'}{w_1} - \frac{P'}{P}$. But simple differentiation and simplification reveals that

$$\begin{aligned}\frac{w_1'}{w_1} &= G(x,\alpha) \equiv \frac{x(1-b)\left(\frac{f(\alpha)}{1+\alpha}\right)^b - \left(1+\frac{\alpha b L_1/L_2}{f(\alpha)}\right)}{x(1+\alpha)\left(\frac{f(\alpha)}{1+\alpha}\right)^b - \alpha} \\ \frac{P'}{P} &= F(x,\alpha) \equiv -b\frac{x\frac{1}{(1+\alpha)^b} - \frac{1}{(f(\alpha))^b}}{x(1+\alpha)^{\theta b} + \frac{L_2}{L_1}(f(\alpha))^{\theta b} + \frac{\delta^\theta \Phi^*}{L_1}}\end{aligned}$$

where $x \equiv \eta^b$, $f(\alpha) = 1-\alpha L_1/L_2$. Let $x_F(\alpha)$ and $x_G(\alpha)$ be defined implicitly by $F(x,\alpha) = 0$ and $G(x,\alpha) = 0$, respectively. The following lemma, whose proof is simple and therefore omitted, summarizes a number of properties of these functions:

Lemma 2 *$F(x,\alpha)$ is decreasing in x, $G(x,\alpha)$ is increasing in x,*

$$x_F(\alpha) = \left(\frac{1+\alpha}{f(\alpha)}\right)^b > 1, \quad \textit{and } x_G(\alpha) = \frac{\left(1+\frac{\alpha b L_1/L_2}{f(\alpha)}\right)}{(1-b)\left(\frac{f(\alpha)}{1+\alpha}\right)^b} > 1.$$

Also, $x_F(\alpha) < x_G(\alpha)$, $x_F'(\alpha) > 0$, $x_G'(\alpha) > 0$.

Let $x_M(\alpha)$ be defined implicitly by $G(x,\alpha) = F(x,\alpha)$. Such a solution necessarily exists since $x_F(\alpha) < x_G(\alpha)$ and $F(x,\alpha)$ is decreasing in x and $G(x,\alpha)$ is increasing in x. Also, it is clear that $1 < x_F(\alpha) < x_M(\alpha) < x_G(\alpha)$. Since $x > x_M(\alpha)$ implies $G > F$ then it also implies that w_1/P is increasing. Similarly, $x < x_M(\alpha)$ implies that w_1/P is decreasing. The following lemma (whose is proof is long and therefore provided in a separate Appendix downloadable from http://www.econ.psu.edu/~aur10/research.htm) is critical:

Lemma 3 *$x_M(\alpha)$ is increasing*

Let $\hat{\eta}$ be equal to $x_M(0)^{1/b}$. If $\eta \le \hat{\eta}$, then $x = \eta^b \le x_M(0) \le x_M(\alpha)$ for any α. This implies that $F(x) > G(x)$ (except the case when $x = \hat{\eta}^b$ and $\alpha = 0$), so w_1/P is decreasing. This establishes the first part of the proposition. To establish the second part, we need the following lemma:

Lemma 4 *For any $\eta > \hat{\eta} = (x_M(0))^{1/b}$ we have $x = \eta^b <\ x_M(\bar{\alpha}(\eta))$.*

Proof. The proof relies on showing that $F(\eta^b, \bar{\alpha}(\eta)) = 0$, which implies that $\eta^b = x_F(\bar{\alpha}(\eta))$. If this is true then $x_M(\bar{\alpha}(\eta)) > \eta^b$, because since $x_F(\alpha) < x_M(\alpha)$ for all α then $\eta^b = x_F(\bar{\alpha}(\eta)) < x_M(\bar{\alpha}(\eta))$, which establishes the result. But from the definition of $\bar{\alpha}$ we see that

$$\eta = \frac{1 + \bar{\alpha}}{f(\bar{\alpha})}$$

and plugging this into $F(\eta^b, \bar{\alpha})$ shows that $F(\eta^b, \bar{\alpha}(\eta)) = 0$. ■

This lemma implies that if $\eta > \hat{\eta}$ then w_1/P is increasing for $\alpha = 0$ and decreasing just before $\alpha = \bar{\alpha}(\eta)$, with a unique point α for which $x_M(\alpha) = x$ at which $G = F$ and hence $(w_1/P)'_\alpha = 0$. This implies that the curve w_1/P as a function of α in the interval $\alpha \in [0, \bar{\alpha}[$ is shaped like an inverted U.

