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### WHAT HAS FINANCED GOVERNMENT DEBT?

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**ABSTRACT**

Dynamic rational expectations models imply that the real value of debt in the hands of the public must be equal to the expected present-value of surpluses. We impose this equilibrium condition on an identified VAR and characterize the way in which the present-value support of debt varies across various types of fiscal policy shocks and between fiscal and non-fiscal shocks. The role of expected primary surpluses in supporting innovations to debt depends on the nature of the shock. For some fiscal policy shocks, debt is supported almost entirely by changes in the present-value of surpluses, however, in the case of other fiscal policy shocks, surpluses fail to adjust and instead leave a large role for expected changes in discount rates. Horizons over which debt innovations are financed are long – on the order of fifty years – while present-values calculated up to any finite horizon up to then fluctuate wildly, particularly following government spending and transfer shocks.

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# WHAT HAS FINANCED GOVERNMENT DEBT?

HESS CHUNG AND ERIC M. LEEPER

## 1. INTRODUCTION

The global recession and financial crisis of 2007-2009 have spurred interest in fiscal policy, particularly in estimates of multipliers associated with various fiscal instruments. Unfortunately, it is difficult to align the thought experiments underlying existing time series evidence on multipliers with the experiments conducted in fully specified general equilibrium models. Theoretical models require that fiscal disturbances that alter the value of outstanding government debt be expected to bring forth adjustments in future net-of-interest surpluses and seigniorage; moreover, theory predicts that the precise source of future fiscal adjustments—taxes, spending, or inflation—matter for the resulting impacts of the fiscal shocks. Identified vector autoregressions, the preeminent tool in empirical fiscal research, do not impose the intertemporal restriction linking current debt to expected future policies and, therefore, are difficult to interpret in light of dynamic theory.

The government's present-value budget constraint is an attractive target for rationalizing macroeconomic responses to fiscal policy because the net taxes component of the present-value relation directly impacts forward-looking households through their own present-value constraints. Accordingly, if we wish to understand why households with rational expectations respond in certain ways to fiscal policy shocks, it is valuable to understand how these households perceive their present-value tax burden as evolving. Moreover, if we wish to understand why forward-looking households are content to hold government debt at prevailing market prices, it is essential to know how violations of the household transversality conditions are avoided, and this means understanding how the government present value relation is satisfied.

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Rational expectations implies that economic agents' beliefs about how future fiscal policy will adjust to innovations in government debt play a crucial role in determining the resulting equilibrium. Prominent examples where theoretical conclusions about macro policy hinge on such beliefs include Ricardian equivalence, "Unpleasant Monetarist Arithmetic," and the fiscal theory of the price level.<sup>1</sup> In striking contrast, empirical studies are either mute, as in Blanchard and Perotti (2002) and the identified VAR work that followed, or build in the assumption that net surpluses or total tax revenues clear the government budget constraint [Bohn (1998), Davig and Leeper (2006), or Favero and Monacelli (2005)]. This paper offers some new empirical findings that connect more tightly to theoretical work.

Our desire to examine the historical sources of fiscal financing leads us to include government debt in an otherwise conventional fiscal VAR, like those estimated by Blanchard and Perotti (2002), Perotti (2004), Canova and Pappas (2007), Mountford and Uhlig (2009), and Caldara and Kamps (2008). By including debt, we confirm the results of Favero and Giavazzi (2007), to the effect that estimates of fiscal policy impacts appear to be sensitive to the choice of information set included in the VAR. Ultimately, however, our baseline model delivers qualitative results similar to those in the literature.

The principal contribution of this paper stems from imposing the government's intertemporal budget constraint on an estimated VAR to answer the question posed by the paper's title. With a consistent accounting framework in hand, we examine how innovations in debt produced by exogenous shocks to government spending, transfers, and taxes have been expected to be financed intertemporally. We find robust evidence in favor of a stabilizing role for the primary surplus following shocks to taxes and transfers. Regarding the role of real interest rates, the evidence is less clear, although there is support for a destabilizing role for the real rate following transfer shocks.

Our work is closely related to Giannitsarou and Scott (2006) and Bohn (1991). The paper by Giannitsarou and Scott, however, is concerned with *testing* present-value balance, an effort initiated by Hamilton and Flavin (1986). Instead, we *impose* the linearized intertemporal government budget constraint on an identified VAR and study its implications for fiscal financing. We focus on describing how present-value balance is achieved, contingent on the realization of certain identified fiscal policy shocks.

This focus on the intertemporal funding mechanism is shared with the paper by Bohn Bohn (1991). Our work differs from Bohn on two dimensions. First, we impose certain strong restrictions deriving from the linearized flow budget constraint. In

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<sup>1</sup>A very partial list of analyses in which intertemporal financing of government debt is pivotal includes Barro (1974), Sargent and Wallace (1981), Leeper (1991), Baxter and King (1993), Sims (1998), Woodford (2001), and Leeper and Yang (2008).

both papers, the flow budget constraint has the form  $b_t = rb_{t-1} - s_t + \delta_t$ , where  $b_t$  is the real value of debt outstanding at time  $t$ ,  $s_t$  is the real primary surplus, including seignorage, and  $r$  is a discount factor. In Bohn (1991),  $\delta$  is an unrestricted process, accounting formally for variations in the real interest rate and for approximation error in the form of the budget constraint. In this paper, we take an explicit stand on the form of  $\delta$ , which is decomposed into a term depending linearly on the real interest rate and an expectation error. The appearance of an expectation error, in turn, arises from a first-order approximation to the flow constraint in the presence of multi-period debt. By constraining this component to be an expectation error, we take the stand that linearization error is negligible (in present-value, at least) and thus shift the burden of intertemporal adjustment to economically interpretable mechanisms.<sup>2</sup> This assumption gives rise to a number of additional constraints beyond the usual co-integration relation, imposed by Bohn. Secondarily, the estimation framework used in Bohn (1991) assumes that present-value balance is the only co-integrating relation among variables in the VAR. This assumption, while reasonable in the context of Bohn's smaller VAR, is less attractive in the much larger VAR that we estimate. Therefore, in addition to our baseline model, which does not impose any unit root restrictions, we present a straightforward estimator which allows for multidimensional co-integration, in addition to the present-value restrictions.

This paper is also closely related to Roberds (1991). Roberds includes a measure of government debt in his empirical work to test whether the government's present-value condition holds in expectation. Although we impose the condition, both Roberds and we avoid the impossibility result of Hansen, Roberds, and Sargent (1991) by including debt in the information set and applying the present-value condition in expectation.

Consistent with previous findings, we find robust evidence in favor of a stabilizing role for the primary surplus following tax shocks and similarly robust evidence of a stabilizing role for taxes following a spending shock. Conversely, our estimates speak strongly against a stabilizing role for spending adjustments in the face of either tax or spending shocks. Our results regarding the role of the real interest rate are rather sensitively dependent on assumptions concerning the long-run behavior of the system. In particular, in the baseline VAR, interest rates are stabilizing following spending shocks, whereas the imposition of unit-root restrictions appears to reverse this conclusion. Finally, regardless of the long-run assumptions, detecting present-value balance requires very extended forecast horizons, on the order of a century, as fiscal shocks generate highly persistent, but ultimately transient, dynamical responses in the primary surplus.

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<sup>2</sup>To the extent that first-order approximation fails, it is likely that the VAR framework used here and in previous papers in the literature would itself have to be substantially re-considered.

## 2. AN ILLUSTRATIVE MODEL

This section uses a conventional dynamic stochastic general equilibrium (DSGE) model—a standard real business cycle model—to derive a typical model’s implications for fiscal financing dynamics and to illustrate the computations we perform in the identified VARs below. Although bare-bones, the model is adequate to our focus on the long-run aspects of fiscal finance. More sophisticated versions of this model which are being fit to data largely consist of modifications of the bare-bones model that are designed to capture short-run dynamics in data. Their long-run implications closely match those of the simpler model we examine [see, for example, Leeper, Plante, and Traum (2009)].

The model shows that in general the impacts of fiscal disturbances depend on how the government budget constraint is expected to be satisfied in the long run, a point that dates back at least to Christ (1968) and has found recent voice in Baxter and King (1993), Sims (1998), and Leeper and Yang (2008). With a simple model in hand, we derive the sources of intertemporal financing of government debt and the horizons at which that funding occurs.

**2.1. Model Specification.** Consider the following real business cycle model. The representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t), \quad 0 < \beta < 1 \quad (1)$$

with  $u(C, 1 - N) = C^{1-\gamma}/(1 - \gamma) + \theta(1 - N)^{1-\gamma_N}/(1 - \gamma_N)$ , subject to

$$C_t + K_t - (1 - \delta)K_{t-1} + B_t = (1 - \tau_t)Y_t + (1 + r_{t-1})B_{t-1} + Z_t, \quad (2)$$

where  $Z_t$  is lump-sum transfers (or taxes when  $Z_t < 0$ ). Goods are produced using a technology that is constant returns to scale in labor,  $N$ , and capital,  $K$ , jointly

$$Y_t = (A_t N_t)^\alpha K_{t-1}^{1-\alpha}. \quad (3)$$

$\{A_t\}$  is the serially correlated technology process.

The aggregate resource constraint is

$$C_t + K_t - (1 - \delta)K_{t-1} + G_t = Y_t. \quad (4)$$

$G_t$  is government purchases of goods at  $t$ , and the government budget constraint is

$$B_t + \tau_t Y_t = G_t + Z_t + (1 + r_{t-1})B_{t-1}, \quad (5)$$

where  $B_t$  is the amount of one-period debt outstanding at  $t$ , which pays  $(1 + r_t)B_t$  at  $t + 1$ . We let  $T_t \equiv \tau_t Y_t$  denote total tax revenues.

Following existing work on fiscal policy, we posit that policy obeys simple rules that make fiscal variables respond contemporaneously to output and with a lag to the state of government debt (written in log deviations from steady state):

$$\hat{g}_t = \varphi_G \hat{y}_t - \gamma_G \hat{b}_{t-1} + u_t^G, \quad (6)$$

$$\hat{t}_t = \varphi_\tau \hat{y}_t + \gamma_\tau \hat{b}_{t-1} + u_t^\tau, \quad (7)$$

and

$$\hat{z}_t = \varphi_Z \hat{y}_t - \gamma_Z \hat{b}_{t-1} + u_t^Z. \quad (8)$$

The  $u$ 's follow AR(1) processes. The output elasticities, the  $\varphi$ 's, are borrowed from the empirical studies of Blanchard and Perotti (2002), Perotti (2004), and Leeper and Yang (2004). We use the baseline parameter values described in table 1.

**2.2. Some Accounting.** Let the equilibrium dynamics be characterized by factors  $f_t$ , evolving according to  $f_t = f_{t-1}A + u_t$ , in terms of which the model variables are  $x_t = f_t C_x$ . In its log-linearized form, the present-value relation is

$$\hat{b}_t = \sum_{j=1}^{\infty} \beta^j \left( \frac{\tau}{B} \hat{t}_{t+j} - \frac{G}{B} \hat{g}_{t+j} - \frac{Z}{B} \hat{z}_{t+j} - \frac{1}{\beta} \hat{R}_{t+j-1} \right), \quad (9)$$

where unsubscripted variables denote deterministic steady state values. This equation gives a decomposition of innovations to real debt into innovations to surplus components, at constant discount rates, and into innovations in the real interest rate. Using the equilibrium law of motion, the infinite sum can be expressed as

$$\hat{b}_t = f_t (\mathbf{I} - \beta A)^{-1} \left[ \left( \frac{\tau}{B} C_T - \frac{G}{B} C_G - \frac{Z}{B} C_Z \right) \beta A - C_R \right]. \quad (10)$$

Denote innovations in  $x_t$  by  $\delta x_t \equiv x_t - E_{t-1} x_t$ . Then

$$\delta \hat{b}_t = \delta f_t (\mathbf{I} - \beta A)^{-1} \left[ \left( \frac{\tau}{B} C_T - \frac{G}{B} C_G - \frac{Z}{B} C_Z \right) \beta A - C_R \right]. \quad (11)$$

It is also possible to compute the horizon over which an innovation to the expected present value of surpluses converges to the innovation in debt. Specifically, suppose that the previous infinite series is truncated at a horizon  $K$ . Again using the equilibrium law of motion, the sum is

$$PV_t(K) = \delta f_t (\mathbf{I} - (\beta A)^K) (\mathbf{I} - \beta A)^{-1} \left[ \left( \frac{\tau}{B} C_T - \frac{G}{B} C_G - \frac{Z}{B} C_Z \right) \beta A - C_R \right]. \quad (12)$$

Expression (12) answers the question, ‘‘What fraction of a \$1 innovation in debt at time  $t$  is financed by period  $t+K$ .’’ Of course, as  $K \rightarrow \infty$ , expression (12) approaches expression (11).

**2.3. Dynamic Impacts of Fiscal Shocks.** Many DSGE models follow the public finance literature in studying the impacts of fiscal disturbances by assuming the government budget clears in some “neutral” manner, for example, through adjustments in lump-sum taxes or transfers. That assumption implies setting  $\gamma_Z > 1/\beta - 1$  and  $\gamma_G = \gamma_\tau = 0$  in policy rules (6)-(8). Solid line in figure 1 report the well-known implications of a standard RBC model. Persistently higher government spending reduces wealth, which reduces consumption and induces greater work effort, initially raising output. Higher taxes reduce output, consumption, and investment. Ricardian equivalence implies that lump-sum taxes do nothing.

Table 2 reports how an increase in debt brought forth by each of the three fiscal shocks is expected to be financed when transfers adjust. Not surprisingly, serially correlated spending and tax shocks create expectations of present values of spending and taxes that move in opposite directions from the initial change in debt. Transfers move with debt, as the fiscal rule would suggest. Discount rate changes account for a trivial fraction of the financing of debt, a result that is ubiquitous in the plain-vanilla RBC model. The discount rate also moves against the change in debt.

When government spending adjusts to clear the budget— $\gamma_G = 1, \gamma_\tau = \gamma_Z = 0$ —important differences emerge in the impulse response functions, as dotted lines in figure 1 show. An expectation that higher spending will reduce future spending eliminates the expansionary effects of higher spending and ameliorates the negative wealth effects on consumption, while it raises the capital stock in the future. When higher current taxes portend higher future government spending, consumption falls more markedly. Higher transfers now create the expectation of lower future spending, which reduces work effort and output, but raises consumption.

Finally, suppose that taxes adjust to ensure fiscal sustainability— $\gamma_\tau = 1, \gamma_G = \gamma_Z = 0$ . Positive spending or transfer shocks, which are expected to generate higher taxes in the future, now sharply reduce output, consumption, and capital [dashed lines in figure 1]. Tax hikes, on the other hand, after initially reducing these variables, raise them with a lag.<sup>3</sup>

The simple policy rules produce monotonic adjustments in funding over horizons after which the serial correlation of the shocks has decayed. Figure 2 illustrates this phenomenon in the case when only taxes adjust to debt. The smaller is the adjustment parameter, the more prolonged is the adjustment process. At horizons beyond about 10 periods, the innovation to the expected present value of surpluses converges monotonically to the innovation in debt for each of the three fiscal disturbances.

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<sup>3</sup>When either spending or taxes adjust to clear the budget, as in table 2, the present value of discount rates accounts for a trivial share of the value of debt.



## 3. A CONSISTENT ACCOUNTING FRAMEWORK

**3.1. The Intertemporal Budget Constraint.** Let government debt in the hands of the public at time  $t$  consist of zero coupon bonds with nominal face value  $B_t(j)$  maturing at  $t+j$ , for all  $j \geq 1$ . Further, let the total nominal value of debt outstanding be  $V_t \equiv \sum_{j=1}^{\infty} B_t(j)Q_t(j)$  and let the nominal primary surplus be  $S_t$ . The surplus is defined as  $S_t = T_t - G_t - Z_t$ , where  $T_t$  is tax receipts,  $G_t$  is government spending, and  $Z_t$  is transfer payments. The government budget identity is then<sup>4</sup>

$$\sum_{j=1}^{\infty} (B_t(j) - B_{t-1}(j+1))Q_t(j) = B_{t-1}(1) - S_t. \quad (13)$$

In real terms the identity is

$$\frac{V_t}{P_t} = \frac{1}{P_t} \sum_{j=1}^{\infty} B_t(j)Q_t(j) = \frac{P_{t-1}}{P_t Q_{t-1}(1)} \frac{1}{P_{t-1}} \sum_{j=1}^{\infty} B_{t-1}(j)Q_{t-1}(j) - \frac{S_t}{P_t} + \omega_t \quad (14)$$

where  $P_t \omega_t \equiv \sum_{j=1}^{\infty} \left( Q_t(j) - \frac{Q_{t-1}(j)}{Q_{t-1}(1)} \right) B_{t-1}(j+1)$  and  $P_t$  is the price level.

The Euler equation for a nominal discount bond implies

$$Q_t(j) = \delta^j E_t \frac{\lambda_{t+j}}{\lambda_t} \frac{P_t}{P_{t+j}}, \quad (15)$$

where  $\delta \in (0, 1)$  is the subjective rate of discount and  $\lambda_t$  is the marginal utility of consumption.

Express  $\omega_t$  as

$$\omega_t \equiv \frac{1}{P_t} \sum_{j=1}^{\infty} \left( Q_t(j) - \frac{Q_{t-1}(j+1)}{Q_{t-1}(1)} \right) B_{t-1}(j+1),$$

from which it follows, after imposing the Euler equation for bond prices, that

$$\omega_t = \frac{1}{\lambda_t} \sum_{j=1}^{\infty} \delta^j \left( E_t \frac{\lambda_{t+j}}{P_{t+j}} - \frac{\lambda_t}{P_t} \frac{E_{t-1} \frac{\lambda_{t+j}}{P_{t+j}}}{E_{t-1} \frac{\lambda_t}{P_t}} \right) B_{t-1}(j+1). \quad (16)$$

$\omega_t$  reflects changes in both the maturity structure of government debt and the term structure of interest rates.

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<sup>4</sup>For expository clarity, we abstract from seigniorage in this section of the paper. The empirical work, however, involves imposing the full budget constraint, including seigniorage, which for the flow budget constraint is defined as  $(M_t - M_{t-1})/P_t$ , where  $M$  is the monetary base. The addition of seigniorage terms introduces no new conceptual issues. We also abstract from modeling population growth, effectively treating it as exogenous with respect to the shocks captured by the VAR.

Thus,  $\lambda_t \omega_t = \eta_t$ , where  $E_t \eta_{t+1} = 0$ . To anticipate slightly, when discounted, the  $\omega_t$  term in (14) will disappear in expectation, so it will not contribute to the present-value expressions below. Innovations in  $\omega_t$  can nonetheless play an important role by revaluing debt.

In equilibrium a transversality condition holds such that

$$\lim_{s \rightarrow \infty} E_t \lambda_{t+s} \frac{V_{t+s}}{P_{t+s}} = 0.$$

Iterate forward on (14) to obtain

$$\frac{V_t}{P_t} = E_t \sum_{j=1}^{\infty} \delta^j \frac{\lambda_{t+j}}{\lambda_t} s_{t+j} \quad (17)$$

where, as usual, the transversality condition implies the absence of a bubble term in (17).

For the purposes of linearization, it is convenient to express (14) and (17) scaled by output. For any nominal variable  $X_t$ , let the corresponding variable  $x_t \equiv \frac{X_t}{Y_t}$ , where  $Y$  is nominal output, and define  $\gamma_t$  as  $\frac{Y_t/P_t}{Y_{t-1}/P_{t-1}}$ , the growth rate of real output. Scaled versions of the flow and intertemporal government budget constraints are:

$$v_t = \frac{1}{\gamma_t \pi_t Q_{t-1}(1)} v_{t-1} - s_t + \frac{P_t \omega_t}{Y_t} \quad (18)$$

and

$$v_t = E_t \sum_{j=1}^{\infty} \delta^j \frac{\lambda_{t+j}}{\lambda_t} \left( \prod_{k=1}^j \gamma_{t+k} \pi_{t+k} \right) s_t, \quad (19)$$

where  $v_t$  is the market value of debt as a share of output.

If it were feasible to estimate an empirical model that included government bonds and bond prices at *all* maturities, we would work directly with the flow constraint in (13). Because such a model is not practicable, we introduce  $\omega_t$  to express the budget constraint in terms of the value of debt,  $v$ , the one-period interest rate,  $Q_{t-1}(1)$ , the surplus, and other variables, as in (18). In the estimated model,  $\omega_t$  is a residual, which is a linear combination of the VAR errors, that clears the period-by-period government budget constraint.