Proof of Proposition 6

The only thing left to show is that steady state P is decreasing in α. It is sufficient to show that $\Phi_{mt} = T_{1t}c_{1t}^{-\theta} + T_{2t}w_{2t}^{-\theta}$ is decreasing in α. But

$$\Phi_{mt} = (1/L_{2t}^F g_L)\left(\phi_1 r_1 \varphi c_{1t}^{-\theta} + \phi_2 r_2 w_2^{-\theta}\right)$$

Using $c_1^{1-b} = \left(\frac{\phi_1/\phi_N}{w_1}\right)^b$ and $w_2 = (\phi_2/\phi_N)^b$ then

$$\begin{aligned}
\Phi_{mt} &= (1/L_{2t}^F g_L)\left(\phi_1 r_1 \varphi \left(\frac{\phi_1/\phi_N}{w_{1t}}\right)^{-b\theta/(1-b)} + \phi_2 r_2 \left(\phi_2/\phi_N\right)^{-b\theta}\right) \\
&= (\phi_N/L_{2t}^F g_L)\left(\varphi r_1 w_1 + w_2 r_2\right)
\end{aligned}$$

But plugging in from the equations (31) and (32) we get that

$$\varphi r_1 w_1 + w_2 r_2 = \varphi w_1 + w_2$$

which is increasing in α. **Q.E.D.**

Proof of Proposition 7

I first show that $x = \alpha w_2/w_1$ is increasing in α. From (36) we get $(\phi_1/\phi_N)^b = z\,(1+x)^{1-b} = z^b(z + w_2\beta(1-\beta)^{-b})^{1-b}$, where $z \equiv (1-\beta)^{1-b}w_1$. Since $\beta(1-\beta)^{-b}$ is increasing in α then z must be decreasing in α. In turn, this implies that x must be increasing in α.

Now, recall that r_1 is determined as the solution of $r_1 = r\,(1 + \alpha\,(1 - r_1)\,w_2/w_1)$. Both the LHS and the RHS are linear functions in r_1, with the LHS increasing and the RHS decreasing.

An increase in α moves the RHS schedule upward because $\alpha w_2/w_1$ increases with α, while the LHS schedule remains the same. This implies that r_1 increases.

In the text, before stating proposition **7** I also stated that $\alpha(1-r_1)w_2/w_1$ is increasing in α. To see this, note that since r_1 is increasing in alpha, then the RHS of $r_1 = r\left(1+\alpha\left(1-r_1\right)w_2/w_1\right)$ must be increasing in α, so $\alpha\left(1-r_1\right)w_2/w_1$ is increasing in α.

Finally, to prove that r_2 is decreasing in α, from (32) I need to show that $\alpha(1-r_1)$ is increasing in α. But we know that $\alpha(1-r_1)w_2/w_1$ is increasing in α while w_2 is constant and w_1 is increasing. This implies that $\alpha(1-r_1)$ must be increasing in α. **Q.E.D.**

Equilibrium with offshoring costs: 2 countries

Let

$$C(w_1, w_2, \lambda) \equiv w_2 F(1,\lambda) + \int_{w_2}^{w_1} x dF(x, \lambda/w_2)dx + w_1(1 - F(w_1, \lambda/w_2))$$

Integration and simplification yields

$$C(w_1, w_2, \lambda) = w_2 - (w_2/\lambda)\exp(-\lambda w_1/w_2) + (w_2/\lambda)\exp(-\lambda)$$

Letting $s(w_1, w_2, \lambda) \equiv F(w_1/w_2, \lambda)$ and $\sigma(w_1, w_2, \lambda) = \Gamma(w_1, w_2, \lambda)s(w_1, w_2, \lambda)$, or

$$\sigma(w_1, w_2, \lambda) \equiv 1 - (1/\lambda)\exp(-\lambda w_1/w_2) - w_1/w_2 \exp(-\lambda w_1/w_2) + (1/\lambda)\exp(-\lambda)$$

then an equilibrium for $\lambda < \lambda_m$ is determined by the following equations:

$$C(w_1, w_2, \lambda) = \delta\left(\frac{T_1}{L_1/(1 - s(w_1, w_2, \lambda))}\right)^b \tag{44}$$

and

$$w_2 = \delta\left(\frac{T_2}{L_2 - \sigma(w_1, w_2, \lambda)L_1/(1 - s(w_1, w_2, \lambda))}\right)^b \tag{45}$$

Totally differentiating above yields equation (44) yields

$$\frac{dC}{d\lambda} = \frac{\partial C}{\partial w_1}\frac{dw_1}{d\lambda} + \frac{\partial C}{\partial w_2}\frac{dw_2}{d\lambda} + \frac{\partial C}{\partial \lambda} = -\delta(T_1/L_1)^b b(1-s)^{b-1}\frac{ds}{d\lambda}$$