**3.2. The VAR and Its Constraints.** Now consider log-linearizations of (18) and (19) around fixed values for  $(v, \gamma, s, P\omega/Y)$  and the growth rate of  $\lambda$ . Denote  $\hat{x}_t \equiv \log(x_t) - \log(x)$ . Define the one-period nominal interest rate at date  $t-1$  as  $R_{t-1} =$

$Q_{t-1}^{-1}(1)$ , the linearized form of the flow constraint is:

$$\hat{v}_t = \frac{1}{\beta} \left( \hat{v}_{t-1} - \hat{\gamma}_t - \hat{\pi}_t + \hat{R}_{t-1} \right) - \frac{s}{v} \hat{s}_t + \frac{1}{v} d \left( \frac{P_t \omega_t}{Y_t} \right) \quad (20)$$

where for steady state values of variables we use sample means. (Because  $\omega$  can be negative, we do not log-linearize it—the term  $d \left( \frac{P_t \omega_t}{Y_t} \right)$  represents deviations of  $\frac{P_t \omega_t}{Y_t}$  from its linearization point.) The linearized form of the present-value constraint is:

$$\hat{v}_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{s}{v} \left( \sum_{s=1}^j -\hat{R}_{t+s-1} + \hat{\gamma}_{t+s} + \hat{\pi}_{t+s} + \hat{s}_{t+j} \right) \quad (21)$$

where  $\beta \equiv \delta \gamma \frac{\lambda_{t+1}}{\lambda_t}$ ,  $\frac{\lambda_{t+1}}{\lambda_t}$  is the constant steady state growth rate of the marginal utility of consumption, and the deviations of the net surplus are given by  $s \hat{s}_t = \tau \hat{\tau}_t - g \hat{g}_t - z \hat{z}_t$ .

Note that  $\sum_{j=1}^{\infty} \beta^j \sum_{s=1}^j X_{t+s} = \frac{1}{1-\beta} \sum_{j=1}^{\infty} \beta^j X_{t+j}$ . Therefore, equation (21) can be written as

$$\hat{v}_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{s}{v} \left( \frac{-\hat{R}_{t+j-1} + \hat{\gamma}_{t+j} + \hat{\pi}_{t+j}}{1-\beta} + \hat{s}_{t+j} \right). \quad (22)$$

Ultimately, we wish to express the real quantity variables in levels, rather than as fractions of output. Define  $\tilde{v}_t \equiv \log(V_t/P_t)$ ,  $\tilde{\pi}_t \equiv \log(\pi_t)$ , and  $\tilde{R}_t \equiv \log(R_t)$ . Using the steady-state relations and eliminating output growth, equation (22) implies that

$$\tilde{v}_t = k + E_t \sum_{j=1}^{\infty} \beta^j \frac{s}{v} \left( \frac{-\tilde{R}_{t+j-1} + \tilde{\pi}_{t+j}}{1-\beta} + \tilde{s}_{t+j} \right). \quad (23)$$

where  $k = -\frac{\beta}{1-\beta} (\ln(1/\beta)/\beta + \tau \ln(\tau)/v - g \ln(g)/v - z \ln(z)/v) + \ln(v)$  and  $s \tilde{s}_t = \tau \tilde{\tau}_t - g \tilde{g}_t - z \tilde{z}_t$ . Thus, fluctuations in real debt must be balanced by expected changes in the present-value expression on the right-hand side. Moreover, this present-value can itself be thought of as consisting of two components: the present-value of surpluses at constant steady-state discount rates and a term which measures changes in the expected path of those discount rates. These two components can play very different roles in restoring present-value balance.

Suppose that the state of the model economy is characterized by the  $M$ -dimensional factors  $f_t$  which, in companion form, evolve according to the VAR process

$$f_t = B_0 + f_{t-1} B + u_t. \quad (24)$$

Let a model variable  $x_t$  be related to the underlying factors by a coefficient matrix  $C_x$  such that

$$\tilde{x}_t = f_t C_x \quad (25)$$

The government budget constraint in equation (23) implies the following restrictions on the VAR in (24):<sup>5</sup>

$$\beta B \left( C_v + \frac{1}{\beta} C_\pi + \frac{\tau}{v} C_\tau - \frac{g}{v} C_g - \frac{z}{v} C_z \right) - (C_v + C_R) = 0 \quad (26)$$

and

$$k + B_0 \frac{\beta}{1 - \beta} (1 - \beta B)^{-1} \left( \frac{1}{\beta} C_\pi + \frac{\tau}{v} C_\tau - \frac{g}{v} C_g - \frac{z}{v} C_z - C_R \right) = 0 \quad (27)$$

Expression (27) imposes restrictions on the deterministic growth components of the VAR. Because our focus is on innovation accounting and the deterministic components are irrelevant, we do not impose (27) on the estimated VAR.

In addition, if the matrix  $\beta B$  possesses explosive eigenvalues, we must impose conditions which guarantee that the infinite sum in (23) exists. Specifically, let  $V$  be the matrix of right eigenvectors of  $\beta B$  and suppose that  $\mu_j$  is an eigenvalue such that  $|\mu_j| \geq 1$ . If  $V^{-1}(j)$  denotes the corresponding row of the inverse of  $V$ , we require that

$$V^{-1}(j) \left( \frac{1}{\beta} C_\pi - C_R + \frac{\tau}{v} C_\tau - \frac{g}{v} C_g - \frac{z}{v} C_z \right) = 0 \quad (28)$$

Note that the VAR does not contain the term  $\omega_t$  in equation (14). Variations in this variable thus implicitly maintain the government's flow budget constraint period by period. As is evident above, however, the present-value constraint implies that the flow constraint holds in expectation.

**3.3. Estimation Procedure.** We conduct estimation and inference in a least-squares framework. As before, let the VAR be  $f_t = B_0 + f_{t-1}B + u_t$ , for  $t = 0, \dots, T$ . Then the objective function is  $\max \sum_t u_t' \Sigma u_t$ . Let  $f$  be a data matrix whose rows consist of the variables  $f_t$ , for each  $t > 0$  and define  $f_-$  as the corresponding lagged data matrix. Define  $W_- \equiv [\mathbf{1}, f_-]$  and  $b \equiv \text{vec}([B'_0, B']')$ . The part of the objective function relating to  $b$  can be re-written  $(b - \hat{b})' S (b - \hat{b})$ , where  $\hat{b} \equiv I \otimes (W_-' W_-)^{-1} (W_-' f)$  and  $S = (\Sigma \otimes Z' Z)$ . We maximize this objective function subject to a constraint  $C_0 b = C_-$  (defined below). The first-order condition for this problem is  $S(b - \hat{b}) = C'_0 \lambda'$ . It follows that  $C_0 b = C_- = C_0 \hat{b} + C_0 S^{-1} C'_0 \lambda'$ , or that  $b = \hat{b} + S^{-1} C'_0 (C_0 S^{-1} C'_0)^{-1} (C_- - C_0 \hat{b})$ .

From equation (26), we have that  $[\mathbf{0}, \mathbf{I}][B'_0, B']' C_0 = C_-$ , so that the constraint is  $(C'_0 \otimes [\mathbf{0}, \mathbf{I}])b = \text{vec}(C_-)$ , where, explicitly,

$$C_0 \equiv \beta \left( C_v + \frac{1}{\beta} C_\pi + \frac{\tau}{v} C_\tau - \frac{g}{v} C_g - \frac{z}{v} C_z \right) \quad (29)$$

<sup>5</sup>Appendix A details this derivation.

and

$$C_- \equiv (C_v + C_R) \quad (30)$$

Estimates presented here are from a feasible GLS procedure, iterated until convergence, in which  $\Sigma^{-1}$  is a consistent estimate of the residual covariance matrix.

The discount factor  $\beta$  plays an important role in computing present values. We compute this from the steady state government budget constraint after imposing that  $1/\beta = R/\pi$  and using the sample means for taxes, spending, transfers, and debt as a share of GDP. The calculated value is  $\beta = .9967$ .

#### 4. THE VAR SPECIFICATION

This section discusses the identification of the VAR models and the data used in the estimation.

**4.1. Identifying Fiscal Policy Shocks.** As is well known, the reduced-form residuals do not necessarily have economically meaningful interpretations. In order to identify the linear combinations of reduced-form residuals that reflect exogenous fiscal policy disturbances, we follow the method of Blanchard and Perotti (2002), as extended by Perotti (2004).

Suppose that the reduced form VAR is  $f_t = B_0 + f_{t-1}B + u_t$  and that one is interested in recovering the structural form  $f_t A_0 = \bar{A} + f_{t-1}A + \epsilon_t J$ , where  $J$  is a block diagonal matrix with the first block at the upper left corner a  $k$ -dimensional matrix coupling the  $k$  fiscal policy shocks. Following Blanchard and Perotti, assume that the reduced-form innovations to a fiscal policy instrument, for example, taxes,  $u_t(T)$ , can be modeled as

$$u_t(T) = u_t(Y)A_{Y,T} + u_t(\pi)A_{\pi,T} + u_t(R)A_{R,T} + \epsilon_t(T) + \epsilon_t(G)J_{G,T} + \epsilon_t(Z)J_{Z,T}, \quad (31)$$

where  $u_t(Y)$  is the residual associated with output,  $u_t(\pi)$  the inflation residual,  $u_t(R)$  the interest rate residual and  $\epsilon_t(T)$ ,  $\epsilon_t(G)$ , and  $\epsilon_t(Z)$  are the identified exogenous fiscal shocks to taxes, government spending, and transfers. The coefficients of the first  $k$  columns in  $A$  in (31) are identified from institutional information about automatic responses in the policy instrument in question. It then is possible to obtain the covariance matrix of the fiscal policy matrix  $J$ . In order to determine impact responses to the individual shocks, it is necessary to make assumptions concerning the relations among the fiscal shocks themselves. We assume that the fiscal shocks are recursively ordered with taxes first, followed by spending and transfers. We refer to the shocks as “tax,” “spending,” and “transfer” shocks, but it is important to bear in mind that each class of shock will entail substantial movements in the other fiscal policy instruments via endogenous propagation mechanisms.

We draw on unpublished results from Leeper and Yang (2004) to calibrate the elasticities in (31). Previous work in this literature has typically entered taxes net of transfers into the VAR. We prefer to disaggregate taxes and transfers, because, on theoretical grounds, distortionary taxes may lead to behavioral responses not characterized simply by their impact on the present-value of lifetime resources. We follow the assumptions in Perotti (2004); specifically, the price elasticity of real transfers is  $-1$  and the output elasticity of transfers is  $-0.15$ . The output elasticity of taxes is therefore given by  $\alpha_{TY} = (1 - \frac{Z}{T})\alpha_{netTY} + \alpha_{ZY}\frac{Z}{T}$ , with a similar equation for the inflation elasticity, where  $Z/T$  is the steady state ratio of transfers to taxes. These elasticities appear in table 3.

**4.2. The Data.** The empirical model is a quarterly VAR using U.S. data on the following variables in log levels: real GDP, the GDP deflator, gross private domestic investment, the three-month Treasury bill rate, the 10-year Treasury bond yield, the monetary base, and fiscal variables. Fiscal data, which are for the Federal government only, include taxes, transfers, spending, and debt (all NIPA).<sup>6</sup> The data cover the period from 1947:2 to 2006:2. Federal spending is defined as the sum of Federal consumption expenditure, gross investment and consumption of fixed capital. Federal taxes include all current tax receipts and contributions for social insurance. Finally, net transfers include net current transfers, capital transfers, income from assets and subsidies. The three-month T-bill rate is used for the sake of consistency with the theoretical model, while the monetary base is necessary to complete the specification of the government budget constraint. Finally, the surplus components are adjusted to better match the conceptual model described above. In particular, adjustments are made to convert corporate income taxes from accrual to cash basis, to include spending and revenue from U.S. territories and Puerto Rico and to include contributions to Federal employee retirement funds. The quantitative importance of these adjustments is small.<sup>7</sup>

To obtain a Federal debt series which obeys a flow budget constraint, we accumulate debt with the NIPA-defined Federal net borrowing figure using the equation  $V_t - V_{t-1} = Net\ Borrowing - Seignorage$ , where  $V$  is total nominal debt outstanding.