As $\lambda \to 0$ then $s \to 0$, hence $\partial C/\partial w_2 \to 0$ and

$$\frac{\partial C}{\partial w_1}\frac{dw_1}{d\lambda} + \frac{\partial C}{\partial \lambda} = -\delta(T_1/L_1)^b b\frac{ds}{d\lambda}$$

Now,

$$\begin{aligned}\frac{ds}{d\lambda} &= \frac{\partial s}{\partial w_1}\frac{dw_1}{d\lambda}+\frac{\partial s}{\partial w_2}\frac{dw_2}{d\lambda}+\frac{\partial s}{\partial \lambda} \\ &= (\lambda/w_2)\exp(-\lambda w_1/w_2)-\left(\lambda w_1/w_2^2\right)\exp(-\lambda w_1/w_2)+(w_1/w_2)\exp(-\lambda w_1/w_2)\end{aligned}$$

When $\lambda \to 0$ then $ds/d\lambda \to w_1/w_2$. Noting that $w_1 \to \delta(T_1/L_1)^b$ as $\lambda \to 0$, we see that

$$\frac{\partial C}{\partial w_1}\frac{dw_1}{d\lambda} = -b\frac{w_1^2}{w_2} - \frac{\partial C}{\partial \lambda}$$

Simple differentiation reveals that

$$\frac{\partial C}{\partial \lambda} = \frac{w_2\exp(-\lambda w_1/w_2) - w_2\exp(-\lambda) + w_1\lambda\exp(-\lambda w_1/w_2) - w_2\lambda\exp(-\lambda)}{\lambda^2}$$

Since both the numerator and denominators converge to 0 as $\lambda \to 0$ then we can use L'Hopital's Theorem to find that

$$\begin{aligned}\lim_{\lambda\to 0}\frac{\partial C}{\partial \lambda} &= \lim_{\lambda\to 0}\frac{w_1(-\lambda)\,(w_1/w_2)\exp(-\lambda w_1/w_2)+w_2\lambda\exp(-\lambda)}{2\lambda} \\ &= \frac{w_2 - w_1^2/w_2}{2}\end{aligned}$$

Thus,

$$\begin{aligned}\lim\left(-b\frac{w_1^2}{w_2}-\frac{\partial C}{\partial \lambda}\right) &= -bw_2\left(\frac{w_1}{w_2}\right)^2 - (w_2/2)\left(1-\left(\frac{w_1}{w_2}\right)^2\right) \\ &= (w_2/2)\left[(w_1/w_2)^2(1-2b)-1\right]\end{aligned}$$

This implies that $\lim_{\lambda\to 0} dw_1/d\lambda > 0$ if and only if $(w_1/w_2)^2 > 1/(1-2b)$.

To show that $\partial C/\partial\lambda < 0$, note that this is equivalent to

$$(1+w_1\lambda/w_2)\exp(-\lambda w_1/w_2) < (1+\lambda)\exp(-\lambda)$$

This inequality holds since the function $f(x) \equiv (1+x)\exp(-x)$ is clearly decreasing.

Equilibrium with offshoring costs: 3 countries

I first characterize an equilibrium in which $w_1 > w_3 > w_2$. In this case the distribution of c_1 is

$$\Pr(C_1 \le c_1) = \begin{cases} 0 & if \quad c_1 < w_2 \\ F(c_1,\lambda/w_2) & if \quad c_1 \in [w_2, w_3[\\ F(c_1,\varphi) & if \quad c_1 \in [w_3, w_1[\\ 1 & if \quad c_1 \ge w_1 \end{cases}$$

where $\varphi \equiv \lambda/w_2 + \lambda/w_3$, and unit cost of the common input in country 1 is

$$\begin{aligned} c_1(w_1,w_2,w_3) &\equiv w_2 F(1,\lambda) + \int_{w_2}^{w_3} dF(x,\lambda/w_2) \\ &\quad + w_3\left(F(w_3,\varphi) - F(w_3,\lambda/w_2)\right) + \int_{w_3}^{w_1} dF(x,\varphi) + w_1(1-F(w_1,\varphi)) \end{aligned}$$

The equilibrium condition for country 1 is

$$c_1(w_1,w_2,w_3) = \delta\left(\frac{T_1(1-s}{L_1}\right)^b$$

where $s = 1-\exp(-\varphi w_1)$ is the total share of services offshored. This share is distributed between countries 2 and 3 as follows:

$$s_{12} = F(w_3,\lambda/w_2) + [F(w_1,\varphi) - F(w_3,\varphi)]\left(\frac{\lambda/w_2}{\varphi}\right)$$

and

$$s_{13} = F(w_3,\lambda/w_2+\lambda/w_3) - F(w_3,\lambda/w_2) + [F(w_1,\varphi) - F(w_3,\varphi)]\left(\frac{\lambda/w_3}{\varphi}\right)$$

Note that $s = s_{12} + s_{13}$.