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<sup>6</sup>Ideally, fiscal variables would include Federal and state and local variables, as is typical in this literature. But state and local governments generally have balanced-budget rules, while debt financing is permitted only for certain capital expenditures. This suggests that fiscal-financing decisions are likely to differ substantially across Federal and state and local governments, so separating the two levels of government is reasonable.

<sup>7</sup>These adjustments are derived from NIPA table 3.18B. The data in this table are not seasonally adjusted, unlike the data series that we employ elsewhere. We have used these corrections without seasonal adjustment largely because they slightly improve the fit between our generated series and the Dallas Fed's data set, as discussed in footnote 8.

We validate this series by comparing it to the market value data produced by Cox and Hirschhorn (1983).<sup>8</sup>

The VAR features five lags of each endogenous variable and a constant, in order to maintain, as much as possible, the framework of Perotti (2004), which serves as our point of comparison to the existing literature. Relative to other VARs estimated in the literature [for example, Perotti (2004)], this system is on the large side. However, as has been recognized in other contexts [see Favero and Giavazzi (2007)], the estimated dynamics of a VAR system including debt and investment can differ consequentially from estimates derived from a model not including these variables. Moreover, a sizable literature, including Hansen and Sargent (1991), Lippi and Reichlin (1994), Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007), demonstrates that studying present-value restrictions such as (26) and (27) in VAR systems which are “too small” can lead to severe difficulties due to non-invertibility.<sup>9</sup> When all components of the linearized budget identity, (20), are included in the information set, however, our previous derivation of the present-value restrictions shows that equations (26) and (27) must be satisfied, even with an information set coarser than that used by private agents. We are thus entitled to impose these restrictions directly on our baseline VAR.

We examine informally the impact of enlarging the information set in figure 3, which presents multipliers from three models. The smallest model is a six variable VAR typical of the literature, except for the separate inclusion of transfers, and includes Federal taxes, transfers and spending as well as the 3-month T-bill rate, the GDP deflator and output. The second model then adds investment and the Federal debt stock to the information set, while the third model is our baseline ten-variable VAR. Over the short-run, point estimates of impulse responses from the six variable model are qualitatively quite similar to those obtained from the larger models, although appreciable quantitative differences for output and price responses do emerge between 10 and 20 quarters after the shock. In particular, the fall in prices following a spending shock is around twice as large in the baseline as compared to the smallest model, while, in the smallest model, the fall in prices following a tax shock is considerably below the baseline (indeed, outside the lower 90 percent confidence interval by the end of the first decade). Much of the difference between

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<sup>8</sup> The Cox and Hirschhorn data are available at <http://www.dallasfed.org/data/data/natdebt.htm>. They construct a market value of debt series by computing  $V_t \equiv \sum_{j=1}^J B_t(j)Q_t(j)$  for maturities  $j = 1, 2, \dots, J$ . Because our empirical model includes NIPA-based measures of receipts and expenditures, the Cox-Hirschhorn debt series will not generally be consistent with net borrowing as defined by NIPA.

<sup>9</sup> That is, small VARs can imply that the econometrician’s information set is strictly smaller than economic agents’, making it impossible to recover the exogenous fiscal shocks— $\epsilon_t(T)$ ,  $\epsilon_t(G)$ , and  $\epsilon_t(Z)$ —from current and past data— $f_{t-j}$ ,  $j \geq 0$ . Leeper, Walker, and Yang (2009) describe this problem in detail.

the smallest model and the baseline appears to arise from the inclusion of investment and debt, insofar as the inclusion of these two variables alone eliminates any important quantitative difference relative to the baseline.

## 5. BASELINE ESTIMATES

**5.1. Baseline Multipliers.** Figures 4 and 5 report multipliers for both the constrained and unconstrained VAR following fiscal policy shocks which lead to a one-percent of trend GDP innovation in the corresponding policy instrument.<sup>10</sup> Confidence intervals are computed from 5000 Monte-Carlo draws using the unconstrained baseline VAR as the data-generating process and assuming normal innovations.

The baseline VAR reproduces many of the standard findings in the literature. Upon a surprise tax increase, output falls, while a spending increase generates a rise in output. Also consistent with the literature, spending shocks generate short-lived crowding out of investment and a prolonged period of lower prices. For each shock, the initial impact on the primary surplus is rapidly reversed within two to three years of the shock. However, due to the presence of highly persistent transients, the surplus then oscillates around zero for a prolonged period (on the order of 50 years) before beginning to converge more or less monotonically. Once convergence sets in, the surplus supports the initial debt innovation for each type of shock.

Broadly speaking, imposing the present-value leaves the impulse responses quantitatively unperturbed. Perhaps surprisingly, the most dramatic effect is on output. After about 10 years, the output response leaves the 68 percent confidence interval for every fiscal policy shock. In the most extreme cases, for tax and spending shocks, the difference in output is around half a percentage point, relative to the unconstrained estimates. As figure 5 reports, imposing present-value balance also has relatively large effects on point estimates for the trajectory of debt, although, in this case, the effect is to restrain the absolute value of deviations from baseline. Ultimately, however, the impulse responses of the two models do not appear sharply distinguishable, given the (im)precision of the baseline point estimates.

## 6. HOW DEBT-FINANCED FISCAL SHOCKS HAVE BEEN FINANCED

At this point, we turn to the forward-looking aspect of government finance. In particular, we wish to ascertain what combination of adjustments in the expected

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<sup>10</sup>Following Blanchard and Perotti (2002), the multipliers for quantity variables are calculated by multiplying the impulse response functions for the log variable by the share of that variable in output. The interpretation is that the multiplier gives the change in the trajectory for that variable, as a share of trend output.



path of fiscal policy instruments and discount rates rationalizes the decision of private agents to hold government debt at prevailing market prices. In addition, we would like to learn something about the dynamics of adjustment. This section addresses the following questions: How does adjustment depend on the nature of the fiscal policy shock? Over what horizon must one forecast in order to see present-value balance? Are there important differences in the way that transient and persistent movements in debt are financed?

**6.1. Fiscal Finance: Present Values.** The basic present-value decomposition previously described is displayed in table 4. Consistent with the convention for impulse response functions, the table shows present-value components following one-percent of trend GDP responses from the policy instruments associated with each type of shock. The components are scaled so that they add to the initial debt innovation, shown in the first row. Present-value components help to stabilize debt when the sign of their contribution in the table is the same as the sign of the initial change in debt.

From the table, one sees immediately that, in the baseline VAR, the different fiscal policy shocks are financed very differently in present-value terms. For tax shocks, both the discount rate and the present-value of surpluses at constant rates move to support debt, with the lion's share of the work done by changes in the present-value of surpluses. This is, presumably, the intuitive picture of how debt is financed in present-value. By contrast, the role of the discount rate is much less intuitive for spending and transfer shocks. In the case of surprise spending increases, in fact, while taxes do rise sharply and persistently, their contribution is swamped by the combination of higher spending and transfers. The present-value of surpluses at constant discount rates actually *falls*. Present-value balance is maintained only by drastic and prolonged fall in real interest rates, the bulk of which is accounted for a fall in the nominal interest rate. The story is reversed for surprise increases in transfers. The initial transfer increase is quite transient and, in the longer horizon, lower expected transfers account for most of the present-value of surpluses, with taxes and spending offsetting each other. The discount rate resists present-value balance, again largely due to changes in the path of nominal interest rates, in this case a sharp rise.

For each type of fiscal policy shock, taxes and transfers experience sizeable but offsetting movements in present value, as was apparent from the impulse response functions. Moreover, except in the case of transfers, the present value of taxes moves to support the innovation in debt, while the present-value of transfers moves against it. In the case of transfers, it is spending and taxes which move in offsetting fashion, while transfers bears the burden of adjustment.

The table presents 68 percent confidence intervals for each present-value component. However, for the purposes of assessing the role in stabilization of a given component, it is perhaps also informative to report the fraction of draws in which each component moves in the same direction as the initial debt innovation, that is, the probability that the component supports the change in debt. Taxes support the change in debt in over 90 percent of draws for both tax and spending shocks. On the other side, the present-value of spending moves with the change in debt in 13 percent and 8 percent of cases, following those shocks. Transfers support debt in less than 10 percent of draws for either shock. Relative imprecision of the spending response estimate prevents sharp characterization of the role of the primary surplus, but net taxes (taxes less transfers) do support debt in over 90 percent of draws, both for tax and for spending shocks. Regarding the role of the real interest rate trajectory, less can be said with any confidence, but the real rate is stabilizing, following a spending shock, in 86 percent of draws.

**6.2. Fiscal Finance: Dynamics.** The summary accounting in the previous section, while informative, does not reveal much of the dynamic structure that supports present-value balance. In this section, we aim to illuminate this topic by examining the horizon over which present-value balance is attained. The funding horizon, which is computed using the analog of 12 for the estimated VAR, addresses the issue of how far into the future one must forecast in order to see present-value balance. For some classes of shocks, the answer is “quite a long time.”

Truncated present-values for each type of fiscal policy shock are illustrated by figure 6, where the solid lines represent truncated present-values of discounted surpluses, up to the date indicated. Each series is scaled by the initial debt innovation, so that asymptotically, each must converge to minus one for a tax cut that lowers debt by one unit and plus one for a spending or transfers increase that raises debt by one unit. The estimated truncated present-value series undergo dramatic fluctuations during the first half-century following the shock. Ultimately, for instance, following a tax shock, roughly sixty years is required before the truncated series is permanently within ten percent of its long-run value, while, following a transfer shock, more than a century is needed. In the intervening period, the truncated present-values exhibit wide and persistent deviations from long-run balance.

## 7. ROBUSTNESS TO ALTERNATIVE MODELING OF THE LONG-RUN

Estimates of long-run funding components are inherently sensitive to the long-run properties of the estimated VAR. In this section, we examine the extent to which our baseline results are affected by the imposition of restrictions reflecting prior beliefs about the non-stationary components of the system. In particular, we re-estimate

the VAR system subject to the intertemporal government budget constraint as well as additional restrictions enforcing the presence of unit-root modes in the dynamics. See appendix B for the derivation of the resulting estimator.