On the other hand, we have that

$$\Pr(C_3 \le c_3) = \begin{cases} 0 & if \quad c_3 < w_2 \\ F(c_3,\lambda/w_2) & if \quad c_3 \in [w_2,w_3[\\ 1 & if \quad c_3 \ge w_3 \end{cases}$$

and

$$c_3(w_1,w_2,w_3) \equiv w_2 F(1,\lambda) + \int_{w_2}^{w_3} x dF(x,\lambda_2/w_2) + w_3(1-F(w_3,\lambda_2/w_2)$$

and

$$s_{32}(w_2,w_3) = F(w_3,\lambda_2/w_2)$$

The equilibrium conditions for countries 2 and 3 are $c_i(w_1,w_2,w_3) = \delta\left(T_i/\widetilde{L}_i\right)^b$ for $i = 2,3$, with $c_2(w_1,w_2,w_3) = w_2$. To derive $\widetilde{L}_3$, note that country 1 uses $L_1/(1-s)$ of every service, and if the offshoring cost of a service in country 3 is ζ_3 then it takes $\zeta_3 L_1/(1-s)$ units of labor to produce $L_1/(1-s)$ units of a service delivered in country 1. The expectation of ζ_3 for services offshored by country 1 to country 3 is

$$\sigma_{13}(w_1,w_2,w_3) \equiv F(w_3,\varphi) - F(w_3,\lambda/w_2) + (1/w_3)\left(\frac{\lambda/w_3}{\varphi}\right)\int_{w_3}^{w_1} x dF(x,\varphi)$$

This implies that $L_3 - \left(\frac{\sigma_{13}}{1-s}\right) L_1$ units of labor are left in country 3 for final good production. But this country will offshore s_{32} services to country 2, so it will use $\left(L_3 - \left(\frac{\sigma_{13}}{1-s}\right) L_1\right)\left(\frac{1}{1-s_{32}}\right)$ of each service for domestic production. This implies that

$$\widetilde{L}_3 = \left(L_3 - \left(\frac{\sigma_{13}}{1-s}\right) L_1\right)\left(\frac{1}{1-s_{32}}\right)$$

On the other hand, country 2 export services to countries 1 and 3. The labor it takes to export services to country 1 is $\left(\frac{\sigma_{12}}{1-s}\right) L_1$, where σ_{12} is the expectation of ζ_2 for services offshored by country 1 to country 2, and is given by

$$\begin{aligned}\sigma_{12}(w_1, w_2, w_3) \ \ &\equiv \ \ (1-\exp(-\lambda)) + (1/w_2)\int_{w_2}^{w_3} x dF(x, \lambda/w_2) \\ &\quad +(1/w_2)\left(\frac{\lambda/w_2}{\varphi}\right)\int_{w_3}^{w_1} x dF(x,\varphi)\end{aligned}$$

Country 2 also exports services to country 3, and by the reasoning above, we know that it takes $\left(L_3 - \left(\frac{\sigma_{13}}{1-s}\right) L_1\right)\left(\frac{\sigma_{32}}{1-s_{32}}\right)$ units of labor to do so, with σ_{32} representing the expectation of ζ_2 for services offshored by country 3 to country 2, given by

$$\sigma_{32}(w_1, w_2, w_3) \equiv (1-\exp(-\lambda)) + (1/w_2)\int_{w_2}^{w_3} x dF(x, \lambda/w_2)$$

Thus, we find that

$$\widetilde{L}_2 = L_2 - \left(\frac{\sigma_{12}}{1-s}\right) L_1 - \left(L_3 - \left(\frac{\sigma_{13}}{1-s}\right) L_1\right)\left(\frac{\sigma_{32}}{1-s_{32}}\right)$$