Johansen (1988, 1991) tests on the unconstrained VAR suggest a two to three dimensional unit-root block. Accordingly, figures 7 and 8 display the impact of imposing a two-dimensional unit-root block on both the unconstrained and the constrained model. Imposing unit-root behavior has little effect on the short-run dynamics (within the first decade, say) of any variable except the short nominal rate. However, over longer horizons, important differences do manifest, most consequentially, in the real interest rate, which displays notably less stabilizing behavior in the non-stationary model. Following a spending shock, the real interest rate is permanently higher and, indeed, is above its initial level forever, after 20 years. Consequently, as table 5 shows, the contribution of changes in the real interest rate trajectory moves against the initial debt innovation for all fiscal shocks.

Moreover, for this model, the primary surplus is stabilizing for each shock and the contribution of the seigniorage term is now no longer negligible. The sharper results for the primary surplus arise from the more consistent behavior of this series following each shock. In the case of tax shocks, the effect of the shock on the primary surplus is reversed within 2-3 years, just as in the baseline model. However, this reversal is now persistent. A surprise tax increase raises the primary surplus almost one-for-one on impact, but, by the end of the second year, the primary surplus has fallen into deficit, where it remains, with a few very brief, slightly positive periods, forever. A similar trajectory for the primary surplus is evoked by transfer shocks, where the initial fall in the surplus is reversed permanently at the end of the third year.

The pattern of behavior across primary surplus components is similarly more consistent across the shocks than in the baseline. In the Monte Carlo simulation, net taxes are stabilizing in 95 percent of draws following a tax shock, 92 percent of draws following a spending shock and 82 percent of draws following a transfer shock. By contrast, spending is stabilizing only in 13 percent of draws following a tax shock and 6 percent of draws following a spending shock. The additional stabilization provided by taxes results from permanent level changes, while much of the change in the contribution from transfers is accounted for by different short- to medium-run behavior during the few decades after the shock.

## 8. CONCLUSION

Although the previous section suggests that our present-value decompositions display considerable sensitivity to the modeling of long-run dynamics, we can nevertheless come to several robust conclusions. In response to either tax or spending

shocks, we find strong and robust evidence that the tax response is stabilizing, as is the response of net taxes. By contrast, in response to either tax or spending shocks, the change in the spending trajectory does not appear to support the initial debt innovation.

Decisions about the modeling of long-run dynamics appear most important for estimates of the role of the real interest rate. In the baseline, there is some evidence that the real interest rate is stabilizing for tax and spending shocks, particularly in the later case. However, when unit restrictions are imposed, this conclusion is reversed, with the preponderance of evidence, if anything, showing that the real interest rate response fails to support debt. Consequently, the imposition of unit-root restrictions appears to attenuate rather substantially the considerable differences in the intertemporal financing mechanisms for tax and spending shocks evident in the baseline. Regardless of the long-run modeling framework, however, present-value balance is only achieved over a long forecast horizon, on the order of 50 to 100 years, depending on the shock, and subject to wide swings away from present-value balance before settling in to convergence.

APPENDIX A. DERIVING THE RESTRICTIONS IMPLIED BY THE GOVERNMENT'S  
PRESENT-VALUE CONSTRAINT

Recall that to first order, the present-value constraint, expression (22), is

$$\hat{v}_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{s}{v} \left( \frac{-\hat{R}_{t+j-1} + \hat{\gamma}_{t+j} + \hat{\pi}_{t+j}}{1 - \beta} + \hat{s}_{t+j} \right). \quad (32)$$

and also that the variables in this equation are assumed to be spanned by factors  $f_t$  which evolve as

$$f_t = B_0 + \sum_{l=1}^L f_{t-l} B_l + u_t \quad (33)$$

Then one has that

$$\begin{aligned} f_t C_v = k + f_t \left[ \sum_{j=1}^{\infty} (\beta B)^j \left( \frac{C_\pi}{\beta} + \frac{\tau}{v} C_T - \frac{g}{v} C_G \right) - \sum_{j=0}^{\infty} (\beta B)^j C_R \right] + \\ B_0 \left[ \sum_{j=1}^{\infty} \beta^j \sum_{k=1}^j B^{j-k} \left( \frac{1}{\beta} C_\pi + \frac{\tau}{v} C_T - \frac{g}{v} C_G \right) - \sum_{j=0}^{\infty} \beta^j \sum_{k=1}^j B^{j-k} C_R \right] \end{aligned} \quad (34)$$

Note that the double summation in the second line can be simplified as

$$\begin{aligned} \sum_{j=0}^{\infty} \sum_{k=1}^j B^{j-k} &= \sum_{j=1}^{\infty} (\beta B)^j (\mathbf{I} - B^{-1})^{-1} (\mathbf{I} - B^{-j}) B^{-1} \\ &= (\mathbf{I} - B^{-1})^{-1} \left[ (\mathbf{I} - \beta B)^{-1} \beta B - \frac{\beta}{1 - \beta} \right] B^{-1} \\ &= \frac{\beta}{1 - \beta} (\mathbf{I} - \beta B)^{-1} \end{aligned} \quad (35)$$

Therefore, the present-value constraint can be written

$$\begin{aligned} f_t C_v = k + f_t \left[ (\mathbf{I} - \beta B)^{-1} \beta B \left( \frac{C_\pi}{\beta} + \frac{\tau}{v} C_T - \frac{g}{v} C_G \right) - (\mathbf{I} - \beta B)^{-1} C_R \right] + \\ B_0 \frac{\beta}{1 - \beta} (\mathbf{I} - \beta B)^{-1} \left[ \left( \frac{1}{\beta} C_\pi + \frac{\tau}{v} C_T - \frac{g}{v} C_G \right) - C_R \right] \end{aligned} \quad (36)$$

Assuming that  $f_t$  has a full rank covariance matrix, this equation implies restrictions on the factor dynamics

$$C_v = \left[ (\mathbf{I} - \beta B)^{-1} \beta B \left( \frac{C_\pi}{\beta} + \frac{\tau}{v} C_T - \frac{g}{v} C_G \right) - (\mathbf{I} - \beta B)^{-1} C_R \right] \quad (37)$$

and

$$0 = k + B_0 \frac{\beta}{1-\beta} (\mathbf{I} - \beta B)^{-1} \left[ \left( \frac{1}{\beta} C_\pi + \frac{\tau}{v} C_T - \frac{g}{v} C_G \right) - C_R \right] \quad (38)$$

For estimation purposes, it is convenient to rewrite the first of these restrictions as a linear restriction on  $B$ . To this end, multiply equation (37) by  $\mathbf{I} - \beta B$  to obtain

$$C_v + C_R = \beta B \left( \frac{C_\pi}{\beta} + \frac{\tau}{v} C_T - \frac{g}{v} C_G + C_v \right) \quad (39)$$

which is equation (26) in the text.

Accounting for seigniorage modifies the present-value constraint (17) such that the value of liabilities outstanding (debt plus the monetary base) is equated to the present value of the primary surplus plus seigniorage (formally  $m_{t-1}(R_{t-1} - 1)/\pi_t$ ). Upon linearization of this augmented present-value relation, a suitable VAR constraint can then be derived exactly parallel to the line of reasoning above.

## APPENDIX B. IMPOSING THE GOVERNMENT BUDGET CONSTRAINT WITH UNIT ROOT DYNAMICS

This appendix describes how to estimate VAR models subject to the intertemporal budget constraint in presence of unit-root dynamics. Specifically, as before, suppose that the underlying factor dynamics is

$$f_t = B_0 + \sum_{l=1}^L f_{t-l} B_l + u_t \quad (40)$$

and that this VAR system is subject to the intertemporal budget constraint, given in equation (29),

$$BC = C_0 \quad (41)$$

where  $B$  is composed of the stacked coefficient matrices  $B_l$ , i.e.,  $B \equiv [B_1; \dots; B_L]$ . As it happens, the restriction on the constant term, equation (30), can be written in similar form, using equation (29). To see this, rewrite equation (29) as

$$\begin{aligned} (1 - \beta B)^{-1} \beta B \left( C_v + \frac{1}{\beta} C_\pi + \frac{\tau}{v} C_\tau - \frac{g}{v} C_g - \frac{z}{v} C_z \right) = \\ ((1 - \beta B)^{-1} - 1) \left( C_v + \frac{1}{\beta} C_\pi + \frac{\tau}{v} C_\tau - \frac{g}{v} C_g - \frac{z}{v} C_z \right) = \\ (1 - \beta B)^{-1} (C_v + C_R) \end{aligned} \quad (42)$$

It follows that equation (30) can be written

$$B_0 \left( C_v + \frac{1}{\beta} C_\pi + \frac{\tau}{v} C_\tau - \frac{g}{v} C_g - \frac{z}{v} C_z \right) = -k \frac{1 - \beta}{\beta} \quad (43)$$

A useful feature of this constraint is that the matrix multiplying  $B_0$  is the same matrix which appears in the constraint for  $B$ .

Assume that the dynamics for  $f_t$  are non-stationary, in the sense that there is a  $k$ -dimensional subspace spanned by vectors  $v$  such that  $\sum_{j=l}^L B_j v = v$ . We wish to estimate (40) subject to this unit-root requirement, as well as the intertemporal constraints (41) and (43). Let the matrix  $Y$  denote the data matrix for the  $f_t$ , and let  $X$  be the data matrix containing a constant (in first position) and the stacked lags of  $f_t$ . Further, using  $;$  to denote vertical concatenation, let  $B \equiv [B_0; B]$  and let  $G_0 \equiv [-k\frac{1-\beta}{\beta}; C_0]$ . Then, suppose that we choose  $\hat{B}$ ,  $\hat{B}_0$  and  $\text{span}(v)$  to minimize

$$\text{Trace} \left( (Y - X\hat{B})'(Y - X\hat{B}) \right) \quad (44)$$

subject to

$$\hat{B}C = G_0 \quad (45)$$

and

$$W'\hat{B}v = v \quad (46)$$

where  $W'$  is a known matrix which maps  $B$  into  $\sum_l B_l$ . Concretely, the matrix  $W$  is an  $NL + 1$  by  $N$  matrix, whose first row is zero, followed by  $L$  copies of the  $N$ -dimensional identity matrix.