If the previous system yields wages that do not respect $w_1 > w_3 > w_2$ then it is not an equilibrium. Other possible equilibrium configurations have $w_1 > w_2 = w_3$, $w_1 = w_3 > w_2$, and $w_1 = w_2 = w_3$. The last case entails full offshoring. Such an equilibrium satisfies

$$\frac{T_m}{L_m} = \frac{T_1(1-s)}{L_1} = \frac{T_2}{L_2 - \left(\frac{s_{12}}{1-s}\right) L_1 - \left(L_3 - \left(\frac{s_{13}}{1-s}\right) L_1\right)\left(\frac{s_{32}}{1-s_{32}}\right)} = \frac{T_3(1-s_{32})}{L_3 - \left(\frac{s_{13}}{1-s}\right) L_1}$$

with $s = s_{12} + s_{13}$. For every s_{32} these equations determine s_{12} and s_{13}, so these variables are not uniquely pinned down in equilibrium: there is no uniqueness because of the absence of offshoring costs for the services that are traded, but all equilibria entail the same wages.[38] If one can find a solution with $s < F(1, 2\lambda)$ and $s_{12}, s_{13}, s_{32} < F(1, \lambda)$, then this solution corresponds

[38] Although there are 3 equations for 3 unknowns (s_{12}, s_{13} and s_{32}), these equations are linearly dependent, so they determine only two unknowns.

to an equilibrium with full offshoring. Since $F(1,2\lambda)$ and $F(1,\lambda)$ both converge to 1 as $\lambda \to \infty$ then necessarily there is some critical value of λ such that for higher values of λ the equilibrium entails full offshoring.

I now establish the equilibrium conditions when wages entail $w_1 > w_2 = w_3$ and $w_1 = w_3 > w_2$. If $w_1 > w_2 = w_3$ then countries 2 and 3 are integrated and their wage should be the same as the one we would would get in a two country system, with

$$w_2 = w_3 = w_{23} = \delta \left(\frac{T_2 + T_3}{L_2 + L_3 - \left(\frac{\sigma}{1-s}\right) L_1} \right)^b$$

but with countries 2 and 3 drawing their offshoring cost from $F(\zeta, 2\lambda)$, and with $s = F(w_1/w_{23}, 2\lambda)$ and

$$\sigma = F(1, 2\lambda) + \int_1^{w_1/w_{23}} x dF(x, 2\lambda)$$

This equilibrium has σ_{12} and σ_{13} and s_{32} such that

$$w_{23} = \delta \left(\frac{T_2}{L_2 - \left(\frac{\sigma_{12}}{1-s}\right) L_1 - \left(L_3 - \left(\frac{\sigma_{13}}{1-s}\right) L_1\right) \left(\frac{s_{32}}{1-s_{32}}\right)} \right)^b$$

and

$$w_{23} = \delta \left(\frac{T_3}{L_3 - \left(\frac{\sigma_{13}}{1-s}\right) L_1 \left(\frac{1}{1-s_{32}}\right)} \right)^b$$

with the restriction that $\sigma_{32} = s_{32} \leq F(1,\lambda)$ and

$$\sigma_{12}, \sigma_{13} \leq F(1,\lambda) + (1/2) \int_1^{w_1/w_{23}} x dF(x, 2\lambda)$$

The first term on the RHS is the measure of services for which ζ_2 or ζ_3 are equal to 1, while the second term is the offshoring cost for services with $\zeta_i \in]1, w_1/w_3]$.

Now consider the case with $w_1 = w_3 > w_2$. This entails

$$w_1 = w_3 = w_{13} = \delta \left(\frac{(T_1 + T_3)(1-s)}{L_1 + L_3} \right)^b$$

and

$$w_2 = \delta \left(\frac{T_2}{L_2 - \left(\frac{\sigma}{1-s}\right)(L_1 + L_3)} \right)^b$$

where $s = F(w_{13}/w_2, \lambda)$ and

$$\sigma = F(1, \lambda) + \int_1^{w_{13}/w_2} x dF(x, \lambda)$$

For this to be an equilibrium, we need that $\sigma_{13} = s_{13} \leq F(1, \lambda)$, have $s_{32} = s_{12} = F(w_{13}/w_2, \lambda)$ and

$$\sigma_{32} = \sigma_{12} = F(1, \lambda) + \int_1^{w_{13}/w_2} x dF(x, \lambda)$$

and the equations of the full system.

For the long run equilibrium, we have $c_i(w_1, w_2, w_3)^{1-b} = \left(\frac{\phi_i/\phi_N}{w_i}\right)^b$ for $i = 1, 2, 3$, with $c_2(w_1, w_2, w_3) = w_2$. This equation for $i = 2$ can be solved directly to yield $w_2 = (\phi_2/\phi_N)^b$, as in (35). Plugging this into the equation for $i = 3$ yields an equation that can be solved for the equilibrium wage in country 3, w_3. We can easily check that w_3 is increasing in λ. Finally, plugging the solution $w_3(\lambda)$ into the equation for $i = 1$ yields the equilibrium $w_1(\lambda)$.

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