The resulting estimator is most easily motivated by considering first the problem subject only to equation (46). The resulting first-order necessary conditions are

$$X'(Y - X\hat{B}) + W\mu v' = \quad (47)$$

$$\mu'(W'B - \mathbf{1}_N) = 0 \quad (48)$$

where  $\mu$  is the matrix of Lagrange multipliers on equation (46). Imposing the constraint to eliminate  $\mu$  from equation (47), one finds that  $\hat{B}$  is given by

$$\hat{B} = B_{OLS} + (X'X)^{-1}W \left( W'(X'X)^{-1}W \right)^{-1} (\mathbf{1}_N - W'B_{OLS}) P_v \quad (49)$$

where  $P_v$  is the orthogonal projector onto the subspace spanned by  $v$ , while  $\mu$  satisfies

$$\mu = \left( W'(X'X)^{-1}W \right)^{-1} (\mathbf{1}_N - W'B_{OLS}) P_v \quad (50)$$

Accordingly, equation (48) can be written

$$P_v \Gamma_0 (\mathbf{1}_N - P_v) \quad (51)$$

where

$$\Gamma_0 \equiv (\mathbf{1}_N - W'B_{OLS})' \left( W'(X'X)^{-1}W \right)^{-1} (\mathbf{1}_N - W'B_{OLS}) \quad (52)$$

Equation (51) implies that  $\Gamma_0$  leaves the kernel of  $P_v$  invariant, which, in turn, implies that some subset of eigenvalues of  $\Gamma_0$  is a basis for the span of  $v$ . To settle which subset, we re-write the  $\hat{B}$ -dependent part of the criterion function (44) as

$$\text{Trace} \left( \hat{B} - B_{OLS} \right)' (X'X) \left( \hat{B} - B_{OLS} \right) \quad (53)$$

which is equivalent to minimizing

$$\text{Trace} (P_v \Gamma_0 P_v) \quad (54)$$

Given that, without loss of generality,  $v$  is a matrix of eigenvectors of  $\Gamma_0$ , the solution of the minimization problem amounts to selecting  $v$  to be the  $k$  eigenvectors of  $\Gamma_0$  whose eigenvalues have the least magnitude.

When the intertemporal budget constraint is imposed, the strategy is similar. In this case, if  $\lambda$  is a matrix of Lagrange multipliers on equation (45), the first-order necessary conditions are

$$X'(Y - X\hat{B}) + W\mu v' + \lambda C' = 0 \quad (55)$$

$$\mu' (W'B - \mathbf{1}_N) = 0 \quad (56)$$

Again, imposing the constraints and eliminating the Lagrange multipliers from (55), we can solve for  $\hat{B}$  explicitly:

$$\hat{B} = B_R + (X'X)^{-1} W \left( W' (X'X)^{-1} W \right)^{-1} (\mathbf{1}_N - W'B_R) J_v \quad (57)$$

where  $B_R$  is the least-squares estimator subject to the budget constraint

$$B_R \equiv B_{OLS} + (G_0 - B_{OLS}C) (C'C)^{-1} C' \quad (58)$$

and  $J_v$  is

$$J \equiv v (v' (\mathbf{1}_N - P_R) v)^{-1} v' (\mathbf{1}_N - P_R) \quad (59)$$

Accordingly, equation (56) can be written

$$J' \Gamma (\mathbf{1}_N - J) \quad (60)$$

where

$$\Gamma \equiv (\mathbf{1}_N - W'B_R)' \left( W' (X'X)^{-1} W \right)^{-1} (\mathbf{1}_N - W'B_R) \quad (61)$$

The matrix  $J$  satisfies

$$Jv = v \quad (62)$$

so, taking the transpose, equation (60) holds if and only if

$$J' \Gamma v = \Gamma v \quad (63)$$

The structure of  $J'$  implies then that  $\Gamma v$  lies in the span of  $(\mathbf{1}_N - P_R) v$ , i.e., there is a matrix  $H$  such that  $\Gamma v H = \Gamma v$ . Moreover,  $H$  must be invertible, for, if not,



there would be a matrix  $H_{\perp}$  such that  $0 = \Gamma^{-1}(\mathbf{1}_N - P_R)vHH_{\perp} = vH_{\perp}$ , which is impossible if  $v$  is a set of linear independent vectors.

Suppose then that  $H^{-1}$  exists and has Jordan decomposition  $U_H\Lambda_H U_H^{-1}$ . One has that

$$vU_H\Lambda_H^{-1} = \Gamma^{-1}(\mathbf{1}_N - P_R)vU_H \quad (64)$$

Without loss of generality, therefore,  $v$  can be chosen as a  $k$ -dimensional set of the eigenvectors of  $\Gamma^{-1}(\mathbf{1}_N - P_R)$ , such that the corresponding eigenvalue is non-zero. As in the unconstrained case, the choice of eigenspace minimizes the criterion function (44).

In this case, the criterion function is

*Trace* :

$$\begin{aligned} & \left( (G_0 - B_{OLS}C)(C'C)^{-1}C' + (X'X)^{-1}W \left( W'(X'X)^{-1}W \right)^{-1} (\mathbf{1}_N - W'B_R) J_v \right)' (X'X) \\ & \left( (G_0 - B_{OLS}C)(C'C)^{-1}C' + (X'X)^{-1}W \left( W'(X'X)^{-1}W \right)^{-1} (\mathbf{1}_N - W'B_R) J_v \right) \end{aligned} \quad (65)$$

The structure of  $J$ , with the projector  $(\mathbf{1}_N - P_R)$  on the right, implies that cross-terms in this quadratic are zero. Hence, the  $v$ -dependent part of the criterion function reduces to  $Trace(JJ'\Gamma)$ , or  $Trace((v'(\mathbf{1}_N - P_R)v)^{-1}v'\Gamma v)$ . Using the fact that  $v\Lambda_H^{-1} = \Gamma^{-1}(\mathbf{1}_N - P_R)v$ , conclude that  $v'\Gamma v = v'\Gamma(\Gamma^{-1}(\mathbf{1}_N - P_R)v\Lambda_H^{-1})$ . Ultimately, the criterion function is minimized by minimizing  $Trace(\Lambda_H)$ , and therefore by choosing  $v$  to be the  $k$  eigenvectors of  $\Gamma^{-1}(\mathbf{1}_N - P_R)$  with the *lowest* non-zero eigenvalues.

Parameters						
$\beta$	$\gamma$	$\gamma_N$	$\theta$	$A$	$\alpha$	$\delta$
.99	1	1	3.48	1	2/3	.025
Policy Rule Output Elasticities						
	$\varphi_G$	$\varphi_\tau$	$\varphi_Z$			
	0	1.5065	-.15			
Steady State Policy Variables						
	$G/Y$	$T/Y$	$Z/Y$	$B/Y$		
	.0839	.1171	.0882	.495		

TABLE 1. Parameter settings in the real business cycle model. Steady state policy variables calibrated to match U.S. data used to estimated identified VARs. Policy shocks have first-order serial correlation parameter of .80. Policy rule output elasticities calibrated from Perotti (2004) and steady state policy variables.

Shock to	Financed by (Present Value)				
	$T$	$G$	$Z$	$R$	$S$
$G$	-.03	-4.03	5.14	-.08	1.08
$T$	-3.24	0	4.25	-.004	1.004
$Z$	0	0	1	0	1

TABLE 2. Real business cycle model. The fraction of positive government debt innovations, due to shocks listed in the first column, that are financed by each of the components of the government budget. Simulation assumes that transfers clear the government budget:  $\gamma_Z = 1, \gamma_G = \gamma_\tau = 0$ .  $R$  denotes the stochastic discount factor;  $S$  denotes net surplus, derived by summing columns labeled  $T$ ,  $G$ , and  $Z$ .

	$Y$	$\pi$
Tax Elasticity	3.15	1.64
Spending Elasticity	0	-.5
Transfer Elasticity	-.15	-1

TABLE 3. Calibrated elasticities in identified VAR with taxes and transfers separated.

Present-Value Component	Tax Shock	Spending Shock	Transfer Shock
Govt Liabilities	-0.24 (-0.26,-0.21)	0.04 (0.02,0.06)	0.00 (-0.01,0.02)
Taxes	-1.93 (-2.38,-1.39)	2.35 (1.94,2.65)	-0.20 (-0.36,0.04)
Spending	0.43 (0.06,0.61)	-0.81 (-1.05,-0.63)	0.19 (0.07,0.32)
Transfers	1.38 (1.00,1.79)	-1.68 (-1.90,-1.30)	0.11 (-0.09,0.24)
Primary Surplus	-0.13 (-0.28,-0.01)	-0.15 (-0.24,-0.04)	0.10 (0.05,0.17)
Real Rate	-0.08 (-0.22,0.09)	0.17 (0.05,0.28)	-0.12 (-0.20,-0.06)

TABLE 4. Present-value funding components from a constrained VAR, along with 68 percent confidence intervals, in units of goods. The confidence intervals are obtained from 5000 draws of a Monte Carlo simulation, using the unrestricted VAR point estimates as a data-generating process and with the median centered at the 2-dimensional unit root VAR point estimates.

Present-Value Component	Tax Shock	Spending Shock	Transfer Shock
Govt Liabilities	-0.15 (-0.17,-0.14)	0.06 (0.05,0.07)	0.05 (0.04,0.06)
Taxes	-2.86 (-5.35,-0.86)	3.06 (1.53,5.02)	0.78 (-0.17,1.81)
Spending	0.86 (0.14,2.06)	-1.08 (-2.12,-0.49)	-0.11 (-0.55,0.43)
Transfers	0.24 (-1.15,1.31)	-1.30 (-2.21,-0.26)	0.04 (-0.57,0.52)
Primary Surplus	-1.75 (-3.31,-0.70)	0.68 (-0.25,1.76)	0.71 (0.23,1.34)
Real Rate	1.90 (0.65,3.71)	-0.77 (-2.06,0.31)	-0.77 (-1.52,-0.20)

TABLE 5. Present-value funding components from a constrained VAR with 2-dimensional unit root block, along with 68 percent confidence intervals, in goods units. The confidence intervals are obtained from 5000 draws of a Monte-Carlo simulation, using the unconstrained 2-dimensional unit root VAR as the data-generating process and with the median centered at the 2-dimensional unit root VAR point estimates.

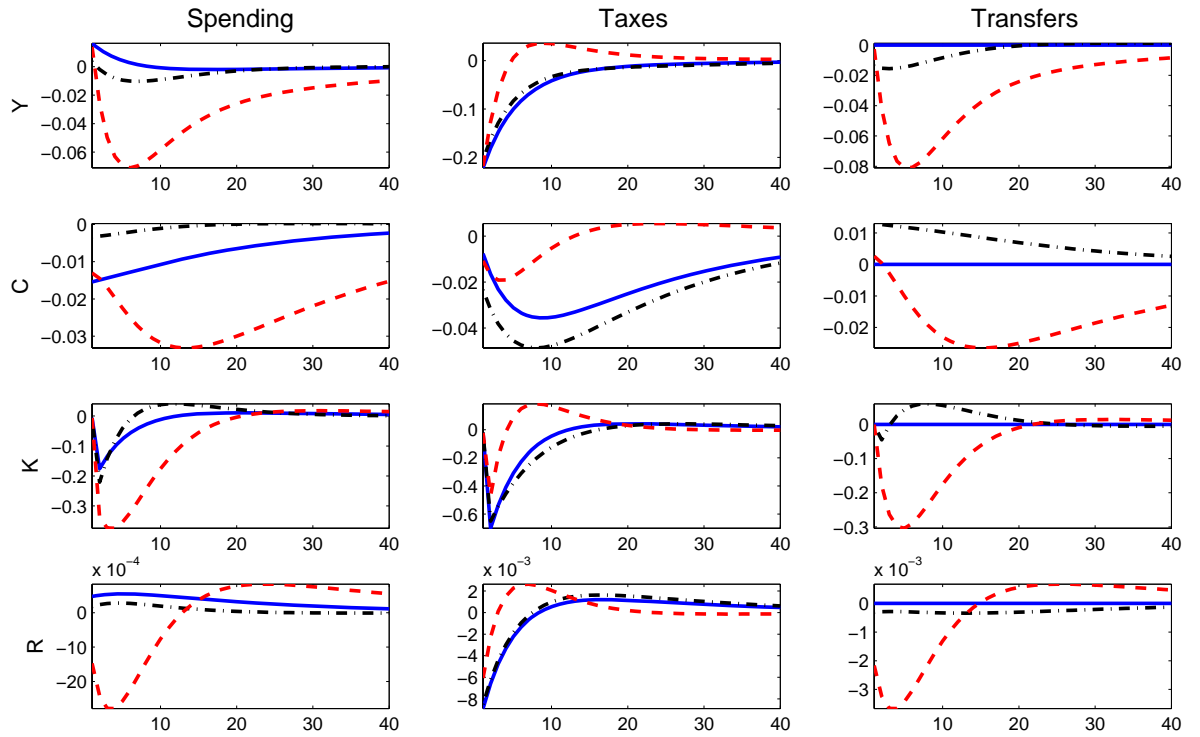


FIGURE 1. Responses to fiscal shocks when taxes clear the budget in calibrated RBC model:  $\gamma_\tau = 1, \gamma_Z = \gamma_G = 0$ . Solid line: transfers adjust; dashed line: taxes adjust; dotted line: spending adjusts. Time units in quarters.

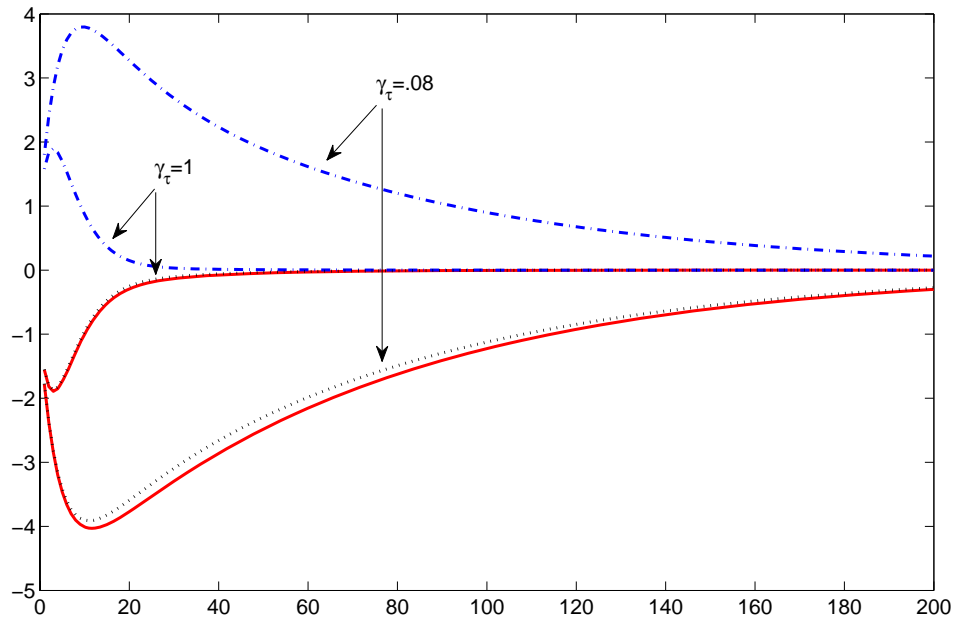


FIGURE 2. Government debt funding horizons for fiscal shocks when taxes clear the budget in calibrated RBC model. Solid line: tax shock; dotted-dashed line: spending shock; dotted line: transfers shock.  $\gamma_\tau = 1$  is rapid adjustment of taxes;  $\gamma_\tau = .08$  is slow adjustment of taxes. Time units in quarters.

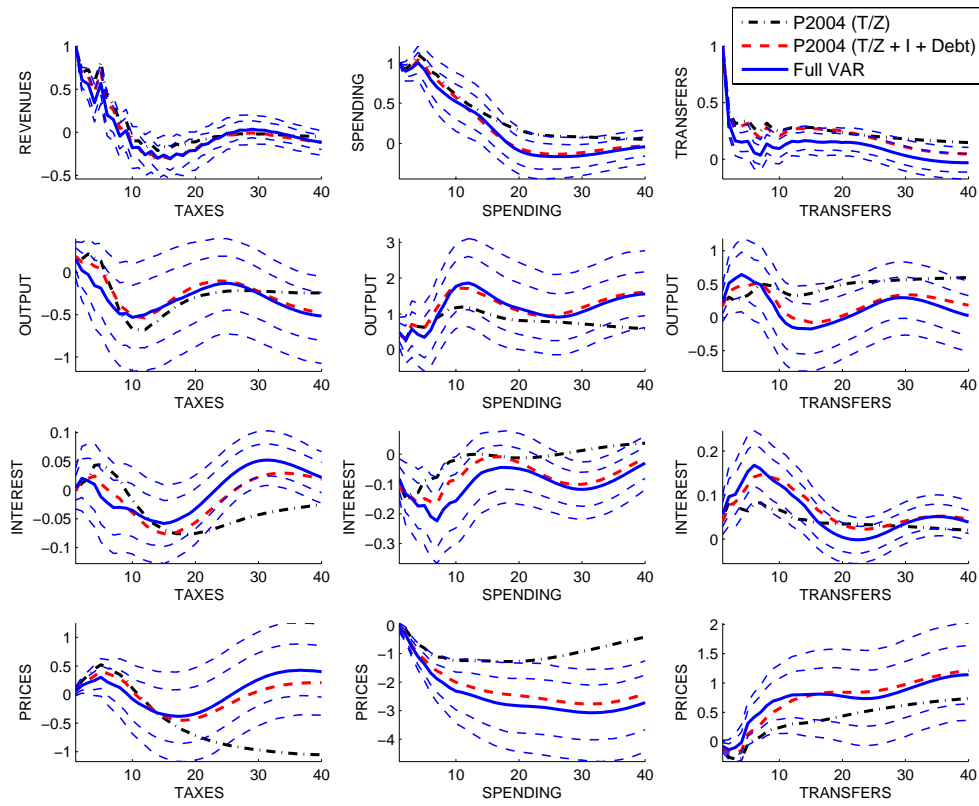


FIGURE 3. Multipliers from VARs with various information sets. P2004 (T/Z), dotted-dashed lines, is close to the specification in Perotti (2004) and includes Federal taxes, spending, transfers, 3-month T-bill rate, GDP deflator, and GDP; P2004 (T/Z+I+Debt), dashed lines, adds investment and Federal government debt; Full VAR is the baseline ten-variable system, solid lines with 68 percent and 90 percent confidence intervals, adds 10-year Treasury bond rate and monetary base. Time units in quarters.

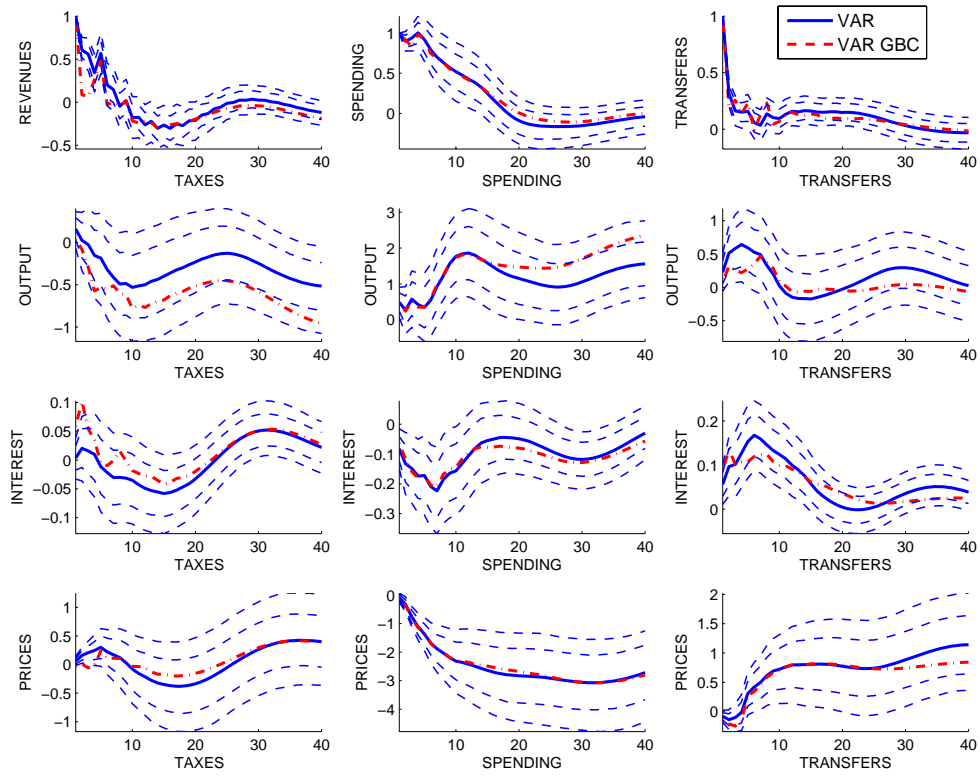


FIGURE 4. Effects of imposing the government budget constraint on baseline VAR. VAR, solid lines with 68 percent and 90 percent confidence intervals, is the unconstrained baseline ten-variable system; VAR GBC, dashed-dotted lines, imposes the government’s intertemporal budget constraint. Time units in quarters.



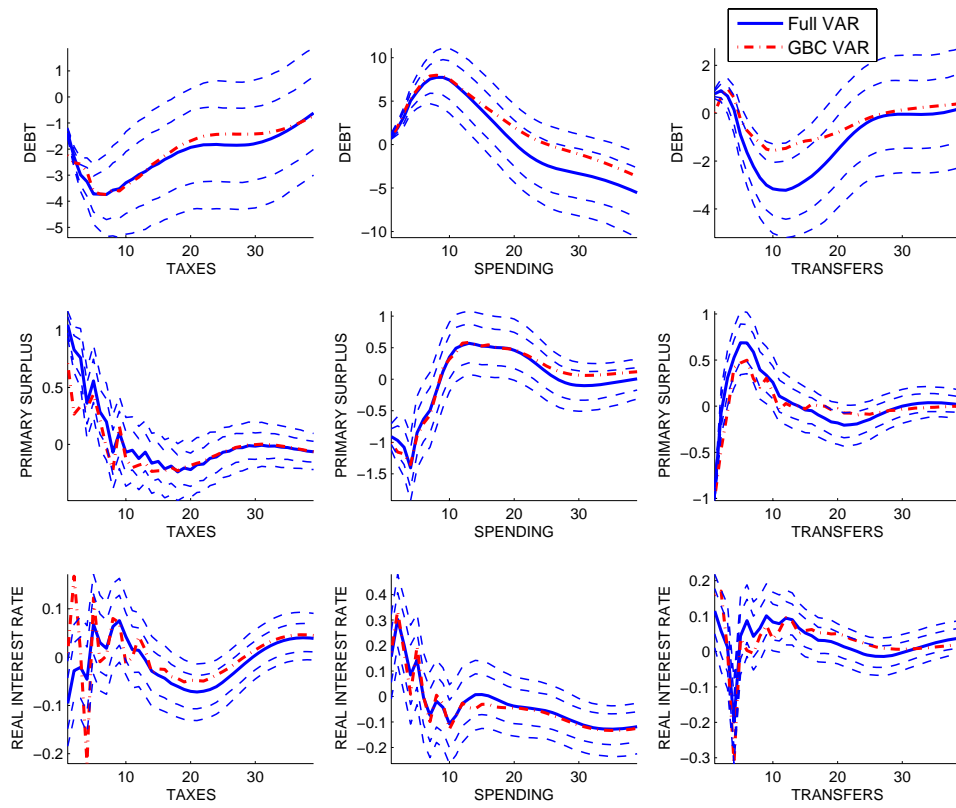


FIGURE 5. Effects of imposing the government budget constraint on baseline VAR. Components of the government’s intertemporal budget constraint. VAR, solid lines with 68 percent and 90 percent confidence intervals, is the unconstrained baseline ten-variable system; VAR GBC, dashed-dotted lines, imposes the government’s intertemporal budget constraint. Time units in quarters.

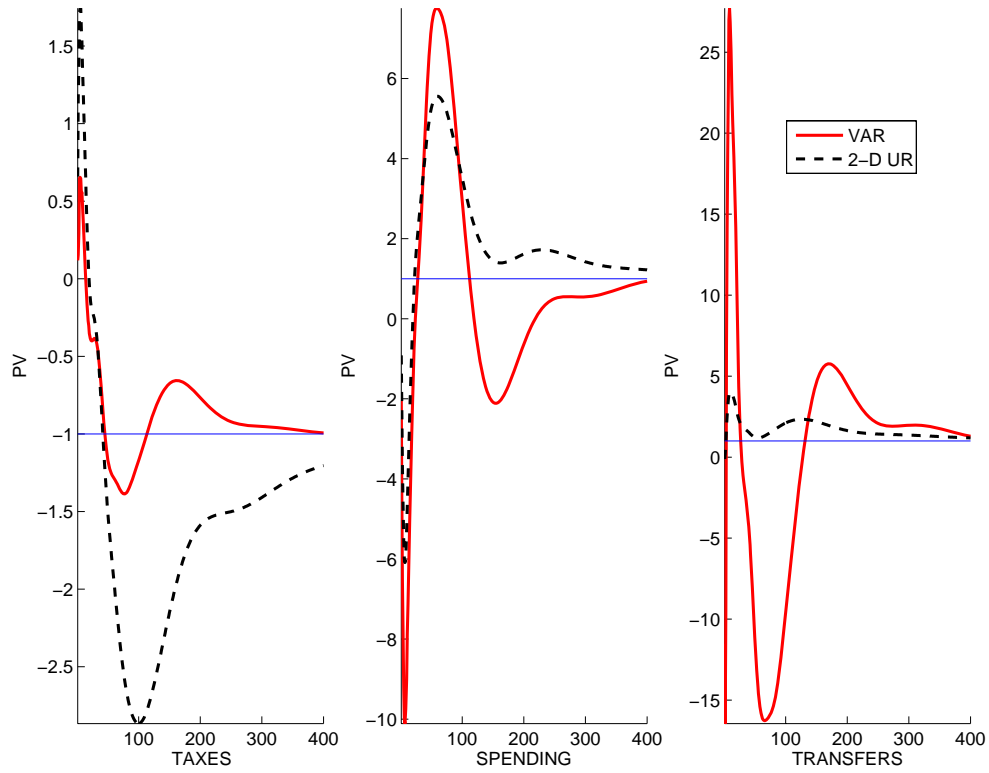


FIGURE 6. Effects of truncating the forecasting horizon on present-value calculation. Uses the analog of expression (12), as applied to the estimated baseline ten-variable VAR, solid lines; 2-dimensional unit root VAR, dashed lines, with the government's intertemporal budget constraint imposed. Time units in quarters.

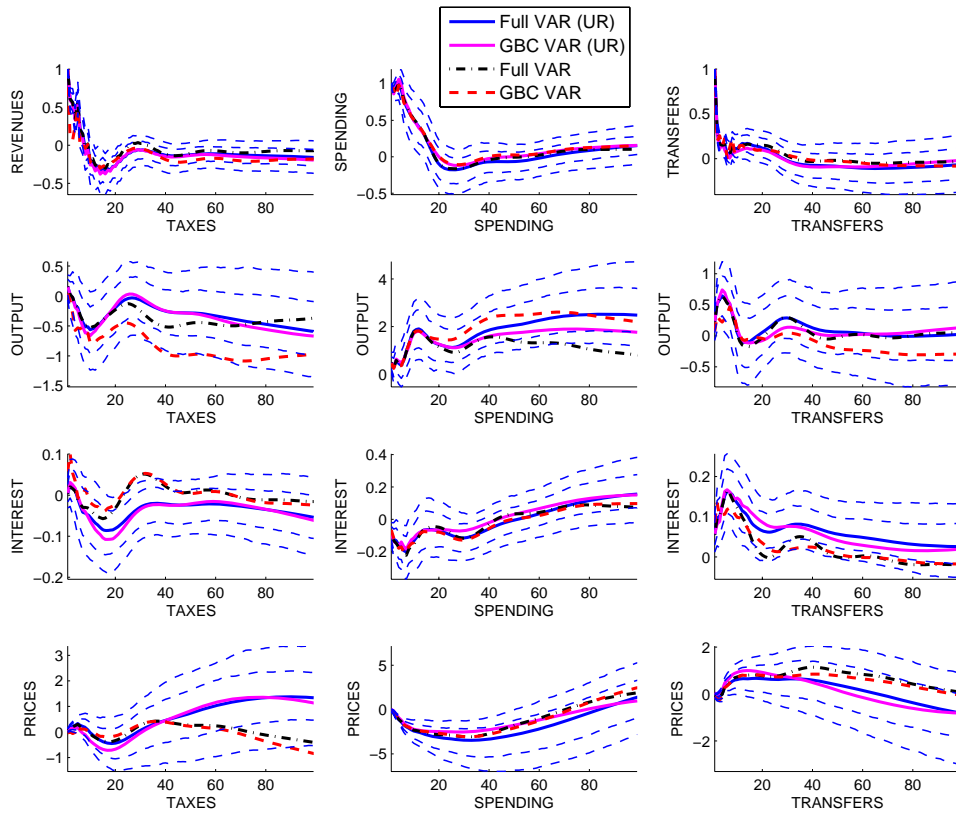


FIGURE 7. Effects of imposing the government budget constraint. Unconstrained 2-dimensional unit root VAR, dark solid lines and dashed 68 percent and 90 percent confidence intervals; 2-dimensional unit root VAR with government's intertemporal budget constraint imposed, light solid lines; unconstrained baseline ten-variable VAR, dark dotted-dashed lines; and baseline ten-variable VAR with government's intertemporal budget constraint imposed, light dashed lines. Time units in quarters.

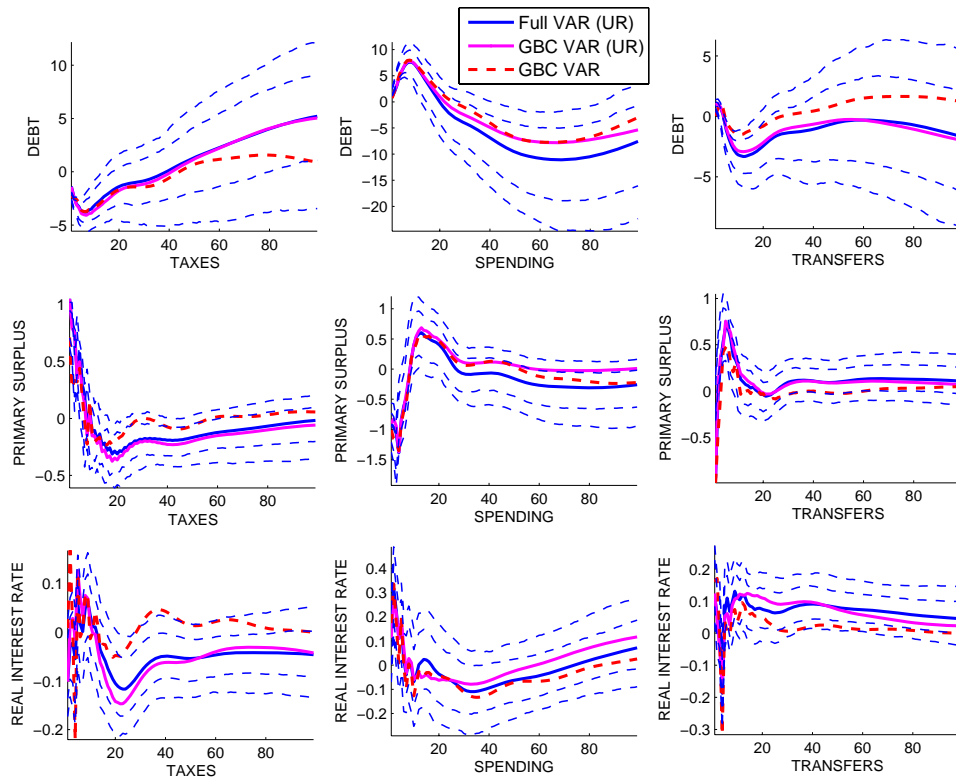


FIGURE 8. Effects of imposing the government budget constraint. Components of the government's intertemporal budget constraint. Unconstrained 2-dimensional unit root VAR, dark solid lines and dashed 68 percent and 90 percent confidence intervals; 2-dimensional unit root VAR with government's intertemporal budget constraint imposed, light dashed lines; baseline ten-variable VAR with government's intertemporal budget constraint imposed, light dotted-dashed lines. Time units in quarters.

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